P18: THE FRESNEL EQUATIONS

Aims

- Gain an understanding of reflection/refraction phenomena.
- Use Snell's Law to derive critical angles, Brewster's angle and the refractive index for silica glass.
- Record and analyse data on the reflectance and transmittance of silica glass using the Fresnel Equations.

Safety

This experiment uses a Class II Laser. This is eye safe but you should not stare into the beam. Optical components must not be handled directly as this damages them, wear gloves to prevent damage. The silica glass half cylinder is fragile so take care not to drop it or knock it off the platform.

1. Introduction

The boundary (optical interface) between any two transparent optical materials of different refractive index acts as a partial transmitter/reflector of light. The reflectance and transmittance vary as a function of the angle of incidence and depend on the relative refractive indices of the two materials, the polarisation of the incident light, and the side of the interface the light is coming from. Reflected light returns at an angle equal to the angle of incidence and transmitted light is bent relative to the surface normal, emerging at an angle determined by the relative indices of the materials and the incidence angle. Particular features of the transmittance/reflectance functions include the critical angle, total internal reflection and Brewster's angle. Indeed, the foundations of optical waveguiding, Brewster angle laser windows, Glan Thompson polarisers and thin film reflection and anti-reflection filters are based on simple reflection/ refraction phenomena.

Clearly, physicists must have a thorough understanding of reflection/refraction phenomena. The experiments detailed below provide the opportunity to investigate many aspects of reflection and refraction of light at an optical interface and thus lay the foundations for the study of applications such as the design of polarisers, reflection elements and optical waveguides as part of the photonics theme in physics.

2. Theory

2.1 Snell's law

Light incident on the boundary (interface) between two transparent optical materials is partially transmitted and partially reflected where the fractional transmittance and reflectance are given by the Fresnel equations (explained in detail later). The reflected angle is equal to the incidence angle, and as shown in Figure 1, the angle of transmission is bent relative to the normal depending upon the relative indices of the two materials and on the side of the surface from which the beam is incident.

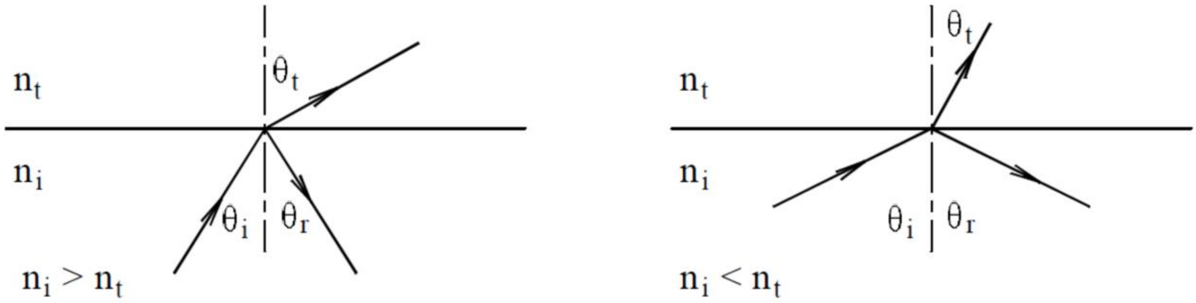

Figure 1: Reflection and refraction at an optical interface - Snell's Law.

The transmitted angle θ_t is related to the incident angle θ_i by Snell's law:

$$n_i \sin\theta_i = n_t \sin\theta_t \,, \tag{1}$$

where n_i and n_t are the refractive indices of the materials on the incidence and transmission side of the interface respectively and θ_i and θ_t are the incidence and transmission angles respectively relative to the surface normal as shown in Figure 1.

As the path of the light is reversible, it is also possible to write this in terms of the angles in the materials ignoring the incident and transmitted labels. So, for an air/glass boundary:

$$n_A \sin\theta_A = n_G \sin\theta_G \,, \tag{2}$$

where the A and G suffixes denote air and glass respectively.

