Evolution of EROIs of Electricity Until 2050: Estimation Using the Input-Output Model THEMIS

Adrien Fabre¹, **PRELIMINARY VERSION**

Abstract

The EROI –for Energy Returned On Invested– of an energy technology measures its ability to extract energy efficiently. Previous studies draw a link between the affluence of a society and the EROI of its energy system, and show that EROIs of renewables are lower than those of conventional fossil fuels. Logically, concerns have been expressed that system-wide EROI may decrease during an energy transition. First, I explain theoretically that the EROIs of renewables themselves could decrease as energy-efficient fossil fuels would be replaced by less energy-efficient renewables in the chain of production. Then, using the multiregional input-output model THEMIS, I estimate the evolution of EROIs and prices of electric technologies from 2010 to 2050 for the baseline and the Blue Map scenarios of the International Energy Agency, and for the 100% renewable Greenpeace's electricity [r] evolution scenario. Global EROI of electricity is predicted to remain quite stable, going from 8 in 2010 to 6 or 7 in 2050, depending on the scenario. Finally, I study the relation between the EROI and the price of a technology, and show that in theory they can both decrease at the same time, against the intuition that they are inversely related; but that in practice they move in opposing directions.

Keywords: EROI; input-output; THEMIS; MRIO; sustainability; energy transition

Acknowledgments. I am grateful to Thomas Pregger and his team at DLR for graciously providing me the tables of future energy consumption in the Greenpeace [R]evolution scenarios. I am indebted to Thomas Gibon, Anders Aversen and the Norwegian University of Science and Technology (NTNU) for providing me the data of THEMIS and helping me using it. I am thankful Arjan de Koning for answering my questions related to Cecilia 2050. I hail the work of Konstantin Stadler on pymrio, an invaluable open-source python library when dealing with Multi-Regional Input-Output Tables (MRIO). I am grateful to Cyril François for his thoughtful comments on this project. I am thankful to the Isterre for providing me an office at Grenoble to conduct this research. I am grateful to Olivier Vidal and Mouez Fodha for their support.

Disclaimer. This work is preliminary.

All the code is on-line, and can be found inside a note-book at: bit.ly/future eroi code.

1. Introduction

As the harmful impacts of climate change call for a prompt energy transition away from fossil fuels —not to mention their depletion that shall ultimately make this transition unavoidable, concerns have been expressed that, in a decarbonized energy system, the lower efficiency of renewable energy might not allow to sustain advanced standards of living [26, 33]. The Energy Returned On Invested (EROI) is the key concept to characterize the efficiency of a technology or of an energy system at delivering more useful energy than it requires during its construction, operation and dismantling, as it is the ratio between these two quantities. A minimal requirement for a technology or energy system to be energetically sustainable is to have an EROI above 1, meaning that it provides more energy than it requires.

One issue to assess future energy systems is that the future EROI of a given technology cannot be readily deduced from current estimates. Indeed, as King [22] remarked, the EROI of a technology is not intrinsic, but depends on the whole technological structure of the economy. To see this, let us suppose that plants currently used to build solar panels employed renewable electricity instead of electricity from coal

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 $^{^2}$ The energy expert Jean-Marc Jancovici also expressed concerns over this subject during a speech at the École Normale Supérieure in 2018; this is what motivated me to study the topic.

as their sources of energy. Then, provided that the EROI of renewable electricity is lower than that from fossils, the energy required to build solar panels will increase, and their EROI will decrease. Surprisingly, it seems that no study has aimed at estimating future EROIs while taking this system dependency into account, although some have called for such studies[4].³ Yet, granted that EROIs of renewables are lower than EROIs of fossils and that decreasing EROIs jeopardize prosperity, the evolution of EROIs during the energy transition is of critical importance: let us review these two motivations in turn.

Many estimations of EROIs have been made, and among the various different figures derived from diverse data sets and methodologies, none stands out as singularly authoritative. Dale [9] review all EROI estimates until 2010; while Hall et al. [16] aggregate the estimates of the literature in a metanalysis. I choose to present the results of Weißbach et al. [34] (see Figure 1), because they compute the EROIs of different technologies in a comparable manner. In addition, the buffered EROIs of Weißbach et al. [34] take into account the supplementary capacity, grid and storage required for the deployment of renewable technologies, which yields lower but presumably more accurate estimates for their EROIs. As anticipated, the EROIs of renewable electricity sectors they find are significantly lower than those of electricity from fossil fuels, except for hydro.

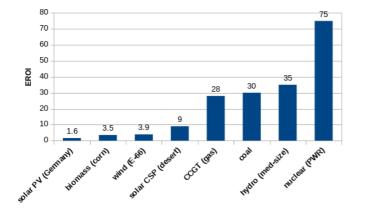


Figure 1: Estimates of EROIs of different electricity technologies, from Weißbach et al. [34], where supplementary capacity and storage required for the deployment of these technologies is accounted for.

Furthermore, some authors argue that the value of EROI is of primary relevance, as they draw a link between the systemwide EROI and affluence of a society [15, 25, 26, 11]. Here is how Hall [14] summarizes the argument:

Think of a society dependent upon one resource: its domestic oil. If the EROI for this oil was 1.1:1

then one could pump the oil out of the ground and look at it. (...) Hall et al. [15] examined the EROI required to actually run a truck and found that if the energy included was enough to build and maintain the truck and the roads and bridges required to use it (i.e., depreciation), one would need at least a 3:1 EROI at the wellhead. Now if you wanted to put something in the truck, say some grain, and deliver it that would require an EROI of, say, 5:1 to grow the grain. (...) 7 or 8:1 to support the families. If the children were to be educated you would need perhaps 9 or 10:1, have health care 12:1, have arts in their life maybe 14:1 and so on.

The reasoning of Hall relies on the observation that all sectors of the economy require energy, and that the more efficient is the energy production (i.e. the higher is the EROI), the more energy is available to the rest of the economy. In strict logic, Hall's argument relies on two questionable assumptions: that factors of production (and especially the labor force) are used at their full capacity, and that technical and organizational progress will not be sufficient to sustain current level of prosperity with significantly less labor (or other factors of production in limited supply). In rejection of these assumptions, one can imagine a sustained level of prosperity with a lower system-wide EROI, provided that a higher share of factors of production be devoted to the energy sector. That being said, given that current system-wide EROI is already declining due to the decline in fossil fuels quality [8, 30, 7] and that technical progress is incremental, the aforementioned analyses should not be neglected. Under the current system of production, which will persist in the short term, EROI should stay largely above 1 and not decrease too much for prosperous standards of living to be sustained. Admittedly, a systemwide EROI close to the theoretical lower bound of 1 might fuel an industrial civilization in the very long run as long as the resources required for the massive deployment of energy systems remain available (this is not guaranteed, as human labor, land and materials are in limited supply). However, in the medium term and with more sensible scenarios, a diminishing EROI would probably imply that a substantial share of the labor force will shift their occupation to the energy sector.

