Squeezing water from a stone

A brief overview of lattice QCD

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A bit about me

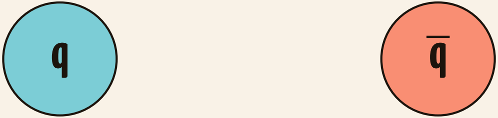
Background

- NTNU: Master’s in physics
- Frankfurt: PhD in Lattice QCD
- Swansea: Postdoc Lattice QCD
- Oslo: Software development

Lattice QCD

Quantum Chromo Dynamics

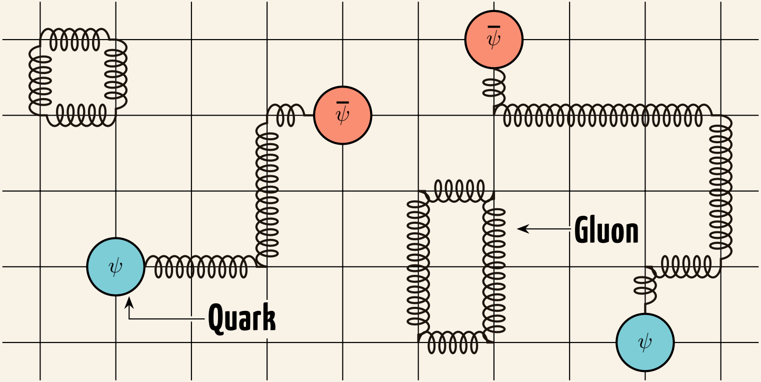
Theoretical description of the strong nuclear force



It binds matter together at the subatomic level

Lattice QCD

Basically we just put on a (HUGE) lattice



The important equations

$$S = \int \mathrm{d}^4 x \, \bar{\psi}(x) Q \psi(x) + \mathcal{L}_{\mathrm{g}}[U(x)]$$

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$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathrm{D}\psi \mathrm{D}U \, \mathcal{O}[\psi, U] \, e^{-S[\psi, U]}$$

Discretisation

$$S \to \sum_{i,j} \bar{\psi}(x_i) Q_{i,j}[U]\, \psi(x_j) + \mathcal{L}_{\mathrm{g}}[U(x_i)]$$

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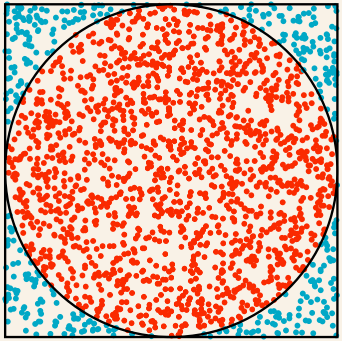
$$\mathcal{Z} = \int \mathrm{D}\psi \mathrm{D}U\, e^{-S[\psi,U]} = \int \mathrm{D}U \det(Q) e^{-\sum \mathcal{L}_{\mathrm{g}}[U(x)]}$$

