1 amu = 931.50 MeV/c^2	Particle rest masses:	Electron	5.486 x 10^{-4} u
		Proton	1.007276 u
		Neutron	1.008665 u

1. *Which of the following reactions is forbidden because it would involve the non-conservation of charge?*

   (a) $p + p \rightarrow p + p + \pi^0$
   (b) $p + p \rightarrow p + p + \pi^+ + \pi^-$
   (c) $p + p \rightarrow p + p + \pi^+ + \pi^- + \pi^+$

(a) charge(LHS) = +2 ; charge(RHS) = +2
(b) charge(LHS) = +2 ; charge(RHS) = +2
(c) charge(LHS) = +2 ; charge(RHS) = +3
This reaction is forbidden because of the non-conservation of charge.

2. *Which of the following reactions is forbidden because baryon number is not conserved?*

   (a) $p + p \rightarrow \pi^+ + p + n$
   (b) $p + p \rightarrow \pi^+ + p + n + n$
   (c) $p + p \rightarrow p + p + \pi^0 + \pi^0$

(a) B(LHS) = +2 ; B(RHS) = +2
(b) B(LHS) = +2 ; B(RHS) = +3
This reaction is forbidden because of the non-conservation of the baryon number.
(c) B(LHS) = +2 ; B(RHS) = +2

3. *Why is the decay $\Lambda^0 \rightarrow p + \pi^- + \pi^0$ forbidden?*

LHS: the rest mass is 1115.6 MeV/c^2. (from the Particle Data Sheet)
RHS: the rest mass is 938.3 + 139.6 + 135 = 1212.9 MeV/c^2
The final rest mass is larger than the intitial rest mass. The decay is forbidden because of the non-conservation of energy.

4. *The strong interaction conserves quark flavour. Analyse each of the following strong interactions in terms of their constituent quarks and confirm that quark flavour is conserved.*

   (a) $\pi^- + p \rightarrow \mathrm{K}^0 + \Lambda^0$
   (b) $\pi^+ + p \rightarrow \mathrm{K}^+ + \Sigma^+$

Expressing the interactions with the constituent quarks (from the Particle Data Sheet), we get:
(a) $d\bar{u} + uud \rightarrow d\bar{s} + uds$
Net: $udd \rightarrow udd$ Quark flavour is indeed conserved.
(b) $u\bar{d} + uud \rightarrow u\bar{s} + uus$

Net: *uuu* $\rightarrow$ *uuu* Quark flavour is indeed conserved.

5. *An electron-positron pair bound by their Coulomb attraction is called positronium. Show that when positronium decays from rest to two photons, the photons have equal energy. What is the wavelength of each photon?*

[2.42 x 10^{-12} m]

The initial momentum equals 0. The final momentum therefore equals 0.
Therefore, the photons must have equal and opposite momentum.

For a photon, $p = \dfrac{E}{c}$

Hence the photons must have equal energies.

The energy of a photon is given by: $E = h\,f = \dfrac{h\,c}{\lambda}$

$$\Leftrightarrow \lambda = \frac{h\quad c}{E} = \frac{6.62\;10^{-34} \times 3\quad 10^{8}}{0.51 \times 1.6 \times 10^{-13}} = 2.42\;10^{-12}\;m$$

6. *Find:*
   (a) *An approximate expression for the mass of a nucleus of mass number A.*
   (b) *An expression for the volume of the nucleus in terms of A.*
   (c) *An estimate of the nuclear density (in kg m^{-3}).*

[3 x 10^{17} Kgm^{-3}]

(a) From the Particle Data Sheet, or any textbook, one can find that the mass of the proton and the mass of the neutron are very close. We can therefore assume: $m_p \approx m_n = m$

Hence: mass of nucleus $\approx$ *A* amu (atomic mass units)
Id est: mass of nucleus $\approx$ *A m* kg, where $m = 1.66\;10^{-27}$ kg is the nucleon mass.