In view of the potential implications of a declining EROI, this paper provides an assessment of the EROI of different electricity technologies in various prospective scenarios, which includes a 100% renewable electricity system. To this end, I employ input-output analysis and I rely on a prospective series of multi-regional Input-Output Tables (IOT): THEMIS [13], which models two scenarios from the International Energy Agency [20]: Baseline and Blue Map. Then, I further modify THEMIS' IOTs to embed two decarbonized scenario of power generation: Greenpeace's Energy [R]evolution (ER) and Advanced Energy [R]evolution (ADV) [32]. To my knowledge, this work is the first to compute EROIs in a 100% renewable scenario (the ADV one).

But more than EROIs, the evolution of energy prices is

³Admittedly, King [22] provided a numerical application of the system dependency of EROI (see his Table 4), but his computations had a purely illustrative purpose and his input values were not supposed to be accurate: this is why he did not even comment on the result that the EROI fell incidentally below 1 in his 100% renewable mix. Although Although Arvesen et al. [2] also compute energy requirements similar to mine in some respects, but they do not assess the evolution of EROIs in a 100% renewable scenario.

pivotal for economists and practitioners. Admittedly, previous studies suggest an inverse relation between EROIs and energy prices. Yet, such a simple rule does not withstand a careful analysis. Indeed, while explaining to what extent EROI and price are related, I will show that they do not necessarily move in opposite directions. This does not mean that the kind of input-output analysis that is used to estimate EROIs cannot be used to predict prices. In fact, I will present such predictions; but one should take them with more caution than EROI estimates, because IOT is better suited to handle physical notions than economic ones.

Finally, section 2 explains theoretically why the EROI of a technology is not an intrinsic property; section 3 presents the methodology and the results; section 4 studies the implications of declining EROIs on prices and GDP; section 5 concludes.

2. The EROI of a Technology Is Not Intrinsic

2.1. A Simple Model With A Unique Energy Technology

The element $a_{i,j}$ of the technology matrix A represents the quantity of input i required to produce one unit of output j. Below is an illustrative technology matrix with three inputs (and the same three outputs): an energy technology, materials, and energy. m_e denotes the quantity of materials (m) required to produce one unit of energy technology (e), and this notation extends naturally to all elements of A. The numerical values of the coefficients have a purely pedagogical purpose and have been arbitrarily chosen; taking other figures would not change qualitatively the results.

$$A = \begin{pmatrix} 0 & 0 & 1 \\ m_e & m_m & 0 \\ E_e & E_m & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ m_e & 0.2 & 0 \\ 0.1 & 0.5 & 0 \end{pmatrix} \text{ energy techno.}$$

The system-wide EROI, or Energy Returned On Invested, is the ratio between the energy delivered by the system, and the energy required to build, operate, maintain and dismantle it. In other words, it is the inverse of the amount of energy required to produce one unit of energy, when the series of all embodied inputs are taken into account.

The embodied inputs *x* required for a final demand *y* can be calculated using the well known formula [27, 10, 28]:

$$x(y) = (I - A)^{-1} \cdot y.$$

We denote by $\mathbb{1}_S$ the vector with 1 at the positions of the sectors $s \in S$, and zeros everywhere else. As energy E is the

last input of our list,
$$\mathbb{1}_E = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$
 and the gross embodied energy

required for a final demand y is the last element of x: $\mathbb{1}_E^T \cdot (I_n - A)^{-1} \cdot y$. Thus, the EROI is

$$\begin{aligned} \text{EROI} &= \frac{\text{delivered energy}}{\text{net embodied energy}} \\ &= \frac{1}{\mathbb{1}_{E}^{T} \cdot \left((I - A)^{-1} \cdot \mathbb{1}_{E} - \mathbb{1}_{E} \right)}. \end{aligned}$$

After some calculations, we find:⁴

EROI =
$$\frac{(1 - E_e)(m_m - 1) + E_m m_e}{E_e(m_m - 1) - E_m m_e}$$
$$= \frac{0.72 - 0.5 m_e}{0.08 + 0.5 m_e}$$

Unsurprisingly, one can see in Figure 2 that the EROI decreases with the material intensity of the energy technology, because extracting and processing material requires energy.

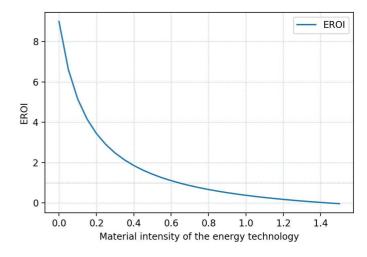


Figure 2: EROI in the simple model in function of the material intensity m_e of the energy technology.

For an intensity above 0.6, the EROI is below 1. An EROI below 1 means that the energy technology is not worth developing, because (in net) it consumes energy rather than providing it. Such a system is not sustainable (and not realistic): for it to happen the society should have accumulated energy in the past from an energy source no more accessible, and would waste this energy in that absurd technology.

For even higher intensities, the EROI falls below 0, which means that the energy (recursively) required to produce one unit of energy is infinite. Here, free energy coming from the past would not suffice to build the energy technology: one would also need to have free materials (i.e. materials requiring no energy to access them). Such a world is physically impossible.

2.2. A Simple Model With A Mix of Two Technologies

Now, let us consider two energy technologies, with the same energy intensity, but different materials intensities.

Even if this example is purely illustrative, let us call them PV (for solar photovoltaic) and gas (for gas power-plant electricity) to grasp the motivation for this paper. The numbers are completely made up, but they respect the fact that PV is more material intensive than gas [18]. Here is our new technology matrix, where *p* represents the share of PV in the energy (or electricity) mix.

 $^{^4\}mbox{All}$ the code and computations can be found on-line, at: bit.ly/futureEROI

$$A = \begin{pmatrix} 0 & 0 & 0 & p \\ 0 & 0 & 0 & 1-p \\ m_{PV} & m_g & m_m & 0 \\ E_{PV} & E_g & E_m & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 & p \\ 0 & 0 & 0 & 1-p \\ 0.7 & 0.1 & 0.2 & 0 \\ 0.1 & 0.1 & 0.5 & 0 \end{pmatrix} \text{ pV}$$
gas materials energy

With some calculus, we obtain:

$$EROI = \frac{0.67 - 0.3p}{0.13 + 0.3p}$$

This corresponds to the system-wide EROI. But now that we have two technologies, we can compute the EROI of each of them: 5

$$EROI_{PV} = 1.558 - 0.698p$$

 $EROI_{gas} = 5.154 - 2.308p$

Notice that they depend on the energy mix *p*: the EROI of a technology is not an intrinsic property. Indeed, it depends on the whole economic system, or more precisely, of all technologies used in their chain of production.⁶ Here, the higher the share of PV in the mix, the lower the EROI of both technologies: this comes from the higher material intensity of PV.

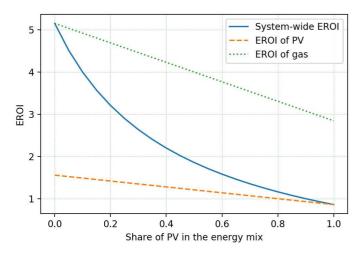


Figure 3: EROIs in the two-technology model in function of the share p of PV in the energy mix.

