PH20014: Electromagnetism 1

Problem Sheet 3

1. A plane wave in air is incident normally on an infinite lossless dielectric material having $\varepsilon_r = 3$, $\mu_r = 1$. If the incident wave is $\vec{E}_j = 10\cos(\omega t - x)\,\hat{j}$ V/m, find
(a) the wavelength λ_i and the value of ω of the incident wave;
(6.28 m, 3x10^8 rad/s)
(b) the wavelength λ_t of the transmitted wave; (3.63 m)
(c) the amplitude of the incident magnetic field; (0.026 Am^{-1})
(d) the amplitude of the transmitted electric field; (7.32 Vm^{-1})
(e) the time-averaged transmitted power. (0.123 Wm^{-2})

2. A plane wave propagates through a dielectric medium in the region $z \le 0$ with $\varepsilon_r = 9$ and $\mu_r = 1$ with $\vec{H}_j = -0.2\cos\left(10^9 t - Kx - K\sqrt{8}z\right)\hat{j}$ A/m. The wave is incident on a boundary with air, which forms the *xy* plane, at $z = 0$. *[Note that $K = k\sin\theta_i$ and that $\sqrt{8}K = k\cos\theta_i$ in the usual notation, where k is the wavevector in the dielectric material]* Find:
(a) θ_i and θ_t; (19.47º; 90º - incident wave at critical angle)
(b) the values of *K* and *k*; (3.33 m-1; 10 m-1)
(c) the wavelengths in the dielectric and in the air; (0.628m, 1.88 m)
(d) the incident electric field; $\left(-23.7\hat{i} + 8.38\hat{k}\right)\cos\left(10^9 t - Kx - K\sqrt{8}z\right)$ Vm-1
(e) the transmitted and reflected electric fields;
$150.7\cos\left(10^9 t - 3.333x\right)\hat{k}$ V/m and $\left(23.7\hat{i} + 8.38\hat{k}\right)\cos\left(10^9 t - Kx + K\sqrt{8}z\right)\hat{k}$ V/m (f) the Brewster angle. (18.43 º)

3. Consider the case of a plane wave polarised with its E-field perpendicular to the plane of incidence. The wave propagates in vacuum and reflects from a non-magnetic material with a relative permittivity > 1. Show that the reflected wave is π radians out of phase with the incident wave for any angle of incidence. [Hint: You may wish to use Snell's Law]

4. *[2016 Exam question]* A uniform plane wave is propagating in air with an electric field given by $\vec{E} = 10\cos\left[\left(19.21\times10^8\right)t - 4x - 5z\right]\hat{y}$ [V/m], with *x* and *z* measured in metres and *t* in seconds. It hits the surface of a lossless dielectric slab with $\varepsilon_r = 4$, $\mu_r = 1$ occupying the half-space $z \ge 0$. Find
(a) the polarisation of the incident wave with respect to the plane of incidence, (2)
(b) the angle of incidence, (1)
(c) the reflected electric field $\vec{E}_r$ (6)

5. *[HARDER]* This question is about refraction and reflection of light at the surface of a good conductor. You may assume that the refractive index is >>1 for a good conductor *(More on this in electromagnetism 2)*. [the material is non-magnetic]
(a) Summarise how light transmitted into a good conductor is refracted.
(b) Using the expressions for the impedance of free space and the impedance of a good conductor find a good approximation for the reflectivity (*R*) of a metal surface. You can assume normal incidence to the surface to simplify the calculation.