The Python based statistics and LaTeX report coursework # 1

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This is my submission for the Python statistical analysis and LaTeX report writing. Here I work through the instructions outlined in the booklet using Python and LaTeX hence gaining a better understanding of how to use them. I'll report a range of findings based on the supplied data. This course-work has really helped me gain an insight into how all this works and will aid me in my future lab reports across all time and space. I could do with a really strong cup of coffee.

I. STATS AND LaTeX #1

A. Data Set 1

Figure 1 presents a graph of the first 50 electrical measurements including, where appropriate, good use the SI-Prefixes. Note to be nice the 'measurement number' axis ranges from 0 to 51. Also note, the nice scale on the y-axis which is symmetric about the mean value of the measurements.

What's important was to produce a good legible graph that allows the casual reader to get a good feel for my experimental results. It was easy to read font size for both the axis labels and the numerical text. The choice of unit is also important to keep the numerical values presented in a nice human range, typically from 1 to 1000.

Table I presents the numerical results required for the analysis of the electrical current measurements. They show as the number of data point analysed increased so the mean of the current $\langle I \rangle$ and the standard deviation converge, whereas the standard error on the mean drops

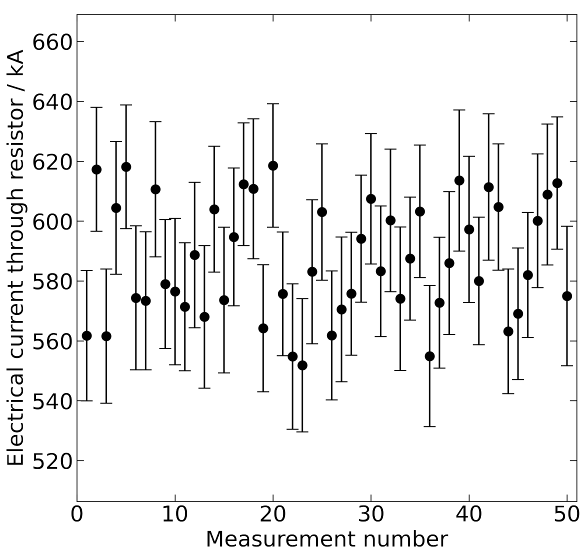

FIG. 1. The first 50 electrical current measurements. See table I for statistical values.

TABLE I. The mean $\langle I \rangle$, standard deviation σ and standard error on the mean s_{mean} for a set of n electrical current measurements.

n	$\langle I \rangle$ / kA	σ / kA	s_{mean} / kA
3	580	32	19
5	593	29	13
10	587.7	22	7.1
50	586.8	19	2.8
100	585.8	20	2.0
10000	584.56	20	0.20

, displaying its $s_{\mathrm{mean}} \propto \sqrt{N}$ relationship. Also note the use of the SI Prefix so as to present these numerical values nicely. This analysis also highlights that taking more and more data does help the precision of the final mean results, but at ever diminishing return.

If I was so inclined (not here) I could also have investigated the dependence of the standard error s_{mean} as a function of the number of data points analysed. A plot should show a square-root dependence. I could have gone further and used logarithms to show that the power law was indeed near 0.5.

The current was flowing thought a $R = 100\ \Omega$ resistor. The power dissipated in the resistor P is given by $P = I^2 R$. Hence, have $P = (34.172 \pm 0.024)$ TW.

B. Data Set 2

This section presents experimental results and analyses of a Hook's law spring obeying the relationship

$$F = -k(x - x_0) \tag{1}$$

where F is the measured force, k is the spring constant, x is the experimentally controlled position of one end of the spring as measured in the laboratory, and x_0 is the equilibrium length of the spring. Forty pairs of F and x were measured, each with their own standard deviation in the force measurement. Figure 2 presents these data.

To determine k and the intercept with the Force axis ($F_{x=0}$,) a linear least squares fit was performed giving $k = (642 \pm 13)$ nN/m and $F_{x=0} = (467 \pm 10)$ N (see figure 2).

