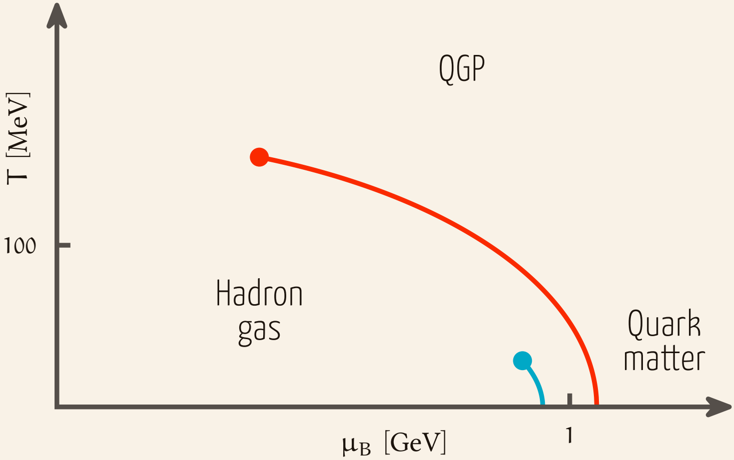
# Exploring the sign problem with Complex Langevin

**Aleksandra Rylund Glesaaen**
with S. Zafeiropoulos (Frankfurt U.),
J. Verbaarschot (Stony Brook U.)

Palaver
July 4th 2016

# Introduction

# QCD in the dense regime



# The sign problem
## Conceptual challenges

### The QCD integration measure:

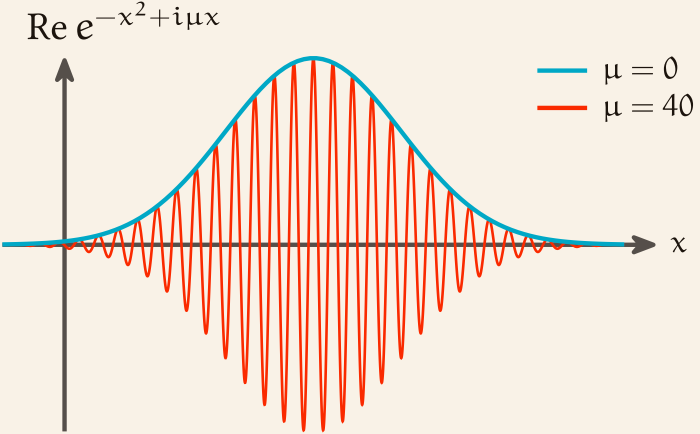
$$\exp(-S_{\mathrm{QCD}}) = \det Q_{\mathrm{uark}}(\mu_B)\ \exp(-S_{\mathrm{g}})$$

### Monte Carlo methods rely on sampling configuration with a probability distribution

$$P(\mathrm{conf}) \sim \exp\left(-S(\mathrm{conf})\right)$$

**However for** $\mathrm{Re}(\mu_B) \neq 0:\ \det Q_{\mathrm{uark}}(\mu_B) \in \mathbb{C}$

# The sign problem
## Conceptual challenges



# The sign problem
## Numerical approaches

Conventional/Monte Carlo methods:

- Reweighting
- Taylor expansion around $\mu_B = 0$
- Imaginary chemical potential
- Strong coupling expansions
- Mean field analyses

# The sign problem
## Numerical approaches

**Alternative methods:**

- Stochastic quantisation / Complex Langevin
- Lefschetz thimble
- Canonical ensembles
- Dual variables
- Density of states

# The sign problem
## Numerical approaches

**Alternative methods:**

- Stochastic quantisation / Complex Langevin
- Lefschetz thimble
- Canonical ensembles
- Dual variables
- Density of states

Current study

# Complex Langevin and Random Matrix Theory

# Stochastic quantisation

**Introduce a fictional flow time $\alpha$**

$$\phi_i(x) \to \phi_i(x, \alpha)$$

**and the stochastic flow in $\alpha$**

$$\frac{\partial \phi_i(x,\alpha)}{\partial \alpha} = -\frac{\delta S[\phi_i]}{\delta \phi_i} + \eta(x,\alpha)$$

