Analytic calculation of an effective lattice theory

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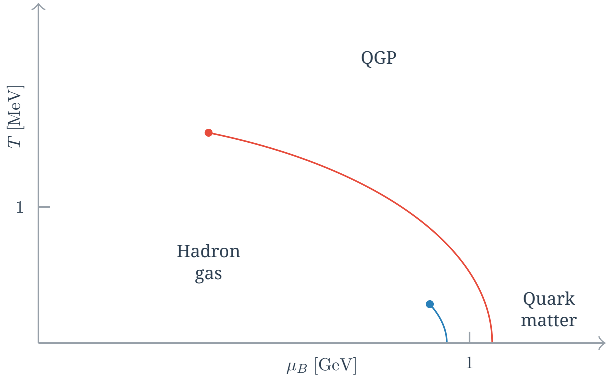
Our Group

- Me: Started August 2013
- Group leader: Owe Philipsen
- Collaborator: Mathias Neuman

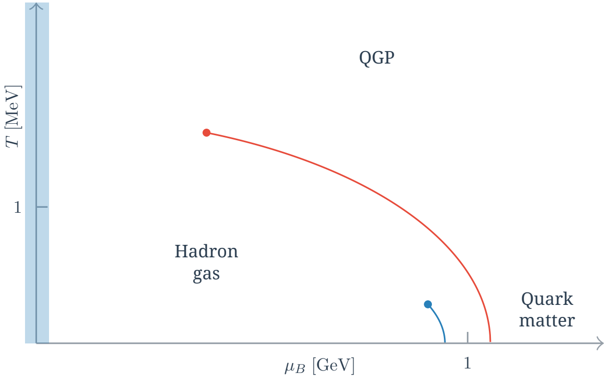
Introduction

The QCD Phase Diagram



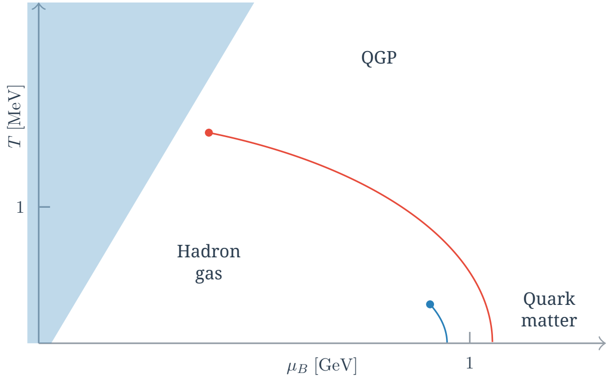
- Different approaches can access different regions

The QCD Phase Diagram



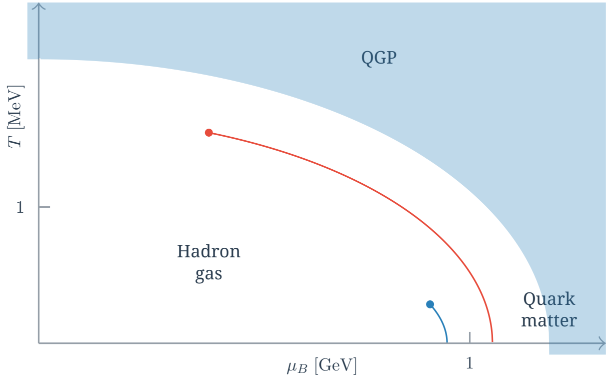
- Naive reach of Lattice QCD

The QCD Phase Diagram



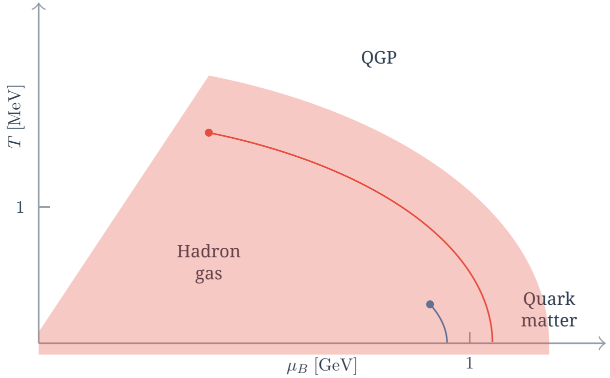
- Lattice QCD with additional methods (analytic continuation, reweighting,...)

