PH20014: Electromagnetism 1

Problem Sheet 1

Electric Fields

1. At a particular temperature and pressure, a helium gas contains 5×10^{25} atoms per cubic metre. If an electric field of 10 kV m^{-1} applied to the gas causes an average electron cloud shift of 10^{-18} m, find the relative permittivity of the helium.

[ε_r=1.00018]

2. A material has an electric susceptibility of 3.5. Calculate the electric dipole moment, *P*, and the *D*-field if the electric field *E* is 15 Vm^{-1}.

[P = 4.6 x 10^{-10} Cm^{-2}, D = 6.0 x 10^{-10} Cm^{-2}]

3. A slab of dielectric of relative permittivity ε_r fills the space between $z = \pm a$ in the *x*-*y* plane, and contains a uniform density of free charges ρ_f per unit volume. Using Gauss' Law, find $\vec{E}$, $\vec{D}$ and $\vec{P}$ as functions of *z.* What is the surface density of polarisation charge on the surface of the dielectric?

[σ=P=$\pm(\varepsilon_r$-1)ρ_f a/ ε_r]

4. A charge *Q* is placed at the centre of a spherical shell of LIH dielectric material with relative permittivity ε_r. The shell has an inner radius *a,* and an outer radius *b*, and the rest of space is a vacuum. Using Gauss' Law, find (a) the displacement field $\vec{D}$ at any distance, *r*, from the charge, (b) Find the electric field for (i) r <a, (ii) $a< r <b$ and (iii) $r > b$ and (c) find the electrostatic energy stored in the dielectric.

[(a) D=Q/(4πR^2), (b) (i),(iii) E =Q/(4$\pi\varepsilon_0$R^2), (ii) E =Q/(4$\pi\varepsilon_r\varepsilon_0$R^2), (c) U=Q^2/(8$\pi\varepsilon_r\varepsilon_0$) (1/$a$ – 1/b)]

Magnetic Fields

5. A material has a magnetic susceptibility of 0.01. Calculate the magnetic dipole moment per unit volume *M* and the *B*-field if the *H*-field is 10^3 A m^{-1}.

[M=10 Am^{-1}, B=1.3 x 10^{-3} T]

6. An infinitely long cylindrical conductor of radius *a* and permeability $\mu_0\mu_r$ is placed along the *z*-axis. The conductor carries a uniformly distributed current *I* along *z*. Use Ampère's Law to find *H* and then *M* for 0 < r < a.

[M= (μ_r-1) I R /(2πa^2)]

7. In a certain homogeneous isotropic medium for which $\mu_r = 4$, the magnetic field (B-field) is given in mT by $\vec{B} = 2\hat{i} - 5\hat{j} + 4\hat{k}$, where $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors. Calculate (a) the magnetic susceptibility χ_m, (b) the *H*-field – the magnetic field intensity, (c) the magnetisation $\vec{M}$ and (d) the magnetic energy stored per unit volume.

[(a) χ=3, (b) H=(2, -5, 4)/4μ_0 mAm^{-1}, (c) M= 3(2, -5, 4)/4μ_0 mAm^{-1}, (d) W = 4.5 J m^{-3}]

8. Use your new definition of $\vec{B}$ and a Maxwell equation to show that $\nabla \cdot \vec{H} = -\nabla \cdot \vec{M}$. Using your understanding of divergence, explain the meaning of this result. What does this tell you about the fields around a bar magnet?

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More General Question

9. **The Dielectric Relaxation Time**. Use the following equations,

$$\nabla \cdot \vec{J}_f = -\frac{\partial \rho_f}{\partial t} \qquad \text{(The continuity Equation),}$$

$$\vec{J}_f = \sigma \vec{E} \qquad \text{(Ohm's law) and}$$

$$\nabla \cdot \vec{E} = \frac{\rho_f}{\varepsilon_0} \qquad \text{(Gauss' Law),}$$

to find a differential equation which has the solution $\rho_f = \rho_{f0} e^{-t/\tau}$, where $\tau = \frac{\varepsilon_0}{\sigma}$, ρ_{f0} is the initial charge density at $t = 0$ and τ is the relaxation or rearrangement time.