

# Regime Shifts and Tipping Points (Part I)



### Layman Definitions

#### **Tipping point**

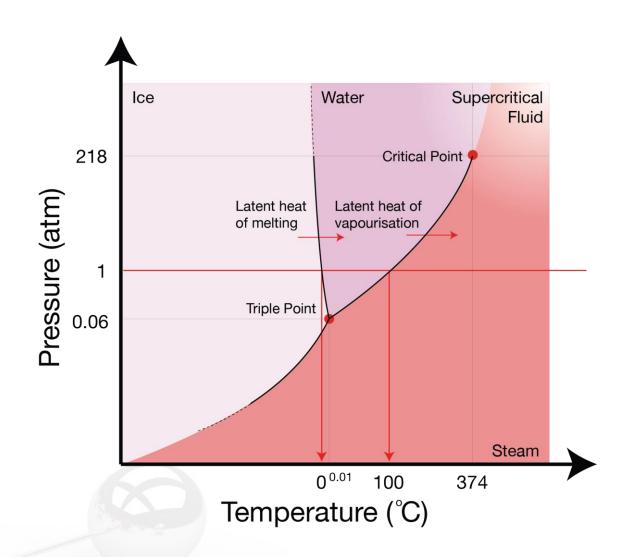
- Dictionary.com
  - the point at which an issue, idea, product, etc., crosses a certain threshold and gains significant momentum, triggered by some minor factor or change
  - the point in a situation at which a minor development precipitates a crisis
- Merriam-Webster
  - the critical point in a situation, process, or system beyond which a significant and often unstoppable effect or change takes place

#### **Sudden Change!**

#### Nomenclature

- Physical science
  - Phase transitions
  - Critical transitions
- Ecological science
  - Regime shifts
  - Critical transitions
- Socio-economic science
  - Regime shifts
  - Regime switches

#### **Phase Transitions**

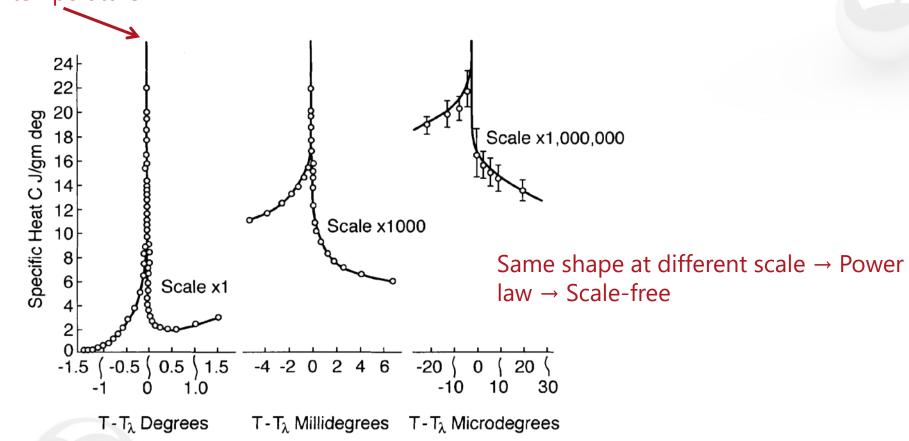




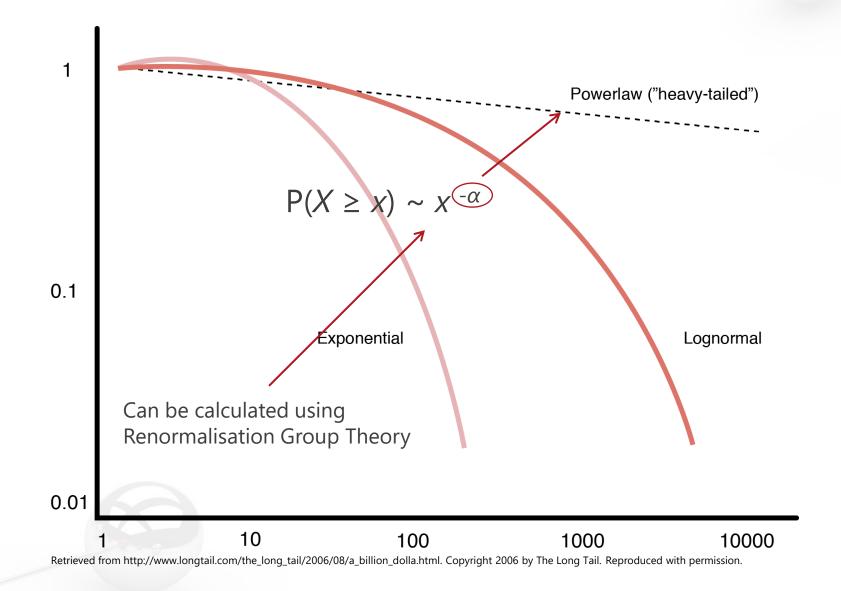
### Criticality

#### Diverge at critical temperature

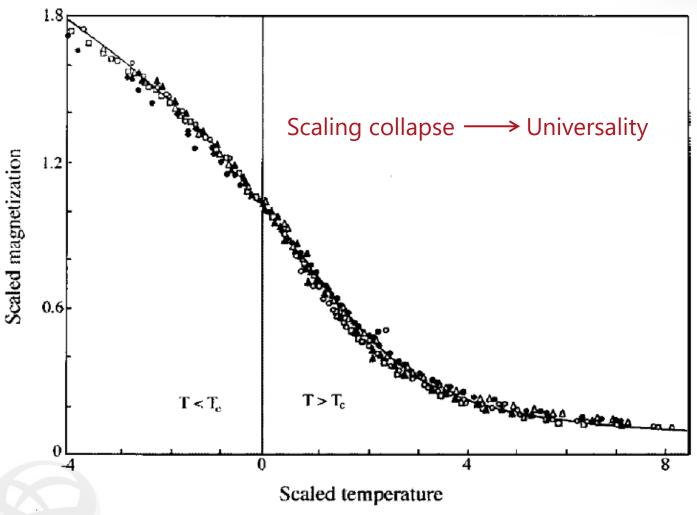
(U.S. Patent No. US 20050229609 A1, 2005)



