Thermodynamics of Fermi gases:

Chemical potential μ: $N_{\text{particle}} = \int_0^\infty \frac{dN}{dE}\, dE$

$$= \int_0^\infty \underbrace{\frac{g}{4\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} V E^{1/2}}_{\frac{dG}{dE}\text{ for gas}} n_{FD}(E)\, dE$$

at $T \to 0$ $\qquad n_{FD}(E) = \begin{cases} 1 & E < \mu \\ 0 & E > \mu \end{cases}$

$$\mu(T=0) = E_F$$

$n_{FD}(E) = 1$

$$N = \frac{g}{4\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} V \int_0^{E_F} E^{1/2}\, dE$$

$$\int_0^{E_F} E^{1/2}\, dE = \frac{2}{3} E^{3/2}\Big|_0^{E_F} = \frac{2}{3} E_F^{3/2}$$

$$N = \frac{g}{\cancel{4}_{2}\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} V \frac{\cancel{2}}{3} E_F^{3/2} = \frac{g}{6\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} V E_F^{3/2}$$

$$E_F = \left(\frac{6\pi^2}{g}\underbrace{\frac{N}{V}}_{\rho\text{ - number density}}\right)^{3/2}\frac{\hbar^2}{2m} \qquad E_F \sim \rho^{2/3} \text{ at } T=0$$

$\mu(T=0) > 0$ (1)

For a classical gas $\mu \to 0, T \to 0)$

Internal energy U

$$U = \int_0^\infty E \frac{dN}{dE}\, dE = \frac{g}{10\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} V E_F^{5/2}$$

$U = 3/5\, N E_F$ (cf $T \to 0$)

$U = \frac{3}{2} N k_B T \to 0$ for classical ideal gas at $T \to 0$

pressure: (at $T=0$)

$$p = -\left(\frac{\partial U}{\partial V}\right)_{S,N}$$ (from thermo identity: $dU = \cdots$)

$$= \frac{2}{5} \frac{N}{V^{5/3}} \left(\frac{6\pi^2 N}{g}\right)^{2/3} \frac{\hbar^2}{2m}$$

$$= \frac{2}{5} \rho^{5/3} \left(\frac{6\pi^2}{g}\right)^{2/3} \frac{\hbar^2}{2m}$$

$$= 2/5 \frac{N}{V} E_F$$

g for Helium = 1
~ no degeneracy

$$\left[\frac{\hbar^2}{2mL^2}\right] = E = \frac{\hbar^2 k^2}{2m} \qquad p = \hbar k$$

Classical gas $p \to 0$ $T \to 0$

Degeneracy pressure p due to Pauli exclus^n.

Ideal Boson gas: Photons in a black body.
Phonons in a solid

$T \to 0$ $$n_{BE}(E) = \frac{1}{e^{(E-\mu)/k_BT} - 1} \xrightarrow{T=0} \begin{cases} N & \text{at } E = 0 \\ 0 & \text{at } E > 0 \end{cases}$$

Boson-Einstein condensate