1 A.U

Rocky

snow line : cold enough volatile components to condense into ice grains ~5AU

Sun, Mercury, Venus, Earth, Mars, | Jupiter, Saturn, Uranus, Neptune

- Mercury: • smallest • thin atmosphere (O_2, Na, H) • mag field 1% Earth • water ice pols
- Venus: • retrograde spin • thick atmosphere (CO_2, clouds, sulfuric acid) • greenhouse effect • volcanism 2 tectonis
- Earth: 72% N_2, 21% O_2, 1% other
- Mars: • ½ size earth • thin atmosph CO_2, N_2, Ar • no magnetosphere
- Jupiter: • x11 earth • atmosph H_2, He • great red spot • magnetosphere 16-54 x Earth's
- Saturn: • x9 earth • atmsp H_2, He • winds > 500m/s • dense metal core • magnetosphere 20x Saturn diameter • rings mostly water
- Uranus: • x4 earth • atmosph H_2, He, H_2O / CH_4 (blue), amonia • huge winds
- Neptune: • small rocky core • Bath 1781

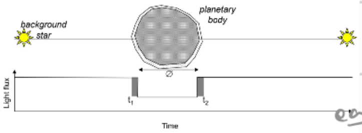
Mass variat° inertia contribut°

$$I = \int dI = \int_0^M r^2 dm$$

- solid sphere : $I = \frac{2}{5}MR^2$
- hollow sphere : $I = \frac{2}{3}Mr^2$

Newton's Laws

1) rst unless ext F
2) F = ma
3) Act° => = React°



Diameters can be measured:

-directly (angular diameter)	*prone to errors (small values)*
- with radar	*only for close planets*
- most commonly: stellar occultation	*also reveals planetary atmospheres wavelength-dependent*

$\varnothing = V \times (t_1 - t_2)$

$\lambda = \frac{c}{f}$ 2 diff waves : $10 \lg_{10} \frac{A_1}{A_2} = \Delta A\,(dB)$

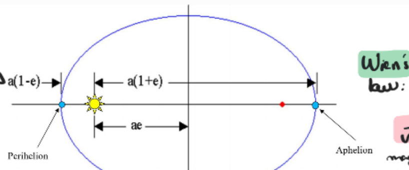
Coherence $E_V * E_H$

Kepler's Laws

1) Each planet's path around sun ellipse
2) line to sun sweeps equal distance at all intervals of time
3) $\frac{a^3}{T^2} = cnst$

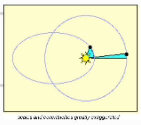
earth $a = 1$ A.U
$T = 1$ year

1 focus



a = size of orbit (semi-major axis)
e = eccentricity
θ = polar angle (0 at perihelion, +π at aphelion)
r = heliocentric distance

$$r = a \times \frac{(1-e^2)}{(1+e\cos\theta)}$$



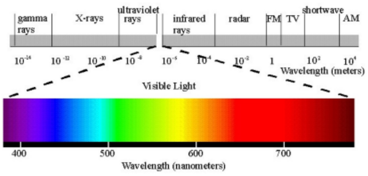
A planet moves fastest at the nearest point to the Sun (*perihelion*)

It moves most slowly at the farthest point from the Sun (*aphelion*)

$$\frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt} = const.$$

Wien's displacement law: $\lambda_{peak} = \frac{b}{T}$ ← const prop

$\frac{T}{V_{max}} = 0.5099$



Parameters of interest:

energy absorbed/transmitted/reflected $E = h\nu = \Delta E$

molecular/atomic level transitions $\lambda = \frac{c}{\nu}$

wavelengths/frequencies

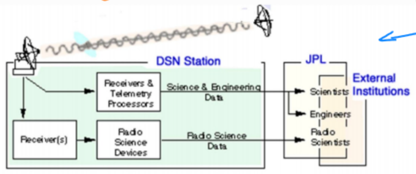
$\lambda_{max} \times T = 2898\ \mu m \cdot K$

environment: mag field, solar rad; particles, near planets; on planets – atmospheres, gravity, temp