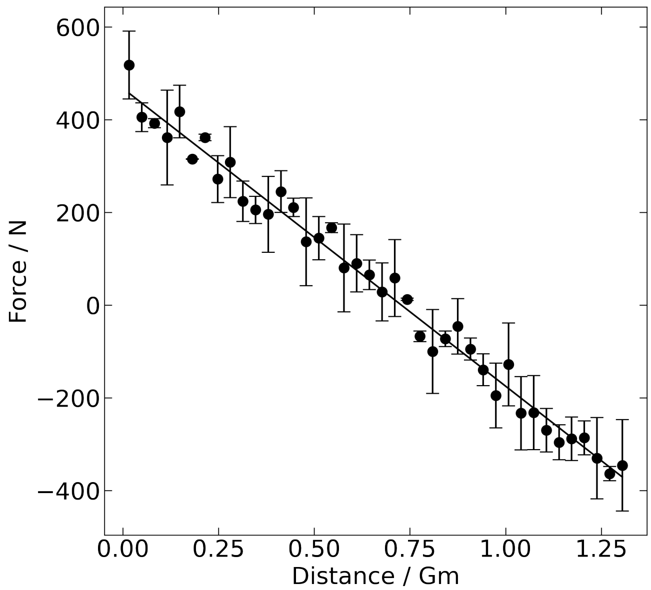

FIG. 2. The force generated by the controlled expansion and compression of a Hook's law spring. Line indicates linear last square fit with parameters as presented in the main text. Error bars indicate the standard deviation of each individual measurement.

II. STATS AND LATEX #2

A. Data Set 1

Table II presents the numerical results required for the analysis of the electrical current measurements. Here

TABLE II. The weighted mean $\langle I \rangle$ and weighted standard error on the mean s_{mean} for a set of n electrical current measurements.

n	$\langle I \rangle$ / kA	s_{mean} / kA
3	582	12
5	594.0	9.6
10	588.9	7.0
50	587.1	3.2
100	586.2	2.2
10000	584.56	0.22

the uncertainties σ_i of each data point was used to generate a weight $w_i = 1/\sigma_i^2$ to compute weighted values of the mean of the current $\langle I \rangle$ and standard error on the mean s_{mean}. Where as usual,

$$\langle I \rangle = \frac{\sum_{i=1}^{i=n} w_i I_i}{\sum_{i=1}^{i=n} w_i} \tag{2}$$

and

$$s_{\mathrm{mean}} = \frac{1}{\sqrt{\sum_{i=1}^{i=n} w_i}}. \tag{3}$$

Note from Dr Sloan: The analysis of the uncertainties in your experiments can nearly be as subtle and complex as you wish. It takes time and practise to work out what uncertainty goes where and what's important and what's not. These labs and this assessment allows you to practice the process. What we ask for in these [WL] labs is that you get all the basic uncertainty analysis right and use the most appropriate techniques. As you go further then you can delve deeper into the statistical underpinning of all this work.

B. Uncertainty propagations equations

Here I present an uncertainty propagation analysis of the following equations. This has given me good practice both of the propagation mathematics, and in using LATEX to present equations in a report. Specifically an uncertainty Δx in x is propagated through to an uncertainty Δy in y.

1. *Equation 1*

Given that a and b are constants, for the equation,

$$y = \frac{a}{b+x} \tag{4}$$

the uncertainty Δx in x propagates to y as

$$\Delta y = -\frac{a}{(b+x)^2}\Delta x. \tag{5}$$

Equation 2

Given that a and b are constants, for the equation,

$$y = a\cos(b+x) \tag{6}$$

the uncertainty Δx in x propagates to y as

$$\Delta y = -a\sin(b+x)\Delta x. \tag{7}$$

2. *Equilibrium Spring position*

By combining the spring constant k, force $F_{x=0}$ at $x = 0$ with their associated uncertainties, the x position of the equilibrium spring position is $x_0 = (728 \pm 22)$ Mm.

