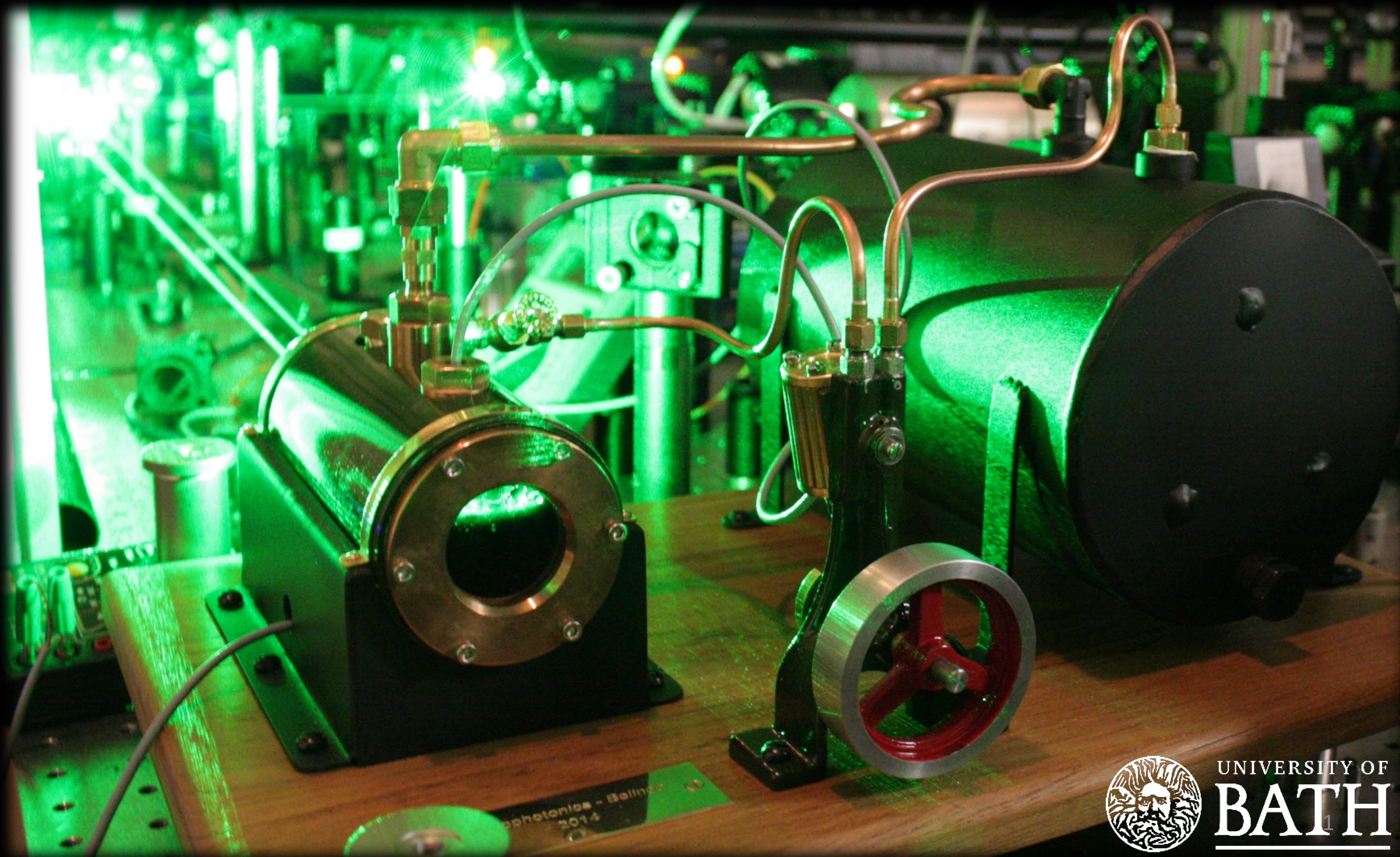
Lecture 20

Lasers



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Last time we saw

At the boundary between two materials, EM wave is partially reflected and partially transmitted.

The plane of incidence is defined by the wave vector of the incident wave and a unit vector normal to the boundary.

On each side of the boundary, the electric and magnetic fields can be resolved into normal and tangential components.

For LIH materials, assuming no surface charges and no surface currents.

	Electric fields	Magnetic fields
Normal components	$\vec{D}_{1n} = \vec{D}_{2n}$	$\vec{B}_{1n} = \vec{B}_{2n}$
Tangential components	$\vec{E}_{1t} = \vec{E}_{2t}$	$\vec{H}_{1t} = \vec{H}_{2t}$

Last time we saw

At normal incidence, the reflection and transmission coefficients are:

$$r_{\parallel/\perp} \equiv \frac{E_{r0}}{E_{i0}} = \frac{Z_2 - Z_1}{Z_1 + Z_2} \qquad t_{\parallel/\perp} \equiv \frac{E_{t0}}{E_{i0}} = \frac{2Z_2}{Z_1 + Z_2}$$

Snell's law results from conservation of moment at the interface:

For a general angle of incidence, we distinguish two cases of light polarisation:

(i) in the plane of incidence (this is P-polarized light)

(ii) perpendicular to the plane of incidence (this is S-polarized light)

The Fresnel coefficients for P- and S-polarized light are:

$$r_{\parallel} \equiv \frac{E_{r0}}{E_{i0}} = \frac{Z_2 \cos\theta_t - Z_1 \cos\theta_i}{Z_2 \cos\theta_t + Z_1 \cos\theta_i} \qquad t_{\parallel} \equiv \frac{E_{t0}}{E_{i0}} = \frac{2Z_2 \cos\theta_i}{Z_2 \cos\theta_t + Z_1 \cos\theta_i}$$

$$r_{\perp} \equiv \frac{E_{r0}}{E_{i0}} = \frac{Z_2 \cos\theta_i - Z_1 \cos\theta_t}{Z_2 \cos\theta_i + Z_1 \cos\theta_t} \qquad t_{\perp} \equiv \frac{E_{t0}}{E_{i0}} = \frac{2Z_2 \cos\theta_i}{Z_2 \cos\theta_i + Z_1 \cos\theta_t}$$

Last time we saw

For P-polarized incident lights the Brewster angle ($\tan\theta_B = n_2/n_1$) determines the incidence angle for which the reflected beam is polarized perpendicular to the plane of incidence.

