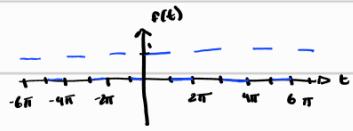
Q1

a) $f(t) = \begin{cases} 1, & |t| < \pi \\ 0, & \pi < |t| < 2\pi \end{cases}$



b) $T = 4\pi$ ✓

$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{4\pi} = \frac{1}{2}$ ✓

c) $b_n = \frac{2}{T}\int_0^{0+T} f(t)\sin(n\omega_0 t)\,dt$

$= \frac{2}{4\pi}\int_0^{4\pi} f(t)\sin\left(\frac{nt}{2}\right)dt$

$= \frac{1}{2\pi}\left[\int_\pi^{2\pi} dt + \int_0^{\pi}\sin\left(\frac{nt}{2}\right)dt\right]$

$= \frac{1}{\pi}(\pi - \pi) = 0$

$e^{\frac{in\pi}{2} + in\frac{\pi}{2}t} : e^{n\left(\frac{i\pi}{2} + \frac{i\pi}{2}t\right)}$

$\frac{in\pi}{2}$ $in\frac{\pi}{2}t$

Q2

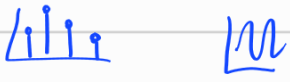
a) $f(t) = 1 + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{1}{n\pi} e^{\quad} e$

$\omega_0 = \frac{\pi}{2t} + \frac{\pi}{2} = \frac{\pi}{2}\left(\frac{1}{t} + 1\right)$ ignore

not sure if to just ignore this

$T = \frac{2\pi}{\omega_0} = \frac{4}{i\left(\frac{1}{t}+1\right)} = 4$

b) Plot amplitude & spectra?



Q3 ~second attempt

a) $F_1(\omega) = -i\omega e^{-|2\omega|}$

Time shift: $g(t+\gamma) \xrightarrow{FT} e^{i\omega\gamma}G(\omega)$

$-i\omega e^{-|2\omega|} = e^{i\omega\gamma}(-i\omega)$ $\begin{cases} \gamma = 2i \\ G(\omega) = -i\omega \end{cases}$

$G(\omega) = -i\omega$

$\mathcal{F}^{-1}[G(\omega)] = \frac{1}{2\pi}\int -i\omega e^{i\omega t}\,d\omega$

$= \frac{1}{2\pi}\left(\omega^2 e^{i\omega t} + \omega e^{i\omega t}\right)$

$= \frac{\omega^2}{\pi} e^{i\omega t}$

b) $f_2(t) = \frac{\sin(2t)}{t}$

$\int \frac{\sin(2t)}{t} e^{-i\omega t}\,dt =$

$\frac{d}{dt}$	$\int dt$
$\frac{\sin(2t)}{t}$	$e^{-i\omega t}$
$\frac{2\cos(2t)}{t} + \frac{\sin(2t)}{t}$	$-i\omega e^{-i\omega t}$
↓ not going to 0	$-\omega^2 e^{-i\omega t}$

try 2: $\frac{\sin(\omega_0 t)}{\pi t} \to H(\omega+\omega_0) - H(\omega-\omega_0)$

$\frac{1}{\pi} f_2(t) = \frac{1}{\pi}\frac{\sin(2t)}{t}$ $\therefore \omega_0 = 2$

$\mathcal{F} = \frac{1}{\pi}\left[H(\omega+2) - H(\omega-2)\right]$

c) $F_3(\omega) = \int e^{-|y|}e^{-(\omega-y)^2}\,dy$

Frequency $\frac{1}{2\pi}\int F(y)G(x-y)\,dy \xrightarrow{IFT} f(t)g(t)$

$\frac{1}{2\pi}F_3(\omega) = \int e^{-|y|}e^{-|\omega-y|^2}\,dy$ $\therefore$ $F(y) = e^{-|y|}$, $G(x-y) = e^{-|\omega-y|^2}$

$F(y) = e^{-|y|} \xrightarrow{IFT}$?

$G(x) = e^{-|x|^2}$

$\frac{\pi}{a}e^{-a|\omega|} \xrightarrow{IFT} \frac{1}{a^2+t^2}$ $\qquad \sqrt{\frac{\pi}{a}}e^{-\omega^2/4a} \xrightarrow{IFT} e^{-at^2}$

a) $F_1(\omega) = -i\omega e^{-|2\omega|}$

Derivative: $i\omega G(\omega) \xrightarrow{IFT} \frac{dg(t)}{dt}$

$F_1(\omega) = i\omega G(\omega)$ with $G(\omega) = -e^{-|2\omega|}$

$\frac{d}{d\omega}$	$\int d\omega$
$-i\omega$	$e^{i\omega t}$
$-i$	$i\omega e^{i\omega t}$
0	$-\omega^2 e^{i\omega t}$

also $\frac{\pi}{a}e^{-a|\omega|} \xrightarrow{IFT} \frac{1}{a^2+t^2}$

$a = 2$ $\quad \frac{\pi}{2}G(\omega) \xrightarrow{IFT} \frac{\pi}{2}\frac{1}{4+t^2} = g(t)$

