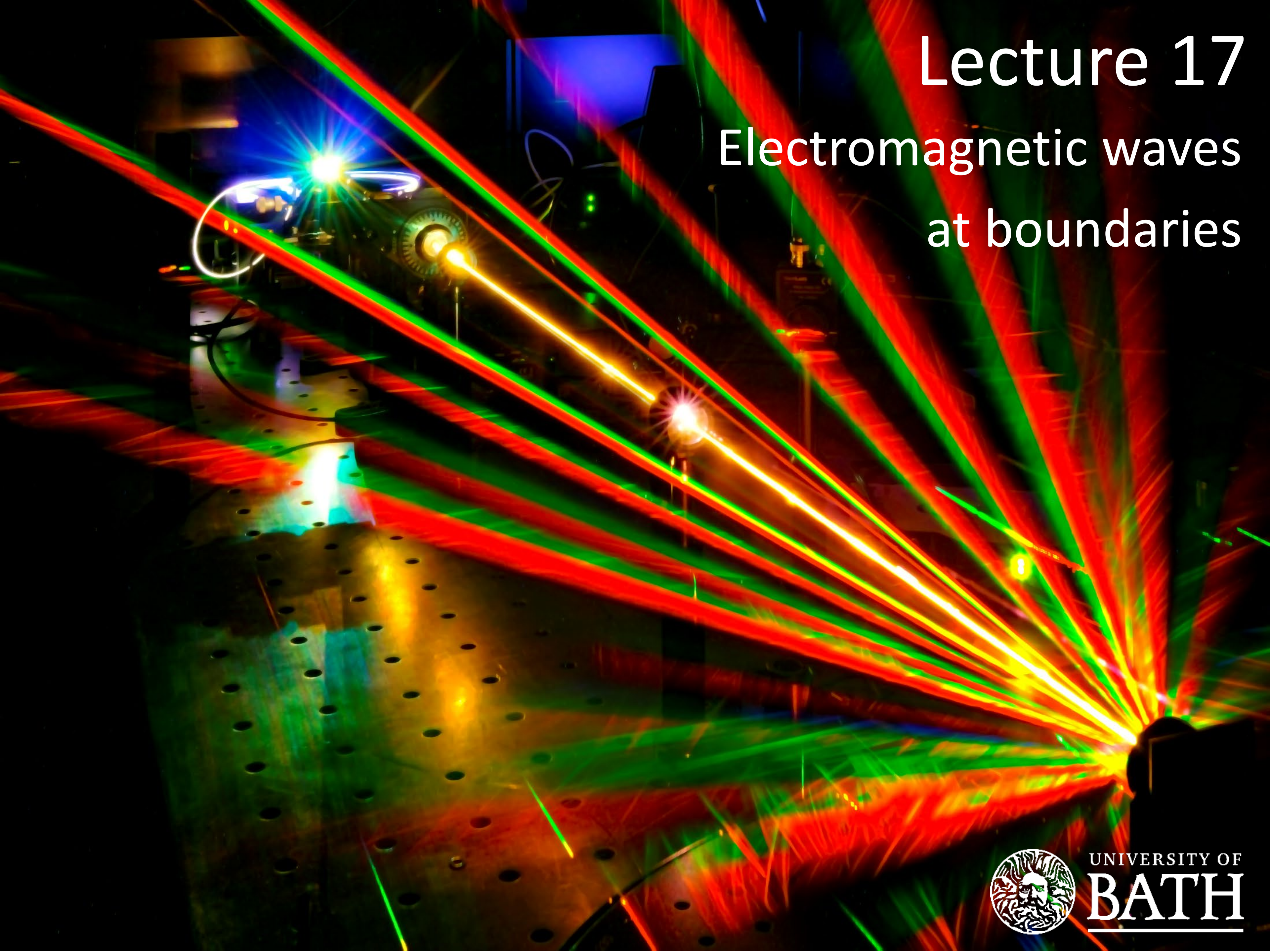
Lecture 17

Electromagnetic waves at boundaries



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We saw that

In LIH materials, Maxwell's equations become: $\nabla \cdot \vec{D} = \rho_f \qquad \nabla \cdot \vec{B} = 0$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \nabla \times \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t}$$

In an ideal LIH dielectric, the electromagnetic wave propagates similarly to the way it does in free space – it remains a plane wave but its phase velocity is lowered.

$$v_p = \frac{c}{\sqrt{\mu_r \varepsilon_r}}$$

The wave equation $\nabla^2 \vec{E} = \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}$ becomes $\nabla^2 \vec{E} = \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} + \mu\sigma \frac{\partial \vec{E}}{\partial t}$, which is the modified wave equation.

A solution to the modified wave equation is $\vec{E} = \vec{E}_0 e^{(\gamma x - i\omega t)}$ with $\gamma = -\alpha + i\beta$

So: $\vec{E} = \vec{E}_0 e^{-\alpha x} e^{i(\beta x - \omega t)}$ Upon solving, an important ratio appears: $\sigma/\varepsilon\omega$

This ratio is part of the dielectric function of the material: $\varepsilon(\omega) = \varepsilon_r(\omega) + i\frac{\sigma(\omega)}{\varepsilon_0 \omega}$

We saw that

and of its complex refractive index: $\tilde{n} = n + i\eta = \sqrt{\varepsilon(\omega)} = \sqrt{\varepsilon' + i\varepsilon''}$

This ratio is the loss tangent: $\tan\theta = \tan 2\varphi = \dfrac{\sigma}{\omega\varepsilon} = \left|\dfrac{\text{ohmic current}}{\text{displacement current}}\right|$

When an EM wave propagates in a lossy dielectric, its amplitude decays at a rate α. The electric and magnetic fields oscillate with a phase shift described by the angle φ in $\tan\varphi = \alpha/\beta$.

Altogether we have: $\begin{cases} \vec{E}(x) = |Z|\vec{H}_0 e^{-\alpha x} e^{i(\beta x - \omega t)} \\ \vec{H}(x) = \vec{H}_0 e^{-\alpha x} e^{i(\beta x - \omega t + \varphi)} \end{cases}$

In poor conductors, we have $\dfrac{\sigma}{\varepsilon\omega} \ll 1$. The EM plane wave decays slowly. The *E*- and *H*-fields are in phase.

In good conductors, we have $\dfrac{\sigma}{\varepsilon\omega} \gg 1$. The EM wave decays rapidly. The *E*- and *H*-fields are no longer in phase.

