1) a) $\psi(x) = \begin{cases} Cx(2-x) & , 0 \le x \le 2 \\ 0 & , \text{otherwise} \end{cases}$

$$\int_{-\infty}^{\infty} |\psi|^2 \, dx = 1$$

$A \le \text{cnst}$

$$\int_0^2 |Cx(2-x)|^2 + \cancel{\int_{-\infty}^{0} |0|^2}^{\,\to 0}$$

$$+ \cancel{\int_2^{+\infty} |0|^2} \, dx = 1$$

$$\int_0^2 \left[\sqrt{C^2 x^2 (2-x)^2}\right]^2 dx + A = 1$$

$$C^2 \int_0^2 4x^2 - 4x^3 + x^4 + A = 1$$

Assume $A = 0$

$$c^2\left[\frac{4x^3}{3} - x^4 + \frac{x^5}{5}\right]_0 = 1$$

$$c^2\left(\cancel{\frac{32}{3}}^{\,10 + 1/3} \cancel{-16} + \cancel{\frac{32}{5}}^{\,6 + 2/5}\right) = 1$$

$$c^2 = \frac{1}{1/3 + \cancel{2/5}}$$

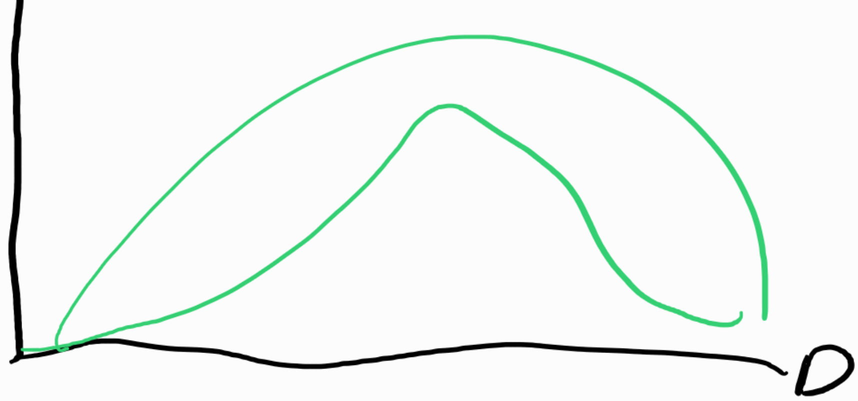
$1/3 \rightarrow$ ~~5/15~~ ~~6/15~~ $\quad 12/15$

$$c = \sqrt{\frac{15}{12}} = \sqrt{\frac{5}{4}}$$

↓ saw one

b)

4



c) $\displaystyle\int_{0.55}^{1.05} P(x,t)\,dx = \int_{0.55}^{1.05} \left|\sqrt{\tfrac{5}{4}}\,x\,(2-x)\right|^2$

$\displaystyle = \frac{5}{4}\int_{0.55}^{1.05} x^4 - 2x^3 + 4x^2 \, dx$

$= 0.6005$

d) $\displaystyle \langle x \rangle = \int^{\infty} x\,P(x,t)\,dx$

$_{-\infty}$

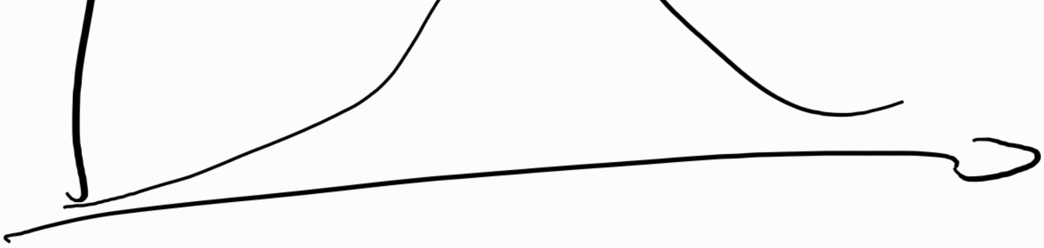
$$= \int_0^2 0.6 \, x \, dx$$

$$= \left[\frac{0.\cancel{6}^{3} x^2}{\cancel{2}} \right]_0^2$$

$$\langle x \rangle = 0.6$$

2) $\psi(x) = A e^{-\frac{|x|}{a}}$

a)



