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# 1 Introduction

This is the work-in-progress specification of the Cardano ledger. The Agda source code with which we formalize the ledger specification and which generates this pdf document is open source and resides at the following

repository: https://qithub.com/IntersectMBO/formal-ledger-specifications

The current status of each individual era is described in Table 1.

Era	Figures	Prose	Cleanup
Shelley [1]	Partial	Partial	Not started
Shelley-MA [2]	Partial	Partial	Not started
Alonzo [3]	Partial	Partial	Not started
Babbage [4]	Not started	Not started	Not started
Conway [5]	Complete	Partial	Partial

Table 1: Specification progress

#### 1.1 Overview

This document describes, in a precise and executable way, the behavior of the Cardano ledger that can be updated in response to a series of events. Because of the precise nature of the document, it can be dense and difficult to read at times, and it can be helpful to have a high-level understanding of what it is trying to describe, which we present below. Keep in mind that this section focuses on intuition, using terms (set in *italics*) which may be unfamiliar to some readers, but rest assured that later sections of the document will make the intuition and italicized terms precise.

### 1.2 A Note on Agda

This specification is written using the Agda programming language and proof assistant [6]. We have made a considerable effort to ensure that this document is readable by people unfamiliar with Agda (or other proof assistants, functional programming languages, etc.). However, by the nature of working in a formal language we have to play by its rules, meaning that some instances of uncommon notation are very difficult or impossible to avoid. Some are explained in Secs. 2 and B, but there is no guarantee that those sections are complete. If the meaning of an expression is confusing or unclear, please open an issue in the formal ledger repository with the 'notation' label.

# 1.3 Separation of Concerns

The Cardano Node consists of three pieces,

- a networking layer responsible for sending messages across the internet,
- a consensus layer establishing a common order of valid blocks, and
- a ledger layer which determines whether a sequence of blocks is valid.

Because of this separation, the ledger can be modeled as a state machine,

$$s \xrightarrow{b} s'$$
.

More generally, we will consider state machines with an environment,

$$\Gamma \vdash s \xrightarrow{b} s'$$
.

These are modelled as 4-ary relations between the environment  $\Gamma$ , an initial state s, a signal b and a final state s'. The ledger consists of roughly 25 (depending on the version) such relations that depend on each other, forming a directed graph that is almost a tree. (See Fig. 1.) Thus each such relation represents the transition rule of the state machine; X is simply a placeholder for the name of the transition rule.

# 1.4 Ledger State Transition Rules

By a *ledger* we mean a structure that contains information about how funds in the system are distributed accross accounts—that is, account balances, how such balances should be adjusted when transactions and proposals are processed, the ADA currently held in the treasury reserve, a list of *stake pools* operating the network, and so on.

The ledger can be updated in response to certain events, such as receiving a new transaction, time passing and crossing an *epoch boundary*, enacting a *governance proposal*, to name a few. This document defines, as part of the behaior of the ledger, a set of rules that determine which events are valid and exactly how the state of the ledger should be updated in response to those events. The primary aim of this document is to provide a precise description of this system—the ledger state, valid events and the rules for processing them.

We will model this via a number of *state transition systems* (STS) which from now on we refer to as "transition rules" or just "rules." These rules describe the different behaviors that determine how the whole system evolves and, taken together, they comprise a full description of the ledger protocol. Each transition rule consists of the following components:

- an *environment* consisting of data, read from the ledger state or the outside world, which should be considered constant for the purposes of the rule;
- an *initial state*, consisting of the subset of the full ledger state that is relevant to the rule and which the rule can update;
- a signal or event, with associated data, that the rule can receive or observe;
- a set of preconditions that must be met in order for the transition to be valid;
- a new state that results from the transition rule.

For example, the UTXOW transition rule defined in Fig. 34 of Sec. 12.2 checks that, among other things, a given transaction is signed by the appropriate parties.

The transition rules can be composed in the sense that they may require other transition rules to hold as part of their preconditions. For example, the UTXOW rule mentioned above requires the UTXO rule, which checks that the inputs to the transaction exist, that the transaction is balanced, and several other conditions.

A brief description of each transition rule is provided below, with a link to an Agda module and reference to a section where the rule is formally defined.

- CHAIN is the top level transition in response to a new block that applies the NEWEPOCH transition when crossing an epoch boundary, and the LEDGERS transition on the list of transactions in the body (Sec. 20).
- NEWEPOCH computes the new state as of the start of a new epoch; includes the previous EPOCH transition (Sec. 19).
- EPOCH computes the new state as of the end of an epoch; includes the ENACT, RATIFY, and SNAP transition rules (Sec. 19).
- RATIFY decides whether a pending governance action has reached the thresholds it needs to be ratified (Sec. 17).

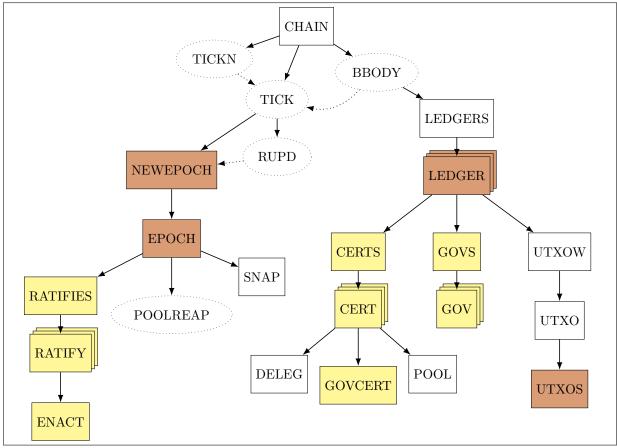


Figure 1: State transition rules of the ledger specification, presented as a directed graph; each node represents a transition rule; an arrow from rule A to rule B indicates that B appears among the premises of A; a dotted arrow represents a dependency in the sense that the output of the target node is an input to the source node, either as part of the source state, the environment or the event ( rules added in Conway; rules modified in Conway; dotted ellipses represent rules that are not yet formalized in Agda).

- ENACT applies the result of a previously ratified governance action, such as triggering a hard fork or updating the protocol parameters (Sec. 16).
- SNAP computes new stake distribution snapshots (Sec. 19).
- LEDGERS applies LEDGER repeatedly as needed, for each transaction in a list of transactions (Sec. 15).
- LEDGER is the full state update in response to a single transaction; it includes the UTXOW, GOV, and CERTS rules (Sec. 15).
- CERTS applies CERT repeatedly for each certificate in the transaction (Sec. 14).
- CERT combines DELEG, POOL, GOVCERT transition rules, as well as some additional rules shared by all three (Sec. 14).
- DELEG handles registering stake addresses and delegating to a stake pool (Sec. 14).
- GOVCERT handles registering and delegating to DReps (Sec. 14).
- POOL handles registering and retiring stake pools (Sec. 14).
- GOV handles voting and submitting governance proposals (Sec. 13).
- UTXOW checks that a transaction is witnessed correctly with the appropriate signatures, datums, and scripts; includes the UTXO transition rule (Sec. 12.2).

- UTXO checks core invariants for an individual transaction to be valid, such as the transaction being balanced, fees being paid, etc; include the UTXOS transition rule (Sec. 12).
- UTXOS checks that any relevant scripts needed by the transaction evaluate to true (Sec. 12).

#### 1.5 Reflexive-transitive Closure

Some state transition rules need to be applied as many times as possible to arrive at a final state. Since we use this pattern multiple times, we define a closure operation which takes a transition rule and applies it as many times as possible.

The closure  $\_\vdash\_ \rightharpoonup \llbracket\_\rrbracket *\_$  of a relation  $\_\vdash\_ \rightharpoonup \llbracket\_\rrbracket\_$  is defined in Fig. 2. In the remainder of the text, the closure operation is called ReflexiveTransitiveClosure.

```
Closure type

data \_\vdash\_ \rightharpoonup [\_] *\_ : C \rightarrow S \rightarrow List Sig \rightarrow S \rightarrow Type \text{ where}

Closure rules

RTC-base : \Gamma \vdash s \rightharpoonup [[]] * s

RTC-ind : \bullet \Gamma \vdash s \rightharpoonup [sig]  s' \bullet \Gamma \vdash s' \rightharpoonup [sigs] * s''

\vdash \vdash s \rightharpoonup [sigs] * s''

\vdash \vdash s \rightharpoonup [sigs] * s''
```

Figure 2: Reflexive transitive closure

#### 1.6 Computational

Since all such state machines need to be evaluated by the nodes and all nodes should compute the same states, the relations specified by them should be computable by functions. This can be captured by the definition in Fig. 3 which is parametrized over the state transition relation.

```
record Computational (\_\vdash\_ \rightharpoonup \emptyset\_, X \Vdash\_ : C \rightarrow S \rightarrow Sig \rightarrow S \rightarrow Type): Type where field compute : C \rightarrow S \rightarrow Sig \rightarrow Maybe S \equiv -just \Leftrightarrow STS: compute \Gamma s b \equiv just s' \Leftrightarrow \Gamma \vdash s \rightharpoonup \emptyset b , X \Vdash s' nothing\Rightarrow \forall \neg STS: compute \Gamma s b \equiv nothing \Rightarrow \forall s' \Rightarrow \neg \Gamma \vdash s \rightharpoonup \emptyset b , X \Vdash s'
```

Figure 3: Computational relations

Unpacking this, we have a compute function that computes a final state from a given environment, state and signal. The second piece is correctness: compute succeeds with some final state if and only if that final state is in relation to the inputs.

This has two further implications:

• Since compute is a function, the state transition relation is necessarily a (partial) function; i.e., there is at most one possible final state for each input data. Otherwise, we could

prove that compute could evaluates to two different states on the same inputs, which is impossible since it is a function.

• The actual definition of compute is irrelevant—any two implementations of compute have to produce the same result on any input. This is because we can simply chain the equivalences for two different compute functions together.

What this all means in the end is that if we give a Computational instance for every relation defined in the ledger, we also have an executable version of the rules which is guaranteed to be correct. This is indeed something we have done, and the same source code that generates this document also generates a Haskell library that lets anyone run this code.

### 1.7 Sets & Maps

The ledger heavily uses set theory. For various reasons it was necessary to implement our own set theory (there will be a paper on this some time in the future). Crucially, the set theory is completely abstract (in a technical sense—Agda has an abstract keyword) meaning that implementation details of the set theory are irrelevant. Additionally, all sets in this specification are finite.

We use this set theory to define maps as seen below, which are used in many places. We usually think of maps as partial functions (i.e., functions not necessarily defined everywhere—equivalently, "left-unique" relations) and we use the harpoon arrow  $\rightharpoonup$  to distinguish such maps from standard Agda functions which use  $\rightarrow$ . The figure below also gives notation for the powerset operation,  $\mathbb{P}$ , used to form a type of sets with elements in a given type, as well as the subset relation and the equality relation for sets.

When we need to convert a list 1 to its set of elements, we write from List 1.

### 1.8 Propositions as Types, Properties and Relations

In type theory we represent propositions as types and proofs of a proposition as elements of the corresponding type. A unary predicate is a function that takes each x (of some type A) and returns a proposition P(x). Thus, a predicate is a function of type  $A \to Type$ . A binary relation P(x) between P(x) and P(x) are a function that takes a pair of values P(x) and P(x) and returns a proposition asserting that the relation P(x) holds between P(x) and P(x). Thus, such a relation is a function of type P(x) and P(x) are type or P(x) are type or P(x) and P(x) are type or P(x) are type or P(x) are type or P(x) and P(x) are type or P(x) and P(x) are type or P(x) are ty

These relations are typically required to be decidable, which means that there is a boolean-valued function that computes whether the predicate holds or not. This means that it is generally safe to think of predicates simply returning a boolean value instead.

# 2 Notation

This section introduces some of the notation we use in this document and in our Agda formalization.

- **Propositions, sets and types.** See Sec. 1.7. Note that Agda denotes the primitive notion of type by Set. To avoid confusion, throughout this document and in our Agda code we call this primitive Type and use  $\mathbb{P}$  for our set type.
- **Lists** We use the notation a :: as for the list with *head* a and *tail* as; [] denotes the empty list, and 1 :: x appends the element x to the end of the list 1.
- Sums and products. The sum (or disjoint union, coproduct, etc.) of A and B is denoted by A ⊌ B, and their product is denoted by A × B. The projection functions from products are denoted proj₁ and proj₂, and the injections are denoted inj₁ and inj₂ respectively. The properties whether an element of a coproduct is in the left or right component are called isInj₁ and isInj₂.
- Addition of map values. The expression  $\Sigma[x \in m]$  f x denotes the sum of the values obtained by applying the function f to the values of the map m.
- Record types are explained in Sec. B.
- **Postfix projections.** Projections can be written using postfix notation. For example, we may write x .proj<sub>1</sub> instead of proj<sub>1</sub> x.
- **Restriction, corestriction and complements.** The restriction of a function or map f to some domain A is denoted by  $f \mid A$ , and the restriction to the complement of A is written  $f \mid A$ . Corestriction or range restriction is denoted similarly, except that  $\mid$  is replaced by  $\mid \land$ .
- **Inverse image.** The expression  $m^{-1}$  B denotes the inverse image of the set B under the map m.
- **Left-biased union.** For maps m and m', we write  $m \cup^1 m'$  for their left-biased union. This means that key-value pairs in m are guaranteed to be in the union, while key-value pairs in m' will be in the union if and only if the keys don't collide.
- **Map addition.** For maps m and m', we write  $m \cup^* m'$  for their union, where keys that appear in both maps have their corresponding values added.
- **Mapping a partial function.** A partial function is a function on A which may not be defined for all elements of A. We denote such a function by  $f:A \to B$ . If we happen to know that the function is total (defined for all elements of A), then we write  $f:A \to B$ . The mapPartial operation takes such a function f and a set S of elements of A and applies f to the elements of S at which it is defined; the result is the set  $\{f \mid x \mid x \in S \text{ and } f \text{ is defined at } x\}$ .
- The Maybe type represents an optional value and can either be just x (indicating the presence of a value, x) or nothing (indicating the absence of a value). If x has type X, then just x has type Maybe X.
  - The symbol  $\sim$  denotes (pseudo)equality of two values x and y of type Maybe X: if x is of the form just x' and y is of the form just y', then x' and y' have to be equal. Otherwise, they are considered "equal".
- The unit type τ has a single inhabitant tt and may be thought of as a type that carries no information; it is useful for signifying the completion of an action, the presence of a trivial value, a trivially satisfied requirement, etc.

# 2.1 Superscripts and Other Special Notations

In the current version of this specification, superscript letters are sometimes used for things such as disambiguations or type conversions. These are essentially meaningless, only present for technical reasons and can safely be ignored. However there are the two exceptions:

- $U^1$  for left-biased union
- c in the context of set restrictions, where it indicates the complement

Also, non-letter superscripts do carry meaning.<sup>1</sup>

<sup>&</sup>lt;sup>1</sup>At some point in the future we hope to be able to remove all those non-essential superscripts. Since we prefer doing this by changing the Agda source code instead of via hiding them in this document, this is a non-trivial problem that will take some time to address.

# 3 Cryptographic Primitives

This section is part of the Ledger.Conway.Crypto module of the formal ledger specification., in which we rely on a public key signing scheme for verification of spending.

Fig. 4 shows some of the types, functions and properties of this scheme.

```
Types & functions

SKey VKey Sig Ser: Type

isKeyPair : SKey \rightarrow VKey \rightarrow Type

isSigned : VKey \rightarrow Ser \rightarrow Sig \rightarrow Type

sign : SKey \rightarrow Ser \rightarrow Sig

KeyPair = \Sigma[ sk \in SKey ] \Sigma[ vk \in VKey ] isKeyPair sk vk

Property of signatures

isSigned-correct: ((sk, vk, \_) : KeyPair) (d : Ser) (\sigma : Sig)

\rightarrow sign sk \ d \equiv \sigma \rightarrow isSigned \ vk \ d \ \sigma
```

Figure 4: Definitions for the public key signature scheme

# 4 Base Types

This section is part of the Ledger.Conway.BaseTypes module of the formal ledger specification., in which we define some of the most basic types used throughout the ledger.

```
Coin = N
Slot = N
Epoch = N
```

Figure 5: Some basic types used in many places in the ledger

```
UnitInterval = [x \in \mathbb{Q} \mid (0 \le x) \times (x \le 1)]
```

Figure 6: Refinement types used in some places in the ledger

# 5 Token Algebras

This section is part of the Ledger.Conway.TokenAlgebra module of the formal ledger specification..

```
Abstract types
   PolicyId
Derived types
 record TokenAlgebra: Type₁ where
   field
     Value : Set
     { Value-CommutativeMonoid } : CommutativeMonoid 0\ell 0\ell Value
     coin
                                 : Value → Coin
                                 : Coin → Value
     inject
     policies
                                : Value → P PolicyId
                                : Value → MemoryEstimate
     size
     _≤<sup>t</sup>_
                                : Value → Value → Type
     coin∘inject≗id : coin ∘ inject ≗ id
     coinIsMonoidHomomorphism : IsMonoidHomomorphism coin
Helper functions
   sum<sup>v</sup> : List Value → Value
   sumv [] = inject 0
   sum^{v} (x :: 1) = x + sum^{v} 1
```

Figure 7: Token algebras, used for multi-assets

# 6 Addresses

This section is part of the Ledger.Conway.Address module of the formal ledger specification., in which we define credentials and various types of addresses here.

A credential contains a hash, either of a verifying (public) key (isVKey) or of a script (isScript).

N.B. in the Shelley era the type of the stake field of the BaseAddr record was Credential (see Corduan et al. [1, Sec 4]); to specify an address with no stake, we would use an "enterprise" address. In contrast, the type of stake in the Conway era is Maybe Credential, so we can now use BaseAddr to specify an address with no stake by setting stake to nothing.

```
Abstract types
   Network
   KeyHash
   ScriptHash
Derived types
 data Credential : Type where
   KeyHashObj : KeyHash → Credential
   ScriptObj : ScriptHash → Credential
 record BaseAddr : Type where
   field net : Network
         pav : Credential
         stake : Maybe Credential
 record BootstrapAddr : Type where
   field net
                   : Network
                   : Credential
         pay
         attrsSize : N
 record RwdAddr : Type where
   field net : Network
         stake : Credential
 VKeyBaseAddr = \Sigma[ addr \in BaseAddr
                                               ]isVKey
                                                         (addr .pay)
 VKeyBootstrapAddr = \Sigma[ addr \in BootstrapAddr ] isVKey (addr .pay)
 ScriptBaseAddr = \Sigma \lceil addr \in BaseAddr
                                          lisScript (addr .pay)
 ScriptBootstrapAddr = \Sigma[ addr \in BootstrapAddr ] isScript (addr .pay)
 Addr
            = BaseAddr
                             ⊎ BootstrapAddr
 VKeyAddr = VKeyBaseAddr ⊌ VKeyBootstrapAddr
 ScriptAddr = ScriptBaseAddr ⊎ ScriptBootstrapAddr
Helper functions
 payCred
            : Addr → Credential
 stakeCred : Addr → Maybe Credential
 netId
             : Addr → Network
 isVKeyAddr : Addr → Type
 isScriptAddr : Addr → Type
 isVKeyAddr
                = isVKey ∘ payCred
 isScriptAddr = isScript o payCred
 isScriptRwdAddr = isScript o CredentialOf
```

Figure 8: Definitions used in Addresses

# 7 Scripts

This section is part of the Ledger.Conway.Script module of the formal ledger specification., in which we define Timelock scripts.

