The Chrial Phase Transition in QCD

$\hookrightarrow$ Mean-Field $\leftrightarrow$ Funtional Renormalisation Group

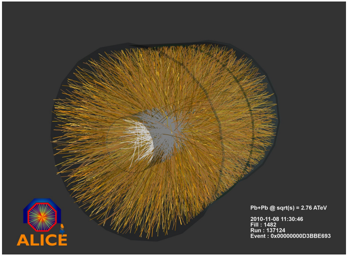
Jonas R. Glesaaen - Goethe Universität Frankfurt am Main

in collaboration with Prof. J.O. Andersen (NTNU)

3. February 2014

Motivation

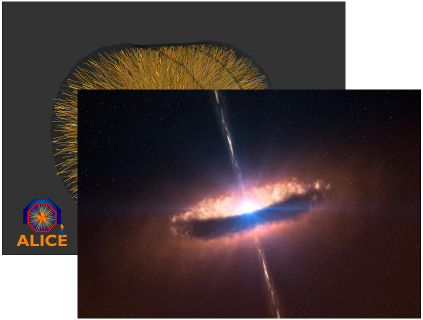
- Heavy Ion Collisions (RHIC, LHC, FAIR)



Picture taken from `cern.ch`

Motivation

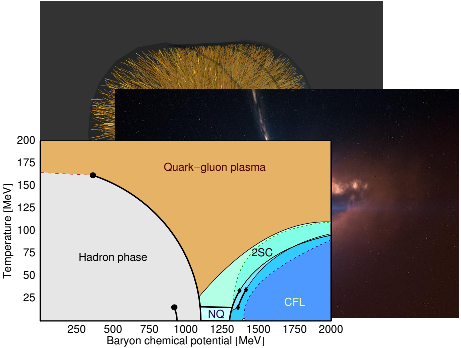
- Heavy Ion Collisions (RHIC, LHC, FAIR)
- Dense-massive stars



Picture taken from `nasa.gov`

Motivation

- Heavy Ion Collisions (RHIC, LHC, FAIR)
- Dense-massive stars
- The T-μ phase diagram of QCD



*Phys.Rev.*D72,n.3,p.1 (2005)

The Linear Sigma Model

$$\mathcal{L}_{\mathsf{LSM}} = \frac{1}{2}\mathsf{tr}\left[\partial_\mu \Phi^\dagger \partial^\mu \Phi\right] + U(\Phi)$$
$$U(\Phi) = \frac{1}{2}m^2\mathsf{tr}\left[\Phi^\dagger\Phi\right] + \frac{\lambda_1}{4!}\left(\mathsf{tr}\left[\Phi^\dagger\Phi\right]\right)^2 + \mathsf{tr}\left[h(\Phi^\dagger + \Phi)\right] + \dots$$

- The linear h term explicitly breaks the $\mathcal{O}(N)$ symmetry of the Φ field
- Φ is composite of the mesons, scalar (σ) and pseudoscalar (π), $\Phi = \sigma + i\pi$

The Linear Sigma Model

$$\mathcal{L}_{\mathsf{LSM}} = \frac{1}{2}\mathrm{tr}\left[\partial_\mu \Phi^\dagger \partial^\mu \Phi\right] + U(\Phi)$$

$$U(\Phi) = \frac{1}{2}m^2 \mathrm{tr}\left[\Phi^\dagger \Phi\right] + \frac{\lambda_1}{4!}\left(\mathrm{tr}\left[\Phi^\dagger\Phi\right]\right)^2 + \mathrm{tr}\left[h(\Phi^\dagger + \Phi)\right] + \dots$$

- The linear h term explicitly breaks the $\mathcal{O}(N)$ symmetry of the Φ field
- Φ is composite of the mesons, scalar (σ) and pseudoscalar (π), $\Phi = \sigma + i\pi$

$N_F = 2$

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} \sigma & 0 \\ 0 & \sigma \end{pmatrix} + i\frac{1}{\sqrt{2}}\begin{pmatrix} \pi_0 & \pi_- \\ \pi_+ & -\pi_0 \end{pmatrix}$$

