



# DCM3: Quantum Computing Challenge

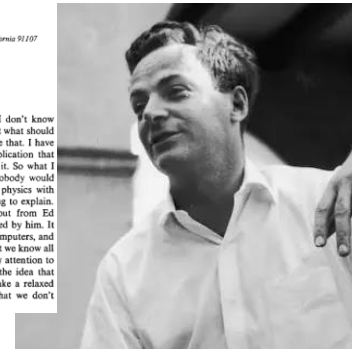
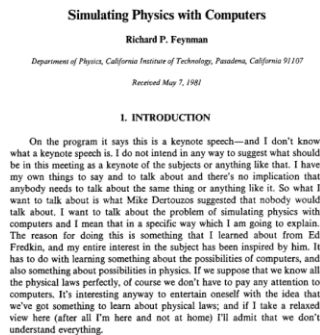
## Training

Infleqtion Quantum Software Org

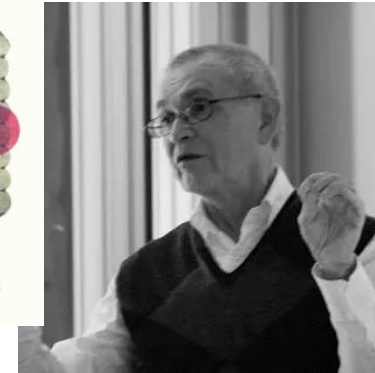
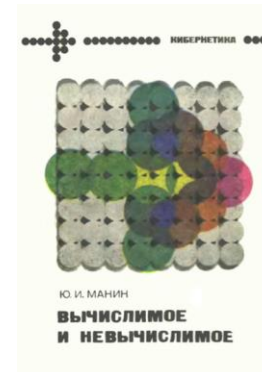


# What is a Quantum Computer?

- A **quantum computer** is a device that processes information by leveraging quantum properties, namely *superposition* and *entanglement*.



*Richard Feynman*



*Yuri Manin*

- Fundamental operations are performed by **quantum logic gates** acting on **qubits**, which store information in quantum states.



# What is a Quantum Computer?

- A **qubit** (quantum bit) is the quantum analogue to a classical bit; while a bit can be in two states (0 or 1), a qubit can be in a *superposition* of the *basis states*  $|0\rangle$  and  $|1\rangle$ .

- We describe a qubit's **superposition** state by

$$|q\rangle = c_0|0\rangle + c_1|1\rangle$$

where the *amplitudes*  $c_0$  and  $c_1$  are complex numbers such that

$$|c_0|^2 + |c_1|^2 = 1$$

- Note: the normalisation condition  $|c_0|^2 + |c_1|^2 = 1$  must hold for  $|q\rangle$  to be a valid quantum state (i.e., satisfy the Schrödinger equation).



# Complex Numbers: A brief aside...

We are familiar with **real numbers** like

$$0, 1, e, \pi, 1.23, 3/4, -10...$$

Complex numbers can be seen as an extension of real numbers, by introducing the *imaginary* unit  $i$ , and are essential to the field of quantum mechanics. The imaginary unit's defining property is that

$$i^2 = -1$$

A **complex number** has a **real** component and an **imaginary** component, which we define as

$$c = a + i \times b$$



# Complex Numbers: A brief aside...

One quantity associated with a complex number  $c$  is its *magnitude*  $|c|$ , defined as

$$|c| = \sqrt{a^2 + b^2}$$

The magnitude is the “**length**” of the complex number (i.e., the length of the associated vector in complex vector space).

E.g., consider the following complex numbers and their respective magnitudes:

$c$	$ c $
-3	3
$4i$	4
$4 - 3i$	5
$2 + 2i$	$2\sqrt{2}$



# Quantum Computing: Qubits

- A **qubit** in a **superposition** of the states  $|0\rangle$  and  $|1\rangle$ :

$$|q\rangle = c_0|0\rangle + c_1|1\rangle \quad |c_0|^2 + |c_1|^2 = 1$$

- We cannot observe this superposition directly, and only have access to **measurement** outcomes that will return  $|0\rangle$  or  $|1\rangle$ , each with some probability.
- However, the measurement outcome is **correlated** to the qubit's quantum state: probabilities of measuring ( $|0\rangle, |1\rangle$ ) are ( $|c_0|^2, |c_1|^2$ ) (i.e., the amplitude's absolute square  $|c|^2 = c^*c = (a - bi)(a + bi)$  from the **Born rule**).