**then if $\frac{\delta S}{\delta \phi} \in \mathbb{R}$ [Damgaard and Huffel, 1987]**

$$\lim_{\alpha \to \infty} \left\langle O[\phi(x,\alpha)]_i \right\rangle_\eta = \left\langle O[\phi(x)]_i \right\rangle$$

# Stochastic quantisation

**Introduce a fictional flow time $\alpha$**

$$\phi_i(x) \to \phi_i(x, \alpha)$$

**and the stochastic flow in $\alpha$**

Stochastic noise

$$\frac{\partial \phi_i(x,\alpha)}{\partial \alpha} = -\frac{\delta S[\phi_i]}{\delta \phi_i} + \eta(x,\alpha)$$

**then if** $\frac{\delta S}{\delta \phi} \in \mathbb{R}$ [Damgaard and Huffel, 1987]

$$\lim_{\alpha \to \infty} \left\langle O[\phi(x,\alpha)]_i \right\rangle_\eta = \left\langle O[\phi(x)]_i \right\rangle$$

# Complex Langevin

Much the same as Real Langevin, however

$$\frac{\delta S}{\delta \phi} \in \mathbb{C} \quad \Rightarrow \quad \phi_i(x, \alpha) \in \mathbb{C}$$

proof for convergence to the right theory no longer holds, exceptions have been found [Ambjørn and Yang, 1985]

# Why stochastic quantisation?

How the CL converges to the wrong theory not well understood

Much interest in methods for ”repairing” CL

- Gauge cooling
- Utilising the Lefschetz thimbles
- ... ?

Need a signal for failure applicable to QCD

# Random matrix theory

Start with a simpler system that also suffer from a strong sign problem

Same flavour symmetries as QCD (in the $\epsilon$-regime)

Numerically a lot cheaper than full lattice studies

First attempts studying the Osborn model
[Mollgaard and Splittorff, 2013-2014; Nagata, Nishimura and Shimasaki, 2015-2016]

# The Stephanov model

## Definition [Stephanov, 1996]

$$\mathcal{Z} = \int [dW]\, e^{-N\Sigma^2 \operatorname{tr} W^\dagger W} \det{}^{N_f} \begin{pmatrix} M & iW + \mu \\ iW^\dagger + \mu & M \end{pmatrix}$$

**For random matrices** $W \in M_{\mathbb{C}}(N, N)$

**Define chiral condensate and baryon number density**

$$\langle \bar{\eta}\eta \rangle = \partial_M \log \mathcal{Z}$$
$$\langle \eta^\dagger \eta \rangle = \partial_\mu \log \mathcal{Z}$$

# The Stephanov model
## Properties

### Solvable via bosonisation

[Stephanov, 1996; Halasz, Jackson and Verbaarschot, 1997]

$$\mathcal{Z} = \int \mathrm{d}\sigma \mathrm{d}\sigma^* \, e^{-N\sigma^2} (\sigma^*\sigma + m(\sigma + \sigma^*) + m^2 - \mu^2)^N$$

**where** $\sigma \in M_{\mathbb{C}}(N_f, N_f)$, **for** $N_f = 1$

$$\mathcal{Z} = \pi e^{-Nm^2} \int_0^\infty \mathrm{d}u \, (u - \mu^2)^N I_0(2mN\sqrt{u}) e^{-Nu}$$