Timelock scripts can verify the presence of keys and whether a transaction happens in a certain slot interval. The scripts are executed as part of the regular witnessing.

```
data Timelock: Type where
  RequireAllOf
                    : List Timelock
                                        → Timelock
  RequireAnyOf
                    : List Timelock
                                        → Timelock
                    : N → List Timelock → Timelock
  RequireMOf
  RequireSig
                    : KeyHash
                                        → Timelock
                                        → Timelock
  RequireTimeStart : Slot
  RequireTimeExpire : Slot
                                         → Timelock
evalTimelock (khs: P KeyHash) (I: Maybe Slot \times Maybe Slot): Timelock \rightarrow Type where
evalAll : All (evalTimelock khs I) ss
        → (evalTimelock khs I) (RequireAllOf ss)
evalAny : Any (evalTimelock khs I) ss
        → (evalTimelock khs I) (RequireAnyOf ss)
evalMOf : MOf m (evalTimelock khs I) ss
        → (evalTimelock khs I) (RequireMOf m ss)
evalSig : x \in khs
        → (evalTimelock khs I) (RequireSig x)
evalTSt : M.Any (a \leq_{-}) (I .proj_1)
        → (evalTimelock khs I) (RequireTimeStart a)
evalTEx: M.Any (\leq a) (I.proj_2)
        → (evalTimelock khs I) (RequireTimeExpire α)
```

Figure 9: Timelock scripts and their evaluation

### 8 Protocol Parameters

This section is part of the Ledger.Conway.PParams module of the formal ledger specification., in which we define the adjustable protocol parameters of the Cardano ledger.

Protocol parameters are used in block validation and can affect various features of the system, such as minimum fees, maximum and minimum sizes of certain components, and more.

The Acnt record has two fields, treasury and reserves, so the acnt field in NewEpochState keeps track of the total assets that remain in treasury and reserves.

```
record Acnt : Type where
field
treasury reserves : Coin

record Hastreasury {a} (A : Type a) : Type a where
field treasury0f : A → Coin
open Hastreasury {...} public

ProtVer : Type
ProtVer = N × N

instance
Show-ProtVer : Show ProtVer
Show-ProtVer = Show-×

data pvCanFollow : ProtVer → ProtVer → Type where
canFollowMajor : pvCanFollow (m , n) (m + 1 , 0)
canFollowMinor : pvCanFollow (m , n) (m , n + 1)
```

Figure 10: Definitions related to protocol parameters

```
data PParamGroup: Type where

NetworkGroup: PParamGroup

EconomicGroup: PParamGroup

TechnicalGroup: PParamGroup

GovernanceGroup: PParamGroup

SecurityGroup: PParamGroup
```

Figure 11: Protocol parameter group definition

PParams contains parameters used in the Cardano ledger, which we group according to the general purpose that each parameter serves.

- NetworkGroup: parameters related to the network settings;
- EconomicGroup: parameters related to the economic aspects of the ledger;
- TechnicalGroup: parameters related to technical settings;
- GovernanceGroup: parameters related to governance settings;
- SecurityGroup: parameters that can impact the security of the system.

```
record DrepThresholds: Type where
field
P1 P2a P2b P3 P4 P5a P5b P5c P5d P6: 

record PoolThresholds: Type where
field
Q1 Q2a Q2b Q4 Q5: 

Q
```

Figure 12: Protocol parameter threshold definitions

The purpose of these groups is to determine voting thresholds for proposals aiming to change parameters. Given a proposal to change a certain set of parameters, we look at which groups those parameters fall into and from this we determine the voting threshold for that proposal. (The voting threshold calculation is described in detail in Sec. 17.1; in particular, the definition of the threshold function appears in Fig. 56.)

The first four groups have the property that every protocol parameter is associated to precisely one of these groups. The SecurityGroup is special: a protocol parameter may or may not be in the SecurityGroup. So, each protocol parameter belongs to at least one and at most two groups. Note that in CIP-1694 there is no SecurityGroup, but there is the concept of security-relevant protocol parameters (see Corduan et al. [5]). The difference between these notions is only social, so we implement security-relevant protocol parameters as a group.

The new protocol parameters are declared in Fig. 13 and denote the following concepts:

- drepThresholds: governance thresholds for DReps; these are rational numbers named P1, P2a, P2b, P3, P4, P5a, P5b, P5c, P5d, and P6;
- poolThresholds: pool-related governance thresholds; these are rational numbers named Q1, Q2a, Q2b, Q4 and Q5;
- ccMinSize: minimum constitutional committee size;
- ccMaxTermLength: maximum term limit (in epochs) of constitutional committee members;
- govActionLifetime: governance action expiration;
- govActionDeposit: governance action deposit;
- drepDeposit: DRep deposit amount;
- drepActivity: DRep activity period;
- minimumAVS: the minimum active voting threshold.

Fig. 13 also defines the function paramsWellFormed which performs some sanity checks on protocol parameters. Fig. 15 defines types and functions to update parameters. These consist of an abstract type UpdateT and two functions applyUpdate and updateGroups. The type UpdateT is to be instantiated by a type that

- can be used to update parameters, via the function applyUpdate
- can be queried about what parameter groups it updates, via the function updateGroups

An element of the type UpdateT is well formed if it updates at least one group and applying the update preserves well-formedness.

```
record PParams : Type where
   field
Network group
     maxBlockSize
                               : N
     maxTxSize
                               : N
                               : N
     maxHeaderSize
     maxTxExUnits
                               : ExUnits
     maxBlockExUnits
                              : ExUnits
     maxValSize
                               : N
     maxCollateralInputs
                               : N
Economic group
                               : N
     а
     b
                               : N
     keyDeposit
                               : Coin
     poolDeposit
                               : Coin
                            : UnitInterval -- formerly: rho
     monetaryExpansion
                               : UnitInterval -- formerly: tau
     treasuryCut
                              : Coin
     coinsPerUTxOByte
     prices
                               : Prices
     minFeeRefScriptCoinsPerByte : 0
     maxRefScriptSizePerTx : N
     maxRefScriptSizePerBlock
                               : IN
     refScriptCostStride
                              : N
     refScriptCostMultiplier
                              : Q
Technical group
     Emax
                               : Epoch
     nopt
                                : N
     a0
                               : 0
     collateralPercentage
                               : IN
                               : CostModel
     costmdls
Governance group
     poolThresholds
                               : PoolThresholds
     drepThresholds
                               : DrepThresholds
     ccMinSize
                               : IN
     ccMaxTermLength
                               : IN
     govActionLifetime
                               : IN
     govActionDeposit
                              : Coin
     drepDeposit
                              : Coin
     drepActivity
                               : Epoch
Security group
maxBlockSize maxTxSize maxHeaderSize maxValSize maxBlockExUnits a b minFeeRefScript-
CoinsPerByte coinsPerUTxOByte govActionDeposit
```

Figure 13: Protocol parameter definitions

Figure 14: Protocol parameter well-formedness

```
Abstract types & functions

UpdateT : Type
applyUpdate : PParams → UpdateT → PParams
updateGroups : UpdateT → P PParamGroup

Well-formedness condition

ppdWellFormed : UpdateT → Type
ppdWellFormed u = updateGroups u ≠ ∅
x ∀ pp → paramsWellFormed pp → paramsWellFormed (applyUpdate pp u)
```

Figure 15: Abstract type for parameter updates

Figure 16: Calculation of fees for reference scripts

### 9 Fee Calculation

This section is part of the Ledger.Conway.Fees module of the formal ledger specification., where we define the functions used to compute the fees associated with reference scripts.

The function scriptsCost (Fig. 16) calculates the fee for reference scripts in a transaction. It takes as input the total size of the reference scripts in bytes—which can be calculated using refScriptsSize (Fig. 29)—and uses a function (scriptsCostAux) that is piece-wise linear in the size, where the linear constant multiple grows with each refScriptCostStride bytes. In addition, scriptsCost depends on the following constants (which are bundled with the protocol parameters; see Fig. 13):

- refScriptCostMultiplier, a rational number, the growth factor or step multiplier that determines how much the price per byte increases after each increment;
- refScriptCostStride, an integer, the size in bytes at which the price per byte grows linearly;
- minFeeRefScriptCoinsPerByte, a rational number, the base fee or initial price per byte.

For background on this particular choice of fee calculation, see [7].

# 10 Governance Actions

This section is part of the Ledger.Conway.GovernanceActions module of the formal ledger specification..

We introduce the following distinct bodies with specific functions in the new governance framework:

- 1. a constitutional committee (henceforth called CC);
- 2. a group of delegate representatives (henceforth called DReps);
- 3. the stake pool operators (henceforth called SPOs).

Fig. 17 defines several data types used to represent governance actions. The type DocHash is abstract but in the implementation it will be instantiated with a 32-bit hash type (like e.g. ScriptHash). We keep it separate because it is used for a different purpose.

- GovActionID: a unique identifier for a governance action, consisting of the TxId of the proposing transaction and an index to identify a proposal within a transaction;
- GovRole (governance role): one of three available voter roles defined above (CC, DRep, SPO);
- VDeleg (voter delegation): one of three ways to delegate votes: by credential, abstention, or no confidence (credVoter, abstainRep, or noConfidenceRep);
- Anchor: a url and a document hash;
- GovAction (governance action): one of seven possible actions (see Fig. 18 for definitions);
- The governance actions carry the following information:
- UpdateCommittee: a map of credentials and terms to add and a set of credentials to remove from the committee;
- NewConstitution: a hash of the new constitution document and an optional proposal policy;
- TriggerHF: the protocol version of the epoch to hard fork into;
- ChangePParams: the updates to the parameters; and
- TreasuryWdrl: a map of withdrawals.

#### 10.1 Hash Protection

For some governance actions, in addition to obtaining the necessary votes, enactment requires that the following condition is also satisfied: the state obtained by enacting the proposal is in fact the state that was intended when the proposal was submitted. This is achieved by requiring actions to unambiguously link to the state they are modifying via a pointer to the previous modification. A proposal can only be enacted if it contains the <code>GovActionID</code> of the previously enacted proposal modifying the same piece of state. NoConfidence and UpdateCommittee modify the same state, while every other type of governance action has its own state that isn't shared with any other action. This means that the enactibility of a proposal can change when other proposals are enacted.

However, not all types of governance actions require this strict protection. For TreasuryWdrl and Info, enacting them does not change the state in non-commutative ways, so they can always be enacted.

Types related to this hash protection scheme are defined in Fig. 19.

<sup>&</sup>lt;sup>2</sup>There are many varying definitions of the term "hard fork" in the blockchain industry. Hard forks typically refer to non-backwards compatible updates of a network. In Cardano, we attach a bit more meaning to the definition by calling any upgrade that would lead to *more blocks* being validated a "hard fork" and force nodes to comply with the new protocol version, effectively rendering a node obsolete if it is unable to handle the upgrade.

# 10.2 Votes and Proposals

The data type Vote represents the different voting options: yes, no, or abstain. For a Vote to be cast, it must be packaged together with further information, such as who votes and for which governance action. This information is combined in the GovVote record. An optional Anchor can be provided to give context about why a vote was cast in a certain manner.

To propose a governance action, a GovProposal needs to be submitted. Beside the proposed action, it contains:

- a pointer to the previous action if required (see Sec. 10.1),
- a pointer to the proposal policy if one is required,
- a deposit, which will be returned to returnAddr, and
- an Anchor, providing further information about the proposal.

While the deposit is held, it is added to the deposit pot, similar to stake key deposits. It is also counted towards the voting stake (but not the block production stake) of the reward address to which it will be returned, so as not to reduce the submitter's voting power when voting on their own (and competing) actions. For a proposal to be valid, the deposit must be set to the current value of <code>govActionDeposit</code>. The deposit will be returned when the action is removed from the state in any way.

GovActionState is the state of an individual governance action. It contains the individual votes, its lifetime, and information necessary to enact the action and to repay the deposit.

```
data GovRole: Type where
 CC DRep SPO : GovRole
           = GovRole × Credential
GovActionID = TxId \times N
data VDeleg : Type where
 credVoter : GovRole → Credential → VDeleg
 abstainRep
                                         VDeleg
 noConfidenceRep:
                                         VDeleg
record Anchor : Type where
 field
   url : String
   hash: DocHash
data GovActionType : Type where
 NoConfidence : GovActionType
 UpdateCommittee : GovActionType
 NewConstitution: GovActionType
 TriggerHF : GovActionType
 ChangePParams : GovActionType
 TreasuryWdrl : GovActionType
 Info
                 : GovActionType
GovActionData : GovActionType → Type
GovActionData NoConfidence
                           = T
GovActionData UpdateCommittee = (Credential → Epoch) × P Credential × ℚ
GovActionData NewConstitution = DocHash × Maybe ScriptHash
GovActionData TriggerHF
                             = ProtVer
GovActionData ChangePParams = PParamsUpdate
GovActionData TreasuryWdrl = RwdAddr → Coin
GovActionData Info
                             = T
record GovAction : Type where
 field
   gaType : GovActionType
   gaData : GovActionData gaType
```

Figure 17: Governance actions

Action	Description
NoConfidence	a motion to create a <i>state of no-confidence</i> in the current constitutional committee
UpdateCommittee	changes to the members of the constitutional committee and/or to its signature threshold and/or terms
NewConstitution	a modification to the off-chain Constitution and the proposal policy script
TriggerHF <sup>2</sup>	triggers a non-backwards compatible upgrade of the network; requires a prior software upgrade
ChangePParams	a change to <i>one or more</i> updatable protocol parameters, excluding changes to major protocol versions ("hard forks")
TreasuryWdrl Info	movements from the treasury an action that has no effect on-chain, other than an on-chain record

Figure 18: Types of governance actions

```
NeedsHash : GovActionType → Type
NeedsHash NoConfidence = GovActionID
NeedsHash UpdateCommittee = GovActionID
NeedsHash NewConstitution = GovActionID
NeedsHash TriggerHF = GovActionID
NeedsHash ChangePParams = GovActionID
NeedsHash TreasuryWdrl = T
NeedsHash Info = T

HashProtected : Type → Type
HashProtected A = A × GovActionID
```

Figure 19: NeedsHash and HashProtected types

```
data Vote: Type where
 yes no abstain : Vote
record GovVote: Type where
 field
   gid
             : GovActionID
             : Voter
   voter
   vote
             : Vote
   anchor : Maybe Anchor
record GovProposal : Type where
 field
            : GovAction
   action
   prevAction : NeedsHash (gaType action)
   policy : Maybe ScriptHash
   deposit
             : Coin
   returnAddr : RwdAddr
            : Anchor
   anchor
record GovActionState : Type where
 field
   votes
            : Voter → Vote
   returnAddr : RwdAddr
   expiresIn : Epoch
            : GovAction
   action
   prevAction : NeedsHash (gaType action)
```

Figure 20: Vote and proposal types

```
getDRepVote : GovVote → Maybe Credential
getDRepVote record { voter = (DRep , credential) } = just credential
getDRepVote _ = nothing

proposedCC : GovAction → P Credential
proposedCC [ UpdateCommittee , (x , _ , _ ) ] ga = dom x
proposedCC _ = ∅
```

Figure 21: Governance helper function

### 11 Transactions

This section is part of the Ledger.Conway.Transaction module of the formal ledger specification., where we define transactions.

A transaction consists of a transaction body, a collection of witnesses and some optional auxiliary data.

Some key ingredients in the transaction body are:

- A set txins of transaction inputs, each of which identifies an output from a previous transaction. A transaction input consists of a transaction id and an index to uniquely identify the output.
- An indexed collection txouts of transaction outputs. The TxOut type is an address paired with a coin value.
- A transaction fee. This value will be added to the fee pot.
- The size txsize and the hash txid of the serialized form of the transaction that was included in the block.

```
Abstract types
 Ix TxId AuxiliaryData : Type
Derived types
 TxIn
       = TxId \times Ix
 TxOut = Addr × Value × Maybe (Datum ⊎ DataHash) × Maybe Script
 UTx0
       = TxIn → TxOut
 Wdrl
        = RwdAddr → Coin
 RdmrPtr = Tag \times Ix
 ProposedPPUpdates = KeyHash → PParamsUpdate
                   = ProposedPPUpdates × Epoch
Transaction types
 record TxBody: Type where
   field
                 : ℙTxIn
     refInputs : P TxIn
     txouts : Ix → TxOut
     txfee
                 : Coin
                 : Value
     mint
     txvldt : Maybe Slot x Maybe Slot
txcerts : List DCert
txwdrls : Wdrl
     txvote     : List GovVote
txprop     : List GovProposal
     txdonation : Coin
     txNetworkId : Maybe Network
     curTreasury : Maybe Coin
               : IN
     txsize
                 : TxId
     txid
     collateral : ℙ TxIn
     reqSigHash : ℙ KeyHash
     scriptIntHash : Maybe ScriptHash
 record TxWitnesses: Type where
   field
     vkSigs : VKey → Sig
     scripts : P Script
     txdats : DataHash → Datum
     txrdmrs : RdmrPtr → Redeemer × ExUnits
   scriptsP1 : ℙ P1Script
   scriptsP1 = mapPartial isInj<sub>1</sub> scripts
 record Tx: Type where
   field
     body : TxBody
     wits : TxWitnesses
     isValid : Bool
                                          28
     txAD : Maybe AuxiliaryData
```

Figure 22: Transactions and related types

```
getValue : TxOut → Value
getValue (_{-}, v,_{-}) = v
TxOut<sup>h</sup> = Addr × Value × Maybe (Datum ⊎ DataHash) × Maybe ScriptHash
txOutHash: TxOut → TxOuth
txOutHash(a, v, d, s) = a, (v, (d, M.map hash s))
getValue<sup>h</sup> : TxOut<sup>h</sup> → Value
getValue^h (\_, v, \_) = v
txinsVKey: P TxIn → UTxO → P TxIn
txinsVKey\ txins\ utxo = txins \cap dom\ (utxo | ^' (isVKeyAddr \circ proj_1))
scriptOuts : UTxO → UTxO
scriptOuts utxo = filter(\lambda(_, addr,_) \rightarrow isScriptAddr addr) utxo
txinsScript : P TxIn → UTxO → P TxIn
txinsScript \ txins \ utxo = txins \cap dom \ (proj_1 \ (scriptOuts \ utxo))
refScripts : Tx → UTxO → List Script
refScripts tx utxo =
  mapMaybe (proj₂ ∘ proj₂ ∘ proj₂) $ setToList (range (utxo | (txins ∪ refInputs)))
  where open Tx; open TxBody (tx.body)
txscripts : Tx \rightarrow UTx0 \rightarrow P Script
txscripts tx utxo = scripts (tx .wits) U fromList (refScripts tx utxo)
  where open Tx; open TxWitnesses
lookupScriptHash : ScriptHash → Tx → UTxO → Maybe Script
lookupScriptHash sh tx utxo =
  if sh \in map^{s} proj_{1} (m^{s}) then
    just (lookup<sup>m</sup> m sh)
  else
    nothing
  where m = setToMap (map<sup>s</sup> < hash , id > (txscripts tx utxo))
```

Figure 23: Functions related to transactions

### 12 UTxO

This section is part of the Ledger.Conway.Utxo module of the formal ledger specification., where we define types and functions needed for the UTxO transition system.

# 12.1 Accounting

```
isTwoPhaseScriptAddress : Tx → UTxO → Addr → Type
isTwoPhaseScriptAddress tx utxo α =
  if isScriptAddr a then
     (\lambda \{p\} \rightarrow \text{if lookupScriptHash } (\text{getScriptHash } \alpha p) \text{ } tx \text{ } utxo)
                     then (\lambda \{s\} \rightarrow isP2Script s)
                     else 1)
  else
    1
  getDataHashes : P TxOut → P DataHash
  getDataHashes txo = mapPartial isInj_2 (mapPartial (proj_1 <math>\circ proj_2 \circ proj_2) txo)
  getInputHashes : Tx → UTxO → P DataHash
  getInputHashes tx utxo = getDataHashes
     (filter<sup>s</sup> (\lambda (a, _ ) \rightarrow isTwoPhaseScriptAddress' tx utxo a)
                (range (utxo | txins)))
     where open Tx; open TxBody (tx.body)
totExUnits : Tx → ExUnits
totExUnits tx = \sum [(-, eu) \leftarrow tx \cdot wits \cdot txrdmrs] eu
  where open Tx; open TxWitnesses
```

Figure 24: Functions supporting UTxO rules

Figs. 24 to 28 define functions needed for the UTxO transition system.

