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# Experimental study on the correlation of Incident angles and wave Frequency in Underwater Solid materials

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*Year 2, Semester 2, [WL]*

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This report investigates how the transmission of ultrasonic waves in a liquid medium is affected when a solid plate is put in the wave's path.This relation was evaluated by connecting an acoustic wave-function generator to one side of a water tank, with a receiving oscilloscope on the other side. Tests were ran with and without solid plate materials at different angles and frequencies. The obtained baseline wave velocity in the water medium was of $1500 \pm 350 ms^{-1}$ which decreased to $1440 \pm 190 ms^{-1}$ and $1452 \pm 190$ ms$^{-1}$ when a brass and a perspex plate where respectively input in the set up. These results lead to investigate how the input wave frequency and incident angle on the plates correlated to the observed transmission loss.

## I. INTRODUCTION

With the rise of underwater construction and the development of technologies which require the transmission of waves through liquid materials, such as ultrasounds, understanding of wave propagation in liquid materials has become a key engineering and physical area of research. Understanding how waves propagate in liquid mediums can allow us to detect the presence solid objects within the liquid, as well as permit us to build safe and solid underwater infrastructure.

Underwater ultrasonic waves travel as longitudinal pressure waves, meaning that they propel particle displacement in the same direction as their original propagation direction. This investigation evaluates the effects arising from ultrasonic waves propagating in a finite material underwater. The report will contribute to the existing research on wave incidence and propagation in underwater solid materials[4][5], and therefore allow us to better understand how solid materials might propagate ultrasonic waves under-water in comparison to their theoretical counterparts.

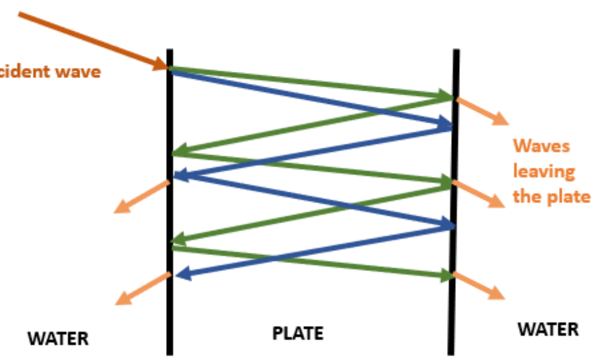



FIG. 1: Theoretical representation of the behaviour of longitudinal waves when incident in an underwater solid plate, at an oblique angle. The figure illustrates the reflection effects that occur within the material as well as their consequent output waves into the water medium in which the plate is present.

## II. THEORY

Ultrasonic waves are acoustic waves with a frequency too high for human hearing. The velocity of acoustic waves in fluids is correlated to the elastic moduli of the material in which they are propagated. This relation can be represented by the equation:

$$c^2 = \frac{f(moduli)}{p_0} \tag{1}$$

In this experiment, observations were made on the behaviour of acoustic waves in underwater plates at different angles of incidence. It is important to note that a 90∘ incidence corresponds to the so called "normal" incidence at which shear waves are not generated and the transmission through the plate is as a pure longitudinal wave. The words "shear" and "longitudinal" refer to the behaviour of the particle motion in association to the direction of the wave front. Longitudinal particle motion in a medium is parallel to the direction of the wave front, whereas shear particle motion in a medium is perpendicular to wave direction.

As the angle of incidence moves away from the normal angle, the input acoustic pulses generate longitudinal and shear waves when they meet the interface of the plate in the water. The interaction of these waves with the plate triggers complicated interference effects within the plate, which leads to the creation of a lamb wave. Lamb waves propagate within the plate and are bounded by the sheet or plate surface causing a wave-guide effect. Figure 1 illustrates this behaviour within the plate. As one can infer from Figure 1, the angle of incidence for lamb waves has a strong correlation with their behaviour and propagation properties such as velocity. In fact, the velocity of lamb waves is a function of the incident angle and the product of the frequency and plate thickness. Other material related variables are also taken into account when evaluating the velocity of lamb waves, but the focus of this report will be the frequency and incident angle relation. The vibrations resulting from different set ups can be grouped into two classes, asymmetric and symmetric lamb waves. These two lamb modes are represented by the Rayleigh-Lamb equations (2)(3) for symmetric and asymmetric lamb waves respectively[3].