Equations 1 and 2 indicate that the light is bent towards the normal when transmitting from low to high index and is bent away from the normal in transmitting from high to low index, as shown in Figure 2.

In the case of transmitting from high to low refractive index, illustrated in Figure 2, as we increase the angle of incidence, the transmission angle moves ever closer to the interface and eventually it becomes 90°.

At precisely the point where the transmission angle is 90°, the incidence angle is referred to as the critical angle, θ_c. At incidence angles greater than θ_c, the light undergoes total internal reflection (TIR) with zero transmission.

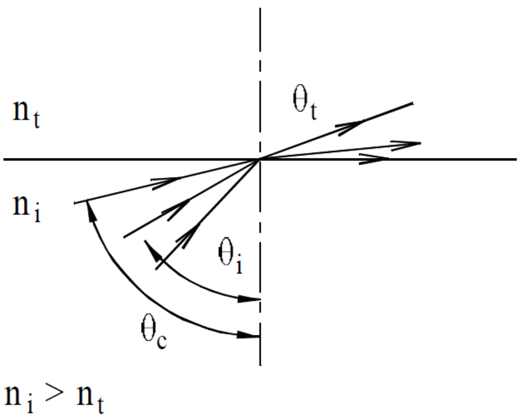

Figure 2: Critical angle and total internal reflection (TIR).

The critical angle is given by Snell's law where θ_t = 90 °, i.e.:

$$\theta_c = \sin^{-1}\left(\frac{n_t}{n_i}\right), \quad (n_i > n_t) \,. \tag{3}$$

Another significant angle is Brewster's angle. When light passes through a material, the oscillating electric field vector in the light wave induces the electrons in the atoms to oscillate relative to their nuclei, thus creating arrays of dipoles oscillating in a direction perpendicular to the propagation axis. At the interface

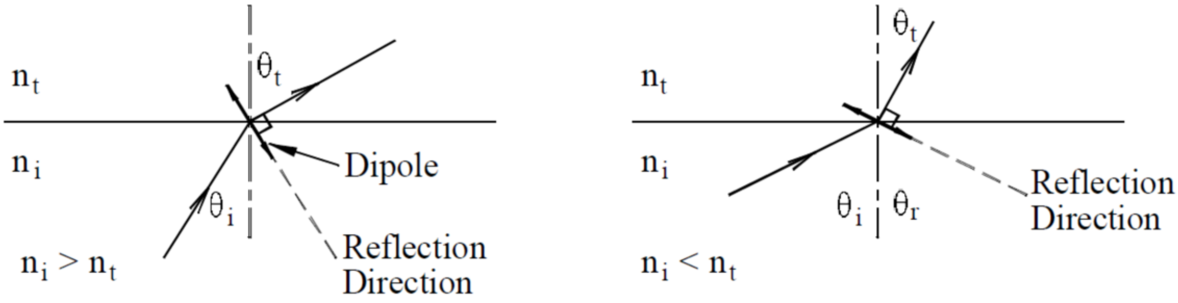

Figure 3: Illustration of Brewster's angle.

between two materials of differing refractive index, the reflection is fed with power from the dipoles induced into oscillation by the refracted light ray. Dipoles cannot radiate power from their ends, in a direction that is collinear with their oscillation axis, and this leads to an interesting case for incident light linearly polarised in the incidence plane.

For this situation, shown in Figure 3, there is a particular incidence angle for which the dipole axis, perpendicular to the refracted ray, is collinear with the reflection direction and hence, the reflectance is zero. This occurs when the incidence angle is such that the refracted ray is at 90° to the reflected ray, i.e. when $\theta_i + \theta_t = 90°$, and hence $\sin\theta_t = \cos\theta_i$. This means that, from Snell's law, the angle of zero reflectance known as Brewster's angle, θ_B, is given by:

$$\theta_B = \tan^{-1}\left(\frac{n_t}{n_i}\right). \tag{4}$$

Note that there is a Brewster's angle for incident light at the boundary for high-to-low and low-to-high index change. Mathematically, there is a solution to arctan for any number from zero to infinity.