Fig. 25 defines the types needed for the UTxO transition system. The UTxO transition system is given in Fig. 31.

- The function outs creates the unspent outputs generated by a transaction. It maps the transaction id and output index to the output.
- The balance function calculates sum total of all the coin in a given UTxO.

The deposits have been reworked since the original Shelley design. We now track the amount of every deposit individually. This fixes an issue in the original design: An increase in deposit amounts would allow an attacker to make lots of deposits before that change and refund them after the change. The additional funds necessary would have been provided by the treasury. Since changes to protocol parameters were (and still are) known publicly and guaranteed before they are enacted, this comes at zero risk for an attacker. This means the deposit amounts could realistically never be increased. This issue is gone with the new design. See also [8].

Similar to ScriptPurpose, DepositPurpose carries the information what the deposit is being made for. The deposits are stored in the deposits field of UTxOState (the type Deposits is defined in Fig. 41). updateDeposits is responsible for updating this map, which is split into updateCertDeposits and updateProposalDeposits, responsible for certificates and proposals respectively. Both of these functions iterate over the relevant fields of the transaction body and

insert or remove deposits depending on the information seen. Note that some deposits can only be refunded at the epoch boundary and are not removed by these functions.

There are two equivalent ways to introduce this tracking of the deposits. One option would be to populate the deposits field of UTxOState with the correct keys and values that can be extracted from the state of the previous era at the transition into the Conway era. Alternatively, we can effectively treat the old handling of deposits as an erratum in the Shelley specification, which we fix by implementing the new deposits logic in older eras and then replaying the chain. (The handling of deposits in the Shelley era is discussed in Corduan et al. [1, Sec 8] and IOHK Formal Methods Team [9, Sec B.2].)

```
UTxO environment
 record UTxOEnv: Type where
   field
     slot
              : Slot
     pparams : PParams
     treasury: Coin
UTxO states
 record UTxOState : Type where
   field
               : UTx0
     fees
               : Coin
     deposits : Deposits
     donations : Coin
UTxO transitions
   _⊢_→(_,UTXO)_: UTxOEnv → UTxOState → Tx → UTxOState → Type
```

Figure 25: UTxO transition-system types

As seen in Fig. 29, we redefine depositRefunds and newDeposits via depositsChange, which computes the difference between the total deposits before and after their application. This simplifies their definitions and some correctness proofs. We then add the absolute value of depositsChange to consumed or produced depending on its sign. This is done via negPart and posPart, which satisfy the key property that their difference is the identity function.

Fig. 26 defines the function minfee. In Conway, minfee includes the cost for reference scripts. This is calculated using scriptsCost (see Fig. 16).

Fig. 26 also shows the signature of ValidCertDeposits. Inhabitants of this type are constructed in one of eight ways, corresponding to seven certificate types plus one for an empty list of certificates. Suffice it to say that ValidCertDeposits is used to check the validity of the deposits in a transaction so that the function updateCertDeposits can correctly register and deregister deposits in the UTxO state based on the certificates in the transaction.

Fig. 31 ties all the pieces of the UTXO rule together. The symbol **≡?** is explained in Sec. 2.

```
outs : TxBody → UTxO
outs tx = mapKeys (tx .txid ,_) (tx .txouts)
balance : UTxO → Value
balance utxo = \sum [x \in mapValues txOutHash utxo] getValue^h x
cbalance : UTxO → Coin
cbalance utxo = coin (balance utxo)
refScriptsSize : UTxO \rightarrow Tx \rightarrow N
refScriptsSize utxo tx = sum (map scriptSize (refScripts tx utxo))
minfee : PParams \rightarrow UTx0 \rightarrow Tx \rightarrow Coin
minfee pp utxo tx = pp .a * tx .body .txsize + pp .b
                 + txscriptfee (pp .prices) (totExUnits tx)
                 + scriptsCost pp (refScriptsSize utxo tx)
certDeposit : DCert → PParams → Deposits
certDeposit (delegate c _ _ v) _ = { CredentialDeposit c , v }
certDeposit (regpool kh _) pp = { PoolDeposit kh , pp .poolDeposit }
certDeposit (regdrep c v _) _ = { DRepDeposit c , v }
certDeposit _
certRefund : DCert \rightarrow \mathbb{P} DepositPurpose
certRefund (dereg c _) = { CredentialDeposit c }
certRefund (deregdrep c _) = { DRepDeposit c }
certRefund _
data ValidCertDeposits (pp : PParams) (deps : Deposits) : List DCert → Set
```

Figure 26: Functions used in UTxO rules

```
updateCertDeposits : PParams → List DCert → Deposits → Deposits
updateCertDeposits pp [] deposits = deposits
updateCertDeposits pp (delegate c vd khs v :: certs) deposits
  = updateCertDeposits pp certs (deposits U* certDeposit (delegate c vd khs v) pp)
updateCertDeposits pp (regpool kh p :: certs) deposits
  = updateCertDeposits pp certs (deposits U* certDeposit (regpool kh p) pp)
updateCertDeposits pp (regdrep c v a :: certs) deposits
  = updateCertDeposits pp certs (deposits ∪* certDeposit (regdrep c v a) pp)
updateCertDeposits pp (dereg c v :: certs) deposits
  = updateCertDeposits pp certs (deposits | certRefund (dereg c v)°)
updateCertDeposits pp (deregdrep c v :: certs) deposits
  = updateCertDeposits pp certs (deposits | certRefund (deregdrep c v)°)
updateCertDeposits pp (_ :: certs) deposits
  = updateCertDeposits pp certs deposits
updateProposalDeposits: List GovProposal → TxId → Coin → Deposits → Deposits
updateProposalDeposits []
                                            deposits = deposits
updateProposalDeposits (_ :: ps) txid gaDep deposits =
 updateProposalDeposits ps txid gaDep deposits
 U* { GovActionDeposit (txid , length ps) , gaDep }
updateDeposits : PParams → TxBody → Deposits → Deposits
updateDeposits pp txb = updateCertDeposits pp txcerts
                        updateProposalDeposits txprop txid (pp .govActionDeposit)
depositsChange : PParams \rightarrow TxBody \rightarrow Deposits \rightarrow Z
depositsChange pp txb deposits =
  getCoin (updateDeposits pp txb deposits) - getCoin deposits
```

Figure 27: Functions used in UTxO rules, continued

```
data inInterval (slot : Slot) : (Maybe Slot × Maybe Slot) → Type where
  both : \forall \{l \ r\} \rightarrow l \leq slot \times slot \leq r \rightarrow inInterval \ slot \ (just \ l \ , just \ r)
  lower: \forall \{1\} \rightarrow 1 \leq slot
                                            → inInterval slot (just 1 , nothing)
  upper: \forall \{r\}
                                            → inInterval slot (nothing, just r)
                      → slot ≤ r
  none :
                                                inInterval slot (nothing , nothing)
feesOK : PParams \rightarrow Tx \rightarrow UTx0 \rightarrow Type
feesOK pp tx utxo = ( minfee pp utxo tx ≤ txfee × (txrdmrs * ≠ ∅
                        \rightarrow (All (\lambda (addr, _) \rightarrow isVKeyAddr addr) collateralRange
                          x isAdaOnly bal
                          x coin bal * 100 ≥ txfee * pp .collateralPercentage
                          × collateral ≢ ∅
                          )
                        )
                      )
  where
    open Tx tx; open TxBody body; open TxWitnesses wits; open PParams pp
    collateralRange = range ((mapValues txOutHash utxo) | collateral)
                       = balance (utxo | collateral)
```

Figure 28: Functions used in UTxO rules, continued

```
depositRefunds : PParams → UTxOState → TxBody → Coin
depositRefunds pp st txb = negPart (depositsChange pp txb (st .deposits))

newDeposits : PParams → UTxOState → TxBody → Coin
newDeposits pp st txb = posPart (depositsChange pp txb (st .deposits))

consumed : PParams → UTxOState → TxBody → Value
consumed pp st txb
= balance (st .utxo | txb .txins)
+ txb .mint
+ inject (depositRefunds pp st txb)
+ inject (getCoin (txb .txwdrls))

produced : PParams → UTxOState → TxBody → Value
produced pp st txb = balance (outs txb)
+ inject (txb .txfee)
+ inject (newDeposits pp st txb)
+ inject (txb .txdonation)
```

Figure 29: Functions used in UTxO rules, continued

```
Scripts-Yes:

let pp = Γ.pparams

sLst = collectPhaseTwoScriptInputs pp tx utxo

in

• ValidCertDeposits pp deposits txcerts

• evalScripts tx sLst ≡ isValid

• isValid ≡ true

\[ \Gamma \cdot \text{[utxo, fees, deposits, donations]} \rightarrow \text{[tx, utxos)} \text{[(utxo | txins c) ul (outs txb), fees} \]

Scripts-No:

let pp = \Gamma \cdot \text{.pparams}

sLst = \text{collectPhaseTwoScriptInputs pp tx utxo}

in

• evalScripts tx sLst ≡ isValid

• isValid ≡ false

\[ \Gamma \cdot \text{[utxo, fees, deposits, donations]} \rightarrow \delta \text{[tx, utxos)} \text{[utxo | collateral c, fees + cbalance} \]
```

Figure 30: UTXOS rule

```
UTXO-inductive:
  let pp = \Gamma .pparams
       slot
               =\Gamma .slot
       treasury = \Gamma \cdot treasury
                = s .UTxOState.utxo
       txouts h = mapValues txOutHash txouts
       overhead = 160
  in
                              • txins ∪ refInputs ⊆ dom utxo
  • txins ≢ ∅
  • txins ∩ refInputs = ∅ • inInterval slot txvldt
  • feesOK pp tx \ utxo • consumed pp s \ txb \equiv produced pp s \ txb
  • coin mint ≡ 0
                              • txsize ≤ maxTxSize pp
  • refScriptsSize utxo tx ≤ pp .maxRefScriptSizePerTx
  • \forall [ (\_, txout) \in | txouts^h | ]
       inject ((overhead + utxoEntrySize txout) * coinsPerUTxOByte pp) ≤ t getValue txout
  • \forall [(_, txout) \in |txouts^h|]
       serSize (getValue^h txout) \le maxValSize pp
  • ∀[ (a , _) ∈ range txouts h ]
       Sum.All (const \tau) (\lambda a \rightarrow a .BootstrapAddr.attrsSize \leq 64) \alpha
  • \forall [(a, \_) \in \text{range } txouts^h] \text{ netId } a \equiv \text{NetworkId}
  • \forall [a \in dom \ txwdrls]
                                   NetworkIdOf a = NetworkId
  • txNetworkId ~ just NetworkId
  • curTreasury ~ just treasury
  • \Gamma \vdash s \rightharpoonup \emptyset tx ,UTXOSD s'
    \Gamma \vdash s \rightharpoonup \emptyset tx, UTXOD s'
```

Figure 31: UTXO inference rules

### 12.2 Witnessing

This section is part of the Ledger.Conway.Utxow module of the formal ledger specification., in which we define witnessing.

The purpose of witnessing is make sure the intended action is authorized by the holder of the signing key. (For details see Corduan et al. [1, Sec 8.3].) Fig. 32 defines functions used for witnessing. witsVKeyNeeded and scriptsNeeded are now defined by projecting the same information out of credsNeeded. Note that the last component of credsNeeded adds the script in the proposal policy only if it is present.

allowedLanguages has additional conditions for new features in Conway. If a transaction contains any votes, proposals, a treasury donation or asserts the treasury amount, it is only allowed to contain Plutus V3 scripts. Additionally, the presence of reference scripts or inline scripts does not prevent Plutus V1 scripts from being used in a transaction anymore. Only inline datums are now disallowed from appearing together with a Plutus V1 script.

### 12.3 Plutus script context

CIP-0069 unifies the arguments given to all types of Plutus scripts currently available: spending, certifying, rewarding, minting, voting, proposing.

The formal specification permits running spending scripts in the absence datums in the Conway era. However, since the interface with Plutus is kept abstract in this specification, changes to the representation of the script context which are part of CIP-0069 are not included here. To provide a CIP-0069-conformant implementation of Plutus to this specification, an additional step processing the List Data argument we provide would be required.

In Fig. 34, the line *inputHashes*  $\subseteq$  *txdatsHashes* compares two inhabitants of  $\mathbb P$  DataHash. In the Alonzo spec, these two terms would have inhabited  $\mathbb P$  (Maybe DataHash), where a nothing is thrown out [3, Sec 3.1].

```
getVKeys: P Credential → P KeyHash
getVKeys = mapPartial isKeyHashObj
allowedLanguages : Tx \rightarrow UTx0 \rightarrow P Language
allowedLanguages tx utxo =
  if (\exists [o \in os] isBootstrapAddr (proj_1 o))
    then ø
  else if UsesV3Features txb
    then fromList (PlutusV3 :: [])
  else if \exists [o \in os] HasInlineDatum o
    then fromList (PlutusV2 :: PlutusV3 :: [])
    fromList (PlutusV1 :: PlutusV2 :: PlutusV3 :: [])
  where
    txb = tx . Tx.body; open TxBody txb
    os = range (outs txb) U range (utxo | (txins U refInputs))
getScripts : P Credential → P ScriptHash
getScripts = mapPartial isScriptObj
credsNeeded : UTxO → TxBody → P (ScriptPurpose × Credential)
credsNeeded utxo txb
  = map<sup>s</sup> (\lambda (i, o) \rightarrow (Spend i, payCred (proj_1 o))) ((utxo | (txins \cup collateral)) s)
                    \rightarrow (Rwrd \alpha, stake \alpha)) (dom | txwdrls |)
  U mapPartial (\lambda c \rightarrow (Cert c_{,-}) < s > cwitness c) (fromList txcerts)
  U \text{ map}^s (\lambda x)
                    \rightarrow (Mint x, ScriptObj x)) (policies mint)
  U map<sup>s</sup> (λ v
                      → (Vote v, proj₂ v)) (fromList (map voter txvote))
  U mapPartial (\lambda p \rightarrow \text{case } p \cdot \text{policy of}
                              (just sh) \rightarrow just (Propose p, ScriptObj sh)
                              nothing → nothing) (fromList txprop)
witsVKeyNeeded : UTxO → TxBody → P KeyHash
witsVKeyNeeded = getVKeys o map s proj2 o credsNeeded
scriptsNeeded : UTxO → TxBody → P ScriptHash
scriptsNeeded = getScripts o maps proj2 o credsNeeded
```

Figure 32: Functions used for witnessing

```
_⊢_→(_,UTXOW)_: UTxOEnv → UTxOState → Tx → UTxOState → Type
```

Figure 33: UTxOW transition-system types

```
UTXOW-inductive:
  let utxo
                          = s .utxo
      witsKeyHashes = map* hash (dom vkSigs)
      witsScriptHashes = map s hash scripts
      inputHashes
                          = getInputHashes tx utxo
      refScriptHashes = fromList (map hash (refScripts tx utxo))
      neededHashes = scriptsNeeded utxo txb
      txdatsHashes
                         = dom txdats
      allOutHashes
                         = getDataHashes (range txouts)
                          = mapPartial isInj<sub>1</sub> (txscripts tx utxo)
      nativeScripts
  in
  • \forall [ (vk, \sigma) \in vkSigs ] isSigned vk (txidBytes txid) \sigma
  • \forall[ s \in nativeScripts ] (hash s \in neededHashes \rightarrow validP1Script witsKeyHashes txvldt <math>s)
  • witsVKeyNeeded utxo txb ⊆ witsKeyHashes
  • neededHashes \ refScriptHashes ≡ e witsScriptHashes
  • inputHashes ⊆ txdatsHashes
  • txdatsHashes ⊆ inputHashes ∪ allOutHashes ∪ getDataHashes (range (utxo | refInputs))
  • languages tx utxo ⊆ allowedLanguages tx utxo
  • txADhash \equiv map hash txAD
  • \Gamma \vdash s \rightharpoonup \emptyset tx ,UTX00 s'
    \Gamma \vdash s \rightharpoonup \emptyset \ tx \ , UTXOWD \ s'
```

Figure 34: UTXOW inference rules

```
Derived types

GovState = List (GovActionID × GovActionState)

record GovEnv : Type where
  field
    txid : TxId
    epoch : Epoch
    pparams : PParams
    ppolicy : Maybe ScriptHash
    enactState : EnactState
    certState : CertState
    rewardCreds : P Credential
```

Figure 35: Types used in the GOV transition system

# 13 Governance

This section is part of the Ledger.Conway.Gov module of the formal ledger specification., where we define the types required for ledger governance.

The behavior of <code>GovState</code> is similar to that of a queue. New proposals are appended at the end, but any proposal can be removed at the epoch boundary. However, for the purposes of enactment, earlier proposals take priority. Note that <code>EnactState</code> used in <code>GovEnv</code> is defined in <code>Sec. 16</code>.

- addVote inserts (and potentially overrides) a vote made for a particular governance action (identified by its ID) by a credential with a role.
- addAction adds a new proposed action at the end of a given GovState.
- The validHFAction property indicates whether a given proposal, if it is a TriggerHF action, can potentially be enacted in the future. For this to be the case, its prevAction needs to exist, be another TriggerHF action and have a compatible version.

Fig. 38 shows some of the functions used to determine whether certain actions are enactable in a given state. Specifically, allEnactable passes the GovState to getAidPairsList to obtain a list of GovActionID-pairs which is then passed to enactable. The latter uses the \_connects\_to\_function to check whether the list of GovActionID-pairs connects the proposed action to a previously enacted one.