The QCD Phase Diagram



- Perturbation theory of QCD

The QCD Phase Diagram



- Region currently not accessible from first principles with traditional methods

Lattice QCD: Basics 1

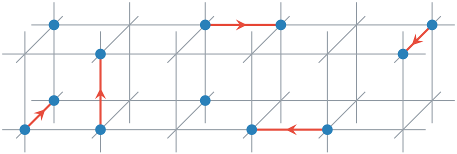
Discretise space and time

Quarks

$$\mathcal{L}_F = \bar{\psi}(x)\big(i\gamma_\mu\partial^\mu + \gamma_\mu A^\mu\big)\psi(x) + m_q\bar{\psi}(x)\psi(x)$$

$$\Big\downarrow \; x \to an$$

$$L_F \sim \sum_\mu \bar{\psi}(n)U_\mu(n)\psi(n+\mu) + m_q\bar{\psi}(n)\psi(n)$$



Lattice QCD: Basics 2

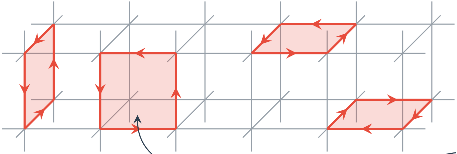
Discretise space and time

Gluons

$$\mathcal{L}_G = \frac{1}{4}\operatorname{tr} F_{\mu\nu}(x) F^{\mu\nu}(x)$$

$$\Big\downarrow \; x \to an$$

$$L_G \sim \beta \sum_{\mu,\nu} \operatorname{tr} U_\mu(n) U_\nu(n+\mu) U_\mu^\dagger(n+\nu) U_\nu^\dagger(n)$$



plaquette

Lattice QCD: Basics 3

Partition function

$$\mathcal{Z} = \int DU_\mu D\bar{\psi} D\psi \ \exp\left\{-\bar{\psi} Q_{\text{uark}} \psi - S_{\text{gluon}}\right\}$$
$$= \int DU_\mu \det Q_{\text{uark}} \exp\left\{-S_{\text{gluon}}\right\}$$

Measure observables by running a Monte Carlo simulation using

Monte Carlo weight

$$\det Q_{\text{uark}} \exp\left\{-S_{\text{gluon}}\right\}$$

Lattice QCD: Challenges

Simulation cost

Counting the degrees of freedom of the system gives a staggering number

$$N = N_t \times N_x \times N_y \times N_z \times N_\gamma \times N_f \times N_c$$

Every simulation point must evaluate $\det Q_{\mathrm{uark}}$, which is a (sparse) $N \times N$ matrix

Lattice QCD: Challenges

The sign problem 1

Lattice QCD relies on Monte Carlo integration for the integral over link variables

$$Z = \int DU_\mu \det Q_{\text{uark}} e^{-S_{\text{gluon}}}$$

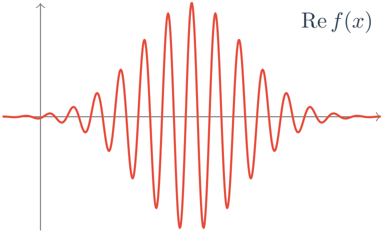
It only makes sense to use $\det Q_{\text{uark}} e^{-S_{\text{gluon}}}$ as a Monte Carlo weight if it is real

Lattice QCD: Challenges
The sign problem 2

The link integral also suffer from exponensial cancellations

Example

$$f(x) = e^{-x^2+i\theta x}$$



The Effective Theory

The Effective Lattice Theory

Our goal

- Integrate out all spatial gauge links

$$\mathcal{Z} = \int DU_\mu \exp\left\{-S_{\text{action}}\right\}$$
$$= \int DU_0 \exp\left\{-S_{\text{effective action}}\right\}$$

Strong Coupling Expansion

Expansion around $\beta = \frac{2N_c}{g^2} = 0$

Recap

$$L_G \sim \beta \sum_{\mu,\nu} \operatorname{tr} U_\mu(n) U_\nu(n+\mu) U_\mu^\dagger(n+\nu) U_\nu^\dagger(n)$$

