

What is a tail call?

$$def f() = g()$$

A call performed as the final action of a procedure.

What is tail recursion?

```
def odd(x) =
if (x == 0) true else even(x-1)
def even(x) =
if (x == 0) false else odd(x-1)
```

When a tail call might lead to the same function being called later in the call chain.

What is tail self-recursion?

```
def factorial(x) = {
  def loop(x,y) =
    if (x == 0) y else loop(x-1, x*y)
  loop(x,1)
}
```

When a function calls itself as its final action.

All of these should be as fast as a GOTO

But they're not :(

None of them should blow the stack

But they do:(



Java

- No support for tail calls
- Dead on arrival



Scala

- Self-recursive tail calls optimised using a goto
- Must be a final method, or a local function
- @tailrec annotation as a safety net



Clojure

- Supports self-recursive tail calls
- recur special form required to trigger optimisation



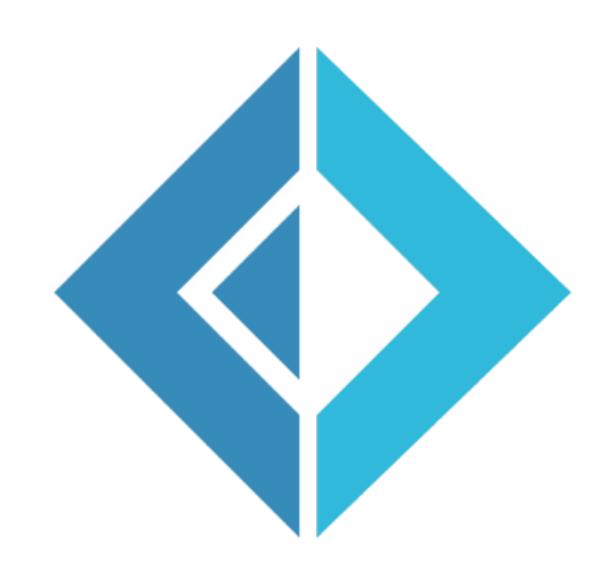
Kawa

- Supports general tail calls when enabled with a compiler flag
- Self-recursion optimised with a goto
- Some mutual recursion optimised with goto
- Uses trampolining for everything else (slow)
- At least it doesn't blow the stack



F#

- Not JVM
- Self-recursion optimised with a goto
- All other tail calls use the .NET .tail opcode
- NET tail call actually slower than a standard call due to extra security checks
- At least it doesn't blow the stack



"Folklore states that GOTO statements are cheap, while procedure calls are expensive.

This myth is largely a result of poorly designed language implementations."

- Guy Steele (1977)



Notation

```
e := v \mid v(vs) \mid
       let x = v in e
       let x = v(vs) in e
       letrec fs in es |
       if v then e_1 else e_2
f ::= x(xs) = e
v := x \mid c
xs ::= x, xs \mid ...
VS ::= V, VS | ...
fs ::= f; fs | ...
x ::= variable
c ::= constant
```

Code generation

- When generating a JVM method
 - If a function is only ever called in tail position
 - And is only called by the JVM method being generated
 - Then the function becomes a block called by a goto
 - Otherwise it becomes its own JVM method

Self recursion

```
letrec factorial(x) =
  letrec loop(x0,r0) =
    if x0 then
      let r1 = mul(r0, x0)
            x1 = sub(x0, 1)
      in loop(x1, r1)
    else
      r0
  in loop(x, 1)
in ...
```