The function govActionPriority assigns a priority to the various types of governance actions. This is useful for ordering lists of governance actions (see insertGovAction in Fig. 36). Priority is also used to check if two actions Overlap; that is, they would modify the same piece of EnactState.

```
govActionPriority : GovActionType → N
govActionPriority NoConfidence
govActionPriority UpdateCommittee = 1
govActionPriority NewConstitution = 2
govActionPriority TriggerHF
govActionPriority ChangePParams = 4
govActionPriority TreasuryWdrl
govActionPriority Info
                                       = 6
Overlap : GovActionType → GovActionType → Type
Overlap NoConfidence UpdateCommittee = T
Overlap UpdateCommittee NoConfidence = T
                       a'
Overlap a
                                          = a = a'
insertGovAction: GovState → GovActionID × GovActionState → GovState
insertGovAction [] gaPr = [ gaPr ]
insertGovAction ((gaID_0, gaSt_0) :: gaPrs) (gaID_1, gaSt_1)
  = if govActionPriority (action gaSt_0 .gaType) \leq govActionPriority (action gaSt_1 .gaType)
    then (gaID_0, gaSt_0) :: insertGovAction gaPrs (gaID_1, gaSt_1)
    else (gaID_1, gaSt_1) :: (gaID_0, gaSt_0) :: gaPrs
mkGovStatePair : Epoch \rightarrow GovActionID \rightarrow RwdAddr \rightarrow (\alpha : GovAction) \rightarrow NeedsHash (\alpha .gaType)
                   → GovActionID × GovActionState
mkGovStatePair e aid addr a prev = (aid , record
  { votes = \( \phi \); returnAddr = addr; expiresIn = e; action = a; prevAction = prev })
addAction: GovState
           \rightarrow Epoch \rightarrow GovActionID \rightarrow RwdAddr \rightarrow (a: GovAction) \rightarrow NeedsHash (a .gaType)
addAction s e aid addr a prev = insertGovAction s (mkGovStatePair e aid addr a prev)
  addVote : GovState → GovActionID → Voter → Vote → GovState
  addVote s aid voter v = map modifyVotes s
    where modifyVotes : GovActionID × GovActionState → GovActionID × GovActionState
           modifyVotes = \lambda (gid , s') \rightarrow gid , record s'
             { votes = if gid ≡ aid then insert (votes s') voter v else votes s'}
  isRegistered : GovEnv → Voter → Type
  isRegistered \Gamma (r, c) = case r of
CC
             \rightarrow just c \in \text{range (gState .ccHotKeys)}
             \rightarrow c \in dom (gState .dreps)
DRep
SP0
            \rightarrow c \in \text{map}^s \text{ KeyHashObj (dom (pState .pools))}
  where
           open CertState (GovEnv.certState Γ) using (gState; pState)
  validHFAction : GovProposal → GovState → EnactState → Type
  validHFAction (record { action = [ TriggerHF , v ] ga ; prevAction = prev }) s e =
    (let (v', aid) = EnactState.pv e in aid = prev \times pvCanFollow v'v)
    \forall \exists_2 [x, v'] (prev, x) \in \text{fromList } s \times x \text{ .action} \equiv [TriggerHF, v']^{ga} \times pvCanFollow v'v
  validHFAction _ _ = T
```

Figure 36: Functions used in the GOV transition system

```
Transition relation types

_⊢_→(_,GOV)_ : GovEnv × N → GovState → GovVote ⊎ GovProposal → GovState → Type
_⊢_→(_,GOVS)_ : GovEnv → GovState → List (GovVote ⊎ GovProposal) → GovState → Type
```

Figure 37: Type signature of the transition relation of the GOV transition system

```
enactable : EnactState → List (GovActionID × GovActionID)
            → GovActionID × GovActionState → Type
enactable e aidPairs = \lambda (aidNew, as) \rightarrow case getHashES e (action as .gaType) of
  nothing
                  \rightarrow T
  (just\ aidOld) \rightarrow \exists [t]\ fromList\ t \subseteq fromList\ aidPairs
                            \times Unique t \times t connects aidNew to aidOld
allEnactable : EnactState → GovState → Type
allEnactable e aid×states = All (enactable e (getAidPairsList aid×states)) aid×states
hasParentE : EnactState → GovActionID → GovActionType → Type
hasParentE e aid gaTy = case getHashES e gaTy of
  nothing → T
  (just id) \rightarrow id \equiv aid
hasParent : EnactState \rightarrow GovState \rightarrow (gaTy : GovActionType) \rightarrow NeedsHash gaTy \rightarrow Type
hasParent e s gaTy aid = case getHash aid of
nothing
             → T
(just aid') → hasParentE e aid' gaTy
                \forall Any (\lambda (gid, gas) \rightarrow gid \equiv aid' \times Overlap (gas .action .gaType) gaTy) s
```

Figure 38: Enactability predicate

```
actionValid: P Credential → Maybe ScriptHash → Maybe ScriptHash → Epoch → GovAction → Type actionValid rewardCreds p ppolicy epoch [ ChangePParams , _ ] ga = p = ppolicy actionValid rewardCreds p ppolicy epoch [ TreasuryWdrl , x ] ga = p = ppolicy × maps RwdAddr.stake (dom x) crewardCreds actionValid rewardCreds p ppolicy epoch [ UpdateCommittee , (new , rem , q) ] ga = p = nothing × (∀[ e ∈ range new ] epoch < e) × (dom new ∩ rem = e) actionValid rewardCreds p ppolicy epoch _ = p = nothing

actionWellFormed: GovAction → Type actionWellFormed [ ChangePParams , x ] ga = ppdWellFormed x actionWellFormed [ TreasuryWdrl , x ] ga = (∀[ a ∈ dom x ] NetworkIdOf a = NetworkId) × (∃[ v ∈ range x ] ¬ (v = 0)) actionWellFormed _ = T
```

Figure 39: Validity and wellformedness predicates

Fig. 39 defines predicates used in the GOV-Propose case of the GOV rule to ensure that a governance action is valid and well-formed.

- actionValid ensures that the proposed action is valid given the current state of the system:
  - a ChangePParams action is valid if the proposal policy is provided;
  - a TreasuryWdrl action is valid if the proposal policy is provided and the reward stake credential is registered;
  - an UpdateCommittee action is valid if credentials of proposed candidates have not expired, and the action does not propose to both add and remove the same candidate.
- actionWellFormed ensures that the proposed action is well-formed:
  - a ChangePParams action must preserve well-formedness of the protocol parameters;
  - a TreasuryWdrl action is well-formed if the network ID is correct and there is at least one non-zero withdrawal amount in the given RwdAddr  $\rightarrow$  Coin map.

```
data _⊢_→ (_,GOV)_ where
  GOV-Vote:
    • (aid, ast) \in fromLists
    • canVote (Γ .pparams) (action ast) (proj<sub>1</sub> voter)
    • isRegistered Γ voter
    • ¬ expired (Γ .epoch) ast
      (\Gamma, k) \vdash s \rightharpoonup \emptyset \text{ inj}_1 \llbracket \text{ aid , voter , } v \text{ , machr } \rrbracket, GOVD \text{ addVote } s \text{ aid voter } v
  GOV-Propose:
    let pp
                       = \Gamma .pparams
                       =\Gamma .epoch
         enactState = \Gamma .enactState
         rewardCreds = Γ .rewardCreds
                       = record { returnAddr = addr ; action = a ; anchor = achr
                                  ; policy = p; deposit = d; prevAction = prev }
    in
    • actionWellFormed a

    actionValid rewardCreds p (Γ .ppolicy) e a

    • d \equiv pp .govActionDeposit
    • validHFAction prop s enactState

    hasParent enactState s (a .gaType) prev

    • NetworkIdOf addr ≡ NetworkId
    • CredentialOf addr ∈ rewardCreds
      (\Gamma, k) \vdash s \rightharpoonup \emptyset \text{ inj}_2 \text{ prop ,GOVD addAction } s \text{ (pp .govActionLifetime } +^e e)
                                                        (\Gamma . txid, k) addr a prev
```

Figure 40: Rules for the GOV transition system

The GOVS transition system is now given as the reflexitive-transitive closure of the system GOV, described in Fig. 40.

For GOV-Vote, we check that the governance action being voted on exists; that the voter's role is allowed to vote (see canVote in Fig. 56); and that the voter's credential is actually associated with their role (see isRegistered in Fig. 37).

For GOV-Propose, we check the correctness of the deposit along with some and some conditions that ensure the action is well-formed and valid; naturally, these checks depend on the type of action being proposed (see Fig. 39).

### 14 Certificates

This section is part of the Ledger.Conway.Certs module of the formal ledger specification..

```
Derived types

data DepositPurpose : Type where
   CredentialDeposit : Credential → DepositPurpose
   PoolDeposit : KeyHash → DepositPurpose
   DRepDeposit : Credential → DepositPurpose
   GovActionDeposit : GovActionID → DepositPurpose

Deposits = DepositPurpose → Coin
   Rewards = Credential → Coin
   DReps = Credential → Epoch
```

Figure 41: Deposit types

### 14.1 Changes Introduced in Conway Era

### 14.1.1 Delegation

Registered credentials can now delegate to a DRep as well as to a stake pool. This is achieved by giving the delegate certificate two optional fields, corresponding to a DRep and stake pool.

Stake can be delegated for voting and block production simultaneously, since these are two separate features. In fact, preventing this could weaken the security of the chain, since security relies on high participation of honest stake holders.

### 14.1.2 Removal of Pointer Addresses, Genesis Delegations and MIR Certificates

Support for pointer addresses, genesis delegations and MIR certificates is removed (see CIP-1694 and Corduan et al. [5]). In DState, this means that the four fields relating to those features are no longer present, and DelegEnv contains none of the fields it used to in the Shelley era (see Corduan et al. [1, Sec 9.2]).

Note that pointer addresses are still usable, only their staking functionality has been retired. So all funds locked behind pointer addresses are still accessible, they just don't count towards the stake distribution anymore. Genesis delegations and MIR certificates have been superceded by the new governance mechanisms, in particular the TreasuryWdrl governance action in case of the MIR certificates.

```
record PoolParams: Type where
field
owners: P KeyHash
cost: Coin
margin: UnitInterval
pledge: Coin
rewardAccount: Credential
```

Figure 42: Stake pool parameter definitions

```
data DCert: Type where
 delegate
             : Credential → Maybe VDeleg → Maybe KeyHash → Coin → DCert
  dereg
             : Credential → Maybe Coin → DCert
             : KeyHash → PoolParams → DCert
  retirepool: KeyHash → Epoch → DCert
  regdrep
             : Credential → Coin → Anchor → DCert
  deregdrep : Credential → Coin → DCert
           : Credential → Maybe Credential → DCert
  ccreghot
cwitness: DCert → Maybe Credential
cwitness (delegate c = 0) = just c
cwitness (dereg c _)
cwitness (regpool kh _)
                           = just $ KeyHashObj kh
cwitness (retirepool kh _) = just $ KeyHashObj kh
cwitness (regdrep c _ _)
                         = just c
cwitness (deregdrep c _)
                        = just c
cwitness (ccreghot c _)
                           = just c
```

Figure 43: Delegation definitions

### 14.1.3 Explicit Deposits

Registration and deregistration of staking credentials are now required to explicitly state the deposit that is being paid or refunded. This deposit is used for checking correctness of transactions with certificates. Including the deposit aligns better with other design decisions such as having explicit transaction fees and helps make this information visible to light clients and hardware wallets.

While not shown in the figures, the old certificates without explicit deposits will still be supported for some time for backwards compatibility.

#### 14.2 Governance Certificate Rules

The rules for transition systems dealing with individual certificates are defined in Figs. 46 to 48.GOVCERT deals with the new certificates relating to DReps and the constitutional committee.

- GOVCERT-regdrep registers (or re-registers) a DRep. In case of registration, a deposit needs to be paid. Either way, the activity period of the DRep is reset.
- GOVCERT-deregdrep deregisters a DRep.
- GOVCERT-ccreghot registers a "hot" credential for constitutional committee members.<sup>3</sup> We check that the cold key did not previously resign from the committee. We allow this delegation for any cold credential that is either part of EnactState or is is a proposal. This allows a newly elected member of the constitutional committee to immediately delegate their vote to a hot key and use it to vote. Since votes are counted after previous actions have been enacted, this allows constitutional committee members to act without a delay of one epoch.

<sup>&</sup>lt;sup>3</sup>By "hot" and "cold" credentials we mean the following: a cold credential is used to register a hot credential, and then the hot credential is used for voting. The idea is that the access to the cold credential is kept in a secure location, while the hot credential is more conveniently accessed. If the hot credential is compromised, it can be changed using the cold credential.

Fig. 49 assembles the CERTS transition system by bundling the previously defined pieces together into the CERT system, and then taking the reflexive-transitive closure of CERT together with CERTBASE as the base case. CERTBASE does the following:

- check the correctness of withdrawals and ensure that withdrawals only happen from credentials that have delegated their voting power;
- set the rewards of the credentials that withdrew funds to zero;
- and set the activity timer of all DReps that voted to drepActivity epochs in the future.

```
record CertEnv: Type where
 field
   epoch : Epoch
           : PParams
   votes : List GovVote
   wdrls : RwdAddr → Coin
   coldCreds : P Credential
record DState : Type where
 field
   voteDelegs : Credential → VDeleg
   stakeDelegs : Credential → KeyHash
             : Credential → Coin
   rewards
record PState: Type where
 field
   pools : KeyHash → PoolParams
   retiring : KeyHash → Epoch
record GState: Type where
 field
   dreps
           : DReps
   ccHotKeys : Credential → Maybe Credential
record CertState : Type where
 field
   dState: DState
   pState: PState
   gState: GState
record DelegEnv : Type where
 field
   pparams
             : PParams
            : KeyHash → PoolParams
   delegatees : P Credential
GovCertEnv = CertEnv
PoolEnv = PParams
```

Figure 44: Types used for CERTS transition system

```
_⊢_→ (_,DELEG)_ : DelegEnv → DState → DCert → DState → Type
_⊢_→ (_,POOL)_ : PoolEnv → PState → DCert → PState → Type
_⊢_→ (_,GOVCERT)_ : GovCertEnv → GState → DCert → GState → Type
_⊢_→ (_,CERT)_ : CertEnv → CertState → DCert → CertState → Type
_⊢_→ (_,CERTBASE)_ : CertEnv → CertState → Type
_⊢_→ (_,CERTS)_ : CertEnv → CertState → List DCert → CertState → Type
_⊢_→ (_,CERTS)_ : CertEnv → CertState → List DCert → CertState → Type
```

Figure 45: Types for the transition systems relating to certificates

```
DELEG-delegate:
  let \Gamma = [pp, pools, delegatees]
  • (c \notin dom \ rwds \rightarrow d \equiv pp \ .keyDeposit)
  • (c \in dom \ rwds \rightarrow d \equiv 0)
  • mv ∈ map s (just ∘ credVoter DRep) delegatees U
       fromList ( nothing :: just abstainRep :: just noConfidenceRep :: [] )
  • mkh ∈ map<sup>s</sup> just (dom pools) ∪ { nothing }
    \Gamma \vdash [vDelegs, sDelegs, rwds] \rightarrow (delegate c mv mkh d, DELEG)
         [ insertIfJust c mv vDelegs , insertIfJust c mkh sDelegs , rwds \cup { c , \bigcirc } ]
DELEG-dereg :
  • (c , 0) ∈ rwds
    [ pp , pools , delegatees ] ⊢ [ vDelegs , sDelegs , rwds ] → ( dereg c md ,DELEG)
       [ vDelegs | { c } c , sDelegs | { c } c , rwds | { c } c ]
DELEG-reg :
  • c ∉ dom rwds
  • d \equiv pp .keyDeposit \forall d \equiv 0
    pp , pools , delegatees | ⊢
       \llbracket vDelegs , sDelegs , rwds \rrbracket \rightharpoonup \emptyset reg c d ,DELEGD
       vDelegs , sDelegs , rwds ∪¹ { c , 0 } ]
```

Figure 46: Auxiliary DELEG transition system

```
POOL-regpool:

• kh ∉ dom pools

pp ⊢ [ pools , retiring ] → ( regpool kh poolParams ,POOL)

[ { kh , poolParams } ∪¹ pools , retiring ]

POOL-retirepool:

pp ⊢ [ pools , retiring ] → ( retirepool kh e ,POOL) [ pools , { kh , e } ∪¹ retiring ]
```

Figure 47: Auxiliary POOL transition system

```
GOVCERT-regdrep:
let Γ = [e , pp , vs , wdrls , cc ]
in

• (d ≡ pp .drepDeposit × c ∉ dom dReps) ⊎ (d ≡ 0 × c ∈ dom dReps)

Γ ⊢ [ dReps , ccKeys ] → ( regdrep c d an ,GOVCERT)
[ { c , e + pp .drepActivity } ∪¹ dReps , ccKeys ]

GOVCERT-deregdrep:
• c ∈ dom dReps

[e , pp , vs , wdrls , cc ] ⊢ [ dReps , ccKeys ] → ( deregdrep c d ,GOVCERT) [ dReps ] { c } c , ccKeys

GOVCERT-ccreghot:
• (c , nothing) ∉ ccKeys
• c ∈ cc

[e , pp , vs , wdrls , cc ] ⊢ [ dReps , ccKeys ] → ( ccreghot c mc ,GOVCERT) [ dReps , { c , mc } ∪¹ cc
```

Figure 48: Auxiliary GOVCERT transition system

```
CERT transitions
  CERT-deleg:
    • [ pp , PState.pools st^p , dom (GState.dreps st^g) ] \vdash st^d \rightharpoonup \emptyset dCert ,DELEGD st^{d'}
       [e, pp, vs, wdrls, cc] \vdash [st<sup>d</sup>, st<sup>p</sup>, st<sup>g</sup>] \rightarrow (dCert, CERT) [st<sup>d'</sup>, st<sup>p</sup>, st<sup>g</sup>]
  CERT-pool :
    • pp \vdash st^p \rightharpoonup \emptyset dCert , POOLD st^p'
       [e, pp, vs, wdrls, cc] \vdash [st<sup>d</sup>, st<sup>p</sup>, st<sup>g</sup>] \rightharpoonup (dCert, CERT) [st<sup>d</sup>, st<sup>p'</sup>, st<sup>g</sup>]
  CERT-vdel:
    • \Gamma \vdash st^g \rightharpoonup \emptyset dCert ,GOVCERTD st^g'
      \Gamma \vdash [st^d, st^p, st^g] \rightarrow (dCert, CERT) [st^d, st^p, st^g']
CERTBASE transition
  CERT-base:
    let refresh
                               = mapPartial getDRepVote (fromList vs)
         refreshedDReps = mapValueRestricted (const (e + pp .drepActivity)) dReps refresh
                              = map s stake (dom wdrls)
         validVoteDelegs = voteDelegs | ^ ( map* (credVoter DRep) (dom dReps)
                                                  U fromList (noConfidenceRep : abstainRep : []) )
    • filter isKeyHash wdrlCreds ⊆ dom voteDelegs

    map<sup>s</sup> (map<sub>1</sub> stake) (wdrls <sup>s</sup>) ⊆ rewards <sup>s</sup>

       [e, pp, vs, wdrls, cc] \vdash
         [ [ voteDelegs , stakeDelegs , rewards ]
          , [ dReps , ccHotKeys ]
          ] → ( _ ,CERTBASE)
          [ [ validVoteDelegs , stakeDelegs , constMap wdrlCreds 0 ∪¹ rewards ]
          , [ refreshedDReps , ccHotKeys ]
```

Figure 49: CERTS rules

# 15 Ledger

This section is part of the Ledger.Conway.Ledger module of the formal ledger specification., where the entire state transformation of the ledger state caused by a valid transaction can now be given as a combination of the previously defined transition systems.

```
record LEnv: Type where
  field
    slot
                : Slot
    ppolicy
               : Maybe ScriptHash
    pparams : PParams
    enactState : EnactState
    treasury : Coin
record LState: Type where
 field
    utxoSt
              : UTxOState
    govSt
              : GovState
    certState : CertState
txgov : TxBody → List (GovVote ⊎ GovProposal)
txgov txb = map inj<sub>2</sub> txprop ++ map inj<sub>1</sub> txvote
  where open TxBody txb
rmOrphanDRepVotes : CertState → GovState → GovState
rmOrphanDRepVotes cs govSt = L.map (map<sub>2</sub> go) govSt
  where
    ifDRepRegistered : Voter → Type
    ifDRepRegistered (r, c) = r \equiv DRep \rightarrow c \in dom (cs.gState.dreps)
    go : GovActionState → GovActionState
    go gas = record gas { votes = filterKeys ifDRepRegistered (gas .votes) }
allColdCreds : GovState → EnactState → P Credential
allColdCreds govSt es =
  ccCreds (es.cc) \cup concatMap<sup>s</sup> (\lambda (_ , st) \rightarrow proposedCC (st.action)) (fromList govSt)
```

Figure 50: Types and functions for the LEDGER transition system

```
data _⊢_→ (_,LEDGER)_ : LEnv → LState → Tx → LState → Type where

LEDGER-V :
    let txb = tx .body
        rewards = certState .dState .rewards
    in
    • isValid tx ≡ true
    • [ slot , pp , treasury ] ⊢ utxoSt → ( tx ,UTXOW) utxoSt'
    • [ epoch slot , pp , txvote , txwdrls , allColdCreds govSt enactState ] ⊢ certState → ( txcerts ,CE
    • [ txid , epoch slot , pp , ppolicy , enactState , certState' , dom rewards ] ⊢ rmOrphanDRepVotes ce
    [ slot , ppolicy , pp , enactState , treasury ] ⊢ [ utxoSt , govSt , certState ] → ( tx ,LEDGER) [ utxoSt , ppolicy , pp , treasury ] ⊢ utxoSt → ( tx ,UTXOW) utxoSt'
    [ slot , ppolicy , pp , enactState , treasury ] ⊢ [ utxoSt , govSt , certState ] → ( tx ,LEDGER) [ utxoSt , ppolicy , pp , enactState , treasury ] ⊢ [ utxoSt , govSt , certState ] → ( tx ,LEDGER) [ utxoSt , ppolicy , pp , enactState , treasury ] ⊢ [ utxoSt , govSt , certState ] → ( tx ,LEDGER) [ utxoSt , ppolicy , pp , enactState , treasury ] ⊢ [ utxoSt , govSt , certState ] → ( tx ,LEDGER) [ utxoSt , ppolicy , pp , enactState , treasury ] ⊢ [ utxoSt , govSt , certState ] → ( tx ,LEDGER) [ utxoSt , ppolicy , pp , enactState , treasury ] ⊢ [ utxoSt , govSt , certState ] → ( tx ,LEDGER) [ utxoSt , ppolicy , pp , enactState , treasury ] ⊢ [ utxoSt , govSt , certState ] → ( tx ,LEDGER) [ utxoSt , ppolicy , pp , enactState , treasury ] ⊢ [ utxoSt , govSt , certState ] → ( tx , LEDGER) [ utxoSt , ppolicy , pp , enactState , treasury ] ⊢ [ utxoSt , govSt , certState ] → ( tx , LEDGER) [ utxoSt , ppolicy , pp , enactState , treasury ] ⊢ [ utxoSt , govSt , certState ] → ( tx , LEDGER) [ utxoSt , ppolicy , pp , enactState , treasury ] ⊢ [ utxoSt , govSt , certState ] → ( tx , LEDGER) [ utxoSt , ppolicy , pp , enactState , treasury ] ⊢ [ utxoSt , ppolicy , pp , enactState , treasury ] ⊢ [ utxoSt , ppolicy , pp , enactState , treasury ] ⊢ [ utxoSt , ppolicy , pp , enactState , treasury ] ⊢ [ utxoSt , ppolicy , pp , enactState , treasury ] ⊢ [ utxoSt , ppolicy , pp , enactState , treasury ] ⊢ [ utxoSt , ppolicy
```

Figure 51: LEDGER transition system

```
_{-\vdash_{-}} (_,LEDGERS)_ : LEnv → LState → List Tx → LState → Type _{-\vdash_{-}} (_,LEDGERS)_ = ReflexiveTransitiveClosure \{sts = \_\vdash_{-} (_,LEDGER)_\}
```

Figure 52: LEDGERS transition system

### 16 Enactment

This section is part of the Ledger.Conway.Enact module of the formal ledger specification.

