PH20016 : Particles, Nuclei and Stars

Lecture 2 Stellar Nucleosynthesis — Dr S R Davies

Stellar Structure

A star is held together by self-gravitation, supported against collapse by internal pressures.

Stars are generally in hydrostatic equilibrium

— inward grav. attraction balances outward pressure at every point within the star

— pressure steadily increases towards the centre

— can show that $\dfrac{dP}{dr} = -g\rho = -\dfrac{GM(r)\,\rho(r)}{r^2}$ (Eqn of hydrostatic eqbm)

Computational modelling can solve this to show that

core pressure, $P_c = 2.5\times10^{16}\ \mathrm{N.m^{-2}}$, core density, $\rho_c \simeq 10^5\ \mathrm{Kg\,m^{-3}}$

Recall that (ideal gas law) : $PV = NkT \Rightarrow P = \dfrac{N}{V}kT \simeq \dfrac{\langle\rho\rangle}{\bar{m}}k\langle T\rangle$

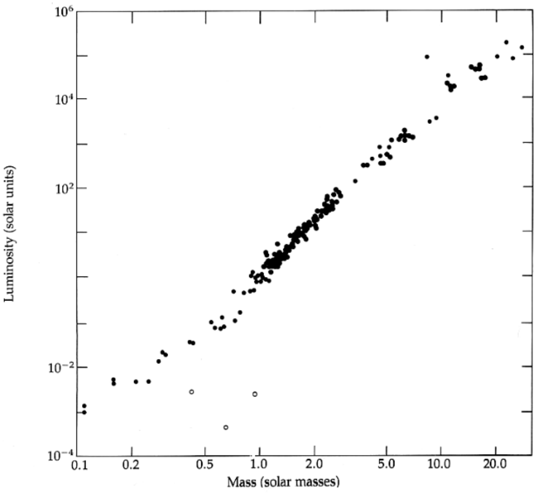
The average particle mass is $\bar{m} \simeq \mu m_p$

where, for pure, ionised hydrogen $\mu = 0.5$. (In practice, it is a little higher due to the He fraction).

The Sun has central temperature $T_c \simeq \dfrac{P_c\,\mu m_p}{k\,\rho_c} \simeq 15\times10^6\ \mathrm{K}$

Main sequence stars closely follow the ***mass-luminosity relationship*** (Eddington, 1924).

Plot of Mass against L for binary stars → get accurate mass



The plot shows a change in slope at a mass of ~ $M_\odot$. The M–L relationship for the two regions is given by

$$\frac{L_*}{L_\odot} = 1.5\left(\frac{M_*}{M_\odot}\right)^{3.5} \qquad M_* > 1.5\,M_\odot$$

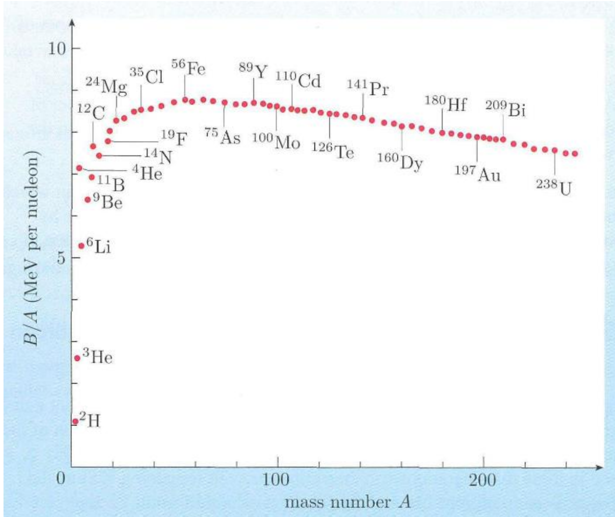
$$\frac{L_*}{L_\odot} = 0.23\left(\frac{M_*}{M_\odot}\right)^{2.3} \qquad M_* < 0.5\,M_\odot$$

The early evolution for most stars follows a similar pattern:

- "star" on Main Sequence when H fusion → E production
- M. Seq. ends when most of H in core → He

Stellar Nucleosynthesis

Thermonuclear fusion reactions maintain a star's luminosity during its lifetime.



Light atomic nuclei collide and fuse to form heavier elements → ***nucleosynthesis***.

Energy will be released if light nuclei fuse to form more tightly bound nuclei.

Binding energy per nucleon for atomic nuclei has a broad maximum at A near 56 (Fe)

→ nuclei near Fe in the periodic table are the most tightly bound.