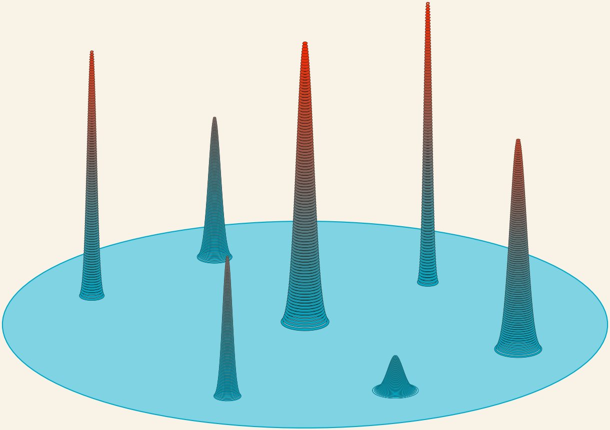
Monte Carlo Integration

Monte Carlo integration

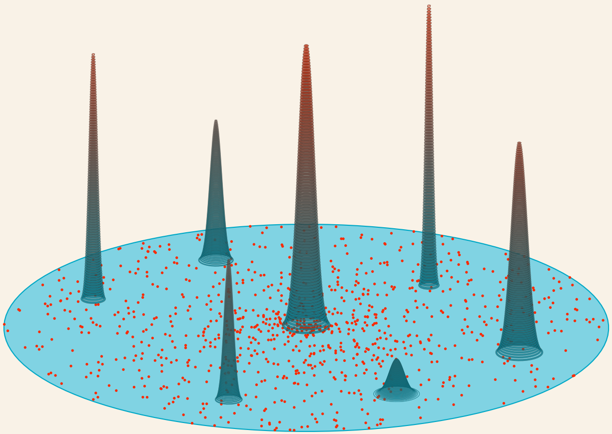


$$\pi \approx 4 \cdot \frac{1609}{2000}$$
$$= 3.21799$$

The QCD Integrand



The QCD Integrand



Redefining the problem

$$\langle \mathcal{O} \rangle = \int \mathrm{D}U \, \mathcal{O}[U] \underbrace{\frac{1}{\mathcal{Z}} e^{-S[U]}}_{\text{Probability density}} = \int \mathrm{D}U \, \mathcal{O}[U] \mathcal{P}[U]$$

Integral over U can be stochastically estimated.

$$\langle \mathcal{O} \rangle \approx \frac{1}{N} \sum_k \mathcal{O}[U_k]$$

Distributed $\propto \mathcal{P}$

Markov Chains

$$U_1 \xrightarrow{\propto \mathcal{P}} U_2 \xrightarrow{\propto \mathcal{P}} U_3 \xrightarrow{\propto \mathcal{P}} U_4 \xrightarrow{\propto \mathcal{P}} \cdots$$

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The distribution can be achieved with a Metropolis accept-reject step

$$p = \min\left\{1, \mathcal{P}[U_k{}'] / \mathcal{P}[U_k]\right\}$$

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But the evaluation if this is very expensive...