Fig. 53 contains some definitions required to define the ENACT transition system. EnactEnv is the environment and EnactState the state of ENACT, which enacts a governance action. All governance actions except TreasuryWdrl and Info modify EnactState permanently, which of course can have further consequences. TreasuryWdrl accumulates withdrawal temporarily in the withdrawals field of EnactState, but this information is applied and reset in EPOCH (see Fig. 78). Also, enacting these governance actions is the *only* way of modifying EnactState.

Note that all other fields of EnactState also contain a GovActionID since they are HashProtected.

```
record EnactEnv: Type where
     gid
               : GovActionID
     treasury: Coin
             : Epoch
     epoch
 record EnactState: Type where
   field
                    : HashProtected (Maybe ((Credential → Epoch) × ℚ))
     constitution : HashProtected (DocHash x Maybe ScriptHash)
                   : HashProtected ProtVer
                   : HashProtected PParams
     pparams
     withdrawals : RwdAddr → Coin
 ccCreds: HashProtected (Maybe ((Credential → Epoch) × ℚ)) → P Credential
 ccCreds (just x , _) = dom (x \cdot proj_1)
 ccCreds (nothing , _) = ∅
 getHash : \forall \{\alpha\} \rightarrow \text{NeedsHash } \alpha \rightarrow \text{Maybe GovActionID}
 getHash {NoConfidence}
                             h = just h
 getHash {UpdateCommittee} h = just h
 getHash {NewConstitution} h = just h
 getHash {TriggerHF}
                             h = just h
 getHash {ChangePParams}
                             h = just h
                             _ = nothing
 getHash {TreasuryWdrl}
 getHash {Info}
                             _ = nothing
 getHashES : EnactState → GovActionType → Maybe GovActionID
 getHashES es NoConfidence
                                 = just (es .cc .proj<sub>2</sub>)
 getHashES es (UpdateCommittee) = just (es .cc .proj2)
 getHashES es (NewConstitution) = just (es .constitution .proj2)
 getHashES es (TriggerHF)
                                 = just (es .pv .proj<sub>2</sub>)
 getHashES es (ChangePParams) = just (es .pparams .proj2)
 getHashES es (TreasuryWdrl)
                                  = nothing
 getHashES es Info
                                  = nothing
Type of the ENACT transition system
   _⊢_→(_,ENACT)_: EnactEnv → EnactState → GovAction → EnactState → Type
```

Figure 53: Types and function used for the ENACT transition system

Figs. 54 and 55 define the rules of the ENACT transition system. Usually no preconditions are checked and the state is simply updated (including the GovActionID for the hash protection scheme, if required). The exceptions are UpdateCommittee and TreasuryWdrl:

- UpdateCommittee requires that maximum terms are respected, and
- TreasuryWdrl requires that the treasury is able to cover the sum of all withdrawals (old and new).

Figure 54: ENACT transition system

```
Enact-HF:

[gid , t , e ] ⊢ s → ( TriggerHF v ), ENACTD record s { pv = v , gid }

Enact-PParams:

[gid , t , e ] ⊢ s → ( ChangePParams up ), ENACTD record s { pv = v , gid }

record s { pparams = applyUpdate (PParamsOf s) up , gid }

Enact-Wdrl: let newWdrls = s .withdrawals U* wdrl in ∑[x ← newWdrls]x ≤ t

[gid , t , e ] ⊢ s → ( TreasuryWdrl wdrl ), ENACTD record s { withdrawals = newWdrls }

Enact-Info:

[gid , t , e ] ⊢ s → ( Info ), ENACTD s
```

Figure 55: ENACT transition system (continued)

### 17 Ratification

This section is part of the Ledger.Conway.Ratify module of the formal ledger specification.

Governance actions are *ratified* through on-chain votes. Different kinds of governance actions have different ratification requirements but always involve at least two of the three governance bodies.

A successful motion of no-confidence, election of a new constitutional committee, a constitutional change, or a hard-fork delays ratification of all other governance actions until the first epoch after their enactment. This gives a new constitutional committee enough time to vote on current proposals, re-evaluate existing proposals with respect to a new constitution, and ensures that the (in principle arbitrary) semantic changes caused by enacting a hard-fork do not have unintended consequences in combination with other actions.

### 17.1 Ratification Requirements

Fig. 56 details the ratification requirements for each governance action scenario. For a governance action to be ratified, all of these requirements must be satisfied, on top of other conditions that are explained further down. The threshold function is defined as a table, with a row for each type of GovAction and the column representing the CC, DRep and SPO roles in that order.

The symbols mean the following:

- vote x: For an action to pass, the fraction of stake associated with yes votes with respect to that associated with yes and no votes must exceed the threshold x.
- -: The body of governance does not participate in voting.
- $\checkmark$ : The constitutional committee needs to approve an action, with the threshold assigned to it.
- $\checkmark$ t: Voting is possible, but the action will never be enacted. This is equivalent to vote 2 (or any other number above 1).

Two rows in this table contain functions that compute the DRep and SPO thresholds simultaneously: the rows for UpdateCommittee and ChangePParams.

For UpdateCommittee, there can be different thresholds depending on whether the system is in a state of no-confidence or not. This information is provided via the ccThreshold argument: if the system is in a state of no-confidence, then ccThreshold is set to nothing.

In case of the ChangePParams action, the thresholds further depend on what groups that action is associated with. pparamThreshold associates a pair of thresholds to each individual group. Since an individual update can contain multiple groups, the actual thresholds are then given by taking the maximum of all those thresholds.

Note that each protocol parameter belongs to exactly one of the four groups that have a DRep threshold, so a DRep vote will always be required. A protocol parameter may or may not be in the SecurityGroup, so an SPO vote may not be required.

Finally, each of the  $P_x$  and  $Q_x$  in Fig. 56 are protocol parameters.

# 17.2 Protocol Parameters and Governance Actions

Voting thresholds for protocol parameters can be set by group, and we do not require that each protocol parameter governance action be confined to a single group. In case a governance action carries updates for multiple parameters from different groups, the maximum threshold of all the groups involved will apply to any given such governance action.

The purpose of the SecurityGroup is to add an additional check to security-relevant protocol parameters. Any proposal that includes a change to a security-relevant protocol parameter must also be accepted by at least half of the SPO stake.

```
threshold: PParams → Maybe Q → GovAction → GovRole → Maybe Q
threshold pp ccThreshold ga =
  case ga ↓ of
        (NoConfidence
                           , -) \rightarrow | - | vote P1 | vote Q1 |
        (UpdateCommittee , _) \rightarrow | - || P/Q2a/b
        (NewConstitution, \_) \rightarrow | \checkmark | vote P3 | -
                           , _{-}) \rightarrow | \checkmark | vote P4 | vote Q4 |
        (TriggerHF
                           , x) \rightarrow | \checkmark | P/Q5 x
        (ChangePParams
                           , _-) \rightarrow | \checkmark | vote P6 | -
        (TreasuryWdrl
        (Info
                           , _) → | √† | √†
         where
         P/Q2a/b: Maybe Q × Maybe Q
         P/Q2a/b = case ccThreshold of
                     (just _) → (vote P2a, vote Q2a)
                     nothing → (vote P2b, vote Q2b)
          pparamThreshold: PParamGroup → Maybe ℚ × Maybe ℚ
          pparamThreshold NetworkGroup
                                              = (vote P5a , -
          pparamThreshold EconomicGroup = (vote P5b , -
          pparamThreshold TechnicalGroup = (vote P5c , -
          pparamThreshold GovernanceGroup = (vote P5d , -
         pparamThreshold SecurityGroup = (-
                                                            , vote Q5 )
          P/Q5 : PParamsUpdate → Maybe @ × Maybe @
          P/Q5 ppu = maxThreshold (map<sup>s</sup> (proj₁ ∘ pparamThreshold) (updateGroups ppu))
                    , maxThreshold (map<sup>s</sup> (proj₂ ∘ pparamThreshold) (updateGroups ppu))
canVote : PParams → GovAction → GovRole → Type
canVote pp \ a \ r = Is-just (threshold pp \ nothing a \ r)
```

Figure 56: Functions related to voting

#### 17.3 Ratification Restrictions

As mentioned earlier, most governance actions must include a GovActionID for the most recently enacted action of its given type. Consequently, two actions of the same type can be enacted at the same time, but they must be *deliberately* designed to do so.

Fig. 57 defines some types and functions used in the RATIFY transition system. CCData is simply an alias to define some functions more easily.

Fig. 58 defines the actualVotes function. Given the current state about votes and other parts of the system it calculates a new mapping of votes, which is the mapping that will actually be used during ratification. Things such as default votes or resignation/expiry are implemented in this way.

actualVotes is defined as the union of four voting maps, corresponding to the constitutional committee, predefined (or auto) DReps, regular DReps and SPOs.

- roleVotes filters the votes based on the given governance role and is a helper for definitions further down.
- if a CC member has not yet registered a hot key, has expired, or has resigned, then actualCCVote returns abstain; if none of these conditions is met, then
  - if the CC member has voted, then that vote is returned;

```
record StakeDistrs : Type where
 field
    stakeDistr : VDeleg → Coin
record RatifyEnv : Type where
 field
    stakeDistrs : StakeDistrs
   currentEpoch : Epoch
                : Credential → Epoch
   dreps
   ccHotKeys : Credential → Maybe Credential
    treasury
                 : Coin
                 : KeyHash → PoolParams
    pools
    delegatees : Credential → VDeleg
record RatifyState : Type where
  field
            : EnactState
   removed : P (GovActionID × GovActionState)
   delay : Bool
CCData: Type
CCData = Maybe ((Credential → Epoch) × ℚ)
govRole : VDeleg → GovRole
govRole (credVoter gv _) = gv
govRole abstainRep
govRole noConfidenceRep = DRep
IsCC IsDRep IsSPO : VDeleg → Type
       v = govRole v \equiv CC
IsDRep v = govRole v \equiv DRep
IsSPO v = govRole v \equiv SPO
```

Figure 57: Types and functions for the RATIFY transition system

- if the CC member has not voted, then the default value of no is returned.
- actual DRepVotes adds a default vote of no to all active DReps that didn't vote.
- actual SPOV otes adds a default vote to all SPOs who didn't vote, with the default depending on the action.

Let us discuss the last item above—the way SPO votes are counted—as the ledger specification's handling of this has evolved in response to community feedback. Previously, if an SPO did not vote, then it would be counted as having voted abstain by default. Members of the SPO community found this behavior counterintuitive and requested that non-voters be assigned a no vote by default, with the caveat that an SPO could change its default setting by delegating its reward account credential to an AlwaysNoConfidence DRep or an AlwaysAbstain DRep. (This change applies only after the bootstrap period; during the bootstrap period the logic is unchanged; see Sec. D.) To be precise, the agreed upon specification is the following: an SPO that did not vote is assumed to have vote no, except under the following circumstances:

• if the SPO has delegated its reward credential to an AlwaysNoConfidence DRep, then their

default vote is yes for NoConfidence proposals and no for other proposals;

• if the SPO has delegated its reward credential to an AlwaysAbstain DRep, then its default vote is abstain for all proposals.

It is important to note that the credential that can now be used to set a default voting behavior is the credential used to withdraw staking rewards, which is not (in general) the same as the credential used for voting.

Fig. 59 defines the accepted and expired functions (together with some helpers) that are used in the rules of RATIFY.

- getStakeDist computes the stake distribution based on the given governance role and the corresponding delegations. Note that every constitutional committe member has a stake of 1, giving them equal voting power. However, just as with other delegation, multiple CC members can delegate to the same hot key, giving that hot key the power of those multiple votes with a single actual vote.
- acceptedStakeRatio is the ratio of accepted stake. It is computed as the ratio of yes votes over the votes that didn't abstain. The latter is equivalent to the sum of yes and no votes. The special division symbol /o indicates that in case of a division by 0, the numbers 0 should be returned. This implies that in the absence of stake, an action can only pass if the threshold is also set to 0.
- acceptedBy looks up the threshold in the threshold table and compares it to the result of acceptedStakeRatio.
- accepted then checks if an action is accepted by all roles; and
- expired checks whether a governance action is expired in a given epoch.

Fig. 60 defines functions that deal with delays and the acceptance criterion for ratification. A given action can either be delayed if the action contained in EnactState isn't the one the given action is building on top of, which is checked by verifyPrev, or if a previous action was a delayingAction. Note that delayingAction affects the future: whenever a delayingAction is accepted all future actions are delayed. delayed then expresses the condition whether an action is delayed. This happens either because the previous action doesn't match the current one, or because the previous action was a delaying one. This information is passed in as an argument.

The RATIFIES transition system is defined as the reflexive-transitive closure of RATIFY, which is defined via three rules, defined in Fig. 62.

- RATIFY-Accept checks if the votes for a given GovAction meet the threshold required for acceptance, that the action is accepted and not delayed, and RATIFY-Accept ratifies the action.
- RATIFY-Reject asserts that the given GovAction is not accepted and expired; it removes the governance action.
- RATIFY-Continue covers the remaining cases and keeps the GovAction around for further voting.

Note that all governance actions eventually either get accepted and enacted via RATIFY-Accept or rejected via RATIFY-Reject. If an action satisfies all criteria to be accepted but cannot be enacted anyway, it is kept around and tried again at the next epoch boundary.

We never remove actions that do not attract sufficient yes votes before they expire, even if it is clear to an outside observer that this action will never be enacted. Such an action will simply keep getting checked every epoch until it expires.

```
actualVotes : RatifyEnv → PParams → CCData → GovActionType
             → (GovRole × Credential → Vote) → (VDeleg → Vote)
actualVotes Γ pparams cc gaTy votes
  = mapKeys (credVoter CC) actualCCVotes U<sup>1</sup> actualPDRepVotes gαTy
  U<sup>l</sup> actualDRepVotes
                                             U<sup>l</sup> actualSPOVotes gaTy
  where
  roleVotes : GovRole → VDeleg → Vote
  roleVotes r = \text{mapKeys} (uncurry credVoter) (filter (\lambda (x, \bot) \rightarrow r \equiv \text{proj}_1 x) votes)
  activeDReps = dom (filter (\lambda (\_, e) \rightarrow currentEpoch \le e) dreps)
  spos = filter* IsSPO (dom (stakeDistr stakeDistrs))
  getCCHotCred : Credential × Epoch → Maybe Credential
  getCCHotCred (c , e) = if currentEpoch > e then nothing
    else case lookup^{m}? ccHotKeys c of
       (just (just c')) \rightarrow just c'
                         → nothing -- no hot key or resigned
  SPODefaultVote : GovActionType → VDeleg → Vote
  SPODefaultVote gaT (credVoter SPO (KeyHashObj kh)) = case lookup<sup>m</sup>? pools kh of
       nothing → Vote.no
       (just p) \rightarrow case lookup^m? delegatees (PoolParams.rewardAccount p), gaTy of
                                      , TriggerHF)
                                                     → Vote.no
              (_
              (just noConfidenceRep , NoConfidence) → Vote.yes
              (just abstainRep , _ )
                                               → Vote.abstain
                                                      → Vote.no
  SPODefaultVote _ _ = Vote.no
  actualCCVote : Credential → Epoch → Vote
  actualCCVote c e = case getCCHotCred (c , e) of
       (just c') → maybe id Vote.no (lookup<sup>m</sup>? votes (CC , c'))
                    → Vote.abstain
  actualCCVotes : Credential → Vote
  actualCCVotes = case cc of
       (just (m, q)) \rightarrow if ccMinSize \leq length ^{s} (mapFromPartialFun getCCHotCred (m^{s}))
                         then mapWithKey actualCCVote m
                         else constMap (dom m) Vote.no
  actualPDRepVotes : GovActionType → VDeleg → Vote
  actualPDRepVotes NoConfidence
                       = { abstainRep , Vote.abstain } U<sup>1</sup> { noConfidenceRep , Vote.yes }
  actualPDRepVotes _ = { abstainRep , Vote.abstain } Ula { noConfidenceRep , Vote.no }
  actualDRepVotes : VDeleg → Vote
  actualDRepVotes = roleVotes DRep
                    U<sup>1</sup> constMap (map<sup>s</sup> (credVoter DRep) activeDReps) Vote.no
  actualSPOVotes : GovActionType → VDeleg → Vote
  actual SPOV otes gaTy = role Votes SPO U^1 map From Fun (SPODe fault Vote <math>gaTy) spos
```

Figure 58: Vote counting

```
getStakeDist : GovRole → P VDeleg → StakeDistrs → VDeleg → Coin
getStakeDist CC     cc sd = constMap (filter* IsCC cc) 1
getStakeDist DRep _ sd = filterKeys IsDRep (sd .stakeDistr)
getStakeDist SPO _ sd = filterKeys IsSPO (sd .stakeDistr)
acceptedStakeRatio : GovRole → P VDeleg → StakeDistrs → (VDeleg → Vote) → ℚ
acceptedStakeRatio r cc dists votes = acceptedStake /0 totalStake
    dist : VDeleg → Coin
    dist = getStakeDist r cc dists
    acceptedStake totalStake : Coin
    acceptedStake = \sum [x \in dist \mid votes^{-1} \ Vote.yes] x
    totalStake = \sum [x \in dist \mid dom (votes \mid^{(\{Vote.yes\})} \cup \{Vote.no\}))]x
acceptedBy : RatifyEnv → EnactState → GovActionState → GovRole → Type
acceptedBy \Gamma (record { cc = cc , _; pparams = pparams , _ }) gs role =
  let open GovActionState gs; open PParams pparams
      votes' = actualVotes Γ pparams cc (gaType action) votes
      mbyT = threshold pparams (proj<sub>2</sub> <$> cc) action role
              = maybe id 00 mbyT
  in acceptedStakeRatio role (dom votes') (stakeDistrs Γ) votes' ≥ t
    \land (role \equiv CC \rightarrow maybe (\land (m, _) \rightarrow length<sup>s</sup> m) 0 cc \ge ccMinSize <math>\forall Is-nothing mbyT)
accepted : RatifyEnv → EnactState → GovActionState → Type
accepted \Gamma es gs = accepted By \Gamma es gs CC \Lambda accepted By \Gamma es gs DRep \Lambda accepted By \Gamma es gs SPO
expired : Epoch → GovActionState → Type
expired current record { expiresIn = expiresIn } = expiresIn < current</pre>
```