We defined the skin depth as: $\delta = \sqrt{\dfrac{2}{\mu\sigma\omega}} = \alpha^{-1}$

Special case II: Good conductors

1. Loss tangent: $\tan\theta = \dfrac{\sigma}{\varepsilon\omega} \gg 1 \rightarrow \theta \approx \dfrac{\pi}{2}$

2. Phase angle: $\varphi = \dfrac{1}{2}\theta \approx \dfrac{\pi}{4}$

3. Propagation parameters: $\alpha \approx \beta \approx \sqrt{\dfrac{\mu\sigma\omega}{2}}$

4. The phase velocity: $v_p = \dfrac{\omega}{\text{wave vector}}$, so $v_p = \dfrac{\omega}{\beta} = \dfrac{\sqrt{\omega^2}}{\sqrt{\dfrac{\mu\sigma\omega}{2}}} = \omega\sqrt{\dfrac{2}{\mu\sigma\omega}} = \omega\delta$

where $\boxed{\delta = \sqrt{\dfrac{2}{\mu\sigma\omega}} = \alpha^{-1}}$ is the **skin depth**, in units of meters.

$$\delta = \sqrt{\frac{2}{\mu\sigma 2\pi f}} \propto \sqrt{\frac{2\left[kg \cdot m^3 \cdot s^{-3} \cdot A^{-2}\right]}{\left[\left(kg \cdot m^2 \cdot s^{-2} \cdot A^{-2}\right)\cdot\left(m^{-1}\right)\right] 2\pi\left[s^{-1}\right]}} \propto \sqrt{\frac{[m]^2}{\pi}} \propto [m]$$

How about the impedance?

Overview

In this Lecture we will look at:

- ❑ Boundary conditions for the electric flux density
- ❑ Boundary conditions for the electric field strength
- ❑ Boundary conditions for the magnetic flux density
- ❑ Boundary conditions for the magnetic field strength
- ❑ Boundary conditions summary

Introduction

We now know how EM waves propagate in a bulk material.

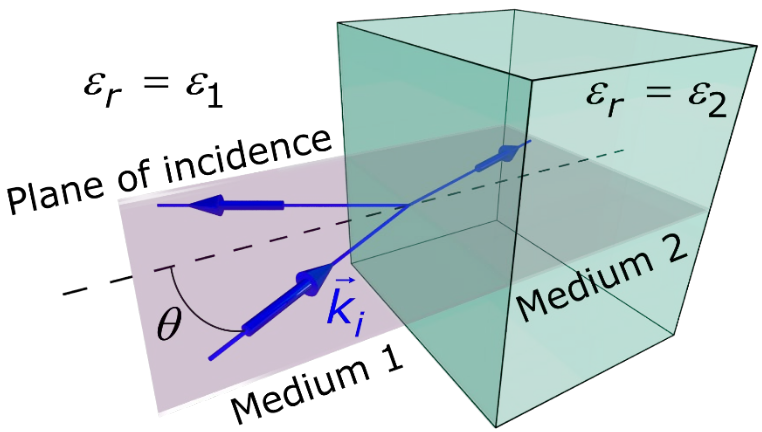
However, what happens when an EM wave encounters a boundary between two materials?

As part of our discussion, we shall see how (some of) the principles of ray (geometrical) optics arise from Maxwell's equations.

In our treatment, we will mostly assume that the boundary contains **no free charges** and that **no surface currents** flow at the boundary.

Let's look at the electric flux density!

Boundary conditions for the electric flux density



The plane of incidence and the boundary between two media.

The interface is free of charge and, in LIH materials, we have

$$\vec{D} = \varepsilon_0 \varepsilon_r \vec{E} = \varepsilon \vec{E}$$

So, we can easily switch between the *E*- and the *D*-fields.

Since we have no free charges:

$$\nabla \cdot \vec{D} = \rho_f$$

becomes:

$$\nabla \cdot \vec{D} = 0$$

We can now integrate both sides over any volume V:

$$\nabla \cdot \vec{D} = 0 \rightarrow \int_V \left(\nabla \cdot \vec{D}\right) dV = \int_V (0)\, dV = 0$$

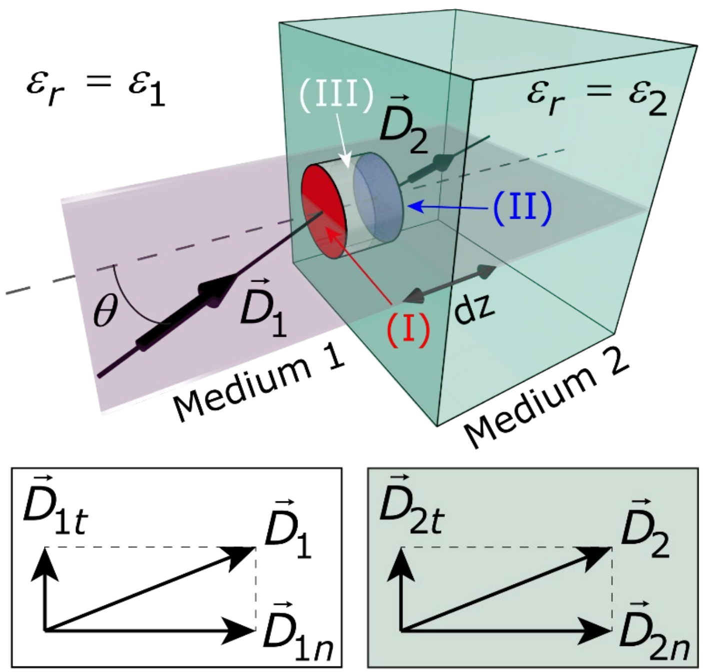
And we can apply the divergence theorem:

$$\int_V \left(\nabla \cdot \vec{D}\right) dV = 0 \rightarrow \int_A \vec{D} \cdot d\vec{A} = 0$$

where A is the surface of V.

This is an expression of Gauss' law. Where can we apply it?

Boundary conditions for the electric flux density



The electric flux density vectors at the boundary between two media. These vectors can be resolved into **normal** and **tangential** components. We consider a cylindrical Gaussian surface crossing the boundary.

We apply Gauss' law to the cylinder:

$$0 = \oint_A \vec{D} \cdot d\vec{A} =$$

$$= \int_{A_I} \left(\vec{D}_{1n} + \vec{D}_{1t}\right) \cdot d\vec{A}_I + \int_{A_{II}} \left(\vec{D}_{2n} + \vec{D}_{2t}\right) \cdot d\vec{A}_{II} +$$

$$+ \int_{A_{III}} \left(\vec{D}_{1n} + \vec{D}_{1t} + \vec{D}_{2n} + \vec{D}_{2t}\right) \cdot d\vec{A}_{III}$$

As they are perpendicular: $\vec{D}_{1t} \cdot d\vec{A}_I = 0$

By integrating:

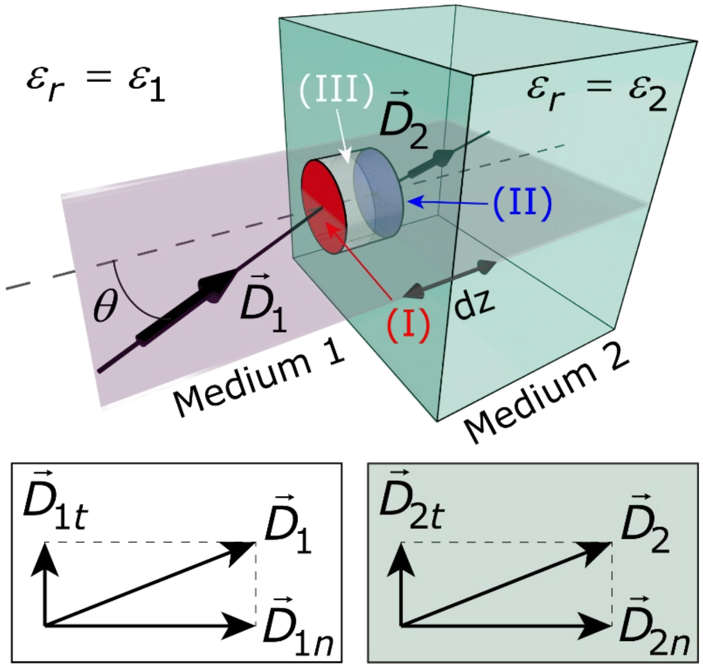
$$\int_{A_I} \vec{D}_{1n} \cdot d\vec{A}_I = \vec{D}_{1n} \cdot \int_{A_I} d\vec{A}_I =$$

$$= \vec{D}_{1n} \cdot \vec{A}_I = -D_{1n} A_I$$

As they are antiparallel.

