Q1

a) $F_1(\omega) = e^{-2\omega^2 + i6\omega}$

time shift property $f(t+\tau) \to e^{i\omega\tau} F(\omega)$

$F(\omega) = e^{-2\omega^2} e^{i6\omega}$ $\therefore \tau = 6$ $F_1(\omega) = F[g(t+6)]$

$= e^{-2\omega^2}$ $g(t) = F^{-1}[G(\omega)] = F^{-1}[G(\omega)]$

$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{i\omega t} d\omega$ $= F^{-1}[\sqrt{\pi} e^{-2\omega^2}] = \sqrt{\frac{a}{\pi}} F^{-1}[\sqrt{\frac{\pi}{a}} e^{-\omega^2/4a}]$

$= \frac{1}{2\pi}[$

$e^{-\omega^2/4a} = e^{-2\omega^2}$

$\frac{-\omega^2}{4a} = -2\omega^2$

$a = \frac{1}{8}$

b) $f_2(t) = \frac{2}{i(t+3)}$

$sgn(t) = \begin{cases} 1, & t>0 \\ -1, & t<0 \end{cases} \xrightarrow{FT} \frac{2}{i\omega}$

Symmetry : $G(t) \xrightarrow{FT} 2\pi g(-\omega)$

$f(t+3) = \frac{2}{i(t+3)}$ $\cancel{\tau = 3}$ cannot make this claim timeshift : $g(t+\tau) \xrightarrow{FT} e^{i\omega t} G(\omega)$

$F = 2\pi\, sgn(\omega) e^{i\omega 3}$

c) $f_3(t) = \int_{-\infty}^{+\infty} y e^{-y^2} e^{-|t-y|} dy$

$= \int_{-\infty}^{\infty} y e^{-|t-y| - y^2} dy = \sqrt{\pi} e^{-t} y e^{y}$

$\int y e^{y} dy$
$= y^2 e^{y} - \cancel{y^2 e^{y}} \to 0$

u.v

$e^{-t} e^{y}$

$\frac{d}{dy}$	$\int dy$
y	$e^{-\lvert t-y \rvert} e^{-y^2}$
1	$e^{-\lvert t-y \rvert} \sqrt{\pi}$
0	$\sqrt{\pi} \int e^{-t+y} dy$

$- \int y e^{y} e^{-t} \sqrt{\pi} dy = e^{-|t-y|} \sqrt{\pi}$

$= \sqrt{\pi} e^{-t} y e^{y}$

$f_3(t) = \sqrt{\pi} e^{-t} e^{y} = (\sqrt{\pi} e^{y}) e^{-t}$

$\frac{1}{\sqrt{\pi} e^{y}} f(t) = e^{-t}$

$F[e^{-t}] = \int_{-\infty}^{\infty} e^{-t} e^{-i\omega t} dt = \int_{-\infty}^{\infty} e^{i\omega t 2} dt$ NOT A ROUTE

CONVOLUTᴼ : $\int_{-\infty}^{\infty} f(y) g(x-y) dy = \int_{-\infty}^{\infty} f(x-y) g(y) dy \xrightarrow{FT} F(\omega) G(\omega)$

$e^{-at^2} \to \sqrt{\frac{\pi}{a}} e^{-\omega^2/4a}$
$a = 1$

$f(y) = y e^{-y^2} \xrightarrow{FT} i\sqrt{\pi} \frac{d e^{-\omega^2/4}}{d\omega} = i\sqrt{\pi}[\frac{1}{2} \omega e^{-\omega^2/4}] = \frac{i}{4}\omega \frac{\sqrt{\pi}}{2} e^{-\omega^2/4}$

$= \frac{\omega}{2} e^{-\omega^2/4}$

$g(y) = e^{-|y|} \xrightarrow{FT} 2\pi\delta(\omega - i)$

$e^{-|t|} = \frac{2}{1+i\omega}$

$i\omega_0 \to \omega_0 = i$

$\therefore F(\omega) G(\omega) = -\frac{\sqrt{\pi}}{\cancel{2}} e^{-\omega^2/4} \frac{\cancel{2}}{1+i\omega}$ $= \frac{\omega \frac{i}{4} \sqrt{\pi} e^{-\omega^2/4}}{1+i\omega}$

Q2

Aliasing $f_s > 2f_m$

$f = \frac{1}{\Delta t}$ ✓

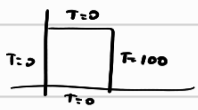
Q3 Linear operator: $L(a\Psi + b\Phi) = aL(\Psi) + bL(\Phi)$

LHS: $L(a\Psi+b\Phi) \rightarrow (\Psi+\Phi)(a\Psi+b\Phi)+1 = a\Psi^2 + b\Phi\Psi + a\Psi\Phi + b\Psi^2 + 1$