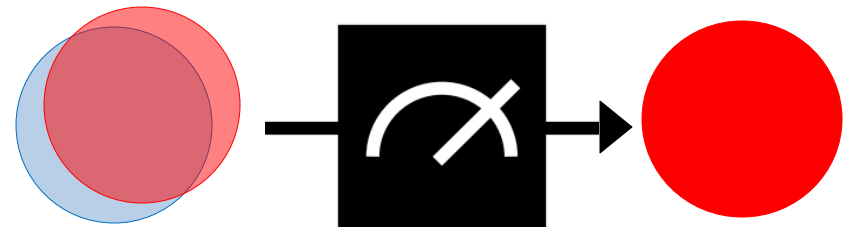
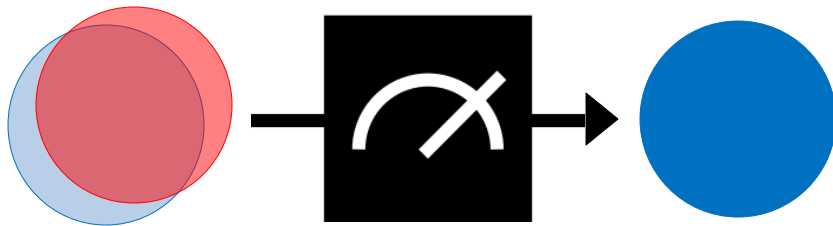


# Quantum Computing: Qubits

- A **qubit** in a **superposition** of the states  $|0\rangle$  and  $|1\rangle$ :

$$|q\rangle = c_0|0\rangle + c_1|1\rangle \quad |c_0|^2 + |c_1|^2 = 1$$

- Following measurement, a qubit's superposition state **collapses** to the measured state





# Quantum Computing: Qubits

- A **qubit** in a **superposition** of the states  $|0\rangle$  and  $|1\rangle$ :

$$|q\rangle = c_0|0\rangle + c_1|1\rangle \quad |c_0|^2 + |c_1|^2 = 1$$

- As previously described, we cannot observe  $|q\rangle$  directly – we can only perform a measurement to obtain 0 and 1, each with some probability.
- E.g., consider the state

$$|q\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle$$

- Measuring  $|q\rangle$  will return 0 with probability  $|c_0|^2 = \left|\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2}$  and 1 with probability  $|c_1|^2 = \left|\frac{i}{\sqrt{2}}\right|^2 = \frac{1}{2}$ .



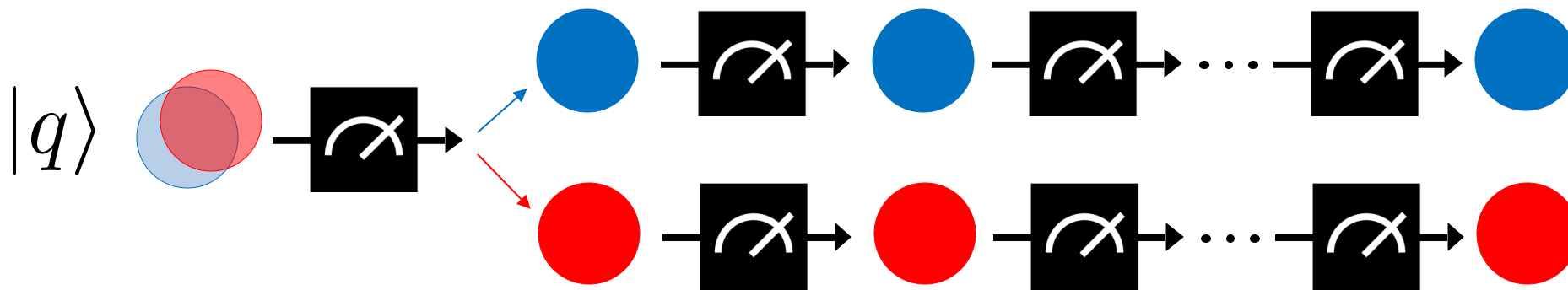


# Quantum Computing: Qubits

- A **qubit** in a **superposition** of the states  $|0\rangle$  and  $|1\rangle$  :

$$|q\rangle = c_0|0\rangle + c_1|1\rangle \quad |c_0|^2 + |c_1|^2 = 1$$

- Immediately after the initial measurement, that state collapses and all subsequent measurements will obtain the same result. If 0 is measured we will have  $|q\rangle = |0\rangle$  but if 1 is measured we will have  $|q\rangle = |1\rangle$  .



Note: we assume the **Copenhagen** interpretation of quantum mechanics (versus Everett, Pilot Wave Theory, etc.) -- before measurement, the qubit is truly in a superposition.