Figure 59: Functions related to ratification

```
verifyPrev : (a : GovActionType) \rightarrow NeedsHash a \rightarrow EnactState \rightarrow Type
verifyPrev NoConfidence
                              h = s = h = es .cc .proj_2
verifyPrev UpdateCommittee h es = h ≡ es .cc .proj₂
verifyPrev NewConstitution h es = h \equiv es .constitution .proj<sub>2</sub>
verifyPrev TriggerHF
                             h = h = es .pv .proj_2
verifyPrev ChangePParams h = es.pparams.proj_2
verifyPrev TreasuryWdrl _ _ = T
verifyPrev Info
                              _{-} _{-} _{-} _{-} _{T}
delayingAction : GovActionType → Bool
delayingAction NoConfidence
delayingAction UpdateCommittee = true
delayingAction NewConstitution = true
delayingAction TriggerHF
delayingAction ChangePParams = false
delayingAction TreasuryWdrl
                                   = false
delayingAction Info
                                   = false
delayed : (\alpha : GovActionType) \rightarrow NeedsHash \alpha \rightarrow EnactState \rightarrow Bool \rightarrow Type
delayed gaTy h es d = \neg verifyPrev gaTy h es \forall d \equiv true
acceptConds: RatifyEnv → RatifyState → GovActionID × GovActionState → Type
acceptConds \Gamma st^{\tau} (id, st) =
    accepted \Gamma es st
  x ¬ delayed (gaType action) prevAction es delay
  x ∃[ es' ] [ id , treasury , currentEpoch ] ⊢ es → ( action ,ENACT) es'
```

Figure 60: Functions related to ratification, continued

```
_⊢_→ ①_,RATIFYD_: RatifyEnv → RatifyState → GovActionID × GovActionState → RatifyState → Type
_⊢_→ ①_,RATIFIESD_: RatifyEnv → RatifyState → List (GovActionID × GovActionState)
→ RatifyState → Type
```

Figure 61: Types of the RATIFY and RATIFIES transition systems

```
RATIFY-Accept:
  let treasury
                  = \Gamma .treasury
                    = Γ .currentEpoch
      (gaId, gaSt) = a
      action
              = gaSt .action
  • acceptConds \Gamma [ es , removed , d ] \alpha
  • [ gaId , treasury , e ] ⊢ es → ( action ,ENACT) es'
   \Gamma \vdash [es, removed, d] \rightarrow (a, RATIFY)
       [ es', { a } ∪ removed , delayingAction (gaType action) ]
RATIFY-Reject:
  let e
                    = Γ .currentEpoch
      (gaId, gaSt) = a
  • ¬ acceptConds Γ [ es , removed , d ] a
  • expired e gaSt
   \Gamma \vdash [es, removed, d] \rightarrow (a, RATIFY) [es, {a} \cup removed, d]
RATIFY-Continue:
 let e
                    = Γ .currentEpoch
      (gaId, gaSt) = a
  • ¬ acceptConds Γ [ es , removed , d ] α
  • ¬ expired e gaSt
   \Gamma \vdash [es, removed, d] \rightarrow (a, RATIFY) [es, removed, d]
_⊢_→(_,RATIFIES)_ = ReflexiveTransitiveClosure {sts = _⊢_→(_,RATIFY)_}
```

Figure 62: The RATIFY transition system

### 18 Rewards

This section is part of the Ledger.Conway.Rewards module of the formal ledger specification.

This section defines how rewards for stake pools and their delegators are calculated and paid out. This calculation has two main aspects:

- The amount of rewards to be paid out. This is defined in Sec. 18.2.
- The *time* when rewards are paid out. This is defined in Sec. 18.3.

#### 18.1 Rewards Motivation

In order to operate, any blockchain needs to attract parties that are willing to spend computational and network resources on processing transactions and producing new blocks. These parties, called *block producers*, are incentivized by monetary *rewards*.

Cardano is a proof-of-stake (PoS) blockchain: through a random lottery, one block producer is selected to produce one particular block. The probability for being select depends on their *stake* of Ada, that is the amount of Ada that they (and their delegators) own relative to the total amount of Ada. (We will explain delegation below.) After successful block production, the block producer is eligible for a share of the rewards.

The rewards for block producers come from two sources: during an initial period, rewards are paid out from the *reserve*, which is an initial allocation of Ada created for this very purpose. Over time, the reserve is depleted, and rewards are sourced from transaction fees.

Rewards are paid out epoch by epoch.

Rewards are collective, but depend on performance: after every epoch, a fraction of the available reserve and the transaction fees accumulated during that epoch are added together. This sum is paid out to the block producers proportionally to how many blocks they have created each. In order to avoid perverse incentives, block producers do not receive individual rewards that depend on the content of their blocks.

Not all people can or want to set up and administer a dedicated computer that produces blocks. However, these people still own Ada, and their stake is relevant for block production. Specifically, these people have the option to delegate their stake to a stake pool, which belongs to a block producer. This stake counts towards the stake of the pool in the block production lottery. In turn, the protocol distributes the rewards for produced blocks to the stake pool owner and their delegators. The owner receives a fixed fee ("cost") and a share of the rewards ("margin"). The remainder is distributed among delegators in proportion to their stake. By design, delegation and ownership are separate — delegation counts towards the stake of the pool, but delegators remain in full control of their Ada, stake pools cannot spend delegated Ada.

Stake pools compete for delegators based on fees and performance. In order to achieve stable blockchain operation, the rewards are chosen such that they incentivize the system to evolve into a large, but fixed number of stake pools that attract most of the stake. For more details about the design and rationale of the rewards and delegation system, see IOHK Formal Methods Team [9].

### 18.2 Amount of Rewards to be Paid Out

#### 18.2.1 Precision of Arithmetic Operations

When computing rewards, all intermediate results are computed using rational numbers,  $\mathbb{Q}$ , and converted to Coin using the floor function at the very end of the computation.

Note for implementors: Values in **Q** can have arbitrarily large nominators and denominators. Please use an appropriate type that represents rational numbers as fractions of unbounded nominators and denominators. Types such as Double, Float, BigDecimal (Java Platform), or

Fixed (fixed-precision arithmetic) do *not* faithfully represent the rational numbers, and are *not* suitable for computing rewards according to this specification!

We use the following arithmetic operations besides basic arithmetic:

- fromN: Interpret a natural number as a rational number.
- floor: Round a rational number to the next smaller integer.
- posPart: Convert an integer to a natural number by mapping all negative numbers to zero.
- :: Division of rational numbers.
- ÷<sub>0</sub>: Division operator that returns zero when the denominator is zero.
- /: Division operator that maps integer arguments to a rational number.
- $/_0$ : Like  $\div_0$ , but with integer arguments.

### 18.2.2 Rewards Distribution Calculation

This section defines the amount of rewards that are paid out to stake pools and their delegators. Fig. 63 defines the function maxPool which gives the maximum reward a stake pool can receive in an epoch. Relevant quantities are:

- rewardPot: Total rewards to be paid out after the epoch.
- stake: Relative stake of the pool.
- pledge: Relative stake that the pool owner has pledged themselves to the pool.
- z0: Relative stake of a fully saturated pool.
- nopt: Protocol parameter, planned number of block producers.
- a0: Protocol parameter that incentivizes higher pledges.
- reward0: Pool rewards as a rational number.
- rewardN: Pool rewards after rounding to a natural number of lovelace.

```
maxPool : PParams → Coin → UnitInterval → UnitInterval → Coin
maxPool pparams rewardPot stake pledge = rewardN
where
    a0 = @.max 0 (pparams .PParams.a0)
    1+a0 = 1 + a0
    nopt = N.max 1 (pparams .PParams.nopt)
    z0 = 1 / nopt
    stake' = @.min (fromUnitInterval stake) z0
    pledge' = @.min (fromUnitInterval pledge) z0
    reward@ =
        ((fromN rewardPot) ÷ 1+a0)
        * (stake' + pledge' * a0 * (stake' - pledge' * (z0 - stake') ÷ z0) ÷ z0)
    rewardN = posPart (floor reward@)
```

Figure 63: Function maxPool used for computing a Reward Update

Fig. 64 defines the function mkApparentPerformance which computes the apparent performance of a stake pool. Relevant quantities are:

• stake: Relative active stake of the pool.

- poolBlocks: Number of blocks that the pool added to the chain in the last epoch.
- totalBlocks: Total number of blocks added in the last epoch.

```
mkApparentPerformance : UnitInterval \rightarrow \mathbb{N} \rightarrow \mathbb{N} \rightarrow \mathbb{Q}
mkApparentPerformance stake poolBlocks totalBlocks = ratioBlocks \div_0 stake'
where
stake' = fromUnitInterval stake
ratioBlocks = (\mathbb{Z}.+ poolBlocks) / (\mathbb{N}.max 1 totalBlocks)
```

Figure 64: Function mkApparentPerformance used for computing a Reward Update

Fig. 65 defines the functions rewardOwners and rewardMember. Their purpose is to divide the reward for one pool between pool owners and individual delegators by taking into account a fixed pool cost, a relative pool margin, and the stake of each member. The rewards will be distributed as follows:

- rewardOwners: These funds will go to the rewardAccount specified in the pool registration certificate.
- rewardMember: These funds will go to the reward accounts of the individual delegators.

Relevant quantities for the functions are:

- rewards: Rewards paid out to this pool.
- pool: Pool parameters, such as cost and margin.
- ownerStake: Stake of the pool owners relative to the total amount of Ada.
- memberStake: Stake of the pool member relative to the total amount of Ada.
- stake: Stake of the whole pool relative to the total amount of Ada.

Fig. 66 defines the function rewardOnePool which calculates the rewards given out to each member of a given pool. Relevant quantities are:

- rewardPot: Total rewards to be paid out for this epoch.
- n: Number of blocks produced by the pool in the last epoch.
- N: Expectation value of the number of blocks to be produced by the pool.
- stakeDistr: Distribution of stake, as mapping from Credential to Coin.
- $\sigma$ : Total relative stake controlled by the pool.
- $\sigma a$ : Total active relative stake controlled by the pool, used for selecting block producers.
- tot: Total amount of Ada in circulation, for computing the relative stake.
- mkRelativeStake: Compute stake relative to the total amount in circulation.
- ownerStake: Total amount of stake controlled by the stake pool operator and owners.
- maxP: Maximum rewards the pool can claim if the pledge is met, and zero otherwise.
- poolReward: Actual rewards to be paid out to this pool.

Fig. 67 defines the function poolStake which filters the stake distribution to one stake pool. Relevant quantities are:

```
rewardOwners : Coin → PoolParams → UnitInterval → UnitInterval → Coin
rewardOwners rewards pool ownerStake stake = if rewards ≤ cost
  then rewards
  else cost + posPart (floor (
         (fromN rewards - fromN cost) * (margin + (1 - margin) * ratioStake)))
  where
   ratioStake = fromUnitInterval ownerStake ÷0 fromUnitInterval stake
   cost
               = pool .PoolParams.cost
               = fromUnitInterval (pool .PoolParams.margin)
   margin
rewardMember : Coin → PoolParams → UnitInterval → UnitInterval → Coin
rewardMember rewards pool memberStake stake = if rewards ≤ cost
  then 0
  else posPart (floor (
         (fromN rewards - fromN cost) * ((1 - margin) * ratioStake)))
  where
    ratioStake = fromUnitInterval memberStake ÷0 fromUnitInterval stake
               = pool .PoolParams.cost
   cost
               = fromUnitInterval (pool .PoolParams.margin)
   margin
```

Figure 65: Functions rewardOwners and rewardMember

- hk: KeyHash of the stake pool to be filtered by.
- delegs: Mapping from Credentials to stake pool that they delegate to.
- stake: Distribution of stake for all Credentials.

Fig. 68 defines the function reward which applies rewardOnePool to each registered stake pool. Relevant quantities are:

- uncurry<sup>m</sup>: Helper function to rearrange a nested mapping.
- blocks: Number of blocks produced by pools in the last epoch, as a mapping from pool KeyHash to number.
- poolParams: Parameters of all known stake pools.
- stake: Distribution of stake, as mapping from Credential to Coin.
- delegs: Mapping from Credentials to stake pool that they delegate to.
- total: Total stake = amount of Ada in circulation, for computing the relative stake.
- active: Active stake = amount of Ada that was used for selecting block producers.
- $\Sigma_{-}$ /total: Sum of stake divided by total stake.
- $\Sigma$ \_/active: Sum of stake divided by active stake.
- N: Total number of blocks produced in the last epoch.
- pdata: Data needed to compute rewards for each pool.

```
Stake = Credential → Coin
rewardOnePool : PParams → Coin → N → N → PoolParams
  → Stake → UnitInterval → UnitInterval → Coin → (Credential → Coin)
rewardOnePool pparams rewardPot n N pool stakeDistr \sigma \sigmaa tot = rewards
  where
    mkRelativeStake = \lambda coin \rightarrow clamp (coin / 0 tot)
    owners = map* KeyHashObj (pool .PoolParams.owners)
    ownerStake = \sum [c \leftarrow stakeDistr \mid owners] c
    pledge = pool .PoolParams.pledge
    maxP = if pledge ≤ ownerStake
      then maxPool pparams rewardPot \sigma (mkRelativeStake pledge)
      else 0
    apparentPerformance = mkApparentPerformance \sigma \alpha n N
    poolReward = posPart (floor (apparentPerformance * fromN maxP))
    memberRewards =
      map Values (\lambda coin \rightarrow reward Member pool Reward pool (mkRelative Stake coin) \sigma)
         (stakeDistr | owners °)
    ownersRewards =
      { pool .PoolParams.rewardAccount
       , rewardOwners poolReward pool (mkRelativeStake ownerStake) σ }<sup>m</sup>
    rewards = memberRewards U* ownersRewards
```

Figure 66: Function rewardOnePool used for computing a Reward Update

```
Delegations = Credential → KeyHash

poolStake : KeyHash → Delegations → Stake → Stake

poolStake hk delegs stake = stake | dom (delegs | ^ { hk })
```

Figure 67: Function poolStake

### 18.2.3 Reward Update

This section defines the RewardUpdate type, which records the net flow of Ada due to paying out rewards after an epoch. This type is defined in Fig. 69. The update consists of four net flows:

- At: The change to the treasury. This will be a positive value.
- Ar: The change to the reserves. We typically expect this to be a negative value.
- $\Delta f$ : The change to the fee pot. This will be a negative value.
- rs: The map of new individual rewards, to be added to the existing rewards.

The function createRUpd calculates the RewardUpdate, but requires the definition of the type EpochState, so we have to defer the definition of this function to Sec. 19.

Fig. 70 captures the potential movement of funds in the entire system that can happen during one transition step as described in this document. Exception: Withdrawals from the "Treasury" are not shown in this diagram, they can move funds into "Reward accounts". Value is moved between accounting pots, but the total amount of value in the system remains constant. In particular, the red subgraph represents the inputs and outputs to the rewardPot, a temporary

```
BlocksMade = KeyHash → N
uncurry<sup>m</sup>:
  A \longrightarrow (B \longrightarrow C) \rightarrow (A \times B) \longrightarrow C
reward : PParams → BlocksMade → Coin → (KeyHash → PoolParams)
  → Stake → Delegations → Coin → (Credential → Coin)
reward pp blocks rewardPot poolParams stake delegs total = rewards
  where
     active = \sum [c \leftarrow stake] c
     \Sigma_{-}/\text{total} = \lambda \ st \rightarrow \text{clamp} \ ((\Sigma[c \leftarrow st]c) /_0 \ total)
     \Sigma_/active = \lambda st \rightarrow clamp ((\Sigma[ c \leftarrow st ] c) /0 active)
     N = \sum [m \leftarrow blocks] m
     mkPoolData = \lambda hk p \rightarrow
        map (\lambda n \rightarrow (n, p, poolStake hk delegs stake)) (lookup<sup>m</sup>? blocks hk)
     pdata = mapMaybeWithKey<sup>m</sup> mkPoolData poolParams
     results : (KeyHash × Credential) → Coin
     results = uncurry<sup>m</sup> (mapValues (\lambda (n, p, s)
        \rightarrow rewardOnePool pp rewardPot n N p s (\Sigma s /total) (\Sigma s /active) total)
        pdata)
     rewards = aggregateBy
        (map^s (\lambda (kh, cred) \rightarrow (kh, cred), cred) (dom results))
        results
```

Figure 68: Function reward used for computing a Reward Update

```
record RewardUpdate : Set where

field

Δt Δr Δf : ℤ

rs : Credential → Coin
```

Figure 69: RewardUpdate type

variable used during the reward update calculation in the function <code>createRUpd</code>. Each red arrow corresponds to one field of the <code>RewardUpdate</code> data type. The blue arrows represent the movement of funds after they have passed through the <code>rewardPot</code>.

#### 18.2.4 Stake Distribution Calculation

This section defines the calculation of the stake distribution for the purpose of calculating rewards.

Fig. 71 defines the type Snapshot that represents a stake distribution snapshot. Such a snapshot contains the essential data needed to compute rewards:

- stake A stake distribution, that is a mapping from credentials to coin.
- delegations: A delegation map, that is a mapping from credentials to the stake pools that they delegate to.
- poolParameters: A mapping that stores the pool parameters of each stake pool.

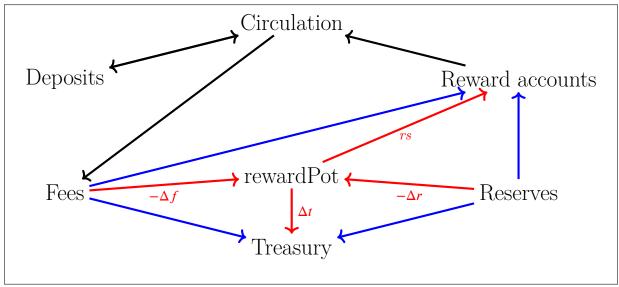


Figure 70: Preservation of funds and rewards

```
record Snapshot : Set where

field

stake : Credential → Coin

delegations : Credential → KeyHash

poolParameters : KeyHash → PoolParams
```

Figure 71: Definitions of the Snapshot type

Fig. 72 defines the calculation of the stake distribution from the data contained in a ledger state. Here,

- aggregate + takes a relation  $R \subset A \times V$ , where V is any monoid with operation +, and returns a mapping  $A \longrightarrow B$  such that any item  $\alpha \in A$  is mapped to the sum (using the operation +) of all  $b \in B$  such that  $(\alpha, b) \in R$ .
- m is the stake relation computed from the UTxO set.
- stakeRelation is the total stake relation obtained by combining the stake from the UTxO set with the stake from the reward accounts.

```
stakeDistr: UTxO \rightarrow DState \rightarrow PState \rightarrow Snapshot stakeDistr \ utxo \ st^d \ pState = [aggregate_+ (stakeRelation \ ^{fs}), stakeDelegs, poolParams] where poolParams = pState \ .PState.pools open \ DState \ st^d \ using \ (stakeDelegs; rewards) m = map^s \ (\lambda \ a \rightarrow (a \ , cbalance \ (utxo \ |^i \ \lambda \ i \rightarrow getStakeCred \ i \equiv just \ a))) \ (dom \ rewards) stakeRelation = m \cup | rewards |
```

Figure 72: Functions for computing stake distributions

### 18.3 Timing when Rewards are Paid Out

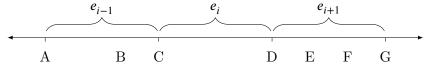
#### 18.3.1 Timeline of the Rewards Calculation

As described in Sec. 18.1, the probability of producing a block depends on the stake delegated to the block producer. However, the stake distribution changes over time, as funds are transferred between parties. This raises the question: What is the point in time from which we take the stake distribution? Right at the moment of producing a block? Some time in the past? How do we deal with the fact that the blockchain is only *eventually consistent*, i.e. blocks can be rolled back before a stable consensus on the chain is formed?