$\frac{dg(t)}{dt} = \frac{\pi}{2}\frac{-2t}{(4+t^2)^2}$

$\therefore \mathcal{F}^{-1}[F_1(\omega)] = \frac{-\pi t}{(4+t^2)^2}$ nope

$\sin(2t) \cdot \frac{1}{t}$

$(u\cdot v)' = u'v + uv'$

$u = \sin(2t)$ $\quad u' = 2\cos(2t)$

$v = \frac{1}{t}$ $\quad v' = -t^{-2}$

$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$

$u = 1$ $\quad u' = 0$

$v = 4+t^2$ $\quad v' = 2t$

$\left(\frac{u}{v}\right)' = \frac{-2t}{(4+t^2)^2}$

b) $f_2(t) = \frac{\sin(2t)}{t}$

Symmetry $G(t) \xrightarrow{FT} 2\pi g(-\omega)$

$\frac{\sin(\omega_0 t)}{\pi t} \xrightarrow{FT} H(\omega+\omega_0) - H(\omega-\omega_0)$ ✓

$\pi F(y) \xrightarrow{IFT} \frac{\pi}{1+t^2}$ ($a=1$) $\qquad a=\frac{1}{4} \therefore \sqrt{4\pi}\,G(x) \xrightarrow{IFT} \sqrt{4\pi}\,e^{-t^2/4}$

$\therefore F^{-1}(F_3(\omega)) = \left(\frac{\pi}{1+t^2}\right)\left(\sqrt{4\pi}\,e^{-t^2/4}\right)$

d) $F_4(\omega) = \sin(3\omega)$

Symmetry: $2\pi g(-\omega) \xrightarrow{IFT} G(t)$

$\frac{\sin(\omega_0 t)}{\pi t} \xrightarrow{FT} H(\omega+\omega_0) - H(\omega-\omega_0)$

$F = \frac{1}{3\pi} F_4 = \frac{\sin(3t)}{\pi t} \therefore \omega_0 = 3 \xrightarrow{FT} \left(H(\omega+3) - H(\omega-3)\right)\frac{1}{3\pi}$

$2\pi F \xrightarrow{IFT} \frac{2}{3}\left(H(\omega+3) - H(\omega-3)\right) \quad \frac{i}{2}\left[\delta(t-3) - \delta(t+3)\right]$

Q4 Aliasing: $f_s > 2f_m$ Take max freq

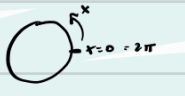
$f = \frac{1}{\Delta t}$ sampling rate = 2 × max freq

$f_1 \rightarrow \Delta t = 9\,s$ $\Delta t = \frac{1}{\text{sampling rate}}$

$f_2 \rightarrow \Delta t = 4.7\,s$

$f_3 \rightarrow \Delta t = 8\,s$

Q5



$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}$ $\quad \alpha > 0$ = const

a) $u(x,t) = X(x)\,T(t)$

$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} T$ $\qquad \frac{\partial^2 u}{\partial x^2} T = \alpha \times \frac{\partial u}{\partial t}$

$\frac{\partial^2 u}{\partial x^2} = X \frac{\partial u}{\partial t}$

possible solut°:

Tutorial → A4!

Q6

$y(x) = \sum_{s=0}^{\infty} a_s x^{s+q}$

a) $q(q-1) = 0$

b) $\sum a_n (n+q)(n+q-1) x^{n+q-2} + \omega^2 \sum a_n x^{n+q} = 0$

$n = n+3 \therefore \sum a_{n+3} (n+q+3)(n+q+2) x^{n+q+1} + \omega^2 a_{n+3} x^{n+q+3}$

$n+q+3 = 0 \therefore n+q = 3$

Q7

s: $\Phi = V_0 \cos^3(\theta)$ $\qquad$ laplace: $\Psi = \left(Ar^{\ell} + Br^{-(\ell+1)}\right) P_{\ell}^{m}(\cos\theta)\left(C\cos(m\phi) + D\sin(m\phi)\right)$

$r > a$: Φ ?

Q8

a) $A = A^{+}$

$Ax = \lambda x$ eigenval

$(A - \lambda I)x = 0$

$\left((A-\lambda I)x\right)^{+} = 0$

$x^{+}(A-\lambda I)^{+} = 0$