This is an expansion in the number of plaquettes on the lattice

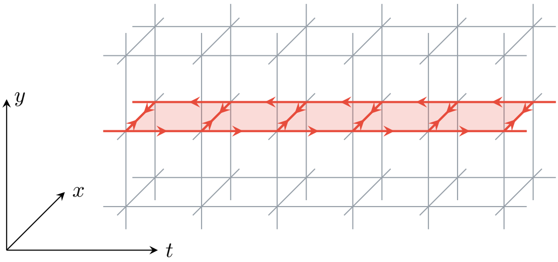
Hopping Parameter Expansion

One can rewrite the fermion matrix Q_{uark} as

$$\det Q_{\mathrm{uark}} = \exp\left\{ -\sum_{n=1}^{\infty} \frac{1}{n} \kappa^n \operatorname{tr} H_{\mathrm{op}}^n \right\}$$

where H_{op} translates the quark one lattice spacing and $\kappa \sim 1/m_q$

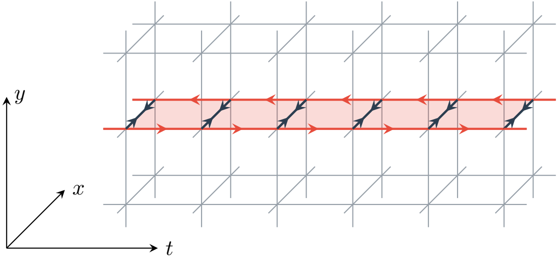
The Effective Lattice Theory
Pure gluon contributions



Put a line of plaquettes in the time direction

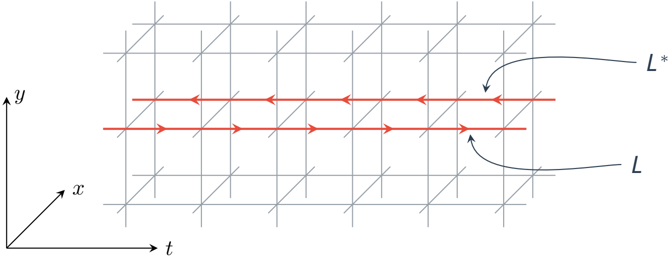
The Effective Lattice Theory

Pure gluon contributions



Integrate over all spatial gauge links

The Effective Lattice Theory
Pure gluon contributions



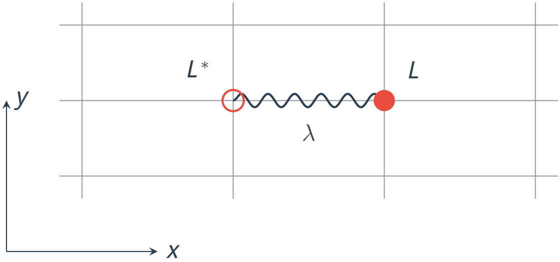
What remains is an interaction between Polyakov Loops