For light with any polarisation, when propagating from a material with higher refractive index into a material with lower refractive index, a critical angle exists ($\sin\theta_c = n_2/n_1$) beyond which the wave experiences total internal reflection.

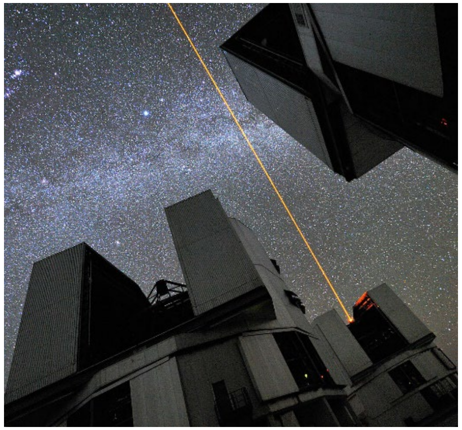
Overview

In this Lecture we will look at:

- ❑ Some applications of lasers
- ❑ Laser – what does it mean?
- ❑ Absorption
- ❑ Spontaneous emission
- ❑ Stimulated emission
- ❑ Planck’s black body spectrum
- ❑ Einstein’s relations
- ❑ Population inversion and lasing
- ❑ Laser properties

Some applications of laser

Astronomy

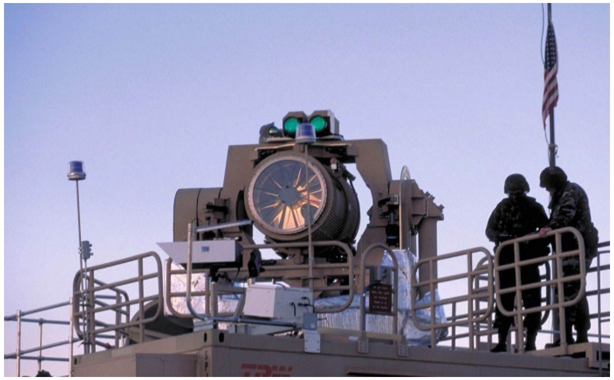


Nuclear fusion research

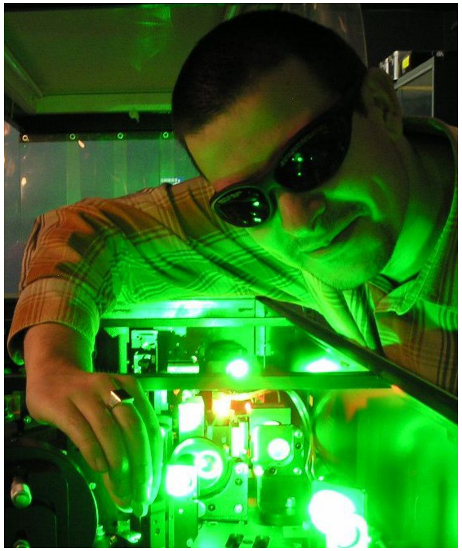
Communications

Welding

Weapons



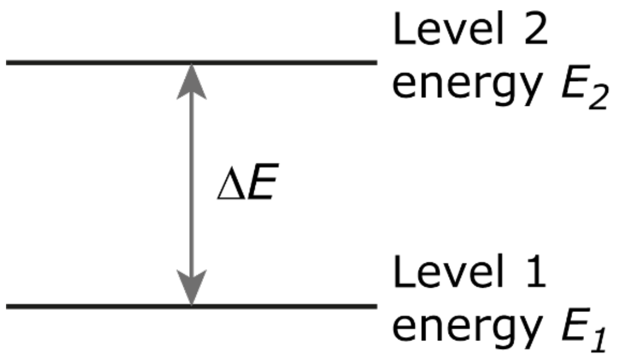
Research



What is a ‘laser’?

Laser – what does it mean?

The word LASER stands for Light Amplification by Stimulated Emission of Radiation. So, what does "stimulated emission" mean?



A two-level energy diagram.

The energy difference between the levels can be written as:

$$\Delta E = E_{21} = E_2 - E_1 = h\nu_{21}$$

Planck's constant is: h

The frequency of absorption is: ν_{21}

There are three interactions to consider:

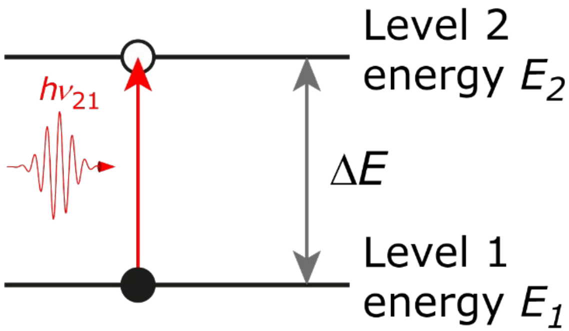
1. Absorption
2. Spontaneous emission
3. Stimulated Emission

These are really important.

Let's consider absorption!

Absorption

Electrons excited to higher energy state.



Absorption of a photon.

Now, we consider a system with many such 2-level atoms.

N_1: number of electrons on level 1

N_2: number of electrons on level 2

dN_1: change of number of electrons in level 1.

To describe the number of photons, we can use the energy density of the radiation: $\rho(\nu_{21})$

with frequency: ν_{21}

We can therefore write:

$$dN_1\big|_{\text{absoprtion}} = -B_{12}\rho(\nu_{21})N_1 dt$$

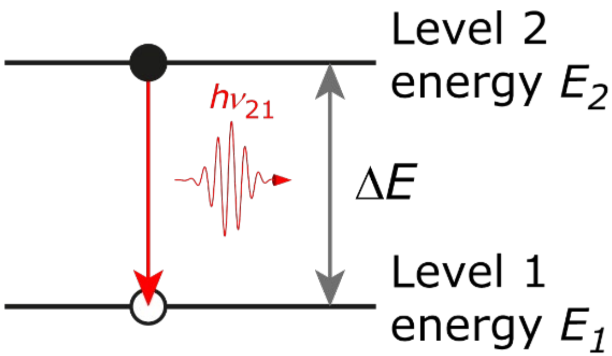
where B_{12} is the **Einstein coefficient of absorption**.