One can see on Figure 3 that for highest penetration of PV, the EROI falls below unity. In other words, a renewable energy mix with 100% PV is not sustainable in this example. Even more worryingly, if one computes the EROI of PV in an energy mix relying mostly on gas, one would find a high-enough

EROI for PV (meaning, above 1). Hence, one cannot conclude that a technology is sufficiently efficient (or sustainable) just by computing its EROI in the current energy mix. Yet, EROIs computations are always done from actual data of our economy, and could falsely represent the efficiencies of energy technologies in another energy mix, say, a 100% renewable one. This uncertainty concerning the sustainability of a decarbonized energy system motivated the core of this paper: the estimation of EROIs after a global energy transition.

3. Estimation of Current and Future EROIs Using THEMIS

3.1. Setting and Data

Different notions of EROIs have been used in the literature, and some papers clarify them all (e.g. 5, 29). The most relevant notion for this research is defined by Brandt & Dale [5] as the Gross Energy Ratio (GER) and called by King [22] the net external energy ratio.⁷ The GER measures the ratio of energy delivered over energy embodied in inputs net of the energy of the fuels used in the process. Thus, for example, the denominator of the GER does not take into account the energy provided by gas in a gas powered plant. The term "gross" is used because all energy output is taken into account; on the contrary Net Energy Ratios subtract from the numerator all "self-use" output that is used in the pathway of production of the technology.⁸ A related indicator that is sometimes used to compute EROI as it is already included in many inputoutput databases is the Cumulated Energy Demand (CED). I don't use it because Arvesen & Hertwich [1] have shown that it is erroneous to use the CED directly for EROI computations, without making adjustments.

In most cases, EROIs (or energy ratios) are defined using quantities of primary energy. However, I adopt a different approach in this paper, and use only secondary energies in my computations. Indeed, as Arvesen & Hertwich [1] put it, "EROI does not need to measure primary energy per se; the crucial point is to measure energy diverted from society in a unit of equivalence". Also, the choice of secondary energy carriers is consistent with an energy system relying on renewable electricity, while for such systems the definition of primary energy is not harmonized and this can lead to inconsistencies: Frischknecht et al. [12] spot for example a factor 6 between the cumulative (primary) energy demand for solar photovoltaic computed according to different methods. Although the sectors bringing energy are not the same in the two approaches (the primary approach uses crude oil when

⁵Similarly to the system-wide EROI, the EROI of a technology is the ratio between the energy delivered by one unit of this technology (over its lifetime), and the energy required to build, operate, maintain and dismantle it.

⁶Chain of production, recursive or embodied inputs are synonyms; their analysis is known as *structural path analysis* in the literature.

⁷The terminologies of these two papers are not compatible. I follow Brandt & Dale [5], as their paper aims at harmonising the terminology.

⁸For King, 'gross' energy is the total energy diverted from Nature while 'net' is the output of energy from the technology, what Brandt and Dale call 'gross'. Furthermore, King would qualify 'external' any notion that subtract direct fuel inputs from the denominator (rather than just subtracting the output), while Brandt and Dale always take this as a base case, and employ 'external' when indirect self-use inputs are also subtracted: it mirrors their notion of 'net' for the denominator.

the secondary approaches uses gasoline, for example), both approaches are equally valid.

Furthermore, practitioners often use a factor of conversion (around 3) to account for the higher quality of electricity as compared to fossil fuels. I follow the recommendation of Murphy et al. [29] by undertaking my computations without and with a quality-adjustment factor of 2.6. However, I prefer not to bring to the fore the quality-adjusted computations, provided in Appendix C, and I focus instead on nonquality adjusted EROIs. The reason for this is that the factor of conversion is not well established: it represents the inverse of the yield of a thermal power station (about 38%), but this yield depends on the technology and on the fuel used. Moreover, for certain usage like heating, the yield of fossil fuels is close to that of electricity, and fossil fuels are disproportionately used for these applications for which they have a higher yield, both of which imply that the difference in quality between fossils and electricity may be smaller than usually assumed.

To avoid the possible ambiguity of sentences, I reproduce below the formulas used to compute the EROI for a technology (or an energy system) t, which I denote GER_t^{2nd} . Let us recall that y is the vector of final demand, given by the scenario, and A is the technology matrix (or input-output table). E^S is the vector of unitary energy supply per sector, meaning that E_t^S is the energy supplied by one unit of sector t, hence $E^S \cdot y_t$ gives the energy supplied by the technology t:

$$\mathrm{supply}_t = E^S \cdot y_t$$

 \odot (resp. \oslash) denotes the Hadamard (or entrywise) product (resp. division), so that $E^S \odot \mathbb{1}_{secondary}$ is the vector of unitary *secondary* energy supply. The main term at the denominator of the *GER* is the secondary energy embodied in inputs, net of the energy supplied by the technology:

net secondary embodied_t =
$$E^S \odot \mathbb{1}_{secondary} \cdot ((I - A)^{-1} \cdot y_t - y_t)$$

To the last term, we also need to subtract the energy supplied by the secondary fuels in the last step of the process¹⁰ (logically, this term is nil for renewables). Indeed, such energy is not used to build or maintain the energy system; rather, it is an energy transformed and delivered by the energy technology.

fuel input
$$_t = E^S \odot \mathbb{1}_{secondary fuel} \cdot A \cdot y_t$$

Finally, we have:

$$y_t = (\operatorname{demand}_t \otimes E_S) \otimes \mathbb{1}_t$$
.

$$GER_t^{2nd} = \frac{\text{supply}_t}{\text{net secondary embodied}_t - \text{fuel input}_t}$$

I apply these formulas to the IOTs (i.e. technology matrices A) and the vectors of unitary energy supply E^S from THEMIS [13]. THEMIS contains hybrid input-output tables: precise data on electricity units (the foreground) is completed with data on other sectors that originates from life cycle inventories and national accounts (the background). Gibon et al. [13] have compiled various life cycle inventories into the 609 sectors of the foreground. The background contains data in physical units for 4,087 sectors from the life cycle inventory ecoinvent and data in monetary units for 203 sectors from the input-output database Exiobase 2 [35]. The 44 Exiobase regions are aggregated into 9 macro-regions that coincides with those of the International Energy Agency (IEA), so that the number of rows and columns in each IOT is 9 times the number of sectors: 44,046. Starting from data of the 2010 IOT, the 2030 and 2050 IOTs of THEMIS embed expected technological efficiency improvements of key sectors. Furthermore, it is worth noting that THEMIS IOTs are constructed as if the whole economy were at a steady-state, contrarily to national accounts, which give the flows between sectors for a given year. This matches perfectly our purpose, because there is no need to adjust the EROI computations for the growth of some sector or for the lifetimes of some technologies.

The two scenarios native in THEMIS are the baseline (BL) and the Blue Map (BM) scenarios of the IEA [20]. While the former posits an almost constant electricity mix, the latter is compatible with a 50% probability to contain global mean temperature anomaly to +2°C in 2100. As Blue Map still relies at 30% on fossil fuels based electricity in 2050 —including 17% with Carbon Capture and Storage (CCS); I wanted to assess more ambitious scenarios. Hence, I combined with THEMIS the scenarios from Greenpeace's Energy [R]evolution report [32]. Greenpeace proposes a business as usual scenario (REF) close to baseline, as well as two scenarios compatible with the 2°C target. Both exclude CCS and phase out from nuclear between 2012 and 2050.11 The first Greenpeace scenario, Energy [R]evolution (ER), comprises 93% of electricity from renewable sources in 2050, while the second one, Advanced Energy [R]evolution (ADV), attains 100% renewable. As the difference is small between these two scenarios, I focus on the 100% renewable one. I describe my methodology for embedding the regional electricity mixes of Greenpeace's scenarios into THEMIS in Appendix A.

