1) a) $E_x = E_y = 0$

- $\nabla \cdot \vec{E} = \nabla \cdot 0 = 0$
  $= \frac{\rho}{\varepsilon_0} = \frac{0}{\varepsilon_0} = 0$
- $\nabla \times \vec{E} = \nabla \times 0$
  $= -\frac{\partial \vec{B}}{\partial t}$
  $= 0$
- $\nabla \cdot \vec{B} = 0$
- $\nabla \times \vec{B} = \mu_0 \vec{J} + \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}$
  $= 0 + 0 = 0$

All good ✓

b) $E_z = \cos\left(y - \frac{1}{\sqrt{\varepsilon_0 \mu_0}} t\right)$

- $\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} = 0$
  $= \frac{1}{y - \frac{1}{\sqrt{\varepsilon_0 \mu_0}}} \cos(\dots t) \neq 0$
  NOPE ✗
- $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
- $\nabla \cdot \vec{B} = 0$
- $\nabla \times \vec{B} = 0$ ...

c) $B_x = \sqrt{\varepsilon_0 \mu_0} \cos\left(y - \frac{1}{\sqrt{\varepsilon_0 \mu_0}} t\right)$

- $\nabla \times \vec{E} = \frac{\rho}{\varepsilon_0} = 0$
- $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = \frac{\varepsilon_0 \mu_0}{y - \frac{1}{\sqrt{\varepsilon_0 \mu_0}}} \cos(\dots t)$
- $\nabla \cdot \vec{B} = 0$
- $\nabla \times \vec{B} = \mu_0 \vec{J} + \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t} = 0$
  $\neq 0$

NOPE ✗

d) $B_y = B_z = 0$

- $\vec{\nabla} \times \vec{B} = 0$ ✓
- $\nabla \cdot \vec{B} = 0$ ✓

All good ✓

2) • Stoke's theorem : $\int_c \vec{F} \, d\vec{r} = \iint (\nabla \times \vec{F}) \cdot (\vec{r}_x \times \vec{r}_y) \, dx \, dy$

• Divergence theorem : $\int_V (\nabla \cdot F) \, dV = \int_{\partial V} F \cdot da$

• Both $\iint (\nabla \cdot F) \, d\dots$ $\iint \dots$

- Both: $\iiint \nabla\cdot\vec{F}\, dV = \oiint_S \vec{F}\cdot\hat{n}\, d\Sigma$

① $\nabla\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$

div $\hookrightarrow \int_V \nabla\cdot\vec{E}\, dV = \int_S \vec{E}\, d\vec{S} = \int \frac{\rho}{\varepsilon_0}\, dV$

$\oint_S \vec{E}\cdot d\vec{S} = \frac{1}{\varepsilon_0}\int_V \rho\, dV$

② $\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$

Stokes $\hookrightarrow \iint (\nabla\times\vec{E})\, dS = \oint_C \vec{E}\, d\vec{r} = \int -\frac{\partial\vec{B}}{\partial t}\, dS$

$\oint_C \vec{E}\, d\vec{r} = -\frac{\partial}{\partial t}\int_S \vec{B}\, ds$

③ $\nabla\cdot\vec{B} = 0$

div $\hookrightarrow \int \nabla\cdot\vec{B}\, dS = \int 0\, dS = 0 = \int_S \vec{B}\, d\vec{S}$

④ $\nabla\times\vec{B} = \mu_0\vec{J} + \varepsilon_0\mu_0\frac{\partial\vec{E}}{\partial t}$

Stokes $\hookrightarrow \iint \nabla\times\vec{B}\, dS = \int_S \vec{B}\, dr = \int \mu_0\vec{J} + \varepsilon_0\mu_0\frac{\partial\vec{E}}{\partial t}\, dS$

$= \mu_0\int_S \vec{J}\, dS + \varepsilon_0\mu_0\frac{\partial}{\partial t}\int_S \vec{E}\, ds$

$\oint_S$ closed : no input?

$\nabla\cdot B = \rho^*$ $\quad J^*$

V

$\rho^*(r,t)$

Net m. current out of V
=
- rate of charge of mag ✓

3) a) $\nabla\cdot\vec{B} = \rho^*$ [T/m]

$\oint B\, dS = \int_V \rho^*\, dV$

$\oint_C B\, dr = \mu_0\int\vec{J}\, d\vec{S} + \varepsilon_0\mu_0\frac{\partial}{\partial t}\int_S \vec{E}\, d\vec{S}$

$B(r) = \frac{\mu_0}{4\pi}\int_{V'} \vec{J}(\vec{r}\,') \times \frac{\hat{R}}{R^2}\, dV'$

$\cancel{\oint \frac{\mu_0}{4\pi}\int_{V'} \vec{J}^*(\vec{r}\,') \times \frac{\hat{R}}{R^2}\, dV'\, dS} = \int_V \rho^*\, dV = \oint_S \vec{J}\, ds$

