Picture taken from https://www.thoughtco.com/helium-facts-606542

Heat in ideal gas

Joule-Kelvin effect

Back to ideal gas

At constant volume $C_V = \left(\frac{\text{đ}Q}{\partial T}\right)_V = \left(\frac{\partial U}{\partial T}\right)_V$

At constant pressure, it is $C_p = \left(\frac{\text{đ}Q}{\partial T}\right)_p = \left(\frac{\partial U}{\partial T}\right)_p + p\left(\frac{\partial V}{\partial T}\right)_p$

for a ideal gas: $U{=}U(T)$ only (*remind yourself why*), which means

$$\left(\frac{\partial U}{\partial T}\right)_p = \left(\frac{\partial U}{\partial T}\right)_V$$

internal energy only dep on temp for an ideal gas

$$C_p = \left(\frac{\text{đ}Q}{\partial T}\right)_p = C_V + p\left(\frac{\partial V}{\partial T}\right)_p$$

For a ideal gas $pV = nRT \leftrightarrow V = \frac{nRT}{p}, \left(\frac{\partial V}{\partial T}\right)_p = \frac{nR}{p}$:

$$C_p = C_V + nR \qquad \boxed{C_p > C_V}$$

Back to ideal gas

$$đQ = \mathrm{d}U - đW = \mathrm{d}U + p\mathrm{d}V$$

$$C_V = \left(\frac{đQ}{dT}\right)_V = \left(\frac{\partial U}{\partial T}\right)_V = \frac{dU}{dT}$$

$$\Rightarrow \mathrm{d}U = C_V \mathrm{d}T = đQ - p\mathrm{d}V$$

$$đQ = C_V \mathrm{d}T + \boxed{p\mathrm{d}V}$$

Ideal gas EoS: $pV = nRT$

$$p\mathrm{d}V + V\mathrm{d}p = nR\mathrm{d}T$$

$$\rightarrow p\mathrm{d}V = nR\mathrm{d}T - V\mathrm{d}p$$

$$đQ = (C_V + nR)\mathrm{d}T - V\mathrm{d}p$$

$$đQ = C_p \mathrm{d}T - V\mathrm{d}p$$

Adiabatics of ideal gas

$$đQ = C_V \mathrm{d}T + p\mathrm{d}V$$
$$đQ = C_p \mathrm{d}T - V\mathrm{d}p$$

for an adiabatic reversible process $đQ = 0$

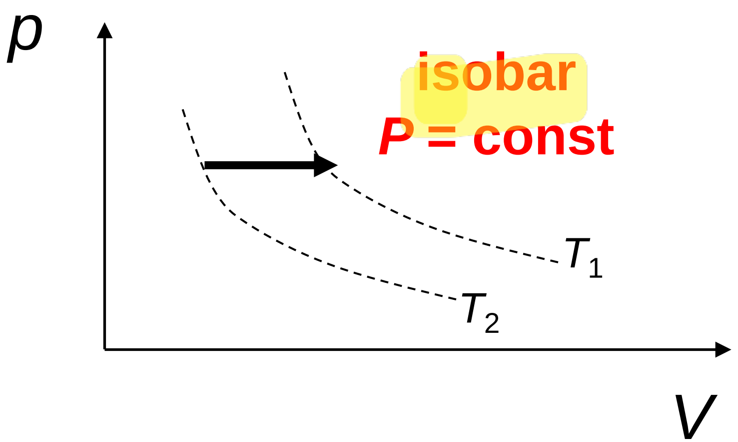
$$p\mathrm{d}V = -C_V \mathrm{d}T$$
$$V\mathrm{d}p = C_p \mathrm{d}T$$

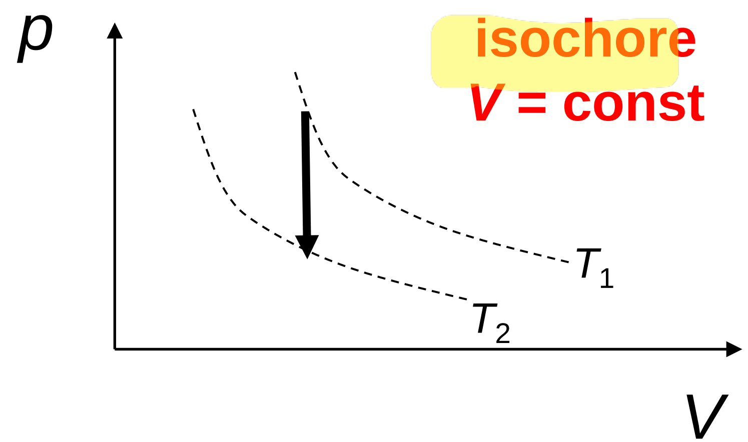
$$\frac{\mathrm{d}p}{p} = -\frac{C_p}{C_V}\frac{\mathrm{d}V}{V}$$
$$\ln p = -\gamma \ln V + const$$

