SmartRail Network Optimization
IEMS 313

Project Phase 2

Group 6

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Contents

Tables

Figures

1 Problem description

1.1 Description

SmartRail, a leading transportation company, has established an extensive network of train stations and interconnected tracks to facilitate the smooth movement of shipments. Each track has a specified capacity for transporting containers, ensuring efficient transportation from origin to destination. However, when a shipment exceeds the track or reloading station's capacity, it becomes necessary to divide the shipment and transport it using multiple routes on different tracks.

In order to enhance the transportation system and optimize the movement of containers, SmartRail faces the critical decision of whether to build additional tracks and reloading zones. This decision is crucial, considering the large volume of containers that need to be transported between certain stations. The ultimate goal is to minimize the biweekly shipping costs, penalty fees incurred for delayed deliveries, and the construction costs associated with new tracks and reloading stations.

This report aims to develop an optimized network expansion plan for SmartRail. This plan ensures that the number of tracks and reloading zones between stations complies with the maximum limit. It also takes into account the track capacity constraints and reloading zone limitations for shipments. Moreover, containers are picked up or delivered only when available, adhering to their specified schedules. The primary focus is minimizing overall network expansion costs while maintaining an efficient and reliable transportation system.

By successfully implementing this optimized network expansion plan, SmartRail aims to streamline its operations, reduce expenses, and provide timely deliveries for its customers. The comprehensive approach considers the interplay between track capacities, reloading zones, and overall costs, leading to an effective and sustainable transportation infrastructure.

1.2 Objective

1.2.1 Phase 2 objective

Our team will design a program to minimize the cost of SmartRail's network expansion plan. We will write a data-independent MILP AMPL model to find a transportation plan for the shipments in the given tables with minimal cost. Our solution model will use the CSV setup with an AMPL model file, CSV files, and a run file.

2 Mathematical program

2.1 Sets

S: Set of stations i

R: Set of all tracks (i, j)

M: Set of shipments k

D: Set of days $d \in 0..T$

U: Set of tracks reversed $(j, i) \in R$, $(i, j) \notin R$

2.2 Variables

N_{ijkd}: The number of containers transported from station i to station j on day d belonging to shipment k (for $(i, j) \in R$, $k \in M$, $d \in D$)

new_tracks_{ij}: The number of new tracks between station i and station j (for $(i, j) \in R$)

$new\_reload\_zones_i$: Number of new reloading zones constructed at station i (for $i \in S$)

$pickup_{ikd}$: Number of containers picked up from station i for shipment k on day d (for $k \in M$, $d \in D$)

$deliver_{ikd}$: Number of containers delivered to station i for shipment k on day d (for $i \in S$, $k \in M$, $d \in D$)

$late_{kd}$: Binary variable that is 0 if day $d < deliv\_day$ for shipment k, otherwise 1 (for $k \in M$, $d \in D$)

2.3 Parameters

x_coord_i: X coordinate of station i, in Cartesian coordinates (for $i \in S$)

y_coord_i: Y coordinate of station i, in Cartesian coordinates (for $i \in S$)

len_track_{ij}: Track length between station i and station j (for $(i, j) \in R$)

num_reload_i: Number of reloading zones of station i (for $i \in S$)

$cost\_reload$: Cost for reloading one container, in $/container

$cost\_ship$: Cost for shipping one container for one mile, in $/container/mile

$capa\_track$: Container capacity of railroad tracks, in # of containers/track

$capa\_reload$: Container capacity of reloading zones, in # of containers/zone

T: Number of days in the biweekly schedule

$cost\_penalty$: Daily late penalty fee per shipment

$cost\_track$: Daily fee for constructing a new track

$cost\_new\_reload$: Daily fee for constructing a new reload station

max_reload: Maximum number of reloading stations at each station

max_tracks: Maximum number of tracks between two stations

$orig_k$: Origin station of shipment k (for $k \in M$)

$dest_k$: Destination station of shipment k (for $k \in M$)

$volume_k$: Volume of shipment k, in # of containers/shipment (for $k \in M$)

$avail\_day_k$: Day that shipment k is available for pickup (for $k \in M$)

$deliv\_day_k$: Day that shipment k should be delivered (for $k \in M$)

2.4 Objective function

$$\begin{aligned}\textbf{min} \sum_{(i,j)\in R}\sum_{k\in M}\sum_{d\in D}(cost\_ship \cdot N_{ijkd} \cdot len\_track_{ij}) + \sum_{i\in S}\sum_{d\in D}(cost\_new\_reload \cdot new\_reload\_zones_i) + \\ \sum_{(i,j)\in R}\sum_{d\in 1..T}(cost\_track \cdot new\_tracks_{ij} \cdot len\_track_{ij}) + \sum_{(i,j)\in R}\sum_{k\in M}\sum_{d\in D}(cost\_penalty \cdot N_{ijkd} \cdot late_{kd})\end{aligned}$$

2.5 Constraints

$$\textbf{s.t.}\ \ new\_tracks_{ij} + 1 \leq max\_tracks \qquad \forall\ (i,j) \in R$$

$$new\_tracks_{i,j} - new\_tracks_{j,i} = 0 \qquad \forall\ (i,j) \in R$$

$$new\_reload\_zones_i + num\_reload_i \leq max\_reload \qquad \forall\ i \in S$$

$$\sum_{k\in M}(N_{ijkd} + N_{jikd}) \leq capa\_track \cdot (new\_tracks_{ij} + 1) \qquad \forall\ (i,j) \in R,\ d \in D$$

$$\sum_{(i,j)\in R}\sum_{k\in M} N_{ijkd} \leq capa\_reload \cdot (new\_reload\_zones_j + num\_reload_j) \qquad \forall\ j \in S,\ d \in D$$

$$\sum_{(i,j)\in R} N_{ijkd} = \begin{cases} 0 & \text{if } d < avail\_day_k \\ N_{ijkd} & \text{otherwise} \end{cases} \qquad \forall\ k \in M$$

$$\sum_{d \in avail\_day_k..T-1} pickup_{jkd} = \begin{cases} volume_k & \text{if } orig_k = j \\ 0 & \text{otherwise} \end{cases} \qquad \forall\, k \in M, j \in S$$

$$\sum_{d \in avail\_day_k..T-1} deliver_{jkd} = \begin{cases} volume_k & \text{if } dest_k = j \\ 0 & \text{otherwise} \end{cases} \qquad \forall\, k \in M, j \in S$$

$$pickup_{ikd} = \begin{cases} \sum_{(orig_k, j) \in R} N_{orig_k jkd} & \text{if } orig_k = i \\ 0 & \text{otherwise} \end{cases} \qquad \forall\, k \in M, i \in S, d \in avail\_day_k..T-1$$

$$deliver_{ikd} = \begin{cases} \sum_{(j, dest_k) \in R} N_{j dest_k kd} & \text{if } dest_k = i \\ 0 & \text{otherwise} \end{cases} \qquad \forall\, k \in M, i \in S, d \in avail\_day_k..T-1$$

$$late_{kd} = \begin{cases} 0 & \text{if } d < deliv\_day_k \\ 1 & \text{otherwise} \end{cases}$$

$$\sum_{(i,g) \in R} N_{igkd} + pickup_{gk(d+1)} = \sum_{(g,j) \in R} N_{gjk(d+1)} + deliver_{gkd} \qquad \forall\, g \in S,\ k \in M,\ d \in D, d < T$$

$$N_{ijkd} \geq 0 \text{ and Integer} \qquad \forall\, (i, j) \in T,\ k \in M$$

$$new\_tracks_{ij} \geq 0 \text{ and Integer} \qquad \forall\, (i, j) \in R$$

$$new\_reload\_zones_i \geq 0 \text{ and Integer} \qquad \forall\, i \in S$$

$$pickup_{ikd} \geq 0 \text{ and Integer} \qquad \forall\, i \in S, k \in M, d \in D$$

$$deliver_{ikd} \geq 0 \text{ and Integer} \qquad \forall\, i \in S, k \in M, d \in D$$

$$late_{kd} \in \{0, 1\} \text{ Binary} \qquad \forall\, k \in M, d \in D$$

3 Model explanation

The following is an overview of the mathematical program designed by our team for SmartRail to minimize operational costs. The program is focused on minimizing both reloading and shipping costs, which are the two main drivers of total operational costs. The mathematical variable used in this model to represent the number of containers moving throughout is N_{ijkd}, which represents the number of containers transported from station i to station j that belong to shipment k on day d. This allows us to model the movements of each container and ensure efficient transportation.

To achieve the goal of minimizing costs, we have designed a function that calculates the total shipping cost, the cost of building new tracks, the cost of building new reloading stations, and the cost of late fees. This function calculates the total shipping cost by multiplying the cost per mile by the length of the track in miles for each shipment for each day between all station pairs. These costs are then summed to achieve the final total shipping cost. The cost of building new tracks is calculated by summing the cost of building each new track per day, per mile multiplied by the number of new tracks built between each station pair by the length of the new track for each station pair for each day. The cost of new reloading stations is calculated in a similar way. It is the sum of the cost of building a new reloading station per day multiplied by the number of new reloading stations for each station for each day. Finally, the total late fee is calculated by summing the late fee per day by the number of shipments between each station that are late for each day between each station pair for all shipments. The model determines if a shipment is late by using a binary variable, *late*. This is 1 if a shipment is late or 0 if a shipment is on time. Therefore, when the value is multiplied by the late fee, it will be 0 if a shipment is on time.

