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ALGEBRAIC FEATURE EXTRACTION OF IMAGE FOR RECOGNITION

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Abstract—The extraction of image features is one of the fundamental tasks in image recognition. Up until now, there have been several kinds of features to be used for the purpose of image recognition as follows: (1) visual features; (2) statistical features of pixel; (3) transform coefficient features. In addition, there is another kind of feature which the author believes is very useful, i.e. (4) algebraic features which represent intrinsic attributions of an image. Singular Values (SV) of image are this kind of feature. In this paper, we prove that SV feature vector has some important properties of algebraic and geometric invariance, and insensitiveness to noise. These properties are very useful for the description and recognition of images. As an example, SV feature vector is used for the problem of recognizing human facial images. In this paper, using SV feature vector samples of facial images, a normal pattern Bayes classification model based on Sammon's optimal descriminant plane is constructed. The experimental result shows that SV feature vector has good performance of class separation.

Image recognition   Algebraic feature extraction   Singular value feature
Facial image recognition   Discriminant vector   Dimensionality reduction

1. INTRODUCTION

Feature extraction is an old problem in the field of pattern recognition, however, it has been the most fundamental and important problem in the field. For one specific recognition problem, finding the efficient features is always the key to solving the problem. And so also is image recognition. Extracting effective features is an important step to complete the task of image recognition. So far, there have been several kinds of image features for recognition which may be divided into four types as follows:

(1) *Visual features.* They include edges, contours, textures and regions of an image. They are all visual features of pixel. So far, there has been much literature concerning the problem of extracting these features;

(2) *Statistical features of pixel.* For example, histogram features belong to this kind of feature. In addition, with the view of taking an image as a two-dimensional stochastical process, various statistical moments may be introduced to describe and analyze images. Reference (1) studied the properties of all kinds of moments. For instance, the ability of describing image, sensitivity to noise and information redundance, etc. Among the properties of moments, we can say, geometric invariance is the most important property.[2] Reference (3) pointed out that the features extracted by means of Zernike moment are very effective for image recognition;

(3) *Transform coefficient features.* We may take the transform coefficients as a kind of image feature using a mathematical transformation. It is well known[4,5] that Fourier descriptors (or Fourier coefficients) have a good ability to describe edges of objects. And Fourier transform can also be used to extract texture features of image.[6] Reference (7) proposed applications of Hadamard transform in image feature extraction. In addition, in references (8, 9), an autoregressive model method is proposed for dealing with the problem of description and recognition of two-dimensional objects. In polar coordinate system, the radial distance $r(t)$ of two-dimensional objects boundary is considered as a time sequence, which means that the radial value of every boundary point is dependent on the values of neighbouring boundary points. Therefore, an autoregressive model can be established. The parameters of the model are used as the feature vector for describing boundaries of the objects. Among various mathematical transforms, many researchers are interested in the transforms of geometric invariance.[10–14]

In addition to the three kinds of image features mentioned above, there is another kind of image feature as follows:

(4) *Algebraic features.* They represent intrinsic attributions of an image. Any image can be considered as a matrix. Therefore, various algebraic transforms or matrix decompositions can be used for algebraic feature extraction of the image. The well-

known Karhunen–Loeve transform is actually one method of algebraic feature extraction which is based on the eigenvectors of covariance matrix. Because eigenvectors of matrix represent algebraic attributions of the matrix and are of geometric invariance, eigenvectors can be used as a kind of image feature.

In the last few years, studies[15–19] have shown that Singular Value Decomposition (SVD) of matrix is a new kind of effective algebraic feature extraction of image. SVD has been used in many fields such as data compression, signal processing and pattern analysis. According to Marinovic's study,[15] after one-dimensional signal is transformed into two-dimensional representation of Wigner distribution, the singular values extracted by SVD of the representation have a good performance for shape description. In the example of shape recognition listed in their paper, only a few dominant singular values, which have high performance of class separation, are sufficient for successful shape classification. Reference (18) reported that a faint electrocardiogram signal of foetus on the background of strong noise signal is extracted using SVD.

In the author's view, the main theoretical backgrounds in image recognition applications of singular value features in image recognition are: (1) the SVs of image have very good stability, i.e. when a small perturbation is added to an image, great variance of its SVs does not occur; (2) singular values represent algebraic attributions of image which are intrinsic and not visual. As a kind of image feature, SV features have the properties of algebraic and geometric invariances.

