Analytic computations of an effective lattice theory for heavy quarks

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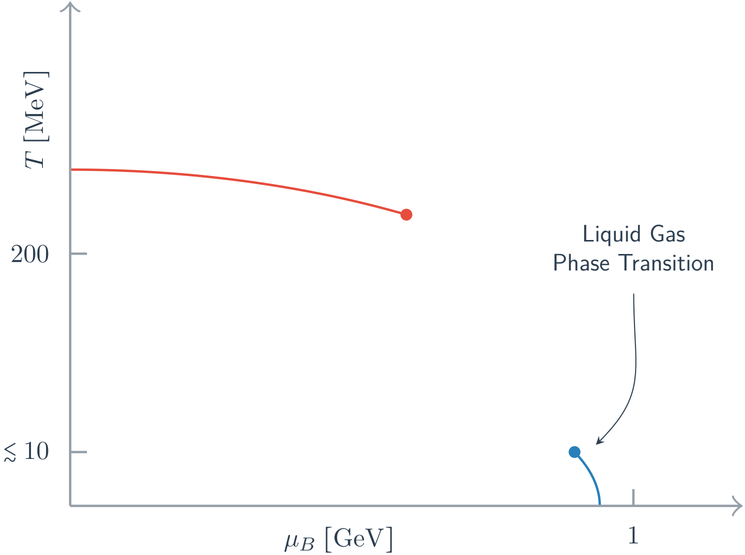
Heavy QCD Phase Diagram



Advantages of the Effective Theory

- Dimensionally reduced theory
  - 4D $\rightarrow$ 3D
  - $U_\mu(x) \rightarrow L(x)$
- Very mild sign problem, most gauge fields integrated analytically
- Want to study the very dense limit, liquid gas transition

The Effective Theory

The Effective Lattice Theory

Effective Theory

- Integrate out all spatial gauge links

$$
\begin{aligned}
\mathcal{Z} &= \int DU_\mu \exp\left\{-S_{\text{action}}\right\} \\
&= \int DU_0 \exp\left\{-S_{\text{effective action}}\right\}
\end{aligned}
$$

Using:

- The strong coupling expansion
- The hopping parameter expansion

Effective Theory

$$\mathcal{Z} = \int \prod_x \mathrm{d}L(x) \exp\left\{-S_{\text{eff action}}\right\} \qquad (\dagger)$$

- Previous Talk: Monte Carlo simulations of $(\dagger)$
- Current Talk: Analytic calculation of $\mathcal{Z}$

The Effective Theory Action

$$S_{\text{eff action}} = S_0\left[L\right] + S_I\left[L\right]$$

Where $S_I\left[L\right]$ is made up of interactions at varying distances

$$S_I\left[L\right] = \sum_{\text{terms}} \sum_{\text{dof}} v_i(1, 2, \ldots, n_i)\phi_1\left[L\right]\phi_2\left[L\right] \cdots \phi_{n_i}\left[L\right]$$

Can be represented with connected graphs

The Effective Theory Action

$$\boxed{S_{\text{eff action}} = S_0[L] + S_I[L]}$$

Where $S_I[L]$ is made up of interactions at varying distances

$$\boxed{S_I[L] = \sum_{\text{terms}} \sum_{\text{dof}} v_i(1, 2, ..., n_i)\phi_1[L]\phi_2[L]\cdots\phi_{n_i}[L]}$$

Can be represented with connected graphs

Remember to say that the effective couplings V_i are themselves functions of κ and β, and we can therefore carry out a consistent expansion.

Talk about the expansion point. What is S_0 physically? Free hadron gas.

The Effective Theory Action

$$S_l[L] = \sum_{\text{terms}} \sum_{\text{dof}} v_i(1,2,...,n_i)\phi_1[L]\phi_2[L]\cdots\phi_{n_i}[L]$$

In our theory:

- $v_i(1,2,...n_i) \rightarrow \{\lambda_i, h_i\} \times \text{geometry}$
- $\phi_i \rightarrow \{L_i, L_i^*, W_i\}$

Analytic Calculations

N-point Linked Cluster Expansion

Classical Linked Cluster Expansion

The action consists of two-point interactions which can be expanded in a set of connected graphs.