# The Stephanov model
## Properties

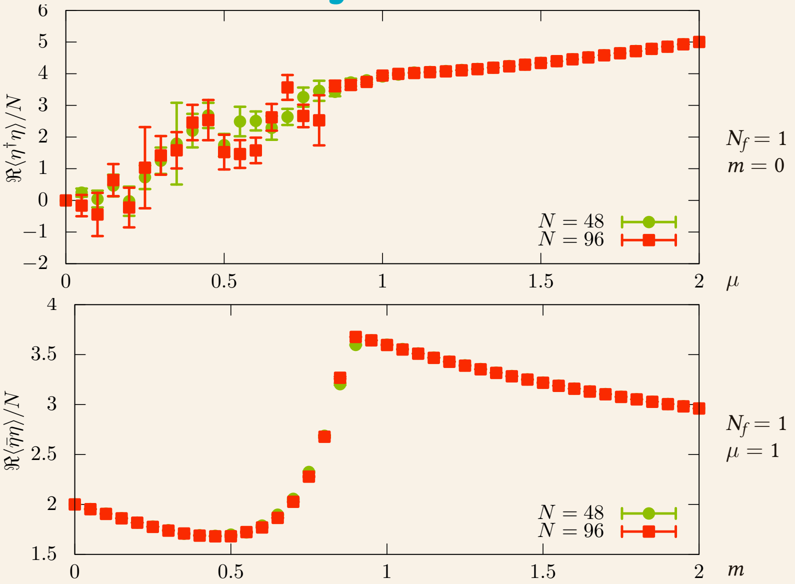
The model has a phase transition as it develops non-zero baryon number density

Solvable in the thermodynamic limit via a saddle point approximation

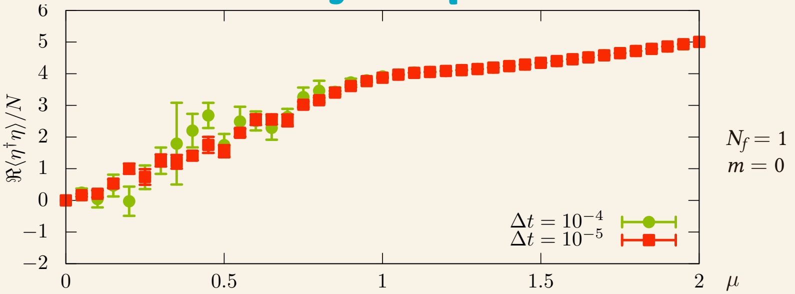
In the chiral limit one finds $\mu_c = 0.572...$ and $\mu_c = i$

