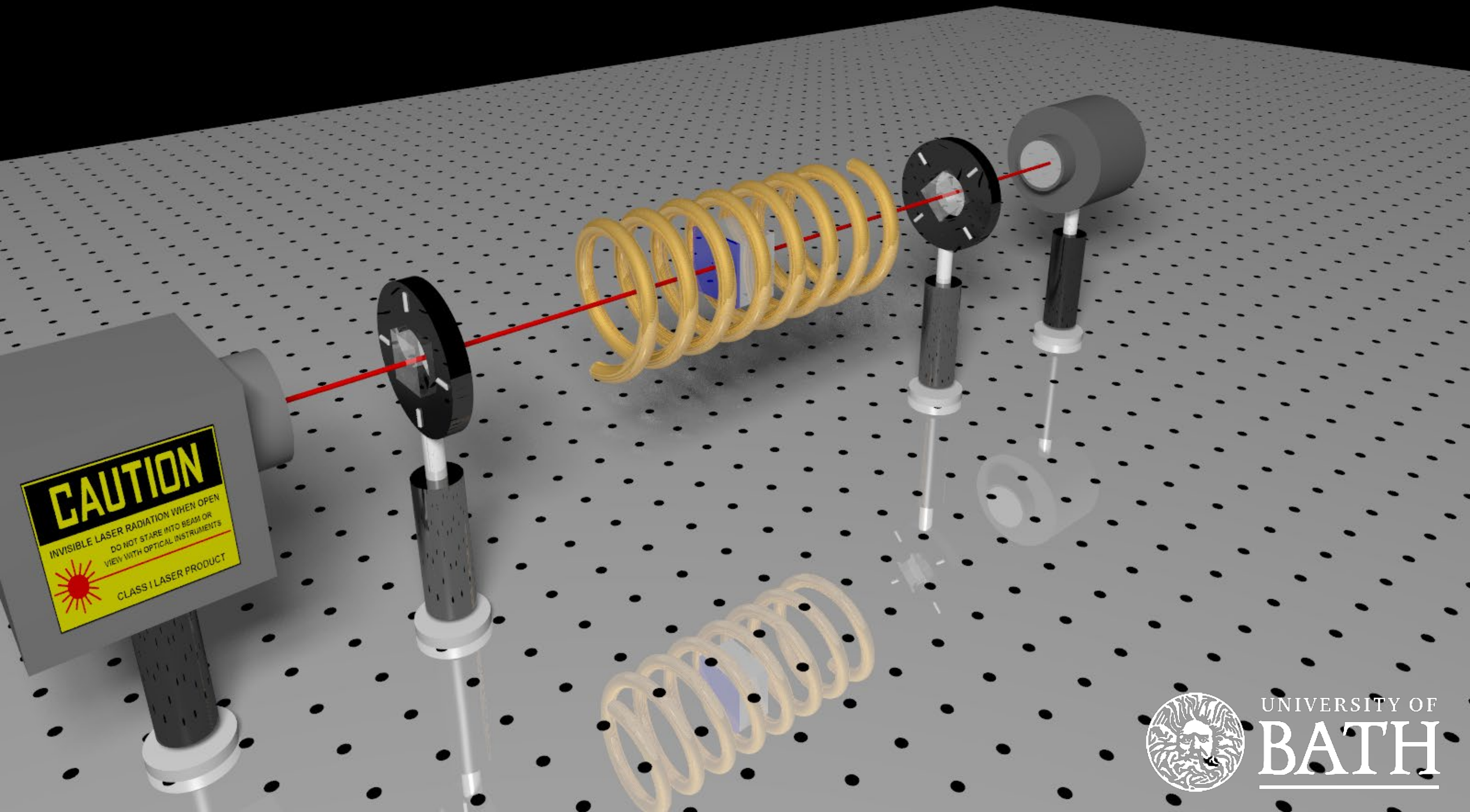
Lecture 14

Magnetic fields in materials



UNIVERSITY OF BATH

Last time we saw

Electric current is a source of magnetic fields.

The magnetic dipole moment is current times area: $m = I \times (\text{area})$

The magnetisation is the magnetic dipole moment per unit volume: $\vec{M} = \frac{\sum \vec{m}}{V}$

When an external magnetic field is applied to a magnetic dipole moment, the torque acts to bring $\vec{m}$ parallel to $\vec{B}$.

The magnetisation induces a surface current density $\vec{k}_m = \vec{M} \times \hat{n}$ and bound current density $\vec{J}_m = \nabla \times \vec{M}$.