How about D_2?

Boundary conditions for the electric flux density



The electric flux density vectors at the boundary between two media. These vectors can be resolved into **normal** and **tangential** components. We consider a cylindrical Gaussian surface crossing the boundary.

Similarly:

$$\vec{D}_{2t} \cdot d\vec{A}_2 = 0$$

By integrating:

$$\int_{A_{II}} \vec{D}_{2n} \cdot d\vec{A}_{II} = \vec{D}_{2n} \cdot \vec{A}_{II} = D_{2n} A_{II}$$

(positive) as they are parallel.

For an infinitely short cylinder A_{III} vanishes, so:

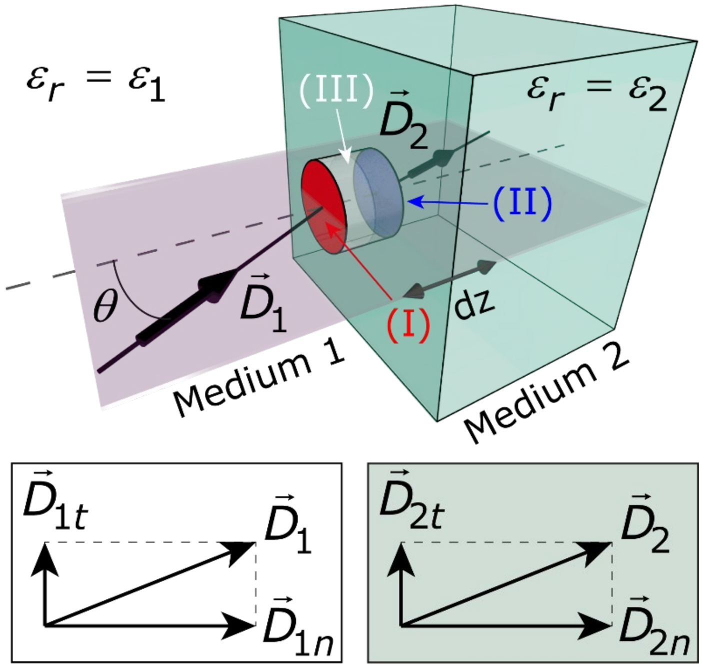
$$\oint_{A_{III}} \left(\vec{D}_{1n} + \vec{D}_{1t} + \vec{D}_{2n} + \vec{D}_{2t} \right) \cdot d\vec{A}_{III} \approx 0$$

Moreover, A_{II} and A_{II} are same size, so:

$$D_{2n} A_{II} = D_{2n} A_{I}$$

What do we now have left for Gauss’ theorem?

Boundary conditions for the electric flux density



The electric flux density vectors at the boundary between two media. These vectors can be resolved into **normal** and **tangential** components. We consider a cylindrical Gaussian surface crossing the boundary.

Altogether, we now have:

$$0 = \oint_A \vec{D} \cdot d\vec{A} = -D_{1n} A_I + D_{2n} A_I$$

Which means:

$$\left(D_{1n} - D_{2n}\right) A_I = 0$$

So,

$$D_{1n} = D_{2n}$$

As those are parallel and similarly oriented, we can write:

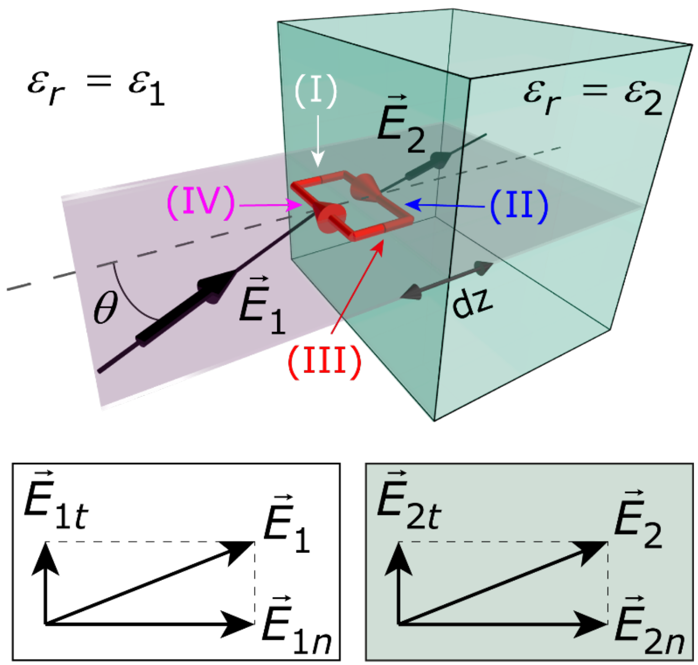
$$\boxed{\vec{D}_{1n} = \vec{D}_{2n}}$$

✓ components preserved at an interface

The **normal component** of $\vec{D}$ is **continuous** across the boundary.

How about the electric field strength?

Boundary conditions for the electric field strength



The electric field strength vectors at the boundary between two media. These vectors can be resolved into **normal** and **tangential** components. We consider a rectangular loop across the boundary, its four sides are labelled with Roman numerals.

We start with Maxwell's equation:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

We integrate over any surface A that is within a loop crossing the boundary

$$\int_A \left(\nabla \times \vec{E}\right) d\vec{A} = \int_A \left(-\frac{\partial \vec{B}}{\partial t}\right) d\vec{A} = -\frac{\partial}{\partial t}\int_A \vec{B} \cdot d\vec{A}$$

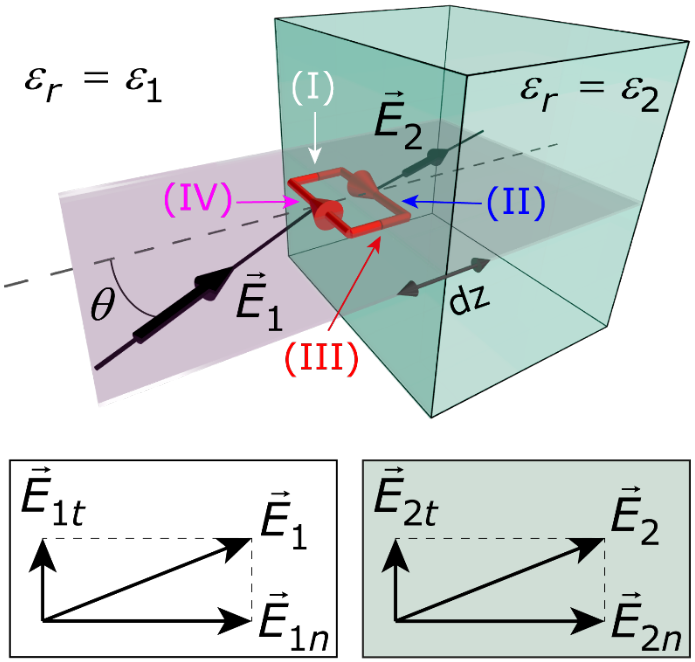
We then apply Stoke's theorem:

$$\oint_L \vec{E} \cdot d\vec{L} = -\frac{\partial}{\partial t}\int_A \vec{B} \cdot d\vec{A}$$

We now consider two vectors $\vec{E}_1$ and $\vec{E}_2$ in the two different media.