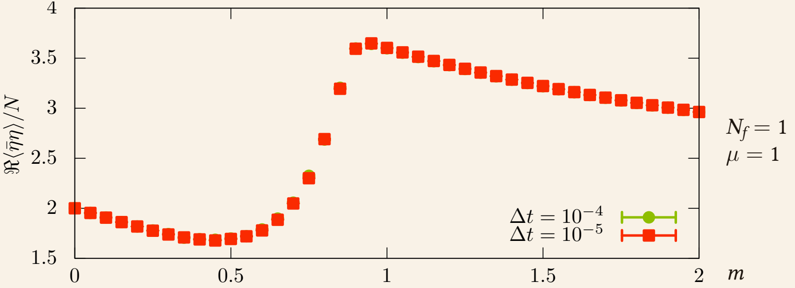
# Results and analysis

# Numerical validity, matrix size

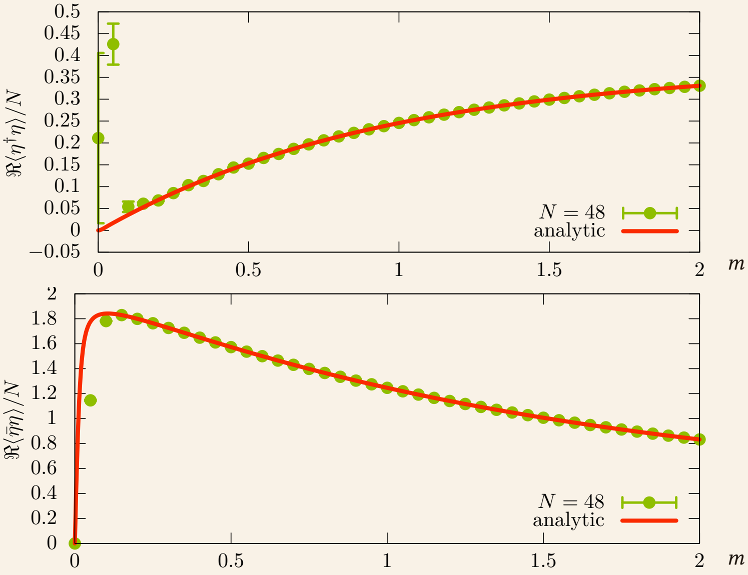


# Numerical validity, step size

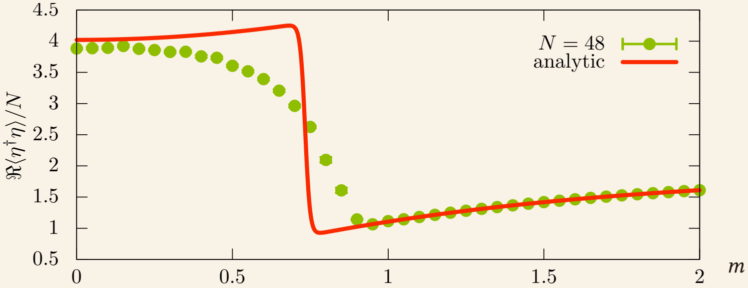


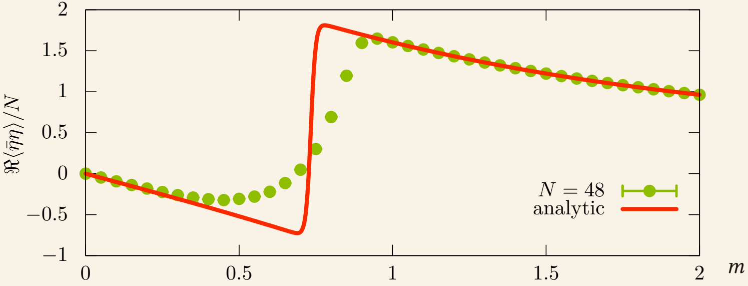



# Mass scans @ $\mu = 0.2$

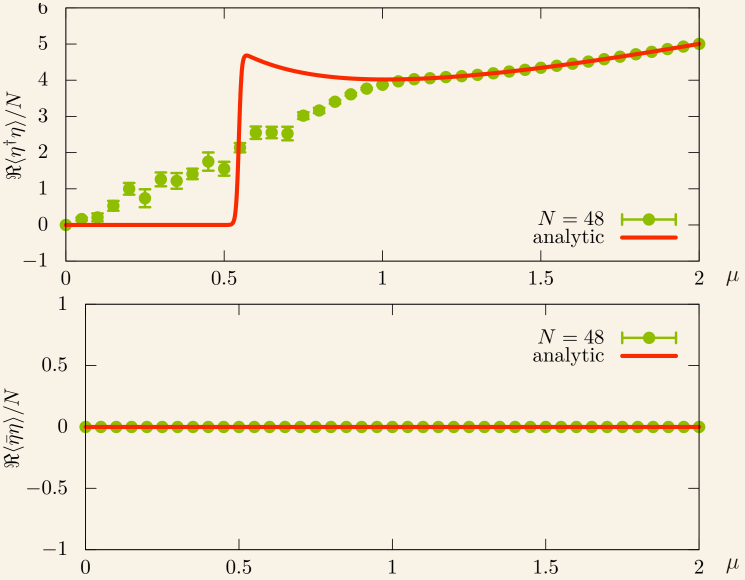


# Mass scans @ $\mu = 1.0$

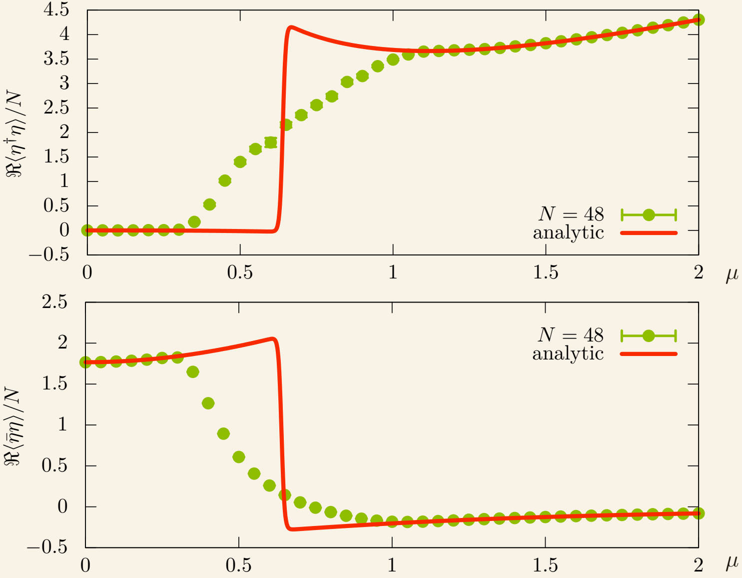


