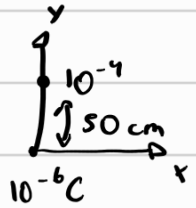
1) a) Y ✓ b) N ✓ c) N ✓ d) Y ✓ e) Y ✓
f) N ✓ g) Y ✓ h) Y ✓ i) Y ✓

2)



$$F = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{R^2}\hat{R}$$

$R^2 = 2500$ cm $= 25$ m

$$F = \frac{1}{4\pi\varepsilon_0}\frac{10^{-4}\,10^{-6}}{25} \approx 3.6 \text{ N}$$

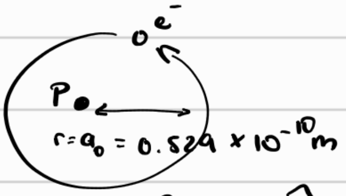
↑ unit for F

$1\text{N} = 1\,\text{kg}\cdot\text{m}\cdot\text{s}^{-2}$

$F_{Q_1} = +3.6$ N

$F_{Q_2} = -3.6$ N

3)



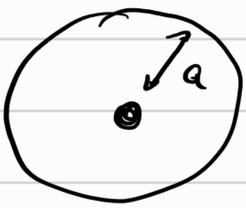
Coulombs F vs Grav F (justif ign)

$$\vec{F}_c = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{R^2}\hat{R} = 8.22 \times 10^{-18} \text{ N}$$

$$\vec{F}_G = G\frac{\overset{\text{kg}}{M}\,\overset{\text{kg}}{m}}{R^2} = 3.63 \times 10^{-47} \text{ N}$$

G: $\text{N m}^2\,\text{kg}^{-2}$; R^2: m^2

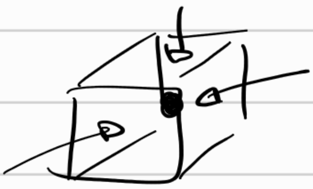
4)



Gauss's $\oint_\sigma \vec{E}\,d\vec{S} = \frac{Q}{\varepsilon_0}$

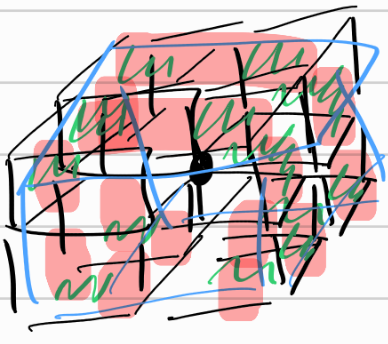
Flux in = Flux out ∴ int not affected outside charge

5)



$$\frac{\text{Tot flux}}{n^\circ \text{ faces}} = \text{Flux per face} = \frac{Q}{6\varepsilon_0}$$ ✓

Tot flux $= \frac{Q}{\varepsilon_0}$



$\frac{24 \text{ faces}}{3} = 8$ ✓ ∴ flux per face $= \frac{Q}{8\varepsilon_0}$

✓

6) $\phi(r) = xyz$

$\vec{E}(r) = -\phi(r) = xyz$

$$\nabla \cdot \vec{E} = \nabla \cdot (-\phi(r))$$
$$= \left(\frac{-1}{x} yz, \frac{-1}{y} xz, \frac{-1}{z} xy\right)$$

$$\nabla \times \vec{E} = \nabla \times (-\phi(r))$$
$$= 0 \quad \therefore \text{no } B$$

$$\nabla^2 \phi = \frac{1}{xyz}\left[\, 0 \,\right] = 0 \quad : \text{Laplace's}$$

$\therefore$ Poisson's $\nabla^2 \phi = \frac{\rho}{\varepsilon_0} = 0$ ~~consistent~~

$\rho = 0$ in source free vacuum

7) a) $-3y\hat{i} - 3x\hat{j}$

div $E = 0$

$$\text{curl } E = \frac{1}{xyz} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -3y & -3x & 0 \end{vmatrix}$$

$$= -3\hat{k} + 3\hat{k} = 0$$

~~Electrostatic potential = 0 ∴ no electrostatic field~~

curl = 0 ∴ could be electrostatic field

b) $-4x\hat{i} - 4y\hat{j} - 4z\hat{k}$

$\text{div } E = -12$

$$\text{curl } E = \frac{1}{xyz}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -4x & -4y & -4z \end{vmatrix}$$

$= 0$

curl E ∴ could be E field ✓

~~$\vec{\rho} = \frac{\varepsilon_0}{-12}$ inconsistent with 0 curl~~

c) $(x^2 + y^2)\hat{i} + 2xy\hat{j} + 2yz\hat{k}$

$\text{div } E = (2x + \cancel{2})\cancel{\hat{i}} + 2x\cancel{\hat{j}} + 2y\cancel{\hat{k}} \rightarrow 4x + 2y$

$$\text{curl } E = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 2x+2 & 2x & 2y \end{vmatrix}$$

$= 2\hat{i} + 2\hat{k}$

$\text{curl } E \neq 0 \Rightarrow$ not an electrostatic field

$\therefore \vec{B} \neq 0$

8) a) $\text{div } E = 0 \quad \therefore \rho = 0$ ✓

~~$\phi = 0$~~ more complicated

$E = -3y\hat{i} - 3x\hat{j}$ $E = -\nabla\phi$ ∴ integrate component by component $x \to$

x component $\to \int 3y \, dx = 3xy + f_1(y,z)$

$y \to \int 3x = 3xy + f_2(x,z)$

$z \to$ cnst $+ f_3(x,y)$

$\therefore \phi = 3xy + c$

b) $\text{div}\, E = -12$ $\quad \therefore \rho = -12\varepsilon_0$ ✓

$\cancel{\phi = 12}$ more complicated

$\vec{E} = -4x\hat{i} - 4y\hat{j} - 4z\hat{k}$

$x \to 2x^2 + f_1(y,z)$

$y \to 2y^2 + f_2(x,z)$ $\quad \therefore \phi = 2(x^2+y^2+z^2) + c$

$z \to 2z^2 + f_3(x,y)$

always there

9) $\nabla \times \vec{B} = \mu_0 \vec{J} + \cancel{\varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}}$ ignore

$B_x = 5y$

$B_y = -5x$ $\quad \therefore \vec{B} = 5y\hat{i} - 5x\hat{j}$

$B_z = 0$

$$\nabla \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 5y & -5x & 0 \end{vmatrix}$$

$$= -5\hat{k} - 5\hat{k} = -10\hat{k}$$

$\therefore \vec{J} = \frac{-10}{\mu_0}\hat{k}$ ✓ respect the component (still a vector field)

10) a) $2x\hat{i} + 2y\hat{j} + 2z\hat{k}$

$\text{div } B = 0$ ✓

Units

B	T (Tesla)
E	Vm^{-1}
ϕ	V
ρ	Cm^{-3}
J	Am^{-2}