The magnetic flux density results from adding up the magnetic field strength and the magnetisation: $\vec{B} = \mu_0 \left(\vec{H} + \vec{M}\right)$

The magnetic field strength is defined as $\vec{H} = \frac{1}{\mu}\vec{B}$.

We also saw the power of levitation!

The power of levitation!

The High Field Magnet Laboratory in Nijmegen (the Netherlands) reaches fields of up to 37.5 T. Currently developing a 45 T field. Current world record is 41.4 T, at the National High Magnetic Field Laboratory, in Florida.

A live frog levitates inside a magnetic field of about 16 tesla at the Nijmegen High Field Magnet Laboratory.

The minimum criterion for levitation:

$$B\frac{dB}{dz} = \mu_0 \rho \frac{g}{\chi_m}$$

Where ρ is the density of the material

And g is the local gravitational acceleration

Frogs are diamagnetic!

Overview

In this Lecture we will look at:

- Linear, isotropic and homogeneous magnetic materials
- Classification of magnetic materials
- Diamagnetism
- Energy stored in a magnetic material

Linear, isotropic and homogeneous magnetic materials

A magnetic material is **linear** if the magnetic permeability and susceptibility do not change with the magnitude of the applied magnetic field.

A magnetic material is **isotropic** if the magnetic permeability and susceptibility do not change with the direction of the magnetic field in the material. In other words, all directions in the material are equivalent.

A dielectric material is **homogeneous** if the magnetic permeability and susceptibility do not change from point to point in the material. In other words, they are independent of the coordinates. We can say that all locations in the material are equivalent.

In vacuum, $\chi_m = 0$ and $\mu_r = 1$. In air, we will take $\mu_r = 1$ as well.

What are the main types of magnetic material?

Classification of magnetic materials

Magnetic materials can be classified according to their magnetic susceptibility χ_m

In **diamagnetic** materials, the magnetic response opposes the externally applied magnetic field. For diamagnets, typically, $\chi_m \approx -10^{-5}$. Examples include gold, silver, copper, bismuth and beryllium. Superconductors form a special group of materials that are also diamagnetic, with $\chi_m \approx -1$.

In **paramagnetic** materials, the magnetic response is weak but aligned parallel with the direction of the externally applied magnetic field. For paramagnets, typically, $\chi_m \approx 10^{-3}$ to 10^{-5}. Examples of paramagnets are platinum, aluminium and manganese.

In **ferromagnetic** materials, the magnetics response is also aligned with the externally applied field but this response is very strong. For ferromagnets, typically, $\chi_m \approx$ 50 to 10,000. Examples of ferromagnets include iron, cobalt, nickel and many rare earth elements.

Examples of some materials?

Classification of magnetic materials

Magnetic type	Element	χ_m
Diamagnets	Au	-2.74×10^{-6}
	Ag	-2.02×10^{-6}
	Cu	-0.77×10^{-6}
	Be	-1.85×10^{-6}
	Bi	-1.31×10^{-6}
	Ge	-0.56×10^{-6}
Paramagnets	Pt	21.04×10^{-6}
	Al	1.65×10^{-6}
	W	6.18×10^{-6}
	Mn	66.10×10^{-6}

Note: There are also other types of magnetic materials, such as ferrimagnets, antiferromagnets, helimagnets and superparamagnets. [you don't need to know them for the exam]

More materials?

Magnetic properties of pure elements in the solid state

Table 3.2 - Low temperature magnetic properties of pure elements in the solid state. In bold framed: the magnetically ordered substances

H																	He dia
Li para	Be dia											B dia	C dia	N dia	**O** **AF**	F dia	Ne dia
Na para	Mg para											Al para	Si dia	P dia	S dia	Cl dia	Ar dia
K para	Ca para	Sc para	Ti para	V para	**Cr** **AF**	**Mn** **AF**	**Fe** **Ferro**	**Co** **Ferro**	**Ni** **Ferro**	Cu dia	Zn dia	Ga dia	Ge dia	As dia	Se dia	Br dia	Kr dia
Rb para	Sr para	Y para	Zr para	Nb para	Mo para	*Tc*	Ru para	Rh para	Pd para	Ag dia	Cd dia	In dia	Sn *	Sb dia	Te dia	I dia	Xe dia
Cs para	Ba	La	Hf	Ta para	W para	Re para	Os para	Ir para	Pt para	Au dia	Hg dia	Tl dia	Pb dia	Bi dia	*Po*	*At*	Rn dia
Fr	*Ra*	*Ac*															
				Ce *	Pr para	**Nd** **AF**	*Pm*	**Sm** **AF**	**Eu** **Ferri**	**Gd** **Ferro**	**Tb** **Ferro**	**Dy** **Ferro**	**Ho** **Ferri**	**Er** **Ferri**	**Tm** **Ferri**	Yb para	Lu para
				Th para	*Pa*	U para	*Np*	*Pu*	*Am*	*Cm*	*Bk*	*Cf*	*Es*	*Fm*	*Md*	*No*	*Lw*

