PH30030: QUANTUM MECHANICS

- non relativistic

time dep Schröd eqⁿ

$$\hat{H}\psi = i\hbar\frac{\partial\psi}{\partial t}$$

:)

Professor Philip S Salmon

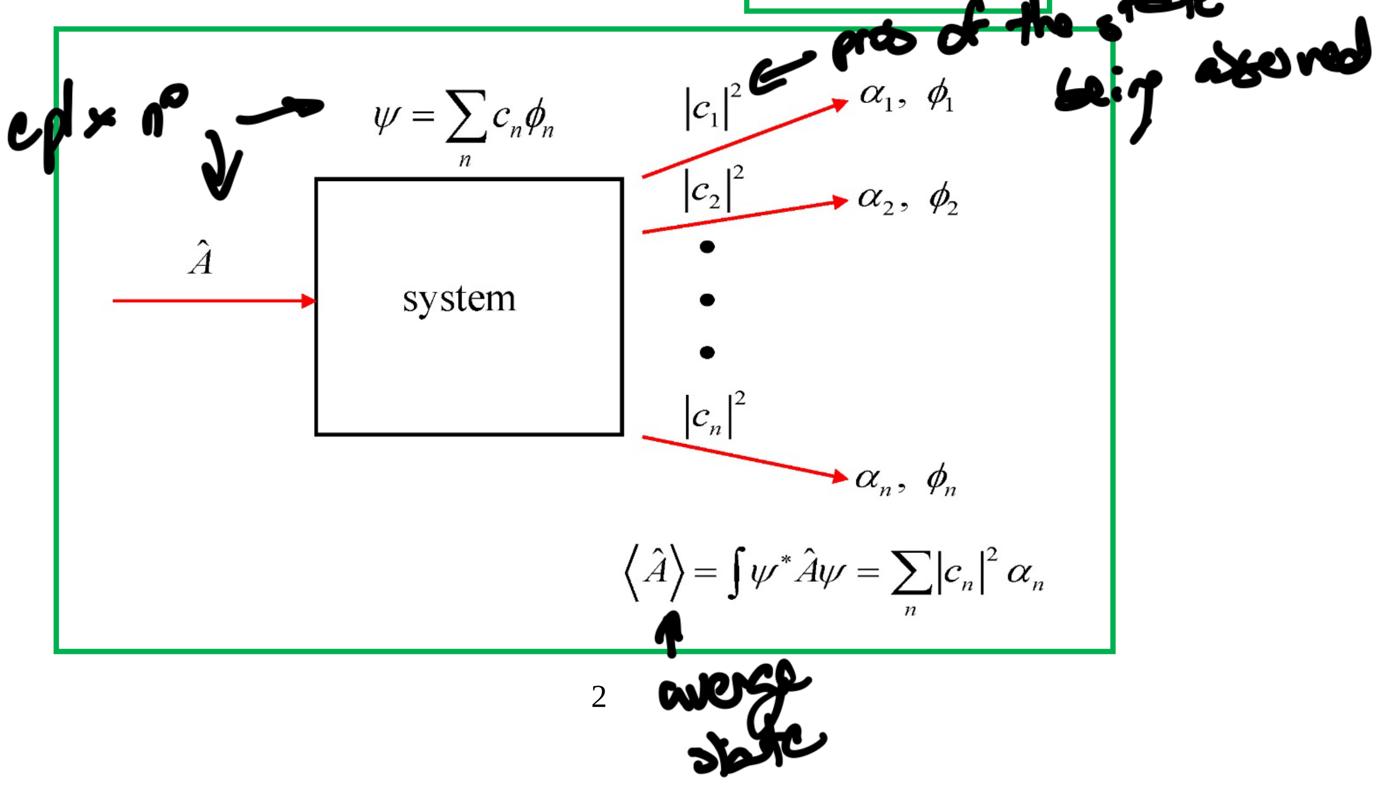
This course builds on the second year quantum course by providing a more formal and mathematical approach to quantum mechanics.