First, this paper proves some important properties of SV feature vector, which we are concerned with in the field of pattern recognition, such as stability, the invariances to orthogonal transformation, rotation, and translation. In order to recognize image, the problem of dimensionality reduction of SV feature vector space is considered. Using SV feature vector samples of images, Sammon's optimal discriminant plane is constructed and then a statistical classification model of normal patterns is designed based on the plane. As an example, we use the model for recognizing human facial image. The experimental results show that SV feature vector of facial image is more effective than commonly used visual geometric features of human face in respect of recognition performance.

2. SINGULAR VALUE FEATURE EXTRACTION

We know, any real symmetric matrix can be transformed into a diagonal matrix by means of orthogonal transformation, and similarly for any general rectangular matrix $A_{m\times n}$ by means of so-called Singular Value Decomposition as follows.

Theorem 1. (SVD) Let $A_{m\times n}$ be a real rectangular matrix (suppose $m>n$, without loss of generality) and $\operatorname{rank}(A)=k$. Then there exist two orthonormal matrices $U_{m\times m}$, $V_{n\times n}$, and a diagonal matrix $\Sigma_{m\times n}$, and the following formula holds,

$$A = U\sum V^T \tag{1}$$

where

$$\sum = diag(\lambda_1, \lambda_2, \ldots, \lambda_k, 0, \ldots, 0). \tag{2}$$

Superscript T denotes transpose; $\lambda_1>\lambda_2>\ldots>\lambda_k$. Each λ_i^2, $i=1,2,\ldots,k$, is the eigenvalue of AA^T as well as A^TA, λ_i is called singular value of matrix A; and

$$U=(u_1,u_2,\ldots,u_k,u_{k+1},\ldots,u_m),$$
$$V=(v_1,v_2,\ldots,v_k,v_{k+1},\ldots,v_n)$$

in which, $\mathbf{u}_i$ and $\mathbf{v}_i$, $i=1,2,\ldots,k$, are column eigenvectors of AA^T and A^TA corresponding to eigenvalue λ_i^2, respectively; u_i, $i=k+1,\ldots,m$, can be seen as column eigenvectors of AA^T corresponding to eigenvalue $\lambda=0$, for the sake of convenience of matrix expression. Similarly, $\mathbf{v}_i$, $i=k+1,\ldots,n$ can also be seen as column eigenvectors of A^TA corresponding to eigenvalue $\lambda=0$.

Formula (1) can be written in the form of product sum expression as follows:

$$A=\sum_{i=1}^{k}\lambda_i u_i v_i^T. \tag{3}$$

With the view of taking an image as a matrix, formula (3) means that the original image A has undergone orthogonal decomposition. We may construct the following column vector consisting of the principal diagonal entries of matrix Σ, i.e. singular values $\lambda_i, i=1,2,\ldots,k$, and the $(n-k)$ remainders are zero of principal diagonal.

$$x_{n\times 1}=\sum e=\begin{bmatrix}\lambda_1\\ \ldots\\ \lambda_k\\ 0\\ \ldots\\ 0\end{bmatrix} \tag{4}$$

where, column vector $e=(1,1,\ldots,1)^T_{n\times 1}$. We call $x_{n\times 1}$ Singular Value (SV) feature vector of image A. For any real rectangular matrix A, under the constraint of $\lambda_1>\lambda_2>\lambda_k$, diagonal matrix Σ of singular values in formula (1) is unique. Therefore, the original image A corresponds to a unique SV feature vector.

Now we prove several important properties of SV feature vector.

(1) *The stability of SV feature vector*

Since the original image and its SV feature vector have a unique corresponding relationship, the SV feature vector can be used to represent image. We further consider the stability of the vector; i.e. when grey values of an image have small change, whether its SV feature vector has a great change or not. If SV feature vector does not have a great change, we call it stable. From the perturbation analysis of singular values,[20] it can be easily concluded that SV feature vector has very good stability, which is shown by the following theorem. Let $R^{m\times n}$ denote the set of all $m\times n$ order real matrices.