In the literature, most EROIs estimations follow a bottomup approach that use data from life cycle inventories. Bottomup studies describe in details the power facilities and the most

 $^{^{9}}$ In practice, y is obtained from the scenario of energy demand from the IEA:

 $^{^{10}}$ Technically, we would need to subtract secondary fuels to all similar steps in the embodied inputs, for each embodied input that uses a thermal power plant. However, I neglect these terms, as this would complicate the presentation for no significant difference in the result: a variation in the value of *fuel input* below 0.1% for all technologies considered.

¹¹ The study funded by Greenpeace was in fact conducted by researchers at the Institute of Engineering Thermodynamics of the German Aerospace Center (DLR), who applied their model REMix. Using the same model, Berrill et al. [3] minimize the cost of European electricity generation under different carbon prices. Interestingly, an outcome of the model was to phase nuclear out, but to select coal with CCS. This indicates that the choices of Greenpeace was not solely motivated by a minimization of costs, but also by expert judgment and ethical considerations.

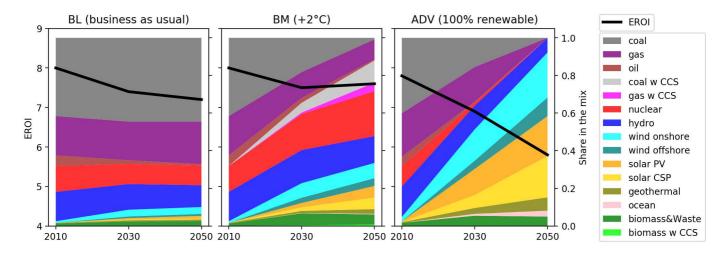


Figure 4: Evolution of global EROIs and mixes of electricity for different scenarios.

direct inputs to the energy technologies —the *foreground*, but they do not cover the *background*: indirect inputs such as clerical work or R&D. On the contrary, the input-output method allows to encompass all embodied inputs exhaustively. As a consequence of this more comprehensive account of embodied energy than usual, we expect estimates of EROIs lower than the average of the literature. That being said, it is not a concern if our estimates are not directly comparable to those of the literature, and we are mainly interested in comparing them internally, among the different years and scenarios, and to scrutinize whether they vary substantially or not.

Because renewable sources are intermittent and dispersed, the capacity, grid extension and storage they require do not increase linearly with the electricity delivered. Hence, as Greenpeace scenarios are not native in THEMIS, they need further adjustments to account for these non-linearities. I explain in Appendix A how the need for surcapacity is addressed. Concerning transmission and storage, however, the requirements are not given by the Greenpeace report [32]. Even if the report does not precise any plan relative to storage, hydrogen produced from renewables seems to play a substantial role in Greenpeace scenarios, as its share in the electricity mix is 5% in ADV 2050. However, as the sector 'Electricity from hydrogen' is absent from THEMIS, hydrogen has been excluded from this analysis. This limitation will be addressed in future work, together with the study of an energy transition in the transportation sector (which also relies on hydrogen). Meanwhile, other sources can inform on orders of magnitude of storage and transmission [3, 24, 31]. Applying the same optimization model that is used in the report, Scholz et al. [31] show that the cost of storage and transmission combined is 4.6% of total cost in a business-as-usual scenario and 10.6% in a 100% renewable one. The adjustment needed for the cost, around 6%, gives a rough estimate of the upward bias of unadjusted EROI estimates (see section 4.2 on the relation between price and EROI).

Finally, data for Concentrated Solar Panels (CSP) had to be adjusted, because the original data mistakenly contained an energy supplied by unit of solar CSP of 0 (leading to abnormally low EROIs, around 2). Backed by Thomas Gibon, the creator of THEMIS, I corrected this error by setting the unitary energy supplied for solar CSP in all regions to its value in *OECD North America* (still letting the value depend on the scenario and the year).

3.2. Main Results

Main results are shown in Table 1, while other results can be found in Appendix C. Some EROIs are missing, because not all technologies already existed on an industrial scale in 2010, and some technologies are discarded in the future by some scenarios. One can notice that, as expected, PV and wind panels have a lower EROI than electricity from fossil fuels. The EROIs of renewables decrease, as anticipated in the previous section. However, they remain largely above 1, suggesting that renewables are truly sustainable energetically. The system-wide EROI for the entire electricity sector is given at the bottom line. It is currently 8.0; it decreases slightly until 7.4 ± 0.2 in 2030 and 2050 in both IEA/THEMIS scenarios. Unsurprisingly, the decrease is a little more pronounced in the 100% renewable scenario: at 6.9 in 2030 and 5.8 in 2050.

4. Implications of a Decreasing EROI on Prices and GDP

Economists and practitioners may wonder whether the decline in EROI predicted in case of an energy transition has implications on future electricity prices. In this section, I review the literature on the relation between EROI and the price of energy, extend a result from Herendeen [17] to characterize this relation, and estimate this relation empirically. Although an inverse relation holds in special cases and has some explanatory power on observations, the theoretical analysis shows that "anything can happen". This theoretical result weakens the idea that decreasing a EROI necessarily leads to a recession.

Table 1: EROIs and share in electricity mix of electric technologies in the model THEMIS for different scenarios and years.

Scenario			Baselir	ne (BL)			Blue Map (BM, +2°)				ADV (100% renewable)			
Year	2010		2030		2050		2030		2050		2030		2050	
Variable	EROI	mix	EROI	mix	EROI	mix	EROI	mix	EROI	mix	EROI	mix	EROI	mix
biomass w CCS	_	0	_	0	_	0	4.6	0	4	0.01	_	0	_	0
biomass&Waste	11.3	0.01	6.2	0.02	5.8	0.03	5.5	0.06	5.2	0.05	5.2	0.05	4.6	0.05
ocean	5.5	0	2.4	0	2.9	0	3.7	0	5.8	0	4.8	0.01	4.9	0.03
geothermal	5.3	0	5.1	0.01	5	0.01	5.2	0.01	5.4	0.02	3.8	0.03	3.9	0.07
solar CSP	21.5	0	8.8	0	9.1	0.01	8.2	0.02	7.9	0.06	9.2	0.07	7.8	0.22
solar PV	9.2	0	7.4	0.01	7.1	0.01	6.3	0.02	6	0.06	5.4	0.14	4.7	0.21
wind offshore	9.3	0	10.9	0.01	10.5	0.01	7.6	0.03	6.2	0.04	6.5	0.04	6.4	0.1
wind onshore	9.4	0.01	9.2	0.04	8	0.04	7	80.0	7.3	80.0	7.1	0.17	5.8	0.24
hydro	13.1	0.16	11.8	0.14	11.8	0.12	12.7	0.18	13.1	0.14	11	0.13	10.9	80.0
nuclear	10.4	0.14	7.2	0.11	7	0.1	7.3	0.19	7.4	0.24	8.3	0.02	_	0
gas w CCS	_	0	_	0	7.4	0	7.9	0.01	9.1	0.05	-	0	_	0
coal w CCS	_	0	_	0	6.1	0	7.1	0.05	7.1	0.12	_	0	_	0
oil	8.2	0.06	9.5	0.02	9.6	0.01	9.4	0.03	7.3	0.01	9.9	0.01	_	0
gas	13.7	0.21	14.8	0.21	14.6	0.23	17.1	0.14	19.6	0.11	16.4	0.18	_	0
coal	12.6	0.42	11.3	0.45	11.3	0.45	11.4	0.18	12.4	0.01	10.3	0.16	11.5	0
Total (PWh/a)	8	19.76	7.4	34.29	7.2	45.97	7.5	28.01	7.6	40.22	6.9	36.74	5.8	64.04