2.2 The Fresnel Equations

The reflectance (R) and the transmittance (T) of an optical interface (i.e. the ratios of the reflected and transmitted power (P_r and P_t) to the incident power (P_i) respectively) depend on the angle of incidence. The precise form of the relationship depends on the polarisation state of the incident light with respect to the incidence plane, the side from which the light is incident (high or low index side) and the refractive indices of the two materials (where the incidence plane is the plane containing the normal to the interface and the incident ray).

The reflectance and transmittance for all possible combinations of the above variables are given by the Fresnel equations that predict the critical angle and Brewster's angle as special cases. For reflectance, the Fresnel equations are as follows:

$$R_I = \left(\frac{n_t\cos\theta_i - n_i\cos\theta_t}{n_t\cos\theta_i + n_i\cos\theta_t}\right)^2, \qquad R_O = \left(\frac{n_i\cos\theta_i - n_t\cos\theta_t}{n_i\cos\theta_i + n_t\cos\theta_t}\right)^2 \tag{5}$$

where R_I is the reflectance for linearly polarised light with its polarisation vector in (i.e. parallel to) the incidence plane and R_O is for linearly polarised light with its polarisation vector out of (i.e. perpendicular to) the incidence plane.

A similar set of equations can be presented for the transmittance but it is usually easier to calculate the transmittance on the basis that:

$$T_{I,O} = 1 - R_{I,O}\ . \tag{6}$$

It should be noted that Equations 5 & 6 are valid for light incident from either side of the interface giving different answers for each case since θ_t is different for each case and must be known or calculated to use the equations.

Before leaving the Fresnel equations, it is worth considering the case of ***normal incidence,*** where θ_i and θ_t are both equal to 0° and hence, the reflectance R_n is given by:

$$R_n = R_I = R_O = \left(\frac{n_i - n_t}{n_i + n_t}\right)^2\ . \tag{7}$$

The Fresnel equations are derived from first principles in PH20014 *Electromagnetism 1* for the general case of the boundary between two materials with electromagnetic characteristic impedance Z_1 and Z_2. The Fresnel equations shown above are for the special, but frequently observed, case of light propagating in non-magnetic material where the refractive index is linked to the dielectric constant of the medium.

2.3 Sample results from the Fresnel equations

Figure 4 shows the Fresnel reflectivity and transmission coefficient for the air/diamond interface. You should be able to see the features discussed in the section above. (Unfortunately, we cannot provide a semi-circular prism of solid diamond for this experiment.)

Preparation Activity: Make notes in your logbook identifying for the figure below: The diamond/air and air/diamond plot, the polarisation associated with each line in the plots below, Brewster's angle(s), and the critical angle.

From these plots, **what is the refractive index of diamond**? Ask a demonstrator for assistance if you cannot complete this task having read the theory section.