Let's replace with the elements of the loop!

Boundary conditions for the electric field strength



The electric field strength vectors at the boundary between two media. These vectors can be resolved into **normal** and **tangential** components. We consider a rectangular loop across the boundary, its four sides are labelled with Roman numerals.

The loop has four segments, so:

$$\oint_L \vec{E}\cdot d\vec{L} = \int_{L_{IV}} \left(\vec{E}_{1n} + \vec{E}_{1t}\right)\cdot d\vec{L}_{IV} +$$

$$+\int_{L_{II}} \left(\vec{E}_{2n} + \vec{E}_{2t}\right)\cdot d\vec{L}_{II} +$$

$$+\int_{L_{I}} \left(\vec{E}_{1n} + \vec{E}_{1t} + \vec{E}_{2n} + \vec{E}_{2t}\right)\cdot d\vec{L}_{I} +$$

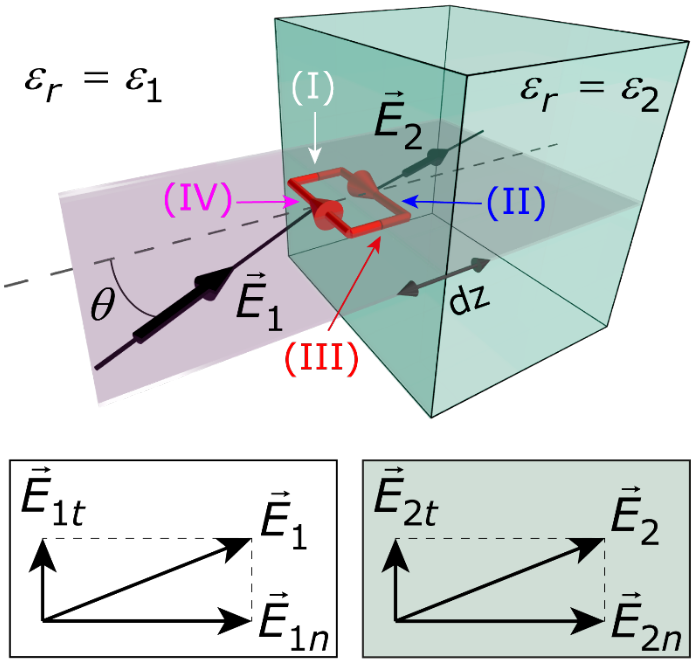
$$+\int_{L_{III}} \left(\vec{E}_{1n} + \vec{E}_{1t} + \vec{E}_{2n} + \vec{E}_{2t}\right)\cdot d\vec{L}_{III}$$

The length vectors are oriented along the loop and following the arrows, so:

$\vec{E}_{1n}\cdot d\vec{L}_{IV} = 0$ and $\vec{E}_{2n}\cdot d\vec{L}_{II} = 0$

What if we shrink the loop?

Boundary conditions for the electric field strength



The electric field strength vectors at the boundary between two media. These vectors can be resolved into **normal** and **tangential** components. We consider a rectangular loop across the boundary, its four sides are labelled with Roman numerals.

The loop has four segments, so:

$$\oint_L \vec{E} \cdot d\vec{L} = \int_{L_{IV}} \vec{E}_{1t} \cdot d\vec{L}_{IV} +$$

$$+ \int_{L_{II}} \vec{E}_{2t} \cdot d\vec{L}_{II} +$$

$$+ \int_{L_I} \left(\vec{E}_{1n} + \vec{E}_{1t} + \vec{E}_{2n} + \vec{E}_{2t}\right) \cdot d\vec{L}_I +$$

$$+ \int_{L_{III}} \left(\vec{E}_{1n} + \vec{E}_{1t} + \vec{E}_{2n} + \vec{E}_{2t}\right) \cdot d\vec{L}_{III}$$

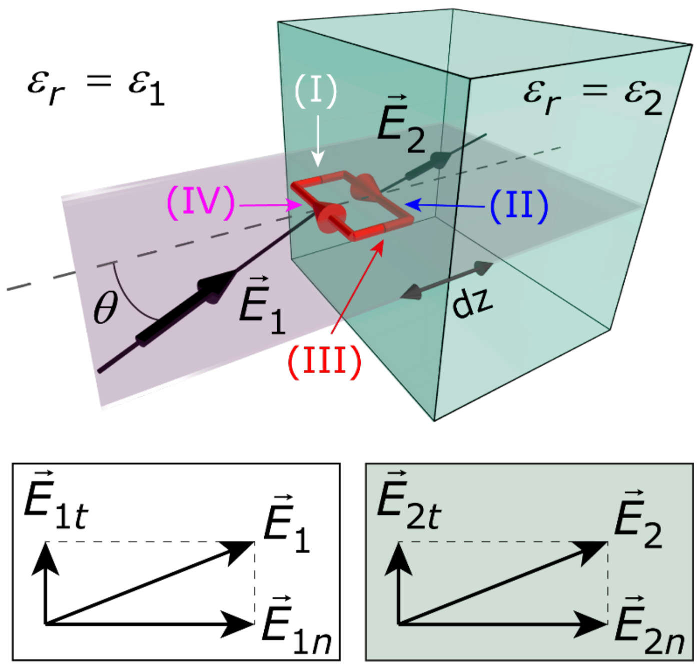
When we shrink the loop I and $III \to 0$

$$\int_{L_I} \left(\vec{E}_{1n} + \vec{E}_{1t} + \vec{E}_{2n} + \vec{E}_{2t}\right) \cdot d\vec{L}_I \approx 0 \quad \text{and}$$

$$\int_{L_{III}} \left(\vec{E}_{1n} + \vec{E}_{1t} + \vec{E}_{2n} + \vec{E}_{2t}\right) \cdot d\vec{L}_{III} \approx 0$$

What is left?

Boundary conditions for the electric field strength



The electric field strength vectors at the boundary between two media. These vectors can be resolved into **normal** and **tangential** components. We consider a rectangular loop across the boundary, its four sides are labelled with Roman numerals.

