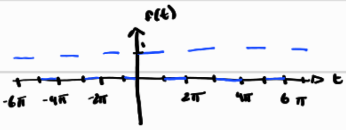
Q1

a) $f(t) = \begin{cases} 1, & |t| < \pi \\ 0, & \pi < |t| < 2\pi \end{cases}$

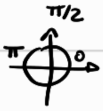


b) $T = 4\pi$

$w_0 = \frac{2\pi}{T} = \frac{2\pi}{4\pi} = \frac{1}{2}$

c) $b_n = \frac{2}{T}\int_{\theta}^{\theta+T} f(t)\sin(nw_0 t)\,dt$

$= \frac{2}{4\pi}\int_0^{4\pi} f(t)\sin\left(\frac{nt}{2}\right)dt$



$= \frac{1}{\cancel{2}\pi}\cancel{2}\left[\int_{\pi}^{2\pi} dt + \int_0^{\pi}\sin\left(\frac{nt}{2}\right)dt\right]$

$= \frac{1}{\pi}(\pi - \pi) = 0$

Q2

$e^{\frac{in\pi}{2} + in\frac{\pi}{2}t} : e^{n\left(\frac{i\pi}{2} + \frac{i\pi}{2}t\right)}$

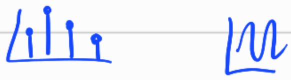
a) $f(t) = 1 + \sum_{\substack{-\infty \\ n \neq 0}}^{\infty} \frac{1}{n\pi}\, \underbrace{e^{\frac{in\pi}{2}}\, e^{in\frac{\pi}{2}t}}$

$w_0 = \cancel{\frac{\pi}{2t}} + \frac{\pi}{2} = \frac{\pi}{2}\left(\cancel{\frac{1}{t}} + 1\right)$ ignore

not sure if to just ignore this

$T = \frac{2\pi}{w_0} = \frac{4}{i\left(\frac{1}{t}+1\right)} = 4$

b) Plot amplitude & spectra?



Q3

Q3 ~second attempt

a) $F_1(w) = -iwe^{-|2w|}$

Time shift: $g(t+\tau) \xrightarrow{FT} e^{iw\tau}G(w)$

$-iwe^{-|2w|} = e^{iw\tau}(-iw)$ $\begin{cases}\tau = 2i \\ G(w) = -iw\end{cases}$

$G(w) = -iw$

$F^{-1}[G(w)] = \frac{1}{2\pi}\int -iwe^{iwt}\,dw$

$= \frac{1}{2\pi}\, w^2 e^{iwt} + we^{2\,iwt}$

$= \cancel{\frac{w^2}{\pi} e^{iwt}}$

$\frac{d}{dw}$	$\int dw$
$-iw$	e^{iwt}
$-i$	iwe^{iwt}
0	$-w^2e^{iwt}$

a) $F_1(w) = -iwe^{-|2w|}$

Derivative: $iwG(w) \xrightarrow{IFT} \frac{dg(t)}{dt}$

$F_1(w) = iwG(w)$

with

$G(w) = -e^{-|2w|}$

also $\frac{\pi}{a}e^{-a|w|} \xrightarrow{IFT} \frac{1}{a^2+t^2}$

$a = 2$ $\quad \frac{\pi}{2}G(w) \xrightarrow{IFT} \frac{\pi}{2}\,\frac{1}{4+t^2} = g(t)$

$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$

$u = 1 \quad u' = 0$

$v = 4+t^2 \quad v' = 2t$

$\left(\frac{u}{v}\right)' = \frac{-2t}{(4+t^2)^2}$

$\frac{dg(t)}{dt} = \frac{\pi}{2}\,\frac{-2t}{(4+t^2)^2}$

$\therefore F^{-1}[F_1(w)] = \cancel{\frac{-\pi t}{(4+t^2)^2}}$ nope

b) $f_2(t) = \frac{\sin(2t)}{t}$

$\int \frac{\sin(2t)}{t}e^{-iwt}\,dt =$

$\frac{d}{dt}$	$\int dt$
$\frac{\sin(2t)}{t}$	e^{-iwt}
$\frac{2\cos(2t)}{t} + \frac{\sin(2t)}{t}$	$-iwe^{-iwt}$
	$-w^2e^{-iwt}$

not going to 0

$\sin(2t)\cdot\frac{1}{t}$

$(u\cdot v)' = u'v + uv'$

$u = \sin(2t)$

$u' = 2\cos(2t)$

$v = \frac{1}{t}$

$v' = -t^{-2}$

try 2: $\frac{\sin(w_0 t)}{\pi t} \to H(w+w_0) - H(w-w_0)$

$\frac{1}{\pi}f_2(t) = \frac{1}{\pi}\,\frac{\sin(2t)}{t}$ $\quad\therefore w_0 = 2$