Complex structures (helimagnetic...) are classified as Ferri; AF means antiferromagnetic. The magnetic state (para or dia) of Sn and Ce depends on their crystallographic structure. The substances painted grey are superconductors at very low temperature, and those in italics are radioactive.

Magnetic moment of atoms?

Magnetic properties of atoms

79 of the 103 first pure elements carry an atomic moment in the atomic ground state, as shown in table 3.1.

Table 3.1 - Magnetic properties of pure elements in the atomic state

H																	**He**
Li	**Be**											B	**C**	N	O	F	**Ne**
Na	**Mg**											Al	**Si**	P	S	Cl	**Ar**
K	**Ca**	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	**Zn**	Ga	**Ge**	As	Se	Br	**Kr**
Rb	**Sr**	Y	Zr	Nb	Mo	*Tc*	Ru	Rh	**Pd**	Ag	**Cd**	In	**Sn**	Sb	Te	I	**Xe**
Cs	**Ba**	La	Hf	Ta	**W**	Re	Os	Ir	Pt	Au	**Hg**	Tl	**Pb**	Bi	Po	At	**Rn**
Fr	***Ra***	*Ac*															
				Ce	Pr	Nd	*Pm*	**Sm**	Eu	Gd	Tb	Dy	Ho	Er	Tm	**Yb**	Lu
				Th	*Pa*	U	*Np*	*Pu*	*Am*	*Cm*	*Bk*	*Cf*	*Es*	*Fm*	*Md*	*No*	*Lw*

In the atomic ground state, the only ***non*** *magnetic elements (bold framed) are those for which $J = 0$ (see chap. 7): Be..., Zn..., He..., Pd, Yb ($^1S_0$), C... ($^3P_0$), W ($^5D_0$), and Sm ($^7F_0$). The radioactive atoms are shown in italics.*

Let us consider an example

Étienne du Trémolet de Lacheisserie, Damien Gignoux, Michel schlenker, “Magnetism Fundamentals”, Springer (2005), p. 84

Example question

In a certain homogeneous isotropic medium for which $\mu_r = 4$, the magnetic field is given by $\vec{B} = 4\hat{x} - 3\hat{y} + 15\hat{z}$ mT. Calculate:

(a) the magnetic susceptibility χ_m,
(b) the magnetic field intensity $\vec{H}$,
(c) the magnetisation $\vec{M}$.

Example question

In a certain homogeneous isotropic medium for which $\mu_r = 4$, the magnetic field is given by $\vec{B} = 4\hat{x} - 3\hat{y} + 15\hat{z}$ mT. Calculate:

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(b) the magnetic field intensity $\vec{H}$,
(c) the magnetisation $\vec{M}$.

(a) From $\mu_r = 1 + \chi_m$, we obtain: $\chi_m = \mu_r - 1 = (4) - 1 = \underline{\underline{3}}$.

(b) From $\vec{B} = \mu_0 \mu_r \vec{H}$, we obtain:

$$\vec{H} = \frac{1}{\mu_0 \mu_r}\vec{B} = \frac{1}{\mu_0 (4)}\left(4\hat{x} - 3\hat{y} + 15\hat{z}\right) = \underline{\underline{\frac{1}{\mu_0}\left(\hat{x} - \frac{3}{4}\hat{y} + \frac{15}{4}\hat{z}\right)\times 10^{-3}\mathrm{Am}^{-1}}}$$

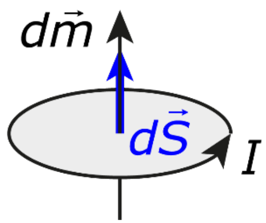
(b) From $\vec{M} = \chi_m \vec{H}$, we obtain:

$$\vec{M} = (3)\frac{1}{\mu_0}\left(\hat{x} - \frac{3}{4}\hat{y} + \frac{15}{4}\hat{z}\right)\times 10^{-3} = \underline{\underline{\frac{1}{\mu_0}\left(3\hat{x} - \frac{9}{4}\hat{y} + \frac{45}{4}\hat{z}\right)\times 10^{-3}\mathrm{Am}^{-1}}}$$

Let's have a closer look at diamagnetism!