$N_F = 3$

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{1}{\sqrt{2}}\sigma_{ud} + \frac{1}{\sqrt{2}}a_0 & a_0^- & K_0^{*-} \\ a_0^+ & \frac{1}{\sqrt{2}}\sigma_{ud} - \frac{1}{\sqrt{2}}a_0 & \bar{K}_0^* \\ K_0^{*+} & K_0^* & \sigma_s \end{pmatrix} + \text{pseudo scalar mesons}$$

. . . with quarks

$$\mathcal{L}_{\mathsf{LSMq}} = \bar{\psi}\Big(i\partial\!\!\!/ - g(\sigma + i\gamma^5\pi)\Big)\psi + \mathcal{L}_{\mathsf{LSM}}$$

- We have chiral symmetry of the quark Lagrangian when the fermionic fields are massless
- $\Rightarrow$ The degree of Chiral symmetry determined by $\langle\Phi\rangle$
- Which itself is determined by the thermodynamic potential

$$\Omega = -\frac{1}{V\beta}\log\mathcal{Z}$$

Simplest first approximation is at one-loop, where all mesonic quantum fluctuations are suppressed.

$$\Omega[\Phi] = U(\Phi) + \Omega_{q\bar{q}}[\Phi]$$

- The physical free energy Ω is given at its minimum, Φ_0,

$$\left.\frac{\partial\Omega}{\partial\Phi}\right|_{\Phi_0} = 0$$

- where $\Omega_{q\bar{q}}$ is the free energy of N_F massive free fermionic fields, with masses given by Φ_0.

Goal

Find the evolution of Gibb's free energy with respect to the renormalisation scale.

- First regularise the action by using Pauli-Villars regularisation:

$$S[\phi] \longrightarrow S[\phi] + \Delta_k[\phi] = S[\phi] + \frac{1}{2}\int \mathrm{d}^d p\, R_{k,i,j}(p)\phi_{p,i}\phi_{-p,j}$$

- Which in turns adds a renormalisation scale dependence to Gibb's free energy[1]
- Can in turn find a PDE for Gibb's free energy

The Wetterich equation

$$\partial_k \Gamma_k = \frac{1}{2}\mathrm{tr}\int_q \partial_k R_{k,i,j}(q)\left(\frac{\delta^2\Gamma_k}{\delta\phi_i(p)\delta\phi_j(p')} + \delta(p+p')R_{k,i,j}(p)\right)^{-1}_{q,-q}$$

- With an identical procedure for fermionic fields

[1] being the Legendre transform of Helmholtz' free energy

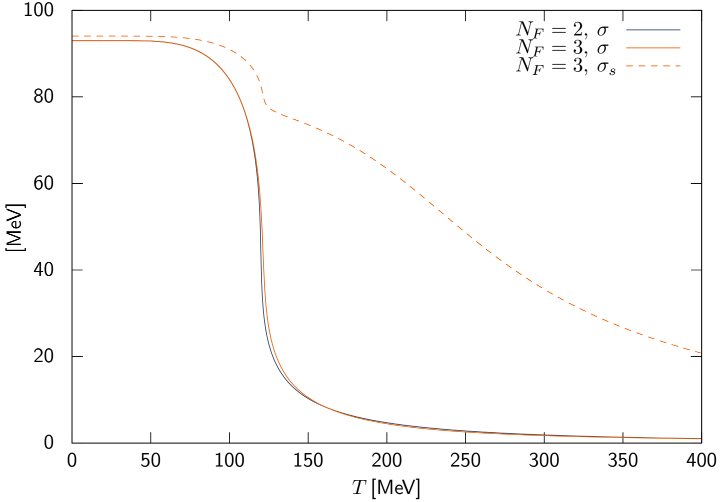
- Expand Gibb's free energy in powers of the ∂ operator, and truncate it

