$$\alpha_t(k) = p(x_{1:t-1}, z_t = k)$$

$$= \sum_{k' \in [K]} p(x_{1:t-1}, z_t = k, z_{t-1} = k')$$

$$= \sum p(x_{1:t-2}, z_{t-1} = k') \, p(x_{t-1}, z_t = k \mid z_{t-1} = k')$$

$$= \sum \alpha_{t-1}(k') \, p(x_{t-1} \mid z_{t-1} = k') \, p(z_t = k \mid z_{t-1} = k$$

$$\delta(u,i) = p(X_{\downarrow u \cap 0} \mid X_u = i)$$

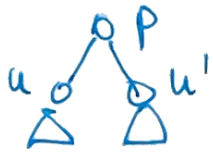
$$= \sum_{j,k} p(X_{\downarrow u \cap 0}, X_v = j, X_w = k \mid X_u = i)$$

$$= \sum_{j,k} p(X_v = j, X_w = k \mid X_u = i)\, p(X_{\downarrow u \cap 0} \mid X_v = j, X_w = k)$$

$$= \sum_{j,k} p(X_v = j \mid X_u = i)\, p(X_w = k \mid X_u = i)\, p(X_{\downarrow u \cap 0} \mid X_v = j)\, p(X_{\downarrow u \cap 0} \mid$$

$$X_w = k)$$

$$= \left(\sum_j p(X_v = j \mid X_u = i)\, p(X_{\downarrow u \cap 0} \mid X_v = j)\right)\left(\sum_k p(X_w = k \mid X_u = i)\, p(X_{\downarrow u \cap 0} \mid X_w = k\right.$$



$$t(u,i)$$

$$= p(X_{\uparrow u \downarrow o}, X_u = i)$$

$$= \sum_{j,k} p(X_{\uparrow u \downarrow o}, X_u = i, X_p = j, X_{u'} = k)$$

$$= \sum_{j,k} p(X_{\uparrow p \downarrow o}, X_p = j)\, p(X_{\downarrow u' \downarrow o}, X_{u'} = k \mid X_p = j)$$

$$= \sum_{j,k} t(p,j)\, p(X_{\downarrow u' \downarrow o} \mid X_{u'} = k)\, p(X_{u'} = k \mid X_p = j)$$

$$= \sum_{j,k} t(p,j)\, s(u',k)\, p(X_{u'} = k \mid X_p = j)$$

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