- defo need to become more confident with Schröd Eq^t° qst^°s : find + pbs

5) $\tilde{\Psi} = A\Psi$

TISE : $\frac{-\hbar^2}{2m}\frac{d^2\psi}{dx} = E\psi$ if $\tilde{\Psi} = A\Psi$ sol:

$$\frac{-\hbar^2}{2m} A \frac{d^2\psi}{dx} = E \cancel{A} \psi \Rightarrow \text{TRUE}$$

normalizat° ψ does not change E level of the associated eigenfunct°

TISE sol $= k\Psi$, $k \in$ int $\Rightarrow E \propto \psi'$

6) a) $E_1 = E2$

TISE $\Rightarrow$ $\frac{-\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = \cancel{E_1\psi(x) = E_2\psi(x)}$

Prove smth Sch Eq^t° : 1) $\psi \Leftrightarrow$ smth

2

Substitute $\Psi = A\Psi_1 + B\Psi_2$

2) $= 0$

$$\frac{-\hbar^2}{2m}\frac{d^2 A\Psi_1 + B\Psi_2}{dx^2} + (A\Psi_1 + B\Psi_2)V(x) = E(A\Psi_1 + B\Psi_2)$$

3) If smth ✓ $\Rightarrow = 0$

$A(\cdots) + B(\cdots) = 0$

$$A\left(\frac{-\hbar^2}{2m}\frac{d^2\psi_1}{dx_2} + \psi_1 V(x) - E\psi_1\right) + B\left(\frac{-\hbar^2}{2m}\frac{d^2\psi_2}{dx_2} + \psi_2 V(x) - E\psi_2\right) = 0$$

$\therefore$ if $E = E_1 = E_2 \Rightarrow \psi_1$ solut° TISE : $\frac{-\hbar^2}{2m}\frac{d^2\psi_1}{dx^2} + V\psi_1 - E\psi_1 = 0$

& similarly for ψ_2

$\therefore$ eq^t° 2 is satisfied & Ψ is a solut° to the TISE

b) $\psi_1 = \psi_2$: if $E_1 = E_2$

c) TDSE: $\frac{-\hbar^2}{2m}\frac{\partial^2 \Psi(x,t)}{\partial x^2} + V(x,t)\Psi(x,t) = i\hbar\frac{\partial \Psi(x,t)}{\partial t}$

$$\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x}\left(A\psi_1 e^{-iE_1 t/h}\right) + VA\psi_1 e^{-iE_1 t/h} \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x}\left(B\psi_2 e^{-iE_2 t/h}\right) + VB\psi_1 e^{-iE_2 t/h}$$

$$= i\hbar \frac{\partial}{\partial t}\left(A \psi_1 e^{-iE_1 t/h}\right) + i\hbar \frac{\partial}{\partial t}\left(B \psi_1 e^{-iE_2 t/h}\right)$$

$$A\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x}\left(\psi_1 e^{-iE_1 t/h}\right) + V\psi_1 e^{-iE_1 t/h} - i\hbar\frac{\partial}{\partial t}\left(\psi_1 e^{-iE_1 t/h}\right)\right)$$

$$+ B\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x}\left(\psi_1 e^{-iE_2 t/h}\right) + V\psi_1 e^{-iE_2 t/h} - i\hbar\frac{\partial}{\partial t}\left(\psi_1 e^{-iE_2 t/h}\right)\right) = 0$$

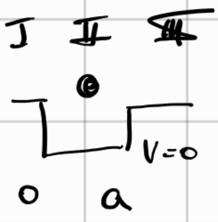
We know that even if $E_1 = E_2$, ψ_1 & ψ_2 are both solut°s to the TISE

$$\Rightarrow \begin{cases} -\dfrac{\hbar^2}{2m}\dfrac{d^2\psi_1}{dx^2} + V\psi_1 - E_1\psi_1 = 0 \\ -\dfrac{\hbar^2}{2m}\dfrac{d^2\psi_2}{dx^2} + V\psi_2 - E_2\psi_2 = 0 \end{cases}$$

Hence $\Rightarrow \Psi = A\psi_1 e^{-iEt/\hbar} + B\psi_2 e^{-iE_2 t/\hbar}$

is a superposit° solut° to the TDSE.

7. a)



$$-\frac{h^2}{2m}\frac{1}{\psi(x)}\frac{d^2\psi(x)}{dx^2} + V(x) = i\hbar\frac{1}{\phi(t)}\frac{d\phi(t)}{dt} = C$$

Schröd eqt° II: $-\dfrac{\hbar^2}{2m}\dfrac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$

b) $x = 0$: $A\sin(0) + B\cos(0) \Rightarrow B = 0$

$x = L$: $A\sin(kL) = 0 \Rightarrow \sin(kL) = 0$

$\therefore kL = n\pi \Rightarrow k = \frac{n\pi}{}$

$$k = \frac{\sqrt{2mE}}{\hbar} \Rightarrow E_n = \frac{\hbar^2 k_n^2}{2m} = \frac{\hbar^2 \pi^2 n^2}{2mL^2}$$