For charged particles, the Coulomb barrier to fusion is

$$E_C = \frac{Z_A Z_B q^2}{4\pi \varepsilon_0 r_N}$$

where r_N = range of strong nuclear force

Typically, E_C > 1 MeV.

For particles with relative energy $E \ll E_C$, there is a small but finite probability of the particle penetrating the Coulomb barrier through QM tunnelling and approaching within r_N.

probability of penetration $\propto \exp\left[-\left(\frac{E_G}{E}\right)^{1/2}\right]$

where Gamow energy,
(m_r = reduced mass)

$$E_G = 2 m_r c^2 \left(\frac{\pi Z_A Z_B}{137}\right)^2$$

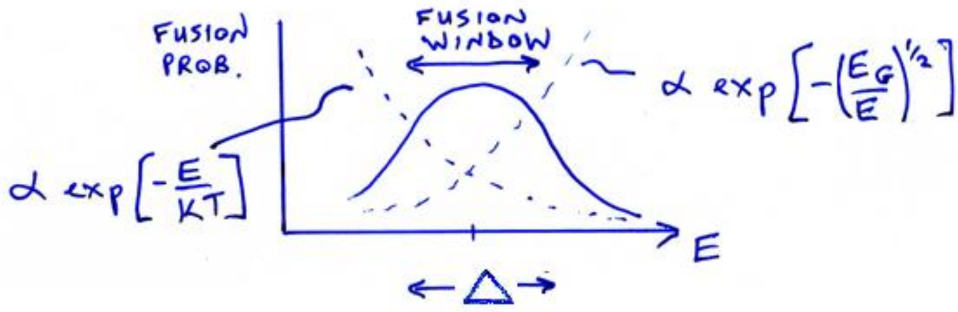
For protons at $T \sim 10^7$ K … fusion proceeds at a leisurely pace.

In general, the nuclei will form a classical, non-relativistic gas, with a Maxwell-Boltzmann distribution of speeds.

Prob. of particle having energy E is $\propto \exp\left[-\frac{E}{KT}\right]$

The fusion reaction rate is proportional to both exponential factors, i.e.

$$R_{AB} \propto \exp\left[-\frac{E}{KT} - \left(\frac{E_G}{E}\right)^{1/2}\right]$$



Fusion predominantly takes place in a narrow energy range around a most likely fusion energy.

We can relate the fusion reaction rate to other physical parameters, such as the mean free path and the fusion cross-section.

In general, one can think of a mean free path as $l = \frac{1}{\sigma n}$ where n = number density (m^{-3}), σ = cross-section (m^2)

The fusion cross-section for nuclei with kinetic energy E between them is

$$\sigma(E) = \frac{S(E)}{E} \exp\left[-\left(\frac{E_G}{E}\right)^{1/2}\right]$$

where the nuclear fusion factor, $S(E)$ is determined by the specific fusion reaction involved.

The nuclei A and B will have a range of speeds, but if we take an average value for the product of cross-section with relative speed, then we can write

$$R_{AB} = n_A n_B \langle \sigma v_r \rangle \quad \text{and} \quad R_{AA} = \frac{n_A^2}{2} \langle \sigma v_r \rangle$$

(For similar particles, we have to avoid double counting – a particle cannot fuse with itself).

The mean time it takes for a nucleus of type A to fuse with one of type B is

$$\tau_A = \frac{1}{n_B \langle \sigma v_r \rangle} = \frac{n_A}{R_{AB}} \quad \left(\text{or } \frac{n_A}{2R_{AA}}\right)$$

In general, the particle speeds will follow a Maxwell-Boltzmann distribution, for which it is possible to show that (not derived here)

$$R_{AB} = n_A n_B \left(\frac{8}{\pi m_r}\right)^{1/2} \left(\frac{1}{KT}\right)^{3/2} \int_0^\infty S(E) \exp\left[-\frac{E}{KT} - \left(\frac{E_G}{E}\right)^{1/2}\right] dE$$

n_A and n_B are the number densities (number per unit volume) of the fusing nuclei, m_r is the reduced mass and $S(E)$ is the nuclear S-factor, which encapsulates the strength of the fusion interaction.

