November 24, 2023

General Information:

You have to submit your solution via Moodle. We allow **groups of two students**. Please list all names and matriculation numbers in the **team_members.txt** file. **Both team members need to upload the solution**. You have **one week of working time** (note the deadline date in the footer). To get the 0.3 bonus, you have to pass at least four exercises, with the fifth one being either completely correct or borderline accepted. The solutions of the exercises are presented the day after the submission deadline respectively. Feel free to use the Moodle forum or use the Q&A session to ask questions!

For this project we use **Eigen** (http://eigen.tuxfamily.org/), **CeresSolver** (http://ceres-solver.org/), **FLANN** (https://github.com/mariusmuja/flann) and **FreeImage** (http://freeimage.sourceforge.net/).

To inspect your result, you need a 3D file viewer like **MeshLab** (http://www.meshlab.net/). The exercise zip-archive contains the source files, a CMake configuration file, and the bunny data. A header-only version of the FLANN library (https://github.com/jlblancoc/nanoflann) is provided with the source code on Moodle. For the other dependencies and the scene data see exercises 4 and 1.

Since this exercise may involve more effort installing libraries than usual, please follow the following suggestions:

1. In case you're using the OS of your computer (not a virtual machine / clean OS for the exercises / Docker / etc.) and encounter issues, please refer to the FAQ published on the Moodle forum. You're also encouraged to ask questions about installing the libraries for your OS setup there.

2. We provide a GitHub Codespace for this exercise: https://github.com/seva100/3DSMC-exercise-5-core. This is a tiny environment that can be run right in your browser and is very easy to set up and requires only one shell command to install all the libraries. We highly recommend trying to solve the exercise in the Codespace!

Expected submission files: ICPOptimizer.h, PointCloud.h, bunny_icp.off, mesh_merged.off

Exercise 5 – Registration with ICP

In this exercise we want to align 3D shapes. We will explore different variants of the Iterative Closest Points (ICP) algorithm and test them on shapes provided as either meshes or depth maps.

Expected file structure:

Exercises/
    Data/
        bunny_part1.off
        bunny_part2_trans.off

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```
Libs/
        /Flann-1.8.4