Given $\psi = A_n \sin(k_n x)$

$$= A_n \sin\left(\frac{n\pi x}{a}\right)$$

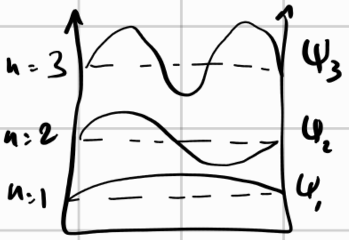
$\int_{-\infty}^{\infty} |\psi|^2 \, dx$ 1 + General sol $\Rightarrow A_n = \sqrt{\frac{2}{L}}$

c) $p = \frac{h}{\lambda} = \hbar k$

$$\lambda_n = \frac{h}{\hbar k_n} = \frac{hL}{\hbar n\pi}$$

$$\lambda_n = \frac{2a}{n}$$

d)



e) Prob: $|\psi(x)|^2 = \left[\sqrt{\frac{\sqrt{2}}{L}\sin\left(\frac{n\pi x}{L}\right)}\right]^2$

$n=1 \Rightarrow \frac{\sqrt{2}}{\sqrt{a}}^2 \sqrt{\sin\left(\frac{0.45\pi a}{a}\right)}^2 =$

$|\psi(0.45a)|^2 = \left|\frac{\sqrt{2}}{L}\sin\left(0.45a\,\frac{n\pi}{a}\right)\right| \longrightarrow$

$|\psi(0.55a)|^2 =$

$$P(x_1 < x < x_2) = \int_{x_1}^{x_2} \frac{2}{a}\sin^2\left(\frac{n\pi x}{a}\right) dx$$

$$= \frac{2}{a}\int_{x_1}^{x_2} \sin^2\left(\frac{n\pi x}{a}\right) dx$$

$\boxed{2\sin^2(x) = 1-\cos(x)}$

$$= \frac{1}{a}\int_{x_1}^{x_2} 1 - \cos\left(\frac{2n\pi x}{a}\right) dx$$

$$= \frac{1}{a}\left[x - \frac{a}{2n\pi}\sin\left(\frac{2n\pi x}{a}\right)\right]_{x_1}^{x_2}$$

$$= \frac{1}{a}(x_2 - x_1) - \frac{1}{2n\pi}\left(\sin\left(\frac{2n\pi x_1}{a}\right) - \sin\left(\frac{2n\pi x_2}{a}\right)\right)$$

$n_1 \Rightarrow 0.1 - \frac{1}{2n\pi}\sin(2\pi 0.45) + \frac{1}{2n\pi}\sin(2\pi 0.55) = 0.102 \Rightarrow 10\%$

$n_2 \Rightarrow$ giving up in this qst', I got the technique

f) $n \to \infty$: ~~range~~ ?

as $n \to \infty$ $P(x) \to 0.1$: classical & quantum match

8) a) $E_n = \frac{\hbar^2 \pi^2 n^2}{2mL^2} \Rightarrow L_1 = \sqrt{\frac{\hbar^2 \pi^2}{2m E}} = 4.33 \times 10^{-10}\ m$

m_e J

b) ~~ΔE 2eV~~ $E_2 = 2^2 \times E_1$

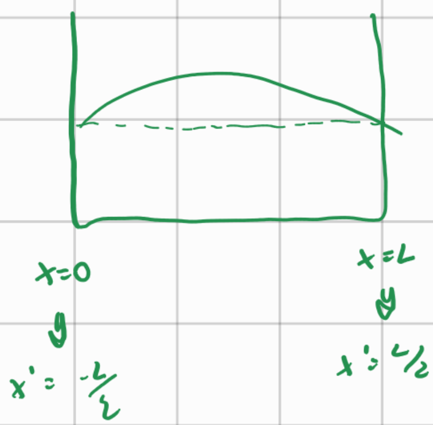
$\boxed{\Delta E = E_2 - E_1}$ ← E necessary promote e to 1^{st} excited state

9) a) 1nm 6eV $n = \sqrt{\frac{2mL^2 E_n}{\hbar^2 \pi^2}} \approx 4$

b) Exact $E_n = \frac{2^n \hbar^2 \pi^2}{2ma^2}$

Go over this qst° properly → whiteboard

10. (4.20) $\psi_n = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right)$, $E_n = \frac{\hbar^2\pi^2 n^2}{2mL^2}$: show (well def) parity



$\psi_n(x) = A\sin(k_n x)$

$k_n = \frac{n\pi}{L}$

Change of variables: $x' = x - \frac{L}{2}$

$\Rightarrow x = x' + \frac{L}{2}$

$\psi_n(x') = A \sin[k(x' + L/2)]$

$= A[\dots$

11) a) ✓ b) Electromagnetic radiat° btw $E_1 \to E_2$ jump : $\lambda_{2,1} = \frac{c}{\nu_{2,1}} = \frac{hc}{\Delta E_{2,1}}$

$\boxed{\text{TODO: 10, 12, 13, 14}}$