PH20014: Electromagnetism 1

Problem Sheet 1

Standard problems

1. The electric field $\vec{E}$ associated with an electromagnetic plane wave propagating in a lossy medium is of the form

$$\vec{E} = 50\hat{y}e^{-\alpha x}e^{i(\beta x-\omega t)} \text{ Vm}^{-1},$$

in our usual notation. If $\varepsilon_r = 3.0$, $\mu_r = 7.0$, $\sigma = 6.0$ ohms^{-1}m^{-1} at the frequency 6.0 MHz, show that the medium can be considered as a good conductor and hence find α and β. Then, calculate

(a) the phase velocity,

(b) the wavelength,

(c) the amplitude of the electric field $\vec{E}$ at $x = 0.1$ m,

(d) the amplitude of the $\vec{H}$ field at $x = 0.1$ m.

Compare the results of (a) and (b) with those that would be obtained if the conductivity were zero.

2. Deduce the conductivity of a medium of relative permittivity equal to five if the magnitude of the conduction and displacement current densities in it are equal when a monochromatic plane wave of frequency 10^8 Hz is propagated. What is the attenuation per metre of the amplitude of a plane wave of frequency 10^6 Hz when propagated in such a medium? The relative permeability is unity.

3. (a) Show that the time-average of the energy density in a monochromatic (linearly-polarized) plane wave moving in an isotropic non-conducting medium is distributed equally between the electric and magnetic fields.

(b) In comparison, show that in a conducting medium, the time-average of energy density in the magnetic field is greater than in the electric field.

[Hint: When comparing the energy densities in the magnetic and electric fields, use the internal impedance of the medium Z which conveniently expresses the ratio of the magnitudes of the two fields.]

4. *[2016 Exam question]* Consider a conducting medium with conductivity σ, with no free charges and obeying Ohm's law.

(a) Using the appropriate Maxwell's equations, relations between $\vec{D}$ and $\vec{E}$, $\vec{B}$ and $\vec{H}$ in a medium and the identity $\nabla\times\left[\nabla\times\vec{F}(\vec{r})\right] = \nabla\left(\nabla\cdot\vec{F}(\vec{r})\right) - \nabla^2\vec{F}(\vec{r})$, derive the modified wave equation, (4)

$$\nabla^2\vec{E} = \mu_r\mu_0\varepsilon_r\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2} + \mu_r\mu_0\sigma\frac{\partial\vec{E}}{\partial t}. \qquad (1)$$

(b) Show that $\vec{E} = \vec{E}_0e^{-\alpha x}e^{i(\omega t-\beta x)}$ is a solution of the above equation with $\dfrac{2\alpha\beta}{\beta^2-\alpha^2} = \dfrac{\sigma}{\omega\varepsilon_0\varepsilon_r}$.(3)

(c) For an electromagnetic wave with frequency 25 kHz, the loss tangent of sea water ($\varepsilon_r = 80$, $\mu_r = 1$) is approximately 36,000. What is the skin depth for this wave? (2)

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Advanced problems

5. From the first half of the unit, you know that the energy flow in electromagnetism is described by Poynting vector $\vec{S} = \vec{E} \times \vec{H}$. Show that for a conducting LIH material, the following (Poynting's) theorem is true:

$$\oiint_S \left(\vec{E} \times \vec{H}\right) \cdot d\vec{a} = -\frac{\partial}{\partial t} \iiint_V \left(\frac{1}{2}\varepsilon E^2 + \frac{1}{2}\mu H^2\right) dV - \iiint_V \sigma E^2 dv .$$

Explain the physical meaning of all the terms in the theorem.