The shipping costs, new track costs, new reloading stations costs, and late fees are then aggregated, representing the total cost of the operation. The mathematical statement that models this is:

$$\begin{aligned}\mathbf{min} \sum_{(i,j)\in R} \sum_{k\in M} \sum_{d\in D} (cost\_ship \cdot N_{ijkd} \cdot len\_track_{ij}\,) \;+\; \sum_{i\in S} \sum_{d\in D} (cost\_new\_reload \cdot new\_reload\_zones_i) \;+ \\ \sum_{(i,j)\in R} \sum_{d\in 1..T} (cost\_track \cdot new\_tracks_{ij} \cdot len\_track_{ij}) \;+\; \sum_{(i,j)\in R} \sum_{k\in M} \sum_{d\in D} (cost\_penalty \cdot N_{ijkd} \cdot late_{kd})\end{aligned}$$

Our Model is also subject to certain constraints, which we have included in our formal program. They are as follows:

The first constraint ensures that the model does not exceed the maximum amount of tracks that can be constructed between each station pair. The number of tracks at a station is calculated by adding 1, for the original track, to the number of tracks constructed. It is then required that this number is less than or equal to the maximum amount of tracks allowed. This constraint is modeled for each track between each station pair. The constraint is as follows:

$$new\_tracks_{ij} + 1 \leq max\_tracks \qquad \forall\ (i,j) \in R$$

The next constraint requires that the number of new tracks built in one direction, for example from station i to station j, is equal to the number of tracks built in the opposite direction, station

j to station i. This is modeled by the following equation:

$$new\_tracks_{i,j} - new\_tracks_{j,i} = 0 \qquad \forall\ (i,j) \in R$$

The next constraint requires that the total number of reloading stations at each station does not exceed the maximum amount of reloading stations allowed. This is calculated by adding the number of new reloading stations built, $new\_reload\_zones_i$, to the original number of reloading stations, num_reload_i for each station. This sum is then required to be less than or equal to the maximum number of reloading stations, max_reload.

$$new\_reload\_zones_i + num\_reload_i \leq max\_reload \qquad \forall\ i \in S$$

The next constraint ensures containers do not exceed the track capacity. This is calculated by summing the number of containers on each track, in both directions and ensuring this value is less than or equal to the capacity of each track. The capacity of the track is calculated by multiplying the capacity of an individual track by the number of tracks between the stations (including the newly built tracks). This constraint is modeled by the following inequality:

$$\sum_{k \in M} (N_{ijkd} + N_{jikd}) \leq capa\_track \cdot (new\_tracks_{ij} + 1) \qquad \forall\ (i,j) \in R,\ d \in D$$

The next constraint ensures containers do not exceed reload capacity. This is modeled by summing the total number of shipments at a station and requiring the value to be less than or equal to the reload capacity at each station. The reload capacity at a station is calculated by summing the total number of reloading stations at a station, including the existing stations and new stations, and multiplying that by the reload capacity of an individual station.

$$\sum_{(i,j) \in R} \sum_{k \in M} N_{ijkd} \leq capa\_reload \cdot (new\_reload\_zones_j\ +\ num\_reload_j) \qquad \forall\ j \in S,\ d \in D$$

The next constraint models the availability of a shipment on a given day. This constraint requires the number of containers moving between stations to be 0 if a shipment is not available yet. This is done by checking if the current day, d, is less than a shipment's availability day, $avail\_day$. If the current day is greater than or equal to a shipment's availability day then the number of containers is allowed to be whatever value the model assigns it.

$$\sum_{(i,j) \in R} N_{ijkd} = \begin{cases} 0 & \text{if } d < avail\_day_k \\ N_{ijkd} & \text{otherwise} \end{cases} \qquad \forall\ k \in M$$

The next constraint ensures the total number of containers picked up at each station is equal to that station's volume. This is done by summing all shipments at each individual station for all days and requiring the sum to be equal to that station's volume if they originate from that station. This is modeled with the following equation:

$$\sum_{d \in avail\_day_k..T-1} pickup_{jkd} = \begin{cases} volume_k & \text{if } orig_k = j \\ 0 & \text{otherwise} \end{cases} \qquad \forall\ k \in M, j \in S$$

The next constraint ensures the total number of containers delivered at a given station is equal to the volume of that station's deliveries. This is calculated by summing the number of shipments at each individual station for all stations for all shipments and ensuring it is equal to the volume at a given station if that station is the destination for a given shipment. This is modeled with the following equation:

$$\sum_{d \in avail\_day_k..T-1} deliver_{jkd} = \begin{cases} volume_k & \text{if } dest_k = j \\ 0 & \text{otherwise} \end{cases} \qquad \forall\, k \in M, j \in S$$

The next constraint ensures the number of containers leaving an origin station equals the number of containers picked up on that day from that same station. This is done by setting the number of containers picked up at each station on each day equal to the sum of all containers that match the same origin station. This gets the ball rolling on moving containers between stations.

$$pickup_{ikd} = \begin{cases} \sum_{(orig_k, j) \in R} N_{orig_k jkd} & \text{if } orig_k = i \\ 0 & \text{otherwise} \end{cases} \qquad \forall\, k \in M, i \in S, d \in avail\_day_k..T-1$$

The next constraint ensures the number of containers arriving at a destination station equals the number of containers delivered on that day at that station. This is done by setting the number of containers delivered at each station on each day equal to the sum of all containers that match the same delivery station. This constraint is also used to ensure that containers start to move.

$$deliver_{ikd} = \begin{cases} \sum_{(j, dest_k) \in R} N_{jdest_k kd} & \text{if } dest_k = i \\ 0 & \text{otherwise} \end{cases} \qquad \forall\, k \in M, i \in S, d \in avail\_day_k..T-1$$

The following constraint sets a binary variable equal to 1 if a shipment is late, otherwise 0 if it is on time. This is done by setting the variable to 0 when the current day, d, is less than the required delivery day, $deliv\_day$. It is considered late if $d = deliv\_day$ since it takes an extra day to count as delivered. This is modeled by the following equation:

$$late_{kd} = \begin{cases} 0 & \text{if } d < deliv\_day_k \\ 1 & \text{otherwise} \end{cases}$$

The last major constraint modeled by our program ensures that the inflow and outflow of each station are logical. This constraint is designed to prevent a situation where a station has more containers leaving it than are technically possible. To ensure logical inflow and outflow, the model requires that the number of containers entering a station combined with the number of containers that originate from that station are equivalent to the number of containers departing from a station summed with the number of containers terminating at that station. This is modeled by the equation:

$$\sum_{(i,g) \in R} N_{igkd} + pickup_{gk(d+1)} = \sum_{(g,j) \in R} N_{gjk(d+1)} + deliver_{gkd} \qquad \forall\, g \in S,\ k \in M,\ d \in D, d < T$$

The remaining constraints are constraints to ensure variables are non-negative and integer. This is because it is not possible to have a negative number of containers, tracks, reloading zones, pickups, deliveries, or late shipments. These numbers also all need to be integers because it is not possible to have a fractional amount of anything. These constraints are modeled with the following equations:

$$N_{ijkd} \geq 0 \text{ and Integer} \qquad \forall\ (i,j) \in T,\ k \in M$$

$$new\_tracks_{ij} \geq 0 \text{ and Integer} \qquad \forall\ (i,j) \in R$$

$$new\_reload\_zones_{i} \geq 0 \text{ and Integer} \qquad \forall\ i \in S$$

$$pickup_{ikd} \geq 0 \text{ and Integer} \qquad \forall\ i \in S, k \in M, d \in D$$

$$deliver_{ikd} \geq 0 \text{ and Integer} \qquad \forall\ i \in S, k \in M, d \in D$$

Finally, the last constraint requires the $late_{kd}$ variable to be binary. This means it can only be 0 or 1. This is because this is used to determine if a shipment is late or not, as explained in the equation for overall cost. This is modeled with the following equation:

$$late_{kd} \in \{0,1\} \text{ Binary} \qquad \forall\ k \in M, d \in D$$

By minimizing the total transportation cost of SmartRail's operations while following a set of well-defined constraints, our mathematical model is able to calculate optimal container paths and volumes. The mathematical program will find the optimal number of containers traveling between each station for all shipments. These numbers can be used in real-world operations to minimize the cost of SmartRail's operation.

4 Case 1 – Optimal solution

4.1 Table solutions

4.1.1 Cost Breakdown

Cost Category	Cost (USD)
Total Shipment Cost	$14,835,500
Total Late Delivery Cost	$1,500,000
New Reloads Cost	$180,000
New Tracks Cost	$21,854,400
Optimal Total Cost	$38,369,969.86

Table 1: Optimized cost breakdown (case1).

4.1.2 Company_A_4_7

Day	From Station	To Station	Container Load	Containers Delivered
1	4	6	10	0
	4	1	10	
2	1	3	10	10
	4	6	10	
	6	7	10	
3	3	7	10	20
	6	7	10	

Table 2: Shipment paths for Company_A_4_7 (case 1).

4.1.3 Company_A_3_2

Day	From Station	To Station	Container Load	Containers Delivered
3	3	5	5	0
4	5	2	5	5

Table 3: Shipment paths for Company_A_3_2 (case 1).

4.1.4 Company_B_1_11

Day	From Station	To Station	Container Load	Containers Delivered
8	1	4	10	0
	1	8	7	
9	1	4	2	0
	1	8	7	
	4	6	7	
	4	8	3	
	8	9	7	
10	1	8	10	0
	4	6	2	
	6	9	7	
	8	9	10	
	9	10	7	
11	6	9	2	7
	8	9	10	
	9	10	17	
	10	11	7	
12	9	10	12	17
	10	11	17	
13	10	11	12	12

Table 4: Shipment paths for Company_B_1_11 (case 1).

4.1.5 Company_C_13_8

Day	From Station	To Station	Container Load	Containers Delivered
5	13	14	3	0
5	13	7	10	
6	13	14	10	0
	14	3	3	
	7	6	10	
	13	7	10	
7	3	1	3	0
	14	3	10	
	6	4	10	
	7	6	10	
	13	7	3	
8	1	8	3	13
	4	8	10	
	6	9	3	
	3	1	10	
	6	4	7	
	7	6	3	
	13	7	6	
9	1	4	7	13
	1	8	3	
	4	8	7	
	6	4	3	
	7	6	6	
	9	8	3	
10	4	8	10	10
	6	4	6	
11	4	8	6	6

Table 5: Shipment paths for Company_C_13_8 (case 1).

4.1.6 Company_C_13_1

Day	From Station	To Station	Container Load	Containers Delivered
4	13	14	10	0
5	13	14	7	0
	14	3	10	
6	3	1	10	10
	14	3	7	
7	3	1	7	7

Table 6: Shipment paths for Company_C_13_1 (case 1).

4.1.7 Company_C_13_7

Day	From Station	To Station	Container Load	Containers Delivered
2	13	7	10	10
3	13	7	10	10
4	13	7	10	10
10	13	7	2	2
12	13	7	10	10

Table 7: Shipment paths for Company_C_13_7 (case 1).

4.1.8 Company_D_5_6

Day	From Station	To Station	Container Load	Containers Delivered
11	5	7	6	0
12	7	6	6	6

Table 8: Shipment paths for Company_D_5_6 (case 1).

4.1.9 Company_D_5_10

Day	From Station	To Station	Container Load	Containers Delivered
5	5	14	3	0
6	5	14	3	0
	14	15	3	
7	5	7	7	0
	14	15	3	
	15	6	3	
8	5	7	3	0
	6	9	3	
	7	6	7	
	15	6	3	
9	5	7	8	3
	6	9	10	
	9	10	3	
	7	6	3	
10	5	7	10	10
	6	9	3	
	9	10	10	
	7	6	8	
11	6	9	8	3
	9	10	3	
	7	6	10	
12	6	9	10	8
	9	10	8	
13	9	10	10	10

Table 9: Shipment paths for Company_D_5_10 (case 1).

4.1.10 Company_E_3_13

Day	From Station	To Station	Container Load	Containers Delivered
1	3	14	9	0
2	3	14	10	9
	14	13	9	
3	14	13	10	10
6	3	7	7	0
7	3	7	4	7
	7	13	7	
8	3	7	10	4
	7	13	4	
9	7	13	10	10

Table 10: Shipment paths for Company_E_3_13 (case 1).

4.1.11 Company_E_3_15

Day	From Station	To Station	Container Load	Containers Delivered
4	3	14	10	0
5	14	15	10	10

Table 11: Shipment paths for Company_E_3_15 (case 1).