$F = \cancel{\frac{1}{\pi}}\pi\left[H(w+2) - H(w-2)\right]$

b) $f_2(t) = \frac{\sin(2t)}{t}$

Symmetry $G(t) \xrightarrow{FT} 2\pi g(-w)$

$\frac{\sin(w_0 t)}{\pi t} \xrightarrow{FT} H(w+w_0) - H(w-w_0)$

c) $F_3(w) = \int e^{-|y| - |w-y|^2}\,dy$

Frequency $\frac{1}{2\pi}\int F(y)\,G(x-y)\,dy \xrightarrow{IFT} f(t)\,g(t)$

$\frac{1}{2\pi}F_3(w) = \int e^{-|y|}\, e^{-|w-y|^2}\,dy$ $\quad\therefore$ $F(y) = e^{-|y|}$, $G(x-y) = e^{-|w-y|^2}$

$F(y) = e^{-|y|} \xrightarrow{IFT} ?$

$G(x) = e^{-|x|^2}$

$\frac{\pi}{a}e^{-a|w|} \xrightarrow{IFT} \frac{1}{\ldots}$ $\quad \boxed{\frac{\pi}{\ldots}}\,e^{-w^2/4a} \xrightarrow{IFT} e^{-at^2}$

$\pi F(y) \xrightarrow[a=1]{IFT} \frac{\pi}{1+t^2}$ $\qquad a=\frac{1}{4} \therefore \sqrt{4\pi}\, G(x) \xrightarrow{IFT} \sqrt{4\pi}\, e^{-t^2/4}$

$\therefore F^{-1}(F_3(\omega)) = \frac{1}{\sqrt{\pi}}\left(\frac{\cancel{\pi}}{1+t^2}\right)\left(\sqrt{4\pi}\, e^{-t^2/4}\right)$

d) $F_4(\omega) = \sin(3\omega)$

Symmetry: $2\pi g(-\omega) \xrightarrow{IFT} G(t)$

$\frac{\sin(\omega_0 t)}{\pi t} \xrightarrow{FT} H(\omega+\omega_0) - H(\omega-\omega_0)$

$F = \frac{1}{3\pi} F_4 = \frac{\sin(3t)}{\pi t} \therefore \omega_0 = 3 \xrightarrow{FT} \left(H(\omega+3) - H(\omega-3)\right)\frac{1}{3\pi}$

$2\pi F \xrightarrow{IFT} \cancel{\frac{2}{3}\left(H(\omega+3) - H(\omega-3)\right)}$ $\quad \frac{i}{2}\left[\delta(t-3) - \delta(t+3)\right]$

Q4 Aliasing: $f_s > 2f_m$

Take max freq ↓ sampling rate = 2 x max freq ↓ $\Delta t = \frac{1}{\text{sampling rate}}$

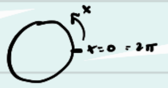
$f = \frac{1}{\Delta t}$

$f_1 \rightarrow \Delta t = 9\,s$

$f_2 \rightarrow \Delta t = 4.7\,s$

$f_3 \rightarrow \Delta t = 5\,s$

Q5



$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}$

$\alpha > 0$ = const

Tutorial → A4!

a) $u(x,t) = X(x)\,T(t)$

$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} T$

$\frac{\partial^2 u}{\partial x^2} = X \frac{\partial u}{\partial t}$

$\frac{\partial^2 u}{\partial x^2} T = \alpha\, x\, \frac{\partial u}{\partial t}$

possible solut°:

Q6

$y(x) = \sum_{s=0}^{\infty} a_s x^{s+q}$

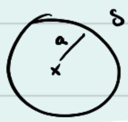
a) $q(q-1) = 0$

b) $\sum a_n (n+q)(n+q-1)\, x^{n+q-2} + \omega^2 \sum a_n x^{n+q} = 0$?

$n = n+3 \therefore \quad \sum a_{n+3}(n+q+3)(n+q+2)\, x^{n+q-1} + \omega^2 a_{n+3} x^{n+q+3}$

$n+q+3 = 0 \quad \therefore \quad n+q = 3$

Q7



S: $\Phi = V_0 \cos^3(\theta)$

$r > a$: Φ ?

Laplace: $\Psi = (A r^{\ell} + B r^{-(\ell+1)})\, P_\ell^m(\cos\theta)\,(C\cos(m\phi) + D\sin(m\phi))$

Q8

a) $A = A^{+}$

$A x = \lambda x$ eigenvalue

$(A - \lambda I)x = 0$

$((A - \lambda I)x)^{+} = 0$

$x^{+}(A - \lambda I)^{+} = 0$