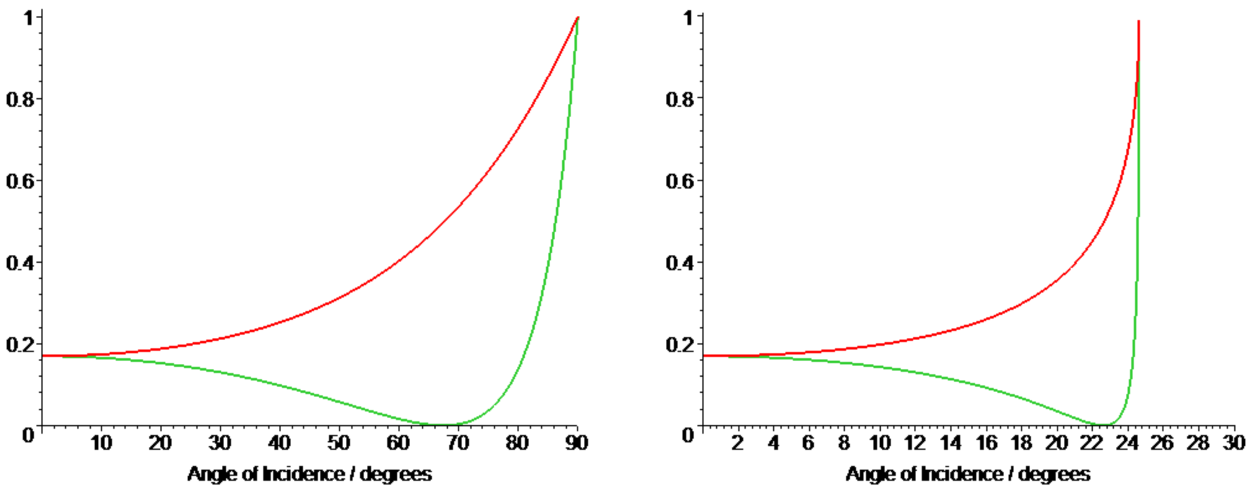

Figure 4: Example of calculated Fresnel coefficients for the air/diamond interface.

3. Apparatus

The apparatus consists of an optical rail bench, shown in Figure 5, fitted with the following hardware elements: a mounted 633nm laser; a polariser with a graduated rotational mount; a rotating table assembly consisting of a circular table with angle graduations and a mounting plate for the glass half cylinder; a rotating indicator arm for measuring beam angles; and an optical detector for measuring the transmitted and reflected power as a function of angle.

The **laser** is controlled by the on/off switch and is indicated as on when the green LED, adjacent to the switch, lights up. A moveable shutter at the laser output can be employed to block the emitted laser beam, although for quick measurements of the background level of light in the room it is easiest to block the path of the laser with a piece of dull card or paper.

The **polariser** is used because the reflectance and transmittance of light at an optical interface are polarisation sensitive (see Section 2). The linear polarisation of the incident beam can either be in the incidence plane of the interface, referred to as '*p*' polarisation, or be perpendicular to the incidence plane, referred to as '*s*' polarisation, where the incidence plane is the plane containing the incident ray and the normal to the interface. The polarisation direction incident on the half cylinder may be adjusted simply by rotating the polariser until its transmission axis is vertical (0°) or horizontal (90°) as required.

The head of the **optical detector** contains a silicon photodiode, which is connected to the photoreceiver module. The detected optical power is displayed on the panel meter. Turn this off when not in use.

A **silica glass half cylinder** (made of fused silica, grade JGS-1) is held on the rotational table by the spring clip. The flat side of this element provides the optical interface to be investigated in terms of reflection and refraction. Be very careful when handling this as it is fragile. Do not touch optical surfaces with bare hands (use the gloves provided).

3.1 Alignment

The alignment of the system needs to be checked before use. **Poor alignment will cause significant errors in the collected data.** The beam from the laser should be incident on the centre of the flat side of the silica glass, which has been arranged to coincide with the rotation axis of its mounting plate. This ensures that the beam incident through the silica glass or any reflected and refracted beams always intersect the curved surface at 90° to its tangent, irrespective of the angular position of the rotation stage. Such an arrangement eliminates errors in angular measurement arising from refraction at this surface, ensures that the reflectance and transmittance of this surface is constant with rotation angle and is known.

To check the alignment:

- Turn the laser on then turn the table so the laser shines on the flat face of the silica glass first.
- Align the table so the flat face of the silica glass is perpendicular to the laser. Use the indicator arm to help you – check on both sides of the silica glass. (See Section 4.1 about the indicator arm.)
- Rotate the platform the silica glass is on and observe the laser spot on the flat face as you are rotating the platform.
- The spot should stay in the centre of the face of the silica glass and not wander to the sides when the platform is rotated though it will spread out at high angles (>75°) of incidence.

If the spot moves or you see two spots of light then the alignment is wrong and needs to be corrected before continuing further. Talk to your demonstrator or a technician.

