Implementation of TLC (Tiny Lambda Calculus)

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Contents

2016 级弘毅班编译原理课程设计第 4 次编程作业 (the parser of TLC)

1 Introduction

Our goal is the effective implementation of the programming language TLC (Tiny Lambda Calculus) by using the closure.

Lambda calculus is a formal system in mathematical logic and computer science for expressing computation by way of variable binding and substitution (see `https://en.wikipedia.org/wiki/Lambda_calculus`).

It is computation model of Functional Programming (see L. Paulson's lecture lambda.pdf, or my lecture lambda_lecture.pdf, and try lambda reducer at `http://www.itu.dk/people/sestoft/lamreduce/index.html`).

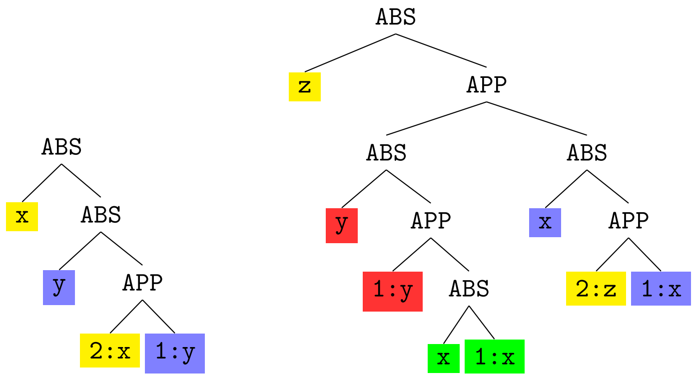
1.1 Specification of the language LAMBDA

the syntax of TLC can be desribed as:

```
lines : lines decl
      | decl
      ;
decl : LET ID '=' expr ';'
     | expr ';'
     ;
expr : INT
     | ID
     | IF expr THEN expr ELSE expr FI
     | '(' expr ')'
     | '@' ID '.' expr
     | expr expr
     ;
```

where `@x.M` is the abstraction (instead of "λ" in lambda calculus for input). `M N` is the application. and the conditional construct is specially added for the lazy evaluation of the conditional lambda terms. the application is left associative. and the precedence from low to high is: conditional construct, abstraction and application.

see `lexer.l` and `grammar.y` in detail.



(a) `@x.@y.x y` (b) `@z.(@y.y(@x.x))(@x.z x)`

Figure 1: AST with binding deepth (the first number of ID node)

1.2 Abstract syntax trees

We use De Brujin index for the AST, it will replace the binding variable by the *binding depth*. Ex. `@x.@y.x` is `@x.@y.2`, `@z.(@y.y(@x.x))(@x.z x)` is `@z.(@y.1(@x.1))(@x.2 1)` (see Figure 1). It will be the key to access the closure environment in the implementation. the free occurence of variable is strictly forbidden in TLC.

```
typedef enum {CONST=1, VAR=2, COND=3, ABS=4, APP=5} Node_kind;

typedef struct Ast {
  Node_kind kind;
  int value; /* for CONST and De Brujin index */
  struct Ast  *lchild, /*  for variable name and
                           abstraction variable
                         & apply function  body*/
    *rchild; /* for abstraction body  and app argument*/
  struct Ast *cond; /* for condition */
} AST;
```

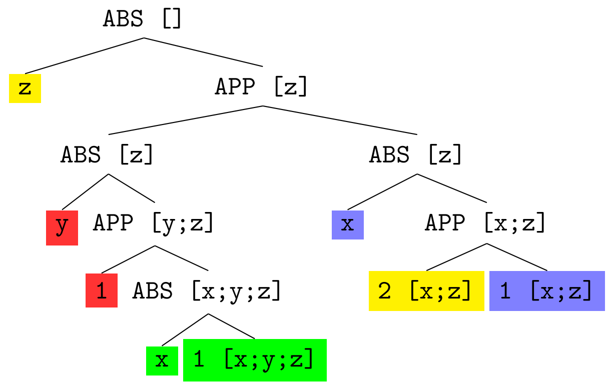
1.3 Binding Deepth

to find the binding deepth, we use the static stack `char *name_env[MAX_ENV]` with the cursor `int current` (`tree.c`) to store the abstraction level. each time enter AST with `ABS` node, we push the abstraction name in the stack, increase `current` for the next, and popup by decreasing `current` after leave the abstraction body. each time a variable encountered in the abstraction body, `find_deepth()` will return the number of the deepth in stack when first occurrence is found, see Figure 2.

```
int find_deepth(char *name)
{
  int i = current - 1;
  while (i + 1) {
    if (strcmp(name, name_env[i]) == 0) return current - i ;
    i--;
  }
  printf("id %s is unbound!\n", name);
  exit (1);
}
```