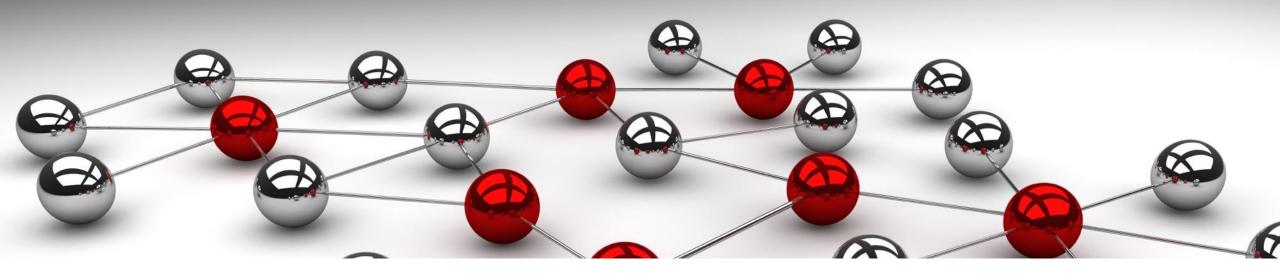
## Criticality



## Criticality



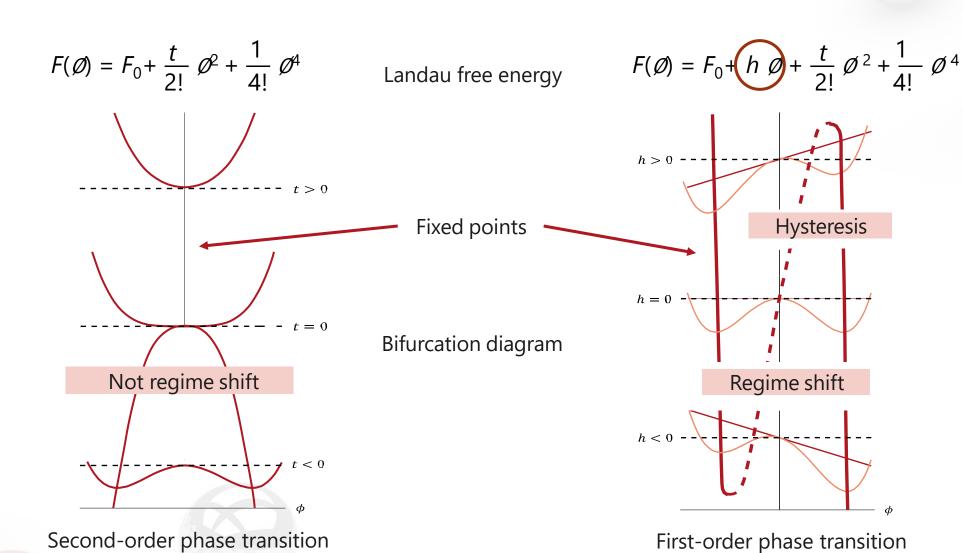
Retrieved from "Three pillars of modern critical phenomena" (doi:10.1103/RevModPhys.71.S35). Copyright 1999 by The American Physical Society. Reproduced with permission.

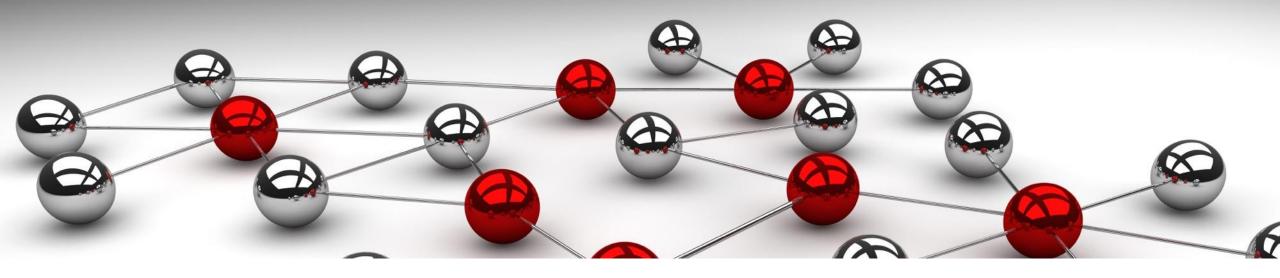


# Regime Shifts and Tipping Points (Part II)



### Landau Theory

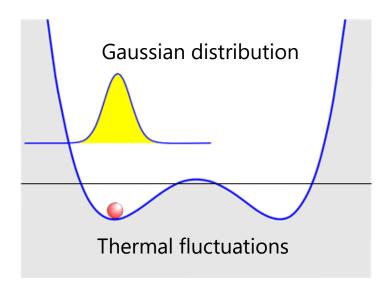




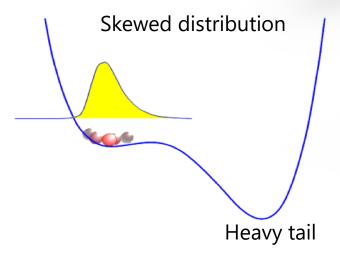
# Regime Shifts and Tipping Points (Part III)