$$\frac{tan(qh)}{tan(ph)} = \frac{4k^2qp}{(k^2-q^2)^2} \tag{2}$$

$$\frac{\tan(qh)}{\tan(ph)} = \frac{(k^2-q^2)^2}{4k^2qp} \tag{3}$$

In the Rayleigh-Lamb equations (2)(3) $2h$ correspond to the thickness of the plate, and $k$ corresponds to the lamb wave wave number. These can be evaluated through equations (4)-(6):

$$k = \frac{2\pi}{\lambda} \tag{4}$$

$$q^2 = \frac{w^2}{C_S^2} - k^2 \tag{5}$$

$$p^2 = \frac{w^2}{C_L^2} - k^2 \tag{6}$$

Equations (2)-(6) permit recognition of which lamb wave modes maximum transmission through the plate is achieved as a result of constructive interference patterns between the reflected waves within the material as seen in Figure 1.

## III. EXPERIMENTAL

Through this experiment the apparatus was set up as shown in Figure 2. A function generator was set to generate sine wave cycles which where delivered to a water basin of 15cm in length. Each pulse had a measured velocity of 1500 ms$^{-1}$, with a bust period of 1.5ms and frequency of 1.5 MHz. These initial values where set as a starting point for the experiment, and were modified through out the various tests. These specific starting values were chosen as they where of reasonable magnitude and length. The generated pulses propagated through the water as ultrasonic waves.

In the middle of the basin, a solid plate is suspended, which the waves will meet on their path to the receiving transducer. In the experiment two plate materials where tested, brass and perspex. The primary solid material used throughout this experiment was the brass plate. The brass probe weighed approximately 28$g$, and had a calculated mass density of $8.36 \pm 0.01$ g/cm$^3$. Brass is metallic alloy of zinc and copper. It is widely used in industry and plumbing, and therefore makes a great testing material for underwater construction. A perspex plate was also briefly tested in order to investigate if the resulting wave speeds and observed behaviours where correlated to the material of the plate, as predicted by theory.

The transmission of the waves through these plates was compared and normalized against a baseline test of the basin without any plate. This permitted the analysis of how the presence of these solid materials affected the wave's behaviour when it found them on it's path.

The measurements were obtained via a receiving transducer at the other end of the basin, which was connected to an oscilloscope.

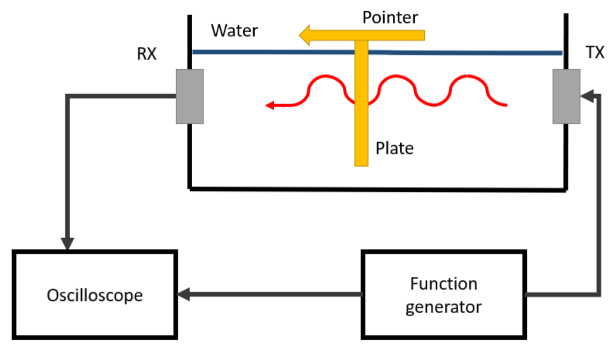



FIG. 2: Schematic Diagram of the experimental set-up used. A water basin with a Transmitting transducer (TX) at one end and a Receiving transducer (RX) at the other, is connected to a function generator which generates ultrasonic waves. These waves, illustrated in red, pass through a metal plate and then continue their path through the basin until reaching the Receiving transducer, which in turn is connected to an Oscilloscope in order to measure the received frequencies.

## IV. RESULTS AND SPECIFIC DISCUSSION

This section details the obtained results throughout the experiment. Their discussion will follow in Section 5.