$VOL \times N_d \times N_c$

$$
\begin{pmatrix}
D(0|0) & D(0|\hat{0}) & 0 & 0 & 0 & \dots \\
D(\hat{0}|0) & D(\hat{0}|\hat{0}) & D(\hat{0}|2\hat{0}) & 0 & 0 & \dots \\
0 & D(2\hat{0}|\hat{0}) & D(2\hat{0}|2\hat{0}) & D(2\hat{0}|3\hat{0}) & 0 & \dots \\
0 & 0 & D(3\hat{0}|2\hat{0}) & D(3\hat{0}|3\hat{0}) & D(3\hat{0}|4\hat{0}) & \dots \\
0 & 0 & 0 & D(4\hat{0}|3\hat{0}) & D(4\hat{0}|4\hat{0}) & \dots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots \\
D(\hat{1}|0) & D(\hat{1}|\hat{0}) & 0 & 0 & 0 & \dots \\
D(\hat{1}|0) & D(\hat{0}\hat{1}|\hat{0}) & D(\hat{0}\hat{1}|2\hat{0}) & 0 & 0 & \dots \\
0 & D(2\hat{0}\hat{1}|\hat{0}) & D(2\hat{0}\hat{1}|2\hat{0}) & D(2\hat{0}\hat{1}|3\hat{0}) & 0 & \dots \\
0 & 0 & D(3\hat{0}\hat{1}|2\hat{0}) & D(3\hat{0}\hat{1}|3\hat{0}) & D(3\hat{0}\hat{1}|4\hat{0}) & \dots \\
0 & 0 & 0 & D(4\hat{0}\hat{1}|3\hat{0}) & D(4\hat{0}\hat{1}|4\hat{0}) & \dots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots \\
D(2\hat{1}|0) & D(2\hat{1}|\hat{0}) & 0 & 0 & 0 & \dots \\
D(2\hat{1}|0) & D(\hat{0}2\hat{1}|\hat{0}) & D(\hat{0}2\hat{1}|2\hat{0}) & 0 & 0 & \dots \\
0 & D(2\hat{0}2\hat{1}|\hat{0}) & D(2\hat{0}2\hat{1}|2\hat{0}) & D(2\hat{0}2\hat{1}|3\hat{0}) & 0 & \dots \\
0 & 0 & D(3\hat{0}2\hat{1}|2\hat{0}) & D(3\hat{0}2\hat{1}|3\hat{0}) & D(3\hat{0}2\hat{1}|4\hat{0}) & \dots \\
0 & 0 & 0 & D(4\hat{0}2\hat{1}|3\hat{0}) & D(4\hat{0}2\hat{1}|4\hat{0}) & \dots \\
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D(3\hat{1}|0) & D(3\hat{1}|\hat{0}) & 0 & 0 & 0 & \dots \\
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0 & D(2\hat{0}3\hat{1}|\hat{0}) & D(2\hat{0}3\hat{1}|2\hat{0}) & D(2\hat{0}3\hat{1}|3\hat{0}) & 0 & \dots \\
0 & 0 & D(3\hat{0}3\hat{1}|2\hat{0}) & D(3\hat{0}3\hat{1}|3\hat{0}) & D(3\hat{0}3\hat{1}|4\hat{0}) & \dots \\
0 & 0 & 0 & D(4\hat{0}3\hat{1}|3\hat{0}) & D(4\hat{0}3\hat{1}|4\hat{0}) & \dots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix}
\begin{pmatrix}
\psi(0) \\ \psi(\hat{0}) \\ \psi(2\hat{0}) \\ \psi(3\hat{0}) \\ \psi(4\hat{0}) \\ \vdots \\
\psi(\hat{1}) \\ \psi(\hat{0}\hat{1}) \\ \psi(2\hat{0}\hat{1}) \\ \psi(3\hat{0}\hat{1}) \\ \psi(4\hat{0}\hat{1}) \\ \vdots \\
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\end{pmatrix}
=
\begin{pmatrix}
\xi(0) \\ \xi(\hat{0}) \\ \xi(2\hat{0}) \\ \xi(3\hat{0}) \\ \xi(4\hat{0}) \\ \vdots \\
\xi(\hat{1}) \\ \xi(\hat{0}\hat{1}) \\ \xi(2\hat{0}\hat{1}) \\ \xi(3\hat{0}\hat{1}) \\ \xi(4\hat{0}\hat{1}) \\ \vdots \\
\xi(2\hat{1}) \\ \xi(\hat{0}2\hat{1}) \\ \xi(2\hat{0}2\hat{1}) \\ \xi(3\hat{0}2\hat{1}) \\ \xi(4\hat{0}2\hat{1}) \\ \vdots \\
\xi(3\hat{1}) \\ \xi(\hat{0}3\hat{1}) \\ \xi(2\hat{0}3\hat{1}) \\ \xi(3\hat{0}3\hat{1}) \\ \xi(4\hat{0}3\hat{1}) \\ \vdots
\end{pmatrix}
$$

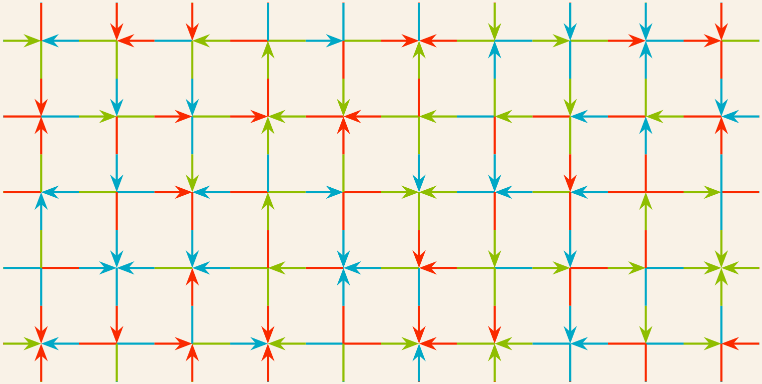
$VOL \times N_d \times N_c$

We need to solve this matrix equation many many many times

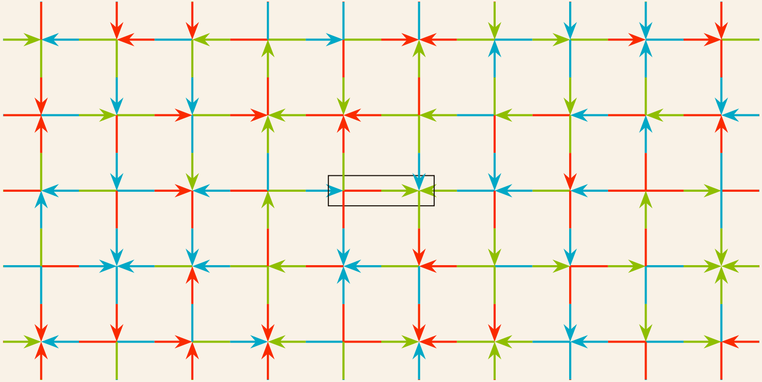
Our low temperature lattices are:

$$256 \times 32^3 \times 4 \times 3 \sim 10^8$$

Single link updates



Single link updates



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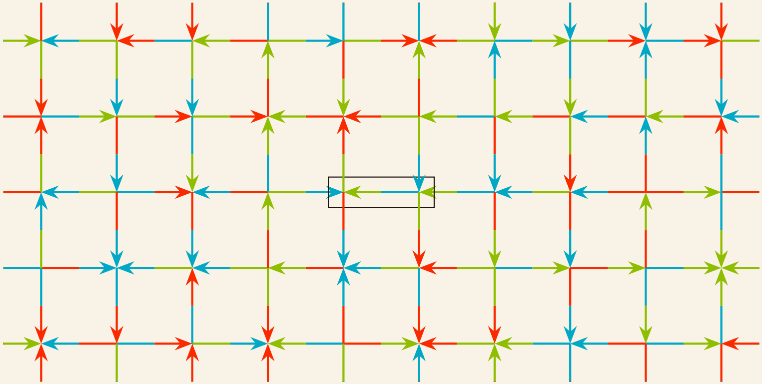

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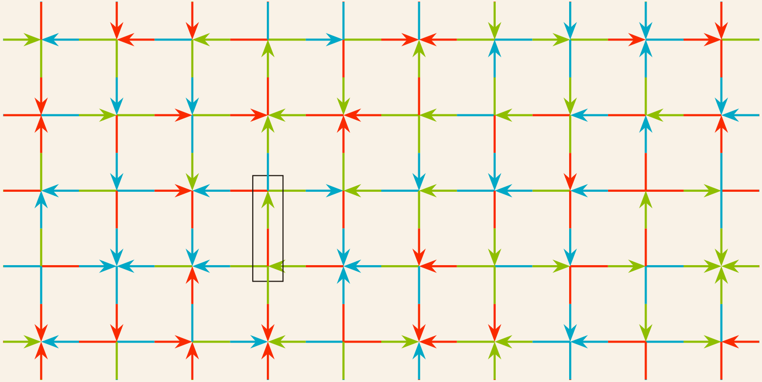

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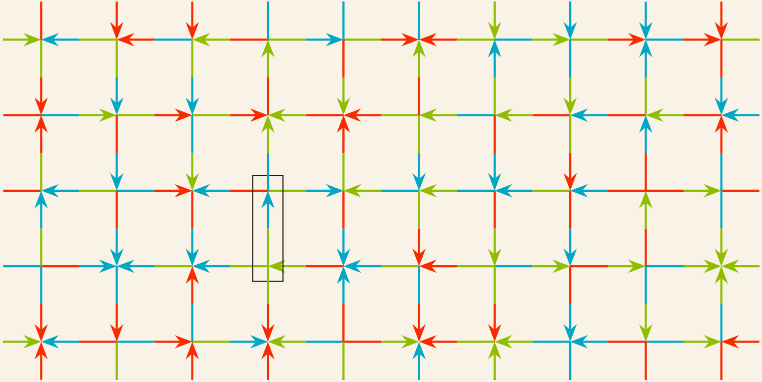

