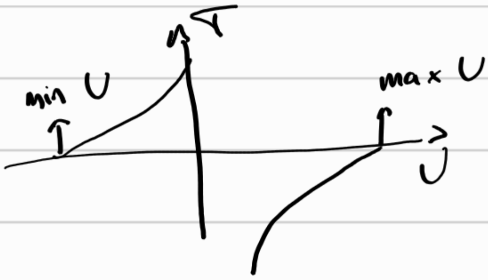
2.



$$\frac{1}{k_B T} = \frac{\partial \ln W}{\partial U}$$

$$T = 1 / \left(k_B \frac{\partial \ln(W)}{\partial U} \right)$$

4. a) $S = k_B \ln(W)$

$W = 6$ microstates

$$\Rightarrow S = k_B \ln(6) \approx 2.47 \times 10^{-23}\, J/K$$

b) $\Delta S = \frac{\Delta Q}{T}$

$\Delta Q = mgh \Rightarrow \Delta S = \frac{mgh}{T}$

$$W = e^{\Delta S / k_B}$$

$$= e^{mgh / k_B T}$$

$$\approx e^{10^{-3} \cdot 9.8 \cdot 1/(1.38 \times 10^{-23} \times 300)}$$

$$\approx e^{2.37 \times 10^{18}}$$

5. N_{sites} $\qquad A_x B_{1-x}$

n atoms A

$(N-n)$ atoms B

$$S = k_B \ln(W)$$

$$W(n) = \frac{N!}{n!(N-n)!}$$

$$\boxed{\ln x! \simeq x \ln x - x}$$

$$\ln W = N \ln N - \cancel{N} - n \ln n + \cancel{n} - (N-n)\ln(N-n) + \cancel{(N-n)}$$

$$n = xN \Rightarrow N - n = N - xN = N(1-x)$$

$$S = k_B \ln W = k_B \left(N \ln N - Nx \ln Nx - N(1-x) \ln [N(1-x)] \right.$$

$$S = k_B \left(\cancel{N \ln N} - Nx \ln x - Nx \cancel{\ln N} - \cancel{N(1-x) \ln N} - N(1-x) \ln(1-x) \right)$$

$$(N - Nx) \ln N = N(1-x) \ln$$

$$S = -k_B N \left(x \ln x + (1-x) \ln(1-x) \right)$$

7) $W(U) = A U^{3N/2}$

$$\frac{1}{k_B T} = \frac{\partial \ln W(U)}{\partial U} = \frac{\partial \ln \left(A U^{3N/2}\right)}{\partial U}$$

$$= \frac{\partial \left(\frac{3}{2} N \ln U + \cancel{\ln A} \right)}{\partial U}$$

$$= \frac{3}{2} N \frac{1}{U}$$

$$T = \frac{2}{3} \frac{U}{N} \frac{1}{k_B}$$

$$U = \frac{3}{2} N k_B T$$