The minus sign indicates that absorption decreases the number of electrons in Level 1.

Next, spontaneous emission!

Spontaneous emission

Electrons relax to lower energy state.



Spontaneous emission of a photon.

Now, we consider a system with many such 2-level atoms.

N_1: number of electrons on level 1

N_2: number of electrons on level 2

dN_2: change of number of electrons in level 2.

Similarly, to the way we did before:

$$dN_2\Big|_{\substack{\text{spontaneous}\\ \text{emission}}} = -A_{21}N_2dt$$

where A_{21} is the **Einstein coefficient of spontaneous emission**.

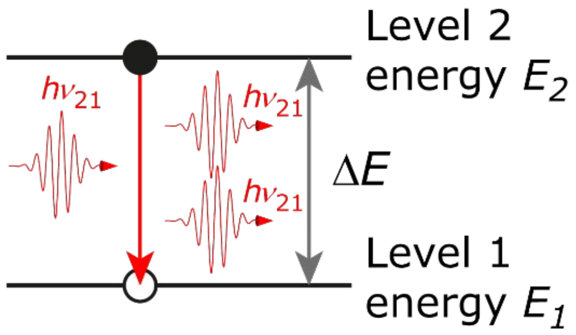
dt is the time it takes to wait for such a phenomenon to occur.

The minus sign indicates that absorption decreases the number of electrons in Level 2.

What about stimulated emission?

Stimulated emission

Electrons relax to lower energy state and emit photons that are in phase with the incident photons.



Stimulated emission of a photon.

For a system with many such 2-level atoms.

N_1: number of electrons on level 1

N_2: number of electrons on level 2

dN_2: change of number of electrons in level 2.

Similarly, to the way we did before:

$$dN_2\Big|_{\substack{\text{stimulated}\\ \text{emission}}} = -B_{21}\rho\left(\nu_{21}\right)N_2dt$$

where B_{21} is the **Einstein coefficient of stimulated emission.**

dt is the time it takes to wait for such a phenomenon to occur.

The minus sign indicates that absorption decreases the number of electrons in Level 2.

Remember Planck's black body?

Planck's black body spectrum

For a perfect black body, the **spectral energy density** (energy per unit volume per unit frequency) is:

$$\boxed{\rho(\nu) = \frac{8\pi\nu^2}{c^3}\frac{h\nu}{e^{\frac{h\nu}{k_B T}} - 1}}$$

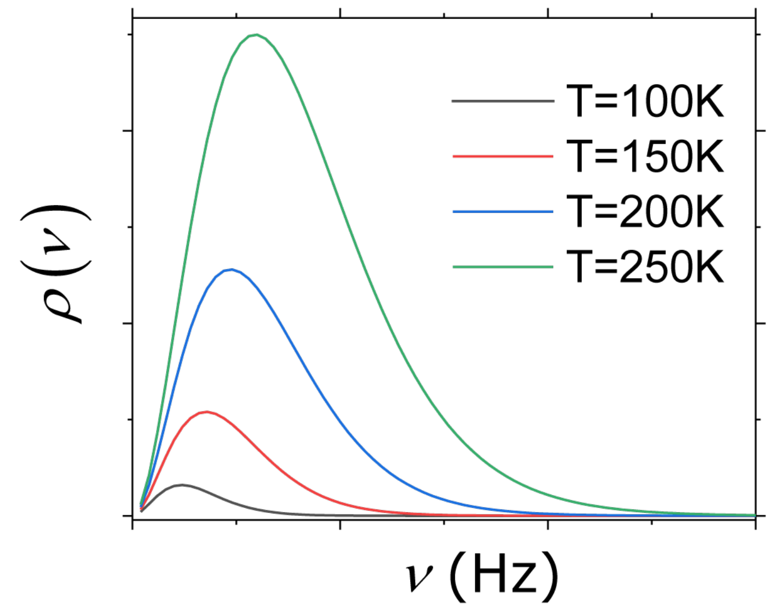
c is the speed of light

h is Planck's constant

ν is the frequency of light

k_B is Boltzmann's constant

T is the temperature.

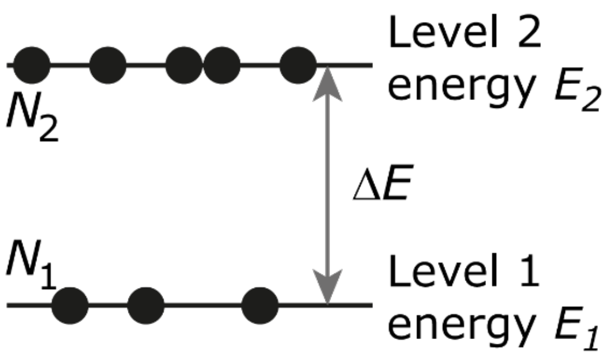


Black body spectra at different temperatures.

This formular appears in Einstein's relations. What are they?

Einstein's relations

We consider an energy diagram:



A two-level energy diagram with several electrons.

We define the upward transition rate as:

$$dN_1\big|_{\text{absoprtion}} = -B_{12}\rho(\nu_{21})N_1 dt$$

So, we can write:

$$\frac{dN_1}{dt} = -B_{12}\rho(\nu_{21})N_1$$

The downward rate has two parts:

$$dN_2\big|_{\substack{\text{spontaneous}\\ \text{emission}}} = -A_{21}N_2 dt$$

and also:

$$dN_2\big|_{\substack{\text{stimulated}\\ \text{emission}}} = -B_{21}\rho(\nu_{21})N_2 dt$$

So, we can write:

$$\frac{dN_2}{dt} = -\left[A_{21} + B_{21}\rho(\nu_{21})\right]N_2$$

In equilibrium, the number of transitions upwards and downwards is the same. So, we can state:

$$B_{12}\rho(\nu_{21})N_1 = \left[A_{21} + B_{21}\rho(\nu_{21})\right]N_2$$

Just a few more steps…

Einstein's relations

We just found that:

$$B_{12}\rho(\nu_{21})N_1 = \left[A_{21} + B_{21}\rho(\nu_{21})\right]N_2$$

Rearranging:

$$A_{21}N_2 = B_{12}N_1\underline{\underline{\rho(\nu_{21})}} - B_{21}\underline{\underline{\rho(\nu_{21})}}N_2 =$$

$$= \underline{\underline{\rho(\nu_{21})}}B_{21}N_2\left[\frac{B_{12}N_1}{B_{21}N_2} - 1\right]$$