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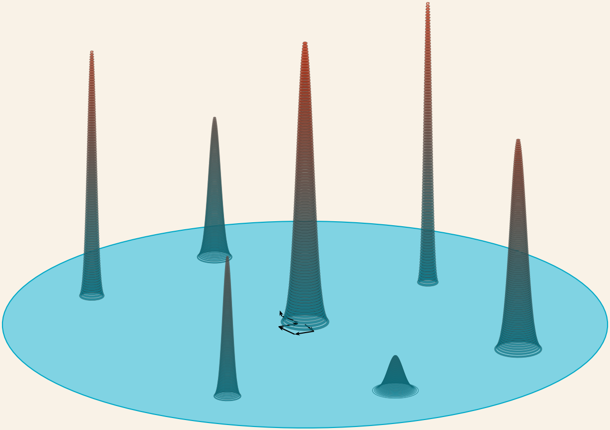
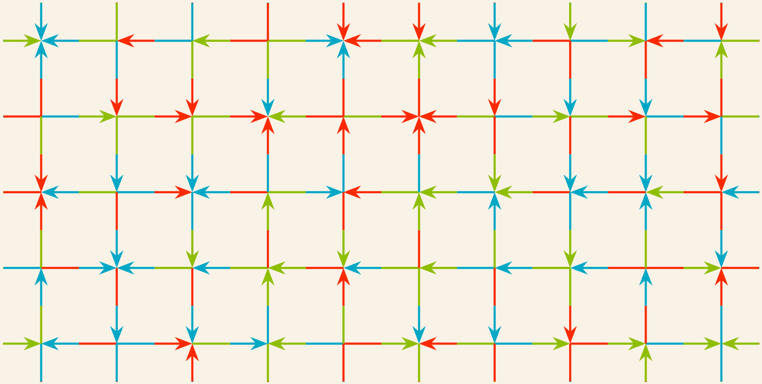
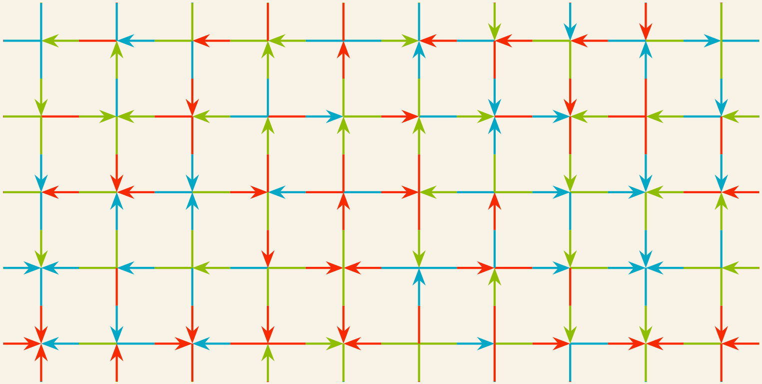


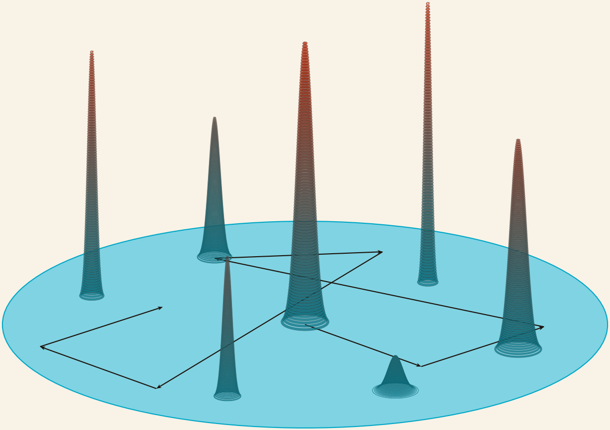
Configuration updates



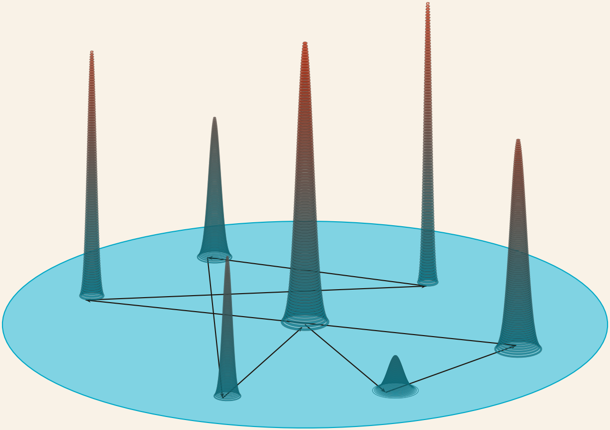
Configuration updates



Configuration updates (random)