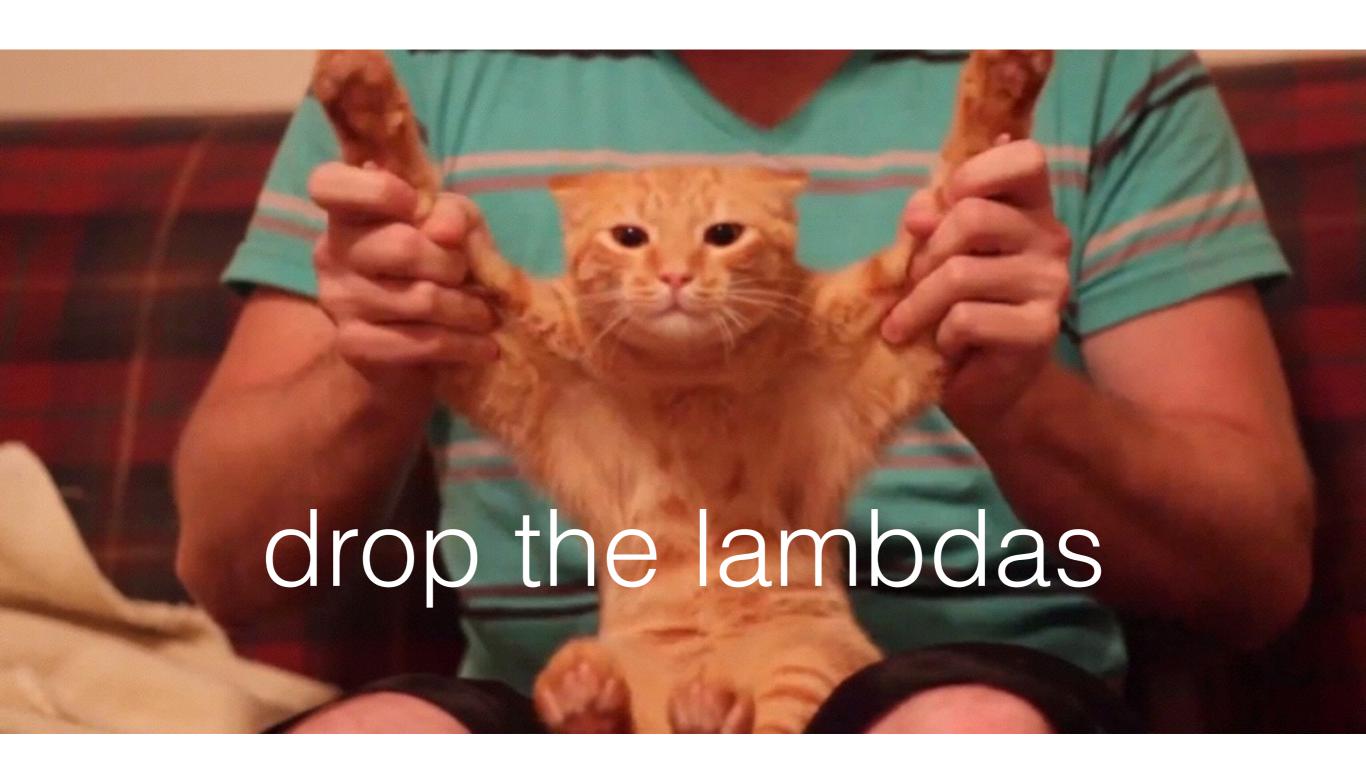
```
static int factorial(int x) {
    int loop_x0 = x;
    int loop_r0 = 1;
    goto loop;
loop:
    if (loop_x0 != 0) {
        int r1 = loop_r0 * loop_x0;
        int x1 = loop_x0 - 1;
        100p_x0 = x1;
        loop_r0 = r1;
        goto loop;
    } else {
        return loop_r0;
```

Mutual recursion

```
letrec
 odd(x) =
    if x then
      let x1 = sub(x, 1)
      in even(x1)
    else
 even(x) =
    if x then
      let x1 = sub(x, 1)
      in odd(x1)
    else
in odd(91)
```

```
static boolean odd(int x) {
    if (x != 0) {
        int x1 = x - 1;
        return even(x1);
    } else {
        return false;
static boolean even(int x) {
    if (x != 0) {
        int x1 = x - 1;
        return odd(x1);
    } else {
        return true;
static boolean f() {
    return odd(91);
```





Lambda dropping

```
letrec
                                       letrec
 odd(x) =
                                         odd(x) =
    if x then
                                           if x then
      let x1 = sub(x, 1)
                                             let x1 = sub(x, 1)
      in even(x1)
                                             in letrec
                                               even(y) =
    else
                                                 if y then
                                                   let y1 = sub(y, 1)
                                                   in odd(y1)
 even(x) =
    if x then
                                                 else
      let x1 = sub(x, 1)
      in odd(x1)
                                             in even(x1)
    else
                                           else
in odd(91)
                                       in odd(91)
```