(b) The nucleus shape is spherical (from common sense <u>and</u> from the lecture notes). Its volume *V* is:

$$V = \frac{4}{3}\pi\, R_{nuc}^{3}$$

We saw (Problem Sheet #1 and lecture notes from a few weeks ago) that: $R_{nuc} = 1.1\,A^{\frac{1}{3}}$ (in fermis)

Therefore: $V = \dfrac{4}{3}\pi\left(1.1\,10^{-15}\right)^{3} \times A$ (in m^3)

(c) The nuclear density *d* is the ratio of the mass and volume:

$$d = \frac{mass}{V} = \frac{A\,m}{\frac{4}{3}\pi\left(1.1\,10^{-15}\right)^{3}\;A} = \frac{m}{\frac{4\pi}{3}(1.1)^{3} \times 10^{-45}} \approx 3\,10^{17}\,\mathrm{kg\;m^{-3}}$$

7. *The compressed core of a star formed in the wake of a supernova explosion can consist of pure nuclear material (neutrons) and is called a pulsar or neutron star.*

Use the result obtained in 6(c) to calculate the mass of a sugar lump sized piece of neutron star.

A sugar lump is a cube with sides of 1 cm. Its volume V is therefore V = 0.01 x 0.01 x 0.01 m^3

The mass of the lump-sized piece of neutron start will have the density $d = 3.\ 10^{17}$ kg m^{-3} found in the previous question.

The mass itself will therefore be: $m = 10^{-6}$ x $3\ 10^{17} = 3\ 10^{11}$ kg

8. $^{60}_{28}Ni$ *has an atomic mass of 59.930789u.*

(a) What is its nuclear mass?

The mass of the nucleus equals the mass of the atom, minus the mass of the electrons (assuming we can neglect the binding energies of the electrons)

The nuclear mass is therefore: 59.930789 u – 28 x 5.486 10^{-4} u = 59.91543 u

(b) What is the binding energy per nucleon?

[59.9154u; 8.78MeV]

$$mass_{nucleus} = mass_{protons} + mass_{neutrons} - \frac{\text{binding energy}}{c^2}$$

And the binding energy BE reads:

$$BE = \left(mass_{protons} + mass_{neutrons} - mass_{nucleus}\right)c^2$$

$$\Rightarrow BE = (28 \times 1.007276 + 32 \times 1.008665 - 59.91543) \times 931.50 \text{ MeV}$$

$$\Leftrightarrow BE = 526.8 \text{ MeV}$$

The binding energy per nucleon is $\frac{526.8}{60} = 8.78$ MeV

*9. * Which of the pair of nuclei,* $^{41}_{20}Ca$ *and* $^{41}_{19}K$ *is unstable with respect to the other (atomic masses 40.962278u and 40.961825u) ? What decay mode/modes are possible and how much energy is released in each allowed mode?*

Since $^{41}_{20}Ca$ is heavier than $^{41}_{19}K$, it will be unstable with respect to $^{41}_{19}K$.

The possible decay modes are β^+ decay and K-capture. Let us put each element on its side of the reaction:

$$^{41}_{20}K \rightarrow {}^{41}_{19}K + \ldots$$

40.962278 u → 40.961825 u

The mass difference between the left-hand side and the right-hand side is:

0.000453 u = 0.42 MeV/c^2 < 2 m_e, where m_e is the mass of the electron.

β^+ decay is therefore not possible, and this reaction can only occur by K-capture, releasing an energy of 0.42 MeV.

(as an aside, one can note that the time for this reaction is $t_{1/2} = 0.1$ million years)

*10. * (a) Show that* $^{8}_{4}Be$ *can decay into two* α *particles with an energy release of 0.1 MeV, but that* $^{12}_{6}C$ *cannot decay into three* α *particles.*

(b) Find the energy released in the following reaction (including the energy of the photon):

$$ {}^{2}_{1}H + {}^{4}_{2}He \rightarrow {}^{6}_{3}Li + \gamma \ . $$

Binding energies: ${}^{8}_{4}Be$ *= 56.50MeV;* ${}^{12}_{6}C$ *= 92.16MeV;* ${}^{2}_{1}H$ *= 2.22MeV;* ${}^{4}_{2}He$ *= 28.30MeV and* ${}^{6}_{3}Li$ *= 31.99MeV.*

