Chapter 0:

Some fundamentals:

Electric field strength E is force per unit of positive charge:

$$E = \frac{F}{Q},$$

with the force provided by Coulomb's law:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{r^2},$$

with r the distance between Q_1 and Q_2.
Remember that gravitational field strength is:

$$g = \frac{F}{m},$$

same form as the electric field strength.
So, the electric field strength is:

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2},$$

where Q is the charge r is the distance between Q and the point where the field is evaluated.
Parallel plate capacitors: in a uniform field, the field strength is given by:

$$E = \frac{V}{d},$$

where V is the voltage and d is the distance between the plates.
Capacitors build up charge on the plates. The capacitance is:

$$C = \frac{Q}{V},$$

where Q is the charge and V is the voltage,
Capacitance is the amount of charge per potential difference.
For parallel plate capacitors:

$$C = \frac{A\varepsilon_0\varepsilon_r}{d},$$

where A is the area of the plates, ε_0 is the permittivity of free space ε_r is the relative permittivity, d is the separation of the plates.
The capacitor energy is:

$$W = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}.$$

Capacitance is the gradient on a Q-V graph and the Energy is the area under the curve.

The magnetic field density (B) is defined as the force (F) on a wire with length (L) of one meter, carrying a current (I) of one amp at right angles to the magnetic field:

$$F = B \cdot I \cdot L \text{ and } B = \frac{F}{I \cdot L}.$$

A force acts on charged particles moving in a magnetic fields. Often these charged particles would be electrons. Their motion is a current:

$$I = Q/t \ .$$

A particle that moves at velocity

$$v = L/t$$

goes a distance such as

$$L = vt \, .$$

So, we get:

$$F = B \cdot I \cdot L = B\frac{Q}{t} v \cdot t = B \cdot Q \cdot v \, ,$$

where Q is often 1.6×10^{-19} C.

In a circular path with radius r, the acceleration is

$$a = v^2/r$$

(defined in A-Level 'Circular Motion'; good to remember that $v = 2\pi r/T$, with T the period) so,

$$F = m \cdot a = m \cdot v^2/r \, .$$

Combining, with

$$F = B \cdot Q \cdot v \, ,$$

we get

$$\frac{mv^2}{r} = B \cdot Q \cdot v \, , \text{ and } r = \frac{mv}{BQ}.$$

The magnetic flux is defined as:

$$\Phi_m = \vec{B} \cdot \vec{A} = BA\cos\theta \, ,$$

where θ is the angle between the field and the normal to the plane of the loop and A is the area. For a coil with number of turns N, we can write:

$$N\Phi_m = N\vec{B} \cdot \vec{A} \, .$$

This is the amount of flux cut by coil, also known as **flux linkage**.

Snell's law is given by:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \, .$$

Useful Maths:

$$\cos(\theta) = \cos(-\theta)$$
$$\sin(a+b) = \sin a \cos b + \cos a \sin b$$
$$\sin(a-b) = \sin a \cos b - \cos a \sin b$$
$$\nabla \times \left(\nabla \times \vec{A}\right) = \nabla\left(\nabla \cdot \vec{A}\right) - \nabla^2 \vec{A}$$

The **cross product** also known as the **vector product** of vectors $\vec{A}$ and $\vec{B}$ is:

$$\vec{A} \times \vec{B} = \left\|\vec{A}\right\|\left\|\vec{B}\right\|(\sin\theta)\hat{u},$$

where $\hat{u}$ is a unit vector perpendicular to the plane containing $\vec{A}$ and $\vec{B}$, and θ is the angle between $\vec{A}$ and $\vec{B}$.

The **dot product** also known as the **scalar product** of vectors $\vec{A}$ and $\vec{B}$ is:

$$\vec{A} \cdot \vec{B} = \left\|\vec{A}\right\|\left\|\vec{B}\right\|\cos\theta,$$

where θ is the angle between $\vec{A}$ and $\vec{B}$.

The divergence theorem states that:

$$\int_A \vec{E} \cdot d\vec{A} = \int_V \left(\nabla \cdot \vec{E}\right) dv,$$

where A is the area of a closed surface that bounds the volume V.

Stoke's theorem states that:

$$\oint_L \vec{E} \cdot d\vec{L} = \int_A \left(\nabla \times \vec{E}\right) \cdot d\vec{A},$$

where L is the closed path that encloses a surface area A.