4.1. Inverse Relation Proposed in First Studies

King & Hall [23] point both theoretically and empirically that the price of a unit of energy p_t and the EROI of a technology t are inversely related. Defining the monetary return on investment MROI (i.e. the financial yield $\frac{\$_{\text{out}}}{\$_{\text{investment}}}$), they derive the formula:

$$p_t = \frac{\$_{\text{out}}}{E_{\text{out}}} = \frac{\text{MROI}_t}{\text{EROI}_t} \cdot \frac{\$_{investment}}{E_{\text{in}}}$$
(1)

Heun & de Wit [19] find an equivalent formula. They call MROI the mark-up m_t , consider production costs per gross output $c_t = \frac{\$_{\text{investment}}}{E_{\text{out}} + E_{\text{in}}}$ and use their own notion of EROI: $\text{EROI}_t^H = \frac{E_{\text{out}} + E_{\text{in}}}{E_{\text{in}}} = \text{EROI}_t + 1, \text{ so that equation (1) rewrites}$

$$p_t = \frac{m_t}{\text{EROI}_t^H - 1} \cdot \frac{\$_{\text{investment}}}{E_{\text{out}} + E_{\text{in}}} \cdot \frac{E_{\text{out}} + E_{\text{in}}}{E_{\text{in}}} = \frac{m_t \cdot c_t}{1 - 1/\text{EROI}_t^H}$$

The problem with these formulas is that all variables move together: when EROI varies, so does the cost of production, so that we cannot predict the future price taking this cost as fixed. Heun & de Wit [19] acknowledge this; and thus study the empirical link between EROI and price.

4.2. Empirical Relation Between EROI and Price

Using US data on oil and EROI from [6], Heun & de Wit [19] regress p_t on the EROI. They obtain a good fit even

Table 2: Predicted average global price of electricity (in €/MWh)

year	2010		2030		2050				
scenario	all	BL	BM	ADV	BL	BM	ADV		
price	27	28	30	30	28	30	32		

in their simplest regression ($R^2 = 0.8$), and find $p_{\rm oil} = \beta_0 \cdot {\rm EROI_{oil}^{-1.4}}$.

This result is interesting, and documents a negative relationship between price and EROI, which is close to an inverse one. As the authors do not regress price on the inverse of EROI, one cannot compare such "inverse fit" with their "loglog fit". To undertake this comparison, I run these two regressions using all estimates of EROI computed using THEMIS, one for each combination of scenario, year, region and sector. To obtain the price corresponding to each EROI, I use the vector \boldsymbol{v} of value-added per unit of each sector provided by THEMIS. Indeed, prices can be seen as emerging from value-added according to

$$p = v \cdot (I - A)^{-1}$$

because the price of s, p_s , is the sum of the value-added of inputs embodied in s: $v \cdot (I-A)^{-1} \cdot \mathbb{1}_s$. To the extent that the physical constituents and processes of a given technology will not change in an unexpected way, and as THEMIS models technical progress but not behaviors nor general equilibrium effects, the prices forecast using the above formula seem less reliable than the EROI estimates. For this reason, I report only the global average electricity prices of the main scenarios (see Table 2), but I do not detail the substantial variations between regions or sectors.

Table 3 reports the results of both the log-log and the in-

¹²Although they claim that their explained and explanatory variables are respectively the cost of production and their notion of EROI, EROI, the former is indeed the producer price and the latter our more standard notion of EROI; according to the source of their data: Cleveland [6].

Table 3: Regressions of price on EROI (both estimated using THEMIS).

Obs.	N	Specification	Coeffic	R ² a	
	11	Specification	a	b	A
All	2079	a = a + b	85	18	0.54
2010	104	$p = \frac{a}{EROI} + b$	71	20	0.53
All	2079	$\log(p) = a \cdot \log(\text{EROI}) + b$	-0.57	2.0	0.57
2010	104	$\log(p) = a \cdot \log(EROI) + b$	-0.44	1.9	0.57

 $^{^{}a}$ The R^{2} given for log-log fits is not the original one, cf. text.

verse fits. I ran each model twice: first, on all 2079 positive observations available, and then on the 104 observations of for year 2010. To make the R^2 of the log-log fit comparable to that of the inverse fit, I computed it as the sum of squared errors between "observed" prices and predicted prices (instead of their respective logarithms). As all R^2 are between 0.53 and 0.57, the inverse fit is almost as accurate than the log-log fit. Moreover, although the elasticity of price on EROI estimated here is different from that found by Heun & de Wit [19] for oil (around -0.5 as compared to -1.4), both figures are close to 1. Overall, empirical findings appear to confirm an inverse relation between price and EROI. That being said, Figure 5 shows that a high share of the variance in price remains unexplained by EROI, even more so for values of EROI around the global averages of 6-8, where the fit is almost flat and the errors substantial. In fact, there is theoretical evidence against a mapping between price and EROI.

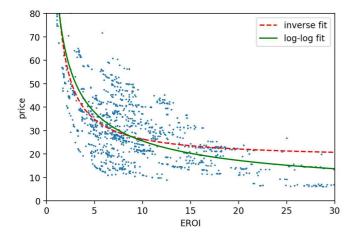


Figure 5: Regressions of price on EROI (all observations, from THEMIS).

4.3. The Case Against Any Simple Relation

Herendeen [17] shed new light on the topic by treating the question from its matrix form and introducing the concept of value-added. Herendeen showed how to express rigorously the price in function of the EROI when the economy is constituted of two sectors (energy and materials), and explained the limits of such exercise. Hereafter, I extend the results of Herendeen to an arbitrary number of sectors, n. First, I need to introduce one concept he uses: energy intensity.

To deliver one unit of energy technology t, the production mobilized is $(I-A)^{-1} \cdot \mathbb{1}_t$, while the energy mobilized is called the energy intensity of t and writes $\varepsilon_t = \mathbb{1}_E^T \cdot (I-A)^{-1} \cdot \mathbb{1}_t$, where E is the set of all energies. $\varepsilon_t = \varepsilon_t =$

In the following, I show that the price a technology t is a certain function of the coefficients of A, ¹⁴ and that each coefficient of A can be expressed as a function of EROI. Composing two such functions, we obtain that the price is inversely related to EROI. However, the relation is not unique (as it depends on the coefficient of A chosen to make the connection), and the other parameters in the relation are not constant. The demonstration starts with a lemma:

Lemma 1. Let A be an invertible matrix and let x be a coefficient of A. Then,

- (i) the determinant of A is a linear function of x, denoted D^A ;
- (ii) each coefficient (i,j) of the adjugate of A is a linear function of x, denoted $P_{i,j}^A$;
- (iii) each coefficient (i,j) of A^{-1} is a rational function in x of degree 1, which writes: $\left(A^{-1}\right)_{i,j} = \frac{P^A_{i,j}(x)}{D^A(x)}$.