4.2 Graphical solutions

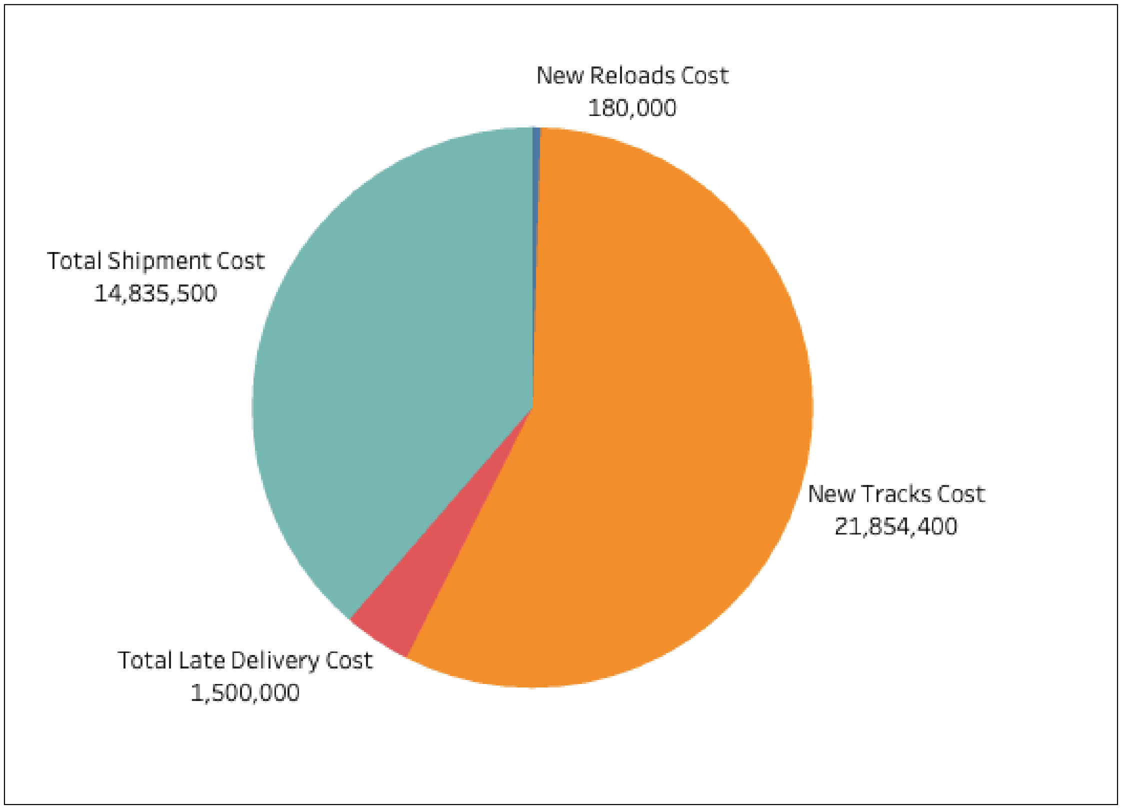


Figure 1: Chart of total cost breakdown of the $38,369,969.86 optimal cost.

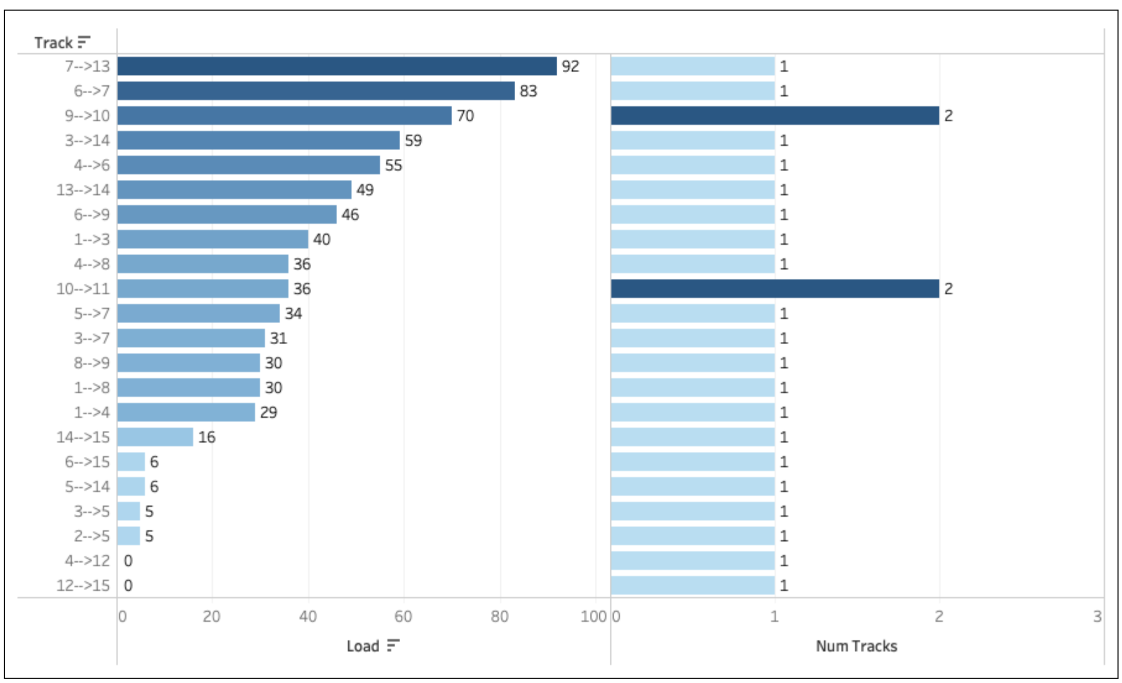


Figure 2: Graph of the number of tracks between stations and the container load (case 1).

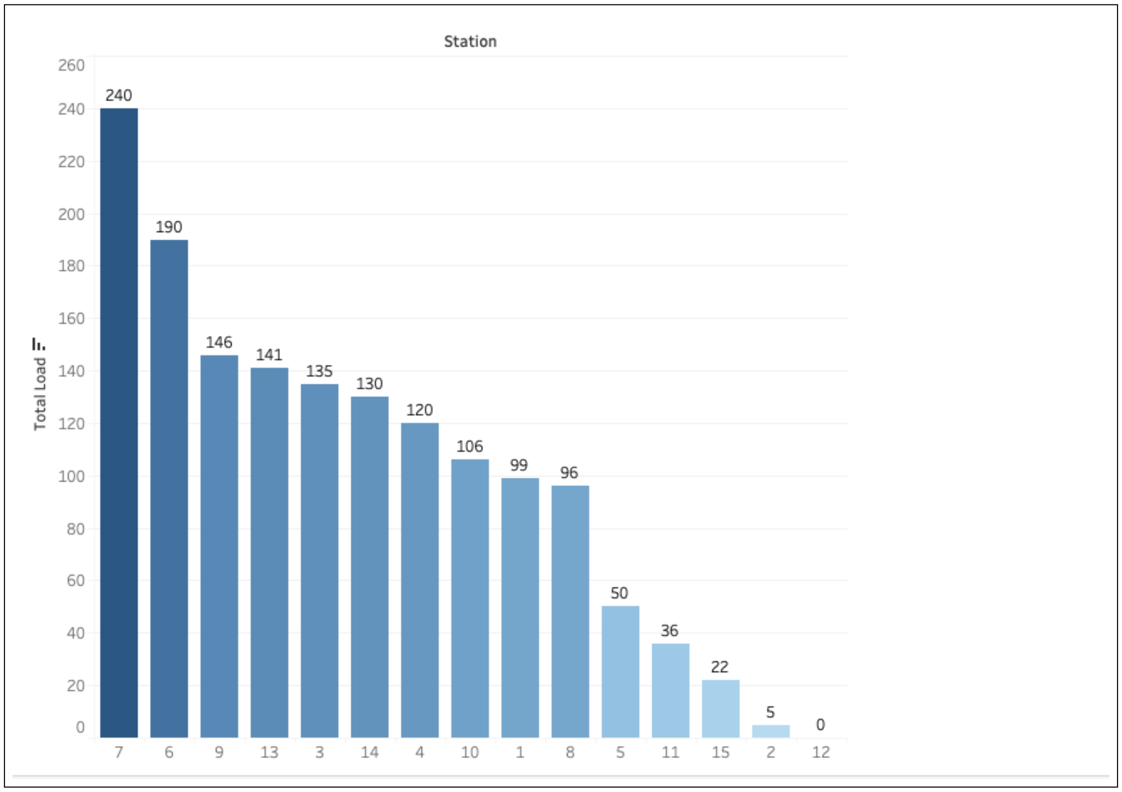


Figure 3: Graph of the container load for each station (case 1).

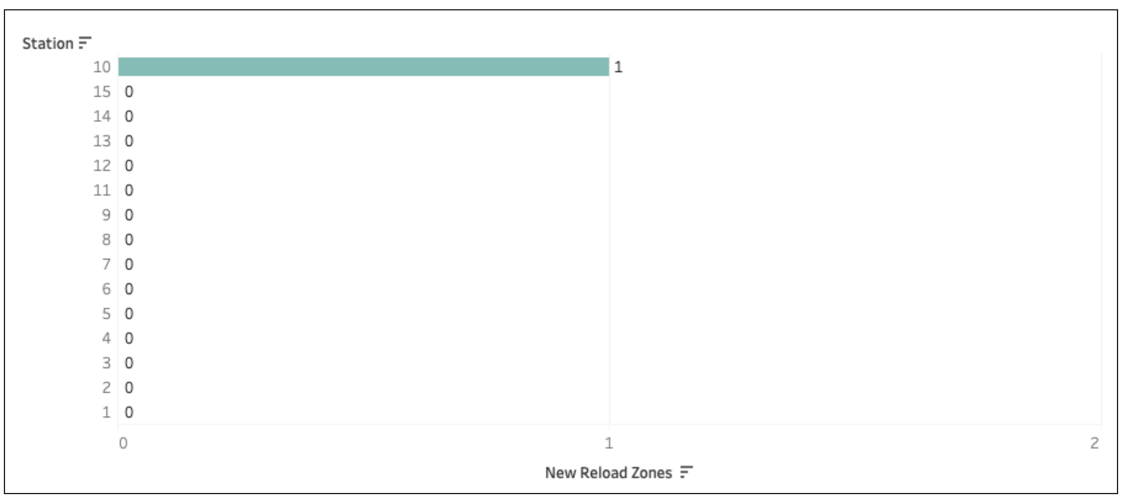


Figure 4: Graph of the number of new reload zones for each station (case 1).

