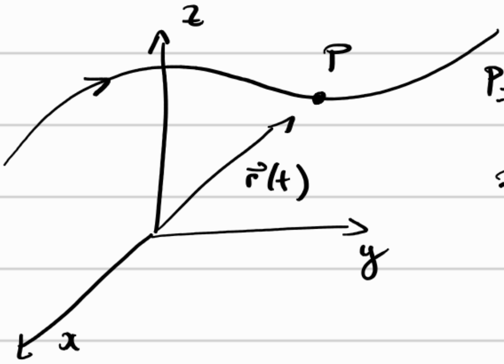
Space curves



$P = \vec{r}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}$

For example

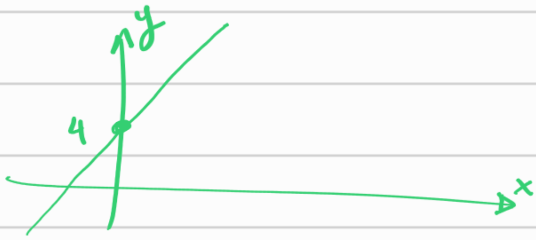
$\vec{r}(t) = \cos(\omega t)\vec{i} + \sin(\omega t)\vec{j} + at\vec{k}$

Parameterisation of curve

Let u be a parameter

need to find expressions for $x(u), y(u)$ & $z(u)$

(e.g.) Parametrise $y = 3x + 4$



$\vec{r}(u) = x(u)\vec{i} + y(u)\vec{j} + z(u)\vec{k}$
(in general)

$\vec{r}(u) = u\vec{i} + (3u + 4)\vec{j} + 0\vec{k}$

Let $v = 3x$ then $y = 3x + 4$

$$\vec{r}(v) = \frac{v}{3}\vec{i} + (v + 4)\vec{j}$$

NO ONE SINGLE WAY TO PARAMETRISE

Express $y = 3x^2$ as $\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{z}$

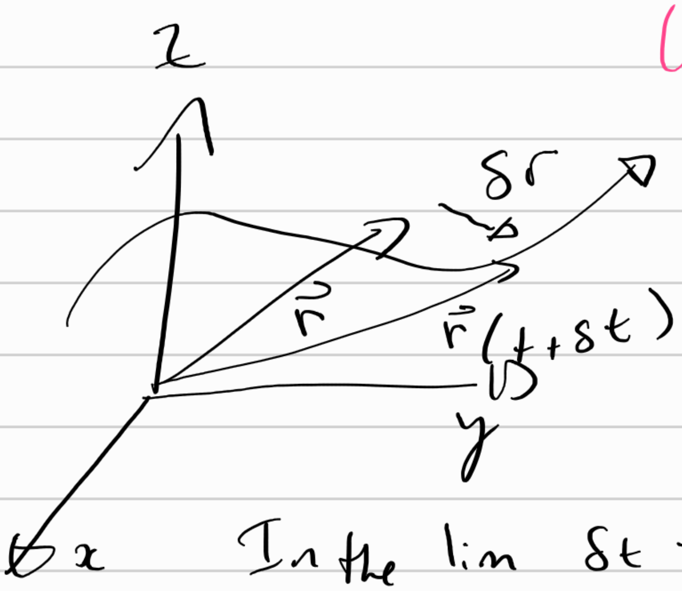
$\vec{r}(t) = x\vec{i} + (3x^2)\vec{j} + z\vec{z}$

$x = \sqrt{3} \qquad y = 3x^2$

$$\vec{r}(t) = \frac{v}{\sqrt{3}}\vec{i} + (v^2)\vec{j}$$

fuck missed it (correct)

check video paused 17:15 ~20 min



$$\delta\vec{r} = \vec{r}(t+\delta t) - \vec{r}(t) \approx \frac{d\vec{r}}{dt}\delta t$$

In the lim $\delta t \to 0$, the approximat° becomes exact

Then $\delta\vec{r} \to //$ to the curve at P

(where P is the point with posit° vector $\vec{r}(t)$)