Section 1. The concepts and postulates of quantum mechanics The wavefunction and its interpretation. Observables and operators: position, momentum, energy operators. Measurements on a quantum system; collapse of the wavefunction. Properties of Hermitian operators: real eigenvalues; orthogonal eigenfunctions; expansion in a complete set of eigenfunctions. Probabilities of measurements; expectation values.

eigenfunction

$$\hat{A}\phi_n = \alpha_n \phi_n$$

eigenvalue



particle: position, momentum

Compatible observables: commutators; the uncertainty principle.

$$\boxed{\Delta A \Delta B \geq \frac{1}{2}\left|\int \psi^* \left[\hat{A}, \hat{B}\right] \psi\right|}$$

$[\hat{A},\hat{B}]\psi = (\hat{A}\hat{B} - \hat{B}\hat{A})\psi$
$= \hat{A}\hat{B}\psi - \hat{B}\hat{A}\psi$

Time evolution of the wavefunction; time-dependent Schrodinger equation.

$$\boxed{\hat{H}\psi = -\frac{\hbar^2}{2m}\nabla^2\psi(\underline{r},t) + V(\underline{r},t)\psi(\underline{r},t) = i\hbar\frac{\partial\psi}{\partial t}}$$

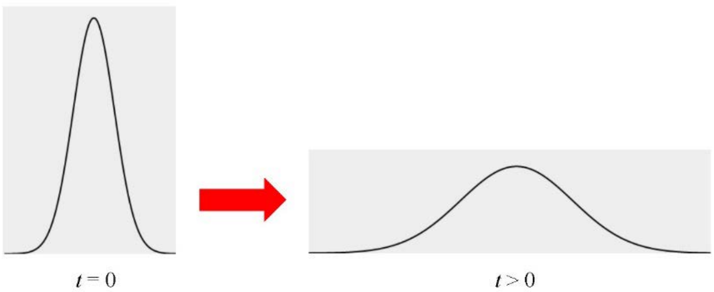
total energy = K.E + Potential energy

Stationary states; time-independent Schrodinger equation.

$V(\underline{r})$ only, i.e., no time dependence

simple harmonic motion

Spreading of a Gaussian wavepacket.



At $t = 0$ we cannot localise the particle more than is allowed by the Uncertainty Principle

Dirac notation. Matrix representation of quantum mechanics.

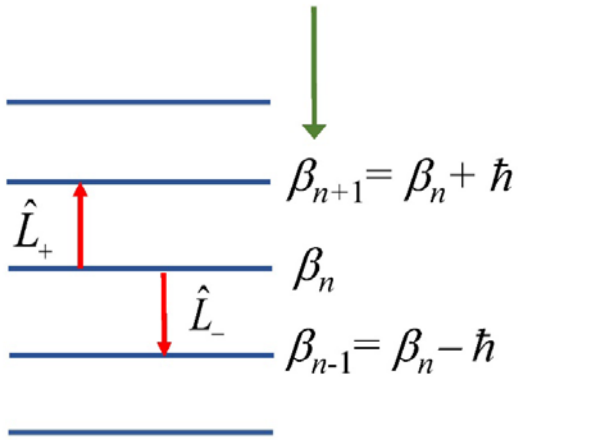
Section 2. Angular momentum Definitions, operators and commutators: $\hat{L}_x$, $\hat{L}_y$, $\hat{L}_z$, $\hat{L}^2$; ladder operators $\hat{L}_+$, $\hat{L}_-$. Eigenvalues and eigenfunctions of $\hat{L}^2$ and $\hat{L}_z$; spherical harmonics.

(total angular mom$^{\mathrm{m}}$)^2

$$\hat{L}^2 \left|\phi_n\right\rangle = \alpha_n \left|\phi_n\right\rangle \quad \text{and} \quad \hat{L}_z \left|\phi_n\right\rangle = \beta_n \left|\phi_n\right\rangle$$

z component of angular mom$^{\mathrm{m}}$

Eigenvalues of $\hat{L}_z$ correspond the same eigenvalue α_n of $\hat{L}^2$



spin: intrinsic "angular momentum"

Spin angular momentum; Pauli spin matrices.

$$\boxed{\hat{S}_x = \frac{\hbar}{2}\sigma_x \qquad \hat{S}_y = \frac{\hbar}{2}\sigma_y \qquad \hat{S}_z = \frac{\hbar}{2}\sigma_z}$$

$$\boxed{\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}}$$

(1) hydrogen -> (2) helium

Section 3. Many particle systems Systems of two particles. Two distinguishable interacting particles; central potentials; the hydrogen atom. Indistinguishable particles. Symmetric and antisymmetric eigenfunctions; bosons and fermions. Pauli exclusion principle. Exchange interaction.

arbitrary choice to distinguish particles => cannot affect the result

$$\boxed{\phi(1,2) = \pm\phi(2,1)}$$

Eigenfunction is symmetric if there is no change of sign on particle interchange (+ sign)

Eigenfunction is antisymmetric if there is change of sign on particle interchange (– sign)