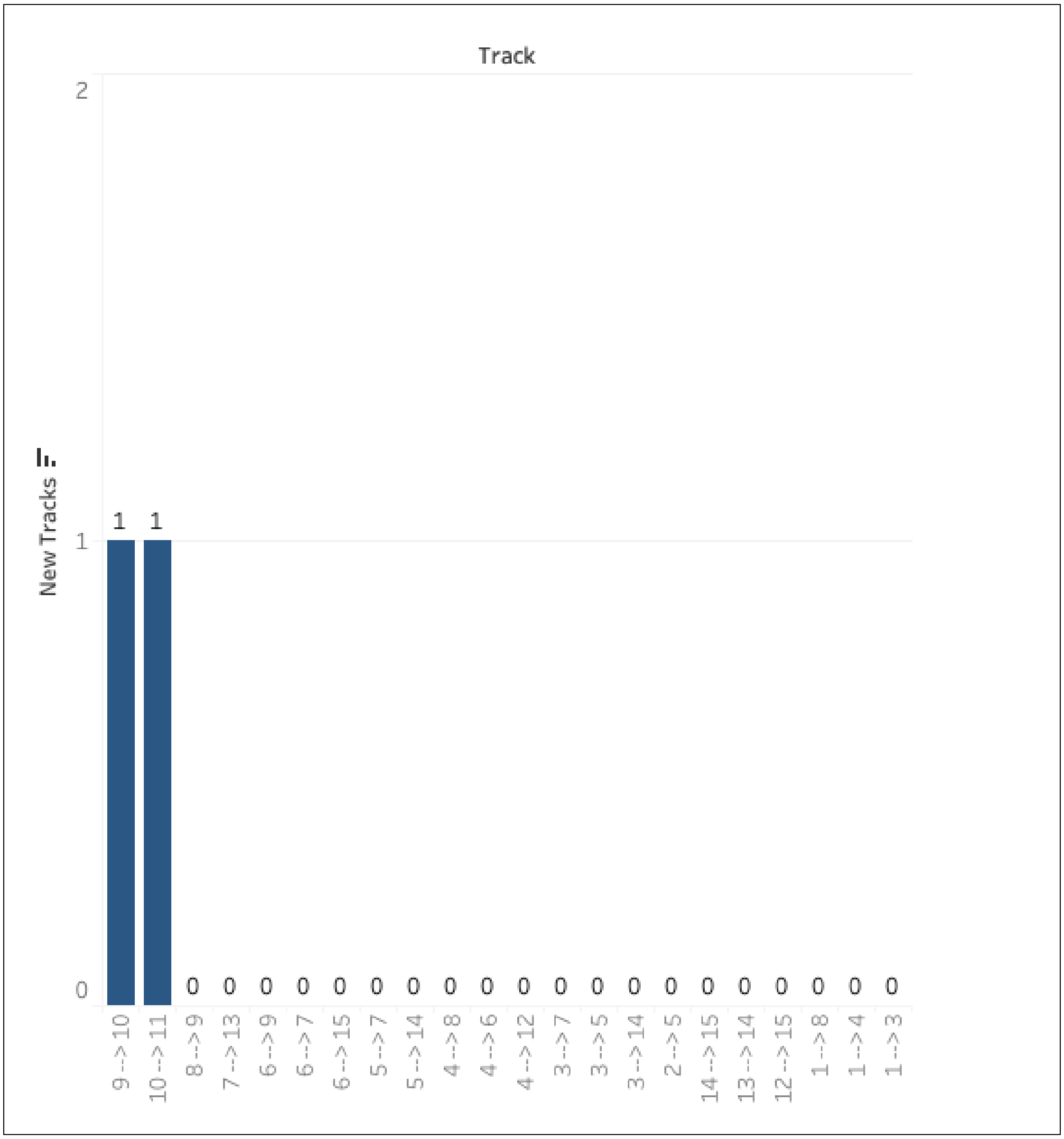


Figure 5: Graph of the number of new tracks between connected stations (case 1).

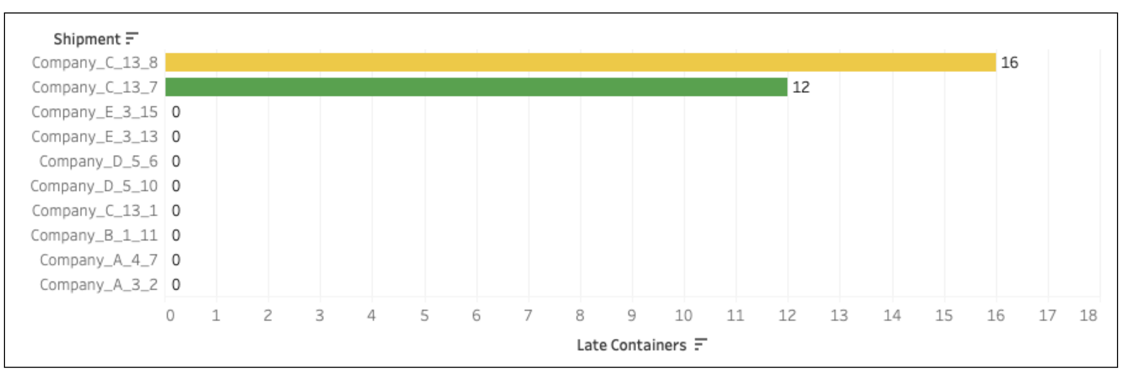


Figure 6: Graph of the number of late containers for each shipment (case 2).

4.3 Case 1 Solution explanation

Case 1 involves minimizing the total cost for 10 shipments by 5 different companies, labeled A through E in our optimal solution. After modeling our proposed MILP above using the provided data from Case 1 in AMPL, our team determined the optimal network expansion plan, represented in the data tables above.

These tables provide detailed information about the transportation plan, including the movement of containers between different stations. It is important to carefully analyze the tables to understand the key aspects of the optimal solutions.

For instance, Table 4.1.1 represents the transportation details for the movement of a shipment of containers from origin Station 4 to destination Station 7. On day 1, a total of 20 containers were loaded at Station 4, with 10 destined for Station 6 and 10 for Station 1. No containers were delivered to Station 7 on this day.

Continuing to day 2, it can be observed that 10 containers were loaded at each of Stations 1, 4, and 6. These containers were subsequently transported to Stations 3, 6, and 7, respectively. As a result, a cumulative total of 10 containers were successfully delivered to the overall destination Station 7 by the end of day 2.

Continuing to day 3, another 10 containers were loaded at both Stations 3 and 6. Both these partial shipments of containers were then transported to Station 7, resulting in the delivery of an additional 20 containers to the overall destination. As a result, a total of 30 containers were successfully transported from Station 4 to Station 7 over the course of 3 days throughout the entire process.

Similarly, we can analyze the other tables and understand the day-by-day transportation plan for every other shipment and its containers en route to their destination stations. The optimized transportation plan ensures efficient utilization of resources, minimizes costs and ensures timely delivery of containers to their designated destinations.

In addition to the table solutions illustrating the optimal transportation plan, several other useful insights can be gleaned from the graphical solutions as well. First and foremost, we can see that the total minimum cost to complete this expansive operation while best satisfying the needs of the companies involved is $38,369,969.86.

However, when this value is broken down into individual components as demonstrated by the 4.2 Figure 1 Pie Chart, we can see that the *new_tracks_cost* is responsible for approximately $21,854,400 or nearly 57% of the total cost. Moreover, the shipment cost is also substantial: $14,835,500 or nearly 39% of the total cost. The late delivery and new reloading station costs add up to about 4% of the total cost making it less significant than the other costs. Given the weight of new tracks cost and shipment cost in contributing to the total cost in comparison to the other 2 cost components, our recommendation to our client, SmartRail, would be to explore possible alternatives that could minimize these construction and shipment costs. Even by reducing the amortized cost to build one mile of track ($7,500/day) by 10%, SmartRail could save millions of dollars that could be better spent elsewhere. In an effort to reduce shipping costs, SmartRail could explore different fuel opportunities to reduce shipping costs by possibly using cheaper or more efficient fuel in their trains. Additionally, if SmartRail employs a large number of employees on each train, SmartRail could also consider lowering costs by reducing the number of employees present on each shipment. SmartRail could also use a more efficient packing method for their trains to reduce the number of train cars needed and the overall number of shipments. Assuming SmartRail can optimize for lowering new track costs and shipment costs, the company would end up saving a lot of money that can be spent elsewhere.

Another insight that can be gleaned from these results is that both Stations 7 and 6 bear the greatest Container Load as demonstrated by the bar graph in 4.2 Figure 3. This suggests that these stations play a crucial role in the transportation network, acting as major hubs for container movement. The bar graph highlights their significance in terms of container handling. Additionally, Figure 2 and Figure 5 indicate that new tracks were only required for the connections between Stations 9 and 10, and 10 and 11. Surprisingly, no new tracks were needed for Stations 6 and 7, indicating their existing infrastructure efficiently manages the shipment load during the bi-weekly period. Station 9 and Station 10 however, do have a fairly high container load too as seen in 4.2 Figure 3. Station 10 requires a new reload zone as well as seen in 4.2 Figure 4. Based on these findings, our recommendation to the client would be to prioritize the expansion of Station 10 followed by Station 9. This approach aims to prevent any potential bottlenecks that may arise from the increasing volume of shipments, ensuring smooth operations and optimal efficiency in the network.

A final insight that can be concluded from 4.2 Figure 6 is the fact that shipments Company_C_13_8 and Company_C_13_7 experience the greatest amount of delays. The shipments start from Station 13 and have their destination stations as Station 8 and 7. The route for shipment C from Station 13 is relatively long and hence this could have caused containers to be delivered late with bottlenecks. SmartRail can build redundant stations that connect these stations to help cut this route short and avoid delay of shipment deliveries.

5 Case 2 – Optimal solution

5.1 Table solutions

5.1.1 Cost Breakdown

Cost Category	Cost (USD)
Total Shipment Cost	$50,049,900
Total Late Delivery Cost	$4,540,000
New Reloads Cost	$1,620,000
New Tracks Cost	$129,802,000
Optimal Total	$186,011,944.6

Table 12: Optimized cost breakdown (case 2).

5.1.2 Company_A_4_7

Day	From Station	To Station	Container Load	Containers Delivered
1	4	6	10	0
	4	1	3	
2	1	3	3	10
	4	6	10	
	6	7	10	
3	3	7	3	13
	4	6	7	
	6	7	10	
4	6	7	7	7

Table 13: Shipment paths for Company_A_4_7 (case 2).

5.1.3 Company_A_3_2

Day	From Station	To Station	Container Load	Containers Delivered
3	3	5	5	0
4	5	2	5	5

Table 14: Shipment paths for Company_A_3_2 (case 2).

5.1.4 Company_B_1_11

Day	From Station	To Station	Container Load	Containers Delivered
8	1	4	10	0
	1	8	9	
9	1	4	10	0
	1	8	7	
	4	6	8	
	4	8	2	
	8	9	9	
10	4	6	10	0
	6	9	8	
	8	9	9	
	9	10	9	
11	6	9	10	9
	9	10	17	
	10	11	9	
12	9	10	10	17
	10	11	17	
13	10	11	10	10

Table 15: Shipment paths for Company_B_1_11 (case 2).

