Scalar field $\phi(\vec{r})$ or $\phi(x,y,z)$
Cartesian coord

vector field $\vec{a}(\vec{r})$ or $\vec{a}(x,y,z)$

↑ PREV

↓ NEW

$\phi(x,y,z)$ gradient of a scalar field

Change in ϕ: $\delta\phi = \phi(x+\delta x, y+\delta y, z+\delta z) - \phi(x,y,z)$

* Def: Partial deriv

$$\frac{\partial \phi}{\partial x} = \lim_{\delta x \to 0} \left[\frac{\phi(x+\delta x, y+\delta y, z+\delta z) - \phi(x,y,z)}{\delta x} \right]$$

so $\phi(x+\delta x, y+\delta y, z+\delta z) - \phi(x,y,z) \approx \frac{\partial \phi}{\partial x}\delta x$

we do the same for the y & z directs

Then the total change in ϕ is the Σ of the changes in the x, y & z directs

$$\delta\phi = \frac{\partial \phi}{\partial x}\delta x + \frac{\partial \phi}{\partial y}\delta y + \frac{\partial \phi}{\partial z}\delta z \quad (1)$$

✱ Vector operator - DEL (NABLA) $\vec{\nabla}$

$$\vec{\nabla} \equiv \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}$$

grad ϕ is a vector field

$$\vec{\nabla}\phi = \frac{\partial}{\partial x}\phi\,\vec{i} + \frac{\partial}{\partial y}\phi\,\vec{j} + \frac{\partial}{\partial z}\phi\,\vec{k} \equiv \text{grad } \phi$$

From (1) $\delta\phi \simeq \text{grad }\phi \cdot \delta\vec{r}$ (2)

we can write $\delta\vec{r} = \hat{s}\,\delta s$

$\vec{s}$ = displacement

$\hat{s}$ = unit vector of displacement

δs = magnitude of the displacement (scalar)

(2) becomes

$$\delta\phi = \text{grad }\phi \cdot \hat{s}\,\delta s$$
$$= |\text{grad }\phi|\,\delta s \cos\theta$$

angle btw $\vec{A}$ & $\vec{B}$

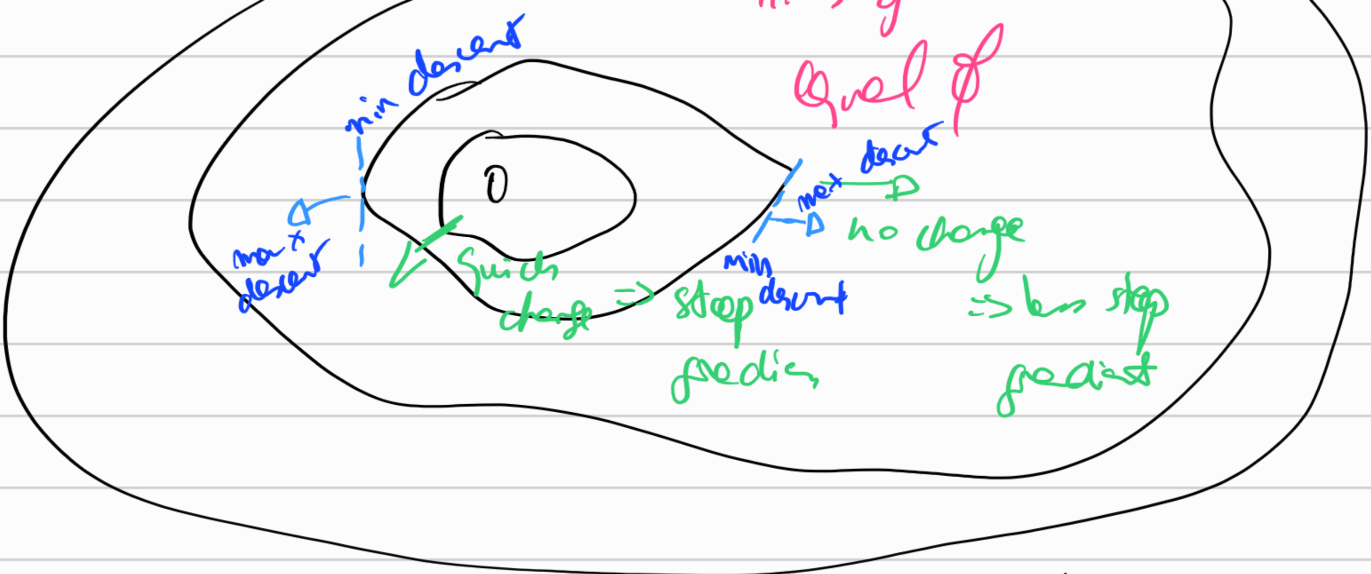
$$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos(\theta)$$

✱ Directional deriv of ϕ in the direct° of $\vec{s}$

$$\frac{d\phi}{ds} = \lim_{\delta s \to 0} \frac{\delta\phi}{\delta s} = \text{grad }\phi \cdot \hat{s} = |\text{grad }\phi|\cos(\theta)$$

this is a scalar

grad ϕ is always normal to $\perp$ lines or surfaces of const ϕ



(plate with flame ~ observe temp / lines = same temp)

ex if $\phi = \sin(x^2 - xy)$, find grad ϕ

$$\vec{\nabla}\phi = \frac{\partial}{\partial x}\phi\,\vec{i} + \frac{\partial}{\partial y}\phi\,\vec{j} + \frac{\partial}{\partial z}\phi\,\vec{k} \equiv \text{grad } \phi$$

$$(2x-y)\cos(x^2-xy)\,\vec{i} - x\cos(x^2-xy)\,\vec{j} = \text{grad } \phi$$

← takeaway point

NB $\phi \longrightarrow \vec{\nabla}\phi \longrightarrow \vec{F}$

scalar field, gradient, vector field

← TODO (weekend)

ex In EM (year 1) you meet $\vec{E}$ (electric field) & V (electric potential)

V is a scalar field $V(x, y, z)$

$\vec{E}$ & V are related by $\vec{E} = \text{grad } V$

Let $V = \dfrac{q}{4\pi\varepsilon_0 r}$ $\qquad r = (x^2 + y^2 + z^2)^{1/2}$

Show that $\vec{E} = -q/4\pi\varepsilon_0 \; \vec{r}/r^3$