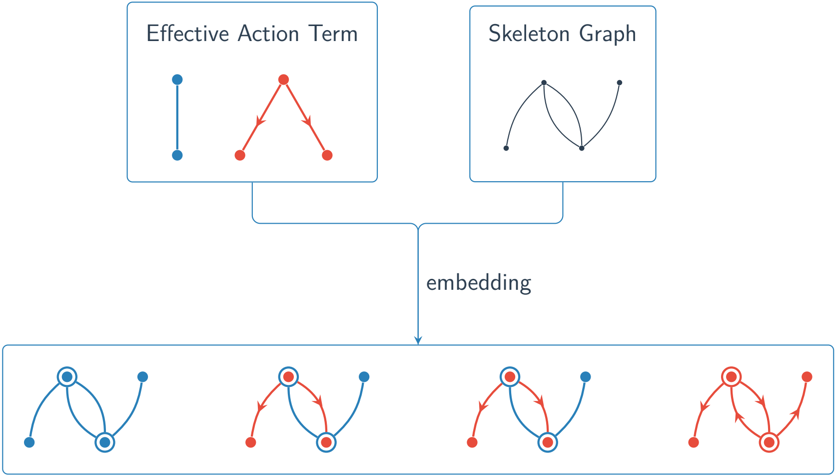
Our Problem

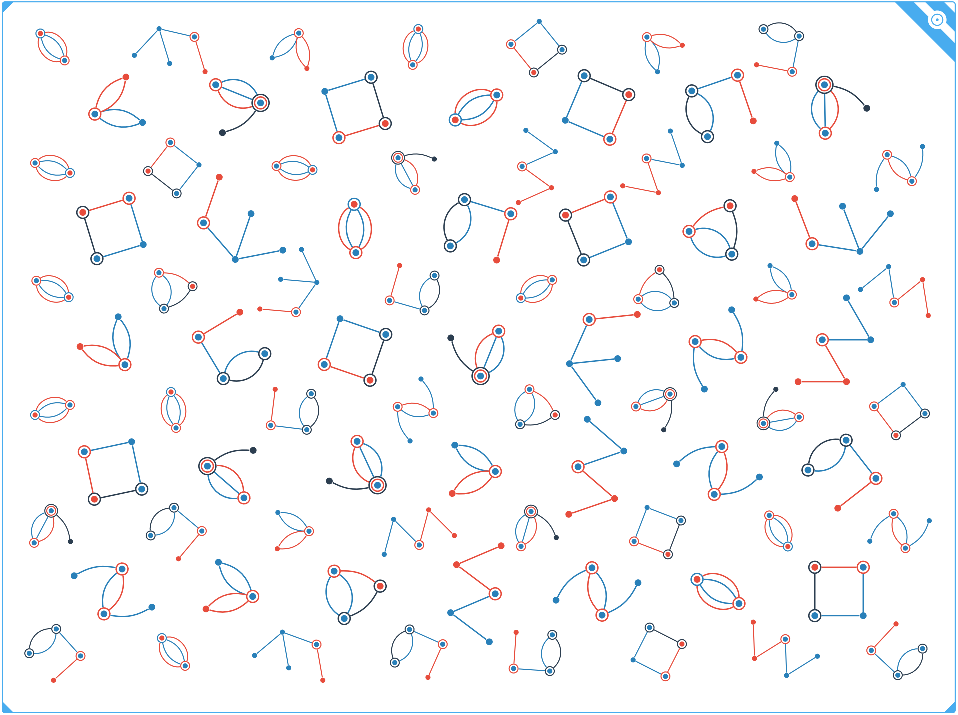
The action contains *n*-point interactions that we can embed on a set of connected graphs.

$\hookrightarrow$ Two step embedding

Analytic Calculations

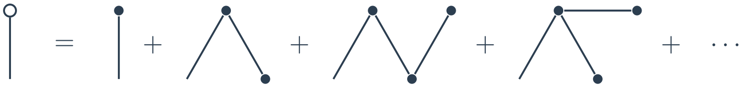
N-point Linked Cluster Expansion





The power of resummations

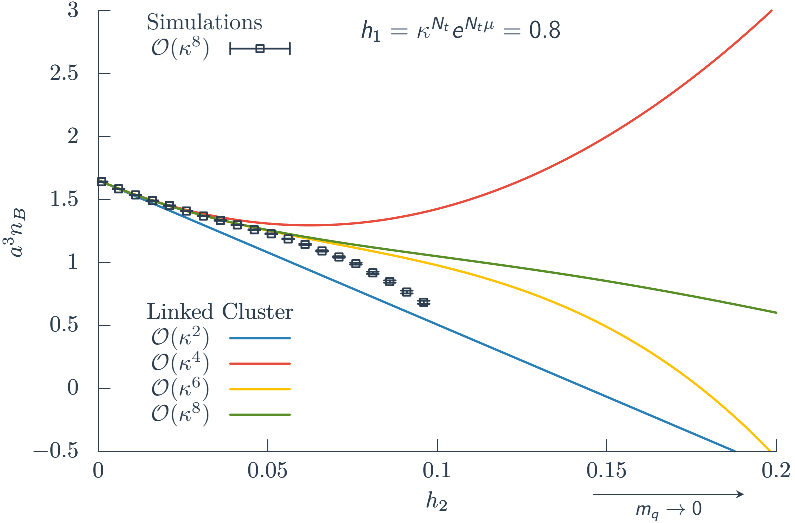
Using the resummed Linked Cluster Expansion as motivation



