1) a) $\psi(x) = \begin{cases} Cx(2-x) & , 0 \le x \le 2 \\ 0 & , \text{otherwise} \end{cases}$

$$\int_{-\infty}^{\infty} |\psi|^2 dx = 1$$

A = const

$$\int_0^2 |Cx(2-x)|^2 + \int_{-\infty}^0 \cancel{|0|^2} \to 0$$

$$+ \int_2^{+\infty} \cancel{|0|^2} \, dx = 1$$

$$\int_0^2 \cancel{\sqrt{C^2 x^2 (2-x)^2}}^{\,2} \, dx + A = 1$$

$$C^2 \int_0^2 4x^2 - 4x^3 + x^4 + A = 1$$

Assume $A = 0$

$$c^2\left[\frac{4x^3}{3} - x^4 + \frac{x^5}{5}\right]_0^2 = 1$$

$10 + 1/3$ $\quad$ $6 + 2/5$

$$c^2\left(\cancel{\frac{32}{3}} - \cancel{16} + \cancel{\frac{32}{5}}\right) = 1$$

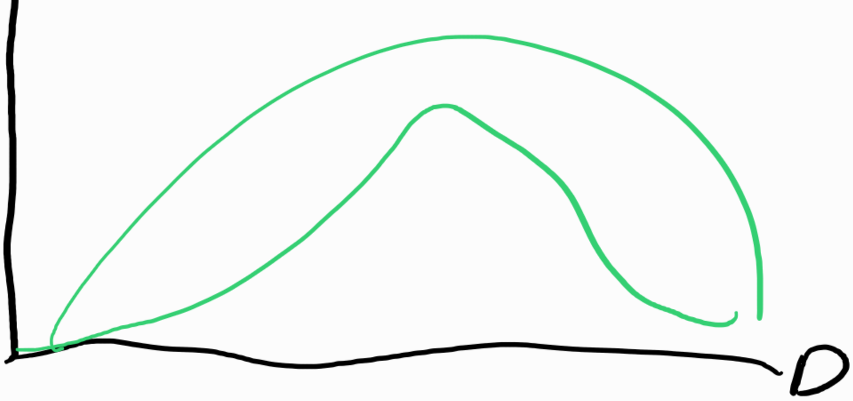
$$c^2 = \frac{1}{1/3 + 2/5}$$

$\cancel{5/15} \quad \cancel{6/15} \quad 12/15$

$$c = \sqrt{\frac{15}{12}} = \sqrt{\frac{5}{4}}$$

↓ saw one

b)



c) $$\int_{0.55}^{1.05} P(x,t)\,dx = \int_{0.55}^{1.05} \left|\sqrt{\tfrac{5}{4}}\,x\,(2-x)\right|^2$$

$$= \frac{5}{4}\int_{0.55}^{1.05} x^4 - 2x^3 + 4x^2 \, dx$$

$$= 0.6005$$

d) $$\langle x \rangle = \int^{\infty} x\,P(x,t)\,dx$$

$-\infty$

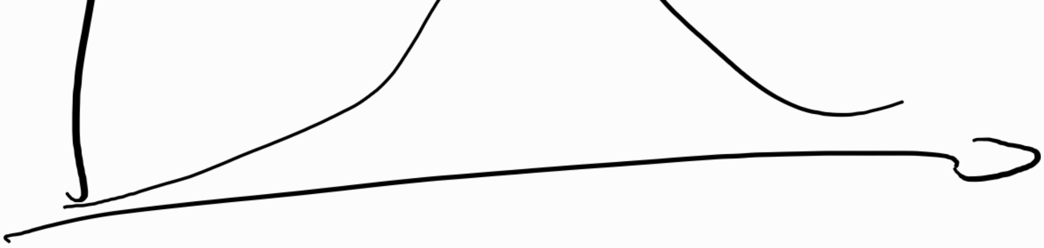
$$= \int_0^2 0.6 \, x \, dx$$

$$= \left[\frac{0.\cancel{6}^{3} x^2}{\cancel{2}} \right]_0^2$$

$$\langle x \rangle = 0.6$$

2) $\psi(x) = A e^{-\frac{|x|}{a}}$

a)