$= \int \nabla\cdot\vec{J}^*\, dV$

$\cancel{\rho^* = \frac{\mu_0}{4\pi}\left[\int_S \vec{J}^*\, dr + \varepsilon\right]}^{0?}$ ← check this

$\nabla\cdot\vec{J}^* = -\frac{\partial\rho^*}{\partial t}$

b) Faraday's law $\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$

$\therefore \oint_C \vec{E}\cdot dr = -\frac{\partial}{\partial t}\int \vec{B}\, d\vec{S}$

$\nabla(\nabla\times B) = \mu_0\, \nabla\cdot J$

$\nabla\times E = -\frac{\partial}{\partial t}$? Faraday's

$\oint_s \vec{B}\, ds \neq 0$

$\oint_s \vec{B}\, ds = \oint_s \int_v \rho^* \, dv\, ds$

$\oint_v J\, dv = I^*$ $\quad \nabla \times E = -\frac{\partial B}{\partial t} - J^*$

c) Current: $\oint \vec{E}_{dr} = \frac{-\partial \Phi_B}{\partial t} - I^*$

$\nabla\left(-\frac{\partial B}{\partial t} - J^*\right)$

$\nabla \times E = -\frac{\partial}{\partial t} B$ [illegible]

$\nabla(\nabla \times E) = -\frac{\partial}{\partial t} \nabla B \neq 0$ monopoles exist! (set up)

$= -\frac{\partial \rho^*}{\partial t} = \nabla \cdot J^*$

$\therefore \nabla\left(\frac{\partial B}{\partial t} + J^*\right) = 0$

$\nabla\left(\frac{-\partial B}{\partial t} - J^*\right) = 0$

4) $\nabla^2 B = \varepsilon_0 \mu_0 \frac{\partial^2 \vec{B}}{\partial t^2}$ ~ wave eqt

Identity: $\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \vec{\nabla}^2 A$

Maxwell-Amp Law: $\nabla \times \vec{B} = \mu_0 \vec{J} + \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}$

$\cancel{\vec{\nabla}(\vec{\nabla} \cdot \vec{B})}^{0} - \vec{\nabla}^2 B = \vec{\nabla} \times (\vec{\nabla} \times \vec{B})$

$-\vec{\nabla}^2 B = \vec{\nabla} \times \left(\mu_0 \vec{J} + \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}\right)$

$\vec{\nabla}^2 B = \varepsilon_0 \mu_0 \frac{\partial^2 \vec{B}}{\partial t^2}$ this jump??

$$\boxed{\begin{array}{l} J = 0 \sim \text{source free vacuum} \\ \vec{\nabla} \times \frac{\partial \vec{E}}{\partial t} = -\frac{\partial^2 B}{\partial t^2} \end{array}}$$

5) $\vec{B} = \mu_0 \vec{H}$ $\qquad \vec{H} = \frac{1}{\mu_0} \vec{B}$

$\frac{|\vec{E}|}{|\vec{H}|} = \frac{\mu_0 \omega}{k_z} = \mu_0 c = \sqrt{\frac{\mu_0}{\varepsilon_0}}$

$\vec{E} \cdot \vec{k} = \vec{B} \cdot \vec{k} = \vec{E} \cdot \vec{B} = 0$ maxwell

$\left.\begin{array}{l} \vec{k} \cdot \vec{E} = \vec{E} \cdot \vec{k} = 0 \\ \vec{k} \cdot \vec{B} = \vec{B} \cdot \vec{k} = 0 \end{array}\right]$ dot product commutative

$\nabla \cdot A = i\underline{k} \cdot A$

$A = A_0 e^{i(k \cdot r - \omega t)}$

$\vec{k} \times \vec{E} - \omega \mu_0 \vec{H} = 0$ ⟸ Faraday's

$\vec{k} \times \vec{H} + \omega \varepsilon_0 \vec{E} = 0$ ⟸ Maxwell-Amp

$= A_{0x} e^{i(k_x x + k_y y + k_z z - \omega t)} + A_{0y} e^{i()} \underline{j} + A_{0z} e^{i()} \underline{k}$

$$\underline{\nabla} \cdot \underline{A} = i\underline{k} \cdot \underline{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$= ik_x A_x + ik_y A_y + ik_z A_z$$

$$\frac{\partial \underline{A}}{\partial t} = -i\omega \underline{A}$$

$$\underline{\nabla} \times \underline{A} = i\underline{k} \times \underline{A}$$

$$\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_x & A_y & A_z \end{vmatrix} = \underline{i}\,(ik_y A_z - ik_z A_y)$$

$$= i\underline{k} \times \underline{A} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ ik_x & ik_y & ik_z \\ A_x & A_y & A_z \end{vmatrix}$$