Sampling states

$$p(z_{1:T+1} \mid x_{1:T})$$

$$= p(z_1 \mid x_{1:T})\, p(z_{2:T+1} \mid z_1, x_{2:T})$$

$$= p(z_1 \mid x_{1:T})\, p(z_2 \mid z_1, x_{2:T})\, p(z_{3:T+1} \mid z_2, x_{3:T})$$

$$= p(z_1 \mid x_{1:T}) \left[\prod_{i=2}^{T-1} p(z_i \mid z_{i-1}, x_{i:T}) \right] p(z_{T+1} \mid z_T)$$

$$p(z_1 = k \mid x_{1:T}) \propto p(z_1 = k, x_{1:T}) = p(z_1 = k, x_1)\, p(x_{2:T} \mid z_1 = k)$$

$$= p(z_1 = k)\, p(x_1 \mid z_1 = k)\, b_1(k)$$

$$p(z_i = k \mid z_{i-1}, x_{i:T}) \propto p(z_i = k, x_{i:T} \mid z_{i-1})$$

$$= p(z_i = k, x_i \mid z_{i-1})\, p(x_{i+1:T} \mid z_i = k)$$

$$= p(z_i = k \mid z_{i-1})\, p(x_i \mid z_i = 1)\, p(x_{i+1:T} \mid z_i = k)$$