# ELECTROMAGNETISM

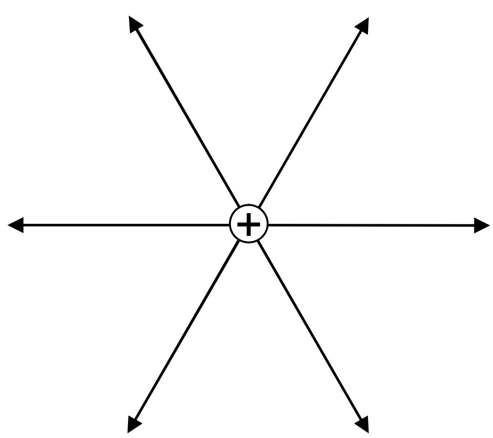
Electromagnetism is primarily concerned with the vector quantities:

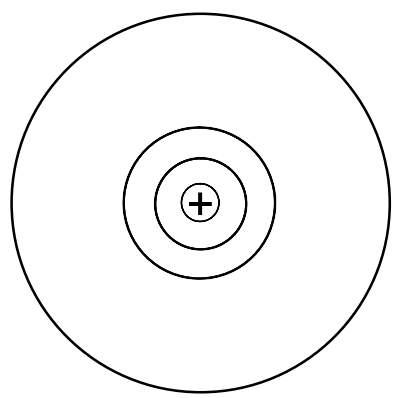
$\mathbf{E}(\mathbf{r},t)$ "The Electric Field" &
$\mathbf{B}(\mathbf{r},t)$ "The Magnetic Field"

They are continuous functions of position $\mathbf{r}$ and time $t$, defined over a given region, or "field".

Other "vector fields", such as the current density $\mathbf{J}(\mathbf{r},t)$ will be used, as will some "scalar fields" such as the electric charge density $\rho(\mathbf{r})$ and the electrostatic potential $\phi(\mathbf{r})$.

**Recall:** Vector fields can be visualised using "field lines" or "flux lines", & scalar fields by contour lines / surfaces.





# 1. Maxwell's Equations

Electromagnetism is governed by 4 differential equations in $\mathbf{E}(\mathbf{r},t)$ and $\mathbf{B}(\mathbf{r},t)$, called Maxwell's equations.

In a vacuum, they are

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} \qquad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \cdot \mathbf{B} = 0 \qquad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \varepsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t}$$

where $t$ is time, $\varepsilon_0$ and $\mu_0$ are constants;

$\rho(\mathbf{r})$ is electric charge density
$\mathbf{J}(\mathbf{r},t)$ is electric current density

- note that these are the only "source terms"

$\Rightarrow$ All electromagnetic fields are caused by CHARGES.

We will assume that the charge-carrying particle is much smaller than any length scale of interest.

(POINT CHARGE)

Hence we have "Classical Electrodynamics", and not "Quantum Electrodynamics" (QED).

Maxwell's equations can almost entirely be derived from the "elementary" Laws of electricity & magnetism; ie

Coulomb's Law
Gauss's Law
Biot-Savart Law
Ampère's Law
Faraday's Law

But we first need these in their most general (integral) form.

So to get started, we will need some

VECTOR CALCULUS

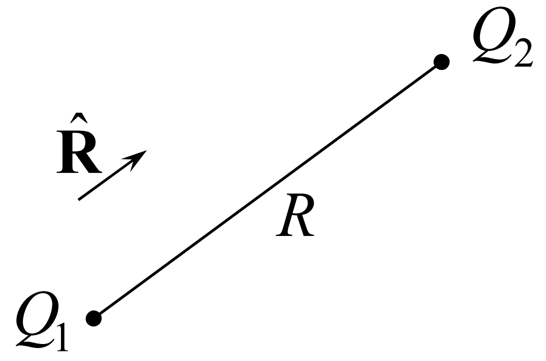
- see "Mathematical operations on field quantities" on moodle

page which gives an overview of volint

## 1.1 Electrostatic fields

**Coulomb's Law** (experimental)

The force between 2 **stationary** point charges is given by



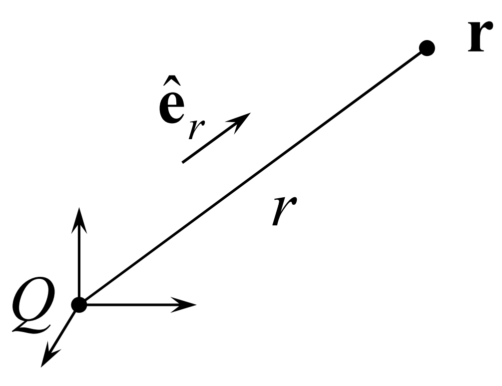

$$\mathbf{F}_e = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{R^2}\hat{\mathbf{R}}.$$

$\varepsilon_0$ is the "permittivity of free space"

$\approx 8.854\times10^{-12}$ $C^2N^{-1}m^{-2}$ or $Fm^{-1}$.

The **Electric Field** $\mathbf{E}(\mathbf{r})$ is the force experienced by a unit positive charge placed at $\mathbf{r}$.

$\therefore$ from Coulomb's Law, the $\mathbf{E}$-field due to a point charge $Q_1 = Q$ at the origin of coordinates is given by




$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{\mathbf{e}}_r.$$

units $Vm^{-1}$

- serves as a definition of $\mathbf{E}$.