Miss° design

- Targets : planetary syst, orbiting / landing, local condit°s
- Instruments ~ need task < power, time; power < radio-isotope (output ↓ with time, useful far from sun), solar panels
- Durat° : transit + study ← consider constrains power / communicat° opt°s

Beam coms



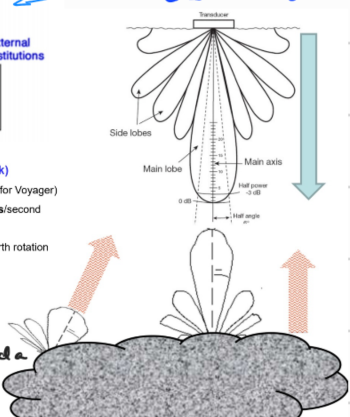
DSN (Deep-Space Network)

Typical radio power: 20 W (e.g. for Voyager)

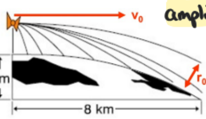
Typical bandwidth: 100's kilo**bits**/second

fixed data rates

Several nodes: accounts for Earth rotation



Orbital mechanics



amplitude accel° : $\frac{v_1^2}{r_0}$

gravity : mass x centripetal a

$G\frac{mM}{r_0^2} = m\frac{v_1^2}{r_0}$

$\therefore v_1 = \sqrt{\frac{GM}{r_0}}$

$v_2 = \sqrt{\frac{2GM}{r_0}}$

Different orbits for different parts of the mission

Example: Psyche (2023 – 2030)

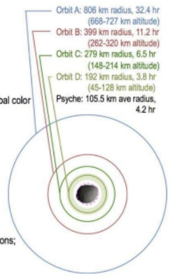
Approach: (100 days)
- Optical navigation and instrument calibration, hazard assessment
- Psyche spin axis and rotation period determination

Orbit A: Characterization: 40 days (29 orbits)
- Nadir mapping for preliminary shape determination, global color mapping, magnetic field detection, and gravity science

Orbit B: Topography: 50 days (90 orbits)
- Nadir and 4 off-nadir imaging cycles for topography
- Global color maps, gravity science, and preliminary magnetic field characterization

Orbit C: Integrated Science: 100 days (369 orbits)
- Gravity science, magnetic field mapping, crater age determination, and preliminary neutron mapping

Orbit D: Elemental Mapping: 70 days (442 orbits)
- Map elemental composition with gamma rays and neutrons; improved imaging, gravity, and magnetic field mapping

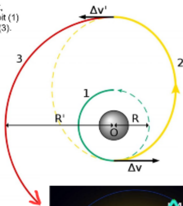


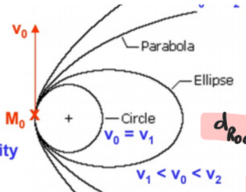
Changing Orbits

Hohmann transfer orbit: part of elliptic orbit, between initial orbit (1) and desired orbit (3).

Transfer (2): changing velocity/direction by firing thruster(s) at beginning and end (to make new orbit circular).

Used to move between orbits or for orbit insertion





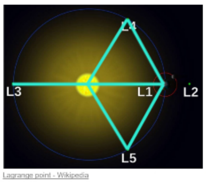
V_1 = circular orbit velocity
V_2 = escape velocity

$d_{Roche} = R\left(2\frac{M}{m}\right)^{1/3}$

d_{Roche}

$d_{Hill} = \sqrt[3]{\frac{3M}{m}}$

$= R \times \sqrt[3]{\frac{2}{3}}$



Lagrange point - Wikipedia

Lagrange points are good for satellites (need for "halo orbits" around L_{1-3}). They are also good for natural objects.