Diamagnetism

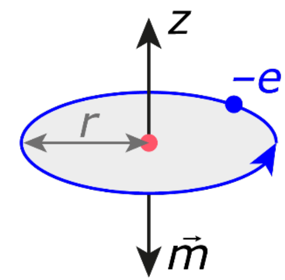
First, we remember the direction of the magnetic moment from a current loop:



The direction of the magnetic dipole moment *m* with respect to the direction of the current I and the surface element *dS*.

Next, we consider an electron circulating around a nucleus.

The current flows in the direction opposite to the electron's motion!



The magnetic dipole *m* with respect to the direction of electron motion.

The current is: $I = \dfrac{Q}{T} = \dfrac{e}{T} = \dfrac{ev}{\frac{2\pi r}{T}T} = \dfrac{ev}{2\pi r}$

Where we have the linear velocity of the electron: $v = 2\pi r/T$

The magnetic dipole moment is then:

$$m = I \times (\text{area}) = \frac{ev}{2\pi r}\pi r^2 = \frac{1}{2}evr$$

With directions: $\vec{m} = -\dfrac{1}{2}evr\hat{z}$

Let's consider the Coulomb force!

Diamagnetism

We just found that:

$$\vec{m} = -\frac{1}{2} e v r \hat{z}$$

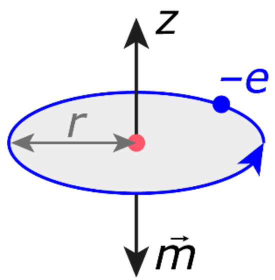
So, *m* points along negative *z*.

In the absence of an externally applied field, the electron experiences the Coulomb force and the centripetal force. They balance:

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} = m_e \frac{v^2}{r}$$

where the right hand term is the mass of the electron times the acceleration.

If an external magnetic field is present…



The magnetic dipole *m* with respect to the direction of electron motion.

Diamagnetism

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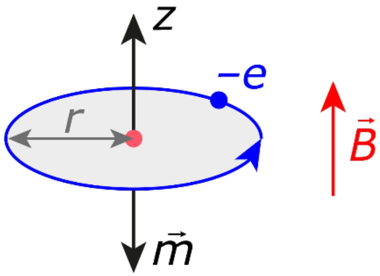


The magnetic dipole *m* and the magnetic flux density *B*.

we need to include the Lorentz force and the velocity of the electron changes from *v* to *v'*:

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} + ev'B = m_e\frac{(v')^2}{r}$$

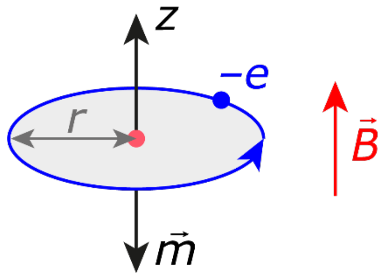
We now combine both equations:

$$\frac{m_e}{r}\left[(v')^2 - v^2\right] = ev'B$$

Which requires *v'*>*v*.

We can rewrite *v'*>*v*.

Diamagnetism



The magnetic dipole *m* and the magnetic flux density *B*.

We obtained:

$$\frac{m_e}{r}\left[(v')^2 - v^2\right] = ev'B$$

And we just found that: $v' > v$

We can write that as: $v' = v + \Delta v$

Equivalently: $v = v' - \Delta v$

We take Δv to be small, so: $(\Delta v)^2 \approx 0$

Then replacing: $\frac{m_e}{r}\left[(v')^2 - (v' - \Delta v)^2\right] \approx \frac{m_e}{r}\left[(v')^2 - (v')^2 + 2v'\Delta v\right] = \frac{m_e}{r} 2v'\Delta v$

And $\frac{m_e}{r} 2v'\Delta v = ev'B$. It follows that $\Delta v = \frac{eBr}{2m_e}$, which we can use in $\vec{m} = -\frac{1}{2} evr\hat{z}$

Therefore: $\Delta\vec{m} = -\frac{1}{2} e\Delta v r\hat{z} = -\frac{1}{2} e\left(\frac{eBr}{2m_e}\right) r\hat{z} = -\frac{1}{4}\frac{e^2r^2}{m_e} B\hat{z} = -\frac{1}{4}\frac{e^2r^2}{m_e}\vec{B}$

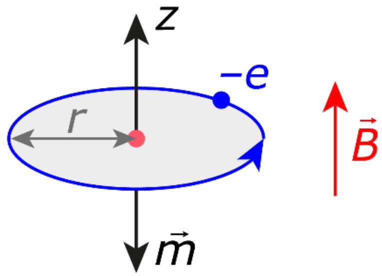
What does this mean?