### Universal Early Warning



- Finite variance
- Finite relaxation time



- Power-law tail at tipping point
- Variance → ∞

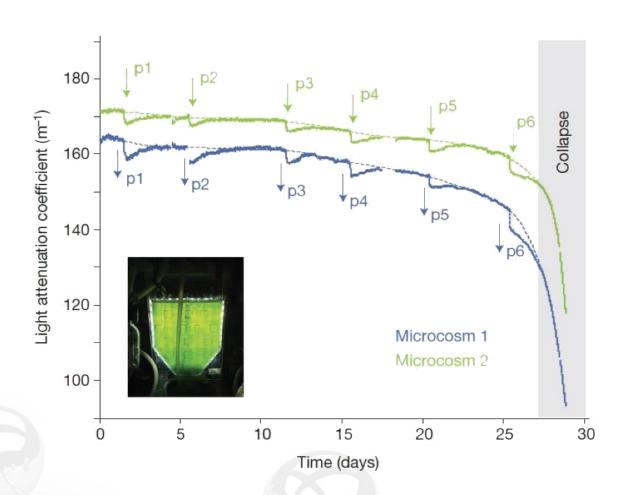
#### **Critical Fluctuations**

- Relaxation time → ∞

#### **Critical slowing down**

- Growing autocorrelation, spectral reddening

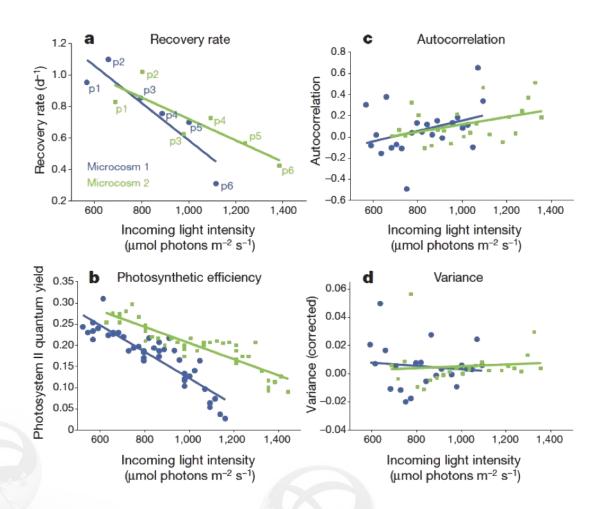
### Slow Recovery



Reprinted by permission from Macmillan Publishers Ltd: Nature Publishing Group (http://www.nature.com/nature/index.html), 2012.

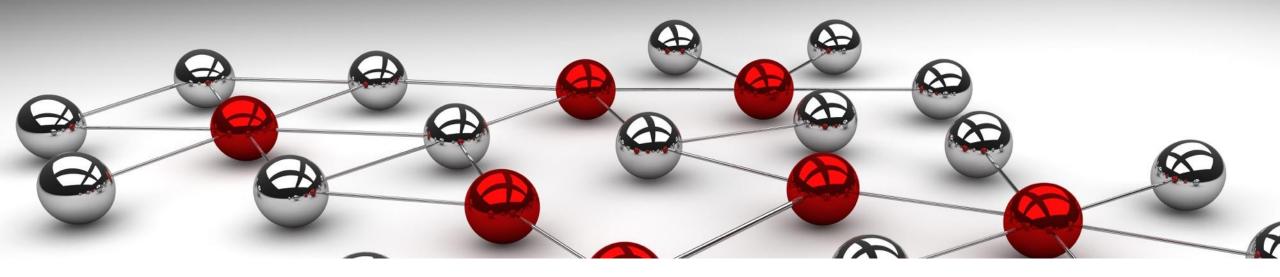
- Cyanobacteria in controlled chemostat microcosm
- Light intensity slowly increased until population collapse
- 10% dilution perturbation every four to five days
- Measure recovery rate

### Slow Recovery



Reprinted by permission from Macmillan Publishers Ltd: Nature Publishing Group (http://www.nature.com/nature/index.html), 2012.

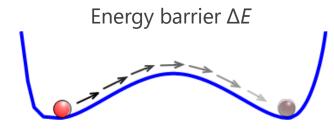
- Cyanobacteria in controlled chemostat microcosm
- Light intensity slowly increased until population collapse
- 10% dilution perturbation every four to five days
- Measure recovery rate



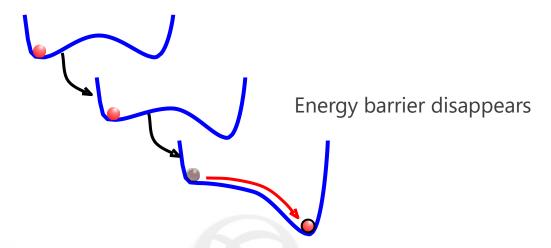
# Regime Shifts and Tipping Points (Part IV)



#### Equilibrium vs Critical Transitions



Equilibrium transition (noise-driven)



Critical transition (control-parameter-driven)

- Equilibrium transition can occur before tipping point is reached. This can be confusing!