Theorem 2. Let $A_{m\times n}$ and $B_{m\times n}\in R^{m\times n}$, and their singular values be $\sigma_1>\ldots>\sigma_n$, $\tau_1>\ldots>\tau_n$, respectively. Then the following formula holds for any norm $\|.\|$ of unitary invariance on $R^{m\times n}$

$$\|\mathrm{diag}(\tau_1-\sigma_1,\ldots,\tau_n-\sigma_n)\|\leqslant\|B-A\|. \quad (5)$$

In particular, if we choose the norm in the above theorem as Frobenius norm $\|.\|_F$, formula (5) becomes

$$\sqrt{\sum_{i=1}^{n}(\tau_i-\sigma_i)^2}\leqslant\|A-B\|_F. \quad (5')$$

Similarly, if we choose the norm in the theorem as spectral norm $\|.\|_2$, formula (5) becomes

$$|\tau_i-\sigma_i|\leqslant\|B-A\|_2, \qquad i=1,2,\ldots,n. \quad (5'')$$

Due to its high stability, SV feature vector has the characteristic of insensitiveness to image noise, or to the small change of grey values of image caused by different illumination conditions. This characteristic relaxes the requirement of image pre-processing. The experimental result presented in reference (24) demonstrated the insensitiveness of SV features to noise.

(2) *The invariance of SV feature vector to the proportional variance of image intensity in the optimal discriminant vector space*

The stability of SV feature vector indicates that when the grey levels of image have small perturbation caused by the stochastic variation, the variance of SV feature vector is less than the norm of difference image, which means that SV feature has higher stability. In the following discussion, another important property of SV vector will be revealed.

Suppose the grey levels of original image to be represented by matrix A. The proportional variance of grey levels of image A is equivalent to multiplying A by a nonzero constant α. So after grey levels of image A are proportionally varied, the new image is αA.

Let $\mathrm{rank}(A)=k$ and the singular values of A and αA be $\lambda_1,\lambda_2,\ldots,\lambda_k$ and $\lambda_1^*,\lambda_2^*,\ldots,\lambda_k^*$, respectively. Thereupon, the eigenequation of $(\alpha A)(\alpha A)^T$ is

$$|(\alpha A)(\alpha A)^T-\lambda^{*2}I|=0$$

which means

$$|AA^T-\frac{1}{\alpha^2}\lambda^{*2}I|=0.$$

Therefore, the relationship between SV feature vectors of αA and A is

$$(\lambda_1^*,\lambda_2^*,\ldots,\lambda_k^*,0,\ldots,0)^T = |\alpha|(\lambda_1,\lambda_2,\ldots,\lambda_k,0,\ldots,0)^T. \quad (6)$$

This relationship indicates that when grey levels of image A are varied by a proportional factor α, its SV feature vector is also multiplied by a constant proportional factor α.

In addition, solving optimal discriminant vector is independent of proportional factor of SV feature vector. Therefore, SV feature vector has following property: SV feature vector is invariant to the proportional variance of image intensity in optimal discriminant space. This property is important for description and recognition of scene.

It should be noted that if proportional factor $\alpha=-1$, then image A and image $-A$ have the same SV feature vector.

(3) *The invariance of SV feature vector to transposition transform*

According to SVD theorem, we have

$$AA^Tu=\lambda^2u,$$
$$A^TAv=\lambda^2v$$

so that A and A^T have same singular values, i.e. they have same SV feature vector.

(4) *The invariance of SV feature vector to rotation transform*

First, elementary orthogonal transformation should be introduced. The square matrix in the form such as

$$Q=I-2uu^T$$

is called elementary orthogonal matrix, or Householder transform, in which I is unit matrix and u, $\|u\|_2=1$, is any real n-dimensional column vector of unit length.

According to matrix theory, the rotation transform can be decomposed into a product of two orthogonal matrices. For example, in the case of two dimensions, we have

$$\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}=(I-2\begin{bmatrix}0\\1\end{bmatrix}\times[0,1])$$
$$\times(I-2\begin{bmatrix}\sin\theta/2\\ -\cos\theta/2\end{bmatrix}\times[\sin\theta/2,\ -\cos\theta/2]). \quad (7)$$

Let the matrix of original image be A. One rotation transform of A is equivalent to A left-multiplied by an orthogonal matrix P. After the transform, the original image A becomes PA. Thus, we obtain

$$(PA)(PA)^T = P(AA^T)P^T$$

where $P^T = P^{-1}$. It can be easily seen that the orthogonal transform of A leads to the orthogonal similarity transform of AA^T (or A^TA). Furthermore, AA^T and $P(AA^T)P^T$ have same eigenvalues. Therefore, the original image A and is rotated image have the same SV feature vector, i.e. SV feature vector is invariant to rotation transform.

