



LAB

CM

m, v_0, E_0

M, v_0

m, v_0

M, v_0

M, v_{CM}

$$\vec{v}_{CM} = \frac{m}{m+M} \vec{v}_0$$

$$\vec{u} = \vec{v} - \vec{v}_{CM}$$

$$\left. \begin{aligned} \vec{v}_{CM} &= \frac{m}{m+M} \vec{v}_0 \\ \vec{u} &= \vec{v} - \vec{v}_{CM} \end{aligned} \right\} u_0 = \frac{M}{m+M} v_0$$

$$E_1 = \frac{1}{2} m |\vec{v}_1|^2$$

$$|\vec{v}_1|^2 = |\vec{u}_1 + \vec{v}_{CM}|^2 = |\vec{u}_1|^2 + |\vec{v}_{CM}|^2 + 2 \vec{u}_1 \cdot \vec{v}_{CM} = u_1^2 + v_{CM}^2 + 2 u_1 v_{CM} \cos \varphi$$

Assume elastic scattering $|\vec{u}_1| = |\vec{u}_0| = u_0$

$$|\vec{v}_1|^2 = \frac{M^2}{(m+M)^2} v_0^2 + \frac{m^2}{(m+M)^2} v_0^2 + 2 \frac{M}{(m+M)} v_0 \frac{m}{(m+M)} v_0 \cos \varphi$$

$$= v_0^2 \frac{M^2 + m^2 + 2 m M \cos \varphi}{(m+M)^2}$$

$\frac{M}{m} \approx A$

$$E_1 = E_0 \frac{A^2 + 1 + 2 A \cos \varphi}{(A+1)^2}$$