1.4 Primitive operations

`char *name_env[]` will also store the name of the declaration. so when the following statement is parsed:



(a) `@z.(@y.y(@x.x))(@x.z x)`

Figure 2: Binding deepth

```
let I = @x.x;
```

I will stored in `name_env[current]`. and we also store the AST of `@x.x` in the global `AST *ast_env[MAX_ENV]` (all defined in `grammar.y`) for the further uses (typing).
to support the arithmetic operations, `name_env[]` is prestored the following prefined functions:

```
char *name_env[MAX_ENV] = {"+", "-", "*", "/", "=", "<"};
```

to the above binary operators work correctly in $λ$-calculus, its should interpret as `@x.@y.op x y`, that is prefix notations! so we will write `+ (* 2 3) 4` instead of `2 * 3 + 4`.
the binding deepth is also the key to access the function defined in the declaration. so when `I` is declared, the `name_env[]` and `ast_env[]` will be

```
name_env[MAX_ENV] = {"+", "-", "*", "/", "=", "<", "I"}
ast_env[MAX_ENV] = {NULL, NULL, NULL, NULL, NULL, NULL, @x.(1:x)}
/* AST of operators is not needed for typing */
```

if we declare `PLUS` by input:

```
let PLUS = @x.@y. + x y;
```

the parser will generate the `(@x.(@y.(((+:9)(x:2))(y:1))))`. see Figure 3. In fact, after the parser enter the abstraction body `+ x y`, `name_env[]` will be:

```
name_env[MAX_ENV] = {"+", "-", "*", "/", "=", "<", "I", "x", "y"}
```

so `find_deepth("+")` will return 9, `find_deepth("x") = 2`, and `find_deepth("y") = 1`. after finish parsing, `name_env[]` changed to:

```
name_env[MAX_ENV] = {"+", "-", "*", "/", "=", "<", "I", "PLUS"}
```

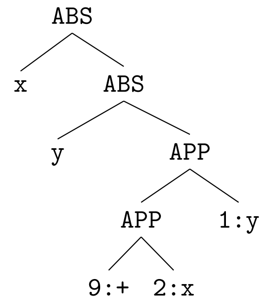
if we continue define `PLUS2` by input:

```
let PLUS = @x.@y + x 2;
```

the parser will generate the `(@x.(@y.(((+:10)(x:2))(y:1))))`. please remark that the binding deepth of `+` changed to `10` (see Figure 4). this is because the parsing of `PLUS2` is based with the new stack top `"PLUS"` of `name_env[]`, the the relative place of `"+"` is increased by `1`.
after `PLUS2`, `name_env[]` changed to:

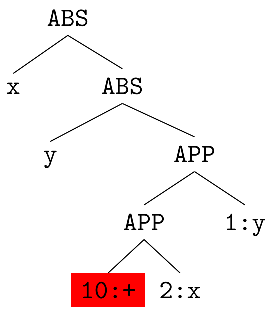
```
name_env[MAX_ENV] = {"+", "-", "*", "/", "=", "<", "I", "PLUS", "PLUS2"}
```

the operators `"+"`, `"-"`, ... must be scanned as normal `ID` with their binding deepth. but `"="` is also used as a single character token in the declaration like `"let I = ..."`. we use a global `int is_decl` (defined in `grammar.y`) to tell the lexer if `"="` should return '=' or `ID`, and add a middle action in the `decl` production to active `is_decl`:



(a) `@x.@y.+ x y`

Figure 3: AST of `PLUS`



(a) `@x.@y.+ x y`

Figure 4: AST of `PLUS2`

```
decl : LET {is_decl = 1; } ID '=' expr ';' {...}
```

deactive each time before return '=' in `lexer.l`:

```
"=" {
      char *id;
      if (is_decl) {is_decl = 0; return '=';}
      id = (char *) smalloc(yyleng + 1);
      strcpy(id, yytext);
      yylval = make_string(id);
      return ID;
}
```

1.5 output

We use the LaTeX graphic system tikz/pgf (`https://sourceforge.net/projects/pgf/`) and tikz-qtree (`https://ctan.org/pkg/tikz-qtree`) to illustrate AST. `printtree(AST *)` transforms the AST to LaTeX commands and store it in the file `expr.tex` which is the included file of `exptree.tex`. "`pdflatex exptree.tex`" generates the pdf of the AST (see exptree.pdf).

2 TODO

Completing `grammar.y` file to generate the AST for each lambda expression input, and output the AST to the file `expr.tex` by call `printtree(AST *)`.
you can use lambda expression in `library.txt` to test your program.
please send `grammar.y` as attached file to `mailto:hanfei.wang@gmail.com?subject=ID(04)` where the ID is your student id number.
–hfwang October 25, 2018