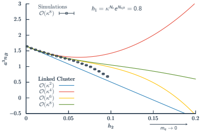
We can do the same resummation for the effective action itself, incorporating long-range effects

Results

Convergence

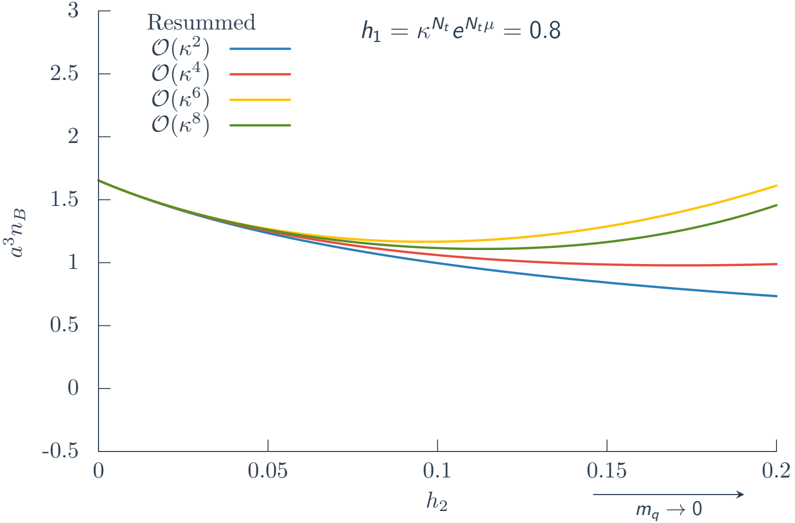


Convergence

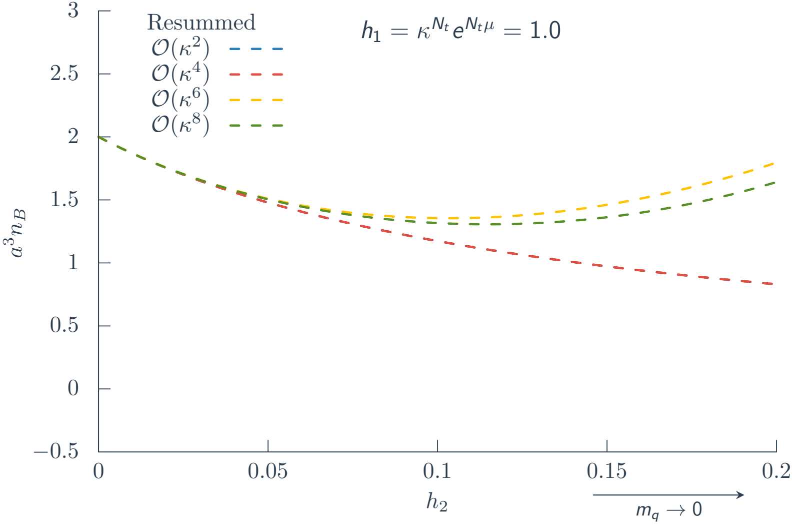


Say something about the fact that higher h_2 means smaller quark masses.