5.1.5 Company_C_13_8

Day	From Station	To Station	Container Load	Containers Delivered
5	13	7	10	0
6	13	14	3	0
	7	6	10	
	13	7	10	
7	14	3	3	0
	6	4	10	
	7	6	10	
	13	7	8	
8	4	8	10	10
	3	1	3	
	7	3	6	
	6	4	10	
	7	6	2	
9	1	8	3	13
	4	8	10	
	3	1	6	
	6	4	2	
	13	7	4	
10	1	8	6	8
	4	8	2	
	7	3	2	
	7	6	2	
	13	7	7	
11	6	9	1	0
	3	1	2	
	6	4	1	
	7	6	7	
12	1	8	2	4
	4	8	1	
	6	9	7	
	9	8	1	
13	9	8	7	7

Table 16: Shipment paths for Company_C_13_8 (case 2).

5.1.6 Company_C_13_1

Day	From Station	To Station	Container Load	Containers Delivered
4	13	14	3	0
	13	7	7	
5	13	14	7	0
	7	3	5	
	14	3	3	
	7	6	2	
6	3	1	8	8
	14	3	7	
	6	4	2	
7	3	1	7	9
	4	1	2	

Table 17: Shipment paths for Company_C_13_1 (case 2).

5.1.7 Company_C_13_7

Day	From Station	To Station	Container Load	Containers Delivered
2	13	7	10	10
3	13	7	10	10
4	13	7	3	3
11	13	7	9	9
12	13	7	10	10

Table 18: Shipment paths for Company_C_13_7 (case 2).

5.1.8 Company_D_5_6

Day	From Station	To Station	Container Load	Containers Delivered
9	5	14	1	0
10	5	14	1	0
	5	3	4	
	14	3	1	
11	3	7	5	0
	14	3	1	
12	3	7	1	5
	7	6	5	
13	7	6	1	1

Table 19: Shipment paths for Company_D_5_6 (case 2).

5.1.9 Company_D_5_10

Day	From Station	To Station	Container Load	Containers Delivered
5	5	3	4	0
6	5	14	3	0
	3	1	4	
	5	3	1	
7	1	8	4	0
	3	7	1	
	5	7	1	
	5	14	7	
	14	3	1	
	14	13	2	
8	3	7	1	0
	5	14	4	
	8	9	4	
	5	3	2	
	14	3	5	
	7	6	2	
	13	7	2	
	14	13	2	
9	3	7	6	4
	5	14	4	
	6	15	2	
	9	10	4	
	3	1	1	
	5	3	8	
	14	3	1	
	7	6	3	
	13	7	2	
	14	13	3	
10	1	4	1	0
	6	9	3	
	14	15	4	
	3	1	9	
	7	6	8	
	15	6	2	
	13	7	3	
11	1	8	9	3
	4	6	1	
	6	9	10	
	9	10	3	

	7	6	3	
	15	6	4	
12	6	9	8	10
	8	9	9	
	9	10	10	
13	9	10	17	17

Table 20: Shipment paths for Company_D_5_10 (case 2).

5.1.10 Company_E_3_13

Day	From Station	To Station	Container Load	Containers Delivered
1	3	14	10	0
2	3	14	10	10
	14	13	10	
3	3	14	7	10
	14	13	10	
4	14	13	7	7
6	3	7	2	0
7	3	7	8	2
	7	13	2	
8	3	7	3	8
	7	13	8	
9	7	13	3	3

Table 21: Shipment paths for Company_E_3_13 (case 2).

5.1.11 Company_E_3_15

Day	From Station	To Station	Container Load	Containers Delivered
3	3	7	3	0
4	3	1	5	0
	7	6	3	
5	1	4	5	3
	6	15	3	
6	4	12	5	0
7	12	15	5	5
12	3	14	2	0
13	14	15	2	2

Table 22: Shipment paths for Company_E_3_15 (case 2).

5.1.12 Company_F_9_2_a

Day	From Station	To Station	Container Load	Containers Delivered
1	9	6	10	0
	9	8	10	
2	6	7	10	0
	8	1	10	
	9	6	10	
	9	8	4	
3	1	3	10	0
	6	7	10	
	8	1	4	
	7	5	10	
	9	6	10	
	9	8	1	
4	1	3	4	10
	3	14	10	
	6	7	10	
	8	1	1	
	5	2	10	
	7	5	10	
5	1	3	1	10
	3	5	4	
	5	2	10	
	7	5	10	
	14	5	10	
6	3	5	1	24
	5	2	24	
7	5	2	1	1

Table 23: Shipment paths for Company_F_9_2_a (case 2).

5.1.13 Company_F_9_2_b

Day	From Station	To Station	Container Load	Containers Delivered
4	9	6	28	0
	9	8	3	
5	6	7	18	0
	6	15	10	
	8	1	3	
	9	8	2	
6	1	3	3	0
	8	1	2	
	7	3	8	
	7	5	10	
	15	14	10	
7	1	3	2	10
	3	5	10	
	3	14	1	
	5	2	10	
	14	5	10	
8	3	5	2	20
	5	2	20	
	14	5	1	
	9	6	2	
9	6	7	2	3
	5	2	3	
10	7	3	2	0
11	3	14	2	0
12	14	5	2	0
13	5	2	2	2

Table 24: Shipment paths for Company_F_9_2_b (case 2).

5.1.14 Company_G_11_7

Day	From Station	To Station	Container Load	Containers Delivered
9	11	10	7	0
10	10	9	7	0
	11	10	5	
11	9	6	7	0
	10	9	5	
12	6	7	7	7
	9	6	5	
13	6	7	5	5

Table 25: Shipment paths for Company_G_11_7 (case 2).

5.1.15 Company_G_11_1

Day	From Station	To Station	Container Load	Containers Delivered
3	11	10	20	0
4	10	9	20	0
	11	10	20	
5	9	6	20	0
	10	9	20	
	11	10	18	
6	6	7	10	0
	6	15	7	
	6	4	3	
	9	6	17	
	9	8	3	
	10	9	18	
	11	10	15	
7	6	7	10	0
	6	15	7	
	4	1	3	
	8	1	3	
	7	3	1	
	7	5	9	
	9	6	17	
	9	8	1	
	10	9	15	
	11	10	14	
	15	14	7	
8	1	3	6	9
	3	5	1	
	6	7	15	
	6	15	2	
	8	1	1	
	5	2	9	
	7	5	10	
	14	5	7	
	9	6	15	
	10	9	14	
	11	10	10	
	15	14	7	
9	1	3	1	18
	3	14	6	
	6	7	15	

	7	13	1	
	5	2	18	
	7	3	4	
	7	5	10	
	14	5	7	
	9	6	13	
	9	8	1	
	10	9	10	
	15	14	2	
10	3	14	5	17
	6	7	10	
	6	15	3	
	13	14	1	
	8	1	1	
	5	2	17	
	7	3	5	
	7	5	10	
	14	5	8	
	9	6	10	
11	1	3	1	18
	3	5	1	
	3	14	4	
	6	7	10	
	5	2	18	
	7	5	10	
	14	5	6	
	15	14	3	
12	3	5	1	17
	5	2	17	
	7	5	10	
	14	5	7	
13	5	2	18	18

Table 26: Shipment paths for Company_G_11_1 (case 2).

5.1.16 Company_H_12_5

Day	From Station	To Station	Container Load	Containers Delivered
6	12	15	3	0
	12	4	2	
7	12	15	1	0
	4	1	2	
	15	14	3	
8	1	3	2	3
	14	5	3	
	15	14	1	
9	3	5	2	3
	14	5	1	
10	12	4	7	0
11	12	15	10	0
	4	1	7	
12	1	3	7	0
	15	14	10	
13	3	5	7	17
	14	5	10	

Table 27: Shipment paths for Company_H_12_5 (case 2).

5.1.17 Company_H_12_2

Day	From Station	To Station	Container Load	Containers Delivered
1	12	15	10	0
	12	4	10	
2	12	15	10	0
	4	1	10	
	12	4	10	
	15	14	10	
3	1	3	10	0
	4	8	1	
	12	15	10	
	4	1	9	
	12	4	10	
	14	5	10	
	15	14	10	
4	1	3	9	10
	3	5	10	
	12	15	10	
	4	1	10	
	8	1	1	
	5	2	10	
	12	4	5	
	14	5	10	
	15	14	10	
5	1	3	11	20
	3	5	2	
	3	14	7	
	4	1	5	
	5	2	20	
	12	4	10	
	14	5	10	
	15	14	10	
6	1	3	5	12
	3	5	8	
	3	14	3	
	4	1	10	
	5	2	12	
	12	4	3	
	14	5	17	
7	1	3	10	25
	3	14	5	
	12	15	2	
	4	1	3	
	5	2	25	

	14	5	3	
8	1	3	3	3
	3	5	5	
	3	14	5	
	12	15	8	
	5	2	3	
	14	5	5	
	15	14	2	
9	3	14	3	10
	12	15	6	
	5	2	10	
	12	4	9	
	14	5	7	
	15	14	8	
10	12	15	7	7
	4	1	9	
	5	2	7	
	14	5	11	
	15	14	6	
11	1	3	9	11
	5	2	11	
	14	5	6	
	15	14	7	
12	3	5	9	6
	5	2	6	
	14	5	7	
13	5	2	16	16

Table 28: Shipment paths for Company_H_12_2 (case 2).

5.1.18 Company_I_1_2

Day	From Station	To Station	Container Load	Containers Delivered
9	1	3	10	0
10	1	3	9	0
	3	5	6	
	3	14	4	
11	3	5	9	6
	5	2	6	
	14	5	4	
12	5	2	13	13

Table 29: Shipment paths for Company_I_1_2 (case 2).

5.1.19 Company_I_1_10

Day	From Station	To Station	Container Load	Containers Delivered
3	1	4	1	0
	1	8	6	
4	1	8	8	0
	4	6	1	
	8	9	6	
5	1	8	7	6
	6	9	1	
	8	9	8	
	9	10	6	
6	1	8	8	9
	8	9	7	
	9	10	9	
7	1	8	3	7
	8	9	8	
	9	10	7	
8	8	9	3	8
	9	10	8	
9	9	10	3	3

Table 30: Shipment paths for Company_I_1_10 (case 2).

5.1.20 Company_J_15_9

Day	From Station	To Station	Container Load	Containers Delivered
6	15	6	13	0
7	6	9	13	13
	15	6	13	
8	6	9	13	13
	15	6	17	
9	6	9	17	17
	15	6	8	
10	6	9	8	8
12	15	6	4	0
13	6	9	4	4

Table 31: Shipment paths for Company_J_15_9 (case 2).

5.1.21 Company_K_8_15

Day	From Station	To Station	Container Load	Containers Delivered
10	8	9	1	0
	8	1	3	
	8	4	18	
11	1	3	3	0
	4	6	8	
	4	12	10	
	8	9	10	
	8	4	20	
	9	6	1	
12	3	14	3	19
	4	6	10	
	4	12	10	
	6	15	9	
	12	15	10	
	9	6	10	
13	6	15	20	33
	12	15	10	
	14	15	3	

Table 32: Shipment paths for Company_K_8_15 (case 2).