[1.47MeV]

(a) The energy released Q reads (cf. lecture notes):

$Q = (mass_{nucleus} - mass_{products}) \times c^2 \Leftrightarrow Q = BE_{products} - BE_{nucleus}$

For ${}^{8}_{4}Be \rightarrow 2\alpha$, we have: Q = 2 x 28.30 – 56.50 MeV = 0.1 MeV

For ${}^{12}_{6}C \rightarrow 3\alpha$, we have: Q = 3 x 28.30 – 92.16 MeV = –7.26 MeV

The energy Q of the last reaction is negative; it is therefore energetically impossible.

(b) The energy released in the reaction ${}^{2}_{1}H + {}^{4}_{2}He \rightarrow {}^{6}_{3}Li + \gamma$ equals:

$$Q = BE\left({}^{6}_{3}Li\right) - BE\left({}^{2}_{1}H\right) - BE\left({}^{4}_{2}He\right)$$

$\Rightarrow Q = 31.99 - 2.22 - 28.30 = 1.47$ MeV

11. * *Two Th isotopes,* ${}^{224}_{90}Th$ *and* ${}^{230}_{90}Th$*, decay by emitting* α *particles with energies of 7.31 MeV and 4.77MeV respectively. For each isotope calculate the range* r_C *at which the* α *particle leaves the Coulomb potential barrier. Estimate the ratio of the half lives for these two isotopes.*

(this last question is optional, but a good practice of what we saw in the lectures)

By definition (cf. lecture notes): $r_C = \dfrac{z\, Z_d\, e^2}{4\pi\, \varepsilon_0\, Q} = \dfrac{2\, Z_d\, e^2}{4\pi\, \varepsilon_0\, Q}$ (as $z = 2$ for the α particle)

For ${}^{224}_{90}Th$, we get: $r_C = \dfrac{2 \times 88 \times \left(1.6\, 10^{-19}\right)^2}{4\pi \times 8.85\, 10^{-12} \times 7.31\, 10^{6} \times 1.6\, 10^{-19}}$ (in metres)

$\Rightarrow r_C = 34.6$ fm

Similarly, for ${}^{230}_{90}Th$, we get: r_C = 53.1 fm

And the half-lives are given by the now (in)famous equation: $t_{1/2} = \ln 2\, \tau_0\, e^{G}$

(cf. lecture notes and the relevant demonstration during the lecture)

The ratio of the two half-lives reads:

$$\frac{t_{1/2}\left({}^{234}_{90}Th\right)}{t_{1/2}\left({}^{230}_{90}Th\right)} = \frac{e^{G_1}}{e^{G_2}} = e^{G_1 - G_2}$$

where G_1 is the Gamow factor for ${}^{224}_{90}Th$ and G_2 the factor for ${}^{230}_{90}Th$.

For $r_C \gg r_{nucleus}$ (cf. lecture notes):

$$G \cong \pi\, r_C \left(\frac{2m\,Q}{\hbar^2}\right)^{\frac{1}{2}} \qquad \text{where } m = m_\alpha = 3730 \text{ MeV/c2}$$

The first factor equals:

$$G_1 = \frac{\pi \times 34.6\,10^{-15}}{1.05\,10^{-34}} \left(\frac{2\times 3730}{\left(3\,10^{8}\right)^2} \times 7.31 \times \left(1.6\,10^{-13}\right)^2\right)^{\frac{1}{2}} = 128.9$$

And $G_2 = 159.8$

The ratio of the half-lives therefore becomes: $e^{(128.9\,-\,159.8)} = e^{-30.9} = 3.8\ 10^{-14}$

More accurate calculations (including the $\mathcal{G}$ term) and experiments show that the half-life of ${}^{224}_{90}Th$ is in fact around 3 10^{12} times longer than that of ${}^{234}_{90}Th$.