Configuration updates (directed)

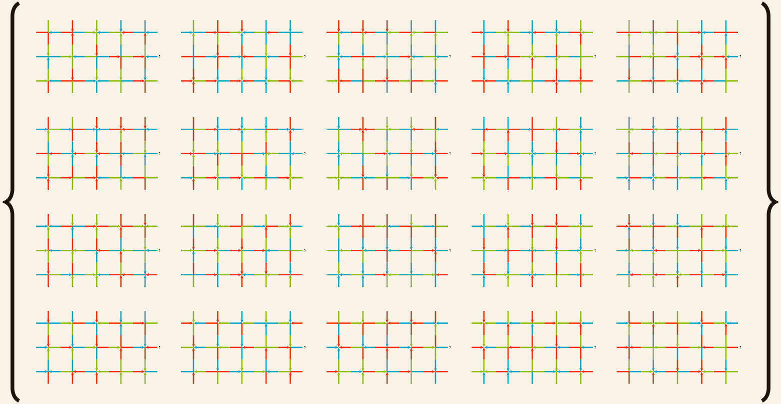


Clever algorithms

- **Langevin algorithm (1981)**
  Seepest descent + Gaussian noise
- **Moleculare Dynamics algorithm (1983)**
  Additional stochastic variables + Hamilton's equations
- **Hybrid Monte Carlo (1987)**
  Combine Langevin and MD with Metropolis accept-reject

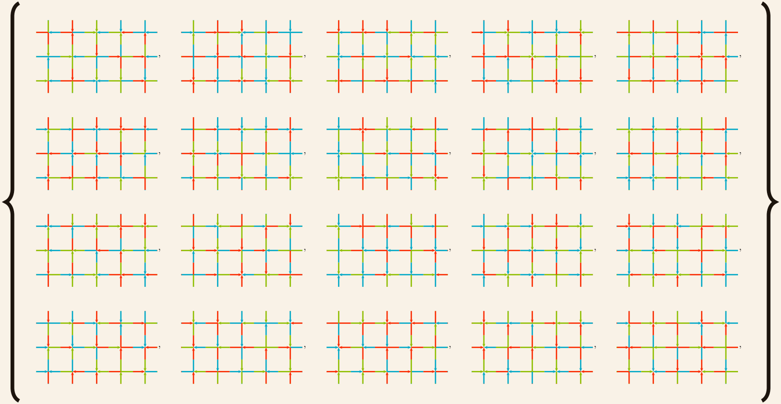
Measurements

Configurations



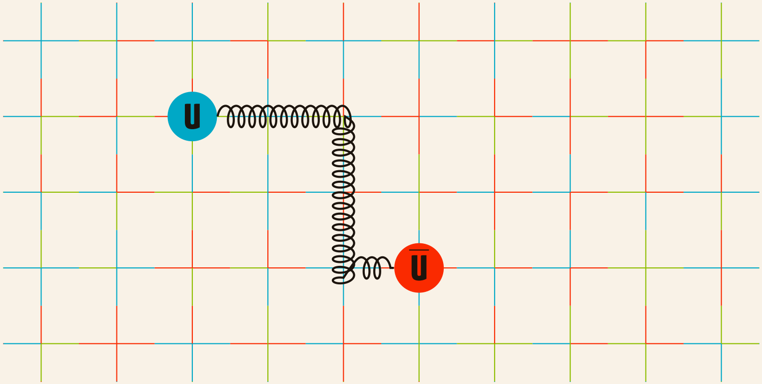
Each configuration consists of $(4 \cdot 18 \cdot \mathrm{VOL})$ **numbers.**