# Quantum Computing: Measurement

- Recall that we can't directly observe a quantum state, and that we can only measure (i.e., read out) a single bit, representing one of the basis states ( $|0\rangle, |1\rangle$ ).
- The **Born rule** states that the probability of obtaining a particular bit outcome equals the *absolute square* of its amplitude in the superposition state being measured.
- E.g., measuring  $|+\rangle$  yields  $|0\rangle$  or  $|1\rangle$  with probability  $\frac{1}{2}$  each.

$$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \rightarrow \begin{aligned} \text{Pr}(|0\rangle) &= \left| \left( \frac{1}{\sqrt{2}} \right) \right|^2 = \frac{1}{2} \\ \text{Pr}(|1\rangle) &= \left| \left( \frac{1}{\sqrt{2}} \right) \right|^2 = \frac{1}{2} \end{aligned}$$



Max Born

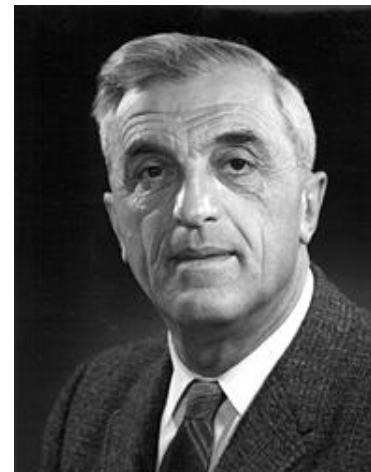
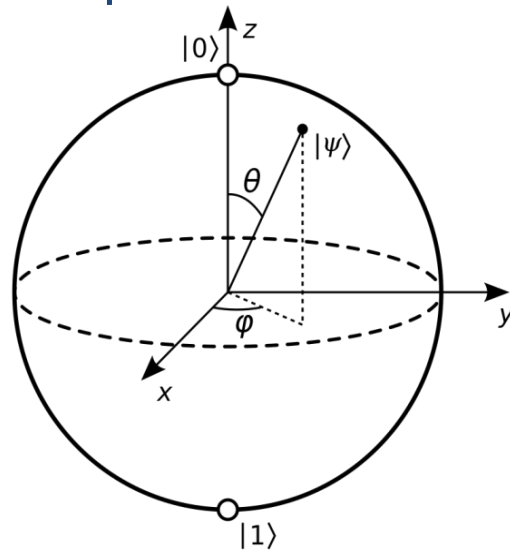


# Quantum Computing: Bloch Sphere

- A **qubit** in a **superposition** of the states  $|0\rangle$  and  $|1\rangle$  :

$$|q\rangle = c_0|0\rangle + c_1|1\rangle \quad |c_0|^2 + |c_1|^2 = 1$$

- The **Bloch** sphere is a geometric representation of a quantum superposition state.



*Felix Bloch*



# Quantum Computing: Bloch Sphere

Deriving the **Bloch sphere**:  $|c_0|^2 + |c_1|^2 = 1$  can be re-expressed using *polar coordinates*

$$c_1 = \sin(\theta/2)e^{i\phi} \quad c_0 = \cos(\theta/2)$$

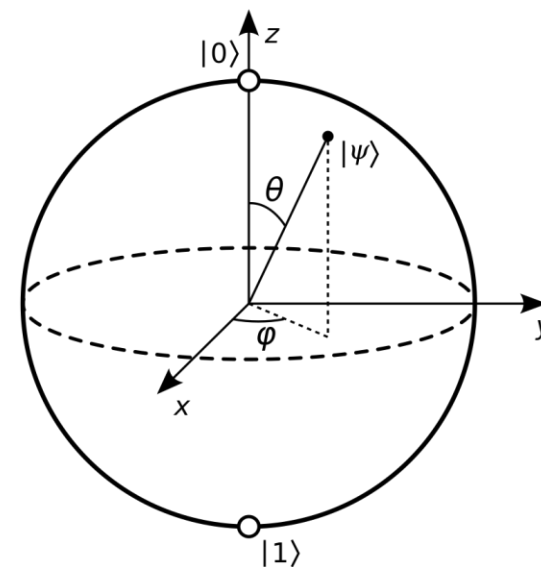
where

$$0 \leq \theta \leq \pi \quad 0 \leq \phi \leq 2\pi$$

and so we rewrite  $|q\rangle \rightarrow |\psi\rangle$  as

$$|\psi\rangle = \cos(\theta/2)|0\rangle + \sin(\theta/2)e^{i\phi}|1\rangle$$

The state  $|\psi\rangle$  then corresponds to a point on the surface of a sphere where the north pole is  $|0\rangle$  and the south pole is  $|1\rangle$  with  $(\theta, \phi)$  as coordinates (colatitude and longitude).





# Extending to Multiple Qubits

- Quantum computers store information on **multiple qubits** (just as classical computers use multiple bits).
- The 2-qubit system  $|q_0\rangle|q_1\rangle = |q_0q_1\rangle$  can be in a superposition of  $2^2 = 4$  *basis states*:  $|00\rangle, |01\rangle, |10\rangle, |11\rangle$

$$|q_0q_1\rangle = c_{00}|00\rangle + c_{01}|01\rangle + c_{10}|10\rangle + c_{11}|11\rangle$$

$$|c_{00}|^2 + |c_{01}|^2 + |c_{10}|^2 + |c_{11}|^2 = 1$$



# Extending to Multiple Qubits

- In general, a system of  $n$  qubits can be in a superposition of  $2^n$  states:

1 qubit

$$|q\rangle = c_0|0\rangle + c_1|1\rangle$$

$$|c_0|^2 + |c_1|^2 = 1$$



2 qubits

$$|q_0q_1\rangle = c_{00}|00\rangle + c_{01}|01\rangle + c_{10}|10\rangle + c_{11}|11\rangle$$

$$|c_{00}|^2 + |c_{01}|^2 + |c_{10}|^2 + |c_{11}|^2 = 1$$



⋮



$n$  qubits

$$|\psi\rangle = |q_0q_1 \cdots q_n\rangle = \sum_{j=0}^{2^n-1} c_{\text{bin}(j)} |\text{bin}(j)\rangle$$

$$\sum_{j=0}^{2^n-1} |c_{\text{bin}(j)}|^2 = 1$$



# Extending to Multiple Qubits

- E.g., a two-qubit state:

$$|q_0q_1\rangle = \frac{1}{3}|00\rangle + \frac{\sqrt{3}i}{3}|01\rangle + \frac{1+i}{3}|10\rangle - \frac{\sqrt{3}}{3}|11\rangle$$

A three-qubit state:

$$|q_0q_1q_2\rangle = -\frac{2}{5}|001\rangle + \frac{3}{5}|010\rangle - \frac{\sqrt{2}i}{5}|011\rangle + \frac{\sqrt{5}}{5}|101\rangle + \frac{\sqrt{3}}{5}|110\rangle + \frac{1-i}{5}|111\rangle$$

where  $c_{000} = c_{100} = 0$ .



# Extending to Multiple Qubits

- Multi-qubit systems can also exhibit **entanglement**. An entangled system exists in a superposition state such that individual qubits cannot be described independently (even if separated in space).
- E.g., consider the separable 2-qubit state  $|q_0q_1\rangle = \frac{1}{2}|00\rangle - \frac{1}{2}|01\rangle + \frac{1}{2}|10\rangle - \frac{1}{2}|11\rangle$  which can be factored:

$$|q_0q_1\rangle = \left(\frac{1}{2}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right) \left(\frac{1}{2}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right) = |q_0\rangle|q_1\rangle$$

- Now consider the entangled 2-qubit state.  $|q_0q_1\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$

This state cannot be factored, and neither qubit's state can be individually described (i.e.,  $|q_i\rangle$  can't be written as a superposition  $c_0|0\rangle + c_1|1\rangle$ ).





# Measuring a Multi-Qubit System

- Similarly, recalling the *absolute square*

$$|c|^2 = c^*c = (a - bi)(a + bi)$$

measuring the state

$$|q_0q_1\rangle = \frac{1}{2}|01\rangle - \frac{i}{2}|10\rangle + \frac{1+i}{2}|11\rangle$$

- Yields the following probability table:

$ \mathbf{ab}\rangle$	$ c_{ab} ^2$	$\text{Pr}[ \mathbf{q_0q_1}\rangle =  \mathbf{ab}\rangle]$
$ 00\rangle$	0	0
$ 01\rangle$	$\left \left(\frac{1}{2}\right)\right ^2$	$\frac{1}{4}$
$ 10\rangle$	$\left \left(\frac{-i}{2}\right)\right ^2$	$\frac{1}{4}$
$ 11\rangle$	$\left \left(\frac{1+i}{2}\right)\right ^2$	$\frac{1}{2}$



# Measuring a Multi-Qubit System

- Also recall that measuring a qubit collapses its quantum state to a classical bitstring, destroying the superposition in the process.
- E.g., suppose measuring the previous 2-qubit system

$$|q_0q_1\rangle = \frac{1}{2}|01\rangle - \frac{i}{2}|10\rangle + \frac{1+i}{2}|11\rangle$$