b) $\int_{-a}^{a} |\psi(x)|^2 \, dx$

$= \int_{-a}^{a} A^2 e^{-2|x|/a} \, dx$

$= A^2 \left(e^{-2\cancel{a/a}} - e^{-2\cancel{a/a}} \right) dx$

$= 0$

3) $P(x,t) = \psi^*(x,t)\,\psi(x,t)$

particle cannot have negative probability of not being somewhere

$$P(x,t) = \underbrace{|\Psi(t)|^2}_{\geqslant 0}$$

$$P(x,t) = |\Psi(t)|^2$$

$$= \Psi^* \Psi$$

4) $\dfrac{i\hbar}{\phi(t)} \dfrac{d\phi(t)}{dt} = E$

$$\int \frac{i\hbar}{\phi(t)}\, d\phi(t) = \int E\, dt$$

$$i\hbar \ln(\phi t) = Et + C, \quad C = \text{const}$$

$$\ln(\phi t) = \frac{Et + c}{i\hbar}$$

$$\phi(t) = e^{Et + c / i\hbar}$$

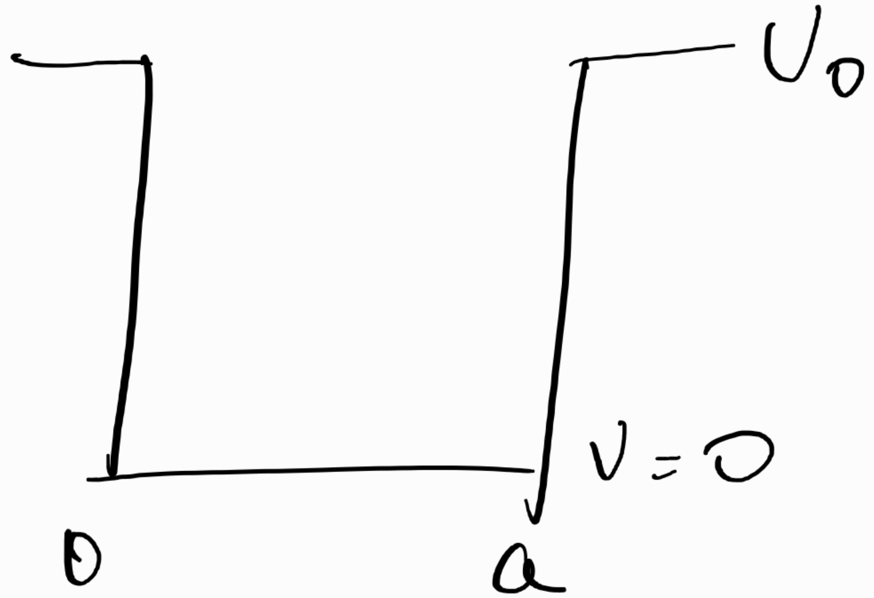
5. ? - come back

6. a) $\psi(x) = A\psi_1 + B\psi_2$

$$E_1 = E_2$$

b) ?

7.



a) $\psi = A\sin(kx) + B\cos(kx)$

b) $x = 0 \Rightarrow 0 = A\sin(0) + B\cos(0)$

$$\Rightarrow B = 0$$

$$x = a \Rightarrow 0 = a\sin(ka) + 0$$

$$\rightarrow \sin(ka) = 0$$

$$\rightarrow ka = n\pi$$

$$k_n = \frac{n\pi}{a}$$

$$\psi_n = A_n \sin(k_n x)$$

$$= A_n \sin\left(\frac{n\pi x}{a}\right)$$

$$\Rightarrow k_n = \frac{n\pi}{a} = \frac{\sqrt{2mE_n}}{\hbar}$$

$$\therefore E_n = \frac{\hbar^2 \pi^2 n^2}{2ma^2}$$

$x = a \qquad 1 = A \sin + 1$

$$A = \frac{2}{\sin(k_n a)}$$

$$= \frac{2}{\sin(ka)}$$

2

struggling to solve this $\sqrt{\frac{2}{a}}$

normalize $|\psi|^2$

$$\int_{-\infty}^{\infty} |\psi|^2 dx =$$

$$\int^{\infty} |A \sin(kx) + B\cos(kx)|^2 dx$$

$$\int_{-\infty}$$

$$1 = A^2 \int_0^a \sin(kx)\,dx$$

$$1 = A\left(\cos(ka) - \cos(k0)\right)$$

θ:

$$1 = A\left(\cos(ka) - 1\right)$$

$$A = \frac{1}{\cos(ka) - 1}$$

2

nope

~~retry~~