NUCLEAR REACTIONS, FUSION AND FISSION

Binding Energies:

${}^{11}_{5}B$	76.21MeV
${}^{4}_{2}He$	28.3MeV
${}^{12}_{6}C$	92.16MeV
${}^{14}_{6}C$	105.29MeV
${}^{14}_{7}N$	104.66MeV
${}^{2}_{1}H$	2.22MeV
${}^{3}_{2}He$	7.72MeV

1. *Calculate the threshold energy needed to make the following reaction occur*

$$ {}^{14}_{6}C + p \rightarrow {}^{14}_{7}N + n \,. $$

Q is the difference between the binding energy of products, and the binding energy of the reactants (cf. lecture notes).
Q = 104.66 MeV – 105.29 MeV = –0.63 MeV
The reaction is endothermic: one needs to provide a threshold energy of 0.63 MeV for the reaction to proceed.

How much kinetic energy is required in the laboratory frame if the stationary target nuclei are:

(a) ${}^{14}_{6}C$ *nuclei?* *[0.68MeV]*

$KE(lab) = KE(CM) \times \left(\dfrac{m+M}{M} \right)$ (cf. lecture notes, and the demonstration of this formula during the relevant lecture).

The KE required for a proton is: $0.63\ MeV \times \left(\dfrac{m+M}{M} \right)$
M = mass of ${}^{14}_{6}C$ ~ 14 amu
m = mass of p ~ 1 amu
Thus: $KE = 0.63 \times \dfrac{15}{14} = 0.68$ MeV

(b) *Protons? (Take nuclear masses to be proportional to mass number).* *[9.45MeV]*

Similarly: $KE = 0.63 \times \dfrac{15}{1} = 9.45$ MeV

2. * *(a)* *Calculate the energy released in the nuclear reaction*

$$p + {}^{11}_{5}B \rightarrow 3\,{}^{4}_{2}He \ .$$

[8.69MeV]

Q is the difference between the binding energy of products, and the binding energy of the reactants (cf. lecture notes).
Q =3 x 28.3 – 76.21 = 8.69 MeV

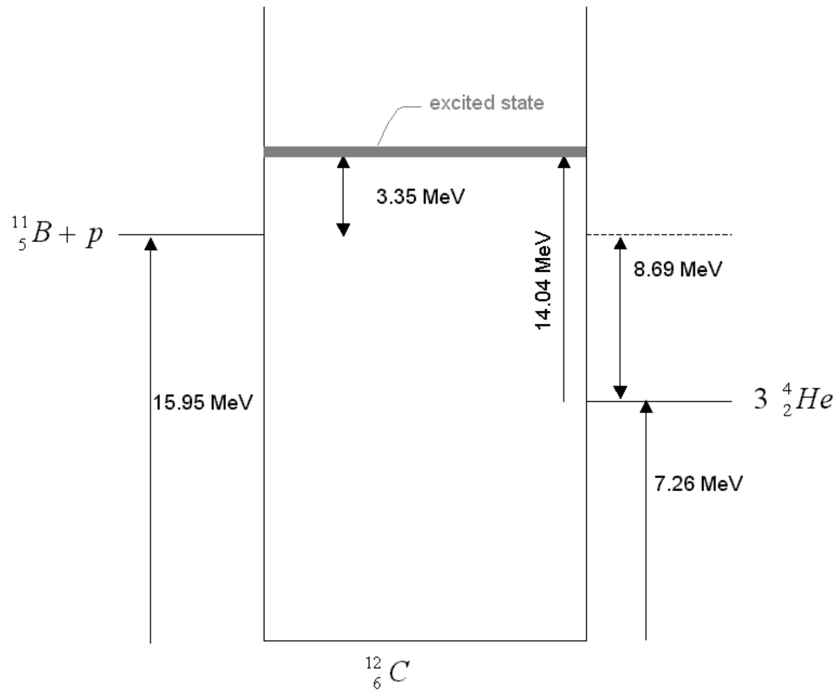
(b) *If the reaction is at a resonance when the incident protons have a kinetic energy of 3.65MeV in the laboratory frame, calculate the excitation energy of the compound nuclear state above the ground state of ${}^{12}_{6}C$ (take nuclear masses to be proportional to mass number).*
[19.3MeV]

Binding energy of ${}^{12}_{6}C$ = 92.16 MeV
Binding energy of ${}^{11}_{5}B + p$ = 76.21 MeV
$\therefore$ ground state of ${}^{12}_{6}C$ is 15.95 MeV below that of ${}^{11}_{5}B$
The kinetic energy of the protons in the Centre of Mass frame is:

$$KE(lab)\times\left(\frac{M}{m+M}\right) = 3.65\times\frac{11}{11+1} = 3.35 \text{ MeV}$$

$\therefore$ excited state will be 15.95 + 3.35 MeV above ground state of ${}^{12}_{6}C$, i.e. 19.3 MeV above ground state.

(c) *Draw an energy level diagram to represent the reaction.*



Only the left-hand part of the diagram is necessary to answer the question.

3. * *Neutrinos, they are very small.*
They have no charge and have no mass
And do not interact at all,
The Earth is just a silly ball
To them, through which they simply pass,
Like dustmaids down a drafty hall
Or photons through a sheet of glass.
[John Updike]

A 500MeV muon neutrino passing through matter interacts mainly with the neutrons in the nuclei. Assuming that half of the mass of a typical piece of matter is due to neutrons, that the neutrino has a total cross-section of 4x10^{-15} barns for interaction with a neutron and that all of the neutrons in a nucleus act independently work out the probability that the neutrino will interact as it passes through the Earth along a diameter.