(5) *The invariance of SV feature vector to translation*

The translation transform of an image equivalent to row (or column) permutation of the image, i.e. such elementary operation as interchange of any two rows (or columns) of the image. It is known that interchanging the entries of the ith and the jth rows of a matrix A is equivalent to left-multiplying A by the following matrix

$$I_{i,j} = I - (e_i - e_j)(e_i - e_j)^T$$

where e_i and e_j denote the ith column and the jth column of unit matrix I, respectively. After the interchange, $I_{i,j}A$ is obtained. The eigenequation of $(I_{i,j}A)(I_{i,j}A)^T$ is

$$|(I_{i,j}A)(I_{i,j}A)^T - \lambda I| = 0.$$

It is noted that $I_{i,j}^T = I_{i,j} = I_{i,j}^{-1}$. The above equation can be simplified as follows:

$$\begin{aligned}&|I_{i,j}AA^TI_{i,j}^T - \lambda I|\\ &= |I_{i,j}| \times |AA^T - \lambda I_{i,j}^{-1}I_{i,j}^{-1}| \times |I_{i,j}|\\ &= |AA^T - \lambda I| = 0.\end{aligned}$$

Thus, the original image A and the image $I_{i,j}A$, in which two rows of A are interchanged, have the same SV feature vector. Similarly, the same conclusion for column permutation can be also obtained. Therefore, SV feature vector is invariant to translation transform.

(6) *The invariance of SV feature vector to mirror transform*

A transform T is called mirror transform if the transform T satisfies the formula $T(y + \alpha x) = y - \alpha x$ for any vector y perpendicular to a fixed vector x, where α is a real constant. We know that the matrix of Householder transform is $T = I - 2uu^T$. For any vector $\mathbf{w}$ perpendicular to vector $\mathbf{u}$, formula $T * (\mathbf{w} + \alpha\mathbf{u}) = \mathbf{w} - \alpha\mathbf{u}$ can be easily obtained. Thus, Householder transform is a kind of mirror transform whose geometric meaning is that the correspondence of vector $\mathbf{w} + \alpha\mathbf{u}$ under the mirror transform is the reversion of vector $\mathbf{w} + \alpha\mathbf{u}$ about superplane $\mathbf{u}^\perp$ perpendicular to $\mathbf{u}$, which is illustrated as Fig. 1.

Whichever scheme of image feature extraction is used, it is always expected that features extracted

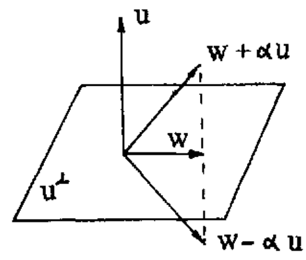

Fig. 1. The geometric meaning of the mirror transform.

are invariant to algebraic and geometric transforms. The above properties of SV feature vector assure it has these invariances, which provide the theoretical basis for using singular values as image features.

In summary, it may be concluded that SV feature vector is a kind of attractive algebraic feature of image.

3. FOLEY-SAMMON TRANSFORM OF SV FEATURE VECTOR

An image $A_{m \times n}$ being represented as an n-dimensional SV feature vector, now, the problem of image recognition can be solved in the n-dimensional feature space. However, the dimensionality of the feature space is usually very high for any image recognition problem. For example, in the problem of facial image recognition presented in this paper, a piece of facial photo of 32 mm × 27 mm needs 70-dimensional SV feature vector to describe it. A lot of studies show that the approach to design classification and model directly in original high dimensional feature space is unfavourable. In order to have a good insight into the distribution of pattern samples and design a classifier of high performance suitable for a specific problem, compressing original high dimensional space into low dimensional space such as two-dimension, even one-dimension, by means of various mathematic transforms is the best way. As in the results of many researchers, our experimental results presented in reference (21) show that the correct recognition rate of a specific classifier in the compressed feature space is greater than one in the original space.