The Effective Lattice Theory

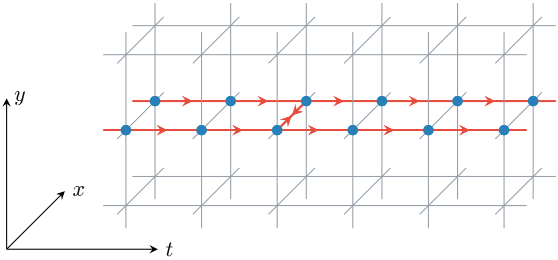
Pure gluon contributions

Effective Gluon Interactions

$$S_{\text{eff gluon}} \sim \lambda \sum_{\langle x,y \rangle} L(x) L^*(y)$$

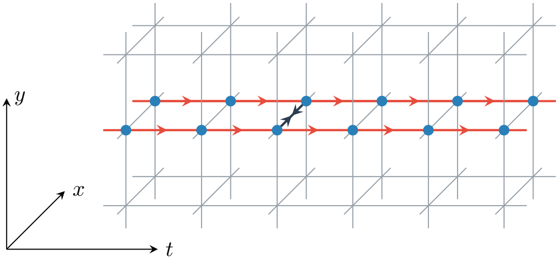


The Effective Lattice Theory
Pure quark contributions



Can produce a closed quark loop with multiple temporal windings

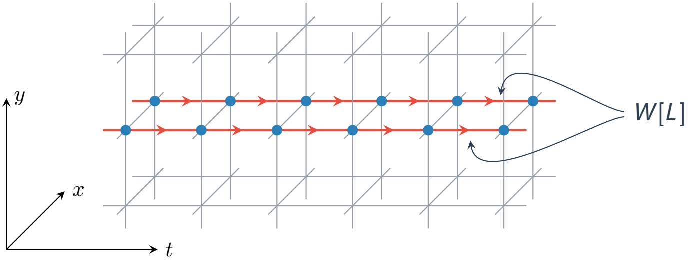
The Effective Lattice Theory
Pure quark contributions



Once again integrate out spatial links

The Effective Lattice Theory

Pure quark contributions



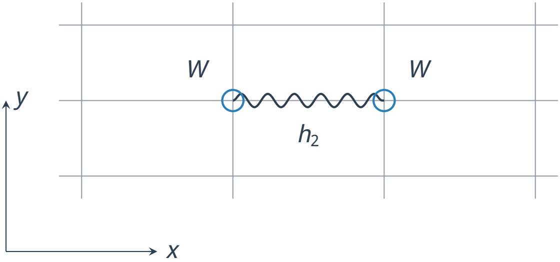
Producing an interaction between the W objects

The Effective Lattice Theory

Pure quark contributions

Effective Gluon Interactions

$$S_{\text{eff quarks}} \sim h_2 \sum_{\langle x,y \rangle} W(x)W(y)$$



The Effective Lattice Theory

Final form

The Effective Action

$$\mathcal{Z} = \int \prod_x \mathrm{d}L(x) \, \exp\left\{-S_{\text{eff action}}\right\}$$

$$S_{\text{eff action}} \sim \lambda \sum_{\langle x,y \rangle} L(x) L^*(y) + h_2 \sum_{\langle x,y \rangle} W(x) W(y)$$