And,

$$\rho(\nu_{21}) = \frac{A_{21}N_2}{B_{21}N_2\left[\frac{B_{12}N_1}{B_{21}N_2} - 1\right]} = \frac{\frac{A_{21}}{B_{21}}}{\frac{B_{12}N_1}{B_{21}N_2} - 1}$$

In thermal equilibrium:

$$\frac{N_1}{N_2} = e^{\frac{E_2 - E_1}{k_B T}} = e^{\frac{h\nu_{21}}{k_B T}}$$

Which leads to:

$$\rho(\nu_{21}) = \frac{A_{21}}{B_{21}}\frac{1}{\frac{B_{12}}{B_{21}}e^{\frac{h\nu_{21}}{k_B T}} - 1}$$

We proceed by identification with Planck's blackbody spectrum:

$$\rho(\nu_{21}) = \frac{A_{21}}{B_{21}}\frac{1}{\frac{B_{12}}{B_{21}}e^{\frac{h\nu_{21}}{k_B T}} - 1} = \frac{8\pi h\nu_{21}^3}{c^3}\frac{1}{1\cdot e^{\frac{h\nu_{21}}{k_B T}} - 1}$$

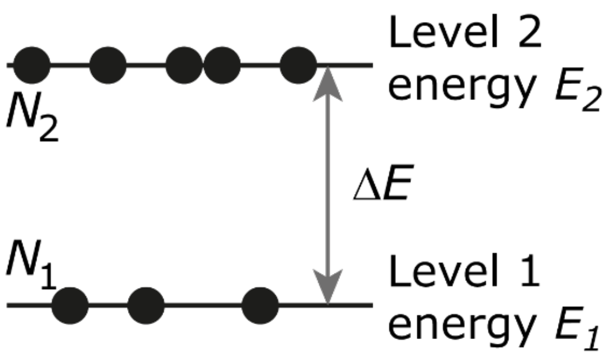
And we obtain **Einstein's relation**:

$$\boxed{\frac{B_{12}}{B_{21}} = 1} \quad \text{and} \quad \boxed{\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu_{21}^3}{c^3}}$$

So, how do we get lasing?

Population inversion and lasing

We consider an energy diagram:



A two-level energy diagram with several electrons.

For the downward rate, we have

$$dN_2\Big|_{\substack{\text{spontaneous}\\\text{emission}}} = -A_{21}N_2 dt$$

and

$$dN_2\Big|_{\substack{\text{stimulated}\\\text{emission}}} = -B_{21}\rho(\nu_{21})N_2 dt$$

Therefore:

$$\frac{\text{rate of spontaneous emission}}{\text{rate of stimulated emission}} = \frac{A_{21}}{B_{21}\rho(\nu_{21})}$$

Then, using Einstein's relation:

$$\frac{A_{21}}{B_{21}}\frac{1}{\rho(\nu_{21})} = \frac{8\pi h\nu_{21}^3}{c^3}\frac{1}{\frac{8\pi h\nu_{21}^3}{c^3}\frac{1}{e^{\frac{h\nu_{21}}{k_BT}}-1}} =$$

$$= e^{\frac{h\nu_{21}}{k_BT}} - 1$$

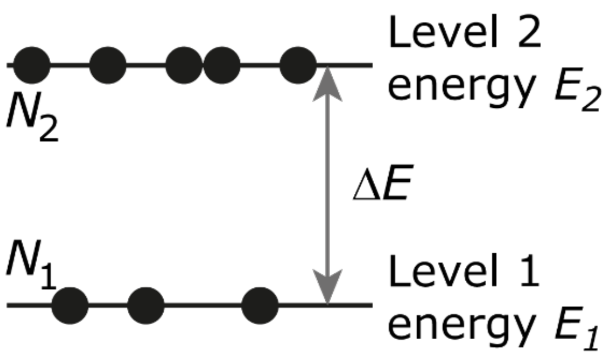
Which means that:

$$\frac{\text{rate of spontaneous emission}}{\text{rate of stimulated emission}} = e^{\frac{h\nu_{21}}{k_BT}} - 1$$

How about absorption?

Population inversion and lasing

We consider an energy diagram:



A two-level energy diagram with several electrons.

Similarly:

$$dN_1\big|_{\text{absoprtion}} = -B_{12}\rho(\nu_{21})N_1 dt$$

Therefore, we can write:

$$\frac{\text{rate of emission}}{\text{rate of absorption}} = \frac{N_2\left[A_{21} + B_{21}\rho(\nu_{21})\right]}{N_1 B_{12}\rho(\nu_{21})} =$$

$$= \frac{N_2}{N_1}\left[\frac{A_{21}}{B_{12}\rho(\nu_{21})} + 1\right]$$

$$\boxed{\frac{B_{12}}{B_{21}} = 1}$$

If the density of incident photons is high:

$$\frac{A_{21}}{B_{12}\rho(\nu_{21})} \ll 1$$

And we can write:

$$\frac{\text{rate of emission}}{\text{rate of absorption}} \approx \frac{N_2}{N_1}$$

if we could inverse the occupation, so that $N_2 > N_1$, then **the rate of emission will exceed the rate of absorption**.

How do we make laser?

Population inversion and lasing

In other terms, the light entering the material will be amplified by laser action.

By definition, **population inversion** refers to a system where more electrons occupy the higher (or ‘excited’) energy state than the lower (or ‘ground’) state.

