



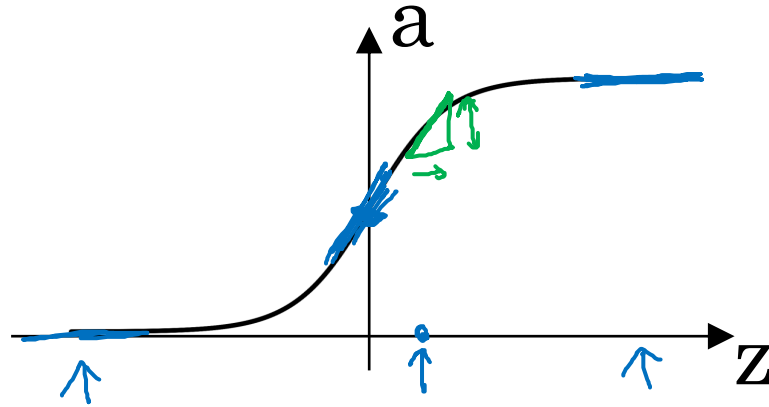
deeplearning.ai

One hidden layer  
Neural Network

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Derivatives of  
activation functions

# Sigmoid activation function



$$\underline{g(z) = \frac{1}{1 + e^{-z}}}$$

$$a = g(z) = \frac{1}{1 + e^{-z}}$$

$$\boxed{g'(z)} = \boxed{\frac{d}{dz} g(z)} = \text{slope of } g(z) \text{ at } z$$

$$= \frac{1}{1 + e^{-z}} \left( 1 - \frac{1}{1 + e^{-z}} \right)$$

$$= g(z) (1 - g(z)) \leftarrow$$

$$= \boxed{a(1-a)} \quad \left| \begin{array}{l} g'(z) = a(1-a) \\ \uparrow \\ a \end{array} \right.$$