Diamagnetism

We obtained:

$$\Delta\vec{m} = -\frac{1}{4}\frac{e^2 r^2}{m_e}\vec{B}$$

The change in magnetic dipole moment is opposite to the direction of the *B*-field.



The magnetic dipole *m* and the magnetic flux density *B*.

Therefore, when the **B**-field is added, it results in a Lorenz force that adds to the Coulomb force (that attracts the electron to the nucleus). As a result, the velocity of the electron changes and, in turn, the magnetic dipole moment is reduced by a small amount. This response is called **diamagnetic**.

Diamagnetism describes the repulsion of a material by an external magnetic field due to inducted magnetic moments opposite to the direction of the magnetic field.

It can be compared to a dielectric materials with electric polarisation, which opposes the external *E*-field.

What energy is stored in a magnetic materials?

Energy stored in a magnetic material

The **Self inductance** [in SI units of henry] is defined as the induction of a voltage in a current-carrying wire when the current in the wire itself is changing.:

$$\boxed{L\left(\text{self inductance}\right) = \frac{\Phi_{m,total}\left(\text{total flux linked through a circuit}\right)}{I\left(\text{current flowing in a circuit}\right)}}$$

The relative permeability of a material is then:

$$\mu_r = \frac{L_{\text{material}}}{L_{\text{vacuum}}}$$

We know that: $\Phi_m = \vec{B} \cdot \vec{A}$

For a solenoid with number of turns N per length l (**do not confuse length *l* with current *I***): $N = nl$

And $\Phi_{m,total} = N\Phi_m = NBA = nBAl$

Moreover, inside a very long solenoid:

$$B = \mu_0 n I$$

Therefore, we can write:

$$\Phi_{m,total} = n\left(\mu_0 n I\right) Al = \mu_0 A n^2 I l$$

So, the self inductance in vacuum is:

$$L_{\text{vacuum}} = \frac{\Phi_{m,total}}{I} = \mu_0 A n^2 l$$

And

$$\boxed{L_{\text{material}} = \mu_r \mu_0 A n^2 l}$$

So how about the energy then?

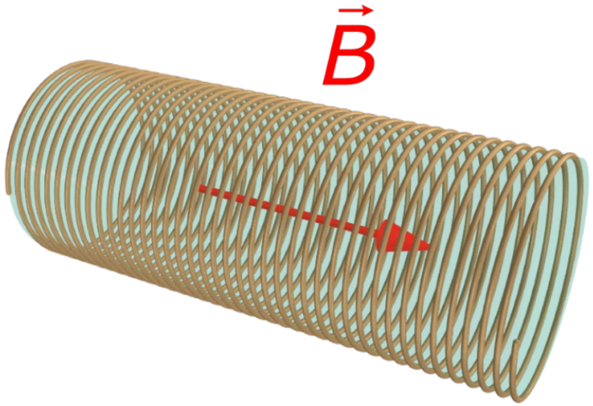
Energy stored in a magnetic material

We obtained:

$$L_{\text{material}} = \mu_r \mu_0 A n^2 l$$

The energy required to establish a current I in the solenoid is:

$$E = \frac{1}{2} L \cdot I^2$$



A solenoid with a magnetic core.

Therefore, the energy stored in the magnetic field:

$$E = \frac{1}{2}\left(\mu A n^2 l\right) I^2 = \frac{1}{2}(nI)(\mu n I)Al = \frac{1}{2}(H)(B)Al$$

So, for a volume such as: $V = A(\text{area}) \times l(\text{length})$, the energy stored is: $w = \dfrac{1}{2}HB$

This equation generalises to: $\boxed{w = \dfrac{1}{2}\vec{H} \cdot \vec{B}}$ $\boxed{w = \dfrac{1}{2}\vec{D} \cdot \vec{E}}$

Another example!

Example question

A very long solenoid with a cross section of 3×3 cm has an iron core ($\mu_r = 1000$) and 3000 turns per meter. If carries a current of 200 mA. Find the following:

(a) Its self-inductance per meter.

(b) The energy per meter stored in its field.

Example question

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(a) Its self-inductance per meter.