Proof. Let $A = (a_{i,j})_{1 \le i,j \le n} \in GL_n(\mathbb{R})$ and let $(i_0,j_0) \in [1;n]^2$ so that, without loss of generality, $x = a_{i_0,j_0}$. (i) From its definition by the Leibniz formula, the determinant of A writes $\det(A) = \sum_{\sigma \in S_n} \operatorname{sgn}(\sigma) \prod_{i=1}^n a_{i,\sigma(i)}$. In this linear combination, each term is a product containing x at most once, it is thus a linear function of x. (ii) A minor being the determinant of a submatrix of A, we know from (i) that it is a linear function of x (which reduces to a constant for submatrices that do not contain x). Each coefficient of the adjugate of A is (plus or minus) a minor of A, hence a linear function of x. (iii) Using (i) and (ii) and the Laplace expansion of A: $A^{-1} = \frac{\operatorname{adj}(A)}{\det(A)}$, we reckon A^{-1} and A^{-1} is A^{-1} in the submatrice of A^{-1} in the laplace expansion of A: A^{-1} in the laplace expansion of A is the laplace expansion of A in the laplace expans

Proposition 1. (Generalization of 17) Assuming that all coefficients of the transformation matrix A are constant except one, noted $x = a_{i_0,j_0}$, and that EROI varies with x; the price of t can be expressed as a linear function of its energy intensity $\varepsilon_t = 1 + \frac{1}{EROI}$, so that:

$$\exists ! \left(\alpha, \beta\right) \in \mathbb{R}^2, \, p_t = \frac{\alpha}{EROI_t} + \beta$$

Remark. With the terminology of Heun & de Wit [19] or Herendeen [17], the relation above would write: $p_t = \alpha \frac{\text{EROI}_t^H}{\text{EROI}_t^H - 1} + \gamma$, with $\gamma = \beta - \alpha$. This is because in their definition of EROI, the numerator is ε_t instead of 1.

 $^{^{13}\}text{I}$ assume here that the unit of an output of an energy sector $e\in E$, hence of $\mathbb{1}^T_E$, is an energy unit, like TWh.

¹⁴More precisely, a function field of a certain algebraic variety.

Proof. From lemma 1, for all $(e,t) \in [1;n]^2$, there is a unique linear function $P_{e,t}^{I-A}$ such that $((I-A)^{-1})_{e,t} = \frac{P_{e,t}^{I-A}(\delta_{i_0,j_0}-x)}{D^{I-A}(\delta_{i_0,j_0}-x)}$, where $\delta_{i,j}$ is the Kronecker delta. As a linear combination of compositions of linear functions, the functions $Q(x) := \sum_{e \in E} P_{e,t}^{I-A}(\delta_{i_0,j_0}-x)$ and $P(x) := \sum_{i=1}^n v_i P_{i,t}^{I-A}(\delta_{i_0,j_0}-x)$ are themselves linear. By definition, we have $\varepsilon_t = \sum_{e \in E} \left((I-A)^{-1}\right)_{e,t}$, so that $Q(x) = \varepsilon_t D^{I-A}\left(\delta_{i_0,j_0}-x\right)$. As P,Q and $x \mapsto D^{I-A}\left(\delta_{i_0,j_0}-x\right)$ are linear, and as ε_t varies with x, it is easy to show that there are unique real numbers α and γ such that $P(x) = \alpha Q(x) + \gamma D^{I-A}\left(\delta_{i_0,j_0}-x\right)$. Finally, observing that $p_t = \sum_{i=1}^n v_i \left((I-A)^{-1}\right)_{i,t} = \frac{P(x)}{D^{I-A}\left(\delta_{i_0,j_0}-x\right)}$, we have:

$$p_t = \frac{\alpha Q(x) + \gamma D^{I-A} \left(\delta_{i_0, j_0} - x\right)}{D^{I-A} \left(\delta_{i_0, j_0} - x\right)} = \alpha \varepsilon_t + \gamma.$$

In the general case, we cannot obtain a *better* result, i.e. a formula that still holds when letting more than one coefficient vary. Indeed, denoting $\omega_{i,t}$ the coefficient (i,t) of $(I-A)^{-1}$, the Laplace expansion of I-A gives us

$$\omega_{i,t} = \frac{(-1)^{i+j}}{\det(I-A)} \det \left((I-A)_{j,k} \right)_{\substack{j \in [\![1;n]\!] \setminus i \\ k \in [\![1;n]\!] \setminus t}} \text{. Hence, we have}$$

$$\varepsilon_t = \sum_{e \in E} \left((I-A)^{-1} \right)_{e,t} = \sum_{e \in E} \omega_{e,t} \text{ and}$$

$$p_t = \sum_{i=1}^n v_i \left((I-A)^{-1} \right)_{i,t} = \sum_{i=1}^n v_i \omega_{i,t} = \sum_{e \in E} v_e \omega_{e,t} + \sum_{i \notin E} v_i \omega_{i,t}$$
Denoting $\tilde{v} = \frac{\sum_{e \in E} v_e \omega_{e,t}}{\sum_{e \in E} \omega_{e,t}}$ and $r = \tilde{v} + \sum_{i \notin E} v_i \omega_{i,t}$, we obtain

$$p_t = \tilde{v}\varepsilon_t + \sum_{i \notin E} v_i \omega_{i,t} = \frac{\tilde{v}}{\text{EROI}_t} + r$$

However, one has to keep in my mind that r, \tilde{v} and EROI_t all depend on the coefficients of A, and vary together when A changes. If there is only one type of energy $(E = \{e\})$ or if value-added is equal for all types of energy $(\forall e \in E, v_e = \tilde{v})$, \tilde{v} does not depend on the coefficients of A anymore, and we obtain a formula close to that of King & Hall [23]: $p_t = \frac{v_e}{\mathrm{EROI}_t} + r$. Still, when the EROI varies because more than one coefficient of A changes, r varies concomitantly, and the EROI cannot be used as a sufficient statistic to infer the price. For this reason, one cannot identify empirically a linear relation between price and the inverse of EROI without strong assumption on the steadiness of A.

Actually, the theoretical relation between EROI and price is so fragile that one cannot even conclude that it is a decreasing relation: I provide in Appendix B a numerical example showing that EROI and price can both increase at the same time when more than one coefficient varies. Such acknowledgment dissuades from predicting long run prices by simply looking at estimations of future EROIs.

Does this mean that EROI is unrelated to any economic concept? Fizaine & Court [11] argue that there is a minimum EROI below which the US economy enters a recession. They first show that energy expenditure Granger causes growth in the US, then determine a threshold of energy expenditure above which the US enters in a recession, and finally use a modified version of equation (1) to relate this to a minimum non-recessionary EROI. However, they misleadingly replace the inverse of the energy intensity of investment $\frac{\$_{investment}}{E_{in}}$ by that

of the whole economy, $\frac{GDP}{E_{\rm out}}$. This prevents them from noticing that cost reductions in the energy production could compensate the effect of a decreasing EROI on prices. As we have seen, EROI, price and energy expenditure can all decrease at the same time, which undermines the idea that a recession caused by a surge in energy expenditure is ineluctable as soon as EROI goes below some threshold. Overall, the analysis of this section tempers the idea that EROI is relevant in economic issues and suggests that this notion is best suited for the physical study of energy systems.