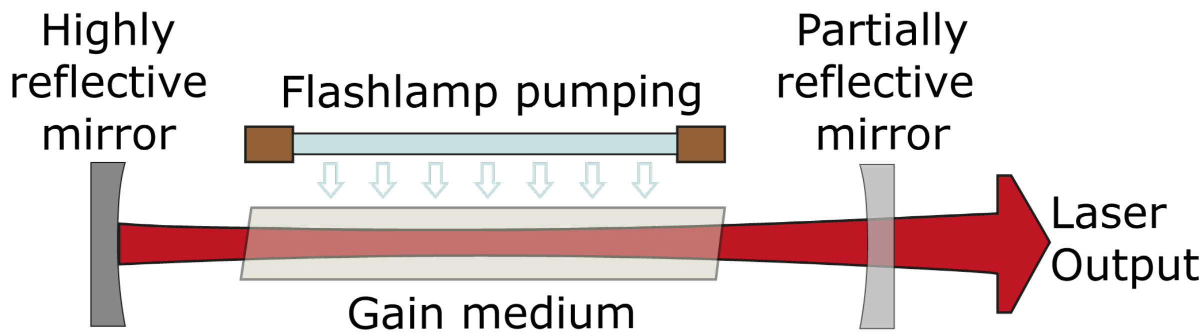


Diagram of the basic components of a laser system.

Transparency: the gain in the material equals the loss in the material (intensity out = intensity in).
Lasing threshold: the gain in the material exceeds all the losses (including the loss of photons through the mirrors) in the laser cavity.

What are the main properties of a laser?

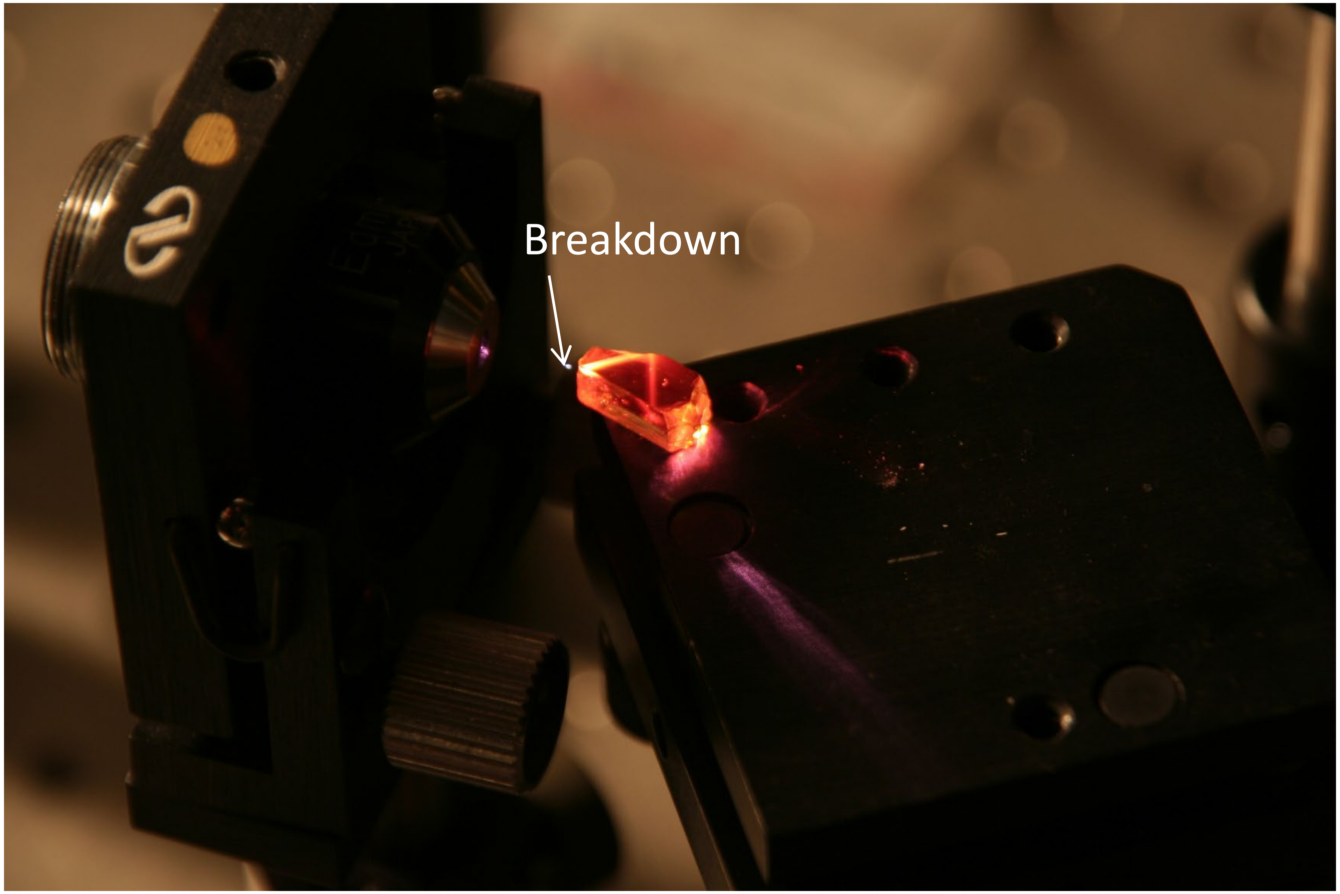
Main laser properties

Lasers have several key properties that distinguish them from other light sources:

- **Coherence:** Laser light is coherent, which means that all the photons in the beam have the same frequency and are in phase with each other. This is a consequence of stimulated emission.
- **Monochromaticity**: Laser light is monochromatic, meaning it consists of a single wavelength or colour. This property is a result of amplification at the same (atomic) transition energy within the gain medium.
- **Directionality**: Laser light is highly directional, meaning it can be focused into a very small spot or beam. This property is useful in applications such as laser cutting, welding and communications.
- **High intensity**: Lasers can produce very high-intensity light beams. Because photons do not interact with each other, we can pack a lot of them in a beam.

Illustrations?