### A. The baseline results

At the start of the experiment a baseline measurement was made for the wave behaviour in the water basin. By setting the output frequency to 1.5 MHz, the peak to peak voltage to 10V, and the output function length to 100 cycles with a burst period of 1.5 ms, three separate pulses were measured and produced an average point to point velocity of 1500 ms$^{-1}$ for the travel through the basin. Figure 3 illustrates the obtained display at the receiving transducer for an individual burst. One can observe the arrival of a main wave bursts followed by less intense wave bursts that have experienced some sort of delay due to reflections or refractions occurring across the basin. Additionally the reading cursor uncertainty was estimated at 20ms; the basin length measurement uncertainty was measured as 1 cm. When considering these uncertainties the point to point velocity can be estimated to $1500 \pm 350$ms$^{-1}$.

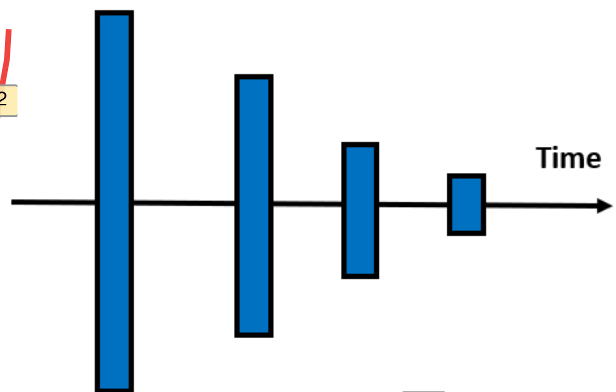



FIG. 3: Receiving transducer scope display at baseline wave transmission of one pulse. An initial wave input was observed followed by reflected and refracted bursts. The time separation between each signal received is approximately the same, but the intensity of the signals dramatically diminishes until no longer being detectable.

### B. Wave velocity

When introducing the brass plate in the basin different receiving patterns were measured which depended on the plate's incidence angle. Figure 1 illustrates this effect.

Using Snell's Law:

$$\frac{V_w}{sin(\theta_i)} = \frac{V_o}{sin(\theta_{LP})} \tag{7}$$

the incident critical angle was evaluated as 15.78° for the arriving wave on the plate. Experimentally the measured critical angle is found at all critical angles between

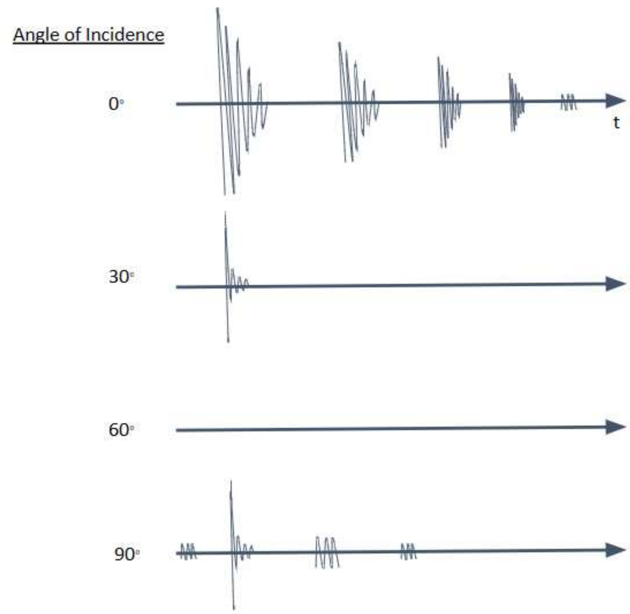



FIG. 4: Received wave-functions observed in the oscilloscope at different angles of incidence.

TABLE I: Table displaying the arrival times of the baseline input sine function associated with the angle of incidence of longitudinal waves at the brass plate.

| Angle of incidence at plate (degrees) | Arrival time (μs) |
|---|---|
| 90 | 96 |
| 60 | 100 |
| 30 | 0 |
| 0 | 101 |
| -30 | 0 |

20° and 30°, with an uncertainty measurement of 3° associated to the pointer accuracy.