5.2 Graphical solutions

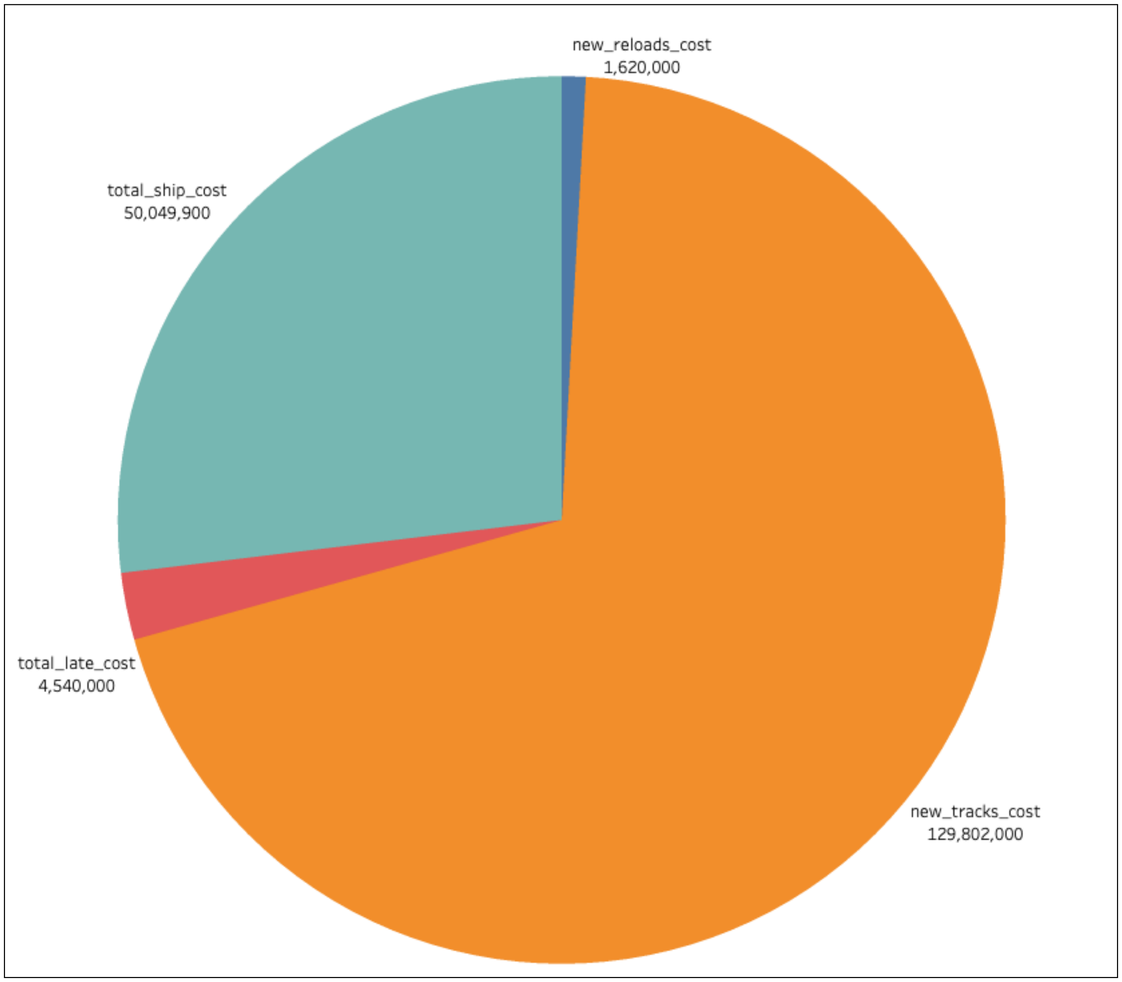


Figure 7: Chart of total cost breakdown of the $186,011,944.6 optimal cost.

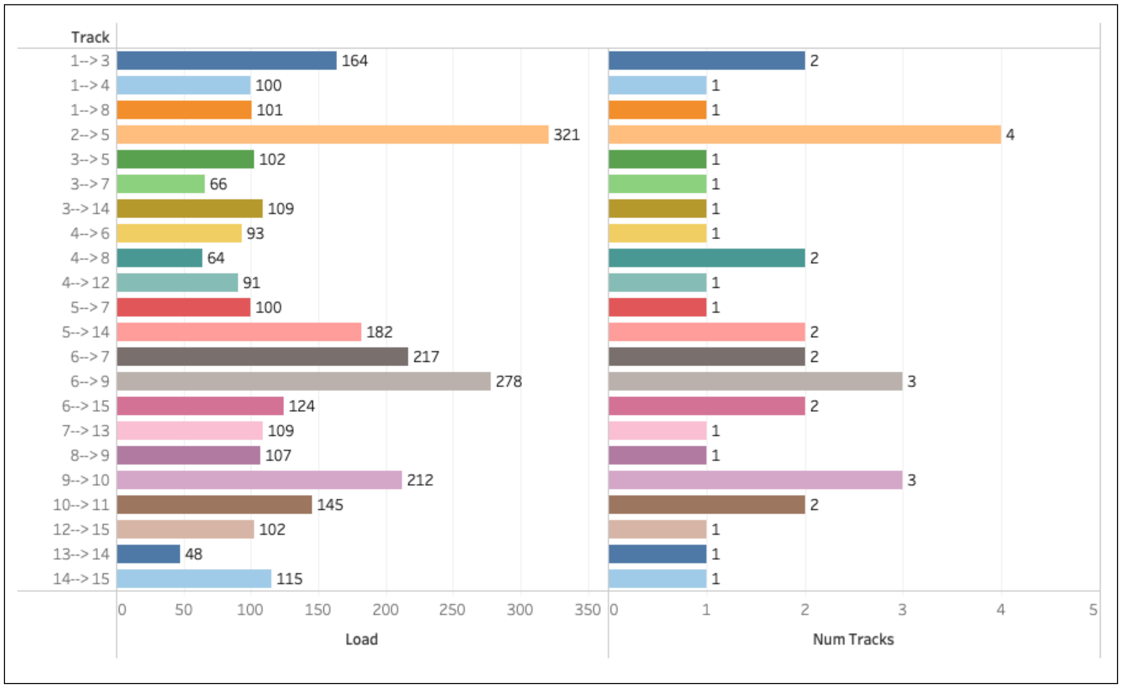


Figure 8: Graph of the number of tracks between stations and the tracks' load (case 2).

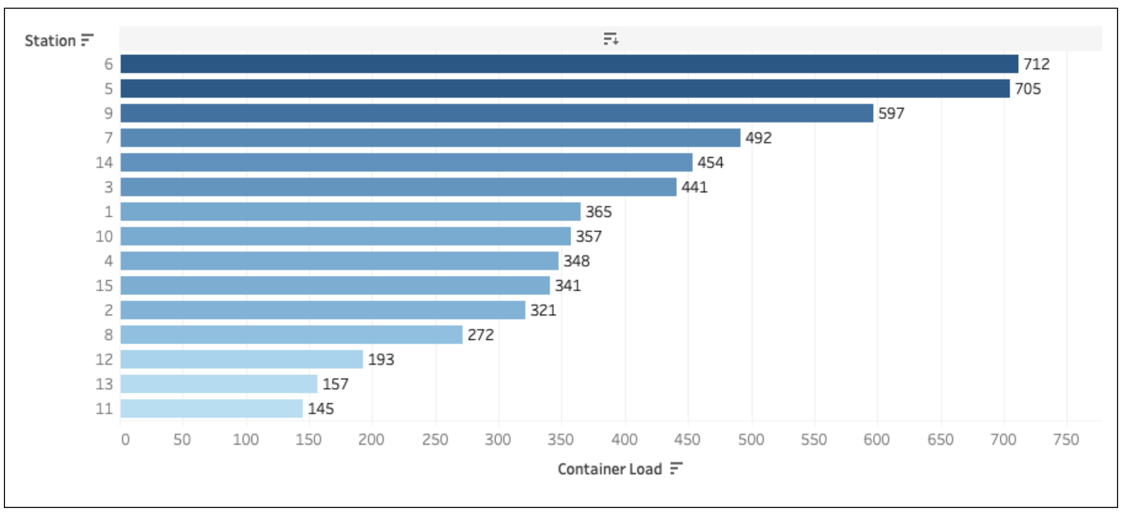


Figure 9: Graph of the container load for each station (case 2).

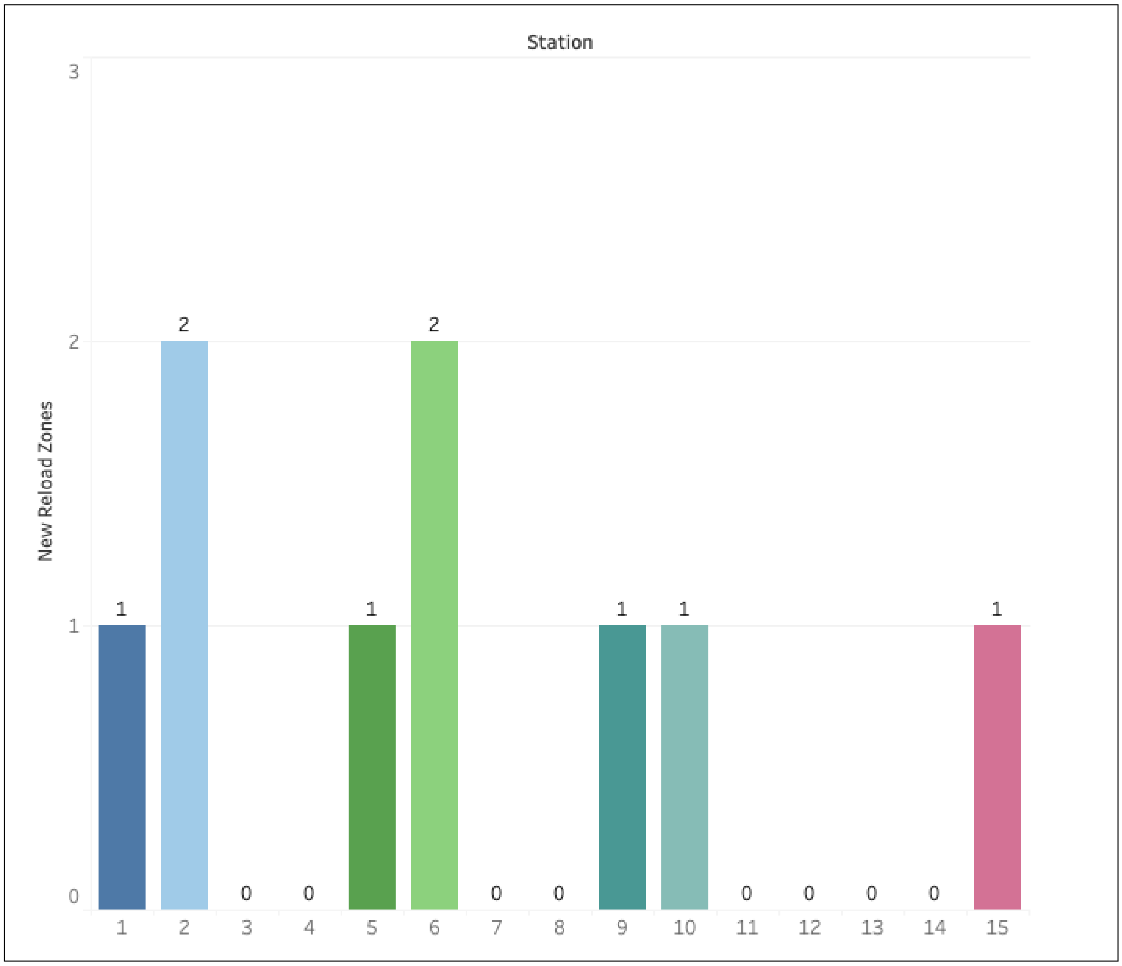


Figure 10: Graph of the number of new reload zones for each station (case 2).

