Particle in a Magnetic Field

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We will develop the trajectory of a charged particle in a uniform magnetic field. We assume that the particle has a charge q, mass m, and an initial velocity $\vec{v} = (v_{0x}, 0, v_{0z})$.

Additionally, we have for the magnetic field vector: $\vec{B} = (0, 0, B)$.

The magnetic force $\vec{F}_m$ on a particle with charge q moving at velocity $\vec{v}$ in a magnetic field $\vec{B}$ is given by the Lorentz force law:

$$\vec{F}_m = q \cdot (\vec{v} \times \vec{B}) \tag{1}$$

By Newton's second law:

$$\dot{\vec{v}} = \frac{q}{m} \cdot (\vec{v} \times \vec{B}) \tag{2}$$

Where $\times$ denotes the vector cross product. We can calculate the cross product as follows:

$$\vec{F}_m = q \cdot \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ v_x & v_y & v_z \\ 0 & 0 & B \end{vmatrix} \tag{3}$$

Then, we have the equation system:

$$\begin{cases} \dot{v_x} = \omega_c v_y \\ \dot{v_y} = -\omega_c v_x \\ \dot{v_z} = 0 \end{cases}$$

Where $\omega_c = \frac{qB}{m}$ is called the cyclotron frequency. Now let's consider $\ddot{v_x}$, and by replacing this we obtain: $\ddot{v_x} + \omega_c^2 v_x = 0$, which has the solution $v_x = v_{0x} \cos(\omega_c t)$. So, the entire system looks like:

$$\begin{cases} v_x = v_{0x} \cos(\omega_c t) \\ v_y = -v_{0x} \sin(\omega_c t) \\ v_z = v_{0z} \end{cases}$$

Finally, we can integrate these equations to obtain $\vec{r}(t)$:

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\begin{cases}
x = \dfrac{v_{0x}}{\omega_c}\sin(\omega_c t) \\
y = \dfrac{v_{0x}}{\omega_c}(\cos(\omega_c t) - 1) \\
z = v_{0z} t
\end{cases}
$$

With these equations, we can plot the trajectory for the particle in a magnetic field.

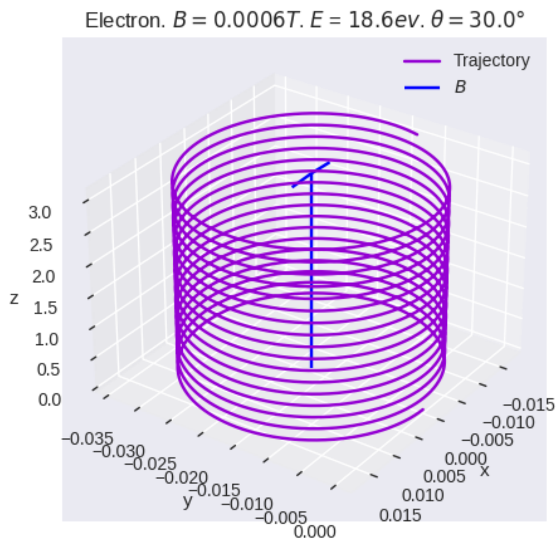

Figure 1: Trajectory for an electron in a magnetic field with intensity $600\mu T$