Among various transformations of compressing dimensionality, Foley-Sammon's transform[22] based on Fisher criterion, i.e. optimal discriminant vectors, is the most popular one. So-called Fisher's discriminant criterion is described as follows:

$$J_F(\varphi) = \frac{\varphi^T S_b \varphi}{\varphi^T S_w \varphi} \tag{8}$$

where S_b and S_w are between-class scatter matrix and within-class scatter matrix of pattern samples, respectively.

The main idea of Foley-Sammon transform is to solve the maximum solution vector φ of the above equation with certain constant conditions on φ.

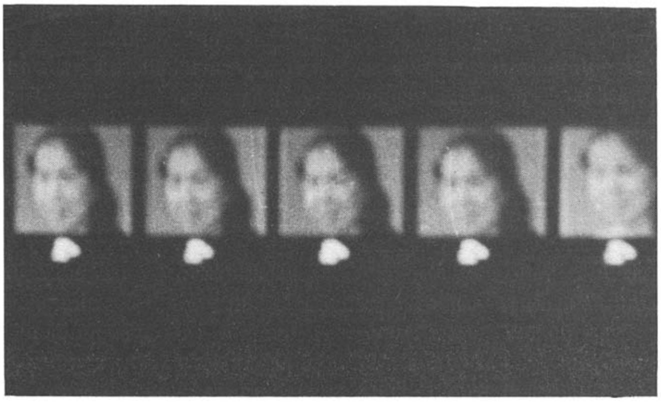

Fig. 2. Five image samples of each photo for constructing model.

In our experiment of facial image recognition, we only use the two former vectors in the set of optimal discriminant vectors, which span so-called Sammon's discriminant plane, in the case of recognizing a small number of facial photos. It may be predicted that more discriminant vectors have to be used to construct a new feature space, in order to avoid the decrease of recognition rate with the increase of the number of photos (i.e. the increase of class number). In reference (23) the author proposed an approach to solve multiclass problems by constructing minimum distance linear classifier on the optimal discriminant plane.

In order to solve these optimal discriminant vectors, first of all, a large number of image samples are needed to calculate the scatter matrices S_b and S_w of pattern samples. Therefore, we have to sample each facial photo several times. For each sampling, the relative positions of the photo and the TV camera have a small change to obtain a good statistical recognition model.

It has been noted in references (19, 24) that it is difficult to obtain the discriminant vectors in the case of a small number of samples, i.e. the number of samples is less than the dimensionality of feature space, because the scatter matrix S_w is singular in this case. Moreover, the case of small sample size often occurs in problems of image recognition. Q. Tian[24] proposed a scheme of using generalized inverse S_w^+ of matrix S_w to replace inverse S_w^- which does not exist because S_w is singular. The authors (19) solved the problem in another way. The main idea is that a singular value perturbation is added to the singular within-class scatter matrix S_w such that the matrix becomes a nonsingular matrix. Therefore, the optimal discriminant vectors can be obtained by existing algorithms. The authors proved that the solutions of the generalized optimal discriminant vectors can be obtained by existing algorithms. The authors proved that the solutions of the generalized eigenequations satisfied by discriminant vectors are indeed the optimal indiscriminant vectors if the perturbation subjecting to some certain conditions and the generalized eigenequations are not ill-conditioned.

4. EXPERIMENTAL RESULTS

We have shown in Section 2 that SV feature vector is invariant to algebraic and geometric transforms, which make the vector have good ability for describing images. In order to examine the ability and the normal pattern classification model of SV feature vector, we produce an experiment of recognizing 9 facial photos of 50mm × 35 mm. In our experiment we use PC Vision image processing system to input photos. Using a window of 84 × 70 to extract a region (32 mm × 27 mm) of facial part shown in Fig. 2, the image size sampled by PC Vision system is 84 × 70, the range of grey levels is 0–255. Each photo is sampled repeatedly by five times with a small change in the relative positions between photo and TV camera. Thus, we obtain five image samples for each class, and 45 image samples in total. Using these samples, a statistical recognition model in the optimal discriminant plane can be constructed.