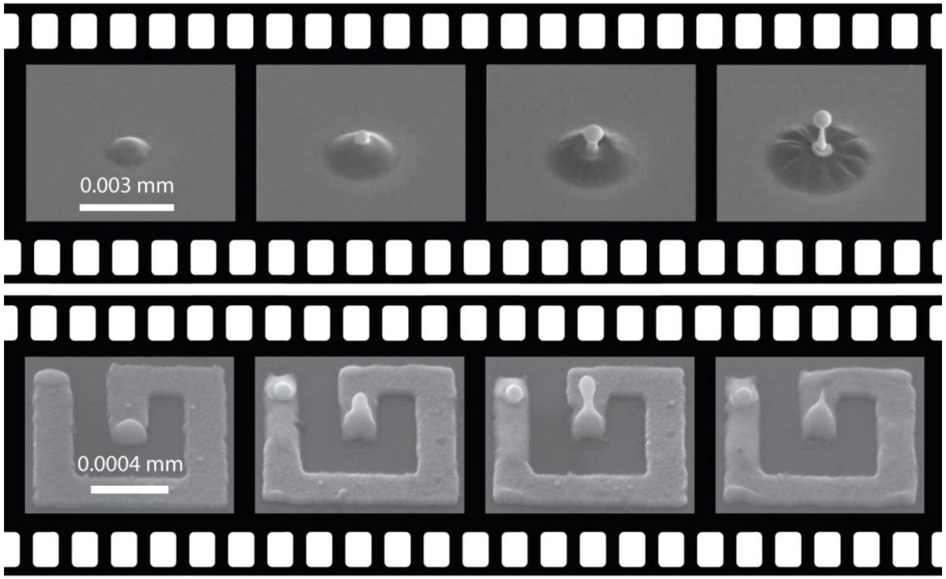
High intensity: laser-induced breakdown in air



The electric field of light within femtosecond pulses can ionize the air molecules, producing a plasma, that is similar to lighting. Wavelength 800 nm. At a repetition rate of 1 KHz, the breakdown is audible.

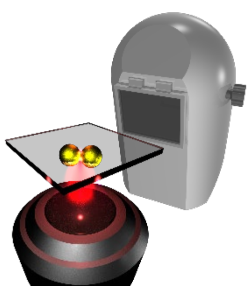
Can be used for welding, even at the nanoscale!

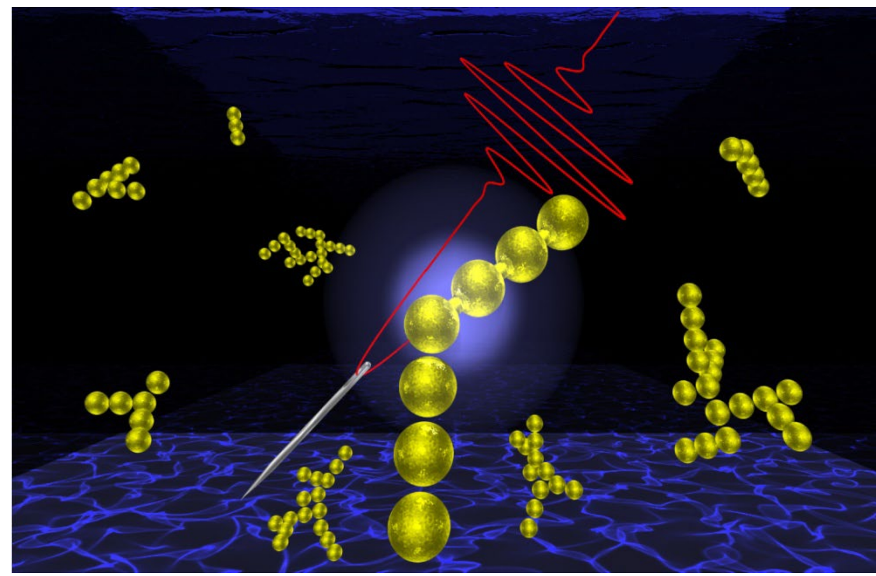
High intensity: nano-welding



Following the impact of a single fs pulse, different stages of hydrodynamic processes can be observed in Au nanostructures. **Ours are the smallest nanojets in the world**.

Nano-welding: a route to manufacturing nanomaterials with fs laser pulses through laser-welding of nanoparticle strings.





Is monochromatic!