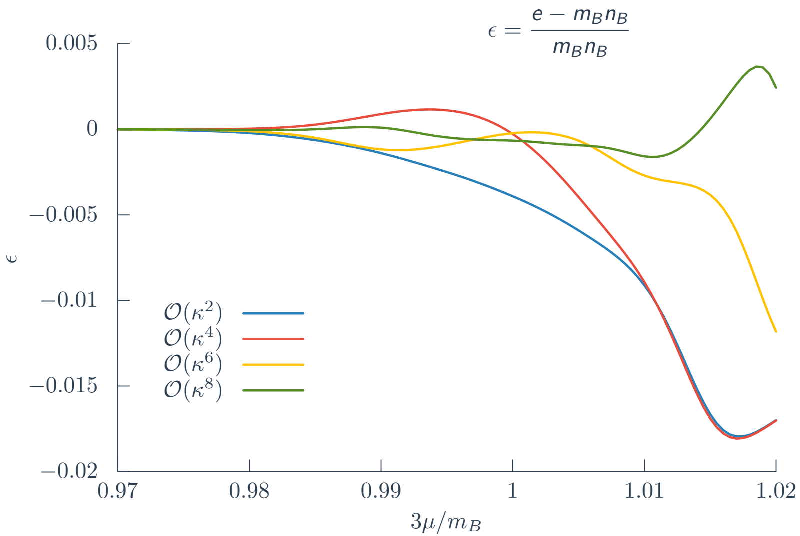
Effect of the resummations



Effect of the resummations



Binding energy



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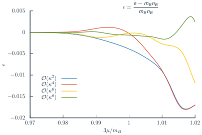
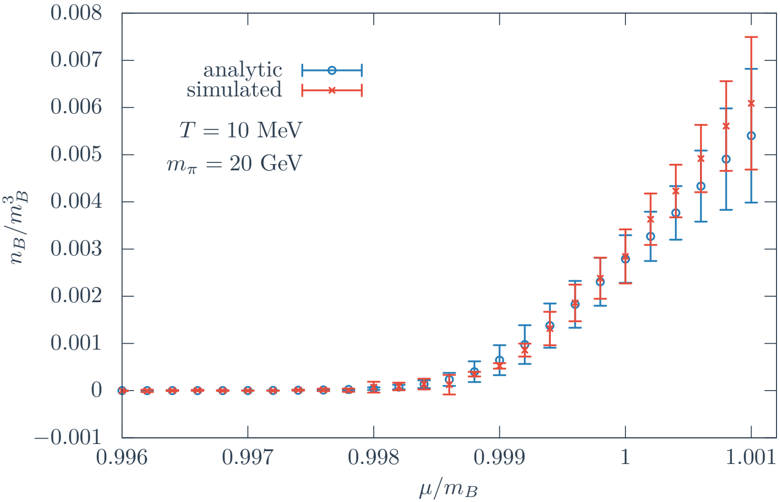
Binding energy

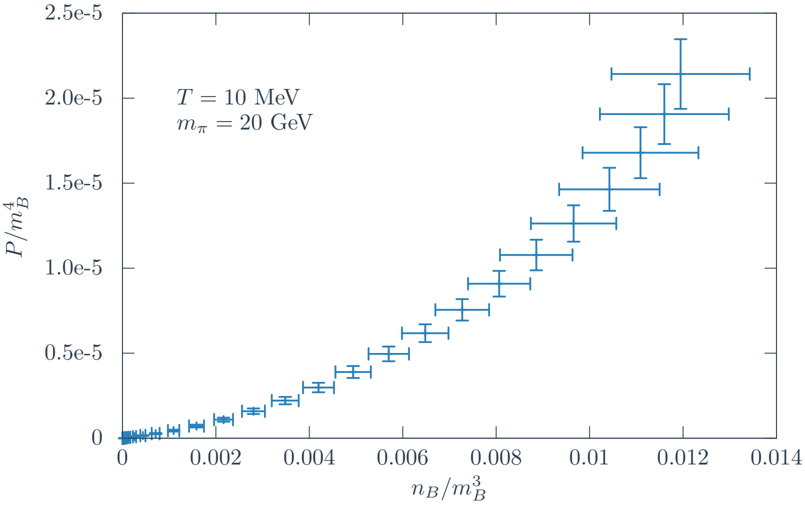


Plot parameters: $\kappa = 0.08$, $N_t = 50$, $\beta = 0$

Continuum comparison



Continuum Equation of State



Conclusion

Summary & Outlook

Summary

- Introduced the effective dimensionally reduced lattice theory
- Looked at how a consistent analytic calculation could be carried out
- Demonstrated convergence and comparisons with numerics

Summary & Outlook

Outlook

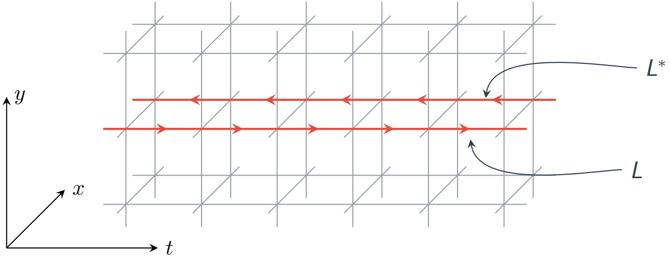
- Use the analytic results as a tool to study the characteristics of the effective theory
- Find analytic resummation schemes to incorporate long-range effects

Thank you!

Backup slides

The Effective Lattice Theory

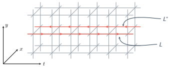
Pure gluon contributions



What remains is an interaction between Polyakov Loops

The Effective Lattice Theory

Pure gluon contributions



What remains is an interaction between Polyakov Loops

As mentioned earlier, we are only interested in quantities that contribute to the thermodynamic of the system. For our system, that means quantities which span the full temporal direction, and thus picks up a temperature dependent component in the infinite volume limit.