yields  $|10\rangle$ . The 2-qubit system will remain in this state

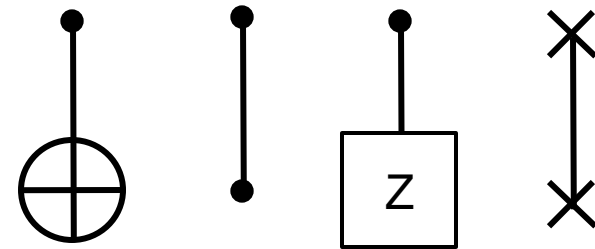
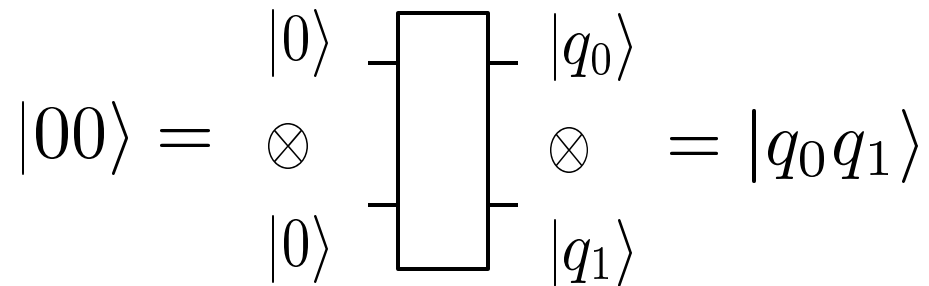
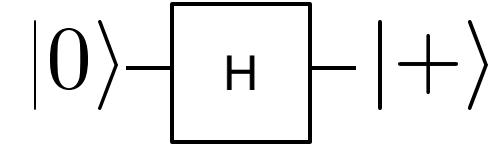
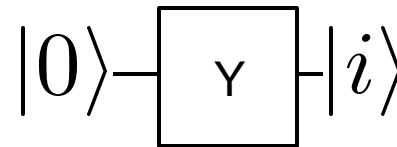
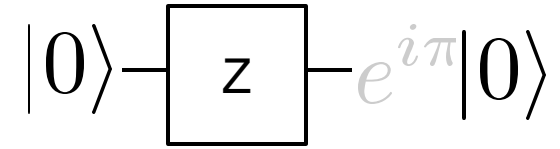
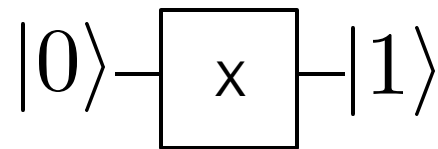
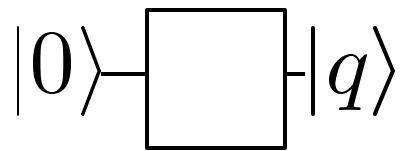
$$|q_0q_1\rangle = |10\rangle$$

such that all subsequent measurement return  $|10\rangle$  with a probability of 1.



# Quantum Logic Gates

- Qubits are typically initialised in the *ground state*  $|0\rangle$ ; we generate superposition states and entanglement using **quantum logic gates**.





# Quantum Logic Gates

- It is common and convenient to adopt a *matrix representation* for quantum operations. We consider the basis states ( $|0\rangle, |1\rangle$ ) as *vectors*

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

- Thus, quantum computation can be expressed in terms of matrix-vector operations. E.g., consider the matrix representation of the commonly used X gate and its effect on a qubit in the *ground state*  $|0\rangle$ :

$$\mathbf{X} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \mathbf{X}|0\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1\rangle$$

- I.e., the X gate is the quantum analogue to a classical NOT that takes the bit  $0 \rightarrow 1$ .



# Quantum Logic Gates

- Another common gate is the Hadamard gate, represented as

$$H = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} = \left( \frac{1}{\sqrt{2}} \right) \cdot \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

- The effect of the Hadamard on a qubit in the ground state  $|0\rangle$  is

$$\mathbf{H}|0\rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)$$

the output of which is commonly labeled as  $|+\rangle = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)$

- Similarly, its effect on a qubit in the *excited state*  $|1\rangle$  is

$$\mathbf{H}|1\rangle = \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle) = |-\rangle$$



# Quantum Logic Gates

- We can also perform gates acting on more than one qubit, e.g., the **controlled-X gate** (CX) receiving as input a *control qubit* and a *target qubit*.

$$CX|00\rangle = |00\rangle \quad CX|10\rangle = |11\rangle$$

$$CX|01\rangle = |01\rangle \quad CX|11\rangle = |10\rangle$$

- CX performs an X on the target when the control is in state  $|1\rangle$
- With matrix representation

$$\mathbf{CX} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad |q_0q_1\rangle = |q_0\rangle \otimes |q_1\rangle = \begin{bmatrix} a_0a_1 \\ a_0b_1 \\ b_0a_1 \\ b_0b_1 \end{bmatrix} \quad |00\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$



# Quantum Logic Gates

Matrix representations of common quantum gates:

$$\mathbf{X} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\mathbf{Y} = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$\mathbf{Z} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\mathbf{H} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\mathbf{S} = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$$

$$\mathbf{T} = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$$

$$\mathbf{CX} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\mathbf{SWAP} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



# Quantum Logic Gates

- Quantum gates are *linear* operations, and so they preserve superpositions: the output of applying a quantum gate to a qubit in superposition will also be a superposition. E.g., for  $|q\rangle = a|0\rangle + b|1\rangle$  we have

$$\mathbf{H}|q\rangle = \mathbf{H}(a|0\rangle + b|1\rangle) = a\mathbf{H}|0\rangle + b\mathbf{H}|1\rangle = \frac{a+b}{\sqrt{2}}|0\rangle + \frac{a-b}{\sqrt{2}}|1\rangle$$

I.e., the output is also in a superposition of  $|0\rangle$  and  $|1\rangle$

- We can also apply multiple gate sequentially:

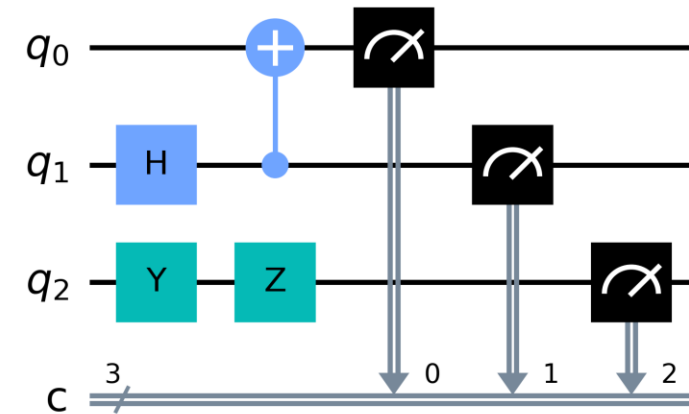
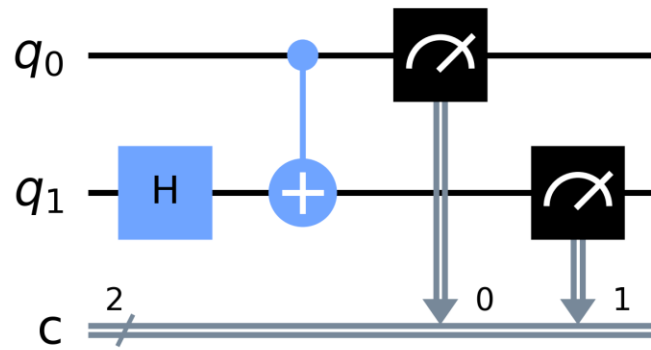
$$\mathbf{CX}(\mathbf{H} \otimes \mathbf{I})|00\rangle = \mathbf{CX}\left(\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|10\rangle\right) = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$





# Quantum Circuits

- **Quantum circuits**, sequences of quantum logic gates acting on qubits, are the quantum analogue to classical (Boolean) logic circuits

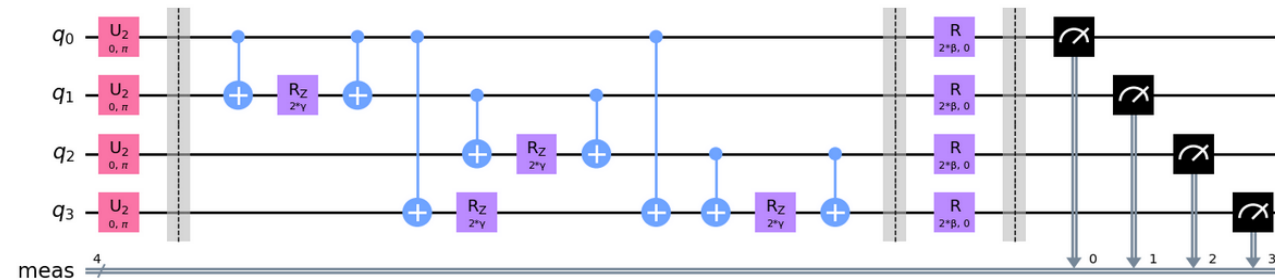


- We represent qubits with wires and gates with blocks placed over the wires corresponding to the qubits they operate on.
- Measurements are represented with meter symbols, and a double wire represents classical bit values.



# Quantum Algorithms

- Quantum computers allow building quantum circuits that create and manipulate quantum phenomena.



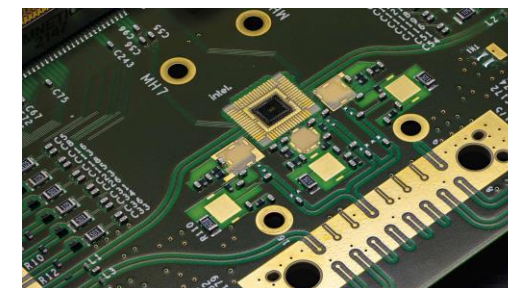
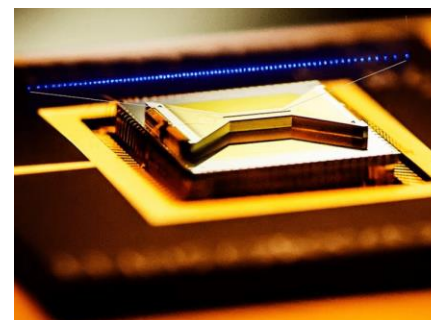
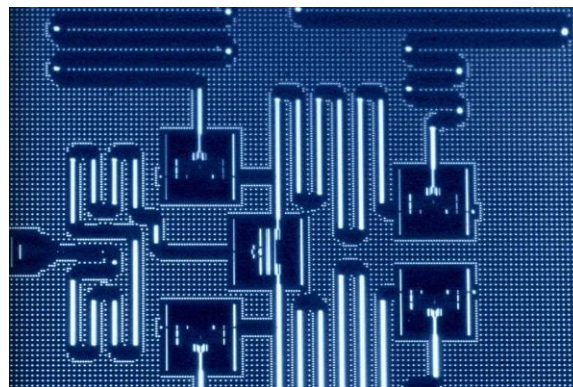
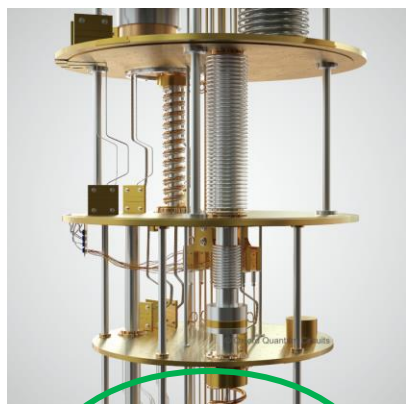
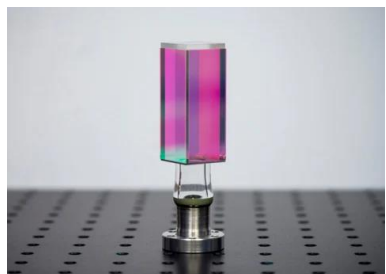
- Quantum algorithms** use quantum circuits to solve a problem more efficiently than classical systems allow.
- E.g., variational quantum algorithms (e.g., QAOA, VQE); Quantum machine learning (QML) algorithms, Shor's algorithm (prime factorisation), Grover's algorithm (database search).