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(a) From the formula $L_{\text{material}} = \mu_0 \mu_r A N^2 l$, we can substitute

$$L_{\text{material}} = \left(4\pi 10^{-7}\right)(1000)\left(9 \times 10^{-4}\right)(3000)^2(1) \approx \underline{\underline{10.18}} \text{ Mm}^{-1}.$$

(b) From the formula $E = \frac{1}{2} L \cdot I^2$, we can substitute

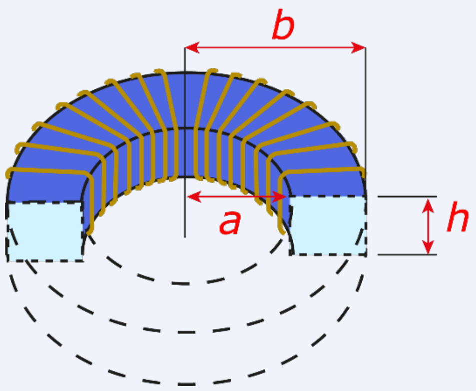
$$E = \frac{1}{2}(10.18)(0.2)^2 \approx \underline{\underline{0.20}} \text{ Jm}^{-1}.$$

Example question

Consider a toroidal solenoid with a rectangular cross-section as shown in the figure and the total number of turns N. It is filled with a material with relative permeability μ_r. The current passing through the circuit is I. Find:

(a) the fields B, H and M,

(b) the energy stored in the magnetic field.



Example question

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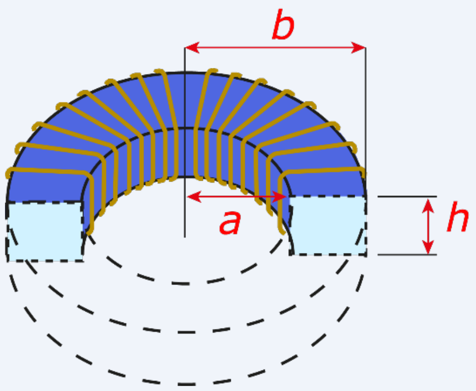

We can use Ampère's + choose a circular loop. It should be inside the solenoid. (Otherwise: if smaller than a, no current enclosed; if larger than b, currents cancel and again no currents enclosed). The number of currents enclosed are set by all the wires perpendicular to a, so they do not depend on h or r. We can write:

$$\oint \vec{H} \cdot d\vec{L} = H 2\pi r = NI \rightarrow \underline{\underline{H = \frac{NI}{2\pi r}}},$$ for $a < r < b$, then for H and M, we have:

$$\vec{B} = \mu_0 \mu_r \vec{H} \rightarrow \underline{\underline{B = \mu_0 \mu_r \frac{NI}{2\pi r}}}$$ and $$\vec{M} = \chi_m \vec{H} \rightarrow \underline{\underline{M = (\mu_r - 1)\frac{NI}{2\pi r}}},$$ so the energy is:

$$W = \frac{1}{2}\int_a^b r dr d\varphi h \frac{NI}{2\pi r}\mu_0 \mu_r \frac{NI}{2\pi r} = \frac{1}{2}\mu_0 \mu_r \left(\frac{NI}{2\pi}\right)^2 2\pi h \int_a^b \frac{1}{r} dr = \underline{\underline{\frac{\mu_0 \mu_r h (NI)^2}{4\pi} \ln \frac{b}{a}}}.$$

Summary

Electric current is a source of magnetic fields.

The magnetic dipole moment is current times area: $m = I \times (\text{area})$

The magnetisation is the magnetic dipole moment per unit volume: $\vec{M} = \dfrac{\sum \vec{m}}{V}$

When an external magnetic field is applied to a magnetic dipole moment, the torque acts to bring $\vec{m}$ parallel to $\vec{B}$.

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The magnetic flux density results from adding up the magnetic field strength and the magnetisation: $\vec{B} = \mu_0 \left(\vec{H} + \vec{M}\right)$

The magnetic field strength is defined as $\vec{H} = \dfrac{1}{\mu}\vec{B}$.

In LIH materials: $\vec{M} = \chi_m \vec{H}$

Examples of magnetic order in magnetic materials include: diamagnetic, paramagnetic and ferromagnetic.

