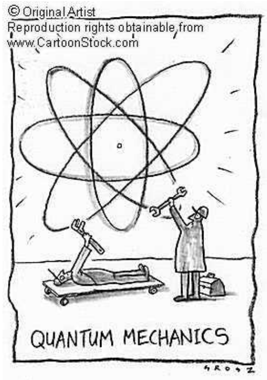
Quantum and Atomic Physics: Course Work # 1: student $\boxed{199058365}$

Answer the questions below. Use the mark-sheet, with your section completed, as the cover sheet by stapling it to the front of your solution submission.

Keep calm, plot and calculate. By the end of this sheet you will be a quantum mechanic.

Dr Peter A. Sloan



(Q1) **Consider the full 1-electron hydrogen wavefunction**

$$\boxed{\psi = A\ r\ e^{-r/2a_0} \sin\theta\ e^{+i\phi}.}$$

where A is a constant of normalization.

(a) Show that the wavefunction is a solution to the Schrödinger equation $\hat{H}\left|\psi\right\rangle = E\left|\psi\right\rangle$ and hence show that its energy $E = -3.4$ eV and its principle quantum number $n = 2$. [9]

(b) The probability of finding the electron somewhere must be 1, therefore show by computing

$$1 = \langle\psi^*|\psi\rangle = \iiint \psi(r,\theta,\phi)^*\psi(r,\theta,\phi)dV$$

that the normalization factor

$$A = \frac{1}{8\sqrt{\pi}} \cdot \frac{1}{a_0^{5/2}}$$

[Hint: Remember your complex conjugate (*) and that the integration is over all space using spherical polar coordinates so get your limits right and your volume element right - see 3D Polar maths video.] [9]

(Q2) **Consider only the radial part of the 1-electron hydrogen wavefunction**

$$\boxed{\psi = A\ \left(6r - \frac{r^2}{a_0}\right)\ e^{-r/3a_0} \sin\theta\ e^{+i\phi}.}$$

where A is a constant of normalization.

(a) Briefly explain what the radial probability distribution $P(r)dr$ represents, and why it has the mathematical form

$$P(r)dr = B\, r^4 \left(36 - 12\frac{r}{a_0} + \frac{r^2}{a_0^2}\right)\, e^{-2r/3a_0} dr.$$

where B is a constant of normalization. [3]

(b) Show that you cannot find the electron at the radial positions $r = 0,\ 6a_0$. [3]

(c) Find the maximum(s), i.e., turning point(s), of $P(r)$. [6]

(d) Given that $B = 8/(81^2 3a_0^5)$ calculate the expectation value of the radial distance r,

$$\bar{r} = \langle R^* |\, r\, | R\rangle = \int_0^\infty r\ P(r)\ dr$$

to show that $\bar{r} = 12.5a_0$ [6]

(e) Using a computer program of your choice plot $P(r)$ indicating the point(s) you have determined. You may indicate these point by hand on your print-out. [3]

(Q3) **Consider only the angular part of the 1-electron hydrogen wavefunction**

$$\boxed{\psi = A\ r^2\ e^{-r/3a_0} \sin\theta\ \cos\theta\ e^{+i\phi}.}$$

where A is a constant of normalization.

(a) Use the appropriate operator to compute the orbital angular momentum and show that $|L| = \sqrt{6}\hbar$ [6]

(b) What is its ℓ quantum number? [3]

(c) What is the magnitude of its orbital magnetic moment (in μ_B)? [3]

(d) What is the value of its angular momentum projected onto the z-axis? [6]

(e) What is its m_ℓ quantum number? [3]

(f) Sketch a semi-classical diagram of the angular momentum of your wavefunction. [3]

Quantum and Atomic Physics: Mark Sheet # 1: candidate 199058365

Please staple this sheet to the front of your submission. Circle what you think of as the appropriate marks and then add them up. I will also mark just to see if we differ on our judgement of your work. There will be prizes for the top 5 entries.

Dr Peter A. Sloan

	Student Mark				Staff Mark			
Grade	Pass	Started	Nearly	Done	Pass	Started	Nearly	Done
Marks	0	1/3	2/3	Full	0	1/3	2/3	Full
Q1(a)	0	3	6	9	0	3	6	9
Q1(b)	0	3	6	9	0	3	6	9
Q2(a)	0	1	2	3	0	1	2	3
Q2(b)	0	1	2	3	0	1	2	3
Q2(c)	0	2	4	6	0	2	4	6
Q2(d)	0	2	4	6	0	2	4	6
Q2(e)	0	1	2	3	0	1	2	3
Q3(a)	0	2	4	6	0	2	4	6
Q3(b)	0	1	2	3	0	1	2	3
Q3(c)	0	1	2	3	0	1	2	3
Q3(d)	0	2	4	6	0	2	4	6
Q3(e)	0	1	2	3	0	1	2	3
Q3(f)	0	1	2	3	0	1	2	3
	Marks/63 =				Marks/63 =			
	% =				% =			