# Chemical potential scans @ $m = 0.0$



# Chemical potential scans @ $m = 0.2$



Converge to the wrong theory

Which theory do we converge to?

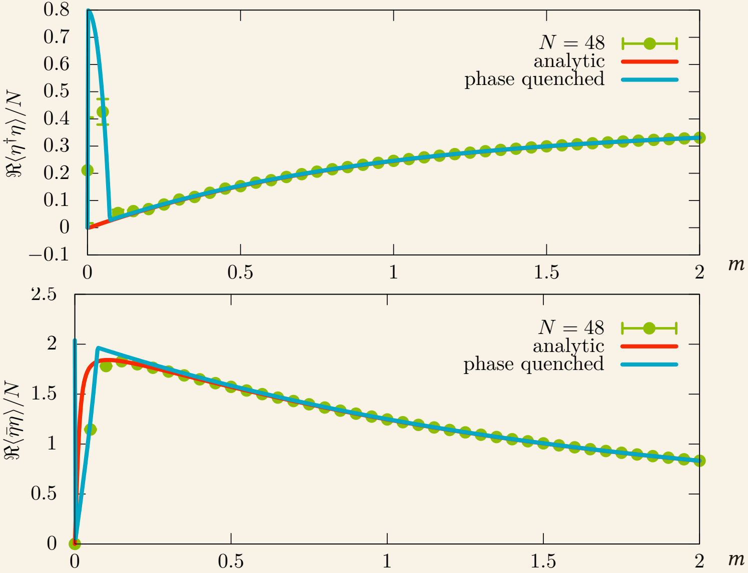
# The phase quenched theory

$$\mathcal{Z}_{\mathrm{PQ}} = \int [\mathrm{d}U] \left| \det Q_{\mathrm{uark}}(\mu_B) \right| \exp(-S_{\mathrm{g}})$$

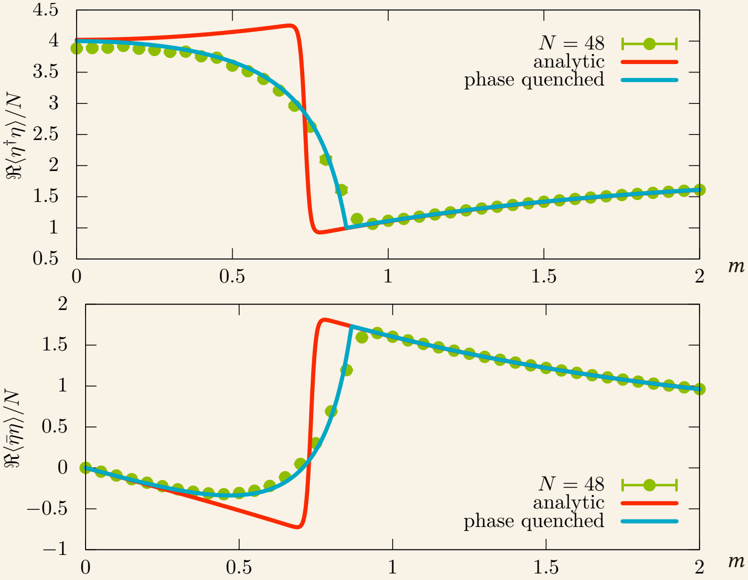
For $N_f = 2$ it corresponds to studying finite isospin chemical potential