A laser is monochromatic

The latest laser we bought in my lab: FWHM is <1 fm

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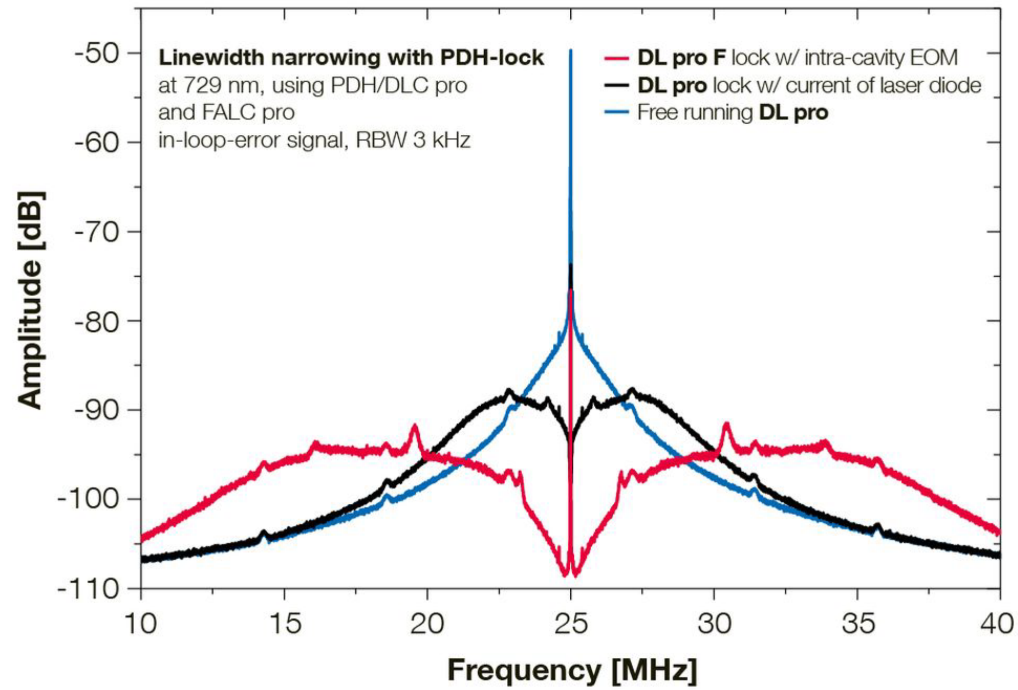
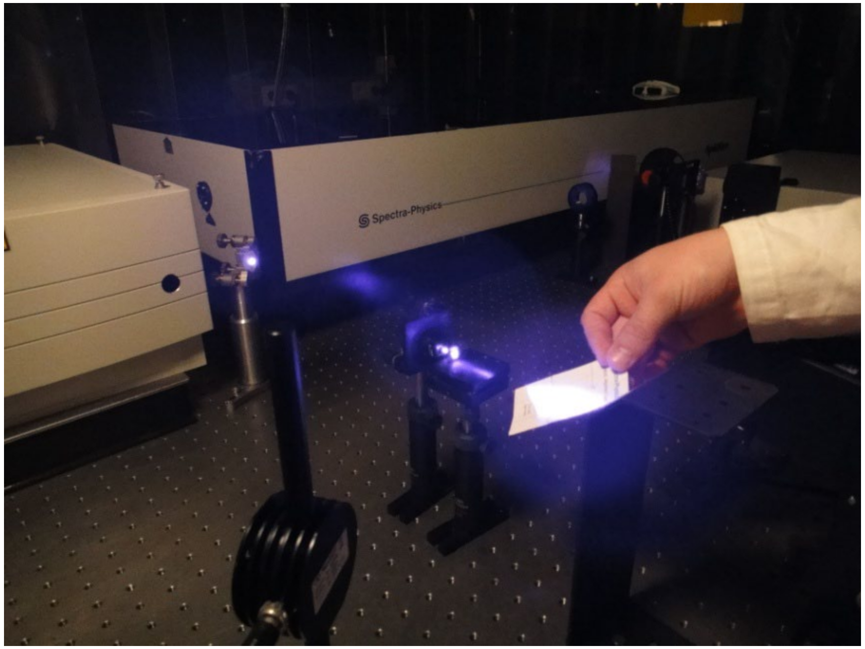
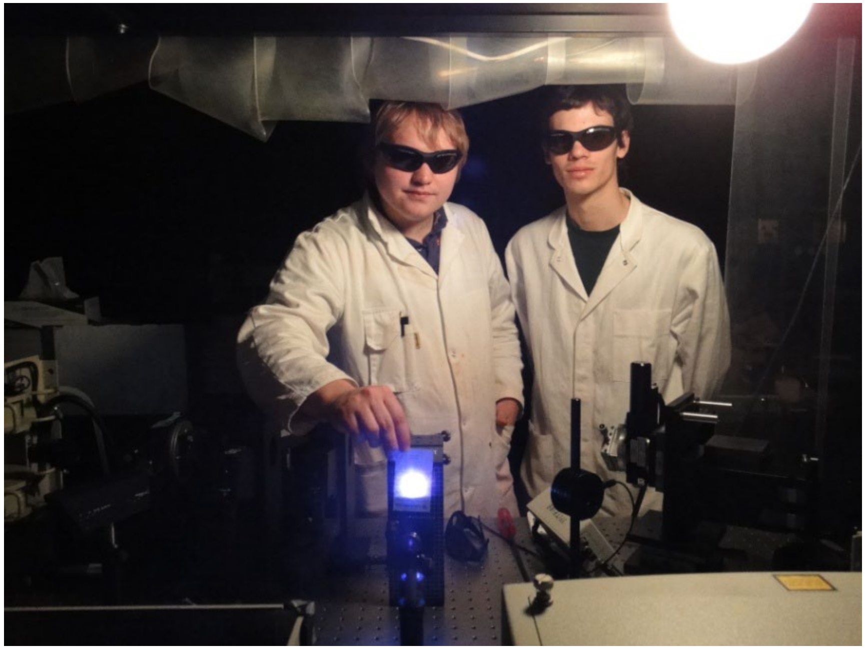


But is it always monochromatic?

A laser is monochromatic, but in nonlinear optics…

Due to the very high power of the laser pulses, a multitude of nonlinear optical effects can occur upon interaction with matter.





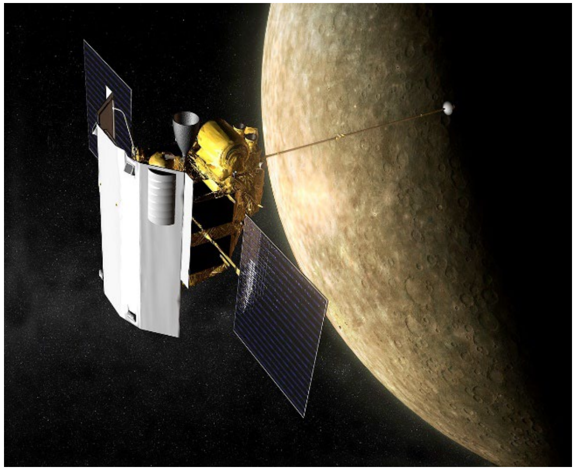
As a result, **white light continuum** generation can be observed, here as a part of a master students' exercise. Using color filters, we could select colors… at the expense of intensity per wavelength.

Directionality?

Lasers have good directionality

Great for communication in space!

Long distance: a two-way distance record for communication was set by the Mercury laser altimeter instrument aboard the MESSENGER spacecraft, and was able to communicate across a distance of 24 million km, as the craft neared Earth on a fly-by in May, 2005.

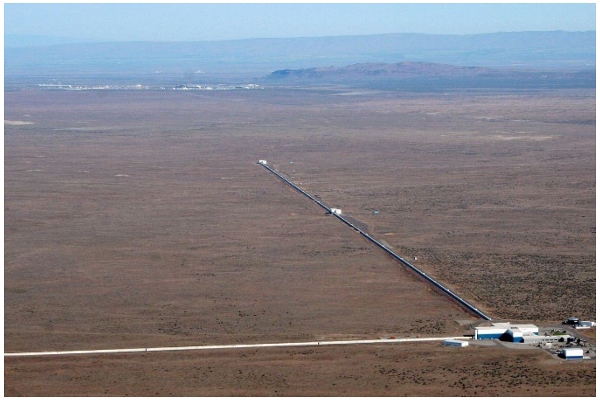


Fast (high bit rate): LunaNet is a current NASA project for internet on the Moon (to avoid scheduling communications). Moonlight is an equivalent ESA project.

