

Analytic computations of an effective lattice theory for heavy quarks

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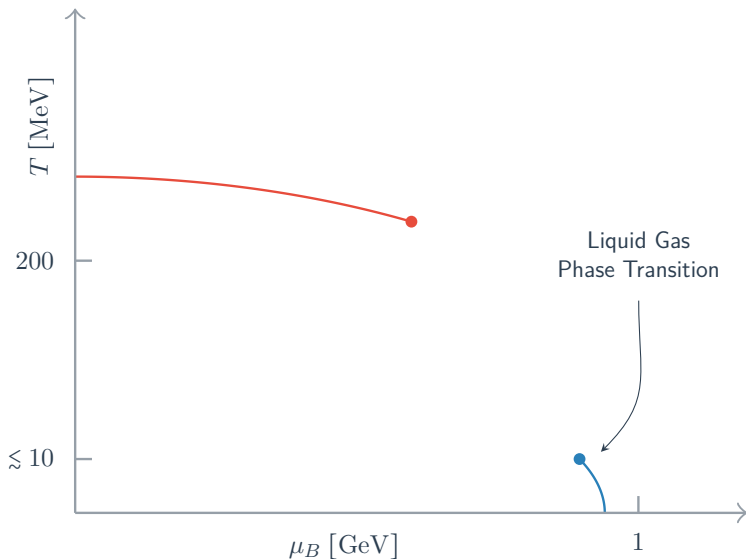


1 The Effective Theory

2 Results

3 Conclusion

Heavy QCD Phase Diagram



Advantages of the Effective Theory

- Dimensionally reduced theory
 - $4D \rightarrow 3D$
 - $U_\mu(x) \rightarrow L(x)$
- Very mild sign problem, most gauge fields integrated analytically
- Want to study the very dense limit, liquid gas transition

The Effective Theory

The Effective Lattice Theory

Effective Theory

- Integrate out all spatial gauge links

$$\begin{aligned}\mathcal{Z} &= \int DU_{\mu} \exp \{ -S_{\text{action}} \} \\ &= \int DU_0 \exp \{ -S_{\text{effective action}} \}\end{aligned}$$

Using:

- The strong coupling expansion
- The hopping parameter expansion

Effective Theory

$$\mathcal{Z} = \int \prod_x dL(x) \exp \{ -S_{\text{eff action}} \} \quad (\dagger)$$

- Previous Talk: Monte Carlo simulations of (\dagger)
- Current Talk: Analytic calculation of \mathcal{Z}

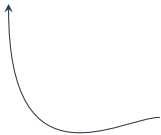
The Effective Theory Action

$$S_{\text{eff action}} = S_0[L] + S_I[L]$$

Where $S_I[L]$ is made up of interactions at varying distances

$$S_I[L] = \sum_{\text{terms}} \sum_{\text{dof}} v_i(1, 2, \dots, n_i) \phi_1[L] \phi_2[L] \cdots \phi_{n_i}[L]$$

Can be represented
with connected graphs



The Effective Theory Action

$$S_I[L] = \sum_{\text{terms}} \sum_{\text{dof}} v_i(1, 2, \dots, n_i) \phi_1[L] \phi_2[L] \cdots \phi_{n_i}[L]$$

In our theory:

- $v_i(1, 2, \dots, n_i) \rightarrow \{\lambda_i, h_i\} \times \text{geometry}$
- $\phi_i \rightarrow \{L_i, L_i^*, W_i\}$

Analytic Calculations

N-point Linked Cluster Expansion

Classical Linked Cluster Expansion

The action consists of two-point interactions which can be expanded in a set of connected graphs.