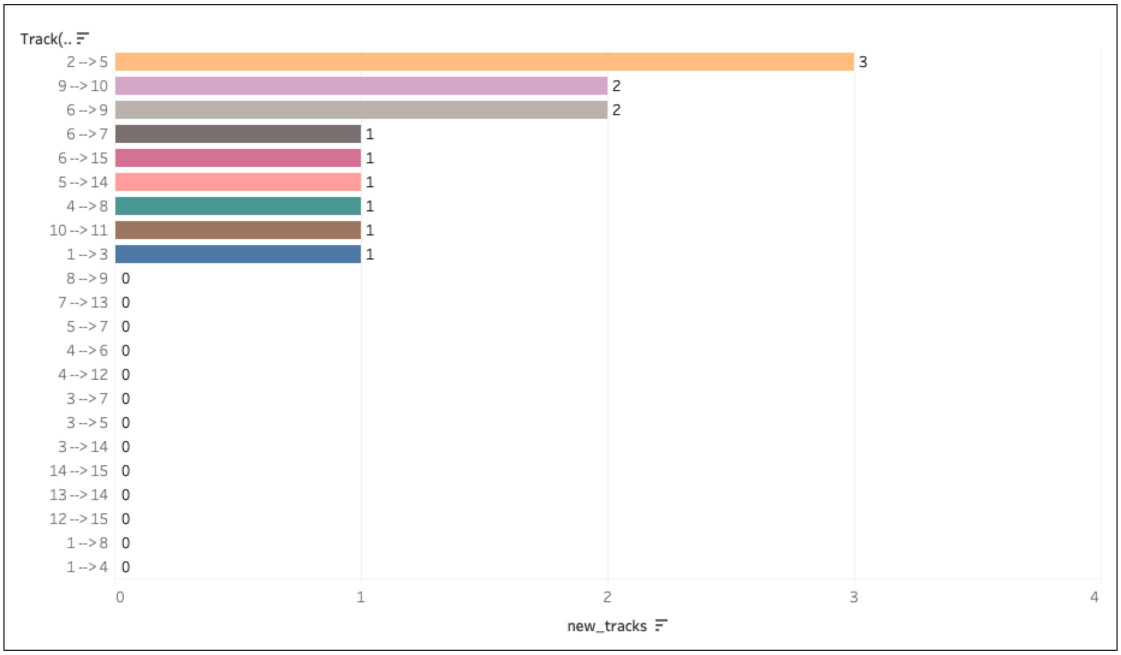


Figure 11: Graph of the number of new tracks between connected stations (case 2).

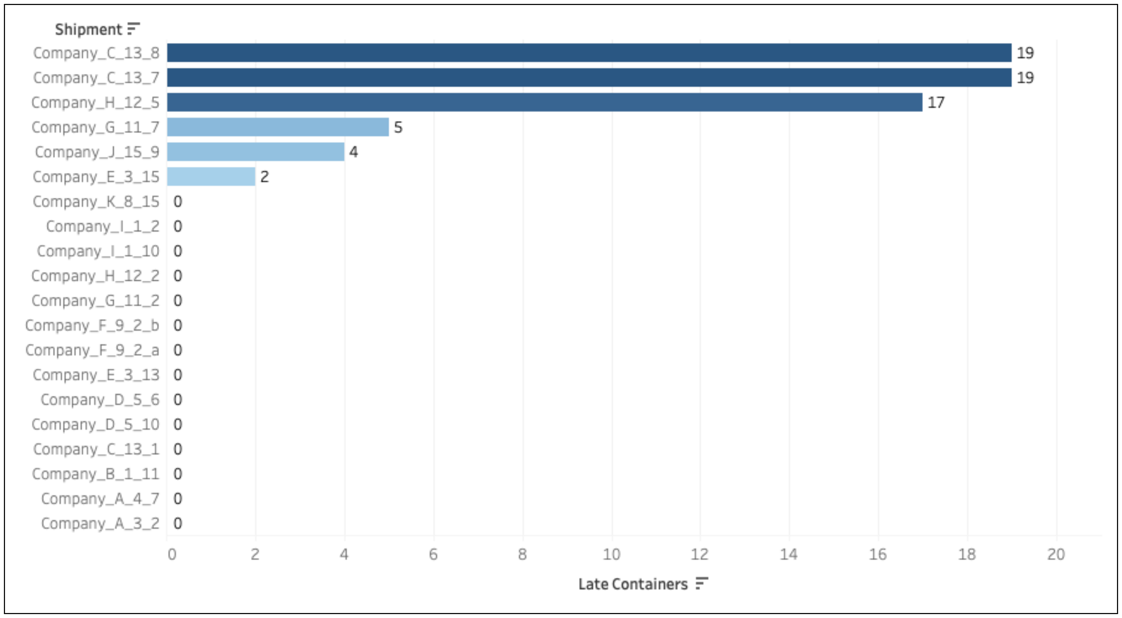


Figure 12: Graph of the number of late containers for each shipment (case 2).

5.3 Case 2 Solution explanation

Case 2 involves minimizing the total cost for 20 shipments by 11 different companies, labeled A through K in our optimal solution. After modeling our proposed MILP above using the provided data from Case 2 in AMPL, our team determined the optimal network expansion plan, represented in the data tables above.

These tables provide detailed information about the transportation plan, including the movement of containers between different stations. It is important to carefully analyze the tables to understand the key aspects of the optimal solutions. The setup is similar to Case 1.

For instance, Table 5.1.1 represents the transportation details for the movement of shipment A of containers from origin Station 4 to destination Station 7. On day 1, a total of 13 containers were loaded at Station 4, with 10 destined for Station 6 and 3 for Station 1. No containers were delivered to Station 7 on this day.

Continuing to day 2, it can be observed that 3 containers were loaded at Station 1 and transported to Station 3. Moreover, 10 containers were loaded at each of Stations 4 and 6. These containers were subsequently transported to Stations 6 and 7, respectively. As a result, a cumulative total of 10 containers were successfully delivered to the overall destination Station 7 by the end of day 2.

Continuing to day 3, 3 containers were loaded on Station 3, another 7 containers were loaded on Station 4, and another 10 containers were loaded on Station 6. All these partial shipments of containers were then transported to Station 7, Station 6, and Station 7 respectively, resulting in the delivery of an additional 13 containers to the final destination.

And on day 4, 7 containers are loaded on Station 6 and transported to Station 7. As a result, a total of 30 containers were successfully transported from Station 4 to Station 7 over the course of 4 days throughout the entire process.

In addition to the table solutions illustrating the optimal transportation plan, several other useful insights can be gleaned from the graphical solutions as well. First and foremost, we can see that the total minimum cost to complete this expansive operation while best satisfying the needs of the companies involved is $186,011,944.60.

However, when this value is broken down into individual components as demonstrated by the 5.2 Figure 7 Pie Chart, we can see that the new_tracks_cost is responsible for approximately $129,802,000 or nearly 75% of the total cost. Given its weight in contributing to the total cost in comparison to the other three cost components, our recommendation to our client, SmartRail, would be to explore possible alternatives that could minimize these construction costs. Even by reducing the amortized cost to build one mile of track ($7,500/day) by 10%, SmartRail could save tens of millions of dollars that could be better spent elsewhere.

Another insight that can be gleaned from these results is that both Stations 5 and 6 bear the greatest Container Load as demonstrated by the histogram in 5.2 Figure 9. Simultaneously, these stations also require the most newly constructed tracks leading in and out of them as indicated by the bar graphs in 5.2 Figure 8 and Figure 11. It can then be inferred from these graphs that Stations 5 and 6 are likely major transportation hubs, considering both station nodes exhibit a high degree of centrality in the transportation network. As such, our recommendation to our client would be to prioritize expanding these two stations first in order to prevent any immediate bottlenecks from the volume of shipments they must support.

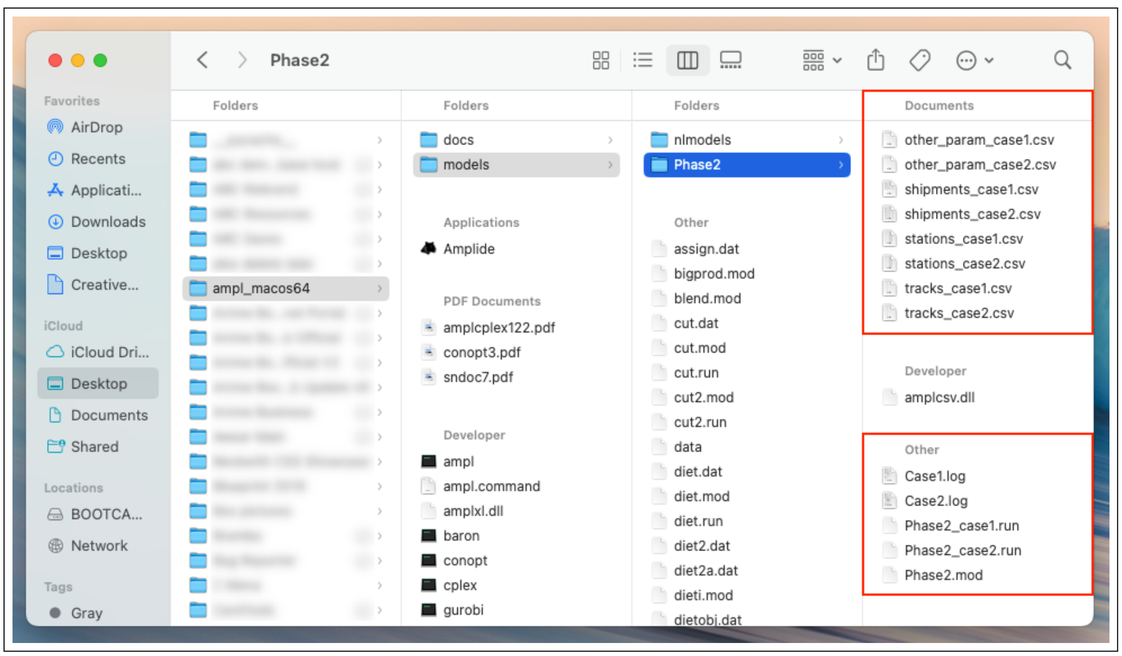
A final insight that can be concluded from 5.2 Figure 12 is the fact that Shipments Company_C_13_8, Company_C_13_7, and Company_H_12_5 experience a substantially greater number of late containers than any other shipment. Combined, these three specific shipments comprise nearly 84% of the total shipment delay penalty cost. This phenomenon can be explained by the fact that, unlike other stations, both Stations 12 and 13 each have only one track connecting them to and from other stations (specifically, 4-12, 12-15, 7-13, and 13-4). Since we can see that these shipments need to travel a longer route in their respective table representations, we can infer that the limited number of tracks leading in/out of these origin stations is causing bottleneck delays. As a result, our recommendation to SmartRail is that they should emphasize building additional stations that connect these stations together to prevent further future delays.

