Fixed-time Online (Inspera) Exam Instruction Sheet

University of Bath
Department of Physics

PH30030: Quantum Mechanics

Please read the Guidance for Students before attempting this exam. The Guidance contains information about submitting your exam attempt.

This is an open book exam. You may refer to your own course and revision notes and look up information in offline or online resources, for example textbooks or online journals.

This exam starts at: 09:30 on 19 January 2022.

This exam is designed to take approximately 2 hours to complete.

You will have an additional 60 minutes of submission time for checking your work, collating your answers and uploading files. You are advised to allow sufficient time for minor technical issues when submitting your work.

The exam will close at the end of the submission time, after which you will not be able to submit an attempt.

Answer all questions

Filenames: If you are required to upload a file as part of your exam attempt, to maintain anonymity please use the following naming convention for your file: CandidateNumberUnitCodeQuestionNumber.pdf (e.g. 01234AR10001Q2a.pdf).If the exam only requires one file to be submitted, you do not need to include the question number(s).

Additional materials needed to complete the assessment: Calculator, University Formula Book (https://www.bath.ac.uk/guides/online-resources-for-mathematics-and-statistics/#university-formula-book0), Fundamental Constants table, a phone with a camera/scanner**.**

Further instructions:

Academic Integrity for Remote Exams

When you registered as a student you agreed to abide by the University's regulations and rules, and agreed that you would access and read your programme handbook. These documents contain references to, and penalties for, unfair practices such as collusion, plagiarism, fabrication or falsification. The University's Quality Assurance Code of Practice, QA53 Examination and Assessment Offences, sets out the consequences of committing an offence and the penalties that might be applied.

By submitting your exam as instructed, you confirm that:

1. You have not impersonated, or allowed yourself to be impersonated by, any person for the purposes of this assessment.

2. This assessment is your original work and no part of it has been copied from any other source except where due acknowledgement is made. The University may submit your work through a plagiarism detection service.

3. You have not previously submitted this work for any other unit/course.

4. You give permission for your assessment response to be reproduced, communicated, compared and archived for plagiarism detection, benchmarking or educational purposes.

5. You understand that plagiarism is the presentation of the work, idea or creation of another person or organisation as though it is your own. It is a form of cheating and is a very serious academic offence that may lead to disciplinary action.

6. You understand that this assessment is undertaken without invigilation, and that you have not communicated with and will not communicate with anyone concerning this assessment before the deadline for submission unless it is expressly permitted by the assessment instructions.

7. No part of this assessment has been produced for, or communicated to, you by any other person, unless it is expressly permitted by the assessment instructions.

If you have any questions about the exam you should contact the exams helpline. Information and contact details can be found on our help and advice webpage.

1. The wavefunction of a particle in a one-dimensional infinite square well potential of width a is given by

$$\psi(x) = c_1\phi_1(x) + \frac{1}{3}\phi_2(x) + c_3\phi_3(x)$$

where ϕ_n (n = 1, 2 or 3) are the orthonormal eigenfunctions of the total energy operator $\hat{H}$ corresponding to eigenvalues $E_n = n^2 \frac{\hbar^2\pi^2}{2ma^2}$.

(a) It has been determined experimentally that the probability of obtaining the energy E_1 is $P(E_1) = 1/2$. Find the coefficients c_1 and c_3 (assume that the phases of these coefficients are zero). $c_1 = \frac{1}{\sqrt{2}}$; $c_2 = \sqrt{\frac{7}{18}}$ (3)

(b) Calculate the expectation value $\langle \hat{H} \rangle$. $\langle \hat{H} \rangle = \frac{40}{9} E_1$ (3)

2. A particle of mass m is in the ground state of a one-dimensional harmonic oscillator of potential energy $V(x) = \frac{1}{2}m\omega^2\left(x - \langle x \rangle\right)^2$ where ω is the angular frequency. The normalised eigenfunction is given by

$$\phi_0(x) = \left(\frac{a}{\pi}\right)^{1/4} \exp\left(-\frac{ax^2}{2}\right)$$

where $a = m\omega/\hbar$.