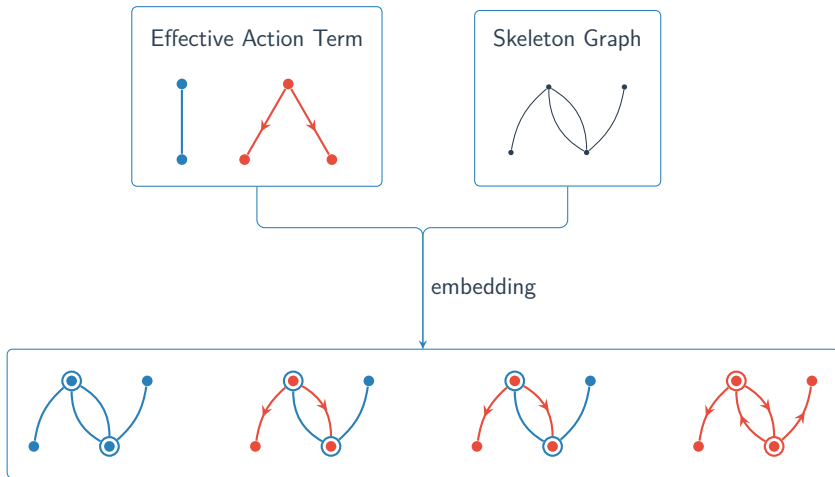
Our Problem

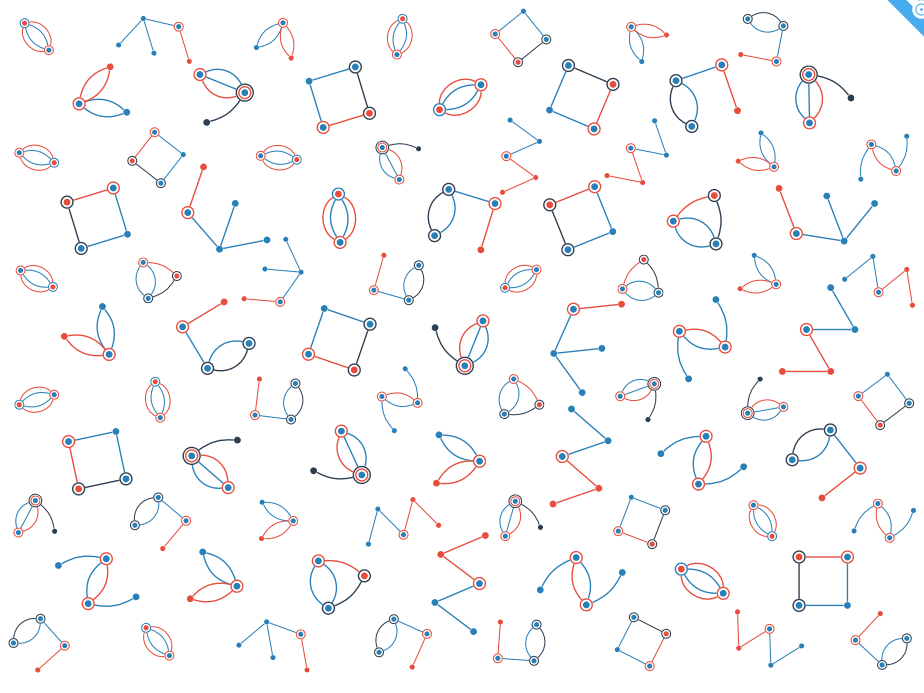
The action contains n -point interactions that we can embed on a set of connected graphs.