5. Concluding Remarks

This work is a first attempt at estimating future EROIs in a decarbonized electricity system. By examining a broad range of scenarios, it concludes that the system-wide EROI of the power sector should not vary much until 2050, albeit a noticeable decrease, from 8 to 7.2 in a business-as-usual scenario, or 6 in a scenario with 100% renewable electricity. As the EROI of each technologies is expected to remain well above 1, our results restore confidence about the energetic sustainability of renewable electricity, which was questioned theoretically.

This paper assessed scenarios of transition in the electricity sector, but further research is still needed to estimate future EROIs in complete energy transitions, which include a mutation of the transportation system, agriculture and industry.

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Appendix A. Updating a Matrix A To a New Given Mix

The technology matrices A for the IEA scenarios are readily available in THEMIS, but these matrices have to be updated to the new electricity mix for the Greenpeace scenarios. To do this, I exploit the fact that both THEMIS and Greenpeace use the world regions of the IEA, and I modify the electricity input of each sector by the regional mix given by Greenpeace. The most accurate algorithm to update an input-output matrix is known as GRAS [21]. Although I implemented this algorithm in python; I could not use it, because this algorithm uses the new sums of rows and columns to balance the matrix, and the vector of final demand y or the vector of production x is necessary to know them. As THEMIS doesn't include such vectors, I had to use a simpler method, which relies on the assumption that the electricity mix of inputs is the same across sectors for a given region. Given the perfect substitutability between electricity produced by different technologies and the uniqueness of electric grids, this assumption seems justified.

There are two different updates to make. First, I modify the vector of second energy demand (used to infer the final demand of technology t, y_t) so that it perfectly matches the demand of the scenario. Second, I modify the submatrix D of A containing the rows of electricity sectors. To convert D in energy units, I multiply each row t of D by the corresponding energy supplied per unit of technology t, E_t^S . I call the result E: the coefficient E_{is} of E gives the electricity from sector i required to make one unit of sector s' output, where i = i(t, r)corresponds to technology t in region r. Then, I premultiply E by a block diagonal matrix with R blocks of size T * T containing only ones (where R = 9 and T = 15 are the number of THEMIS regions and electricity sectors, respectively) to obtain a matrix B. Each row of B gives the total electricity from a given region r required to produced each output, E_r^{tot} , and each row E_r^{tot} is replicated T times:

$$B = \begin{pmatrix} B_1 \\ \vdots \\ B_R \end{pmatrix}, \quad B_r = \begin{pmatrix} E_r^{\text{tot}} \\ \vdots \\ E_r^{\text{tot}} \end{pmatrix}$$

Next, each row of B is multiplied by the share of a technology t in the mix of the corresponding region, which defines a matrix \tilde{E} . Each coefficient \tilde{E}_{is} of \tilde{E} gives the electricity from sector i required to make one unit of sector s output, according to the new mix (by construction, \tilde{E}_{is} is the same for all sectors s of a given region r). Eventually, I obtain the new submatrix \tilde{D} by converting each row of \tilde{E} to the original units of S (by dividing each row by the appropriate unitary energy supplied S).

A last update is needed for Greenpeace scenarios, to account for the extra capacity needed when intermittent sources fail to deliver energy: the ratio of capacity (in GW) over production (in TWh) is a bit higher in Greenpeace scenarios than in IEA/THEMIS ones. Thus, I multiply each column of an energy sector (representing all inputs required for one unit of output of this sector) by the ratio of the capacity-over-production ratios of Greenpeace and IEA/THEMIS. Doing so require relies on the fact that the energy required to operate a power plant is negligible in front of the energy required to build it (see e.g. Arvesen et al. [2]).

Appendix B. Example of Non-Decreasing Relation Between EROI and Price

Herendeen [17] proposes a calibration on US energy data of his toy model with 2 sectors (materials and energy), which yields as realistic results as a two-by-two model can yield. I start from a slightly modified version of his calibration (called *base*), where figures are rounded, and I show how a deviation of two coefficients (in the *new* calibration) leads to an increase of both EROI and price of energy. This proves that in general, nothing can be said of the relation between EROI and price, not even that it is a decreasing relation.

For this, I use the formulas for EROI and price given by Herendeen [17] (where I convert the price to f(g) using the conversion factor 1 Btu = 114,000 gal):

$$EROI = \frac{1}{A_{ee} + \frac{A_{em}A_{me}}{1 - A_{mm}}}$$

$$p = \frac{v_e (1 - A_{mm}) + v_m A_{me}}{(1 - A_{ee}) (1 - A_{mm}) - A_{em}A_{me}} \cdot 114,000$$

Table B.4: Example of sets of coefficients exhibiting a non-decreasing relation between EROI and price in a two sectors model.

	base	new						
v_m	0.5							
v_e	$5 \cdot 1$	0^{-6}						
A_{em}	170	00						
A_{me}	$4 \cdot 10^{-6}$							
A_{mm}	0.5	0.6						
A_{ee}	0.3	0.25						
EROI	3.2	3.7						
price	1.5	1.6						

Appendix C. Complete Results

Results without quality adjustment for IEA/THEMIS scenarios are provided in section 3.2; those for Greenpeace's scenarios are in Table C.5. Quality-adjusted results follows in Table C.6 (IEA/THEMIS) and C.7 (Greenpeace). The quality adjustment consists in separating each energy in the formula of the EROI according to its origin (electric or thermal), and to weight electricity by a factor 2.6. For example, the quality-adjusted (gross) embodied energy for a unit of technology t writes

$$\mathbf{embodied}_t^{\mathrm{qual.\ adj.}} = E^S \odot (2.6 \cdot \mathbb{1}_{\mathrm{electric}} + \mathbb{1}_{\mathrm{thermal}}) \cdot (I - A)^{-1} \cdot \mathbb{1}_{\mathrm{t}}$$

Table C.5: EROIs and share in electricity mix of electric technologies in the model THEMIS for the Greenpeace scenarios.