Has a very different phase diagram than QCD

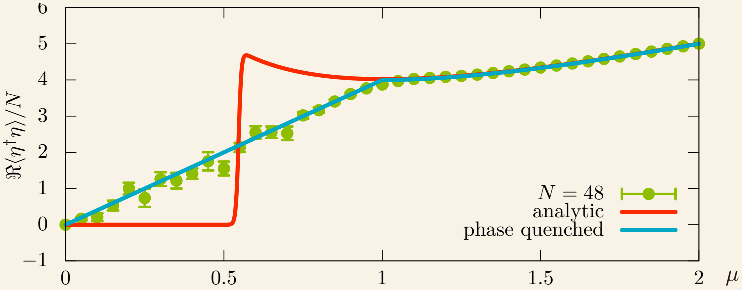
# Mass scans @ $\mu = 0.2$

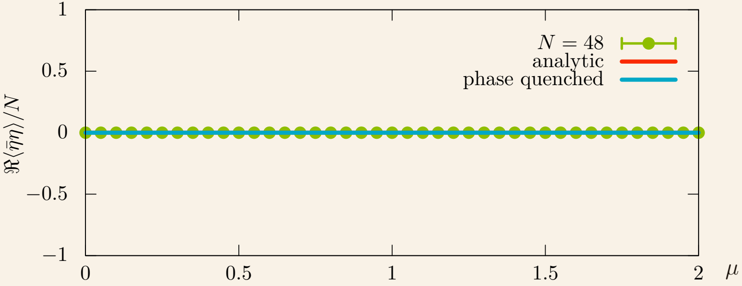


# Mass scans @ $\mu = 1.0$

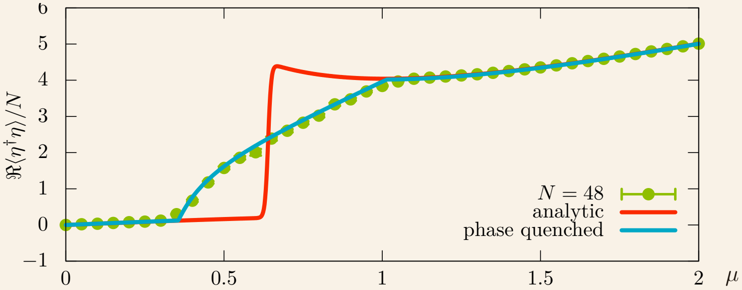


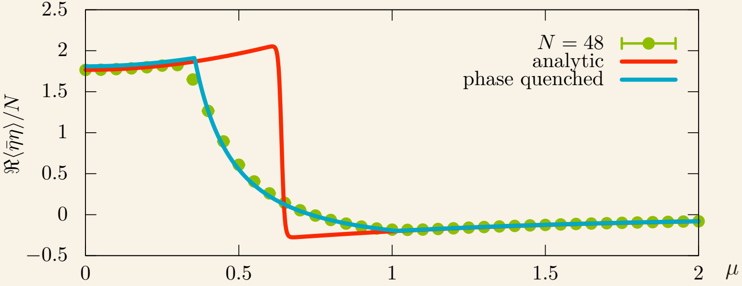
# Chemical potential scans @ $m = 0.0$



# Chemical potential scans @ $m = 0.2$



# Conclusions and outlook

# Conclusion

- Studied Complex Langevin for an interesting random matrix theory
- Compared to analytical results for said theory
- Observed that CL converges to the phase quenched theory

# Outlook

- Still an open question how to repair CL
- Currently working on various gauge cooling methods
- Looking into Lefschetz thimbles
- Open to suggestions!