### Forecasting Regime Shifts

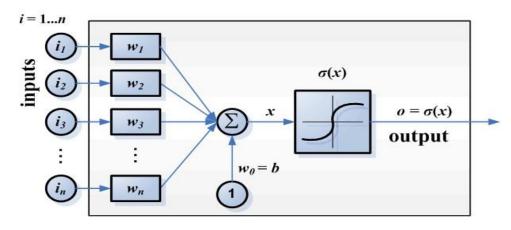


Source: GAO analysis of data from http://www.coindesk.com/price/ (accessed on Apr. 1, 2014).

AR models 
$$x_{t+1} = \sum_{r=1}^{p} a_r x_{t-r} + \epsilon_t$$
 Jump diffusion models

#### Forecasting Regime Shifts

#### ANN

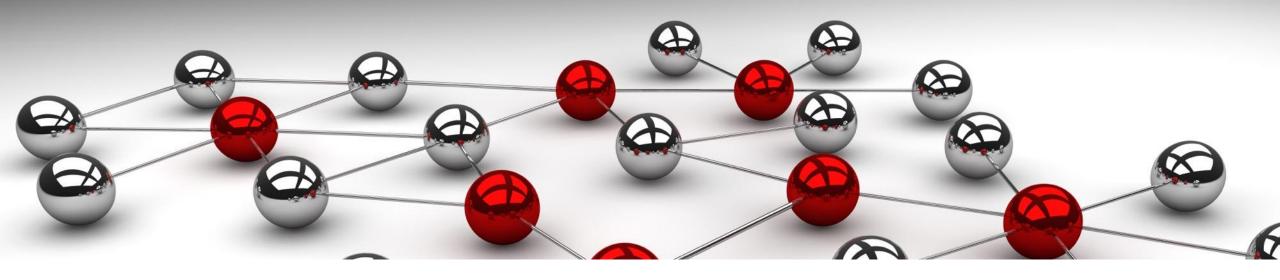


Not regime-shift-aware!

#### Technical trading



Retrieved from http://www.marketoracle.co.uk/Article21209.html. Copyright 2006 by The Market Oracle. Reproduced with permission.

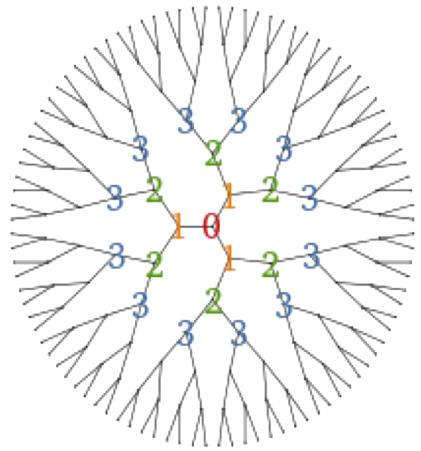


# Regime Shifts and Tipping Points (Part V)





#### Log-Periodic Power Law (LPPL)



**Bethe Lattice** 

Discrete Scale Invariance

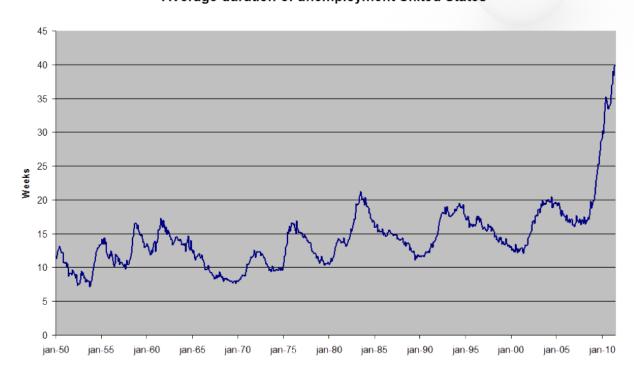
$$I(t) = A + B(t_c - t)^a + C(t_c - t)^a \cos [w(t_c - t) + \Phi]$$

#### Log-Periodic Power Law (LPPL)

#### S&P 500 index around the time of the crash

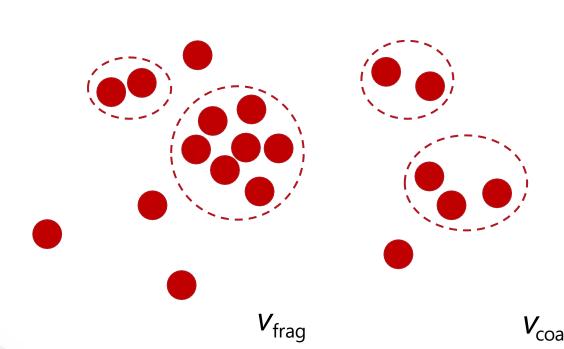


#### Average duration of unemployment United States



Black Monday, 1987

#### Soup-of-Groups (SOG) Model



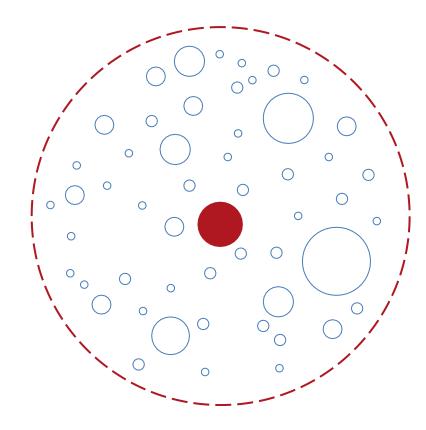
$$\frac{\partial n_s}{\partial t} = \frac{\nu_{\text{coal}}}{N^2} \sum_{k=1}^{s-1} k n_k (s-k) n_{s-k} - \frac{\nu_{\text{frag}} s n_s}{N} - \frac{2\nu_{\text{coal}} s n_s}{N^2} \sum_{k=1}^{\infty} k n_k, \quad s \ge 2,$$

$$\frac{\partial n_1}{\partial t} = \frac{\nu_{\text{frag}}}{N} \sum_{k=2}^{\infty} k^2 n_k - \frac{2\nu_{\text{coal}} n_1}{N^2} \sum_{k=1}^{\infty} k n_k,$$