Configurations

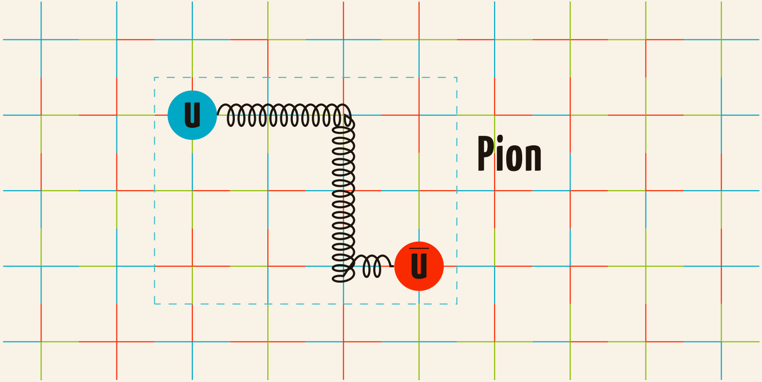


Low temperature configuration is 4.5 GB

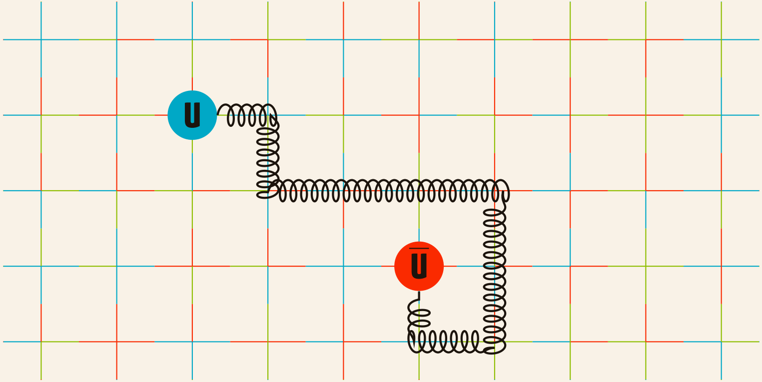
Hadron spectroscopy



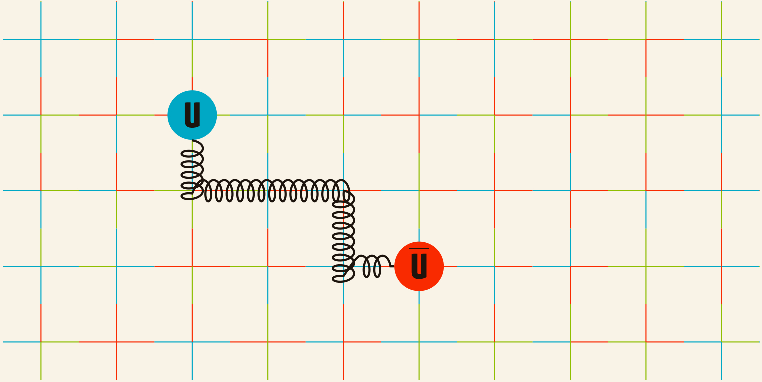
Hadron spectroscopy



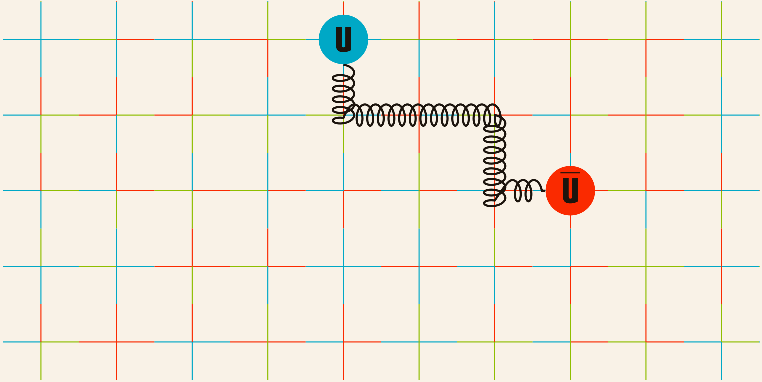
Hadron spectroscopy



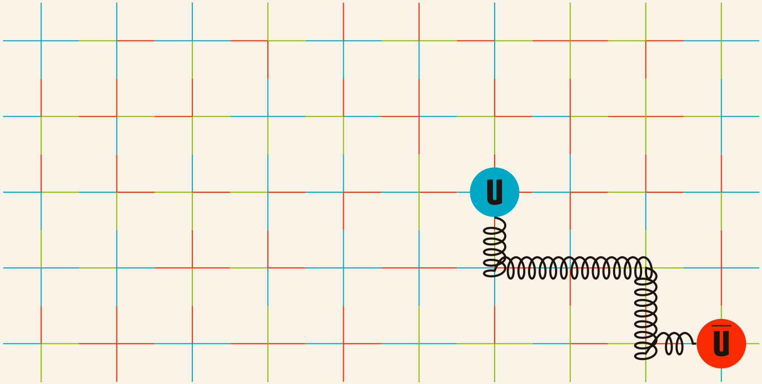
Hadron spectroscopy



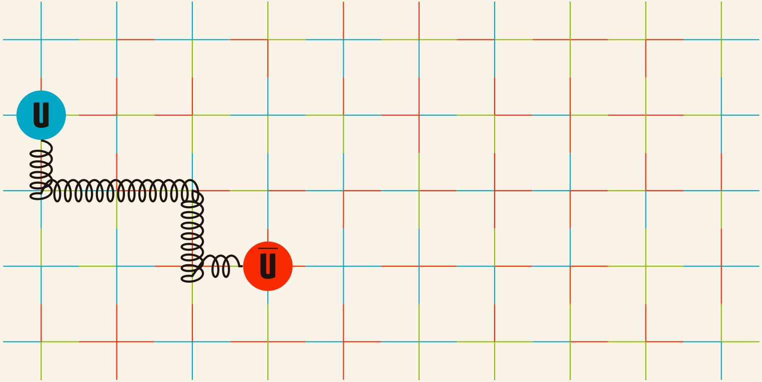