Additionally, measurements were made for the arrival times of transmitted waves at the receiving transducer for different angles of incidence at the plate, in order to obtain their respective wave velocities through the basin when the brass plate was inserted at each of these angles. Table I illustrates these results.

Using the following equation

$$V_0 = V_w \left( \frac{\nabla t V_w}{d} + cos(\theta_i)^2 + sin(\theta_i)^2 \right)^{\frac{1}{2}} \tag{8}$$

measurements of $1440 \pm 190$ ms$^{-1}$ were recorded for the wave velocity at normal incidence for the brass plate. The same experiment was performed with a perspex plate. For which the measured wave velocity was found at $1452 \pm 190$ ms$^{-1}$. These obtained velocities reflect a velocity loss when compared to the baseline. This leads to investigating the possible causes of such loss. As mentioned in previous sections, lamb wave behaviour is associated to the incident angle of the input wave at the tested plate as well as it's frequency. The following subsections discuss tests for both of these values that were carried in order to find any correlations with the obtained transmission loss.

### C. Incident angles

As described in the theory section, as the wave moves away from the angle of incidence both longitudinal and shear waves are generated at each interface of the plate. To observe these reflected waves superpose within the plate's surface, an input of a pulse duration many times longer than the two way travel time between the plate faces was applied. Figure 5 showcases the received amplitude at different incident angles when a constant frequency of 1.32 MHz is input. These results showcase the correlation between the incident angle of the acoustic waves in the brass plate and their resulting measured amplitude at the receiver.

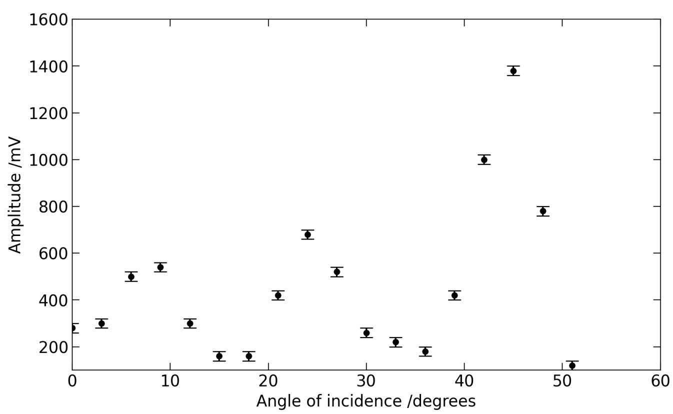



FIG. 5: Plot illustrating the results of input frequency against detected amplitude of waves through an underwater brass plate at a high input frequency. The received amplitude uncertainty is of ±20 mV at all point in this measurement due to hardware associated uncertainties.

### D. Transmitted frequency

Additional measurements were made on the received wave amplitude against the input frequency in the empty basin. Figure 6 illustrates the observed relationship.

When adding a brass plate to the experimental set up the relationship graph goes through some notable changes. Figure 7 illustrates the observed relation between received amplitude and input frequency when a brass plate is inserted.

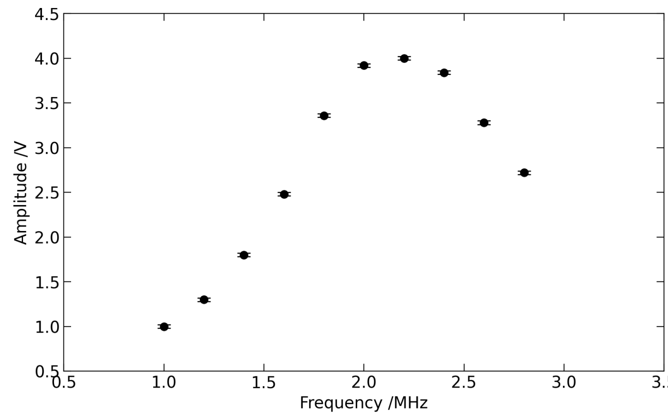



FIG. 6: Plot illustrating the results of input frequency against detected amplitude of waves through an underwater without a plate. The received amplitude uncertainty is of ±20 mV at all point in this measurement due to hardware associated uncertainties.