Coherent?

Lasers are coherent

The Laser Interferometer Gravitational-Wave Observatory (LIGO) is designed to detect cosmic gravitational waves.



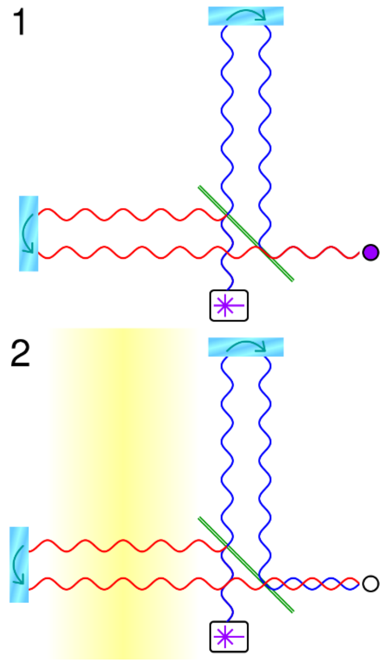
Mirrors spaced 4 km apart

Nd:YAG laser with wavelength 1064 nm

Total laser power in cavity 100 kV

Total distance: about 280 trips down the 4 km length to the far mirrors and back again

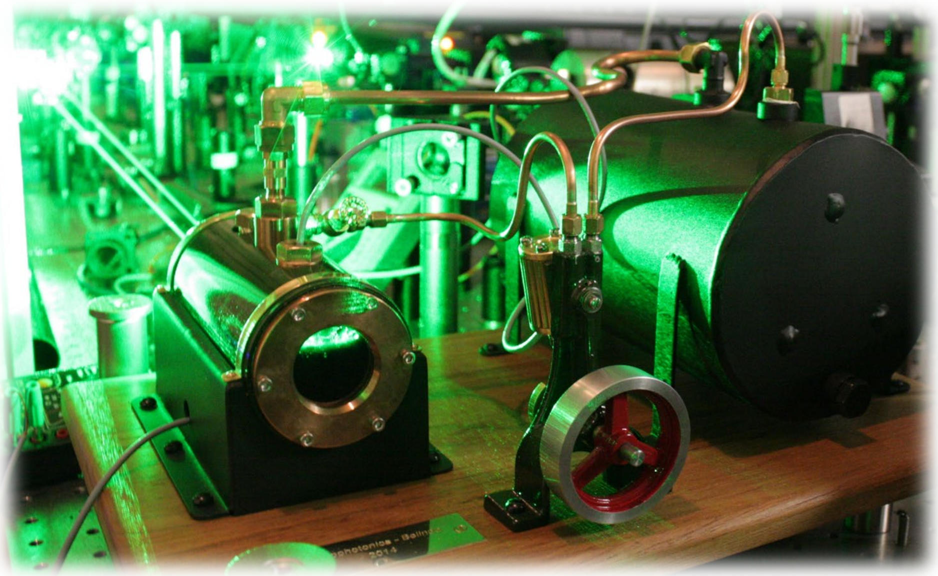
Coherence is preserved, which allows **detecting a space change of 10^{-18} m**.



Most importantly though…

Lasers are fun!

Belinda: the Nanophotonic Steam engine



Laser wavelength of 532 nm

Total power: about 50 W

Gold nanoparticles (20 nm) in diameter are dispersed in the water.

Start time: about 8 hours.

Example question 1

(a) What does the acronym LASER stand for?
(b) Summarise the conditions required for lasing to occur in a system.
(c) Briefly explain the interaction that occurs between a photon and an electron in the following optical transitions: absorption, spontaneous emission and stimulated emission.
(d) Hence, explain how the rate of electron excitation and de-excitation in a system can be written as: $\frac{dN_1}{dt} = -B_{12}\rho\left(\nu_{21}\right)N_1$ and $\frac{dN_2}{dt} = -\left[A_{21} + B_{21}\rho\left(\nu_{21}\right)\right]N_2$.

Example question 1

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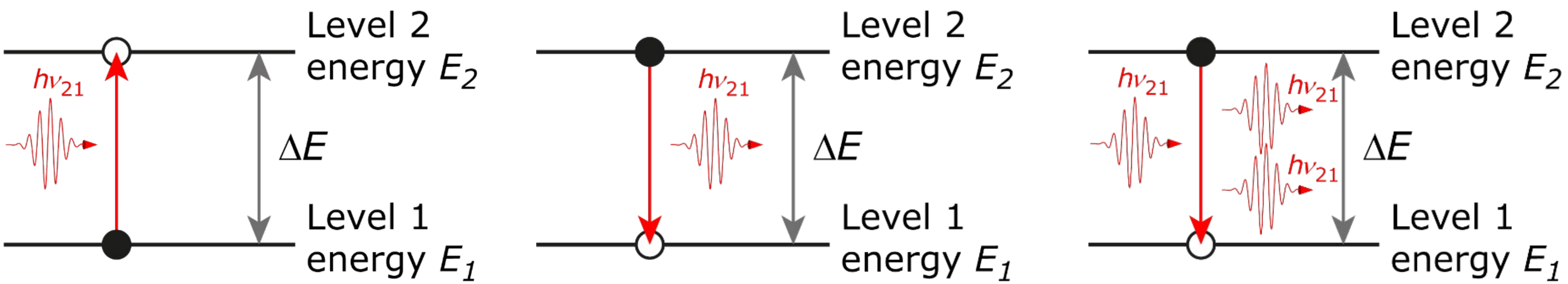
All the answers are in the lecture.

In summary…

Summary

We saw three main processes of light-matter interaction:



Absorption

Spontaneous emission

Stimulated emission

For a perfect black body, the **spectral energy density** is: $\rho(\nu) = \frac{8\pi\nu^2}{c^3}\frac{h\nu}{e^{\frac{h\nu}{k_B T}}-1}$

And we obtain **Einstein's relation**: $\frac{B_{12}}{B_{21}} = 1$ and $\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu_{21}^3}{c^3}$

Electromagnetism beyond this course

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Organized each year on **May 16**, the UNESCO-ratified **International Day of Light** was created as a follow-up to the International Year of Light in 2015.