First of all, we apply SVD operation to each image matrix for extracting SV features and then obtain a SV vector of the facial image such as:

$$
\begin{array}{rlllllllll}
x = (& 0.9632 & 0.1513 & 0.1142 & 0.0893 & 0.0767 & 0.0728 & 0.0661 & 0.0612 & 0.0535 \\
& 0.0351 & 0.0294 & 0.0268 & 0.0251 & 0.0235 & 0.0186 & 0.0165 & 0.0139 & 0.0137 \\
& 0.0115 & 0.0094 & 0.0091 & 0.0084 & 0.0077 & 0.0074 & 0.0072 & 0.0067 & 0.0063 \\
& 0.0061 & 0.0059 & 0.0057 & 0.0056 & 0.0053 & 0.0052 & 0.0050 & 0.0049 & 0.0049 \\
& 0.0046 & 0.0043 & 0.0041 & 0.0039 & 0.0038 & 0.0037 & 0.0035 & 0.0034 & 0.0033 \\
& 0.0033 & 0.0031 & 0.0029 & 0.0029 & 0.0028 & 0.0027 & 0.0026 & 0.0024 & 0.0023 \\
& 0.0022 & 0.0020 & 0.0019 & 0.0017 & 0.0016 & 0.0016 & 0.0015 & 0.0014 & 0.0012 \\
& 0.0011 & 0.0010 & 0.0009 & 0.0008 & 0.0006 & 0.0005 & 0.0004)^T. & &
\end{array}
$$

Table 1. Results of quadratic classifier to recognize 45 training samples

Class and Class center	sample	The output values of quadratic discriminant function									Recog–niton
		d1	d2	d3	d4	d5	d6	d7	d8	d9	
No.1 (−0.0184,−0.0314)	1	13.04	−117.76	−208.13	−6.99	11.59	−792.13	−116.96	−0.44	6.34	1
	2	11.65	−126.38	−177.39	−3.64	−7.74	−625.99	−78.46	−6.72	5.47	1
	3	12.31	−128.59	−217.29	−7.37	11.81	−807.39	−115.00	−0.14	7.44	1
	4	11.32	−6.18	−218.30	−27.45	−20.82	−1193.73	−374.62	4.87	−53.73	1
	5	12.83	−166.44	−195.67	−2.37	−6.33	−654.26	−60.58	−6.50	7.36	1
No.2 (−0.0163,−0.0375)	1	6.58	13.33	−232.63	−52.04	−123.33	−1568.66	−666.79	−1.78	−154.69	2
	2	7.15	13.76	−214.54	−46.31	−68.71	−1403.22	−559.51	−2.74	−117.76	2
	3	−0.62	12.58	−182.44	−54.41	−34.06	−1281.91	−528.66	−14.89	−112.51	2
	4	3.66	12.64	−274.16	−49.97	−208.44	−1759.73	−743.79	6.20	−176.56	2
	5	8.67	12.59	−225.90	−42.00	−75.54	−1417.74	−546.14	1.61	−110.84	2
No.3 (0.0100, −0.0191)	1	−599.58	−126.05	10.88	−269.11	−1456.93	−112.51	−291.48	−343.64	−163.97	3
	2	−432.71	−129.26	11.05	−171.35	−1291.32	−73.22	−226.79	−252.93	−158.64	3
	3	−502.81	−177.68	12.79	−181.04	−1697.99	−60.64	−307.42	−285.21	−229.81	3
	4	−531.04	−207.23	12.39	−182.08	−1908.65	−65.54	−358.23	−298.06	−272.15	3
	5	−529.09	−215.00	12.26	−178.54	−1939.81	−65.49	−366.63	−296.95	−280.81	3
No.4 (−0.0132, −0.0197)	1	−15.25	−406.49	−110.07	11.44	−431.68	−140.95	−30.33	−54.19	−112.31	4
	2	−35.29	−564.06	−91.27	12.93	−753.90	−36.26	−127.43	−81.63	−223.20	4
	3	−61.96	−715.365	−74.90	11.78	−1135.08	9.26	−274.05	−111.07	−353.86	4
	4	−15.42	−649.33	−119.71	11.33	−658.56	−56.84	−117.33	−79.40	−225.50	4
	5	−34.64	−637.99	−94.56	13.00	−829.57	−21.76	−163.51	−88.99	−261.71	4
No.5 (−0.0170,−0.0323)	1	12.78	−90.00	−216.265	−10.40	13.18	−887.54	−162.38	2.77	0.66	5
	2	13.19	−87.71	−201.13	−9.22	13.65	−829.44	−146.94	0.57	1.57	5
	3	12.19	−57.98	−190.73	−12.96	14.45	−862.55	−181.92	−0.12	−6.01	5
	4	7.95	−30.69	−172.03	−18.38	13.34	−854.97	−206.19	−4.85	−13.81	5
	5	9.33	−23.86	−182.07	−21.17	12.90	−939.63	−248.54	−2.35	−22.64	5
No.6 (−0.0086, −0.0162)	1	−109.32	−592.65	−46.35	3.96	−1277.78	12.97	−283.20	−121.74	−341.41	6
	2	−66.58	−824.47	−75.99	10.86	1303.18	12.61	−359.64	−125.10	−428.13	6
	3	−113.06	−679.73	−47.46	5.21	−1431.93	13.41	−355.39	−132.09	−404.15	6
	4	−88.68	−680.71	−58.26	8.86	−1277.86	13.89	−308.20	−120.58	−372.60	6
	5	−111.13	−675.26	−48.08	5.45	−1413.34	13.73	−348.27	−130.73	−398,71	6
No.7 (−0.0229,−0.0260)	1	6.62	−433.22	−204.94	−1.82	−122.11	−381.72	13.71	−35.62	−43.12	7
	2	−4.31	−549.14	−240.85	−15.69	−116.84	−397.61	12.82	−49.88	−58.15	7
	3	−6.01	−487.93	−252.70	−17.18	−69.95	−481.85	13.08	−41.82	−33.17	7
	4	−5.72	−516.01	−248.95	−17.05	−87.49	−447.11	14.11	−45.53	−43.14	7
	5	6.57	−427.11	−206.88	−2.19	114.57	−392.66	13.74	−34.71	−40.08	7
No.8 (−0.0230, −0.0382)	1	−7.30	8.48	−316.72	−47.61	−281.08	−1885.17	−762.43	11.70	−175.92	8
	2	1.86	−30.46	−280.39	−26.97	−79.93	−1371.66	−402.64	11.88	−53.82	8
	3	−28.47	−15.36	−359.86	−45.76	−299.33	−1872.79	−666.27	12.01	−132.56	8
	4	−18.73	−4.45	−341.94	−44.65	−284.08	−1860.38	−689.29	13.06	−143.86	8
	5	−4.03	−9.58	−302.43	−35.31	−161.12	−1598.50	−543.90	13.18	−97.58	8
No.9 (−0.0271, −0.0314)	1	−89.54	−404.68	−406.84	−70.63	−69.40	−1111.25	−113.89	−48.83	11.91	9
	2	−18.78	−298.67	−301.94	−26.18	−4.68	−859.44	−68.06	−18.68	12.77	9
	3	−17.06	−281.36	−299.97	−25.05	−4.26	−876.70	−76.61	−16.21	13.26	9
	4	−9.28	−228.14	−287.63	−20.30	−1.66	−910.74	−101.27	−8.62	12.92	9
	5	−34.79	−265.50	−336.15	−36.09	−29.43	−1042.22	−127.31	−17.16	12.56	9