        …
Exercise-5/
        Build/
                CMake Files and Binaries
        main.cpp
        …
```

Tasks:

1. Setup

1.1. Eigen, Ceres, FreeImage

You should already have Eigen, Ceres, and FreeImage dependencies set up from previous exercises. If not, have a look at exercise 1 for Eigen and FreeImage, and at exercise 4 for Ceres.

1.2. FLANN

You can build FLANN from source using the link above, or you can use the header-only version of the library, which we provide as .zip in moodle. This might be easier, since it does not need to be compiled and installed, and your exercise_05 project only needs to find the ".h" files of the library.

To set up the header-only version, download the .zip from moodle, extract it to the libs folder, and change the Flann_INCLUDE_DIR in your exercise_05 cmake to point to this folder.

Now you should be able to directly build exercise 5 (without having to build and install flann itself).

2. Fine Registration – Iterative Closest Point with Ceres
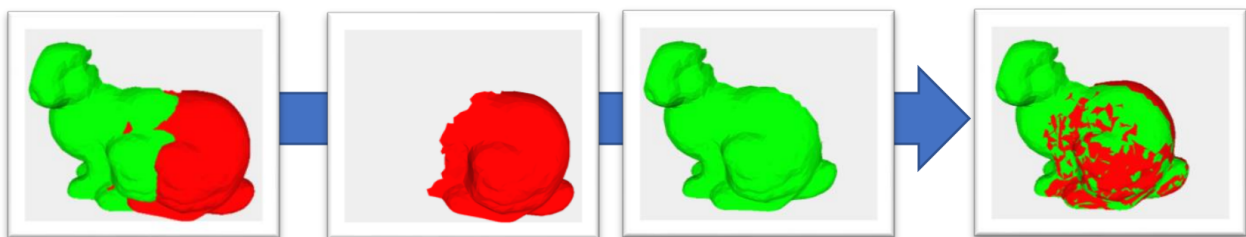
The Iterative Closest Point (ICP) is used for fine registration. Thus, the input meshes or point clouds have to be roughly aligned (e.g. using the Procrustes Algorithm). Our partial bunny meshes are close enough to directly apply ICP. Your task is to implement two variants of the ICP algorithm using the Ceres library to optimize for the best rigid transformation.

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The goal of the ICP is to align a source point cloud to a target point cloud by estimating the rigid transformation (rotation and translation) that transforms the source point cloud into the target point cloud. It is an iterative algorithm, where each iteration consists of two steps:

a) Finding the nearest (closest) target point for each source point, using the current rigid transformation (using the approximate nearest neighbor algorithm);
b) Estimating the relative rigid transformation by minimizing the distance between the matched points with one iteration of the Levenberg-Marquardt algorithm.

After each iteration the current relative rigid transformation is updated. The procedure is repeated until convergence (in our examples we use a fixed number of iterations).

We use two variants of the distance function: point-to-point and point-to-plane distance. If s is a source point with position p_s and t is a target point with position p_t and normal n_t then we define the distance functions as:

a) Point-to-point: $\quad d(s,\ t) = ||Mp_s - p_t||^2$

b) Point-to-plane: $\quad d(s,\ t) = (n_t^T(Mp_s - p_t))^2$

Your task is to implement both distance functions as cost functions in the Ceres solver (in ICPOptimizer.h). The optimization variable is the relative camera pose (rigid transformation), which can be parametrized with 6 parameters: 3 parameters for rotation, represented in axis-angle (SO3) notation, and 3 parameters for translation, given as a 3D vector.

The final mesh bunny_icp.off should be computed using both point-to-point and point-to-plane cost functions. The main difference between using only point-to-point distances compared to using also point-to-plane distances is the speed of convergence. For our partial shapes the variant with only point-to-point constraints needs 20 ICP iterations to converge, while the addition of point-to-plane constraints reduces the number of ICP iterations to 10.

3. Linearized Iterative Closest Point

You might have noticed that the point-to-point variant from the first task is a linear least-squares problem which can be solved with the Procrustes algorithm from the previous exercise. On the other hand, the point-to-plane metric is non-linear in the rotation. However, there exists a method to linearly approximate the point-to-plane metric which would then allow us to use a linear solver for the alignment.

Let $M = T(t_x, t_y, t_z) \cdot R(\alpha, \beta, \gamma)$ be a transformation matrix that transforms the source point cloud to the destination point cloud. For small rotation angles $\alpha, \beta, \gamma \approx 0$ we can use the following approximation for the rotation matrix $R(\alpha, \beta, \gamma)$:

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$$\hat{R}(\alpha,\beta,\gamma) = \begin{pmatrix} 1 & -\gamma & \beta & 0 \\ \gamma & 1 & -\alpha & 0 \\ -\beta & \alpha & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

We now want to find $\hat{M} = T(t_x, t_y, t_z) \bullet \hat{R}(\alpha, \beta, \gamma)$ that minimizes:

$$\sum_i \left((\hat{M}s_i - d_i)n_i \right)^2$$

where each term can be written as a linear expression of rotation and translation parameters:

$$(\hat{M}s_i - d_i)n_i = (\hat{M}\begin{pmatrix} s_{ix} \\ s_{iy} \\ s_{iz} \\ 1 \end{pmatrix} - \begin{pmatrix} d_{ix} \\ d_{iy} \\ d_{iz} \\ 1 \end{pmatrix})\begin{pmatrix} n_{ix} \\ n_{iy} \\ n_{iz} \\ 0 \end{pmatrix}$$

This can further be rewritten in the form $Ax - b$ where x is the vector of unknowns:

$$x = \left(\alpha, \ \beta, \ \gamma, t_x, t_y, t_z \right)^T$$

Thus, our goal is to minimize $|Ax - b|^2$ which is a linear least-squares problem and can be solved using singular value decomposition.

Your task is to implement the point-to-plane constraint function in the LinearICPOptimizer class. For a more detailed analysis please refer to the "Linear Least-Squares Optimization for Point-to-Plane ICP Surface Registration" paper that you can find together with the exercise sheet and code in moodle. Note that the system has $4n$ rows where n is the number of points. For each point, we expect you to add both a point-to-plane constraint (one row) as described in the paper, and a point-to-point constraint (three rows, one for each coordinate). To determine the elements of the rows that correspond to the point-to-point constraints, try expanding $Ms_i = d_i$ by hand and take a look at which coefficients are multiplied by the unknowns and which are free.

You should experiment with different numbers of iterations for all ICP variants. How does the convergence speed change when switching from one method to another?

4. Camera Tracking using Frame-to-Frame ICP

We now want to test your algorithm on real-world data that you already used in the first exercise sheet. If you successfully implemented the ICP algorithm in the previous task, then you are almost done. You still need to implement the computation of point cloud normals in the PointCloud.h header file and set the macro in main.cpp to also run this task. Make sure you run the code in Release mode, otherwise the optimization will take too long.

The Frame-to-Frame ICP produces a depth map mesh with current camera pose every 5 frames. You need to take meshes of frames 1, 11 and 21 and put them into a common mesh mesh_merged.off in MeshLab (Filters -> Mesh Layer -> Flatten Visible Layers).

>>> Submission Deadline: Friday, December 2 2023 23:59 <<<

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Your result should look as shown below:

5.   Submit Your Solution

5.1.   Code Files: **ICPOptimizer.h**, **PointCloud.h**

5.2.   Meshes: **bunny_icp.off**, **mesh_merged.off**

5.3.   (Optional) If you worked in a group, upload **team_members.txt**.