$\underline{Q}$: For a given microstate, what is $P(U_2 = E) = P(E)$.

What is the statistical weight?

$\Rightarrow W(U_2) = 1$ (one microstate)

Use $T = $ const, integrate

$$\frac{1}{k_B T} = \frac{\partial \ln W_1(U_1)}{\partial U_1} \Rightarrow \ln W_1(U_1) = \frac{U_1}{k_B T} + \overset{\text{const}}{B}$$

$$W_1 = e^{B} \underbrace{e^{U_1/k_B T}}_{\text{Important}}$$

$$W(E) = W_1(U_1) \times W_2(U_2)^{=1}$$

$$= W_1(U_0 - E)$$

$$= e^{B} e^{U_0/k_B T} e^{-E/k_B T}$$

$U_0 = U_1 + \overset{=E}{U_2}$

$U_1 = U_0 - E$

$$P(E) \propto W(E) \propto e^{-E/k_B T}$$

Boltzmann distribution

$$\boxed{P(E) = \frac{1}{Z} e^{-E/k_B T}}$$

needs to be in a state

$$\sum_i P(E_i) = 1 = \frac{1}{Z} \sum_i \underbrace{e^{-E_i/k_B T}}$$

all microstates

Boltzmann factor

$$\boxed{Z = \sum_i e^{-E_i/k_B T}}$$

Z = partition function

$$\frac{\partial Z}{\partial T} = \sum_i \frac{\partial}{\partial T} e^{-E_i/k_B T}$$

$$= \sum_i \frac{E_i}{k_B T^2} e^{-E_i/k_B T}$$

$$= \frac{1}{k_B T^2} \sum E_i e^{-E_i/k_B T}$$

$$= \frac{Z}{k_B T^2} \overbrace{\sum E_i P(E_i)}^{=U}$$

$$= \frac{ZU}{k_B T^2}$$

$$U = k_B T^2 \frac{1}{Z} \left(\frac{\partial Z}{\partial T}\right)_{V,N} = k_B T^2 \left(\frac{\partial \ln(Z)}{\partial T}\right)_{V,N}$$

$$\frac{\partial \ln(Z)}{\partial T} = \frac{1}{Z} \frac{\partial Z}{\partial T}$$

Helmholtz free energy (...)

$$F = U - TS$$ Thermodynamics

$$U = F + TS$$

$$= F - T\frac{\partial F}{\partial T} \qquad S = \left(\frac{\partial F}{\partial T}\right)_{V,N}$$

$$= -\cancel{T}^2 \frac{\partial F/T}{\partial T} = k_B \cancel{T}^2 \frac{\partial \ln Z}{\partial T}$$

$$\boxed{F = -k_B T \ln(Z)}$$

$$S = k_B \ln(W)$$

$$W \quad S :: Z : -F/T$$

const V const U

$$S = -\left(\frac{\partial F}{\partial T}\right)_{V,N} = k_B \frac{\partial T \ln(Z)}{\partial T}$$

$$= k_B T\left(\frac{\partial \ln(Z)}{\partial T}\right)_{V,N} + k_B \ln(Z)$$

Pressure $$P = -\left(\frac{\partial F}{\partial V}\right)_{T,N} = k_B T\left(\frac{\partial \ln(Z)}{\partial V}\right)_{T,N}$$

Chem potential $$\mu = \left(\frac{\partial F}{\partial V}\right)_{T,V} = k_B T\left(\frac{\partial \ln(Z)}{\partial N}\right)_{T,V}$$

Two level systems.

Energy levels E_0 (up) & $-E_0$ (down) at const T

Q. What is Z, U, C?

A: $$Z = \sum_i e^{-E_i/k_B T} = e^{E_0/k_B T} + e^{-E_0/k_B T}$$ (down, up)

$$= 2\cosh\left(E_0/k_B T\right)$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$