On Cardano, the answer to these questions is to group time into *epochs*. An epoch is long enough such that at the beginning of a new epoch, the beginning of the previous epoch has become stable. An epoch is also long enough for human users to react to parameter changes, such as stake pool costs or performance. But an epoch is also short enough so that changes to the stake distribution will be reflected in block production within a reasonable timeframe.

The rewards for the blocks produced during a given epoch  $e_i$  involve the two epochs surrounding it. In particular, the stake distribution will come from the previous epoch and the rewards will be calculated in the following epoch. At each epoch boundary, one snapshot of the stake distribution is taken; changes to the stake distribution within an epoch are not considered until the next snapshot is taken. More concretely:

- (A) A stake distribution snapshot is taken at the beginning of epoch  $e_{i-1}$ .
- (B) The randomness for leader election is fixed during epoch  $e_{i-1}$
- (C) Epoch  $e_i$  begins, blocks are produced using the snapshot taken at (A).
- (D) Epoch  $e_i$  ends. A snapshot is taken of the stake pool performance during epoch  $e_i$ . A snapshot is also taken of the fee pot.
- (E) The snapshots from (D) are stable and the reward calculation can begin.
- (F) The reward calculation is finished and an update to the ledger state is ready to be applied.
- (G) Rewards are given out.



In order to specify this logic, we store the last three snapshots of the stake distributions. The mnemonic "mark, set, go" will be used to keep track of the snapshots, where the label "mark" refers to the most recent snapshot, and "go" refers to the snapshot that is ready to be used in the reward calculation.

In the above diagram, the snapshot taken at (A) is labeled "mark" during epoch  $e_{i-1}$ , "set" during epoch  $e_i$  and "go" during epoch  $e_{i+1}$ . At (G) the snapshot taken at (A) is no longer needed and will be discarded.

In other words, blocks will be produced using the snapshot labeled "set", whereas rewards are computed from the snapshot labeled "go".

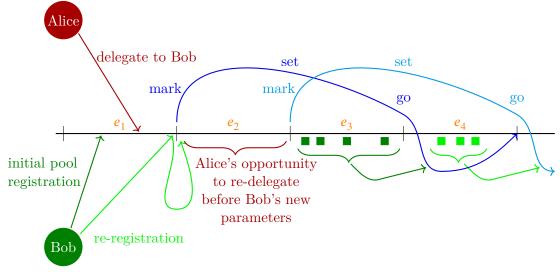


# NOTE

Between time D and E we are concerned with chain growth and stability. Therefore this duration can be stated as 2k blocks (to state it in slots requires details about the particular version of the Ouroboros protocol). The duration between F and G is also 2k blocks. Between E and F a single honest block is enough to ensure a random nonce.

### 18.3.2 Example Illustration of the Reward Cycle

For better understanding, here an example of the logic described in the previous section:



Bob registers his stake pool in epoch  $e_1$ . Alice delegates to Bob's stake pool in epoch  $e_1$ . Just before the end of epoch  $e_1$ , Bob submits a stake pool re-registration, changing his pool parameters. The change in parameters is not immediate, as shown by the curved arrow around the epoch boundary.

A snapshot is taken on the  $e_1/e_2$  boundary. It is labeled "mark" initially. This snapshot includes Alice's delegation to Bob's pool, and Bob's pool parameters and listed in the initial pool registration certificate.

If Alice changes her delegation choice any time during epoch  $e_2$ , she will never be effected by Bob's change of parameters.

A new snapshot is taken on the  $e_2/e_3$  boundary. The previous (darker blue) snapshot is now labeled "set", and the new one labeled "mark". The "set" snapshot is used for leader election in epoch  $e_3$ .

On the  $e_3/e_4$  boundary, the darker blue snapshot is labeled "go" and the lighter blue snapshot is labeled "set". Bob's stake pool performance during epoch  $e_3$  (he produced 4 blocks) will be used with the darker blue snapshot for the rewards which will be handed out at the beginning of epoch  $e_5$ .

### 18.3.3 Stake Distribution Snapshots

This section defines the SNAP transition rule for stake distribution snapshots.

Fig. 73 defines the type Snapshots that contains the data that needs to be saved at the end of an epoch. This data is:

- mark, set, go: Three stake distribution snapshots as explained in Sec. 18.3.1.
- feeSS: stores the fees which are added to the reward pot during the next reward update calculation, which is then subtracted from the fee pot on the epoch boundary.

Fig. 74 defines the snapshot transition rule. This transition has no preconditions and results in the following state change:

- The oldest snapshot is replaced with the penultimate one.
- The penultimate snapshot is replaced with the newest one.
- The newest snapshot is replaced with one just calculated.
- The current fees pot is stored in feeSS. Note that this value will not change during the epoch, unlike the fees value in the UTxO state.

```
record Snapshots : Set where
field
mark set go : Snapshot
feeSS : Coin
```

Figure 73: Definitions for the SNAP transition system

Figure 74: SNAP transition system

## 19 Epoch Boundary

This section is part of the Ledger.Conway.Epoch module of the formal ledger specification.

```
record EpochState : Type where
  field
    acnt : Acnt
    ss : Snapshots
    ls : LState
    es : EnactState
    fut : RatifyState

record NewEpochState : Type where
    field
    lastEpoch : Epoch
    epochState : EpochState
    ru : Maybe RewardUpdate
```

Figure 75: Definitions for the EPOCH and NEWEPOCH transition systems

Fig. 76 defines the function createRUpd which creates a RewardUpdate, i.e. the net flow of Ada due to paying out rewards after an epoch. Relevant quantities are:

- prevPp: Previous protocol parameters, which correspond to the parameters during the epoch for which we are creating rewards.
- ActiveSlotCoeff: Global constant, equal to the probability that a party holding all the stake will be selected to be a leader for given slot. Equals 1/20 during the Shelley era on the Cardano Mainnet.
- Δr<sub>1</sub>: Ada taken out of the reserves for paying rewards, as determined by the monetaryExpansion protocol parameter.
- rewardPot: Total amount of coin available for rewards this epoch, as described in IOHK Formal Methods Team [9, Sec 6.4].
- feeSS: The fee pot which, together with the reserves, funds the rewardPot. We use the fee pot that accumulated during the epoch for which we now compute block production rewards. Note that fees are not explicitly removed from any account: the fees come from transactions paying them and are accounted for whenever transactions are processed.
- Δt<sub>1</sub>: The proportion of the reward pot that will move to the treasury, as determined by the treasuryCut protocol parameter. The remaining pot is called the R, just as in IOHK Formal Methods Team [9, Sec 6.5].
- pstakego: Stake distribution used for calculating the rewards. This is the oldest stake distribution snapshot, labeled go.
- rs: The rewards, as calculated by the function reward.
- $\Delta r_2$ : The difference between the maximal amount of rewards that could have been paid out if pools had been optimal, and the actual rewards paid out. This difference is returned to the reserves.
- ÷<sub>0</sub>: Division operator that returns zero when the denominator is zero.

```
createRUpd : N → BlocksMade → EpochState → Coin → RewardUpdate
createRUpd slotsPerEpoch b es total
  = [\Delta t_1, 0 - \Delta r_1 + \Delta r_2, 0 - feeSS, rs]
  where
                 = PParamsOf (es .EpochState.es)
    prevPp
    reserves = es .EpochState.acnt .Acnt.reserves
    pstakego = es .EpochState.ss .Snapshots.go
    feeSS
                 = es .EpochState.ss .Snapshots.feeSS
    stake
                 = pstakego .Snapshot.stake
                 = pstakego .Snapshot.delegations
    poolParams = pstakego .Snapshot.poolParameters
    blocksMade = \sum [m \leftarrow b] m
    rho = fromUnitInterval (prevPp .PParams.monetaryExpansion)
    \eta = \text{fromN blocksMade} \div_{\theta} (\text{fromN slotsPerEpoch} * ActiveSlotCoeff)
    \Delta r_1 = \text{floor} (\mathbf{Q}.\text{min } 1 \text{ } \eta \text{ * rho * fromN reserves})
    rewardPot = pos feeSS + \Delta r_1
    tau = fromUnitInterval (prevPp .PParams.treasuryCut)
    \Delta t_1 = floor (tau * from \mathbb{Z} rewardPot)
    R = posPart (rewardPot - \Delta t_1)
    circulation = total - reserves
    rs = reward prevPp b R poolParams stake delegs circulation
    \Delta r_2 = R - \sum [c \leftarrow rs]c
```

Figure 76: RewardUpdate Creation

Fig. 78 defines the EPOCH transition rule. Currently, this incorporates logic that was previously handled by POOLREAP in the Shelley specification [1, Sec 11.6]; POOLREAP is not implemented here.

The EPOCH rule now also needs to invoke RATIFIES and properly deal with its results by carrying out each of the following tasks.

- Pay out all the enacted treasury withdrawals.
- Remove expired and enacted governance actions & refund deposits.
- If govSt' is empty, increment the activity counter for DReps.
- Remove all hot keys from the constitutional committee delegation map that do not belong to currently elected members.
- Apply the resulting enact state from the previous epoch boundary fut and store the resulting enact state fut'.

```
applyRUpd : RewardUpdate → EpochState → EpochState
                              treasury
                              reserves
                                  SS
                                utxo
                                fees
                              deposits
                             donations
applyRUpd
                                govSt
                             voteDelegs
                             stakeDelegs
                               rewards
                               pState
                               gState
                                  es
                                 fut
    [ [ posPart (pos treasury + \Delta t + pos unregRU')
      , posPart (pos reserves + \Delta r) ]
    , [ [ utxo , posPart (pos fees + \Delta f) , deposits , donations ]
      , [ [ voteDelegs , stakeDelegs , rewards U^* regRU ] , pState , gState ] ]
    , es
    , fut ]
    where
      regRU = rs | dom rewards
      unregRU = rs | dom rewards c
      unregRU' = \sum [x \leftarrow unregRU] x
  getOrphans : EnactState → GovState → GovState
  getOrphans es govSt = proj_1 $ iterate step ([] , govSt) (length govSt)
      step : GovState × GovState → GovState × GovState
      step (orps , govSt) =
        let
          isOrphan? a prev = ¬? (hasParent? es govSt a prev)
          (orps', govSt') = partition
             (\lambda (\_, record \{action = a ; prevAction = prev\}) \rightarrow isOrphan? (a .gaType) prev) govSt
          (orps ++ orps' , govSt')
```

```
gaDepositStake : GovState → Deposits → Credential → Coin
gaDepositStake govSt ds = aggregateBy
  (map<sup>s</sup> (λ (gaid , addr) → (gaid , addr) , stake addr) govSt')
  (mapFromPartialFun (λ (gaid , _) → lookup<sup>m</sup>? ds (GovActionDeposit gaid)) govSt')
  where govSt' = map<sup>s</sup> (map<sub>2</sub> returnAddr) (fromList govSt)

mkStakeDistrs : Snapshot → GovState → Deposits → (Credential → VDeleg) → StakeDistrs
mkStakeDistrs ss govSt ds delegations .StakeDistrs.stakeDistr =
  aggregateBy | delegations | (Snapshot.stake ss U* gaDepositStake govSt ds)
```

Figure 77: Functions for computing stake distributions

```
data _⊢_→ (_,EPOCH)_: T → EpochState → Epoch → EpochState → Type where
```

```
EPOCH : let
  esW
            = RatifyState.es fut
           = record esW { withdrawals = ∅ }
  tmpGovSt = filter (\lambda x \rightarrow proj_1 x \notin map^s proj_1 removed) govSt
  orphans = fromList (getOrphans es tmpGovSt)
  removed' = removed U orphans
  removedGovActions = flip concatMap<sup>s</sup> removed' \lambda (gaid, gaSt) \rightarrow
    map<sup>s</sup> (returnAddr gaSt ,_) ((utxoSt .deposits | { GovActionDeposit gaid }) *)
  govActionReturns = aggregate_{+} (map^{s} (\lambda (a, _{-}, d) \rightarrow a, d) removedGovActions fs)
  trWithdrawals = esW .withdrawals
  totWithdrawals = \sum [x \leftarrow trWithdrawals] x
  retired = (pState .retiring) -1 e
  payout
             = govActionReturns U⁺ trWithdrawals
             = pullbackMap payout toRwdAddr (dom (dState .rewards))
  refunds
  unclaimed = getCoin payout - getCoin refunds
  govSt' = filter (\lambda x \rightarrow proj_1 x \notin map^s proj_1 removed') govSt
  dState' = [dState .voteDelegs , dState .stakeDelegs , dState .rewards ∪+ refunds ]
  pState' = [ pState .pools | retired c , pState .retiring | retired c ]
  gState' = [ (if null govSt' then mapValues (1 +_) (gState .dreps) else (gState .dreps))
             , gState .ccHotKeys | ccCreds (es .cc) ]
  certState' : CertState
  certState' = [ dState' , pState' , gState' ]
 utxoSt' = [ utxoSt .utxo , utxoSt .fees , utxoSt .deposits | maps (proj1 o proj2) removedGovActions c
 acnt' = record acnt
    { treasury = acnt .treasury - totWithdrawals + utxoSt .donations + unclaimed }
in
record { currentEpoch = e
        ; stakeDistrs = mkStakeDistrs (Snapshots.mark ss') govSt'
                                         (utxoSt' .deposits) (voteDelegs dState)
        ; treasury = acnt .treasury ; GState gState
        ; pools = pState .pools ; delegatees = dState .voteDelegs }
    ⊢ [ es , ∅ , false ] → ( govSt' ,RATIFIES) fut'
  \rightarrow 1s \vdash ss \rightarrow 0 tt ,SNAPD ss'
\_\vdash [acnt, ss, ls, es_0, fut] \rightarrow (e, EPOCH)
    [ acnt' , ss' , [ utxoSt' , govSt' , certState' ] , es , fut' ]
```

Figure 78: EPOCH transition system

```
LF_→ (_,NEWEPOCH)_: T → NewEpochState → Epoch → NewEpochState → Type

NEWEPOCH-New: let

eps' = applyRUpd ru eps
in

• e ≡ lastEpoch + 1

• _ ⊢ eps' → ( e ,EPOCH) eps''

_ ⊢ [ lastEpoch , eps , just ru ] → ( e ,NEWEPOCH) [ e , eps'' , nothing ]

NEWEPOCH-Not-New:
• e ≢ lastEpoch + 1

_ ⊢ [ lastEpoch , eps , mru ] → ( e ,NEWEPOCH) [ lastEpoch , eps , mru ]

NEWEPOCH-No-Reward-Update:
• e ≡ lastEpoch + 1

• _ ⊢ eps → ( e ,EPOCH) eps'

_ ⊢ [ lastEpoch , eps , nothing ] → ( e ,NEWEPOCH) [ e , eps' , nothing ]
```

Figure 79: NEWEPOCH transition system

# 20 Blockchain Layer

This section is part of the Ledger.Conway.Chain module of the formal ledger specification.

```
record ChainState : Type where
   field
    newEpochState : NewEpochState

record Block : Type where
   field
    ts : List Tx
    slot : Slot
```

Figure 80: Definitions CHAIN transition system

```
_⊢_→ (_,CHAIN)_ : τ → ChainState → Block → ChainState → Type
```

Figure 81: Type of the CHAIN transition system

Figure 82: CHAIN transition system

## 21 Properties

This section presents the properties of the ledger that we have formally proved in Agda or plan to do so in the near future. We indicate in which Agda module each property is formally stated and (possibly) proved. A "Claim" is a property that is not yet proved, while a "Theorem" is one for which we have a formal proof.

#### 21.1 Preservation of Value

Theorem 1. (Conway/Ledger/Properties/PoV: LEDGER rule preserves value)

- Informally. Let s, s': LState be ledger states and let tx: Tx be a fresh transaction, that is, a transaction that is not already part of the UTxOState of s. If  $s \rightarrow \emptyset$  tx, LEDGERD s', then the coin values of s and s' are equal, that is, getCoin  $s \equiv getCoin s'$ .
- Formally.

```
LEDGER-pov : {Γ : LEnv} {s s' : LState}

→ txid ∉ map<sup>s</sup> proj₁ (dom (UTxOOf s))

→ Γ ⊢ s → 0 tx ,LEDGERD s' → getCoin s ≡ getCoin s'
```

• *Proof.* See the Conway/Ledger/Properties/PoV module in the formal ledger repository.

Lemma 2. (Conway/Utxo/Properties/PoV: UTXO rule preserves value)

• Informally. Let s and s' be UTxOStates, let tx: Tx be a fresh transaction with withdrawals txwdrls, and suppose  $s \rightarrow \emptyset$  tx, UTXOD s'. If tx is valid, then the coin value of s' is equal to the sum of the coin values of s and txwdrls. If tx is not valid, then the coin values of s and s' are equal. We can express this concisely as follows:

```
getCoin s + getCoin txwdrls \cdot \chi(tx .isValid) \equiv getCoin s',
```

where  $\chi: Bool \to 0, 1$  is the *characteristic function*, which returns 0 for false and 1 for true.

• Formally.

```
UTXOpov : {\Gamma : UTxOEnv} {tx : Tx} {s s' : UTxOState} 

\rightarrow txidOf tx \notin map^s proj_1 (dom (UTxOOf <math>s))
\rightarrow \Gamma \vdash s \rightharpoonup (tx, UTXO) s'
\rightarrow getCoin s + getCoin (wdrlsOf tx) * \chi (tx .isValid) = getCoin s'
```

• Proof. See the Conway/Utxo/Properties/PoV module in the formal ledger repository.

#### Theorem 3. (Conway/Certs/Properties/PoV: CERTS rule preserves value)

- Informally. Let l be a list of DCerts, and let  $s_1$ ,  $s_n$  be CertStates such that  $s_1 \rightarrow 0$  l, CERTS)  $s_n$ . Then, the value of  $s_1$  is equal to the value of  $s_n$  plus the value of the withdrawals in  $\Gamma$ .
- Formally.

```
CERTS-pov : \{\Gamma : \text{CertEnv}\}\ \{s_1 \ s_n : \text{CertState}\}\
\rightarrow \forall [\ a \in \text{dom}\ (\text{CertEnv.wdrls}\ \Gamma)\ ] \text{NetworkIdOf}\ a \equiv \text{NetworkId}\
\rightarrow \Gamma \vdash s_1 \rightharpoonup \emptyset\ 1\ ,\text{CERTS})\ s_n
\rightarrow \text{getCoin}\ s_1 \equiv \text{getCoin}\ s_n + \text{getCoin}\ (\text{wdrlsOf}\ \Gamma)
```

• Proof. See the Conway/Certs/Properties/PoV module in the formal ledger repository.

Lemma 4. (Conway/Certs/Properties/PoVLemmas: CERT rule preserves value)

- Informally. Let s, s' be CertStates such that  $s \rightarrow \emptyset$  deert , CERT $\emptyset$  s' for some deert : DCert. Then, getCoin  $s \equiv getCoin s'$ .
- Formally.

```
CERT-pov : {Γ : CertEnv} {s s' : CertState}
  → Γ ⊢ s → ∅ dCert ,CERT⊅ s'
  → getCoin s ≡ getCoin s'
```

• *Proof.* See the Conway/Certs/Properties/PoVLemmas module in the formal ledger repository.