$\mathcal{O}(\partial^2)$

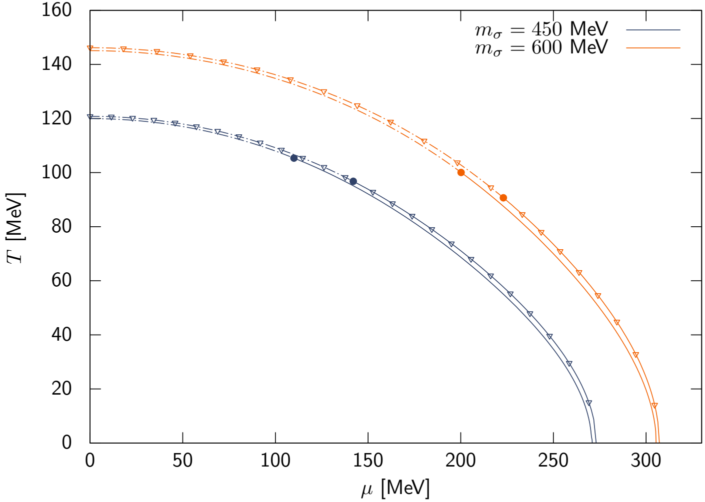
$$\Gamma_k[\phi] = \int \mathrm{d}^d x \left(\frac{1}{2} Z_k(\phi)(\nabla_4 \phi)^2 + U_k(\phi) \right)$$

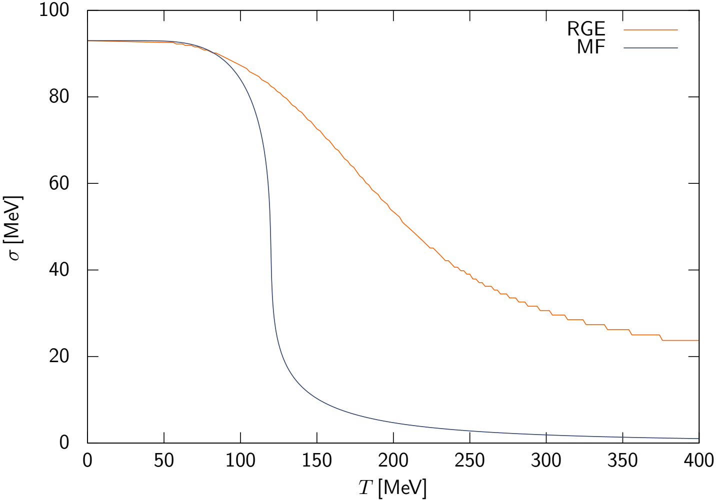
- Without the field renormalisation term $Z_k(\phi)$, the expansion is $\mathcal{O}(\partial^0)$, also known as the Local Area Approximation. In this expansion, the Wetterich eq. is:

$$\partial_k U_k = \frac{1}{2} \int_q \partial_k R_k(q) \left[q^2 + \frac{\partial^2 U_k}{\partial \phi^2} + R_k(q) \right]^{-1}$$

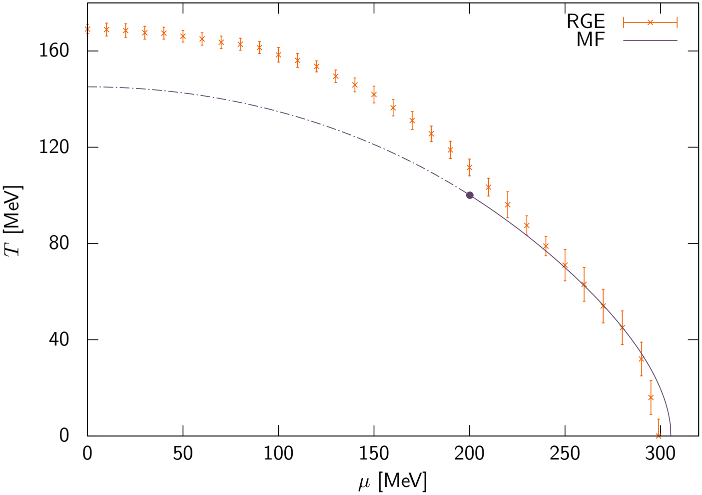


100
80
60
40
20
0
[MeV]
0
50
100
150
200
250
300
350
400
T [MeV]
$N_F = 2,\ \sigma$
$N_F = 3,\ \sigma$
$N_F = 3,\ \sigma_s$


$m_\sigma = 450$ MeV
$m_\sigma = 600$ MeV
160
140
120
100
80
60
40
20
0
T [MeV]
0
50
100
150
200
250
300
μ [MeV]


RGE
MF
100
80
60
40
20
0
σ [MeV]
0
50
100
150
200
250
300
350
400
T [MeV]

Results
⟶ RGE - Chiral phase diagram



Summary

Got a short introduction to:

- The Linear Sigma Model with Quarks
- Symmetry considerations
- The perturbative Mean Field Approximation
- The nonpertbative Local Area Approximation