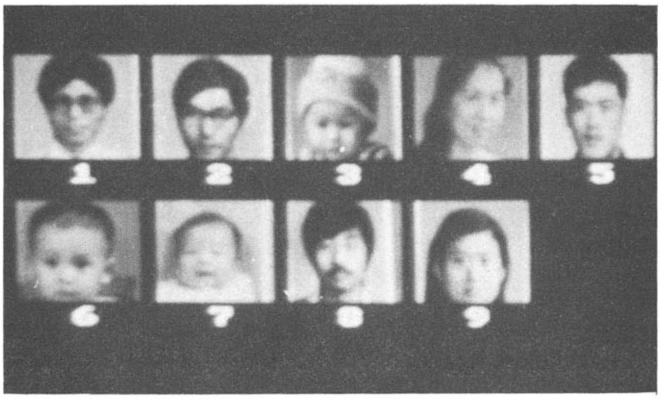

Fig. 3. Nine photos to be recognized and their class numbers.

Secondly, using these 45 SV feature vector samples, the optimal discriminant plane and the quadratic classifier of normal pattern can be constructed in the case of a small number of samples by the method presented in reference (19). Finally, the performance of the classifier should be examined using training samples and testing samples. The experimental results are given as follows. Table 1 gives the recognition results of the quadratic normal pattern classifier. From Table 1 we may see that the classifier correctly recognizes all 45 training samples of the 9 facial photos. The results presented in Table 2 are obtained using 13 testing samples shown in Fig. 4. These 13 testing samples are newly sampled and three photos of which do not belong to the set of photos presented in Fig. 3, which means that the persons presented by these three photos belong to the set of persons presented in Fig. 3, but were taken at different ages. The other 10 photos belong to the set of 9 photos shown in Fig. 3, but they are newly sampled and one of them is sampled two times.