Summary

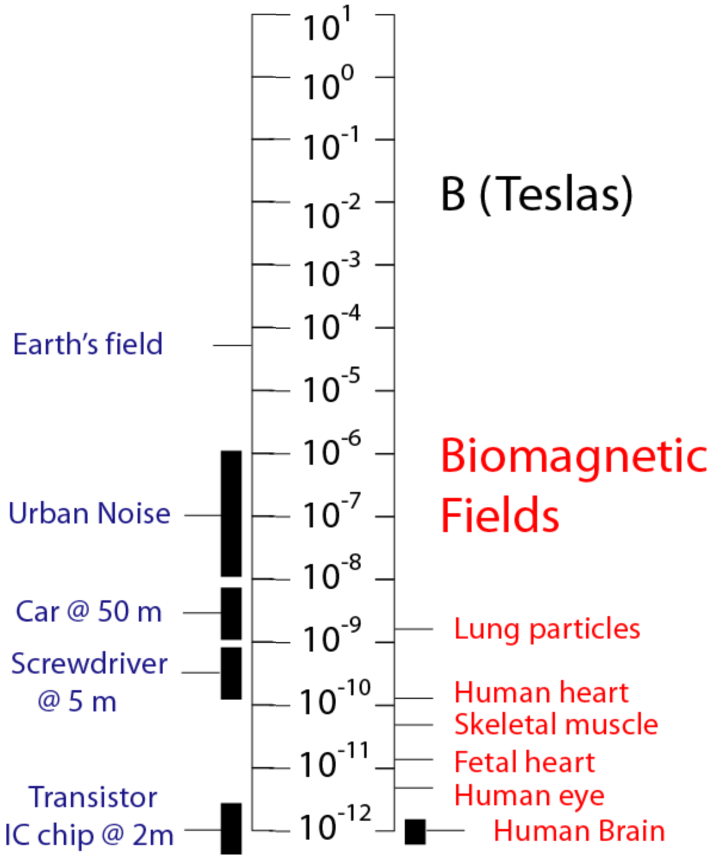
In diamagnetic materials, the magnetic dipole moment $\vec{m}$ is opposite to the direction of $\vec{B}$.

The energy stored in a magnetic material is $w = \dfrac{1}{2}\vec{H} \cdot \vec{B}$.

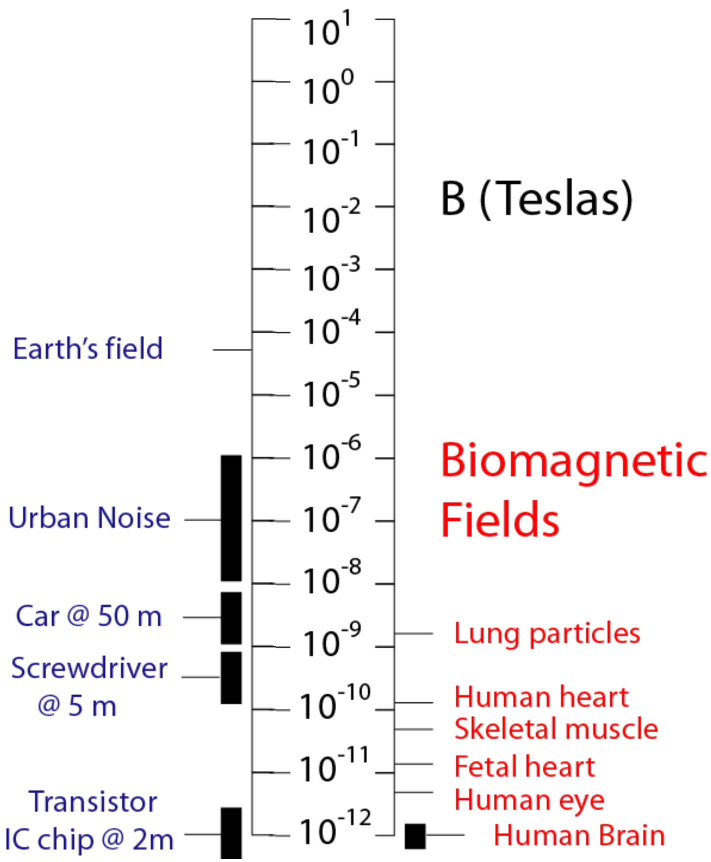
Units: B is in Tesla; M is in A/m; H is in A/m.

Let’s have a look at some B-fields!