Hadron spectroscopy



Hadron spectroscopy

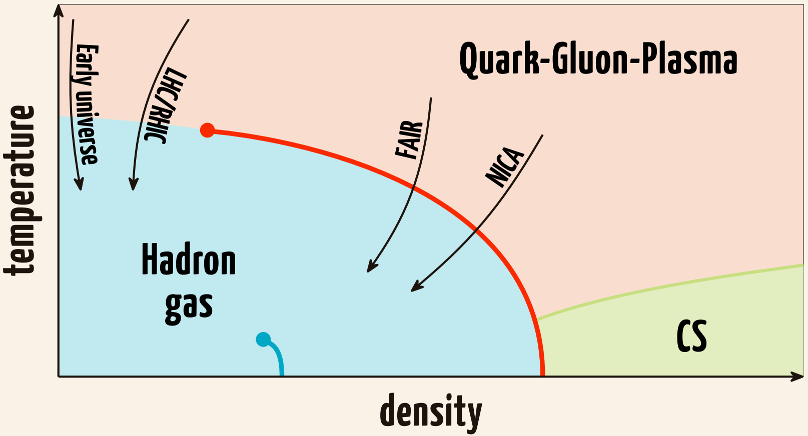


Hadron spectroscopy

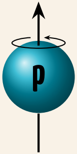


Results

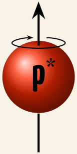
Phase diagram of QCD



Baryon parity breaking

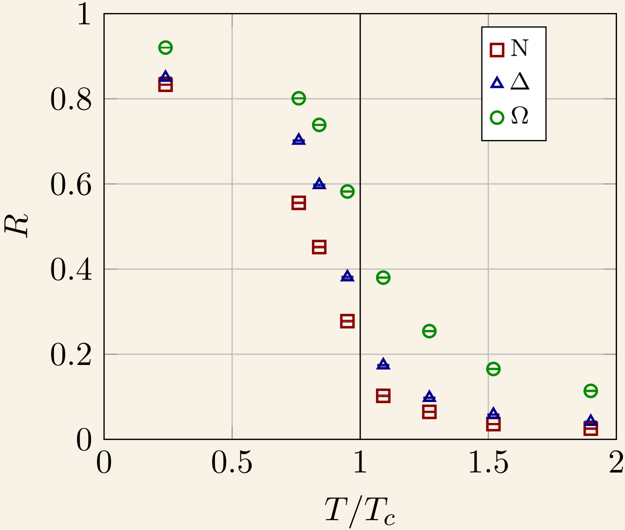


Mass: 938 MeV



Mass: 1535 MeV

Baryon parity restoration



Conclusion

Questions?