In the Sun-Earth system:

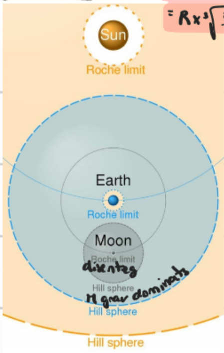
L_1 (1.5 × 10^6 km from Earth, 0.01 AU)
Solar and Heliospheric Observatory (SOHO)
Other solar observation satellites

L_2 (same as L_1)
Herschel Space Telescope (2009-2013)
James Webb Space Telescope (tbc)
Good for stable temperature (50 K)

L_3
Unstable point but space-weather satellites planned (halo orbit, 7-day warning of solar eruptions)

L_4 and L_5
Interplanetary dust, "trojan" asteroid

Closer to the Earth: need to account for gravitational field of the Moon



RADAR p S/35

☆ 101

→ $0.5\,M_\odot \lesssim \star \lesssim 200\,M_\odot$

→ $M_* \sim T_{eff}$ (we know; measured spectroscopy)

$M_* \sim R_*$

not uniform brightness (star spots)

★ spin around an axis with speeds from $V_{*rot} \approx 10 - 200$ km s^{-1}

$V_{*rot} \sim M_*$

Kepler's 3rd: $\frac{a^3}{P^2} = \frac{G(M_* + M_p)}{4\pi^2}$

$M_* >> M_p \qquad M_* + M_p \approx M_*$

Select° effects
→ Hard: we cannot detect (limits instruments)
→ Soft: detectable but rare ~ environment stats any size ppl° may swamp
from infered size] completness correct°

Detecting exoplanets

main:
① Direct imaging — light reflected surface
② Transit — light blocked by planet as it passes from the stars
③ Radial velocity — change in spectrum star light as it passes under grav influence

other:
④ Astronometry: long term repeated patterns of movement of stars in a series of images
→ Micro lensing: repeated short periods of brightening of stars in series of images due to relativistic effects
⑤ Polarimetry → repeated incidents of polarisat° of light as light passes through a planetary atmosphere
⑥ Timing variat°: anomalies in orbital timings of previously discovered planets these may be caused by as yet undetected planets

① Direct imaging

$\theta = \sin\left(\frac{a}{d}\right) \approx \frac{a}{d}$ ← semi-major axis

angular separat° the planet & ★; distance from star to us

$\frac{L_p}{L_*} = A\frac{R_p^2}{4a^2} \qquad \frac{L_p}{L_*} < 10^{-6}$

• L: luminosity < p: planet, ★: star

• A: Bond albedo of planet = fract° light from ★ that is reflected back; dimensionless $0 \le A \le 1$

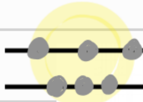
• R_p: rad planet

✓ • possibility doing spectroscopy on planetary atmosph to measure temp of planet or even atmospheric composit°

− • unable constrain mass planet

• most sensitive planets R_p, high A, large a (Jupiter type planets)

② Transit method

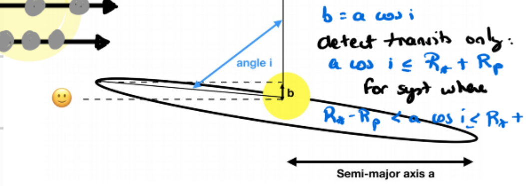


planet passes in front of its star we can measure:
• orbital period planets
• radius of planet
• semi-major axis of orbit

$a \approx \left(G M_* \left(\frac{P}{2\pi}\right)^2\right)^{1/3}$

$\frac{\Delta F}{F} = \frac{R_p^2}{R_*^2}$ fractional reduct° in the flux received during the star in the middle of transit

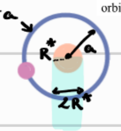
durat° transit → $T_{trans} = P\frac{R_*}{\pi a}$



Sketch showing impact parameter b and orbital inclination angle i alone with the semi-major axis of orbit a.

