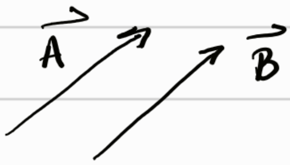
For a vector to b def we need to df a vctor space.
A vctor space is $\begin{cases}\text{homogeneous : no prefered points} \\ \text{istropic : no prefered direct}^{\circ}\end{cases}$



Iff $\vec{A}$ has the same length as $\vec{B}$

$|\vec{A}| = |\vec{B}|$

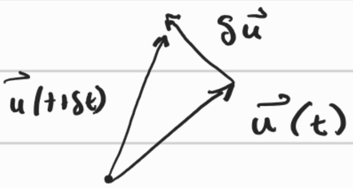
& $\vec{A}$ is in the same direct° as $\vec{B}$

$\vec{A} \parallel \vec{B} \Rightarrow \hat{A} = \hat{B}$

Then $\Rightarrow \vec{A} = \vec{B}$

^ → unit vector

Ex Show that for a vector of fixed magnitude.
The vector must always be $\perp$ to it's 1^{st} derivative.



δ : small case greek delta Δ

d : formal letter 'd'

∂ : 'partial d'

$\vec{u}(t) + \delta\vec{u} = \vec{u}(t+\delta t)$

$$\frac{\delta\vec{u}}{\delta t} = \frac{\vec{u}(t+\delta t) - \vec{u}(t)}{\delta t}$$

$$\frac{d\vec{u}}{dt} = \lim_{\delta t \to 0}\left[\frac{\vec{u}(t+\delta t) - \vec{u}(t)}{\delta t}\right]$$ * Vector Differentiation

if t is time : Newtonian notat°

$$\frac{d\vec{u}(t)}{dt} = \vec{u}'(t) = \dot{\vec{u}}(t)$$

$\vec{A} \cdot \vec{B} = |\vec{B}||\vec{B}|\cos(\theta)$

$\vec{u}(t) \cdot \vec{u}(t) = |\vec{u}(t)||\vec{u}(t)|\cos(\theta)$

therefore

But $\theta = 0$ ← same vctor $\therefore \cos(\theta) = 1$

So $\vec{u}(t) \cdot \vec{u}(t) = |\vec{u}(t)|\,|\vec{u}(t)|$ which must be cnst

$\frac{d}{dt}\left[\vec{u}(t) \cdot \vec{u}(t)\right] = 0$

Now use the Product Rule $\vec{u}(t) \cdot \vec{u}'(t) = 0$

$2(\vec{u}(t) \cdot \vec{u}'(t)) = 0 \quad \therefore \vec{u}(t) \cdot \frac{d\vec{u}}{dt} = 0$

$\vec{u}(t) \cdot \vec{u}'(t) = |\vec{u}(t)||\vec{u}'(t)|\cos(\theta) = 0$

$u'(t) \cdot u(t) = |u'(t)||u(t)|\cos(\theta) = 0$

assuming lengths $\neq 0$ $\Rightarrow$ $\cos(\theta) = 0$

$\Rightarrow$ $\theta = \pi/2$

so $\vec{u}(t) \perp u'(t)$ AS REQUIRED