Altogether, we now have:

$$\oint_L \vec{E} \cdot d\vec{L} = \int_{L_{IV}} \vec{E}_{1t} \cdot d\vec{L}_{IV} + \int_{L_{II}} \vec{E}_{2t} \cdot d\vec{L}_{II}$$

Where *II* and *IV* are the same but opposite. For continuous fields:

$$\int_{L_{II}} \vec{E}_{2t} \cdot d\vec{L}_{II} = \vec{E}_{2t} \cdot \int_{L_{II}} d\vec{L}_{II} =$$

$$= \vec{E}_{2t} \cdot \vec{L}_{II} = -E_{2t} L_{II}$$

Because the vectors are antiparallel. Also:

$$\int_{L_{IV}} \vec{E}_{1t} \cdot d\vec{L}_{IV} = \vec{E}_{1t} \cdot \int_{L_{IV}} d\vec{L}_{IV} =$$

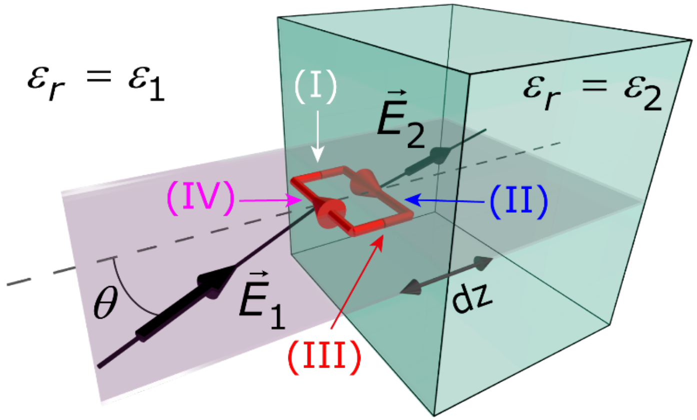
$$= \vec{E}_{1t} \cdot \vec{L}_{IV} = E_{1t} L_{IV}$$

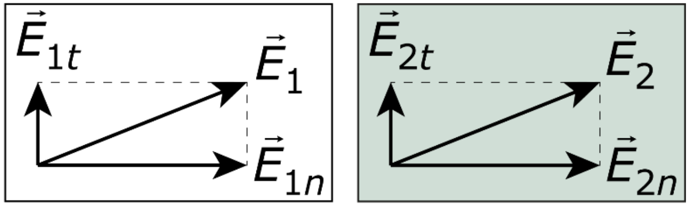
Because the vectors are parallel. So:

$$L_{II} = L_{IV}$$

Now back to Stoke's theorem!

Boundary conditions for the electric field strength





The electric field strength vectors at the boundary between two media. These vectors can be resolved into **normal** and **tangential** components. We consider a rectangular loop across the boundary, its four sides are labelled with Roman numerals.

Using Stoke's theorem:

$$\oint_L \vec{E}\cdot d\vec{L} = -E_{2t}L_{II} + E_{1t}L_{II} = -\frac{\partial}{\partial t}\int_A \vec{B}\cdot d\vec{A}$$

When the loop shrinks $dA \to 0$, so:

$$\int_A \vec{B}\cdot d\vec{A} \approx 0$$

We are left with:

$$-E_{2t}L_{II} + E_{1t}L_{II} = 0 \to \left(E_{2t} - E_{1t}\right)L_{II} = 0$$

Finally: $E_{2t} = E_{1t}$

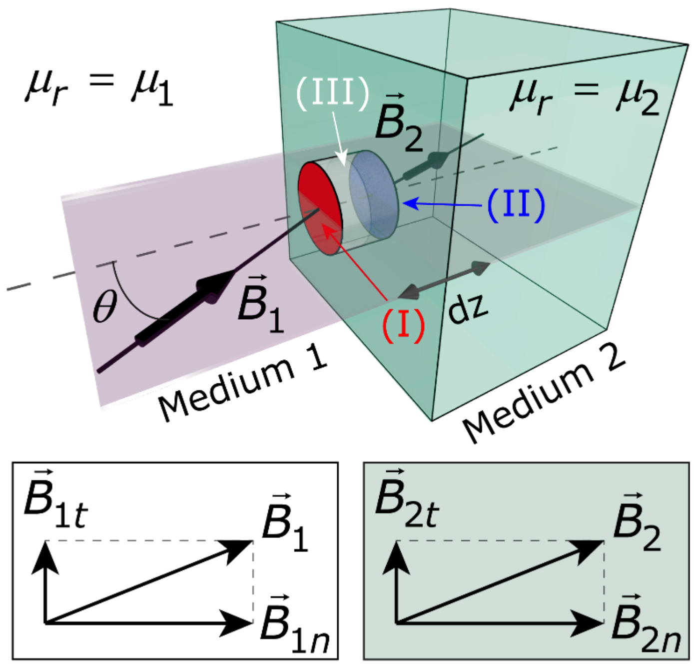
And because they are parallel:

$$\boxed{\vec{E}_{1t} = \vec{E}_{2t}}$$

The tangential component of $\vec{E}$ is continuous across the boundary

How about the magnetic flux density?

Boundary conditions for the magnetic flux density



The magnetic flux density vectors at the boundary between two media. These vectors can be resolved into **normal** and **tangential** components. We consider a cylindrical Gaussian surface crossing the boundary.

From Maxwell's equations: $\nabla \cdot \vec{B} = 0$

We can integrate both sides over a volume V:

$$\int_V \left(\nabla \cdot \vec{B}\right) dV = \int_V (0)\, dV = 0$$

Next, we apply the divergence theorem:

$$\int_V \left(\nabla \cdot \vec{B}\right) dV = 0 \rightarrow \int_A \vec{B} \cdot d\vec{A} = 0$$

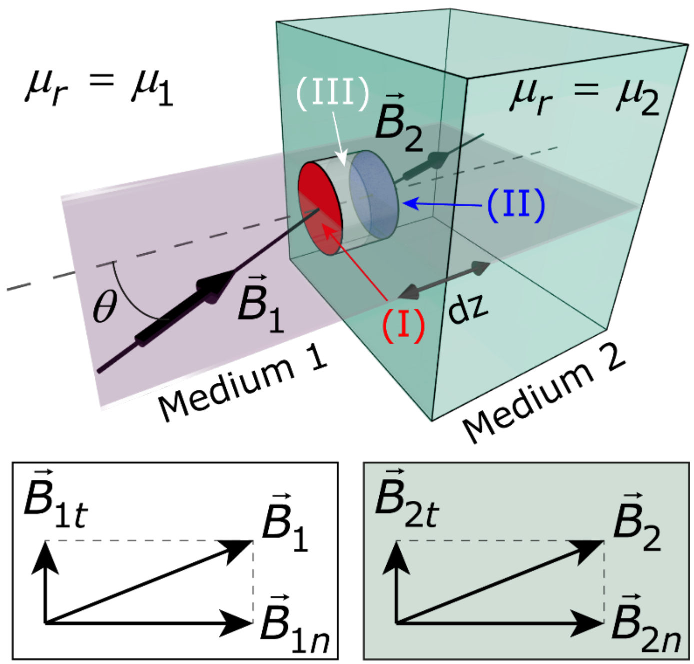
Where A is the surface of V.

As we did with the electric flux density, here we can use Gauss' theorem on the shrinking cylinder.

[You can work it out]

So what do we get?