$b = a\cos i$

detect transits only: $a\cos i \le R_* + R_p$ for syst where $R_* - R_p \le a\cos i \le R_* + R_p$



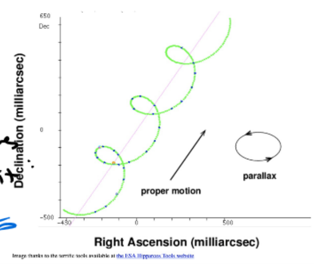
Assuming randomly distributed orbital planes $0° \le i \le 90°$, prob planet observed as transit

P_{tras} = n° transit orbits / n° all possible orbits

④ Astronometry

The other methods of detecting exoplanets

Astrometry



Angular displacement of the star from it's CoM

$\beta = \frac{M_p a_p}{M_* d}$

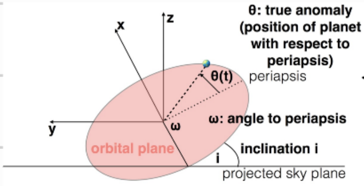
Method requires repeated observations of the same region of the sky for an extended period of time.

Selection effect: Angular resolution $\beta < 1$ arcsec.

③ Radial velocity

→ Instrument: astronomical spectroscopes were originally designed to detect doppler shifts due to stellar & galactic velocities [km s^{-1}]

→ FYI: • Typical Doppler shifts due to orbiting planet ~m s^{-1}
• Mass ratio • Semi-major axis of planetary orbit



Sketch showing the radial velocity method of detecting exoplanets

This is a gravitational method

A star and its planet orbit around their common centre of mass.

We can use spectroscopy to detect the 'Doppler wobble' as the star moves back and forth

Typical velocities are of the order of m s^{-1} i.e. human walking speed

We will derive the equations of motion for the planet in cartesian coordinates first.

ω = angle of periapsis (the point at which the planet is closest to the star) *wrt* line of sight.

θ = position angle of planet *wrt* periapsis.

The velocities in the x, y, & z directions are v_x, v_y & v_z.

$$v_x = -\frac{2\pi a}{P\sqrt{1-e^2}}\left(\sin(\theta+\omega) + e\sin(\omega)\right)$$

$$v_y = -\frac{\cos i\, 2\pi a}{P\sqrt{1-e^2}}\left(\cos(\theta+\omega) + e\cos(\omega)\right)$$

$$v_z = -\frac{\sin i\, 2\pi a}{P\sqrt{1-e^2}}\left(\cos(\theta+\omega) + e\cos(\omega)\right)$$

i = inclination angle, a = semi-major axis, e = eccentricity, ω and θ as defined above.

We now have the planetary motion in the *astrocentric* (*wrt* star) frame, $\overrightarrow{v_{p*}}$.

$$\overrightarrow{v_{p*}} = \overrightarrow{v_p} - \overrightarrow{v_*}$$

where $\overrightarrow{v_p}$ is the velocity of the planet *wrt* the barycentre of the system.

⑤ Polarimetry

− star light unpolarised

Planetary atmosphere can polarise star light → measure changes in polarised° over an orbital period: infers presence of planet

Very difficult in practice

⑥ Transit time variat°

: previously undetected planets in a system may be found by looking for their gravitational effect on the known planets in the syst ~ only works if we already found a planet in the syst

The exoplanet 'zoo'

Is our solar system an outlier?

Wider orbits, light weight planets

Selection effects again:

Transit: Low radius planets are harder to detect
High distance (from Sun) less likely to transit

Radial velocity: Low mass, long orbital period harder to detect.

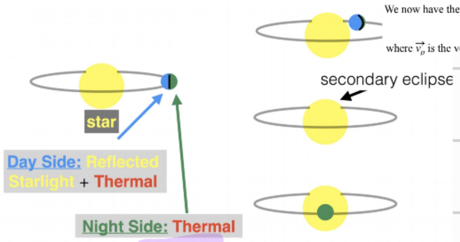
So not possible to claim that Solar System is an outlier. As we continue search for longer times more planets will be found in RV.

As instrumentation improves, smaller planets will be findable in transit.