(a) Find the expectation values $\langle \hat{x} \rangle$, $\langle \hat{x}^2 \rangle$, $\langle \hat{p}_x \rangle$ and $\langle \hat{p}_x^2 \rangle$. (7)

$\langle \hat{H} \rangle = \frac{\hbar\omega}{2}$

(b) Hence find the expectation value for the total energy of the particle. (4)

(c) Show that the uncertainties Δx and Δp_x are consistent with the uncertainty principle. (3)

Note: $\int_{-\infty}^{\infty} dx\, x^2 e^{-ax^2} = \frac{1}{2a}\left(\frac{\pi}{a}\right)^{1/2}$

3. A particle is described by the eigenfunction $\chi = ze^{-\alpha\left(x^2+y^2+z^2\right)} = r\cos\theta\, e^{-\alpha r^2}$ where α is a constant.

(a) Using spherical polar coordinates, show that χ is an eigenfunction of $\hat{L}^2$ and $\hat{L}_z$ and find the corresponding eigenvalues. $\hat{L}^2 : 2\hbar$; $\hat{L}_z : 0$ (6)

(b) Explain the effect of the ladder operators $\hat{L}_+$ and $\hat{L}_-$ on the eigenvalues of $\hat{L}^2$ and $\hat{L}_z$. (3)

(c) Using Cartesian coordinates and the ladder operators, find the other (un-normalised) eigenfunctions of $\hat{L}_z$ that correspond to the same eigenvalue of $\hat{L}^2$.

$\chi_{\pm 1} = k(\mp x - iy)e^{-\alpha(x^2+y^2+z^2)}$ (8)

4. A large number N of non-interacting particles of mass m with antisymmetric wavefunctions are confined to a one-dimensional infinite square well potential of width Na. Calculate the ground state energy. (4)

Note: If M is a large integer, $\sum_{n=1}^{M} n^2 \approx \frac{M^3}{3}$. $E_{ground} = \frac{\hbar^2 \pi^2 N}{24 m a^2}$

5. Consider a one-dimensional anharmonic oscillator of reduced mass μ described by the potential energy

$$V(x) = \frac{1}{2}kx^2 + \frac{1}{6}ax^3$$

where k and a are constants. By employing first-order perturbation theory, calculate the corrections to the harmonic oscillator energy levels. The eigenfunctions of the unperturbed system are given by

$$\phi_{0n}(x) = \left[\left(\frac{\alpha}{\pi}\right)^{1/2} \frac{1}{2^n n!}\right]^{1/2} H_n\left(\alpha^{1/2}x\right)e^{-\alpha x^2/2}$$

where $n = 0,1,2,\cdots$, $\alpha = \left(k\mu/\hbar^2\right)^{1/2}$, and the Hermite polynomials $H_n\left(\alpha^{1/2}x\right)$ are either even or odd real functions. $E_{1n} = 0$ (5)

6. At time $t_0 \to -\infty$, a particle of mass *m* and charge *q* is in the ground state $|0\rangle$ of a one-dimensional harmonic oscillator of angular frequency ω with quantum number $n = 0$. The system is subjected to a weak transient electric field of maximum amplitude *E* such that the potential energy $V(x,t)$ is given by

$$V(x,t) = qE\,x e^{-t^2/\tau^2}.$$

(a) Find the probability that after time *t*, such that $t >> \tau$, the particle is found in an excited state $|n\rangle$ of the harmonic oscillator with quantum number $n > 0$.

The general expression for the coefficient $c_f(t)$ expressing the transition amplitude into the final state *f* at time *t* from the initial state *i* at time t_0 is given by

$$c_f(t) = \frac{1}{i\hbar}\int_{t_0}^{t} H'_{fi}(t')\exp(i\omega_{fi}t')\,dt'$$

where

$$H'_{fi}(t) = \langle f|\hat{H}'(x,t)|i\rangle.$$ (10)

(b) What is the probability that, after time *t*, a transition is made to the first excited state? (1)

(c) What is the probability that, after the same length of time, the particle is found in the second excited state? Briefly comment on the physical meaning of this result.

$P_{0 \to n \neq 1} = 0$

(3)

Note: $\int_{-\infty}^{\infty} dx\, e^{i\alpha x} e^{-x^2/\beta^2} = \sqrt{\pi}\,\beta e^{-\alpha^2\beta^2/4}$

and $|\langle n|x|0\rangle|^2 = \frac{\hbar}{2m\omega}\delta_{n1}$ where δ_{n1} is the Kronecker delta.

THE END