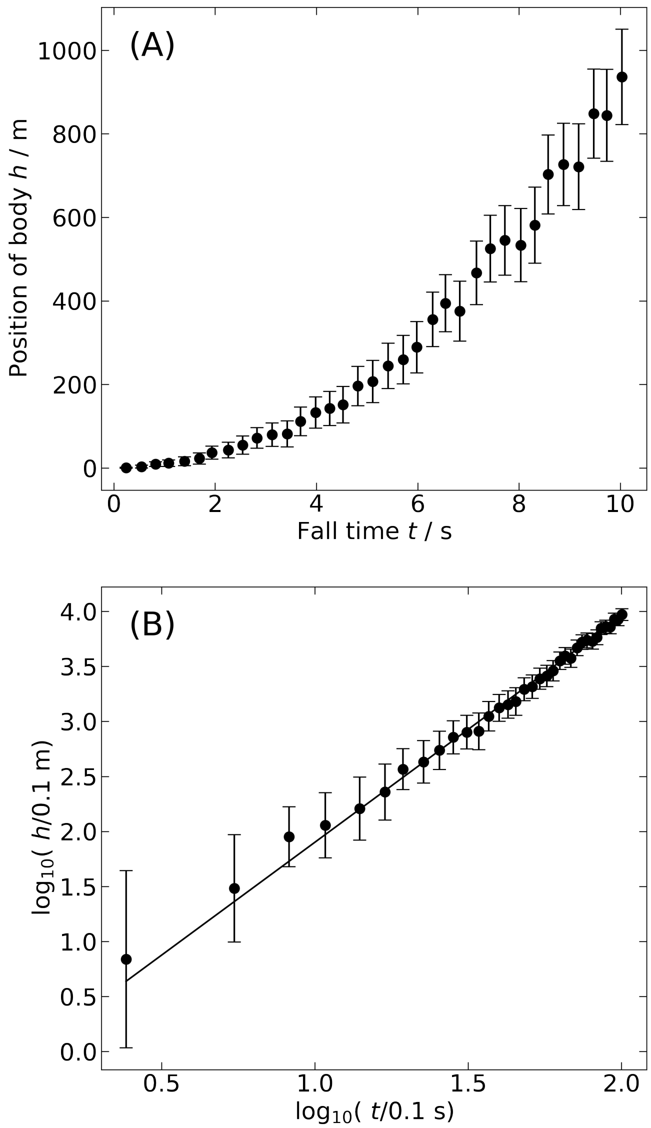

FIG. 3. The position is a body undergoing constant acceleration measured at several time intervals on (A) a linear scale and (B) a logarithmic scale with a weighted linear best fit - see main text for details.

III. STATS AND LaTeX #3

The normal relationship between the position of body h experiencing a constant force with resultant acceleration a with the time t of its fall is

$$h = \frac{1}{2}at^2. \tag{8}$$

Here we measure the position and time for a body undergoing free fall and experiencing the effects of air drag and turbulence. Figure 3A presents 35 measurements and their individual standard deviations. Note, these data are unphysical and just made up by Dr Sloan, hence only good for this statistical analysis practice assessment.

As expected the relationship is nonlinear. But rather than just fitting equation 8 which is just the integrated form of Newton's 2nd law, here we fit a more generic power law

$$h = \frac{1}{2}at^p \tag{9}$$

to determine the power p dependence of the position with the fall-time. To extract the power law here we perform a weighted linear least squares straight line to the data after taking its logarithm.

To ensure the logarithms return positive values (for ease of plotting) the distance measurement is divided by $h_0 = 0.1$ m, and the time measurement by $t_0 = 0.1$ s. Using equation 9 to link h_0 and t_0 gives $a_0 = 2h_0/t_0^p$. Thus dividing both sides of equation 9 by h_0 gives,

$$\frac{h}{h_0} = \frac{a}{a_0}\left(\frac{t}{t_0}\right)^p. \tag{10}$$

Taking base 10 logarithms gives

$$\log_{10}\left(\frac{h}{h_0}\right) = \log_{10}\left(\frac{a}{a_0}\right) + p\log_{10}\left(\frac{t}{t_0}\right). \tag{11}$$

Figure 3B presents the same data as fig. 3A but after taking their logarithms with appropriate divisors. It also shows a weighted linear least squares fit giving a power law exponent $p = 2.051 \pm 0.030$.

The intercept $C = \log_{10}(a/a_0)$ of the logarithmic graph could be used (not here) to determine the acceleration constant a through the relationship

$$a = a_0 10^C \tag{12}$$

leading to

$$a = 2\frac{h_0}{t_0^p}10^C. \tag{13}$$

Here h_0 and t_0 are set, but to determine the uncertainty on a, both the uncertainty on the intercept C and the uncertainty on the power law p would have to be used.

In an actual report I would show that the power law gave strong evidence that the model was a good. Then I could justifiably use that equation to fit $y = h$ and $x = v^2$ to perform another linear least square fit to determine a.