Two options to proceed from here

1. Simulate the effective action
2. Analytically calculate it with a linked cluster expansion

The Effective Lattice Theory

Final form

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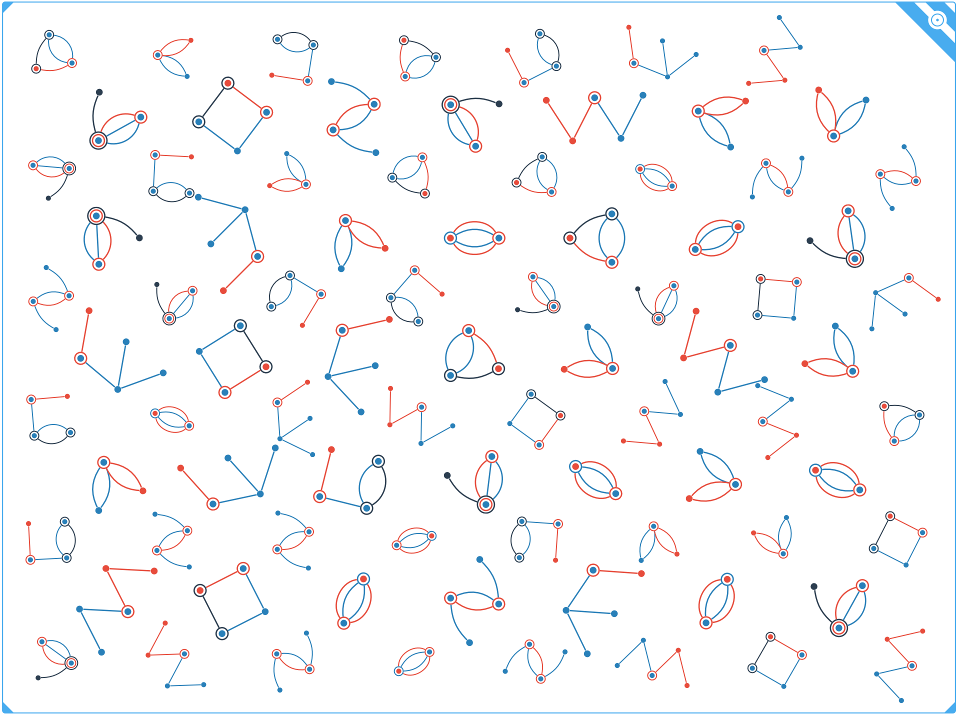
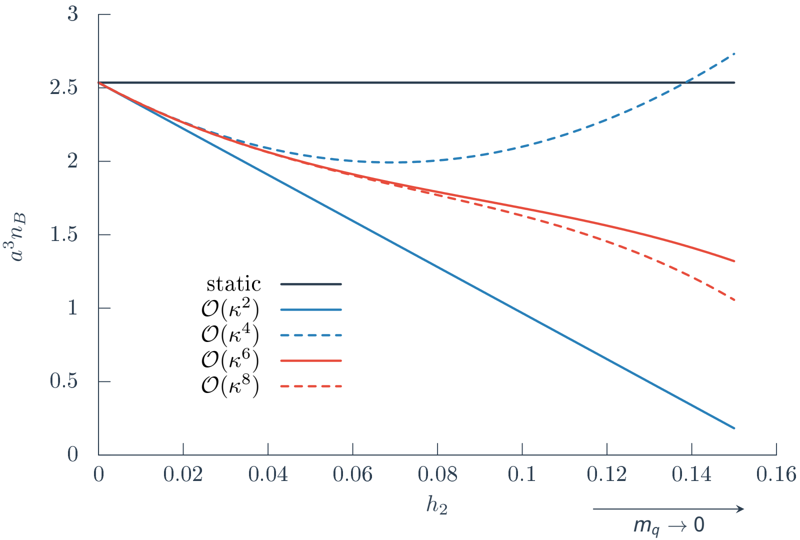
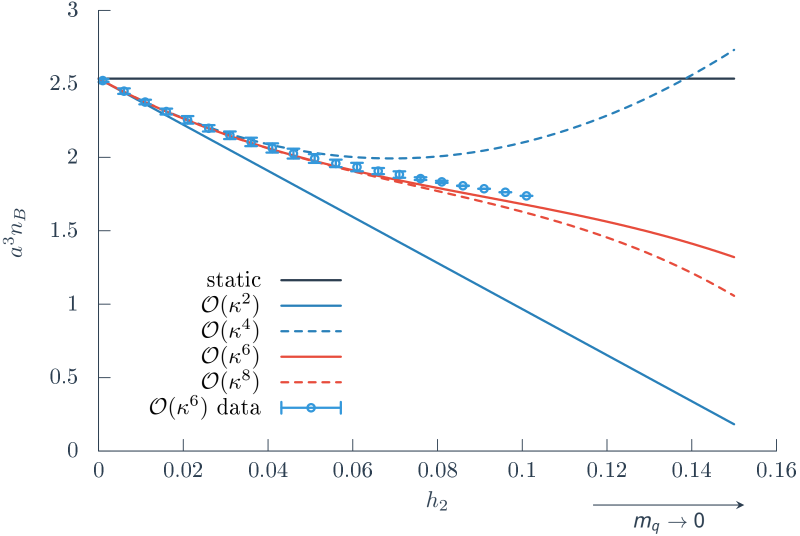