b) $\int_{-a}^{a} |\psi(x)|^2 \, dx$

$= \int_{-a}^{a} A^2 e^{-2|x|/a} \, dx$

$= A^2 \left(e^{-2\cancel{a/a}} - e^{-2\cancel{a/a}} \right) dx$

$= 0$

3) $P(x,t) = \psi^*(x,t)\,\psi(x,t)$

particle cannot have negative probability of not being somewhere

$$P(x,t) = \underbrace{|\psi(t)|^2}_{\geq 0}$$

$$P(x,t) = |\Psi(t)|^2$$

$$= \Psi^* \Psi$$

4) $$\frac{i\hbar}{\phi(t)} \frac{d\phi(t)}{dt} = E$$

$$\int \frac{i\hbar}{\phi(t)}\, d\phi(t) = \int E\, dt$$

$$i\hbar \ln(\phi t) = Et + C,$$

c = const

$$\ln(\phi t) = \frac{Et + c}{i\hbar}$$

$$\phi(t) = e^{Et + c / i\hbar}$$

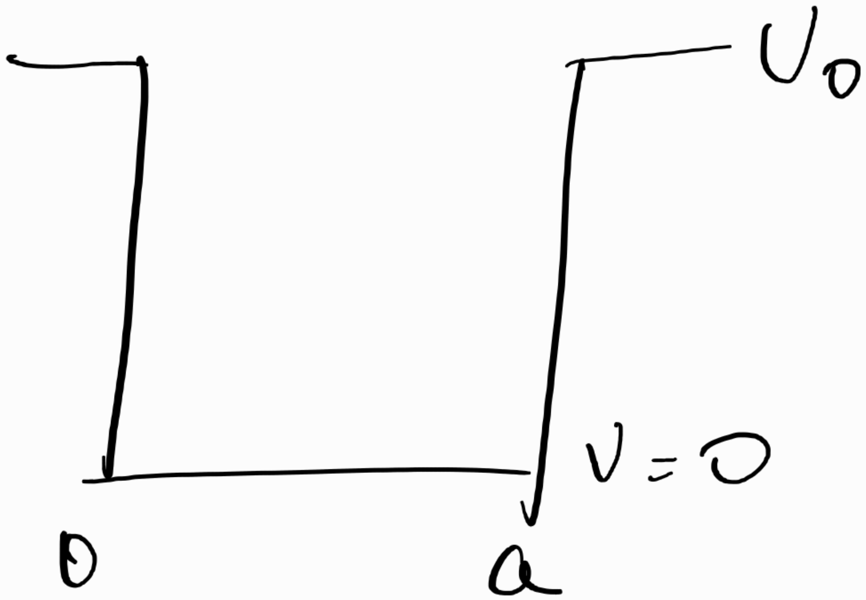
5. ? - come back

6. a) $\psi(x) = A\psi_1 + B\psi_2$

$$E_1 = E_2$$

b) ?

7.



a) $\psi = A\sin(kx) + B\cos(kx)$

b) $x = 0 \Rightarrow 0 = A\sin(0) + B\cos(0)$

$$\Rightarrow B = 0$$

$x = a \Rightarrow 0 = a\sin(ka) + 0$

$\rightarrow \sin(ka) = 0$

$\rightarrow ka = n\pi$

$k_n = \dfrac{n\pi}{a}$

$\psi_n = A_n \sin(k_n x)$

$= A_n \sin\left(\dfrac{n\pi x}{a}\right)$

$\Rightarrow k_n = \dfrac{n\pi}{a} = \dfrac{\sqrt{2mE_n}}{\hbar}$

$\therefore E_n = \dfrac{\hbar^2 \pi^2 n^2}{2ma^2}$

$x = a \qquad 1 = A\sin + 1$

$$A = \frac{2}{\sin(k_n a)}$$

$$= \frac{2}{\sin(ka)}$$

Z

struggling to solve this $\sqrt{\frac{2}{a}}$

normalize $|\psi|^2$

$$\int_{-\infty}^{\infty} |\psi|^2 dx =$$

$$\int^{\infty} |A\sin(kx) + B\cos(kx)|^2 dx$$

$$\int_{-\infty}$$

$$1 = A^2 \int_0^a \sin(kx)\, dx$$

$$1 = A\left(\cos(ka) - \cos(k0)\right)$$

$$0.$$

$$1 = A\left(\cos(ka) - 1\right)$$

$$A = \frac{1}{\cos(ka) - 1}$$

?
nope
retry