It should be noted that the above results are obtained in the case in which all original image matrices are subject to normalized pre-processing in the sense of Frobenius norm. Due to its stability, however, SV feature vector is insensitive to image noise and different illuminations. Therefore, pre-processing of image is not necessary for the image recognition algorithm using SV features.

5. CONCLUSION

This paper proved that SV feature vector of image has the invariance of algebraic and geometric transforms such as orthogonal transform, rotation, translation and mirror transform, etc. These invariances make SV features become very efficient features for describing and recognizing images. The experimental results presented in this paper have shown that the frontier topic of facial image recognition may be well solved using the quadratic normal classification model based on SV feature vector. The method proposed in this paper has general meaning and may be also applied to other recognition problems.

In our recognition model, there are still some statistical limitations due to using a small number of

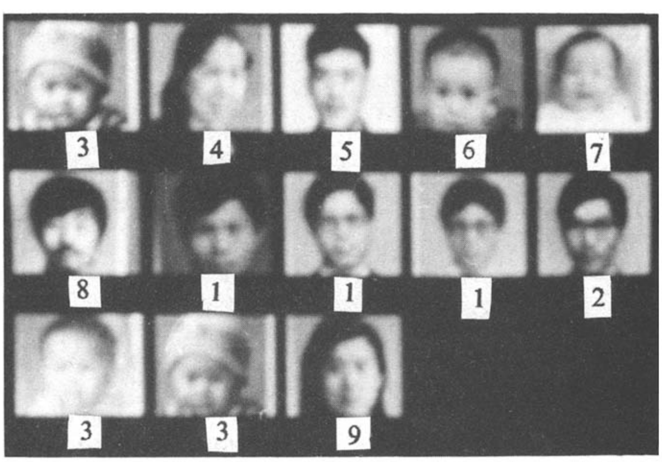

Fig. 4. Thirteen testing image samples. Results of recognition are shown in Table 2.

Table 2. Results of quadratic classifier to recognize 13 testing samples

Sample	Class	The output values of quadratic discriminant function									Recognition
		d1	d2	d3	d4	d5	d6	d7	d8	d9	
1	ω3	−31.7352	−23.0110	10.7938	−14.2865	−26.5584	−5.8977	−36.2276	−44.7727	−35.8063	ω3
2	ω4	11.2819	9.6777	−26.5317	5.4080	11.3821	−4.1338	8.5490	8.5764	8.4281	ω5
3	ω5	11.0999	10.2696	−32.0374	3.1712	11.1303	−8.4696	7.8186	9.9310	8.8872	ω5
4	ω6	9.0166	3.1063	−16.9151	10.3446	8.0569	6.0038	9.2248	1.3847	5.1843	ω4
5	ω7	9.2096	5.4122	−44.2684	2.1618	8.5906	−10.3216	10.2333	8.4983	11.1019	ω9
6	ω8	4.1417	6.7176	−65.1270	−11.3368	3.0680	−34.0116	−0.4660	10.3477	6.7079	ω8
7	ω1	10.4724	5.6091	−22.9579	8.7801	9.8504	2.1609	10.3869	4.9927	7.8135	ω1
8	ω1	11.2146	10.1889	−27.3669	4.5244	11.3855	−5.6217	7.8706	8.9923	8.2232	ω5
9	ω1	11.2794	9.2215	−29.9856	5.0704	11.1955	−5.1794	9.2578	9.0316	9.3012	ω1
10	ω2	9.7525	11.3276	−32.1467	−1.0549	9.9457	−14.3419	2.8534	9.6325	5.5488	ω2
11	ω3	0.6597	−0.9909	1.0937	7.8303	1.3164	8.5577	−1.7425	−9.8672	−5.8899	ω6
12	ω3	−19.8767	−17.5848	10.1345	−2.0295	−18.1917	3.9994	−22.2852	−34.0557	−25.9106	ω3

training samples for constructing this model. In order to make the model more practical, the limit should be overcome and the number of photos should be increased in future.

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