(A4) integrals 1 & 2

PH30030: Quantum Mechanics **Problems Sheet 6**

This problems sheet covers section 6 of the course, on time-dependent perturbation theory. The first two questions involve working through the derivations in the lectures – this is to make sure that you understand exactly where the key results come from.

1. Work carefully through the derivation in section 6.1 of the lecture notes to show that $\frac{dc_m}{dt} = \frac{1}{i\hbar}\sum_n c_n(t)\, H'_{mn}(t)\, \exp(i\,\omega_{mn}\,t)$, where $H'_{mn}(t) = \iiint \phi^*_m(\underline{r})\,\hat{H}'(\underline{r},t)\,\phi_n(\underline{r})\, d^3r$ and $\omega_{mn} = \frac{E_m - E_n}{\hbar}$.

2. Work carefully through the derivation in section 6.2 of the lecture notes to show that, for a periodic perturbation, $c_f(t) = -\frac{H'_{fi}}{2\hbar}\left[\frac{\exp\left(i(-\omega+\omega_{fi})t\right)-1}{-\omega+\omega_{fi}} + \frac{\exp\left(i(\omega+\omega_{fi})t\right)-1}{\omega+\omega_{fi}}\right]$, where $H'_{fi} = \iiint \phi^*_f(\underline{r})\,\hat{H}'(\underline{r})\,\phi_i(\underline{r})\, d^3r$. If only the first term in the bracket is considered, show that $\left|c_f(t)\right|^2 = \frac{\left|H'_{fi}\right|^2}{4\hbar^2}\left(\frac{\sin\left[\left(\omega_{fi}-\omega\right)t/2\right]}{\left(\omega_{fi}-\omega\right)/2}\right)^2$.

(3) a) $l_i = 0$, $l_f = 0$

$$\int_0^{\pi} d\theta \sin\theta \cos\theta \sqrt{\frac{1}{4\pi}}\sqrt{\frac{1}{4\pi}}$$

$$= \frac{1}{4\pi}\int_0^{\pi} d\theta \sin\theta \cos\theta$$