# Quantum Computing Hardware

Quantum computing implementations include various technologies/hardware:

- Cold (neutral) atoms
- Trapped ions
- Superconducting transmons
- Semiconductor spin-qubits (quantum dots)



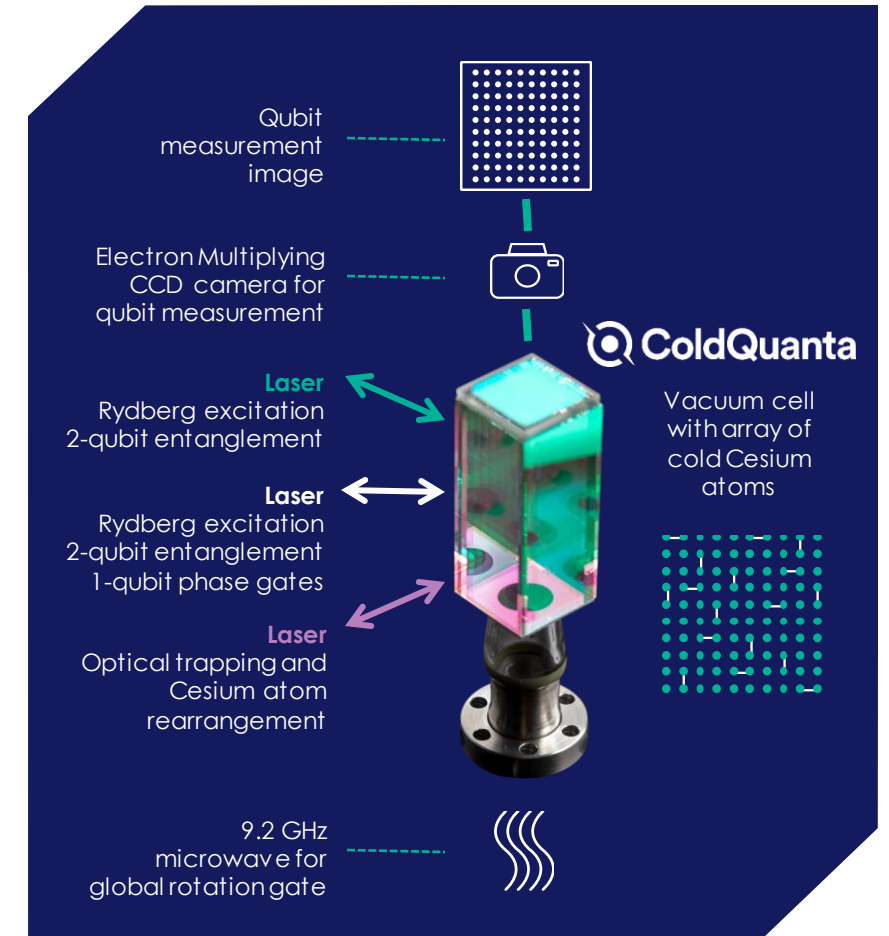
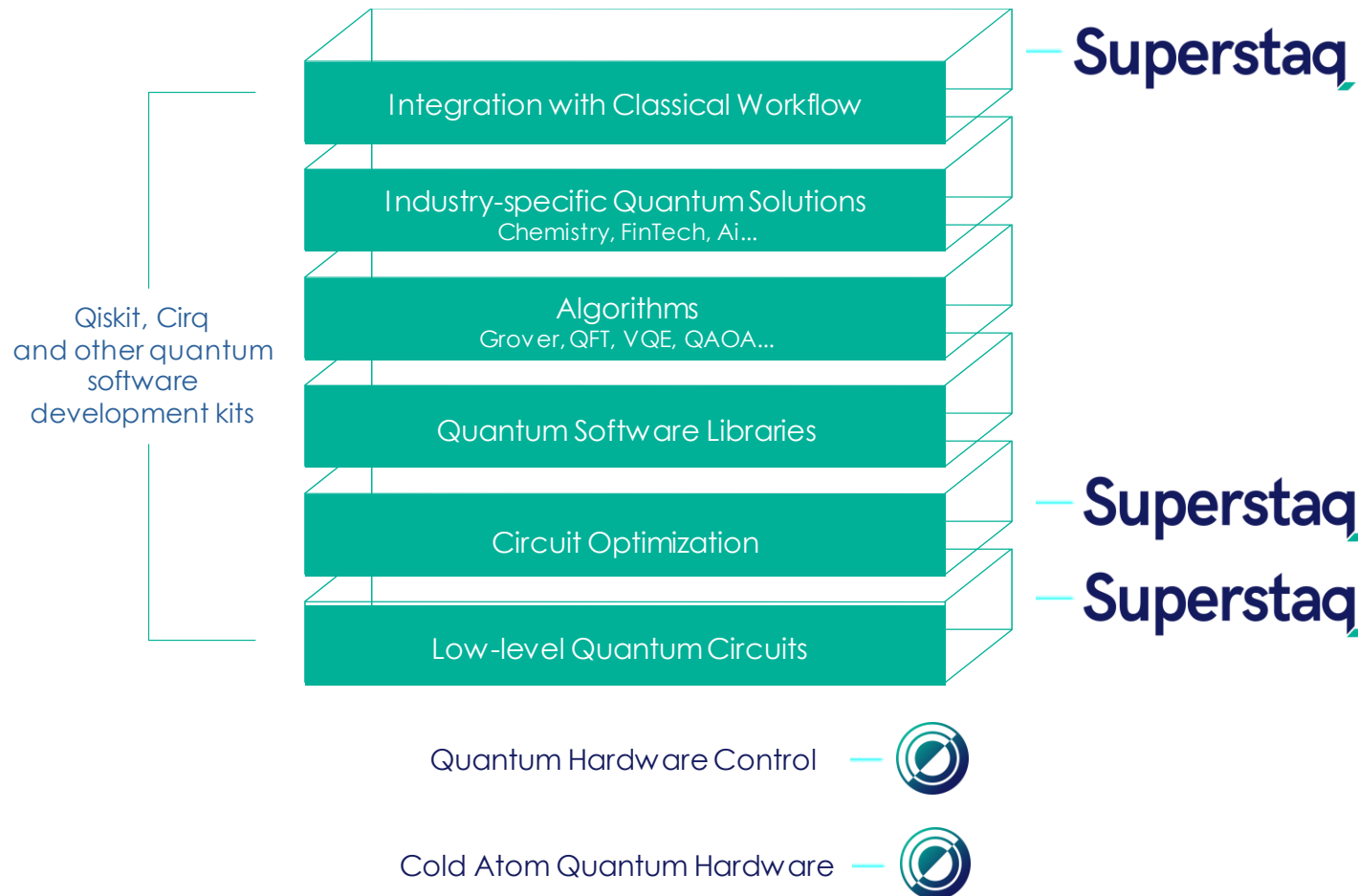
OQC