This equation can be integrated to result in an expression for the total fusion rate per unit volume:

$$R_{AB} = \frac{6.48 \times 10^{-24}}{A_r Z_A Z_B} n_A n_B S(E_0) \left(\frac{E_G}{4KT}\right)^{2/3} \exp\left[-3\left(\frac{E_G}{4KT}\right)^{1/3}\right] \; m^{-3} s^{-1}$$

where the numerical constant has been determined for $S(E_0)$ given in (keV barns) and that the particle densities are given in (m^{-3}). A_r is the reduced mass in atomic mass units ($A_r = m_r / u$)

E_0 is the energy corresponding to the peak in the fusion window and is given by

$$E_0 = \left[E_G \left(\frac{KT}{2}\right)^2\right]^{1/3}$$

and the fusion window has a width given approximately by

$$\Delta \simeq 1.8 \, KT \left(\frac{E_G}{KT}\right)^{1/6}$$

In the Sun, the main nuclear fusion reactions turn 4 protons $\rightarrow$ 1 He + energy

For stars with mass < 1.5 $M_\odot$, main reaction sequence is the **proton-proton chain** (PPI) (energy released in each step given in parentheses) :

$$^1H + ^1H \rightarrow ^2H + e^+ + \nu_e \qquad (1.0 + 0.4 \text{ MeV})$$

$$^2H + ^1H \rightarrow ^3He + \gamma \qquad (5.5 \text{ MeV})$$

$$^3He + ^3He \rightarrow ^4He + ^1H + ^1H + \gamma \qquad (12.9 \text{ MeV})$$

- in 1st reaction, cons. of charge maintained by emission of positron
- e^+ and an e^- annihilate $\rightarrow$ releases 1 MeV energy
- 1st step involves weak nuclear force and is slow
  $\rightarrow$ controls speed at which H processed in star
  $\rightarrow$ p - p fusion rate $\sim 5 \times 10^{13}\ s^{-1} . m^{-3}$

A star of mass 1 $M_\odot$ can exist in stable condition, fusing hydrogen this way for $\sim 10^{10}$ years.

More massive stars have higher central P and T $\rightarrow$ burn their nuclear fuel at a faster rate.

Net result of PPI chain : $4\,^1H + 2e^- \rightarrow {}^4He + 7\gamma + 2\nu_e + 26.7\text{ MeV}$

- core T of $\sim 10^7$ K needed
- only occur in Sun's core ($\sim 0.2 R_\odot$) where T, P sufficient

Energy is carried away by :

γ -rays — absorbed, re-emitted, scattered by matter; mean free path ~ 1 mm

$\rightarrow$ ~ 10^5 yrs to get out ! (~26.2 MeV)

neutrinos — interact ***very weakly*** with matter

$\rightarrow$ ~ 2 seconds to get out ! (~0.5 MeV)

Other reactions can occur instead of the last step of the chain, producing small quantities of ^7Be and ^7Li (PPII and PPIII chains). In the Sun, the PPI chain dominates (85%).

PPII (15%):

$$^3He + ^4He \rightarrow ^7Be + \gamma$$
$$e^- + ^7Be \rightarrow ^7Li + \nu_e$$
$$^1H + ^7Li \rightarrow ^4He + ^4He$$

PPIII (0.1%):

$$^3He + ^4He \rightarrow ^7Be + \gamma$$
$$^1H + ^7Be \rightarrow ^8B$$
$$^8B \rightarrow ^8Be + e^+ + \nu_e$$
$$^8Be \rightarrow ^4He + ^4He$$

Experiments have been carried out to detect the flux of solar neutrinos arriving at the Earth's surface. The measured fluxes are ~ 50% of the flux expected from theoretical models

→ this is known as the ***solar neutrino problem***

In hotter stars ($T_{core} > 2 \times 10^7$ K), the dominant reaction process is the **Carbon-Nitrogen-Oxygen cycle…** uses a carbon nucleus as a catalyst

$$^{12}C + {}^{1}H \rightarrow {}^{13}N + \gamma$$
$$^{13}N \rightarrow {}^{13}C + e^+ + \nu_e$$
$$^{13}C + {}^{1}H \rightarrow {}^{14}N + \gamma$$
$$^{14}N + {}^{1}H \rightarrow {}^{15}O + \gamma$$
$$^{15}O \rightarrow {}^{15}N + e^+ + \nu_e$$
$$^{15}N + {}^{1}H \rightarrow {}^{12}C + {}^{4}He$$

Net result of CNO chain :

$$4\,{}^{1}H + 2e^- \rightarrow {}^{4}He + 7\gamma + 2\nu_e + 23.8\ \text{MeV}$$

- CNO cycle needs higher T than PPI (Coulomb barriers ↑)
- CNO cycle much more T-sensitive
- small traces of C from material formed in previous generations of stars

Energy generation rates for PP and CNO cycles as a function of temperature.

Note the crossover at ~ 18 million K.