Lemma 5. (Conway/Certs/Properties/PoVLemmas: CERTBASE rule preserves value)

• Informally. Let  $\Gamma$ : CertEnv be a certificate environment, and let s, s': CertState be certificate states such that  $s \to \emptyset$ , CERTBASED s'. Then, the value of s is equal to the value of s' plus the value of the withdrawals in  $\Gamma$ . In other terms,

```
getCoin s \equiv getCoin s' + getCoin (\Gamma .wdrls).
```

• Formally.

```
CERTBASE-pov : \{\Gamma : CertEnv\} \{s \ s' : CertState\}

\rightarrow \forall [\ \alpha \in dom\ (CertEnv.wdrls\ \Gamma)\ ]\ NetworkIdOf\ \alpha \equiv NetworkId

\rightarrow \Gamma \vdash s \rightharpoonup \emptyset _ ,CERTBASE\emptyset s'

\rightarrow getCoin\ s \equiv getCoin\ s' + getCoin\ (CertEnv.wdrls\ \Gamma)
```

• *Proof.* See the Conway/Certs/Properties/PoVLemmas module in the formal ledger repository.

Lemma 6. (Conway/Certs/Properties/PoVLemmas: iteration of CERT rule preserves value)

- Informally. Let l be a list of DCerts, and let  $s_1$ ,  $s_n$  be CertStates such that, starting with  $s_1$  and successively applying the CERT rule to with DCerts from the list l, we obtain  $s_n$ . Then, the value of  $s_1$  is equal to the value of  $s_n$ .
- Formally.

```
sts-pov : {\Gamma : CertEnv} {s_1 s_n : CertState}

\rightarrow ReflexiveTransitiveClosure {sts = \_\vdash\_ \rightharpoonup \emptyset\_, CERT\emptyset\_} \Gamma s_1 1 s_n

\rightarrow getCoin s_1 \equiv getCoin s_n
```

Proof. See the Conway/Certs/Properties/PoVLemmas module in the formal ledger repository.

#### 21.2 Invariance Properties

To say that a predicate P is an *invariant* of a transition rule means the following: if the transition rule relates states s and s' and if P holds at state s, then P holds at state s'.

Claim 7. (Conway/Chain/Properties/CredDepsEqualDomRwds: Equality of credential depsoits is a CHAIN invariant)

- Informally. This property concerns two quantities associated with a given ChainState cs,
  - the credential deposits of the UTxOState of cs and
  - the credential deposits of the rewards in the ledger state of cs.

The predicate  $credDeposits \equiv dom-rwds$  cs asserts that these quantities are equal for cs. Formally,

The property credDeposits $\equiv$ dom-rwds-inv asserts that credDeposits $\equiv$ dom-rwds is a chain invariant. That is, if cs and cs' are two ChainStates such that  $cs \rightarrow \emptyset$  tx, CHAIND cs', then credDeposits $\equiv$ dom-rwds cs only if credDeposits $\equiv$ dom-rwds cs'.

• Formally.

```
credDeposits≡dom-rwds-inv : Type
credDeposits≡dom-rwds-inv = LedgerInvariant _⊢_→ (_,CHAIN)_ credDeposits≡dom-rwds
```

• *Proof. To appear* (in the Conway/Chain/Properties/CredDepsEqualDomRwds module of the formal ledger repository).

Claim~8.~(Conway/Chain/Properties/PParamsWellFormed:~Well-formedness~of~PParams~is~a~CHAIN~invariant)

• Informally. We say the PParams of a chain state are well-formed if each of the following parameters is non-zero: maxBlockSize, maxTxSize, maxHeaderSize, maxValSize, refScript-CostStride, coinsPerUTxOByte, poolDeposit, collateralPercentage, ccMaxTermLength, govActionLifetime, govActionDeposit, drepDeposit. Formally,

```
pp-wellFormed : ChainState → Type
pp-wellFormed = paramsWellFormed ∘ PParamsOf
```

This property asserts that pp-wellFormed is a chain invariant. That is, if cs and cs' are chain states such that  $cs \rightarrow \emptyset$  tx, CHAIND cs', and if the PParams of cs are well-formed, then so are the PParams of cs'.

• Formally.

```
pp-wellFormed-invariant : Type
pp-wellFormed-invariant = LedgerInvariant _⊢_→ (_,CHAIN)_ pp-wellFormed
```

• *Proof. To appear* (in the Conway/Chain/Properties/PParamsWellFormed module of the formal ledger repository).

#### 21.2.1 Governance Action Deposits Match

Theorems 9 to 11 assert that a certain predicate is an invariant of the CHAIN, LEDGER, and EPOCH rules, respectively. Given a ledger state s, we focus on deposits in the UTxOState of s that are GovActionDeposits and we compare that set of deposits with the GovActionDeposits of the GovState of s. When these two sets are the same, we write govDepsMatch s and say the govDepsMatch relation holds for s. Formally, the govDepsMatch predicate is defined as follows:

```
govDepsMatch : LState → Type
govDepsMatch ls =
  filter* isGADeposit (dom (DepositsOf ls)) = fromList (dpMap (GovStateOf ls))
```

The assertion, "the govDepsMatch relation is an invariant of the LEDGER rule," means the following: if govDepsMatch s and  $s \rightarrow 0$  tx, LEDGER0 s', then govDepsMatch s'.

Theorem 9. (Conway/Chain/Properties/GovDepsMatch: govDepsMatch is invariant of CHAIN rule)

• Informally. Fix a Block b, a ChainState cs, and a NewEpochState nes. Let csLState be the ledger state of cs. Recall, a ChainState has just one field, newEpochState: NewEpochState. Consider the chain state cs' defined as follows:

```
cs': ChainState
cs' .newEpochState =
  record { lastEpoch = nes .lastEpoch
      ; epochState = record (EpochStateOf cs) {ls = LStateOf nes}
      ; ru = nes .ru }
```

That is cs' is essentially nes, but the EpochState field is set to the epochState of cs with the exception of the LState field, which is set to that of nes.

Let utxoSt and utxoSt' be the respective UTxOStates of the ledger states of cs and cs', respectively, and let govSt and govSt' be their respective GovStates.

Assume the following conditions hold:

- let removed' :  $\mathbb{P}(GovActionID \times GovActionState)$  be the union of
  - \* the governance actions in the removed field of the ratify state of eps, and
  - \* the orphaned governance actions in the GovState of eps.

Let  $\mathcal{G}$  be the set {GovActionDeposit  $id : id \in \operatorname{proj}_1 \operatorname{removed'}$ }.  $\mathcal{G}$  is a subset of the set of deposits of the chain state cs; that is,

```
map (GovActionDeposit \circ proj<sub>1</sub>) removed' \subseteq dom (DepositsOf cs);
```

 the total reference script size of csLState is not greater than the maximum allowed size per block (as specified in PParams);

```
-cs \rightarrow \emptyset b , CHAIND cs'.
```

Under these conditions, if the governance action deposits of utxoSt equal those of govSt, then the same holds for utxoSt' and govSt'. In other terms, govDepsMatch csLState implies govDepsMatch nesState.

• Formally.

```
CHAIN-govDepsMatch :
  map (GovActionDeposit ∘ proj₁) removed' ⊆ dom (DepositsOf cs)
  → totalRefScriptsSize csLState ts ≤ maxRefScriptSizePerBlock
  → _ ⊢ cs → ∅ b ,CHAIND cs'
  → govDepsMatch csLState → govDepsMatch (LStateOf nes)
```

• *Proof.* See the Conway/Chain/Properties/GovDepsMatch module in the formal ledger repository.

- Informally. Suppose s, s' are ledger states such that  $s \to \emptyset$  tx, LEDGERD s'. Let utxoSt and utxoSt' be their respective UTxOStates and let govSt and govSt' be their respective GovStates. If the governance action deposits of utxoSt are equal those of govSt, then the same holds for utxoSt' and govSt'. In other terms, if govDepsMatch s, then govDepsMatch s'.
- Formally.

```
LEDGER-govDepsMatch: LedgerInvariant _⊢_→(_,LEDGER)_ govDepsMatch
```

• *Proof.* See the Conway/Ledger/Properties/GovDepsMatch module in the formal ledger repository.

Lemma 11. (Conway/Epoch/Properties/GovDepsMatch: govDepsMatch is invariant of EPOCH rule)

- Informally. Let eps, eps': EpochState be two epoch states and let e: Epoch be an epoch. Recall, eps .ls denotes the ledger state of eps. If eps @ e,EPOCHD eps', then (under a certain special condition) govDepsMatch (eps .ls) implies govDepsMatch (eps' .ls).

  The special condition under which the property holds is the same as the one in Theorem 9: let removed' be the union of the governance actions in the removed field of the ratify state of eps and the orphaned governance actions in the GovState of eps. Let \$\mathcal{E}\$ be the set
  - of eps and the orphaned governance actions in the GovState of eps. Let  $\mathcal{G}$  be the set {GovActionDeposit  $id : id \in \operatorname{proj}_1 \operatorname{removed}'$ }. Assume:  $\mathcal{G}$  is a subset of the set of deposits of (the governance state of) eps.
- Formally.

• *Proof.* See the Conway/Epoch/Properties/GovDepsMatch module in the formal ledger repository.

#### 21.3 Minimum Spending Conditions

Theorem 12. (Conway/Utxo/Properties/MinSpend: general spend lower bound)

- Informally. Let tx : Tx be a valid transaction and let txcerts be its list of DCerts. Denote by noRefundCert txcerts the assertion that no element in txcerts is one of the two refund types (i.e., an element of l is neither a dereg nor a deregdrep).
  - Let s, s': UTxOState be two UTxO states. If  $s \rightarrow \emptyset$  tx, UTXOD s' and if noRefundCert tx-certs, then the coin consumed by tx is at least the sum of the governance action deposits of the proposals in tx.
- Formally.

```
utxoMinSpend : \{\Gamma : UTxOEnv\} \{tx : Tx\} \{s \ s' : UTxOState\}

\rightarrow \Gamma \vdash s \rightharpoonup \emptyset \ tx \ , UTXO\emptyset \ s'

\rightarrow noRefundCert \ (txcertsOf \ tx)

\rightarrow coin \ (consumed \ \_ s \ (TxBodyOf \ tx)) \ge length \ (txpropOf \ tx) * govActionDepositOf \ \Gamma
```

• Proof. See the Conway/Utxo/Properties/MinSpend module in the formal ledger repository.

#### Theorem 13. (Conway/Utxo/Properties/MinSpend: spend lower bound for proposals)

- Preliminary remarks.
  - 1. Define noRefundCert 1 and pp as in Theorem 12.
  - 2. Given a ledger state ls and a transaction tx, denote by  $validTxIn_2$  tx the assertion that there exists ledger state ls' such that  $ls \rightarrow \emptyset$  tx, LEDGER $\emptyset$  ls'.
  - 3. Assume the following additive property of the U<sup>+</sup> operator holds:

```
\sum \left[ \ x \leftarrow d_1 \ \cup^+ \ d_2 \ \right] \ x \equiv \sum \left[ \ x \leftarrow d_1 \ \right] \ x \diamond \sum \left[ \ x \leftarrow d_2 \ \right] \ x
```

- Informally. Let tx: Tx be a valid transaction and let cs: ChainState be a chain state. If the condition  $validTxIn_2$  tx (described above) holds, then the coin consumed by tx is at least the sum of the governance action deposits of the proposals in tx.
- Formally.

• Proof. See the Conway/Utxo/Properties/MinSpend module in the formal ledger repository.

#### 21.4 Miscellaneous Properties

 ${\it Claim~14.}$  (Conway/GovernanceActions/Properties/ChangePPGroup:  ${\it PParam~updates~have~non-empty~groups}$ )

- Informally. Let p: GovProposal be a governance proposal and suppose the GovActionType of p .action is ChangePParams. If the data field of p—that is pu = p .action .gaData—is denoted by pu ("parameter update"), then the set updateGroups pu is nonempty.
- Formally.

```
ChangePPHasGroup: \{tx: Tx\} \{p: GovProposal\} (pu: PParamsUpdate) \rightarrow p \in Tx.body \ tx \rightarrow p \ .GovProposal.action \equiv [ ChangePParams , pu ] ga \rightarrow Type
ChangePPHasGroup pu = 0 = updateGroups pu \not\equiv 0
```

• *Proof. To appear* (in the Conway/GovernanceActions/Properties/ChangePPGroup module of the formal ledger repository).

Claim 15. (Conway/Chain/Properties/EpochStep: New enact state only if new epoch)

• Informally. Let cs and cs' be ChainStates and b a Block. If  $cs \rightarrow \emptyset$  b, CHAIND cs' and if the enact states of cs and cs' differ, then the epoch of the slot of b is the successor of the last epoch of cs.

• Formally.

```
enact-change\RightarrownewEpoch : (b : Block) {cs cs' : ChainState} \rightarrow \_ \vdash cs \rightharpoonup \emptyset \ b ,CHAIND cs' \rightarrow EnactStateOf \ cs \not\equiv EnactStateOf \ cs' <math>\rightarrow Type enact-change\RightarrownewEpoch b {cs} h es\not\equives' = epoch (b .slot) \equiv suce (LastEpochOf cs)
```

• *Proof. To appear* (in the Conway/Chain/Properties/EpochStep module of the formal ledger repository).

Claim 16. (Conway/Epoch/Properties/ConstRwds: NEWEPOCH rule leaves rewards unchanged)

- Informally. Rewards are left unchanged by the NEWEPOCH rule. That is, if es and es' are two NewEpochStates such that es → (e, NEWEPOCH) es', then the rewards of es and es' are equal.
- Formally.

• *Proof. To appear* (in the Conway/Epoch/Properties/ConstRwds module of the formal ledger repository).

Claim 17. (Conway/Epoch/Properties/NoPropSameDReps: DReps unchanged if no gov proposals)

- Informally. If there are no governance proposals in the GovState of es, then the active-DReps of es in Epoch e are the same as the activeDReps of es' in the next epoch.
- Formally.

```
prop≡∅⇒activeDReps-const : Epoch → (es es' : NewEpochState) → Type
prop≡∅⇒activeDReps-const e es es' =
GovStateOf es ≡ [] → activeDReps e es ≡ activeDReps (suc e) es'
```

• *Proof. To appear* (in the Conway/Epoch/Properties/NoPropSameDReps module of the formal ledger repository).

Claim 18. (Conway/Certs/Properties/VoteDelegsVDeleg: voteDelegs by credVoter constructor)

• Informally. A CertState has a DState, PState, and a GState. The DState contains a field voteDelegs which is a mapping from Credential to VDeleg.

VDeleg is a datatype with three constructors; the one of interest to us here is credVoter, which takes two arguments, a GovRole and a Credential.

Now suppose we have a collection C of credentials—for instance, given d: DState, take C to be the domain of the voteDelegs field of d. We could then obtain a set of VDelegs by applying credVoter DRep to each element of C.

The present property asserts that the set of VDelegs that results from the application of credVoter DRep to the domain of the voteDelegs of d contains the range of the voteDelegs of d.

• Formally.

```
voteDelegsVDeleg : DState \rightarrow Type voteDelegsVDeleg d = range (voteDelegsOf d) \subseteq map^s (credVoter DRep) (dom (voteDelegsOf <math>d))
```

• *Proof.* To appear in the Conway/Certs/Properties/VoteDelegsVDeleg module in the formal ledger repository.

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### A Definitions

To keep this document somewhat self-contained, we define some technical terms that arise when defining and describing the Cardano ledger. This is not meant to be comprehensive and the reader may wish to consult online resources to fill in any gaps. Here are a few such resources that might be helpful.

- Cardano Docs [10];
- (Re)introduction to Cardano [11];
- Ouroboros Chronos blog post [12];
- Cardano Staking: How To Stake ADA [13];
- Glossary from cardano.org [14];
- Glossary from ledger.com [15];
- Glossary from essential cardano.io [16].

### A.1 Cardano Time Handling

For more details, see the Time handling on Cardano section of [17].

In Cardano, the Ouroboros proof-of-stake (PoS) consensus protocol models the passage of physical time as an infinite sequence of time slots and epochs.

- block time The actual time interval between blocks, or *block time*, is the slot length (in seconds) divided by the block coefficient f, which is the expected block frequency (blocks per second). For example, if f is 0.05, then on average 5% of slots contain blocks. If the slot length is 1 second, then the block time is 20 seconds.
  - **epoch** An *epoch* is a period of time, containing some number of slots, used to select block-producing nodes. For example, in Shelley and later eras, an epoch consists of roughly 432,000 slots (or five days if we assume a slot length of 1 second).
- **genesis block** The *genesis block* of Cardano was created on the 23rd of September 2017. As the first block in the blockchain, it set the foundation for the network, it does not reference any previous blocks, and it generated the initial supply of Ada.
  - slot A slot is a discrete time interval in which a single block may be produced; it is the fundamental time unit within the blockchain's consensus protocol. Slots should be long enough for a new block to have a good chance to reach the next slot leader in time. For example, the slot length in the Byron era was 20 seconds, while in Shelley and later eras it is 1 second. Not every slot results in a new block. Indeed, in any given slot, one or more block-producing nodes are nominated (probabilistically based on stake distribution) to be slot leaders and given the opportunity to produce a new block. For example, in Shelley and later eras, on average only 0.05 of slots will produce a block (resulting in 20-second intervals between blocks). Slot number may refer to a slot's position within the current epoch or it may mean the absolute slot count since the genesis block. The context should make clear which meaning is intended.

The parameter values mentioned in the examples above,

- block time = 20 seconds,
- slot length = 1 second,
- block coefficient = 0.05,
- slots/epoch = 432,000,

are unlikely to change in the short-term. However, the longer term plan is to replace the current Ouroboros protocol with Ouroboros Chronos, which addresses timekeeping challenges by providing the first high-resilience cryptographic time source based on blockchain technology (see Hryniuk [12]).

## B Agda Essentials

Here we describe some of the essential concepts and syntax of the Agda programming language and proof assistant. The goal is to provide some background for readers who are not already familiar with Agda, to help them understand the other sections of the specification. For more details, the reader is encouraged to consult the official Agda documentation [6].

### B.1 Record Types

A *record* is a product with named accessors for the individual fields. It provides a way to define a type that groups together inhabitants of other types.

## ${\bf Example}.$

```
record Pair (A B : Type) : Type where
  constructor (_,_)
  field
   fst : A
  snd : B
```

We can construct an element of the type Pair N N (i.e., a pair of natural numbers) as follows:

```
p23 : Pair N N
p23 = record { fst = 2; snd = 3 }
```

Since our definition of the Pair type provides an (optional) constructor (-,-), we can have defined p23 as follows:

```
p23': Pair N N p23' = (2,3)
```

Finally, we can "update" a record by deriving from it a new record whose fields may contain new values. The syntax is best explained by way of example.

```
p24 : Pair N N
p24 = record p23 { snd = 4 }
```

This results a new record, p24, which denotes the pair (2, 4).

See also agda.readthedocs.io/record-types.

# C Bootstrapping EnactState

To form an EnactState, some governance action IDs need to be provided. However, at the time of the initial hard fork into Conway there are no such previous actions. There are effectively two ways to solve this issue:

- populate those fields with IDs chosen in some manner (e.g. random, all zeros, etc.), or
- add a special value to the types to indicate this situation.

In the Haskell implementation the latter solution was chosen. This means that everything that deals with GovActionID needs to be aware of this special case and handle it properly.

This specification could have mirrored this choice, but it is not necessary here: since it is already necessary to assume the absence of hash-collisions (specifically first pre-image resistance) for various properties, we could pick arbitrary initial values to mirror this situation. Then, since GovActionID contains a hash, that arbitrary initial value behaves just like a special case.

## D Bootstrapping the Governance System

As described in CIP-1694, the governance system needs to be bootstrapped. During the bootstrap period, the following changes will be made to the ledger described in this document.

- Transactions containing any proposal except TriggerHF, ChangePParams or Info will be rejected.
- Transactions containing a vote other than a CC vote, a SPO vote on a TriggerHF action or any vote on an Info action will be rejected.
- Q4, P5 and Q5e are set to 0.
- An SPO that does not vote is assumed to have voted abstain.

This allows for a governance mechanism similar to the old, Shelley-era governance during the bootstrap phase, where the constitutional committee is mostly in charge [9]. These restrictions will be removed during a subsequent hard fork, once enough DRep stake is present in the system to properly govern and secure itself.