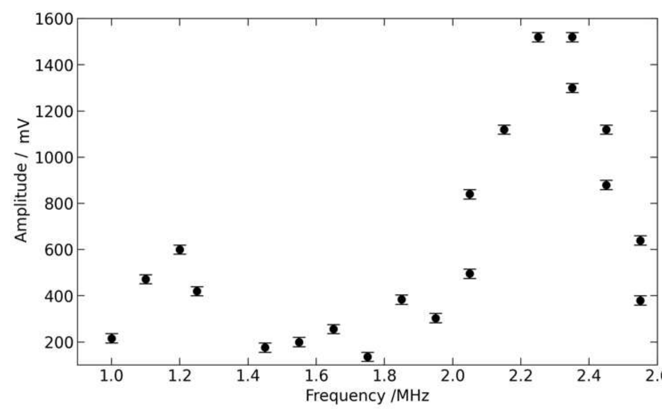



FIG. 7: Plot illustrating the results of input frequency against detected amplitude of waves through an underwater brass plate. The received amplitude uncertainty is of ±20 mV at all point in this measurement due to hardware associated uncertainties.

## V. GENERAL DISCUSSION

This discussion revolves around the results and proceeds to normalize them against the recorded baseline measurements to quantify how the presence of the plate affects the wave behaviour within the liquid medium. As one can observe in the previous section the transmitted frequency of the input wave-function as well as the incident angle at which this wave reaches the plate play an important effect of the observed wave output. In addition to these two factors, the measurements suggest that the plate material affects the wave velocity. As mentioned above, the wave velocity through the basin without any plates present was measured at $1500 \pm 350$ ms$^{-1}$. When adding the brass and perspex plate this velocity is reduce to $1440 \pm 190$ ms$^{-1}$ and $1452 \pm 190$ ms$^{-1}$ respectively. This loss in velocity generated when the wave encounters a plate in it's path is consistent with the theory behind lamb waves. This discussion evaluates how the wave loss correlates to the frequency and incident angle factors which theoretically are directly related to the lamb wave velocity. It is important to note, than in addition to the confirmation of the presence of losses induced by solid plates, these results also illustrate that the losses induced by the plates are correlated to the plates material and therefore it's mass, volume, and atomic structure.

### A. The role of transmitted frequency

To discuss the observed relationship between wave-frequency and the detected wave amplitude the transmission loss coefficient is calculated. Transmission loss of a constant frequency wave within a fluid is defined by

$$T_{loss} = 20\log_{10}\left(\frac{A_r}{A_t}\right) = 8.7x\alpha \qquad (9)$$

with $\alpha$ corresponding to the absorption coefficient, and $x$ referring to the length of the waves' path, in this case the length of the basin used. At room temperature the absorption coefficient of water has been experimentally estimated at 0.135 $cm^{-1}$ [1]. Leading to a predicted $T_{loss} = -0.18$ dB within the water basin.

At normal incidence, shear waves are not generated and therefore one can visualise the transmission through the plate as a pure longitudinal wave, using the following equation to predict the transmission loss coefficient

$$T_{loss} = \frac{1}{1 + \frac{1}{4}\left(\frac{V_{lp}\rho}{V_w\rho_w}\right)^2 sin(K_{L_p}d)^2} \qquad (10)$$

Figure 8 presents the predicted results and obtained results that were obtained using this equation. These results appear to accurately respect the existing predictions. Nevertheless one can note an overall lower transmission coefficient particularly within the first peak. This inconsistency may be due to experimental uncertainties that arise from the resolution of the laboratory equipment, as well as inaccuracies of human handling of the apparatus.