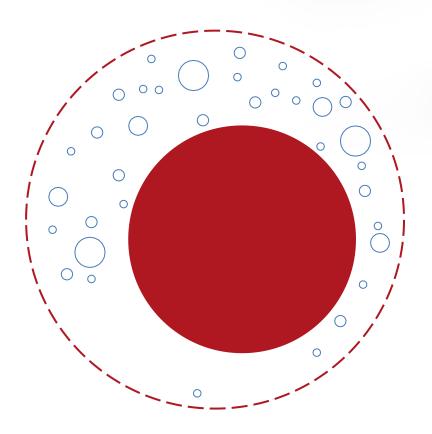
 $p(s) \sim s^{5/2} \rightarrow 2$  for d = 2 Gutenberg-Richter law!

Bohorquez, Gourley, Dixon, Spagat and Johnson. (2009). *Nature, 462*(911) Johnson, Ashkenazi, Zhao and Quiroga. (2011). *AIP Advances, 1*(012114)

#### **Growth of Giant Cluster**

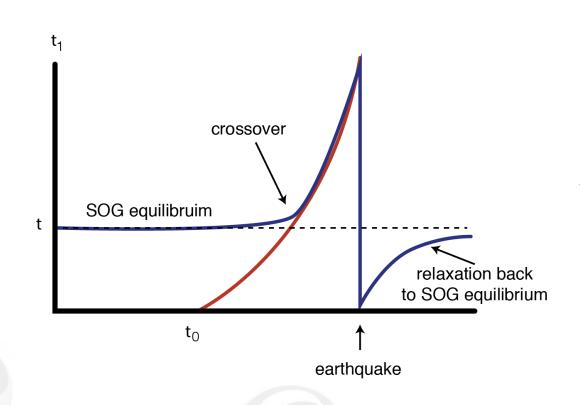


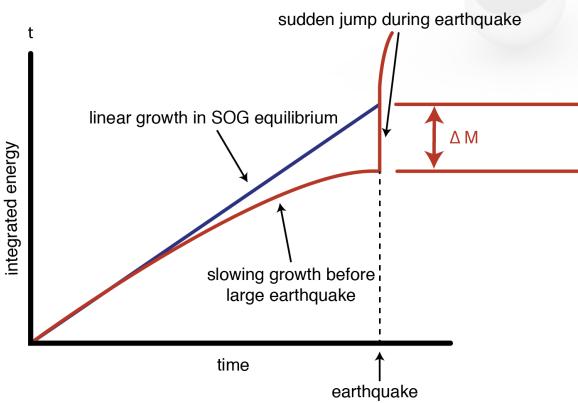
No giant cluster Equilibrium distribution of cluster sizes GR law