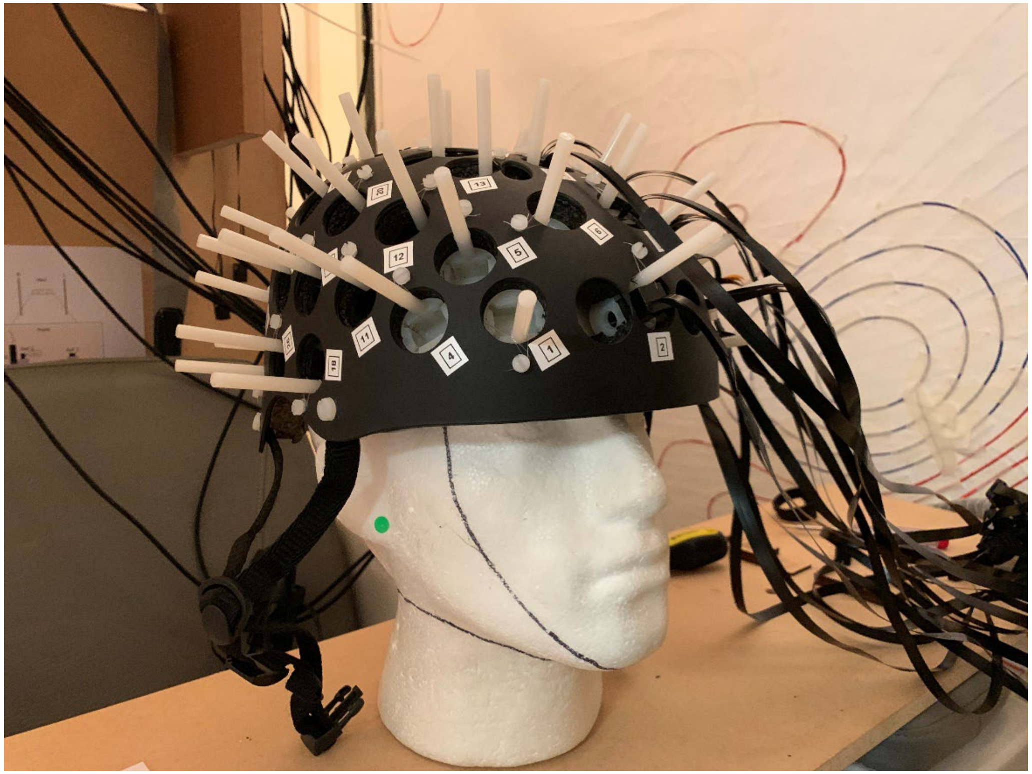
Reading (the magnetic fields of) minds!



Reading (the magnetic fields of) minds!



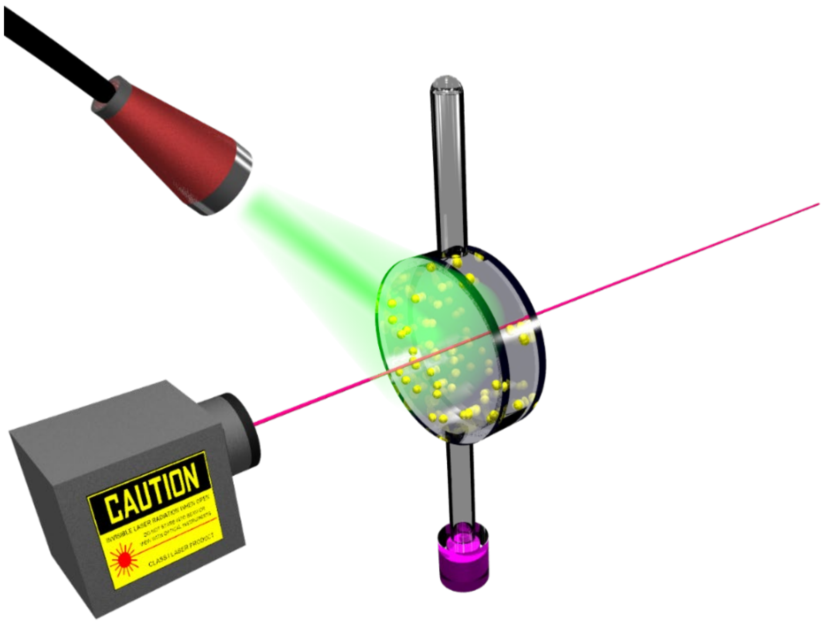
Helmet in the lab of Dr Elena Boto at the University of Nottingham



The helmed uses a quantum optical sensor, but it is very hot.

Enabling quantum optical technologies

We have invented (patented) a new coating for such sensors, that eliminates the need for external heating.



Controlling atomic vapour pressure is key in many quantum technologies that rely of Rb or Cs vapour:

- Magnetometers
- Atomic clocks
- High precision lasers
- Atomic traps (for large quantum objects)
- Quantum computer elements

All are examples of electromagnetism

In the next lecture, we will bring electric and magnetic fields together again