### B. The role of incident angles

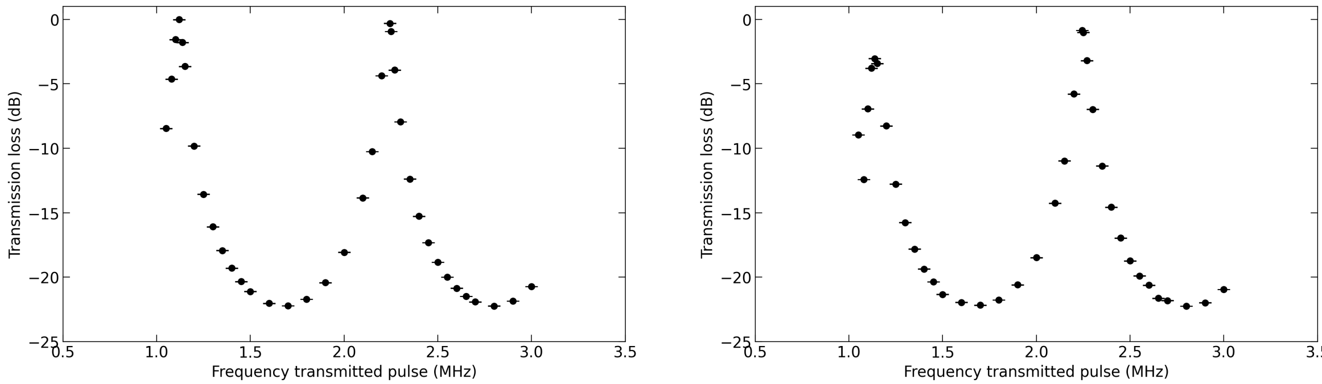
When looking at the measured amplitude against various incident angles in Figure 5, one can observe three clear amplitude peaks at approximately 9, 24, and 46 degrees. The obtained minimum transmission losses at 9, 24, and 46 degrees showcase a minimum transmission loss at the $s_2, a_1$ and $s_0$ generated lamb modes. In order to more accurately analyse these peaks, the results were normalised against the baseline reading, in which the basin did not contain a plate and calculated their associated transmission loss, as per equation 4. Figure 9 illustrates the resulting plot after this normalisation.

Figure 9 shows consistency in the minima and maxima obtained in the normalised graph against the existing theoretical predictions defining the lamb wave behaviour within brass. Nonetheless, the obtained graph behaviour is not as accurate as expected. When comparing the results in Figure 9 to the expected results in Figure 10 there is a shift to the right in the location of the maximas and minimas present in the function. These discrepancies may be caused by the difference in the product of plate thickness and frequency $(f.d)$ between the experimental set up and the theoretical predictions. This once again supports the evaluated correlation between a material's thickness and the incident wave's frequency within the resulting Lamb waves. It confirms that even within the same material and thus atomic structure, these other variable elements play an important role in the wave transmission through a controlled environment.

Ok

## VI. CONCLUSION

This experiment investigated the ~~correlation~~ between incident angles and input wave frequency in an underwater solid material with the resulting transmitted wave within a controlled environment. The results lead to conclude that the observed experimental relationship between frequency of a transmitted pulse and the expected transmission loss of the underwater material is satisfactorily accurate in the experimental setup. Similarly, the correlation between incident angles and transmission losses in the experiment up does respect the characteristic minima and maxima predicted by theory, but it showcases many discrepancies within the overall results. This leads to conclude that theoretical predictions can accurately simulate underwater longitudinal acoustic wave behaviour, although additional experimental tests must be run in specific environments to ensure external factors are taken into account when predicting these behaviours. The specific atomic composition of the material as well as it's effective mass and volume play an important role on the observed transmission losses. Additionally environmental factors such as the presence of salt in water, which can be found in oceanic construction environments, can also affect the transmission behaviour of input waves. It would be interesting to expand this experiment by evaluating the effects of input waves in salty water in order to more accurately understand how materials might experience oceanic waves in underwater construction.




(a) Expected transmission loss

(b) Obtained transmission loss

FIG. 8: On the right you can observe the expected transmission loss in decibels due to brass as a function of transmitted frequency. The graph on the left reflects the obtained results.