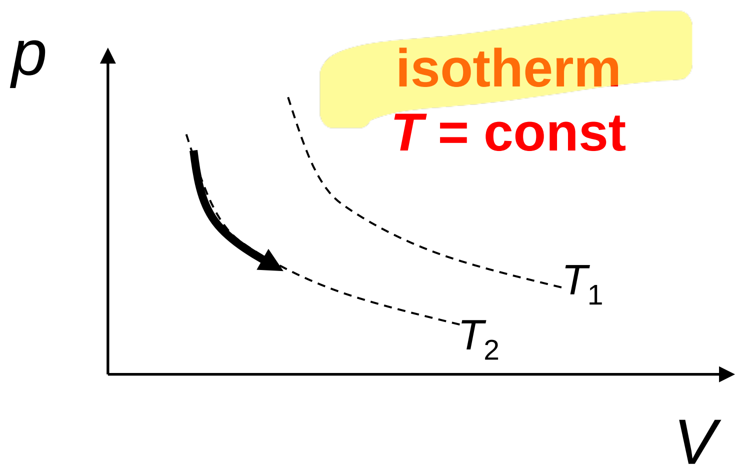
$\gamma \equiv \frac{C_p}{C_V}$ or $pV^{\gamma} = const$ γ = adiabatic index

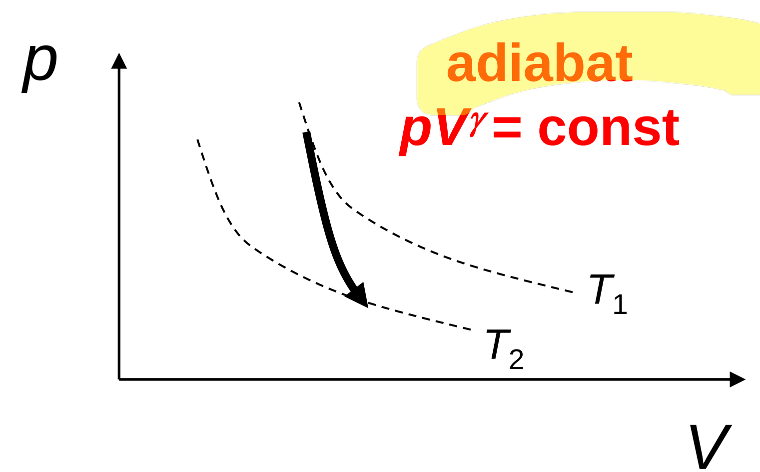
This is valid for an adiabatic, reversible process in an ideal gas.

(Reversible) ways to change the state of a gas

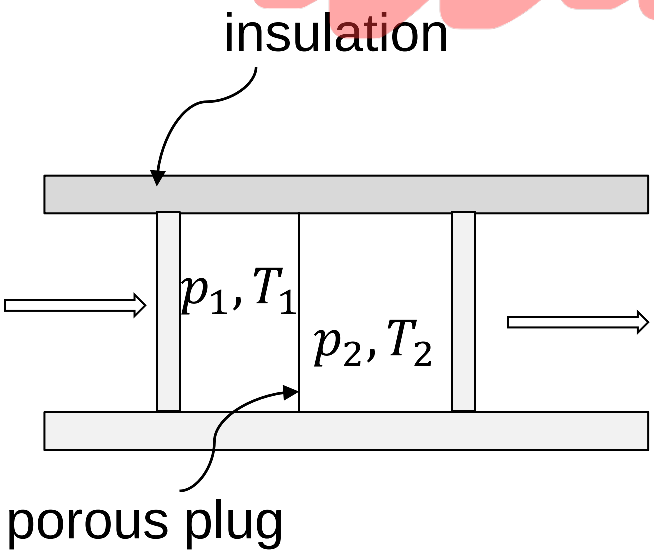








Joule-Kelvin Effect



Push gas through plug very slowly from **1** to **2**

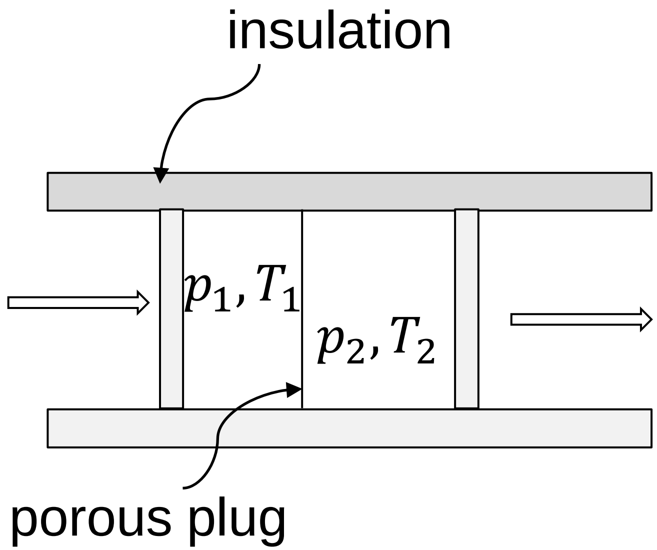
Adiabatic process so

work done on **1** is $\Delta W_1 = -p_1 \int_{V_1}^{0} \mathrm{d}V = p_1 V_1$

work done on **2** is $\Delta W_2 = -p_2 \int_{0}^{V_2} \mathrm{d}V = -p_2 V_2$

Total work done on gas is $\Delta W = p_1 V_1 - p_2 V_2$

Joule-Kelvin Effect



Adiabatic process, so $đQ = 0$; from first law:

$$\Delta U = \Delta W$$

$$U_2 - U_1 = p_1V_1 - p_2V_2$$

$$U_1 + p_1V_1 = U_2 + p_2V_2$$

$$H_1 = H_2$$

isenthalpic process (Enthalpy is conserved)

↑ cnst