Giant cluster, resulting in fewer clusters of all sizes Departure from GR law

#### **Growth of Giant Cluster**



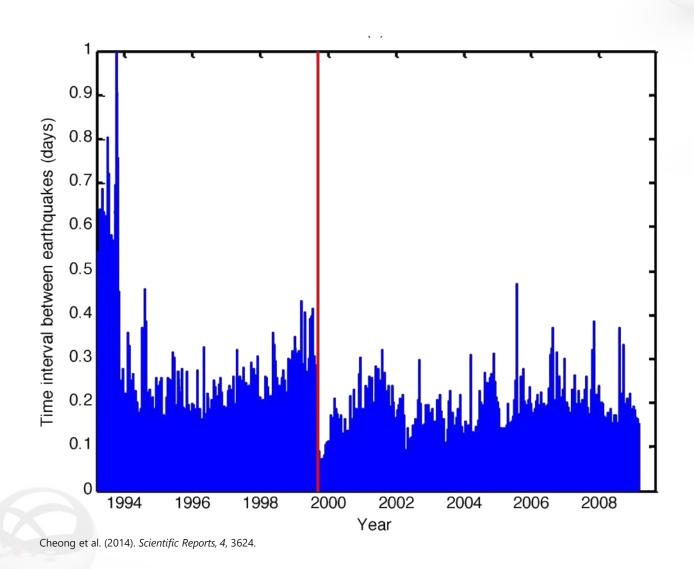


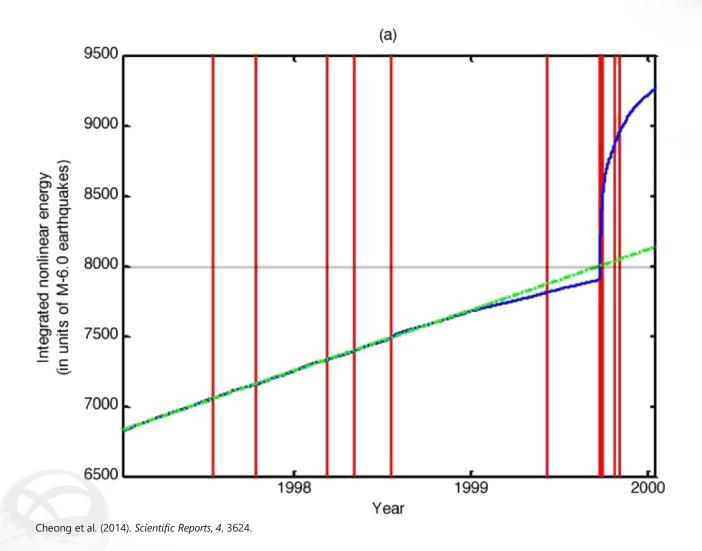
#### **Growth of Giant Cluster**

### Mean Field Theory

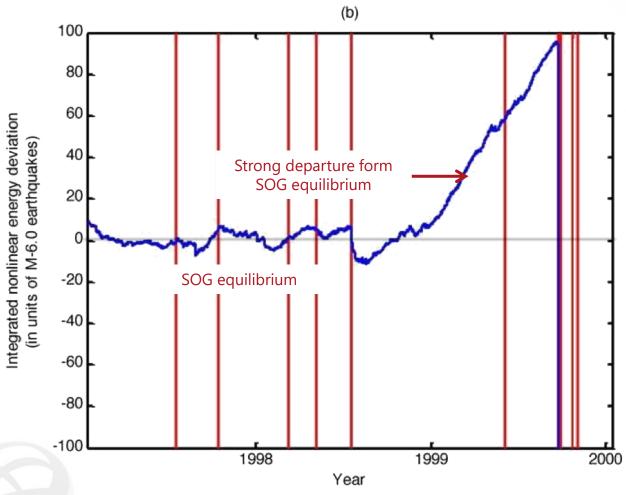
$$E(t) = \int_0^t \int_{m_0}^{\log_{10}(S_0 - \gamma t)} \frac{\text{linear growth}}{10^m R(m) dm \ dt'}$$
GR law

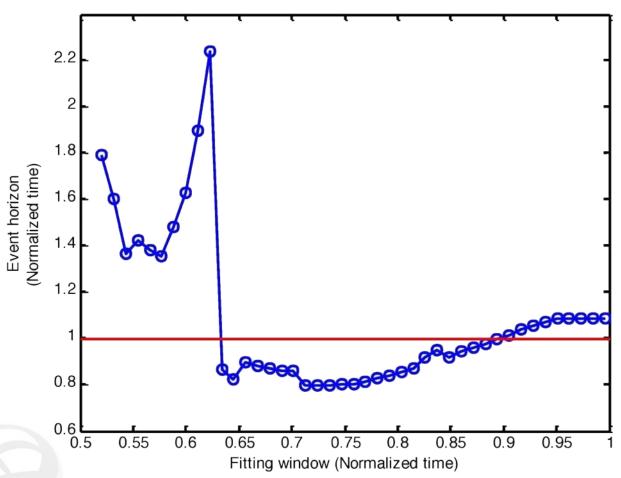
$$= At \left( \log_{10} \frac{S_0 - \gamma t}{S_0} - 1 \right) - \frac{AS_0}{\gamma} \log_{10} \frac{S_0 - \gamma t}{S_0}$$