IBM Quantum





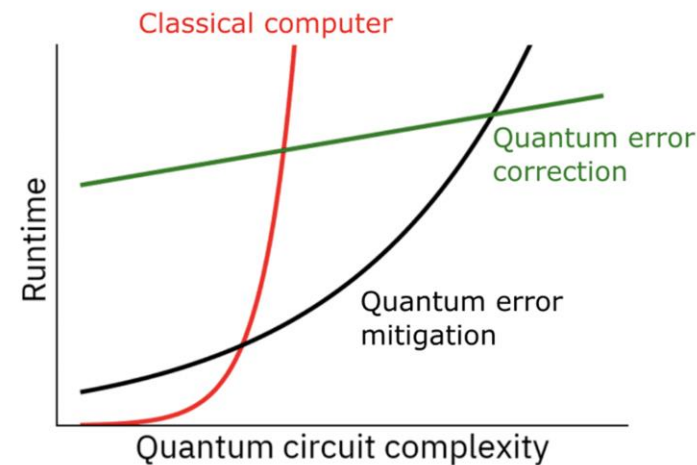
# Infleqtion Quantum Computing Stack





# A footnote on Quantum Circuit Simulation

- **Quantum circuit simulation:** using a classical computer to simulate (digital) quantum computation
- Recall that quantum computation can be represented as matrix-vector operations → use large GPU compute to simulate small quantum compute



<https://research.ibm.com/blog/gammarbar-for-quantum-advantage>



**Infleqtion**

**SUPER.TECH**     **ColdQuanta**

**ColdquantaLabs**