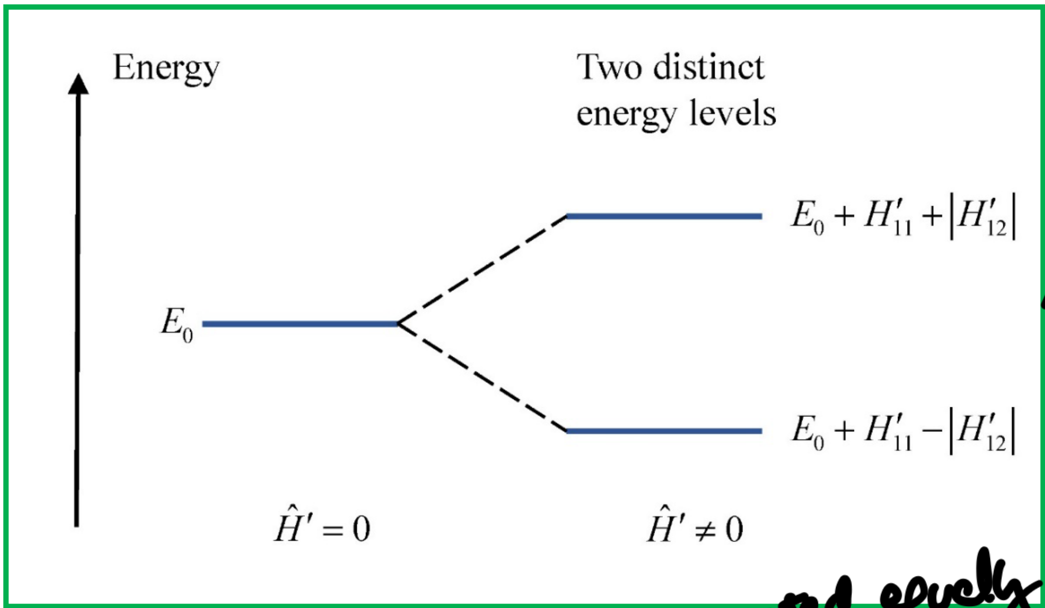
• Turns out that every particle is described by either a symmetric or antisymmetric wavefunction. It is an empirical fact that mixed symmetry does not occur

• Particles with symmetric wavefunctions are called bosons. Bosons have zero or integer spin (s = 0, 1, 2,…). They obey Bose-Einstein statistics. Examples include photons, phonons, pi-mesons, Cooper pairs

• Particles with antisymmetric wavefunctions are called fermions. Fermions have half-integer spin (s = 1/2, 3/2,…). They obey Fermi-Dirac statistics. Examples include electrons, protons, neutrons

Section 4. Approximate methods for stationary states

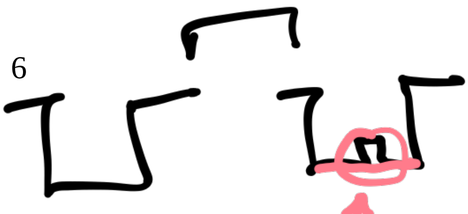
Non-degenerate perturbation theory. Degenerate perturbation theory. Examples.



we can observe transitions

treated equally

we look at perturbations



pesturbat

Section 5. Variational method Upper bound on ground state energy. Example.

another approximate method

KNOW OPERATOR $\hat{H}$

GUESS GROUND STATE WAVEFUNCTION

GROUND STATE ENERGY

IMPROVE WAVEFUNCTION

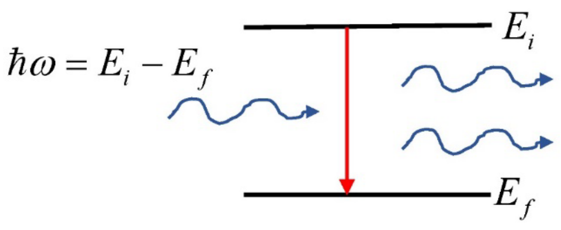
V METHOD

END WITH AN UPPER BOUND ON THE GROUND STATE ENERGY E_1

tential energy & time

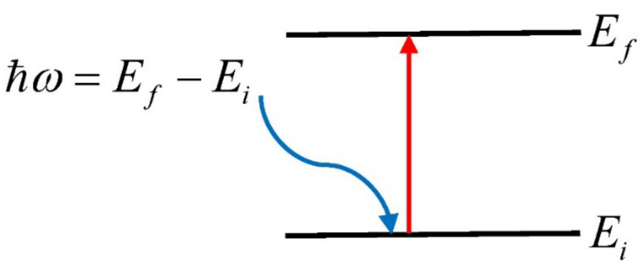
Section 6. Time dependent perturbations First order time-dependent perturbation theory. Periodic perturbations: transitions between energy levels; Fermi's golden rule. Selection rules in atomic spectra.