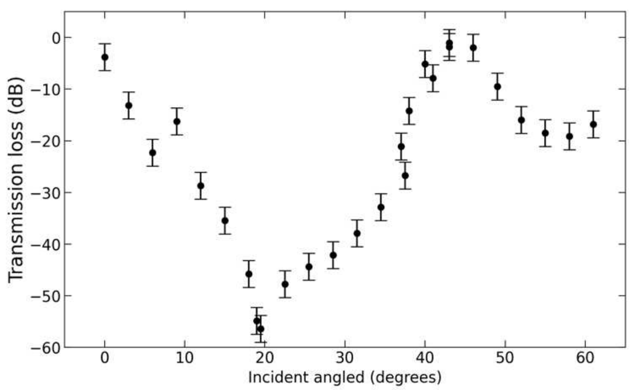



FIG. 9: Measured transmission loss coefficient in dB against the wave incident angle of arrival against the plate.

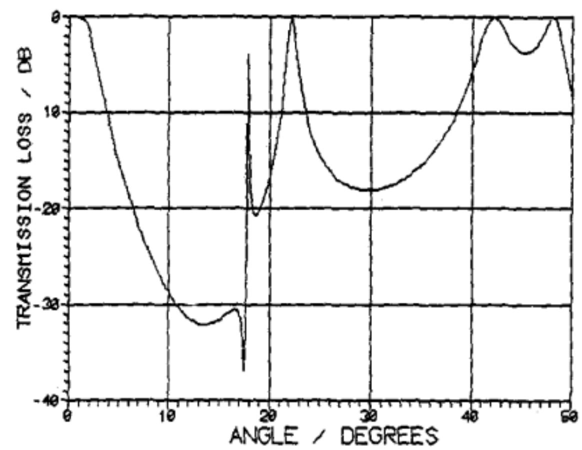



FIG. 10: Expected transmission loss in decibels, against angle of incidence for a brass plate with $f.d = 2.3MHz.mm$ [2]

## VII. REFERENCES

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[5] M. Rucka, B. Zima, R. Kedra. *Application of guided wave propagation in diagnostics of stell bridge components*, Archives of civil engineering, LX, 4, 2014

Sean

# Index of comments

1.1 Title a bit odd and somewhat cryptic...

1.2 phrase sounds analytical (as opposed to experimental)

1.3 fn. generator and oscilloscopes are electronics. "Transducers" send and receive the sound..

1.4 units not in italics in this style - consistent formatting please.

1.5 References?

1.6 Why centre-justified?

1.7 Number references in the order they appear in the body text.

1.8 This introduction to the report is a bit too broad brush.

Provide a bit more context and explicitly state what this report demonstrates.

1.9 You need to, at least, define what the symbols are.

1.10 Is this theory?

2.1 How? I cannot deduce much from the figure...

2.2 Introduce equations as part of sentences.

Only refer to equations after they have been introduced.

Use consistent formatting. ("tan" should not be in italics in this style)

2.3 Usually better to align figure+caption to top/bottom margin.

2.4 convoluted sentence...

2.5 Throughout?

2.6 Good to guide the reader but what you state here is very much the expected norm and not really necessary.

2.7 vague

3.1 Scale?
Vertical axis?

3.2 Vertical axis?

3.3 "schematic representation"?

3.4 Ultra-short paragraph usually symptomatic of inadequate paragraph structure.

4.1 What are we supposed to see? You need to describe the features you want the reader to notice and say what they mean.

4.2 "Empty" sounds like "drained"!

4.3 What are these notable changes? They are of note - so you should describe them explicitly, here, in the body text, in the Results section.

5.1 Some of those figures on the next page would have been better on this page.

5.2 avoid apostrophes..

# Index of comments

5.3 Not a sentence.

5.4 "Correlation" sounds like you are dealing with something statistical in nature. Yes, you might be treating it statistically in order to treat your data impartially, but you need to use language to stress "impartial"-ness or language loaded with meaning in the right places.

5.5 based on what criteria?