RHS: $aL(\Psi) + bL(\Phi) \rightarrow \Psi(a\Psi) + \Phi(b\Phi) + 2 = a\Psi^2 + b\Phi^2 + 2$

Not linear

Q4



a) $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$ $\qquad T(x,y) = \frac{\sin(y)\sinh(x)}{\sinh(\pi)} = X(x)\,Y(y)$ $\qquad X(x) = \frac{\sinh(x)}{\sinh(\pi)}$, $Y(y) = \sin(y)$

b) $\alpha_n = \frac{1}{\sinh(\pi)}$

c) $f(y) = \begin{cases} 100 & 0<y<\pi \\ -100 & -\pi<y<0 \end{cases}$

$f(y+2\pi) = f(y)$

Time shift $g(t+\tau) \rightarrow e^{i\omega\tau} G(\omega)$

$\tau = 2\pi$

$F[f(y)] = \int_{-\pi}^{0} -100\,dy + \int_{0}^{\pi} 100\,dy$

$= 200\pi$

$\therefore F = e^{i\omega 2\pi}\, 200\pi$

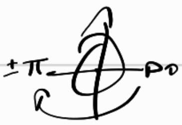
series ≠ transform

$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)\right]$

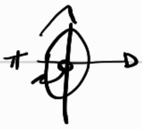
$a_n = \frac{2}{T}\int_{\theta}^{\theta+T} f(t)\cos(n\omega_0 t)\,dt$

$T = 2\pi,\ \theta = -\pi \therefore = \frac{2}{2\pi}\int_{-\pi}^{\pi} f(t)\cos(n\omega_0 t)\,dt$

$= \frac{1}{\pi}\left[-100\int_{-\pi}^{0}\cos(n\omega_0 t)\,dt + 100\int_{0}^{\pi}\cos(n\omega_0 t)\,dt\right]$



$a_n = \frac{100}{\pi n\omega_0}\left(\sin(-n\omega_0\pi) + \sin(n\omega_0\pi)\right)$



$b_n = \frac{2}{2\pi}\int_{-\pi}^{\pi} f(t)\sin(n\omega_0 t)\,dt$

$= \frac{2}{7\pi}\left[\frac{200\cos(n\omega_0\pi)}{n\omega_0}\right]$

Q5

a) $y(x) = \sum_{s=0}^{\infty} a_s x^{s+q}$

$y = a_s x^{s+q}$

$y' = a_s(s+q)x^{s+q-1}$

$y'' = a_s(s+q)(s+q-1)x^{s+q-2}$

$a_s(s+q)(s+q-1)x^{s+q-2} + a_s(s+q)x^{s+q+2} + \alpha a_s x^{s+q+2} y = 0$

$a_s(s+q)(s+q-1)x^{s+q-2} + a_s(s+q)x^{s+q+2} = -\alpha a_s x^{s+q+2} y$

$\frac{\cancel{a_s}(s+q)(s+q-1)x^{\cancel{s+q}-2} + \cancel{a_s}(s+q)x^{\cancel{s+q}+2}}{-\alpha\cancel{a_s}x^{\cancel{s+q}+2}} = y$

$\frac{(s+q)(s+q-1)x^{-2} + (s+q)x^2}{-\alpha x^2} = y$

not what u have to do

a) $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + ky = 0$

trial solutⁿ: $y(x) = \sum_{s=0}^{\infty} a_s x^{s+q}$

$\frac{dy}{dx} = \sum_{s=0}^{\infty} a_s(s+q)x^{s+q-1}$

$\frac{d^2y}{dx^2} = \sum_{s=0}^{\infty} a_s(s+q)(s+q-1)x^{s+q-2}$

$\sum_{s=0}^{\infty}(1-x^2)a_s(s+q)(s+q-1)x^{s+q-2}$

$+ 2a_s(s+q)x^{s+q}$

$+ ka_s x^{s+q} = 0$

$= \sum_{s=0}^{\infty} a_s\left[(s+q)\left[(s+q-1)x^{s+q-2} - (s+q-1)x^{s+q} + 2x^{s+q}\right] + ka_s x^{s+q}\right] = 0$

indicial eqtⁿ: coeff lowest power $\Sigma s=0 \sim x^{s+q-2}$ with $s=0$

$$a_0 q(q-1) x^{q-2}$$

$$a \neq 0$$

$$\downarrow$$

indicial $a \neq 0$: $q(q-1)x^{q-2} = 0 \rightarrow q = 0$, $q = 1$

b) $a_{n+2} = \dfrac{-w^2}{(q+t+2)(q+t+1)} a_t$

Q6 $\dfrac{\partial^2 u}{\partial x^2} = \dfrac{1}{c} \dfrac{\partial u}{\partial t}$

a) i) V = volume

S = surface

dS = over surface

ii) $\nabla^2 T$