Scenario	al	1	RE				ER (+2	°C, no (CCS, no nuclear)		ADV (100%		renewable)	
Year	20	12	20	2030 2050		50	2030		2050		2030		2050	
Variable	EROI	mix	EROI	mix	EROI	mix	EROI	mix	EROI	mix	EROI	mix	EROI	mix
biomass w CCS	_	0	_	0	_	0	_	0	_	0	_	0	_	0
biomass&Waste	8.4	0.02	6.4	0.03	5	0.03	5.3	0.06	4.7	0.06	5.2	0.05	4.6	0.05
ocean	4.6	0	2	0	2.5	0	4.2	0.01	4.5	0.03	4.8	0.01	4.9	0.03
geothermal	5.6	0	3.8	0.01	2.5	0.01	3.6	0.03	3.7	0.07	3.8	0.03	3.9	0.07
solar CSP	35.2	0	9.2	0	7.9	0.01	8.5	0.05	7.7	0.17	9.2	0.07	7.8	0.22
solar PV	13.6	0	6.9	0.02	5.2	0.02	5.5	0.11	4.4	0.2	5.4	0.14	4.7	0.21
wind offshore	9	0	8.6	0.01	7.7	0.01	5.6	0.03	5.9	0.08	6.5	0.04	6.4	0.1
wind onshore	9.6	0.02	9.1	0.05	7.1	0.05	7.2	0.15	6	0.22	7.1	0.17	5.8	0.24
hydro	12.1	0.16	11.3	0.14	11.1	0.13	11	0.14	11	0.1	11	0.13	10.9	80.0
nuclear	12	0.11	7.2	0.1	7	0.08	8.3	0.02	-	0	8.3	0.02	_	0
gas w CCS	_	0	_	0	_	0	-	0	-	0	_	0	_	0
coal w CCS	_	0	_	0	_	0	_	0	-	0	_	0	_	0
oil	8.1	0.05	10.8	0.02	11.1	0.01	9.9	0.01	9.2	0	9.9	0.01	-	0
gas	14.6	0.23	15	0.23	15.4	0.25	16.4	0.21	17.2	0.06	16.4	0.18	_	0
coal	11.6	0.4	11.1	0.4	11.1	0.39	10.6	0.19	10.8	0.01	10.3	0.16	11.5	0
Total (PWh/a)	7.8	22.6	7.4	36.26	7.1	50.11	6.9	33.6	5.7	49.2	6.9	36.74	5.8	64.04

Table C.6: Quality-adjusted EROIs (with a factor of 2.6 for electricity) and share in electricity mix of electric technologies for IEA/THEMIS scenarios.

Scenario	Baseline (BL)							Blue Map (BM, +2°)						
Year	2010		20	30	20	50	20	30	2050					
Variable	EROI	mix	EROI	mix	EROI	mix	EROI	mix	EROI	mix				
biomass w CCS	-	0.00	-	0.00	-	0.00	9.5	0.00	8.5	0.01				
biomass&Waste	20.6	0.01	12.5	0.02	11.7	0.03	11.5	0.06	11.0	0.05				
ocean	9.1	0.00	4.4	0.00	5.5	0.00	7.0	0.00	11.3	0.00				
geothermal	11.4	0.00	10.2	0.01	10.1	0.01	10.5	0.01	11.3	0.02				
solar CSP	44.5	0.00	17.6	0.00	18.1	0.01	17.2	0.02	16.9	0.06				
solar PV	17.4	0.00	14.7	0.01	14.4	0.01	13.2	0.02	12.8	0.06				
wind offshore	19.5	0.00	21.1	0.01	20.3	0.01	15.3	0.03	13.0	0.04				
wind onshore	18.3	0.01	18	0.04	15.6	0.04	14.5	80.0	15.3	80.0				
hydro	25.4	0.16	22.9	0.14	22.8	0.12	25.1	0.18	26.3	0.14				
nuclear	19.1	0.14	13.3	0.11	12.9	0.10	13.5	0.19	13.9	0.24				
gas w CCS	_	0.00	_	0.00	14.3	0.00	16.1	0.01	18.8	0.05				
coal w CCS	_	0.00	_	0.00	11.6	0.00	13.7	0.05	14.2	0.12				
oil	12.7	0.06	15.4	0.02	15.8	0.01	15.4	0.03	11.8	0.01				
gas	22.3	0.21	24.4	0.21	24.1	0.23	28.9	0.14	33.4	0.11				
coal	20	0.42	18	0.45	18	0.45	18.3	0.18	19.6	0.01				
Total (PWh/a)	15.2	19.76	14.2	34.29	14	45.97	14.8	28.01	15.3	40.22				

 $Table \ C.7: \ Quality-adjusted \ EROIs \ (with a factor of 2.6 \ for \ electricity) \ and \ share \ in \ electricity \ mix \ of \ electric \ technologies \ for \ Greenpeace \ scenarios.$

Scenario	al	1		RI	EF		ER (+2°C, no CCS, no nuclear)				ADV (100% renewable)			
Year	20	12	2030		2050		2030		2050		2030		2050	
Variable	EROI	mix	EROI	mix	EROI	mix	EROI	mix	EROI	mix	EROI	mix	EROI	mix
biomass w CCS	_	0	-	0	_	0	_	0	_	0	_	0	_	0
biomass&Waste	15.6	0.02	12.5	0.03	9.8	0.03	10.4	0.06	9.2	0.06	10.2	0.05	9.1	0.05
ocean	7.7	0	3.6	0	4.5	0	8.4	0.01	9.1	0.03	9.3	0.01	9.7	0.03
geothermal	12	0	7.5	0.01	5	0.01	7.2	0.03	7.3	0.07	7.5	0.03	7.7	0.07
solar CSP	72.7	0	18.9	0	16	0.01	17.6	0.05	16.1	0.17	18.9	0.07	16.2	0.22
solar PV	25.6	0	13.6	0.02	10.5	0.02	11.5	0.11	9.2	0.2	11.2	0.14	9.9	0.21
wind offshore	17.2	0	15.9	0.01	15	0.01	11.5	0.03	12.2	80.0	13.2	0.04	13.2	0.1
wind onshore	18.5	0.02	17.5	0.05	14	0.05	14.8	0.15	12.4	0.22	14.7	0.17	12.1	0.24
hydro	23.3	0.16	21.9	0.14	21.7	0.13	21.3	0.14	21.3	0.1	21.3	0.13	21.1	80.0
nuclear	22.6	0.11	13.5	0.1	13.2	0.08	16.3	0.02	_	0	16.4	0.02	_	0
gas w CCS	_	0	_	0	_	0	-	0	-	0	-	0	_	0
coal w CCS	_	0	_	0	_	0	_	0	_	0	_	0	_	0
oil	12.5	0.05	17.5	0.02	17.9	0.01	15.7	0.01	13.3	0	15.7	0.01	_	0
gas	23.4	0.23	24.8	0.23	25.5	0.25	27	0.21	28.2	0.06	27	0.18	_	0
coal	18.3	0.4	17.6	0.4	17.8	0.39	16.5	0.19	16.5	0.01	16.1	0.16	15.3	0
Total (PWh/a)	14.9	22.6	14.3	36.26	13.7	50.11	13.8	33.6	11.8	49.2	13.7	36.74	11.9	64.04

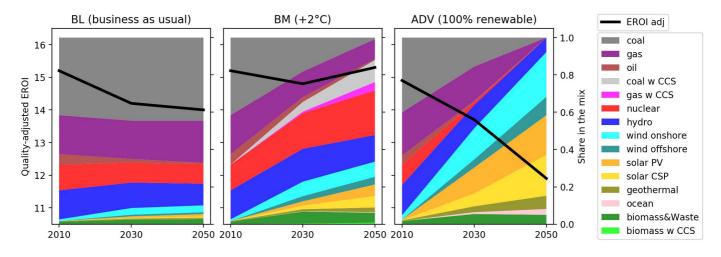


Figure C.6: Evolution of global quality-adjusted EROIs (with a factor 2.6 for electricity) and mixes of electricity for different scenarios.