Mutual recursion (again)

```
letrec
 odd(x) =
    if x then
      let x1 = sub(x, 1)
      in letrec
        even(y) =
          if y then
            let y1 = sub(y, 1)
            in odd(y1)
          else
      in even(x1)
    else
in odd(91)
```

```
static boolean f() {
    int odd_x, even_y;
    odd_x = 91;
    goto odd;
odd:
   if (odd_x != 0) {
        int x1 = odd_x - 1;
        even_y = x1;
        goto even;
    } else {
        return false;
even:
    if (even_y != 0) {
        int y1 = even_y - 1;
        odd_x = y1;
        goto odd;
    } else {
        return true;
```



Mutual recursion (non tail)

```
letrec
  odd(x) =
    if x then
      let x1 = sub(x, 1)
      in even(x1)
    else
  even(x) =
    if x then
      let x1 = sub(x, 1)
      in odd(x1)
    else
in let z = odd(91)
       w = even(92)
in and (z, w)
```

```
static boolean odd(int x) {
    if (x != 0) {
        int x1 = x - 1;
        return even(x1);
    } else {
        return false;
static boolean even(int x) {
    if (x != 0) {
        int x1 = x - 1;
        return odd(x1);
    } else {
        return true;
static boolean f() {
    boolean z = odd(91);
    boolean w = even(92);
    return z && w;
```



Specialisation

letrec

```
odd(x) =
    if x then
      let x1 = sub(x, 1)
      in even(x1)
    else
  even(x) =
    if x then
      let x1 = sub(x, 1)
      in odd(x1)
    else
in let z = odd(91)
       w = even(92)
in and(z, w)
```

```
letrec
 odd(x) =
    if x then
      let x1 = sub(x, 1)
      in odd_even(x1)
    else
       0
  odd_even(x) =
    if x then
      let x1 = sub(x, 1)
      in odd(x1)
    else
```

Specialisation

```
odd(x) =
                                   even(x) =
   if x then
                                      if x then
                                        let x1 = sub(x, 1)
     let x1 = sub(x, 1)
     in odd_even(x1)
                                        in even_odd(x1)
   else
                                      else
 odd_{even}(x) =
                                    even\_odd(x) =
   if x then
                                      if x then
     let x1 = sub(x, 1)
                                        let x1 = sub(x, 1)
     in odd(x1)
                                        in even(x1)
                                      else
   else
```

Lambda dropping

```
odd(x) =
                                  even(x) =
   if x then
                                      if x then
     let x1 = sub(x, 1)
                                        let x1 = sub(x, 1)
     in letrec
                                        in letrec
       odd_{even}(y) =
                                          even\_odd(y) =
                                            if y then
         if y then
           let y1 = sub(y, 1)
                                              let y1 = sub(y, 1)
           in odd(y1)
                                              in even(y1)
         else
                                            else
     in odd_even(y1)
                                        in even_odd(y1)
   else
                                      else
```

Mutual recursion (done!)

```
static boolean odd(int odd_x) {
                                       static boolean even(int even_x) {
odd:
                                       even:
                                          if (even_x != 0) {
   if (odd_x != 0) {
        int x1 = odd_x - 1;
                                               int x1 = even_x - 1;
        odd_even_y = x1;
                                               even\_odd\_y = x1;
                                               goto even_odd;
        goto odd_even;
    } else {
                                           } else {
                                               return true;
        return false;
odd_even:
                                       even_odd:
    if (odd_even_y != 0) {
                                           if (even_odd_y != 0) {
        int y1 = odd_{even_y} - 1;
                                               int y1 = even\_odd\_y - 1;
        odd_x = y1;
                                               even_x = y1;
        goto odd;
                                               goto even;
    } else {
                                           } else {
        return true;
                                               return false;
```





"in F# on .NET (which supports tail calls) there is really nice support for asynchronous programming that depends on tail calls to avoid the stack increasing when you swap between different asynchronous handlers and lightweight software threads."

- Rowan Davies

We still can't do that :(



Trampolines

```
static void trampoline(Cont k) {
interface Cont {
                                       while (k != null) {
   Cont invoke();
                                           k = k.invoke();
class Foo implements Cont {
   Cont invoke() {
        return new Bar(1, 2, 3);
                                             Slow! :(
class Bar implements Cont {
```

Direct tails calls should always be fast and efficient

Even on the JVM!

Acknowledgements

- Patryk Zadarnowski for many long conversations on optimising administrative normal form in the context of the JVM and in particular for pointing me towards the technique of lambda dropping.
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