$\delta\vec{r}$ becomes a tangent to the space curve in the limit $\delta t \to 0$

$$\frac{\frac{d\vec{r}}{dt}}{\left|\frac{d\vec{r}}{dt}\right|} = \hat{T} \text{ unit tangent vectors}$$

$$\hat{G} = \frac{\vec{G}}{|\vec{G}|}$$

From last time we known that

$\frac{d\vec{r}}{dt} = \vec{v}(t)$ & $\left|\frac{d\vec{r}}{dt}\right| = |\vec{v}| = \text{speed}$

$$\hat{T} = \frac{\frac{d\vec{r}}{dt}}{\left|\frac{d\vec{r}}{dt}\right|} = \frac{\vec{v}}{|\vec{v}|} = \hat{v}$$

Let S be distance,

consider $\vec{r}(s)$ instead of $\vec{r}(t)$

$$\hat{T} = \frac{d\vec{r}}{ds}$$

But $\frac{d\vec{r}(s)}{dt} = \frac{d\vec{r}}{ds}\frac{ds}{dt} = \hat{T}\frac{ds}{dt}$

& $\text{speed} = |\vec{v}| = \left|\frac{d\vec{r}}{dt}\right| = \frac{ds}{dt}$

Ex

Find $\hat{T}$ at the point $(0,a)$ on the space curve $\vec{r}(t) = a\cos(\omega t)\vec{i} + a\sin(\omega t)\vec{j}$

$$\hat{T} = \frac{\frac{d\vec{r}}{dt}}{\left|\frac{d\vec{r}}{dt}\right|} \qquad \frac{d\vec{r}}{dt} = -a\omega \sin(\omega t)\vec{i} + a\omega\cos(\omega t)\vec{j}$$

$$\left|\frac{d\vec{r}}{dt}\right| = \left((a\omega)^2 \underbrace{(-\sin^2(\omega t) + \cos^2(\omega t))}_{=1}\right)^{1/2}$$

$$= a\omega$$

so $\hat{T} = -\sin(\omega t)\vec{i} + \cos(\omega t)\vec{j}$

at $(0,a) = \vec{r} = a\cos(\omega t)\vec{i} + a\sin(\omega t)\vec{j}$

$$= 0\vec{i} + a\vec{j}$$

$$\Rightarrow \begin{cases} \cos(\omega t) = 0 \\ \sin(\omega t) = 1 \end{cases} \Rightarrow \omega t = \pi/2$$

so $\hat{T}\Big|_{(0,a)} = -\sin(\pi/2)\vec{i} + \cos\left(\frac{\pi}{2}\right)\vec{j}$

$$= -\vec{i}$$

Newton's 2nd law in vector form

$F = ma$ $\quad F = m \frac{d^2 r}{dt^2}$

$\vec{F} = m\vec{a}$ $\quad \vec{F} = \frac{m d^2 \vec{r}}{dt^2}$

$\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}$

Lorentz Force (Force particle in electromagnetic field)

$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

$\vec{F} = \frac{m d^2 \vec{r}}{dt^2} = q(\vec{E} + \vec{v} \times \vec{B})$

$\vec{F} = q\vec{E} + q\left(\frac{d\vec{r}}{dt} \times \vec{B}\right)$

$\vec{F} = F_x \vec{i} + F_y \vec{j} + F_z \vec{k}$

$F_x = \frac{m d^2 x}{dt^2}$

$F_y = m \frac{d^2 y}{dt^2}$

$$F_z = m\frac{d^2 z}{dt^2}$$

Problem: The force on a particle of charge q moving in a const magnetic field $\vec{B}$ is given by

$$\vec{F} = q(\dot{\vec{r}} \times \vec{B})$$

Let m be the mass of the particle

so $m\ddot{\vec{r}} = q(\dot{\vec{r}} \times \vec{B})$ so $\ddot{\vec{r}} = \frac{q}{m}(\dot{\vec{r}} \times \vec{B})$

Explain why the acceleration of the particle is always (17) perpendicular to the $\vec{B}$ field.