↳ Two step embedding

Analytic Calculations

N-point Linked Cluster Expansion





The power of resummations

Using the resummed Linked Cluster Expansion as motivation

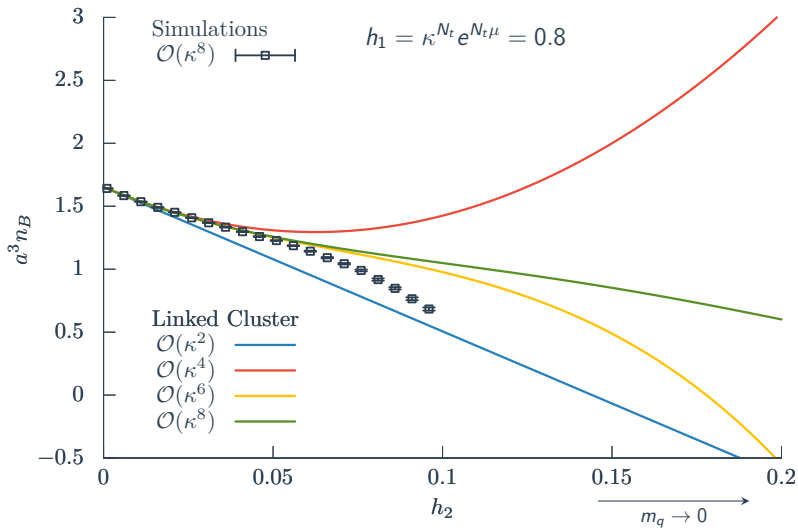
The diagram shows a series of terms in a summation. On the left is a vertical line with an open circle at the top. This is followed by an equals sign, then a series of terms separated by plus signs. The first term is a vertical line with a solid black dot at the top. The second term is a triangle with a solid black dot at each of its three vertices. The third term is a zigzag line with a solid black dot at each of its four vertices. The fourth term is a triangle with a horizontal line extending from its top vertex to a solid black dot, with another solid black dot at the bottom right vertex. The series ends with a plus sign and an ellipsis.

$$\text{Diagram 1} = \text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4} + \text{Diagram 5} + \dots$$

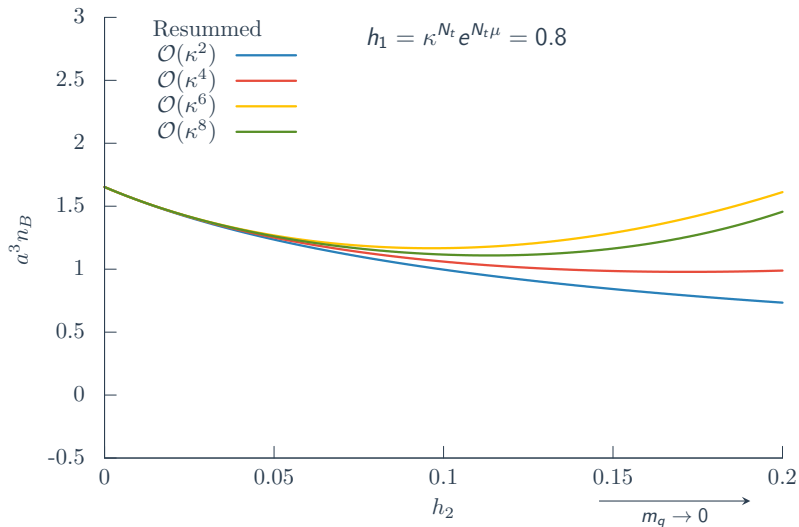
We can do the same resummation for the effective action itself, incorporating long-range effects