Boundary conditions for the magnetic flux density



The magnetic flux density vectors at the boundary between two media. These vectors can be resolved into **normal** and **tangential** components. We consider a cylindrical Gaussian surface crossing the boundary.

Following the same logic as in the case of the electric flux density:

$$0 = \oint_A \vec{B} \cdot d\vec{A} = -B_{1n}A_I + B_{2n}A_I$$

Which leads to:

$$\left(B_{1n} - B_{2n}\right) A_I = 0$$

and

$$B_{1n} = B_{2n}$$

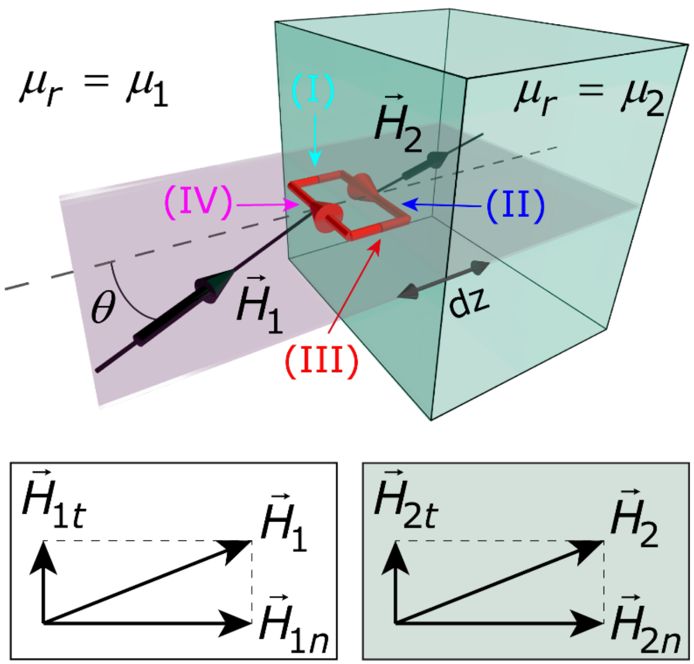
As they are parallel and directed similarly:

$$\boxed{\vec{B}_{1n} = \vec{B}_{2n}}$$

The **normal component** of $\vec{B}$ is **continuous** across the boundary.

How about the magnetic field strength?

Boundary conditions for the magnetic field strength



The magnetic field strength vectors at the boundary between two media. These vectors can be resolved into **normal** and **tangential** components. We consider a rectangular loop across the boundary, its four sides are labelled with Roman numerals

From Maxwell's equations:

$$\nabla \times \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t}$$

We can integrate both sides over a surface that is enclosed by a rectangular loop across the boundary:

$$\int_A \left(\nabla \times \vec{H}\right) \cdot d\vec{A} = \int_A \left(\vec{J}_f + \frac{\partial \vec{D}}{\partial t}\right) \cdot d\vec{A}$$

Then we can apply Stoke's theorem:

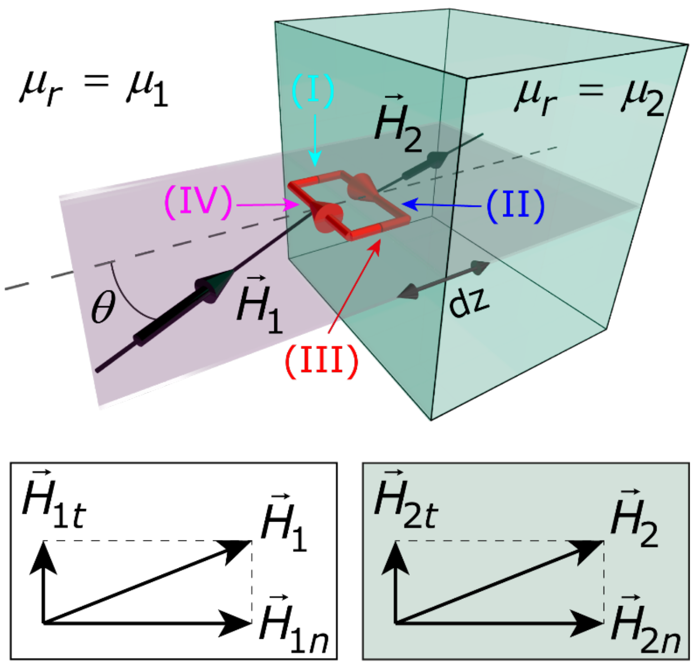
$$\int_A \left(\nabla \times \vec{H}\right) \cdot d\vec{A} = \oint_L \vec{H} \cdot d\vec{L}$$

Then we proceed as we did with the electric field strength.

[You can work it out]

What do we obtain?

Boundary conditions for the magnetic field strength



The magnetic field strength vectors at the boundary between two media. These vectors can be resolved into **normal** and **tangential** components. We consider a rectangular loop across the boundary, its four sides are labelled with Roman numerals

We obtain:

$$\oint_L \vec{H} \cdot d\vec{L} = -H_{2t}L_{II} + H_{1t}L_{IV} =$$

$$= \int_A \left(\vec{J}_f + \frac{\partial \vec{D}}{\partial t} \right) \cdot d\vec{A}$$

When the loop shrinks $dA \to 0$, so:

$$\int_A \left(\vec{J}_f + \frac{\partial \vec{D}}{\partial t} \right) \cdot d\vec{A} \approx 0$$

Therefore: $H_{2t} = H_{1t}$

And since they are parallel and similar:

$$\boxed{\vec{H}_{1t} = \vec{H}_{2t}}$$

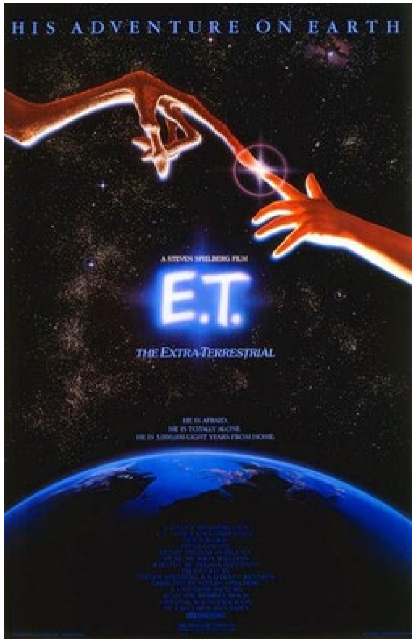
The **tangential component** of $\vec{H}$ is **continuous** across the boundary.

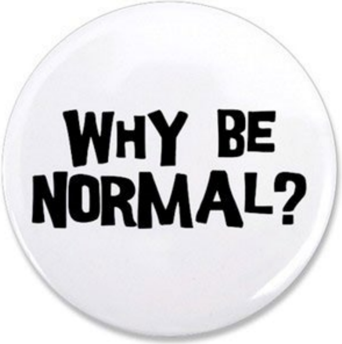
Let's summarise the boundary conditions!