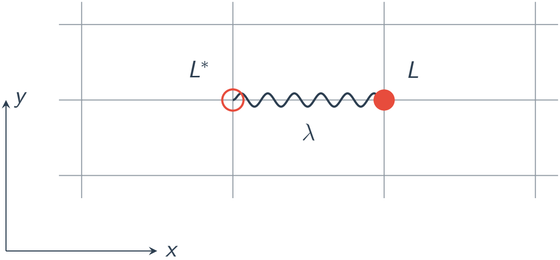
For pure gauge action, in the strong coupling expansion, this means a plane of plaquettes spanning the full temporal direction, as shown in the figure. We then integrate out the spatial links present in this strip of plaquettes, and are left with the Polyakov loops. The final result will thus be a nearest neighbour interaction between two Polyakov loops, or a continuos spin-system on a three dimensional lattice.

The Effective Lattice Theory

Pure gluon contributions

Effective Gluon Interactions

$$S_{\text{eff gluon}} \sim \lambda \sum_{\langle x,y \rangle} L(x) L^*(y)$$



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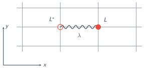

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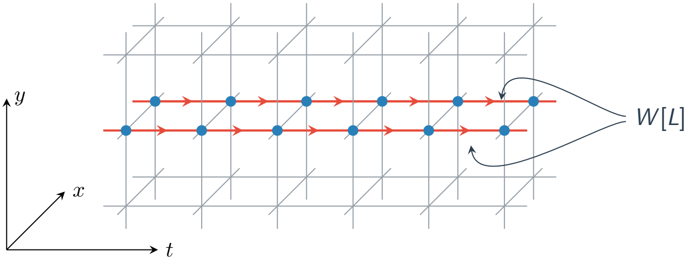
The Effective Lattice Theory

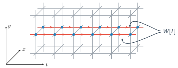
Pure quark contributions



Producing an interaction between the W objects

The Effective Lattice Theory

Pure quark contributions



Producing an interaction between the W objects

For the fermions we are in very much the same situation. We need a quantity that spans the temporal direction of the lattice. The simplest such object is of course a single quark line, exclusively jumping in the temporal direction, going around the lattice. This is the contribution from static quarks, and this is of course included.

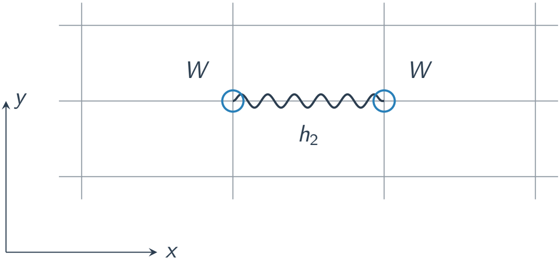
The next order term would be a quark that loops, jumps to a neighbouring site, loops, then jumps back, which is the term depicted in the figure. Here again we integrate out the spatial links, and are left with the interaction of two W-terms, the mathematical structure of which is not important for now.

The Effective Lattice Theory

Pure quark contributions

Effective Quark Interactions

$$S_{\text{eff quarks}} \sim h_2 \sum_{\langle x,y \rangle} W(x)W(y)$$



The Effective Lattice Theory

Pure quark contributions

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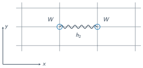

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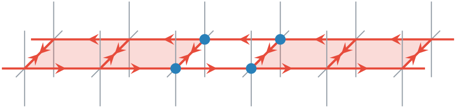
The Effective Lattice Theory

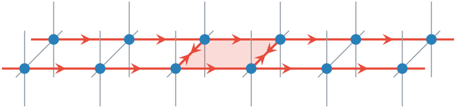
Mixed contributions

Correction to λ

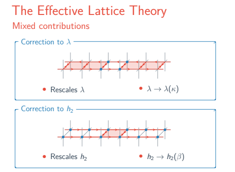


- Rescales λ
- $\lambda \to \lambda(\kappa)$

Correction to h_2



- Rescales h_2
- $h_2 \to h_2(\beta)$



One can of course mix the terms from the two different expansions we are carrying out. Two examples of the possible terms can be seen in the figures. For these two simple cases, the mixing will only contribute as a shift in the nearest neighbour couplings, and can thus be absorbed by those.

There are higher order mixed terms that create entierly new interactions, but those are of much higher order of what we have shown here.

I will probably skip this slide.

EoS in lattice units