Results

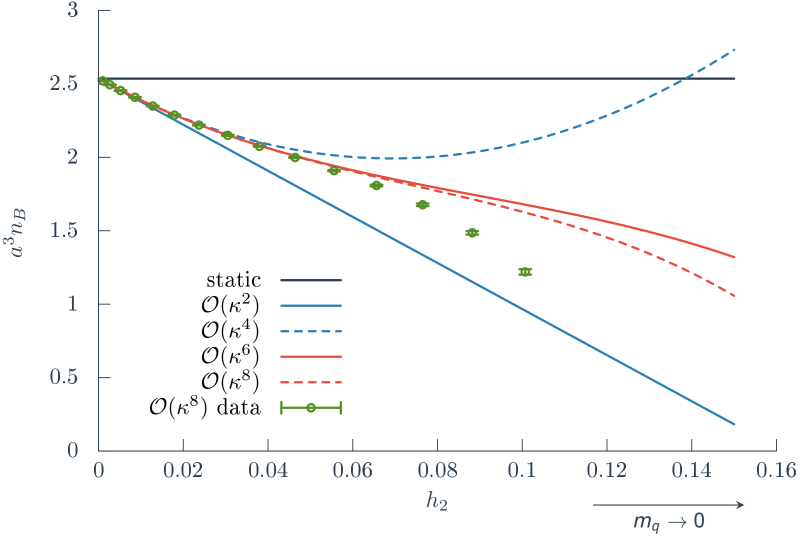
Comparison with Simulation



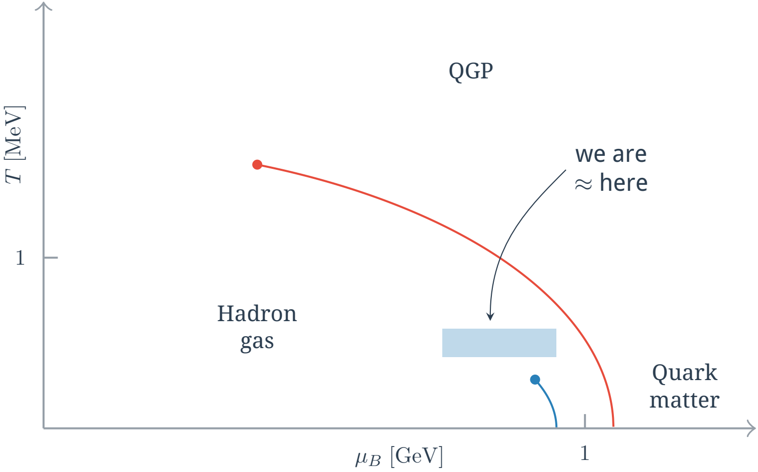
Comparison with Simulation



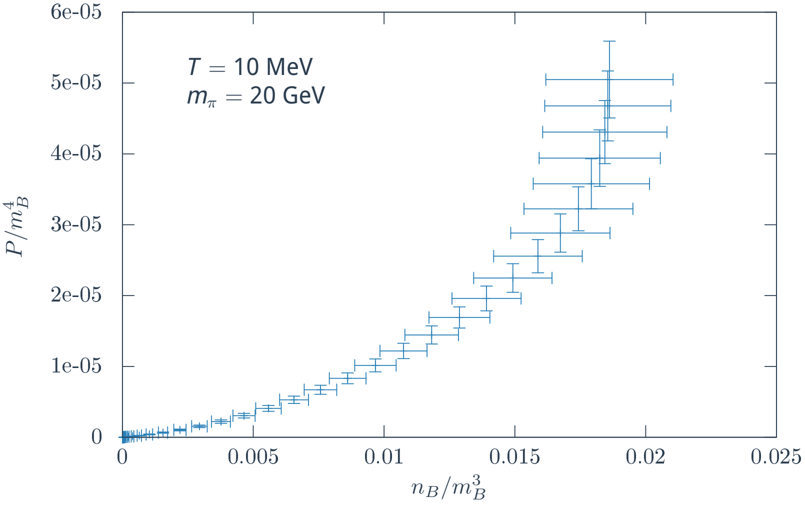
Comparison with Simulation



Equation of State



Equation of State



Conclusion

Summary & Outlook

Summary

- Introduced two expansions for the lattice action
  - Strong coupling expansion
  - Hopping expansion
- Created a dimensionally reduced effective lattice theory

Summary & Outlook

Outlook

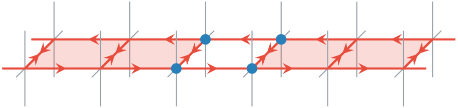
- Look for resummations to help convergence
- Calculate higher order mixing terms
- Move more in the direction of the simulations

Backup slides

The Effective Lattice Theory

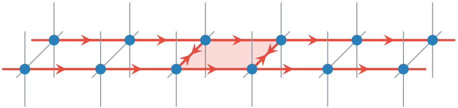
Mixed contributions

Correction to λ



- Rescales λ
- $\lambda \to \lambda(\kappa)$

Correction to h_2



- Rescales h_2
- $h_2 \to h_2(\beta)$

EoS in lattice units