Stimulated emission:

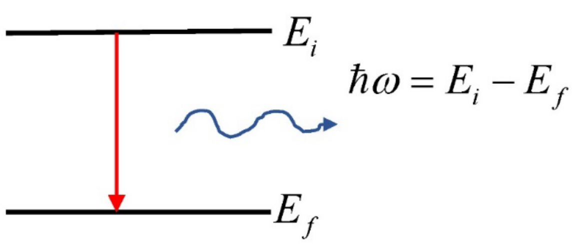


Emitted photons have same energy as incident photon

Absorption:



Spontaneous emission:



Transition is spontaneous – not stimulated by another photon

Pre-requisites: Knowledge of the year 2 quantum course will be assumed, including: normalisation of wavefunctions; solutions of the time-independent Schrodinger equation in 1D (infinite square well, finite square well, harmonic oscillator); quantum numbers of the hydrogen atom (n,l,m) and their interpretation; the Stern-Gerlach experiment and spin.

Assessment: The unit will be assessed by examination.

Problems sheets: There are six problems sheets, one for each part of the course.

yikes!

Problems classes: One per week: -

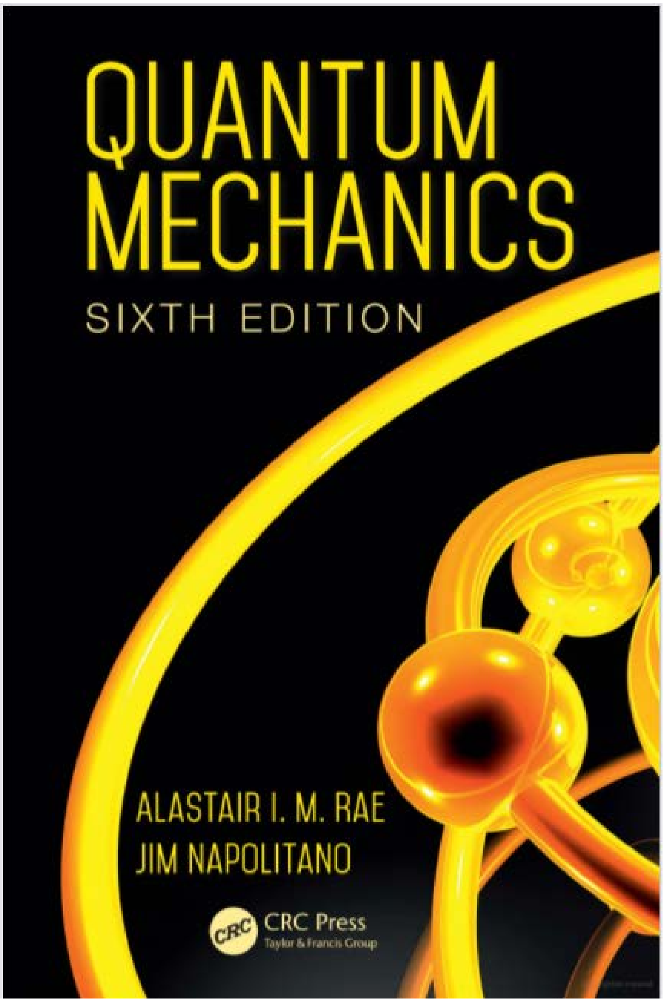
- Friday 09:15

It's far better if you have attempted the problems before each problems class. The focus will be on those problems that are causing the most issues. Solutions will be posted on the Moodle page two or three weeks after the problems are set.

Text books: The course does not follow any one book very closely, but

Quantum Mechanics, A.I.M. Rae & J. Napolitano, Taylor & Francis, 6th Edition, 2008

is a good text. (Earlier editions are fine.)



Other useful references:

Introduction to Quantum Mechanics, D. J. Griffiths and D. F. Schroeter, Cambridge University Press, 3rd Edition, 2018

Quantum Mechanics, S. Gasiorowicz, Wiley,
3rd Edition, 2003

Quantum Mechanics, F. Mandl, Wiley, 1992

Modern Quantum Mechanics, J. J. Sakurai and J. Napoitano, Cambridge University Press, 2nd Edition, 2017

Principles of Quantum Mechanics, R. Shankar, Springer, 2nd Edition, 1994