Secondary transits

We can detect exoplanets by the fact that they can block some of the light of the

But the star can also block radiation coming from the planet



If we assume BB radiation, Planck's law is

$$I(\lambda, T) = \frac{2hc^2}{\lambda^5}\frac{1}{\exp\left(\frac{hc}{\lambda k_B T}\right) - 1}$$

Because the planet will be cooler than the star we can use the Rayleigh Jeans approximation

$$I(\lambda, T) = \frac{2ck_BT}{\lambda^4}$$

So, for a given wavelength $I \propto T$

Taking the surface are of the emitter into account we have

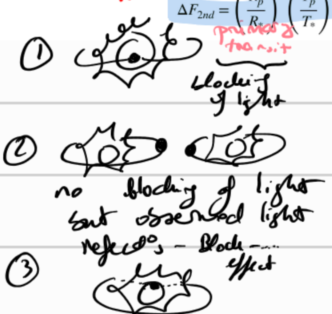
$$I_p \propto A_p \times T_p \text{ and } I_* \propto A_* \times T_*$$

So, in the same way we derived the depth of the primary transit, the depth of the secondary transit will be

Depth second transit:

$$\Delta F_{2nd} = \left(\frac{R_p}{R_*}\right)^2\left(\frac{T_p}{T_*}\right)$$

primary transit

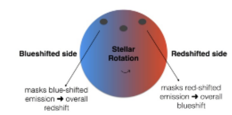


① blocking of light

② no blocking of light but observed light reflect°, - block ... effect

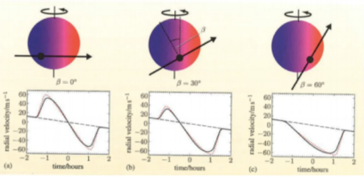
③ secondary transit - removal reflected light

Orbital alignment of exoplanets: The Rossiter-McLaughlin effect



Co-rotating planets begin transit on blueshifted side, and end on redshifted side.

Counter-rotating planets begin on redshifted side, and end on blueshifted side.



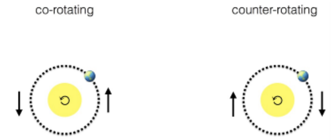
From Transiting Exoplanets, Haswell 2010, Cambridge University Press
Note, Haswell uses β for the spin parameter.

We can combine the radial velocity signal due to star orbiting the common CoM of the system with the apparent signal caused by the planet blocking some of the star's light to estimate the spin parameter α.

The RM effect has been measured for several planets. As expected, $\alpha \sim 0$ in most, But not all cases

Orbital alignment of exoplanets

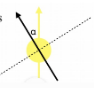
In the solar system, planets co-rotate with the Sun



Planets assumed to form from a rotating 'proto-planetary disc' surrounding newly formed star (See later section for planet formation).

In the solar system, the angular momentum vectors of the sun is ~ perpendicular to the planes of orbit all eight planets. α is the 'spin parameter'.

$\alpha \neq 0$ implies past kinematic interactions



Orbital alignment can be measured via the Rossiter-McLaughlin effect

Orbital alignment provides clues to planet formation processes

Transmission Spectroscopy

While the planet is transiting, it blocks some of the star's light. If the planet has an atmosphere some of the star's light will be absorbed by molecules in the atmosphere.

Therefore, by comparing the spectrum of the star during and between transits, we can detect the presence (or not) of an atmosphere and begin to understand its composition.

Exoplanets and life in the Universe

$\left(\frac{6.4\times10^6}{7\times10^8}\right)^2 \times \frac{300}{5800} = 4\times10^{-6}$ earth vals

The Drake equation

$$N = SFR \times f_p \times n_h \times f_l \times f_i \times f_c \times T_c$$

N = the number of 'communicative' civilisations in the Milky Way Galaxy.

SFR = the star formation rate in the Galaxy (number of stars per year, not mass of stars per year). $1.5 - 3$ stars yr^{-1}

f_p = the fraction of stars around which planets form. $f_p \times n_h \sim 0.4$

n_h = the average number of habitat planets per stellar system.

f_l = the fraction of habitable planets that develop biotic processes. $f_l = f_i = f_c = 1$

Westby, T., Conselice, C. 2020 *ApJ* **896** 58

f_i = the fraction of biotic planets that develop intelligence.

f_c = the fraction of planets with intelligent life that, wittingly or unwittingly, broadcast evidence of their existence.

T_c = the average length of time over which a civilisation is detectable $\quad T_c \sim 300$ yrs