Boundary conditions summary

For **LIH materials**, assuming **no surface charges** and **no surface currents**:

	Electric fields	Magnetic fields
Normal components	$\vec{D}_{1n} = \vec{D}_{2n}$	$\vec{B}_{1n} = \vec{B}_{2n}$
Tangential components	$\vec{E}_{1t} = \vec{E}_{2t}$	$\vec{H}_{1t} = \vec{H}_{2t}$





Now for a couple of question:

Example question

[from Sadiku] The plane z=0 separates air (region 1, z≥0, $\mu = \mu_0$) from iron (region 2, z≤0, $\mu = 200\mu_0$). Given that $\vec{H} = 10\vec{a}_x + 15\vec{a}_y - 3\vec{a}_z$ Am^{-1} in air, find $\vec{B}$ in iron and the angle it makes with the interface.

Example question

[from Sadiku] The plane z=0 separates air (region 1, z≥0, $\mu = \mu_0$) from iron (region 2, z≤0, $\mu = 200\mu_0$). Given that $\vec{H} = 10\vec{a}_x + 15\vec{a}_y - 3\vec{a}_z$ Am^{-1} in air, find $\vec{B}$ in iron and the angle it makes with the interface.

We have $\vec{H}_{1n} = -3\vec{a}_z$ and $\vec{H}_{1t} = \vec{H}_1 - \vec{H}_{1n} = 10\vec{a}_x + 15\vec{a}_y$

$$\vec{H}_{2t} = \vec{H}_{1t} = 10\vec{a}_x + 15\vec{a}_y$$

$\vec{B}_{1n} = \vec{B}_{2n}$ and $\vec{B} = \mu\vec{H}$, so $\vec{H}_{2n} = \dfrac{\mu_1}{\mu_2}\vec{H}_{1n} = \dfrac{1}{200}\left(-3\vec{a}_z\right) = -0.015\vec{a}_z$

$$\vec{H}_2 = \vec{H}_{2n} + \vec{H}_{2t} = 10\vec{a}_x + 15\vec{a}_y - 0.015\vec{a}_z$$

$$\vec{B}_2 = \mu_2\vec{H}_2 = 200 \times 4\pi \times 10^{-7}\left(10, 15, -0.015\right)$$

$$\vec{B}_2 = \underline{\underline{2.51\vec{a}_x + 3.77\vec{a}_y - 0.0037\vec{a}_z}}\ \mathrm{mWbm}^{-2}$$

Next, $\tan\alpha = \dfrac{\left\|\vec{B}_{2n}\right\|}{\left\|\vec{B}_{2t}\right\|}$ and $\alpha = \tan^{-1}\dfrac{\left\|\mu_2\vec{H}_{2n}\right\|}{\left\|\mu_2\vec{H}_{2t}\right\|} = \tan^{-1}\dfrac{(0.015)}{\sqrt{10^2 + 15^2}} \approx \underline{\underline{0.048^\circ}}$

Example question

[from Sadiku] The region 1, described by $3x+4y\geq10$, is free space. The region 2, described by $3x+4y\leq10$ is a magnetic material for which $\mu = 10\mu_0$. Assuming that the boundary between the material and free space is current free, find $\vec{B}_2$ if

$$\vec{B}_1 = 0.1\vec{a}_x + 0.4\vec{a}_y + 0.2\vec{a}_z \text{ Wbm}^{-2}.$$

Example question

[from Sadiku] The region 1, described by 3x+4y≥10, is free space. The region 2, described by 3x+4y≤10 is a magnetic material for which $\mu = 10\mu_0$. Assuming that the boundary between the material and free space is current free, find $\vec{B}_2$ if

$$\vec{B}_1 = 0.1\vec{a}_x + 0.4\vec{a}_y + 0.2\vec{a}_z \text{ Wbm}^{-2}.$$

We set: $f(x,y) = 3x + 4y - 10$, then the unit vector: $\vec{a}_n = \frac{\nabla f}{|\nabla f|} = \frac{3\vec{a}_x + 4\vec{a}_y}{\sqrt{3^2 + 4^2}} = \frac{3\vec{a}_x + 4\vec{a}_y}{5}$

Remembering $\vec{a}_1 = (\vec{a} \cdot \hat{b})\hat{b} = \frac{\vec{a} \cdot \vec{b}}{\|\vec{b}\|}\frac{\vec{b}}{\|\vec{b}\|} = \frac{\vec{a} \cdot \vec{b}}{\|\vec{b}\|^2}\vec{b} = \frac{\vec{a} \cdot \vec{b}}{\vec{b} \cdot \vec{b}}\vec{b}$

$$\vec{B}_{1n} = (\vec{B}_1 \cdot \vec{a}_n)\vec{a}_n = \left[(0.1, 0.4, 0.2) \cdot \frac{(3,4,0)}{5}\right]\frac{(3,4,0)}{5} = 0.228\vec{a}_x + 0.304\vec{a}_y = \underline{\vec{B}_{2n}}$$

$$\vec{B}_{1t} = \vec{B}_1 - \vec{B}_{1n} = (0.1, 0.4, 0.2)(0.228, 0.304, 0) = -0.128\vec{a}_x + 0.096\vec{a}_y + 0.2\vec{a}_z$$

$$\vec{B}_{2t} = \frac{\mu_2}{\mu_1}\vec{B}_{1t} = 10\vec{B}_{1t} = -1.28\vec{a}_x + 0.96\vec{a}_y + 2\vec{a}_z$$

$$\vec{B}_2 = \underline{\vec{B}_{2n}} + \vec{B}_{2t} = \underline{\underline{-1.052\vec{a}_x + 1.264\vec{a}_y + 2\vec{a}_z}} \text{ Wbm}^{-2}.$$

Example question

[from Sadiku] [harder] The region $y \leq 0$ consists of a perfect conductor, while the region $y \geq 0$ is a LIH dielectric medium $\varepsilon_{1r} = 2$. If there is a surface charge (σ_p) of 2 nC/m^2 on the conductor, determine $\vec{E}$ and $\vec{B}$ at: (a) A(3,-2,2) (b) B(-4,1,5).

Example question

[from Sadiku] [harder] The region $y \leq 0$ consists of a perfect conductor, while the region $y \geq 0$ is a LIH dielectric medium $\varepsilon_{1r} = 2$. If there is a surface charge (σ_p) of 2 nC/m^2 on the conductor, determine $\vec{E}$ and $\vec{B}$ at: (a) A(3,-2,2) (b) B(-4,1,5).

(a) The point A(3,-2,2) is in the conductor since y=-2<0 at A. Hence, $\vec{E} = 0 = \vec{D}$.

(b) The point B(-4,1,5) is in the dielectric medium since y=1>0 at B.

If the surface carries charge (Q), we can use the same pillbox (Gaussian cylinder) reasoning as we did for "Boundary conditions for the electric flux density", and we find that

$$\Delta Q = \sigma_p A_I = \oint_A \vec{D} \cdot d\vec{A} = -D_{1n} A_I + D_{2n} A_I = \left(-D_{1n} + D_{2n}\right) A_I$$

Hence, since D=0 on the conductor side, inside the dielectric, $D_n = \sigma_p = 2 \text{ nCm}^{-2}$

Therefore, $\vec{D} = 2\vec{a}_y \text{ nCm}^{-2}$ and

$$\vec{E} = \frac{\vec{D}}{\varepsilon_0 \varepsilon_r} = 2 \times 10^{-9} \times \frac{36\pi}{2} \times 10^9 \vec{a}_y = 36\pi \vec{a}_y = \underline{\underline{113.1\vec{a}_y}} \text{ V/m}$$

Summary

At the boundary between two materials, EM wave is partially reflected and partially transmitted.