An electric field defined as:

$$\vec{E} = \vec{A}e^{i\left(\vec{k}\cdot\vec{r} - \omega t\right)}$$

in Cartesian coordinates, has components $\left(E_x, E_y, E_z\right)$. Similarly, the amplitude is a vector with components $\left(A_x, A_y, A_z\right)$. The wave vector has components $\left(k_x, k_y, k_z\right)$ and the position vector has components (x, y, z). We also have:

$$\vec{k} \cdot \vec{r} = k_x x + k_y y + k_z z\,.$$

Electric flux is the measure of the electric field through a given surface (S), although an electric field in itself cannot flow. It is a way of describing the electric field strength at any distance from the charge causing the field. For a uniform field, like a **parallel plate capacitor**:

$$\Phi_E = \vec{E} \cdot \vec{S} = ES\cos\theta .$$

A **torque** is the turning effect of a force. To calculate it, we use:

$$\vec{\tau} = \vec{r} \times \vec{F} = \|\vec{r}\| \|\vec{F}\| (\sin\theta)\hat{u} ,$$

where $\vec{\tau}$ is the torque, $\vec{F}$ is the force, $\vec{r}$ is the position vector (from the point about which the torque is being measured to the point where the force is being applied), θ is the angle between $\vec{r}$ and $\vec{F}$, and $\hat{u}$ is a unit vector along $\vec{r}$.

The **resolution** (or **projection**) of any vector $\vec{a}$ in the direction of $\vec{b}$ is: $\vec{a}_1 = \left(\vec{a} \cdot \hat{b}\right)\hat{b} = \frac{\vec{a} \cdot \vec{b}}{\|\vec{b}\|} \frac{\vec{b}}{\|\vec{b}\|} = \frac{\vec{a} \cdot \vec{b}}{\|\vec{b}\|^2} \vec{b} = \frac{\vec{a} \cdot \vec{b}}{\vec{b} \cdot \vec{b}} \vec{b}$.

If a surface of a plane is described by $f(x, y) = ax + by + c$, a **unit vector normal to the plane** is given by $\vec{s}_n = \frac{\nabla f}{|\nabla f|} = \frac{a\hat{x} + b\hat{y}}{\sqrt{a^2 + b^2}}$.

Previous physics

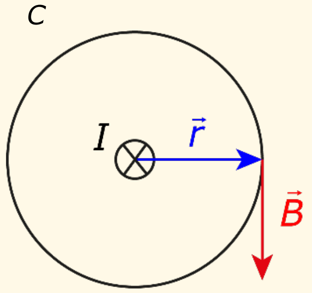
The **Lorentz force** is:

$$\vec{F} = Q\left(\vec{E} + \vec{v} \times \vec{B}\right).$$

While the electric force $Q\vec{E}$ is proportional to Q but independent of $\vec{v}$, the magnetic force $Q\vec{v} \times \vec{B}$ is perpendicular to both $\vec{v}$ and $\vec{B}$.

Ampère's Law states that the total magnetic flux density flowing through a closed current carrying loop is the sum of all currents I:

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 \sum I$$



When applied to a single, circular loop (radius r) of current carrying wire, the length of wire l can be written as $l = r\theta$, and we can differentiate easily to

$$\frac{dl}{d\theta} = r\,,$$

so that

$$dl = rd\theta\,.$$

We can write:

$$\oint_C \vec{B}\cdot d\vec{l} = \oint_C Bdl$$

because $\vec{B}$ and $d\vec{l}$ are parallel. This, we can now integrate:

$$\oint_C Bdl = \oint_C Brd\theta = \int_0^{2\pi} Brd\theta = rB\left[\theta\right]_0^{2\pi} = 2\pi rB\,.$$

So, from Ampère's law, we get:

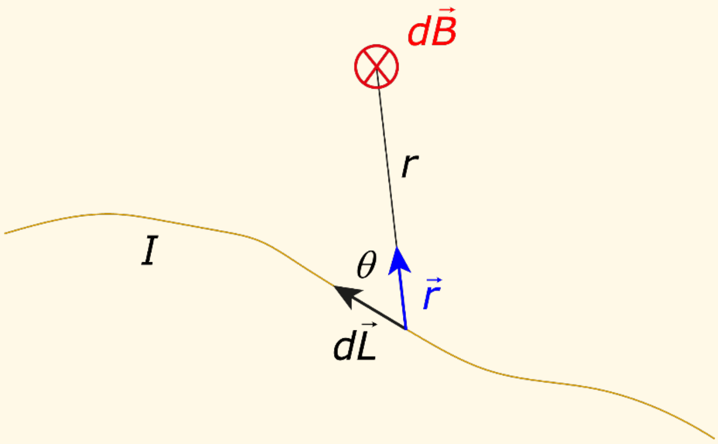
$$2\pi rB = \mu_0 I\,,$$

which is to say:

$$B = \frac{\mu_0 I}{2\pi r}.$$

The **Biot-Savart law** allows us to calculate the magnetic field generated by an electrical current. It is a fundamental law of electromagnetism. It was obtained experimentally. It states:

$$d\vec{B} = \frac{\mu_0 I}{4\pi r^2} d\vec{L}\times\hat{r}\,.$$



By integration over a path C in which the electric current flows (e.g. a length of wire), we obtain:

$$\vec{B} = \frac{\mu_0}{4\pi}\int_C \frac{Id\vec{L}\times\hat{r}}{r^2}.$$