Mean density of Earth = 5.5x10^{3} kg m^{-3}.
Mean diameter of Earth = 12,750 km. *[8x10^{-6}]*

The mean free path is given by: $l_0 = \dfrac{1}{\rho_{nuc}\ \sigma}$

ρ_{nuc}, in this case, is the density of neutrons. It is given by:

$$\rho_{nuc} = \frac{\frac{1}{2}\left(5.5\times10^{3}\right)}{mass\ of\ neutron}\ m^{-3}$$

$$= \frac{1}{2}\frac{5.5\times10^{3}}{1.67\times10^{-27}} = 1.64\times10^{30}\ m^{-3}$$

$$\therefore l_0 = \frac{1}{1.64\times10^{30}\times4\times10^{-15}\times10^{-28}}\ m$$

$$l_0 = 1.52\times10^{12}\ m$$

$\therefore$mean free path is much longer than the diameter of the Earth (and therefore we don't need to consider the attenuation of flux).
Hence the probability of interaction is (d being the Earth's diameter):

$\rho_{nuc}\ \sigma\ d = \dfrac{d}{l_0} = \dfrac{12.750\times10^{6}}{1.52\times10^{12}} \cong 8.4\times10^{-6}$, i.e. less than 0.001%

4. * *The main nuclear reactions which convert hydrogen to helium in the Sun are:*

$$p\ +\ p\ \rightarrow\ {}^{2}_{1}H\ +\ e^{+}\ +\ \nu_e \qquad (1)$$

$$p\ +\ {}^{2}_{1}H\ \rightarrow\ {}^{3}_{2}He\ +\ \gamma \qquad (2)$$

$$ {}^{3}_{2}He\ +\ {}^{3}_{2}He\ \rightarrow\ {}^{4}_{2}He\ +\ 2p \qquad (3)$$

(a) Estimate the mean thermal energy of nuclei at the centre of the Sun (T~10^{7}K). By comparing this to the Coulomb energy of two protons separated by 1fm, explain why these fusion reactions must proceed by quantum mechanical tunnelling.

The mean thermal energy is (by definition) $\approx \frac{3}{2} kT$

$$\approx \frac{3}{2} \times 1.38 \times 10^{-23} \times 10^{7} J$$
$$\approx 2.07 \times 10^{-16} J$$
$$\approx \frac{2.07 \times 10^{-16}}{1.6 \times 10^{-13}} MeV$$
$$\approx 1.3 \, keV$$

The Coulomb potential energy of two protons separated by 1 fm reads:

$PE = \frac{e^2}{4\pi\varepsilon_0 r}$ where $r = 10^{-15} m$

$PE = 1.4 \, MeV$

(Remember that $\frac{e^2}{4\pi\varepsilon_0}$ is given in MeV.fm in the Particle Data Sheet)

$\therefore$ The barrier height is much greater than the mean thermal energy of protons
$\therefore$ The reaction must occur by QM tunnelling

(b) Why is reaction (1) special? Why does it set the overall time scale of hydrogen burning?

(b) occurs via the weak interaction, and is therefore very slow. This is the starting point of the chain, and hence governs the overall rate at which fusion occurs.

(c) How much energy is released in reactions (2) and (3)? *[5.5MeV;12.86MeV]*

$Q_2 = BE({}^3_2 He) - BE({}^2_1 H) = 7.72 - 2.22 = 5.50$ MeV
$Q_3 = BE({}^4_2 He) - 2\,BE({}^3_2 He) = 28.30 - 2 \times 7.72 = 12.86$ MeV

(d) The total power output of the sun is 3.9×10^{26}W. If it assumed that reactions (2) and (3) are the primary energy sources of the Sun, estimate the number of neutrinos that are released each second. [2×10^{38}]

The formation of one ${}^4_2 He$ nucleus requires reaction (3) to occur once, reactions (1) and (2) need to occur twice.
The energy released in the formation of one ${}^4_2 He$ nucleus is: 2 x 5.5 + 12.86 MeV = 23.9 MeV

The number of ${}^4_2 He$ nuclei formed per second is: $\frac{3.9 \times 10^{26}}{23.9 \times 1.6 \times 10^{-13}} = 1.02 \times 10^{38}$

$\therefore$ The number of neutrinos released per second is $\approx 2. \times 10^{38}$

(e) Considering reactions (2) and (3), what proportion of the mass of the reactants is converted into energy? (Take nuclear masses to be proportional to the mass number)

This will be the ratio of the energy released in the reaction to the sum of the masses of the reactants (converting the amu to the proper energy units):
23.9 MeV / (2 × (1 + 2) × 931.502 MeV) = 0.0043

(f) Estimate the number of solar neutrinos that pass through you every day. Should you worry about this? (The distance from the Earth to the Sun is about $1.5x10^{11}$m).

[~$3x10^{19}$]

The average cross-section of a person is estimated to be ~ 0.5 m^2 (that depends of course on their orientation, and other factors).
At the range of the Earth, the neutrinos will be spread over a sphere of surface area $4\pi R^2$, where R = 1.5 x 10^{11} m.
$\therefore$ The number of neutrinos passing through a person, per second, is:

$$\frac{2\times 10^{38}\times 0.5}{4\pi\times\left(1.5\times 10^{11}\right)^2} = 3.54\times 10^{14}$$

Per day, this number becomes: 3.54 x 10^{14} x 60 x 60 x 24 ~ 3 x 10^{19}
Because the cross-section for the neutrino interaction with nucleons is very small, we shouldn't worry about it …