The plane of incidence is defined by the wave vector of the incident wave and a unit vector normal to the boundary.

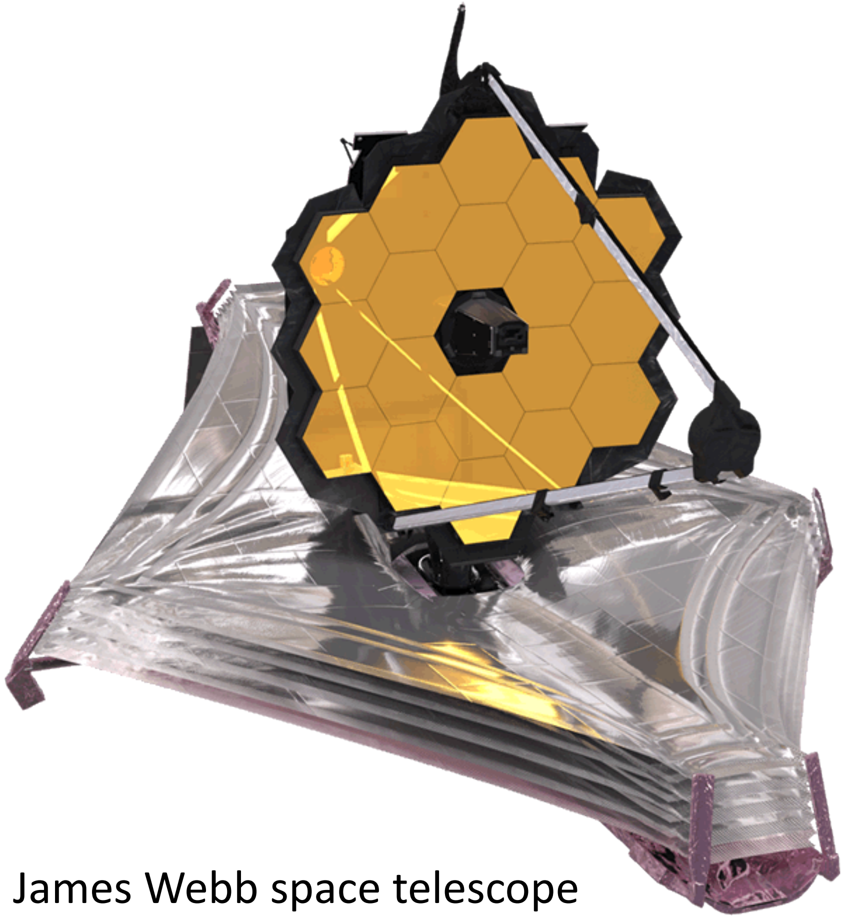
On each side of the boundary, the electric and magnetic fields can be resolved into normal and tangential components.

For LIH materials, assuming no surface charges and no surface currents.

	Electric fields	Magnetic fields
Normal components	$\vec{D}_{1n} = \vec{D}_{2n}$	$\vec{B}_{1n} = \vec{B}_{2n}$
Tangential components	$\vec{E}_{1t} = \vec{E}_{2t}$	$\vec{H}_{1t} = \vec{H}_{2t}$

Next lecture, we will see the law of reflectivity.

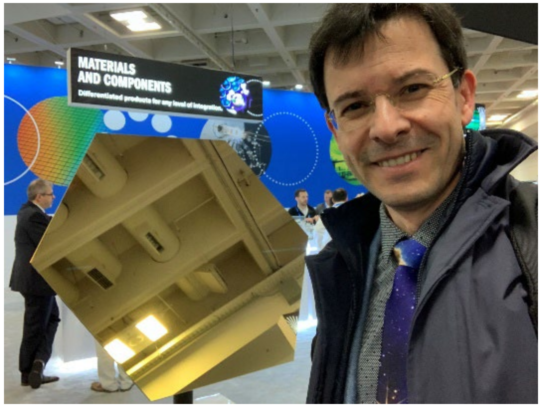
Mirrors and the law of reflectivity



James Webb space telescope

The mirror is made of **beryllium** (light and strong) and has a **gold coating** to provide infrared reflectivity and this is covered by a **thin layer of glass** for durability.

- 18 hexagonal mirror segments
- a 6.5-meter-diameter mirror,
- light-collecting area of about 25 m^2



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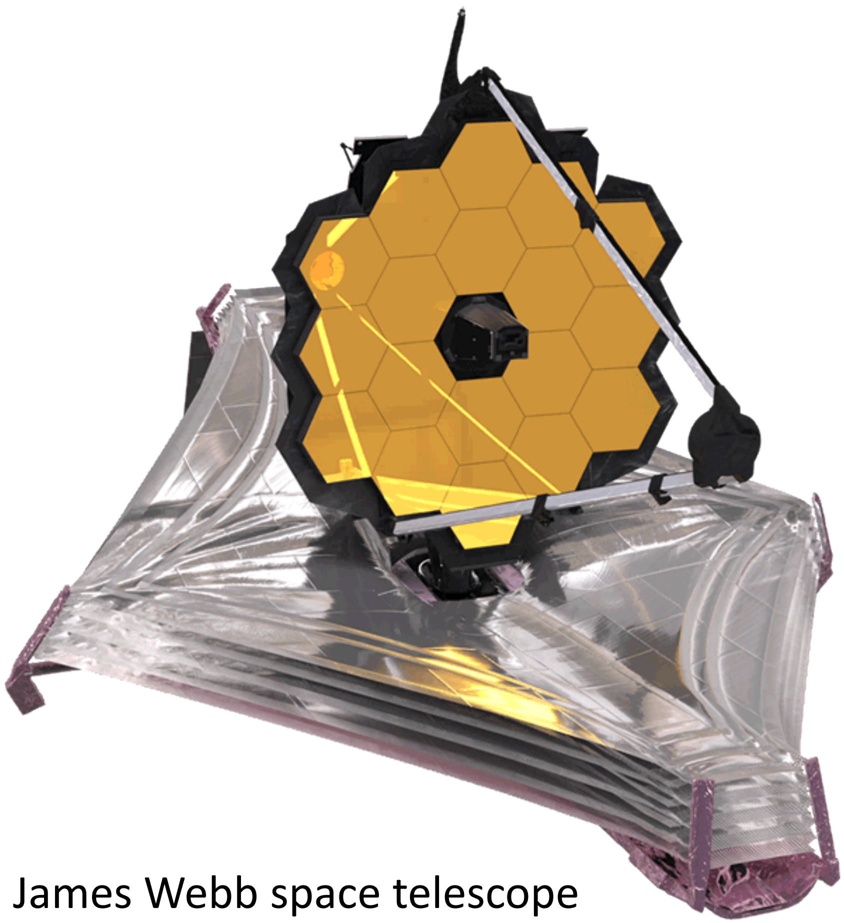

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