Results

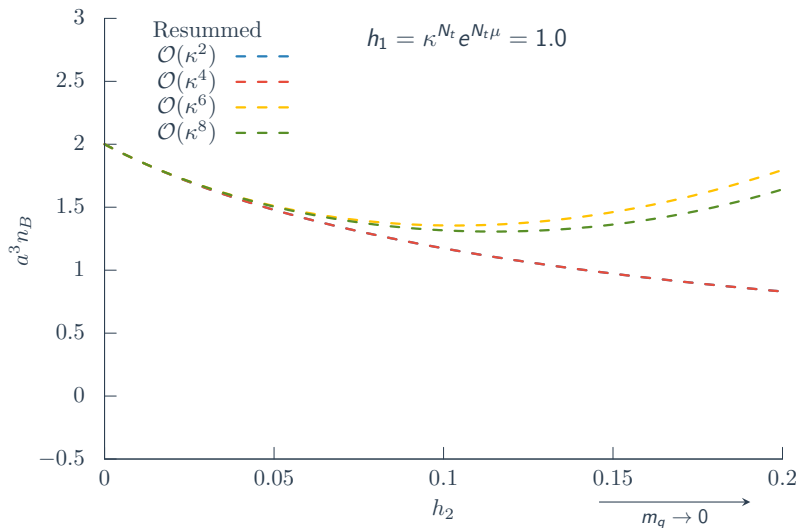
Convergence



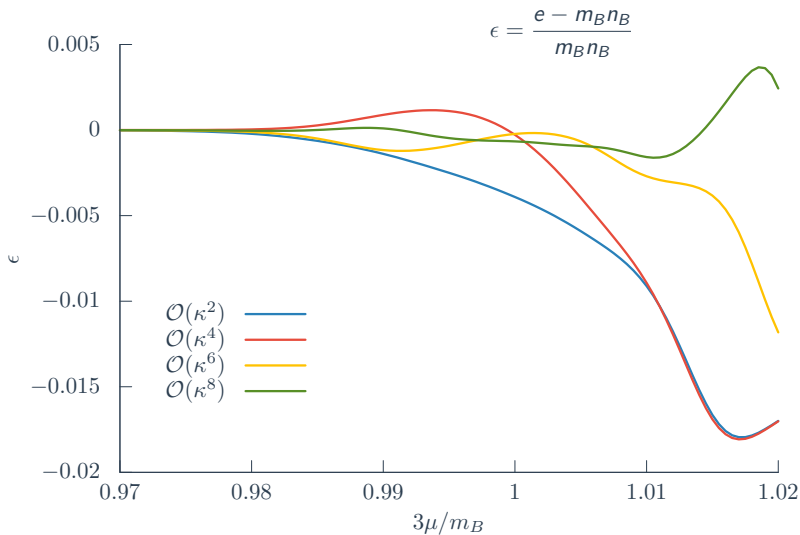
Effect of the resummations



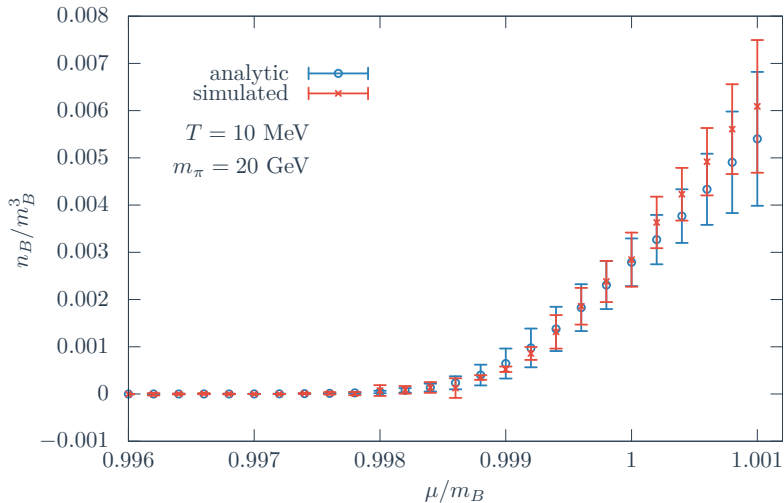
Effect of the resummations



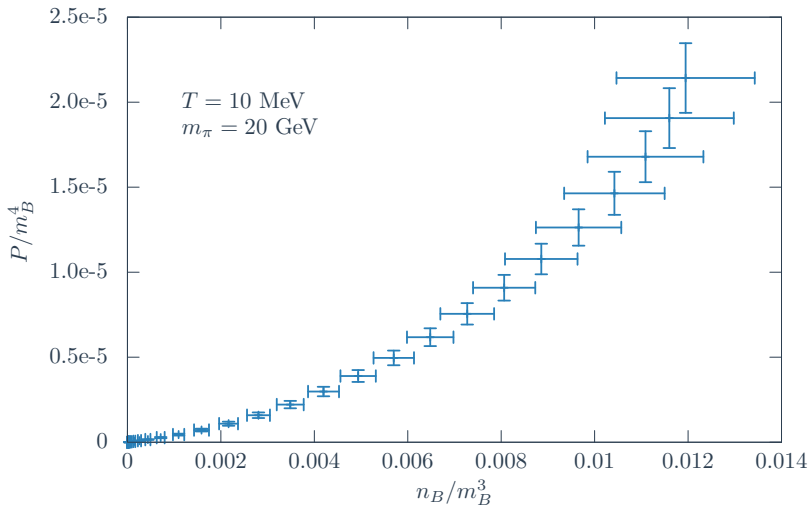
Binding energy



Continuum comparison



Continuum Equation of State



Conclusion

Summary & Outlook

Summary

- Introduced the effective dimensionally reduced lattice theory
- Looked at how a consistent analytic calculation could be carried out
- Demonstrated convergence and comparisons with numerics