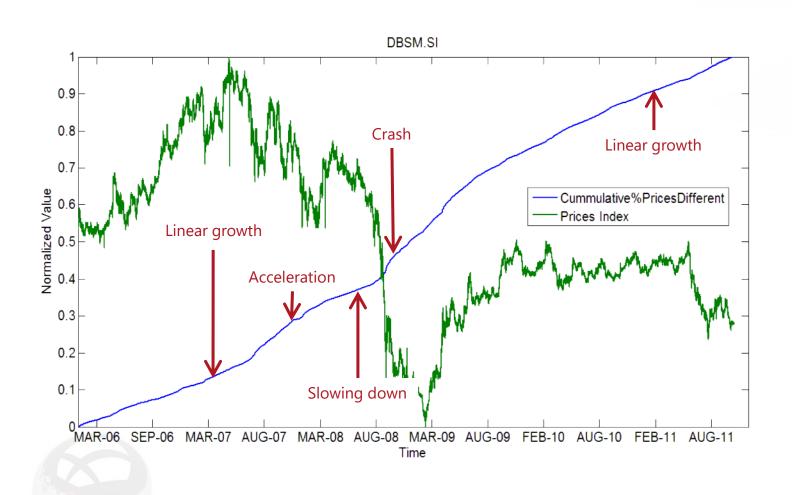


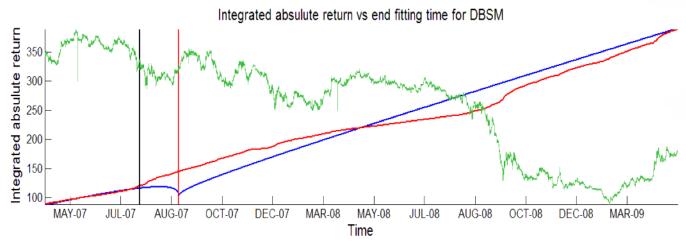
27

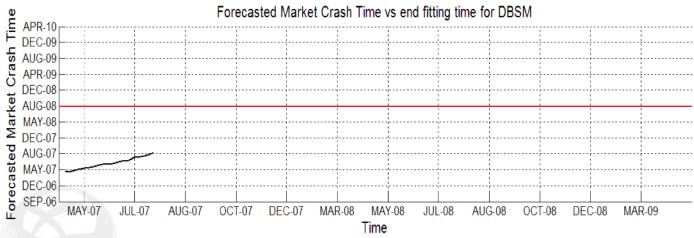




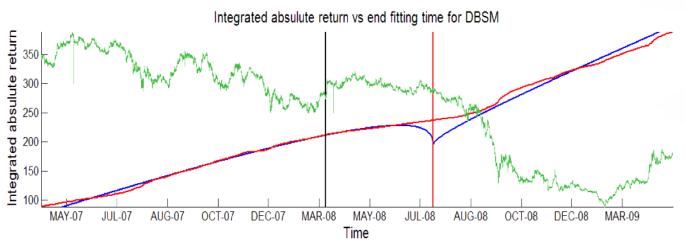
#### Oct 2008 SGX Crash: Precursors

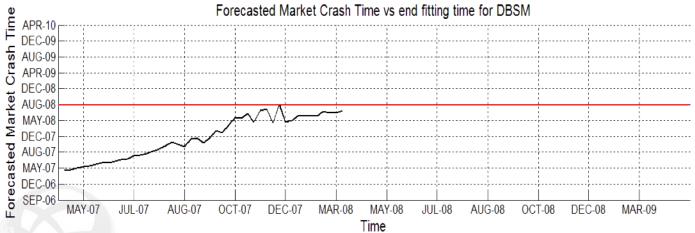




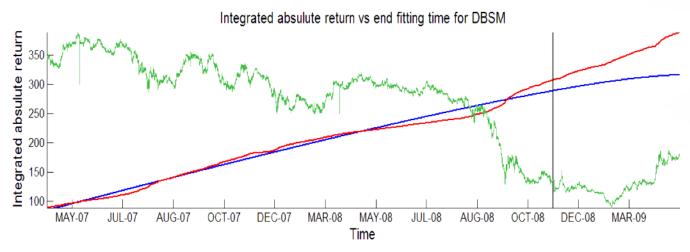


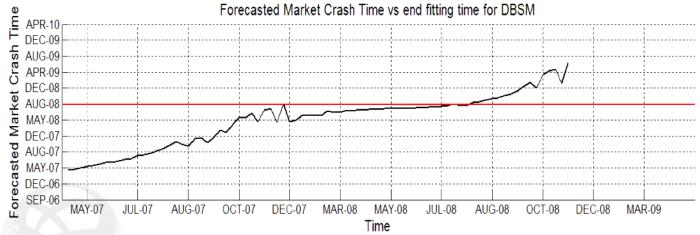
Adapted from Teh and Cheong. (2016). PLoS ONE 11(10), e0163842.



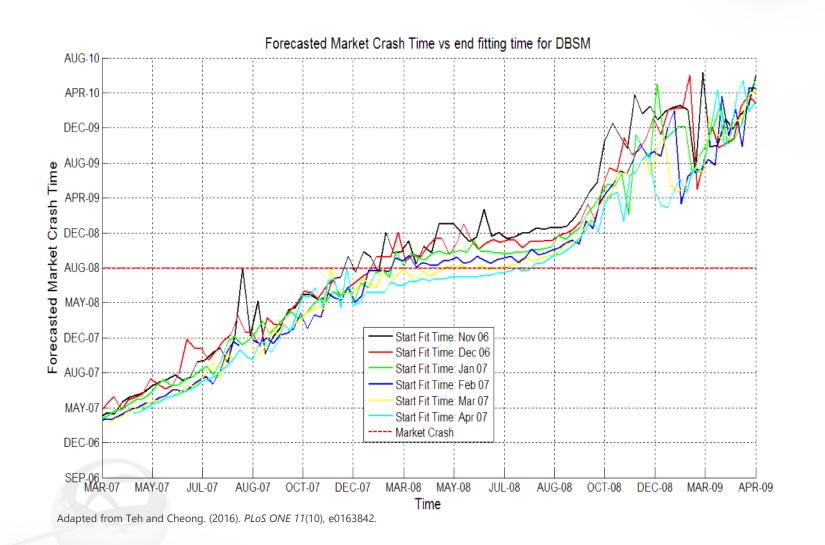


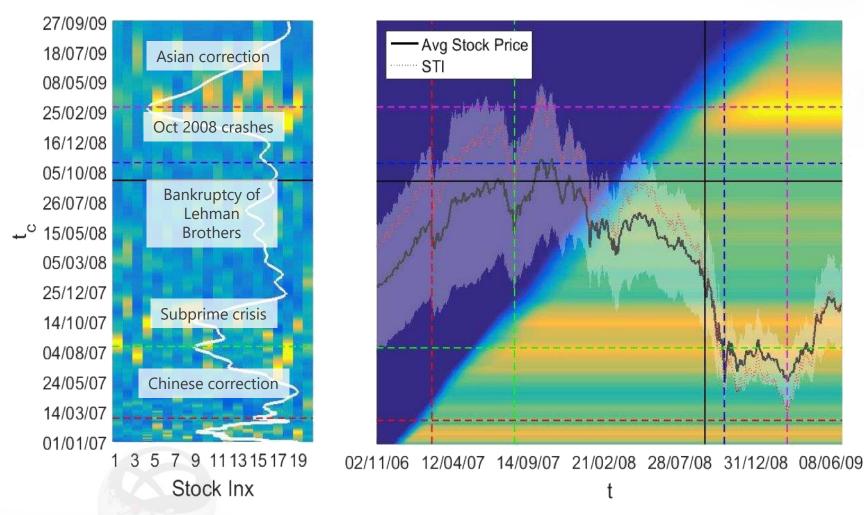
Adapted from Teh and Cheong. (2016). PLoS ONE 11(10), e0163842.





Adapted from Teh and Cheong. (2016). PLoS ONE 11(10), e0163842.