If SmartRail were to implement these changes to the transportation network, there would be massive benefits in cost reduction that would help improve company logistics.

6 AMPL setup instructions

These setup instructions assume you already have the AMPL root directory folder (ex. `ampl_macos64`) containing the Amplide application installed on your device. To use our AMPL setup with CSV files, please follow the steps below:

a. Download and move the folder containing this project's AMPL and CSV files to the `models/` folder of your AMPL installation directory as outlined in red in the screenshot below.



b. Install the AMPL CSV Plugin using the download link that corresponds to your device's operating system as pictured in the screenshot below.

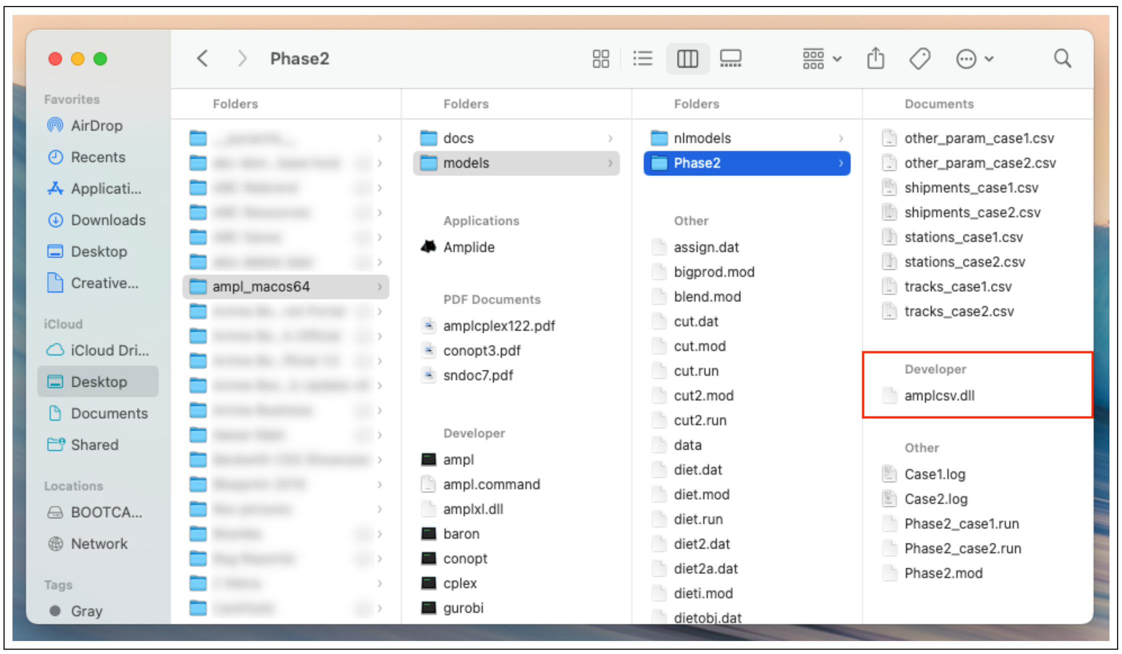
Installation

Use one of the following links to download the csv table handler zipfile appropriate to your computer:

- Windows: amplcsv.win64.zip (64-bit), amplcsv.win32.zip (32-bit)
- Linux: amplcsv.linux64.zip (64-bit), amplcsv.linux32.zip (32-bit)
- macOS: amplcsv.macos.zip

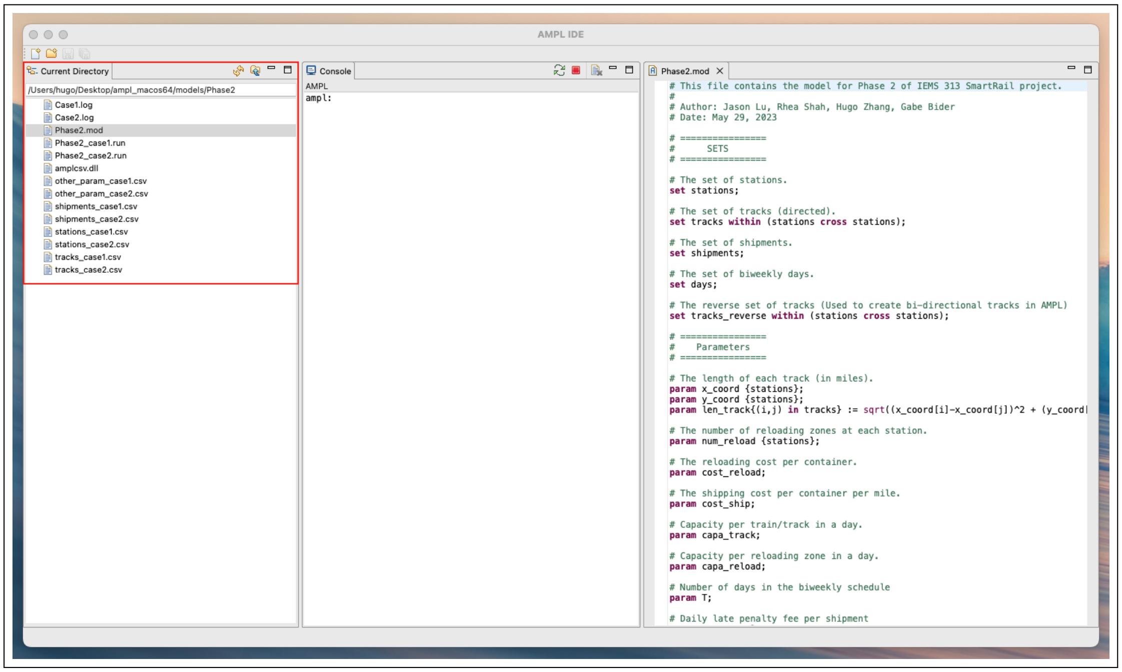
Double-click the zipfile or use an unzip utility to extract the file **`amplcsv.dll`**. Then move **`amplcsv.dll`** into the same Windows/macOS folder or Linux directory as your AMPL program file. (The AMPL program file is **`ampl.exe`** on Windows systems, and **`ampl`** on Linux and macOS systems.)

c. Move the downloaded `amplcsv.dll` file to the project directory folder as outlined in red in the screenshot below.

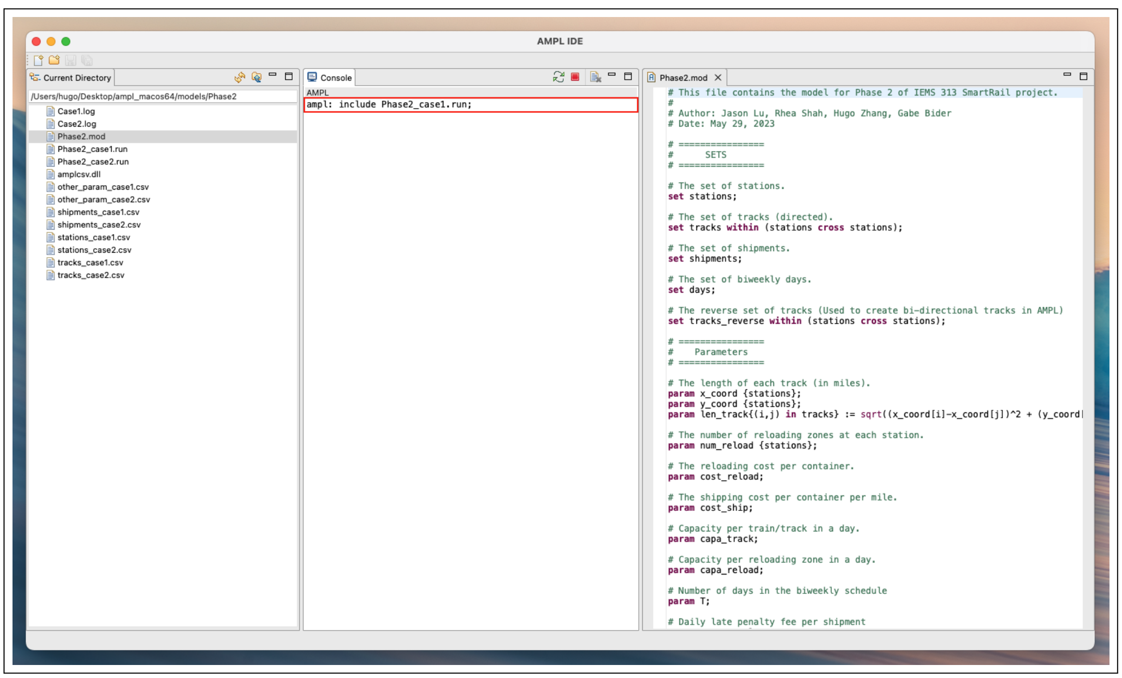


d. Open the Amplide application in the installation directory and navigate to the project folder in the `models/` folder using the Current Directory panel on the left-hand side.

Confirm you are in the correct directory by examining the area outlined in red in the screenshot below. You should see the same files.



e. To run the code for solving Phase 2, type either `include Phase2_case1.run;` for Case 1 or `include Phase2_case1.run;` for Case 2 in the Amplide console and hit enter.



If executed correctly, you should see the results of the corresponding optimization model displayed in the console. Results are also saved and immediately viewable in the corresponding generated `.log` file. In the case that the error of being unable to read tables for CSVs occurs, type `load amplcsv.dll;` into the Amplide console before executing the run file.

References

Fourer, R., D. M. Gay, and B. W. Kernighan (2003). *AMPL A Modeling Language for Mathematical Programming*. 2nd. Duxbury Thomson.

Inc., AMPL Optimization (2023). *AMPL File CSV Interface*. URL: https://amplplugins.readthedocs.io/en/latest/rst/amplcsv.html (visited on 05/06/2023).