Summary & Outlook

Outlook

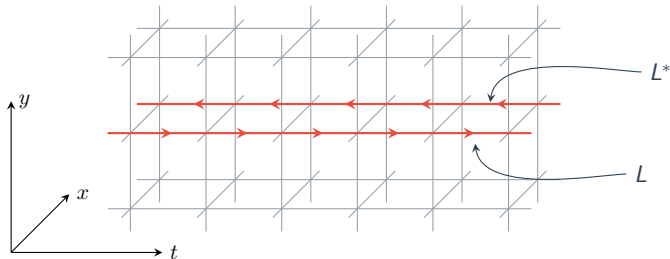
- Use the analytic results as a tool to study the characteristics of the effective theory
- Find analytic resummation schemes to incorporate long-range effects

Thank you!

Backup slides

The Effective Lattice Theory

Pure gluon contributions



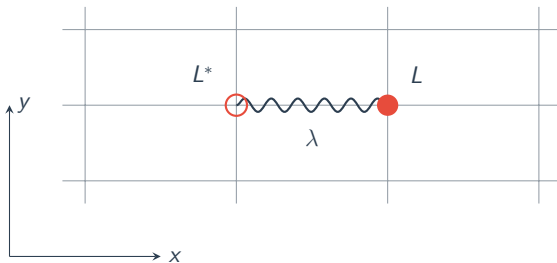
What remains is an interaction between Polyakov Loops

The Effective Lattice Theory

Pure gluon contributions

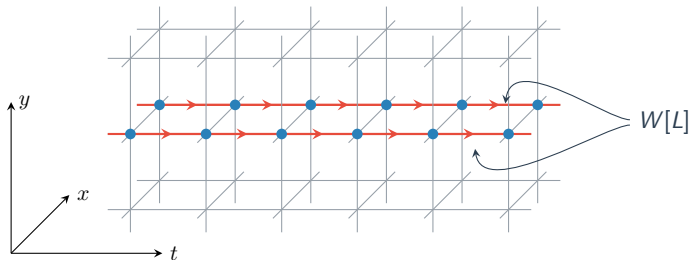
Effective Gluon Interactions

$$S_{\text{eff gluon}} \sim \lambda \sum_{\langle x,y \rangle} L(x) L^*(y)$$



The Effective Lattice Theory

Pure quark contributions



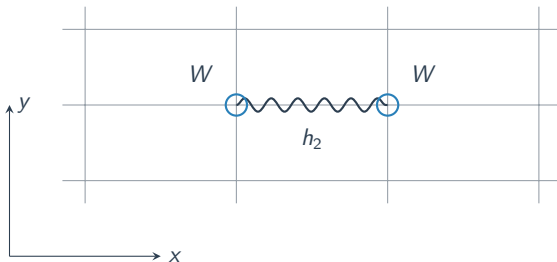
Producing an interaction between the W objects

The Effective Lattice Theory

Pure quark contributions

Effective Quark Interactions

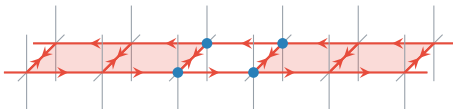
$$S_{\text{eff quarks}} \sim h_2 \sum_{\langle x,y \rangle} W(x)W(y)$$



The Effective Lattice Theory

Mixed contributions

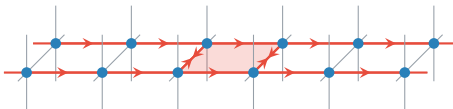
Correction to λ



- Rescales λ

- $\lambda \rightarrow \lambda(\kappa)$

Correction to h_2



- Rescales h_2

- $h_2 \rightarrow h_2(\beta)$

EoS in lattice units