$$z = 10. \quad g(z) \approx 1$$

$$\frac{d}{dz} g(z) \approx 1(1-1) \approx 0$$

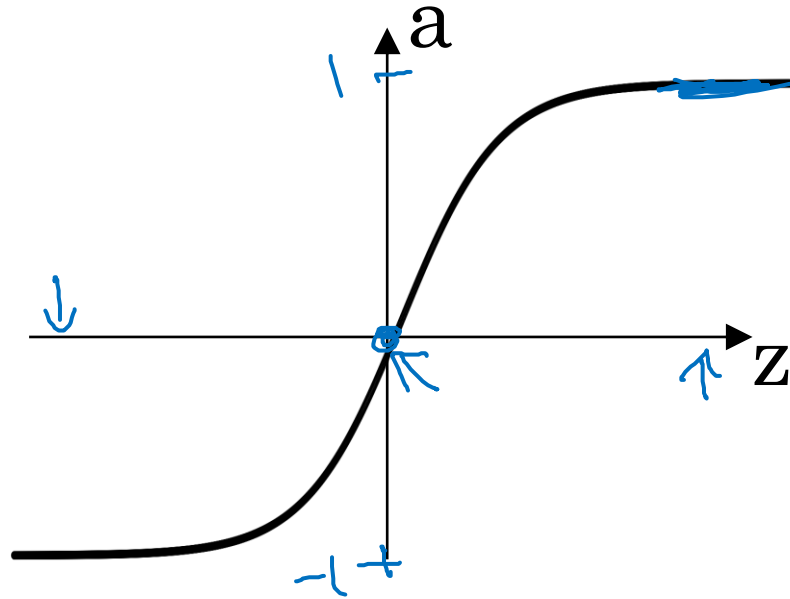
$$z = -10 \quad g(z) \approx 0$$

$$\frac{d}{dz} g(z) \approx 0 \cdot (1-0) \approx 0$$

$$z = 0 \quad g(z) = \frac{1}{2}$$

$$\frac{d}{dz} g(z) = \frac{1}{2} \left( 1 - \frac{1}{2} \right) = \frac{1}{4}$$

# Tanh activation function



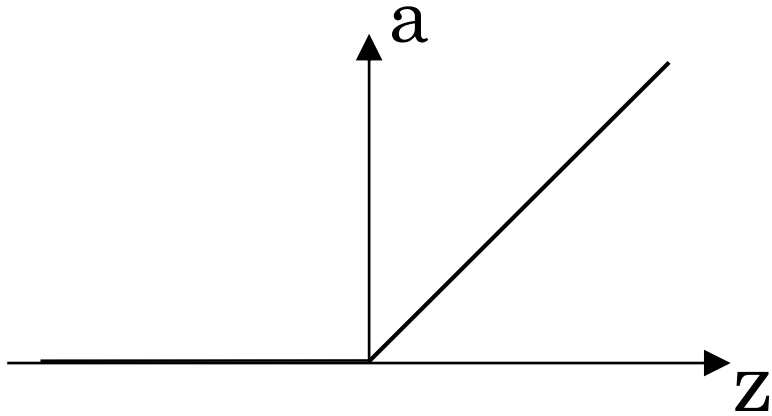
$$g(z) = \tanh(z) \\ = \frac{e^z - e^{-z}}{e^z + e^{-z}}$$

$$g'(z) = \frac{d}{dz} g(z) = \text{slope of } g(z) \text{ at } z \\ = \underline{1 - (\tanh(z))^2} \leftarrow$$

$$a = g(z), \quad g'(z) = 1 - a^2$$

$$\left| \begin{array}{ll} z=10 & \tanh(z) \approx 1 \\ & g'(z) \approx 0 \\ z=-10 & \tanh(z) \approx -1 \\ & g'(z) \approx 0 \\ z=0 & \tanh(z) = 0 \\ & g'(z) = 1 \end{array} \right.$$

# ReLU and Leaky ReLU



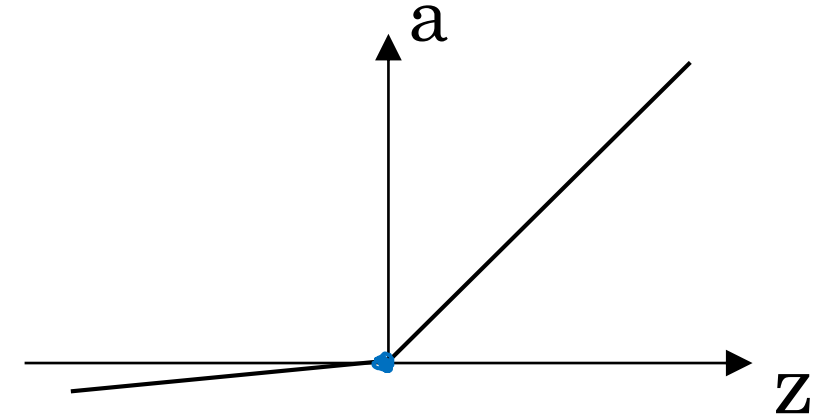
ReLU

$$g(z) = \max(0, z)$$

$$\rightarrow g'(z) = \begin{cases} 0 & \text{if } z < 0 \\ 1 & \text{if } z \geq 0 \\ \text{undefined} & \text{if } z = 0 \end{cases}$$

$z = 0.0000000000$

$g'$  prime then becomes what's called a sub gradient of the activation function  $g(z)$ , which is why gradient descent still works. But you can think of that the chance of  $z$  being you know zero point exactly 0 0 0 0 0 is so small that it almost doesn't matter what you set the derivative to be equal to when  $z$  equal to zero. So in practice this is what people implement for the derivative of  $z$ .



Leaky ReLU

$$g(z) = \max(0.01z, z)$$
$$g'(z) = \begin{cases} 0.01 & \text{if } z < 0 \\ 1 & \text{if } z \geq 0 \end{cases}$$