Consider a vector $\vec{u}(t) = u_x(x)\,\vec{i} + u_y(t)\,\vec{j} + u_z(t)\,\vec{k}$

N.B. $\vec{i}, \vec{j}$ & $\vec{k}$ are not f° of time $\iff$ Fixe length $(=1)$
Fixed direct°

$\vec{r}(t) = x(t)\,\vec{i} + y(t)\,\vec{j} + z(t)\,\vec{k}$ ← the posit° vector

$\vec{v}(t) = V_x(t)\,\vec{i} + V_y(t)\,\vec{j} + V_z(t)\,\vec{k}$ ← the velocity vector

$\vec{a}(t) = a_x(t)\,\vec{i} + a_y(t)\,\vec{j} + a_z(t)\,\vec{k}$ ← the accelerat° vector

Given $\vec{r}(t)$ (on prev page), then $\vec{v}(t) = \frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j} + \frac{dz}{dt}\vec{k} = \frac{d\vec{r}(t)}{dt}$

& $\vec{a}(t) = \frac{dv_x}{dt}\vec{i} + \frac{dv_y}{dt}\vec{j} + \frac{dv_x}{dt}\vec{k} = \frac{d\vec{v}(t)}{dt}$

$= \frac{d^2x}{dt^2}\vec{i} + \frac{d^2y}{dt^2}\vec{j} + \frac{d^2z}{dx^2}\vec{k} = \frac{d^2\vec{r}(t)}{dt^2}$

Example Let $\vec{r}(t) = \cos(\omega t)\,\vec{i} + \sin(\omega t)\,\vec{j}$

$\vec{v}(t) = -\omega\left(\sin(\omega t)\,\vec{i} - \cos(\omega t)\,\vec{j}\right)$

$\vec{a}(t) = -\omega^2\left(\cos(\omega t)\,\vec{i} + \sin(\omega t)\,\vec{j}\right)$

i) $\frac{d}{dt} c\vec{a} = c\frac{d\vec{a}}{dt}$ where c = a cnst scalar

ii) $\frac{d}{dt}(\vec{a} + \vec{b}) = \frac{d\vec{a}}{dt} + \frac{d\vec{b}}{dt}$

iii) $\frac{d}{dt}(\phi(t)\,\vec{a}(t)) = \phi\frac{d\vec{a}}{dt} + \vec{a}\frac{d\phi}{dt}$

iv) $\frac{d(\vec{a}\cdot\vec{b})}{dt} = \vec{a}\cdot\frac{d\vec{b}}{dt} + b\cdot\frac{d\vec{a}}{dt} = \frac{d}{dt}(\vec{b}\cdot\vec{a})$ • commutes ✓

v) $\frac{d}{dt}(\vec{a}\times\vec{b}) = \vec{a}\times\frac{d\vec{b}}{dt} + \frac{d\vec{a}}{dt}\times\vec{b} \neq \frac{d}{dt}(\vec{b}\times\vec{a})$ ✗ non-commutable

vi) $\frac{d}{dt}\vec{a}(s) = \frac{d\vec{a}}{ds}\frac{ds}{dt}$

Problem

Let $\vec{a}(t) = (2t)\vec{i} + \vec{j} + \vec{k}$ & $\vec{b} = (2t)^2 \vec{j}$

Find $\frac{d}{dt}(\vec{a} \times \vec{b})$

$$= \begin{pmatrix} 2t \\ 1 \\ 1 \end{pmatrix} \times \begin{pmatrix} 0 \\ 8t \\ 0 \end{pmatrix} + \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 4t^2 \\ 0 \end{pmatrix}$$

$$= -8t\,\vec{i} + 0\,\vec{j} + 24t^2\,\vec{k}$$