### Summary

- Regime and regime shifts
  - Universal phenomenology
  - Landau theory
- Early warnings
  - Critical fluctuation
  - Critical slowing down
- Quantitative forecasting
  - Log-Periodic Power Law
  - SOG forecasting

#### **Acknowledgements**

- Slide 4: Photo of ice cubes, extracted from Pixabay: <a href="https://pixabay.com/en/ice-cubes-cold-water-melt-drink-1224804/">https://pixabay.com/en/users/ColiN00B-346653/</a> (Public Domain)
- Slide 4: Photo of water glass, extracted from Pixabay: <a href="https://pixabay.com/en/water-freeze-photography-glass-1263493/">https://pixabay.com/en/water-freeze-photography-glass-1263493/</a> by Aman21: <a href="https://pixabay.com/en/users/Aman21-2237083/">https://pixabay.com/en/users/Aman21-2237083/</a> (Public Domain)
- Slide 4: Photo of steaming pot, extracted from Pixabay: <a href="https://pixabay.com/en/pot-steaming-hot-cooking-kitchen-820012/">https://pixabay.com/en/pot-steaming-hot-cooking-kitchen-820012/</a> by Republica: <a href="https://pixabay.com/en/users/Republica-24347/">https://pixabay.com/en/users/Republica-24347/</a> (Public Domain)
- Slide 5: Kirichek, O. & Kouzmenko, G. (2005). U.S. Patent No. US 20050229609 A1. Washington, DC: Google Patents.
- Slide 6: Chris Anderson. (2006). Lognormal distribution [Graph]. *The Long Tail*. Retrieved May 9, 2017, from <a href="http://www.longtail.com/the\_long\_tail/2006/08/a\_billion\_dolla.html">http://www.longtail.com/the\_long\_tail/2006/08/a\_billion\_dolla.html</a>.
- Slide 7: Stanley, H. Eugene. (1999). Scaling, universality, and renormalization: Three pillars of modern critical phenomena. Reviews of Modern Physics, 71(2), S358-S366. doi:10.1103/RevModPhys.71.S358
- Slides 12-13: Veraart, A.J., Faassen, E.J., Dakos, V., van Nes, E.H., Lürling, M. and Scheffer, M. (2012). Recovery rates reflect distance to a tipping point in a living system. *Nature*, 481(7381), 357-359. doi:10.1038/nature10723
- Slide 16: Graph showing GAO analysis of data, extracted from Wikimedia Commons:

  <a href="https://commons.wikimedia.org/wiki/File:Bitcoin\_Price\_Index\_in\_U.S.\_Dollars, January 1, 2013\_through\_March\_31, 2014.jpg">https://commons.wikimedia.org/wiki/File:Bitcoin\_Price\_Index\_in\_U.S.\_Dollars, January 1, 2013\_through\_March\_31, 2014.jpg</a> by U.S. Government Accountability Office: (<a href="https://www.flickr.com/photos/usgao/14534919596/">https://www.flickr.com/photos/usgao/14534919596/</a> (Public Domain)
- Slide 17: Chris Vermeulen. (2006). Gold ETF Price Action [Graph]. The Market Oracle. Retrieved May 9, 2017, from http://www.marketoracle.co.uk/Article21209.html.
- Slide 19: Photo of Didier Sornette, reproduced with permission from Didier Sornette.
- Slide 20: Image of Bethe Lattice, extracted from Wikimedia Commons: <a href="https://commons.wikimedia.org/w/index.php?curid=38258207">https://commons.wikimedia.org/wiki/User:Kilom691</a> under CC BY-SA 4.0: <a href="https://creativecommons.org/licenses/by-sa/4.0/deed.en">https://creativecommons.org/licenses/by-sa/4.0/deed.en</a>
- Slide 21: Graph showing S&P 500 index, extracted from Wikimedia Commons: <a href="https://en.wikipedia.org/wiki/File:S%26P\_500\_index\_around\_the\_time\_of\_the\_crash.png#filelinks">https://en.wikipedia.org/wiki/File:S%26P\_500\_index\_around\_the\_time\_of\_the\_crash.png#filelinks</a> by US Federal Reserve (Public Domain)
- Slide 21: Graph showing United State's unemployment rate, extracted from Wikimedia Commons:

  <a href="https://en.wikipedia.org/wiki/File:US\_average\_duration\_of\_unemployment.png">https://en.wikipedia.org/wiki/File:US\_average\_duration\_of\_unemployment.png</a> by MartinD: <a href="https://commons.wikimedia.org/wiki/User:MartinD">https://commons.wikimedia.org/wiki/User:MartinD</a> under CC BY-SA 3.0:

  <a href="https://creativecommons.org/licenses/by-sa/3.0/deed.en">https://creativecommons.org/licenses/by-sa/3.0/deed.en</a>

Slides 26-29: Cheong, S. A., Tan, T. L., Chen, C.-C., Chang, W.-L., Liu, Z., Chew, L. Y., ... Johnson, N. F. (2014). Short-Term Forecasting of Taiwanese Earthquakes Using a Universal Model of Fusion-Fission Processes. Scientific Reports, 4, 3624. http://doi.org/10.1038/srep03624

Slides 31-35: Teh, B.K. and Cheong, S.A. (2016) The Asian Correction Can Be Quantitatively Forecasted Using a Statistical Model of Fusion-Fission Processes. *PLoS ONE* 11(10), e0163842. doi: https://doi.org/10.1371/journal.pone.0163842 (Licensed under CC BY 4.0: https://creativecommons.org/licenses/by/4.0/)