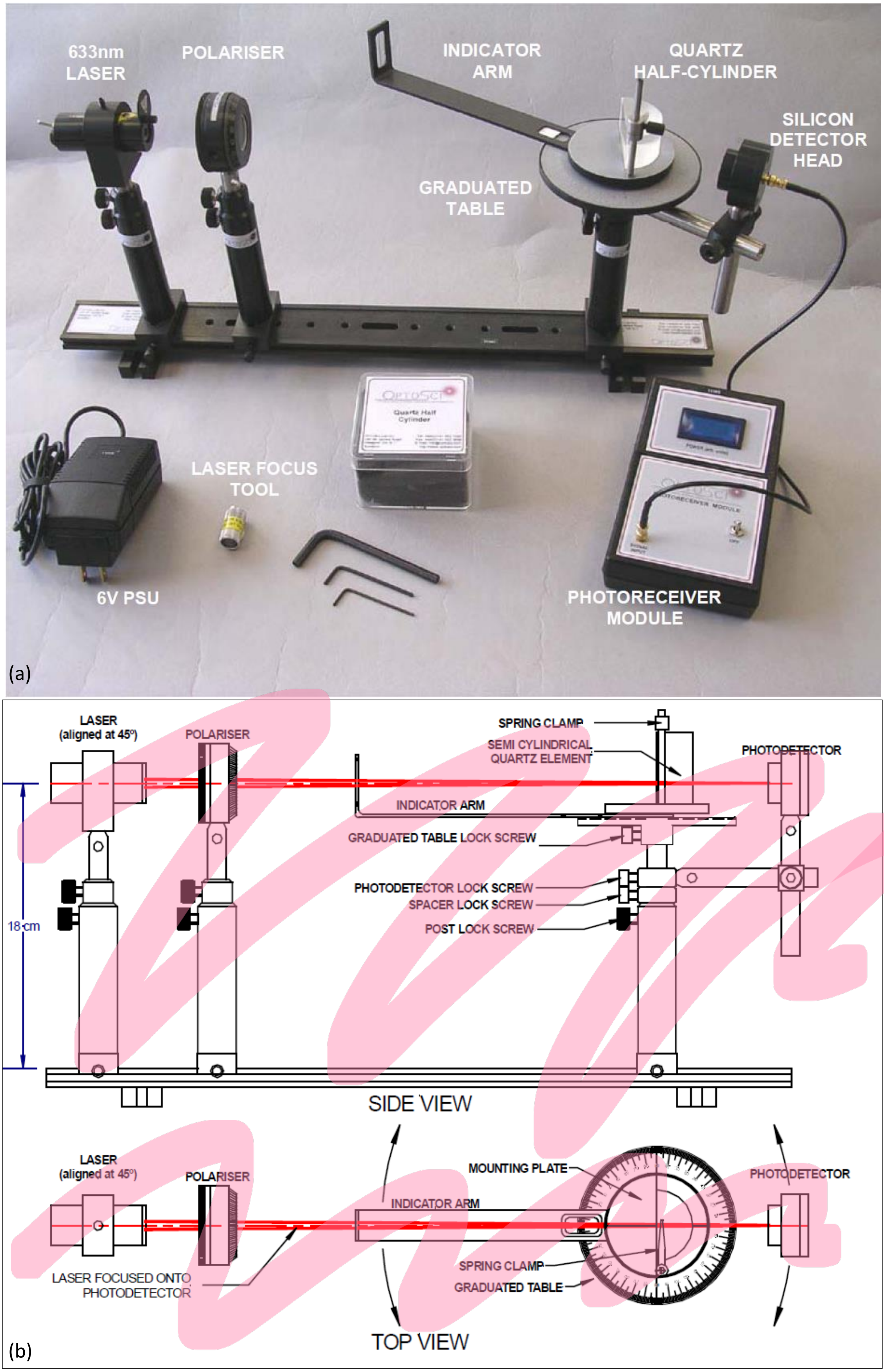

Figure 5: Apparatus. (a) Photograph of apparatus. (b) Labelled schematic of side/top view.

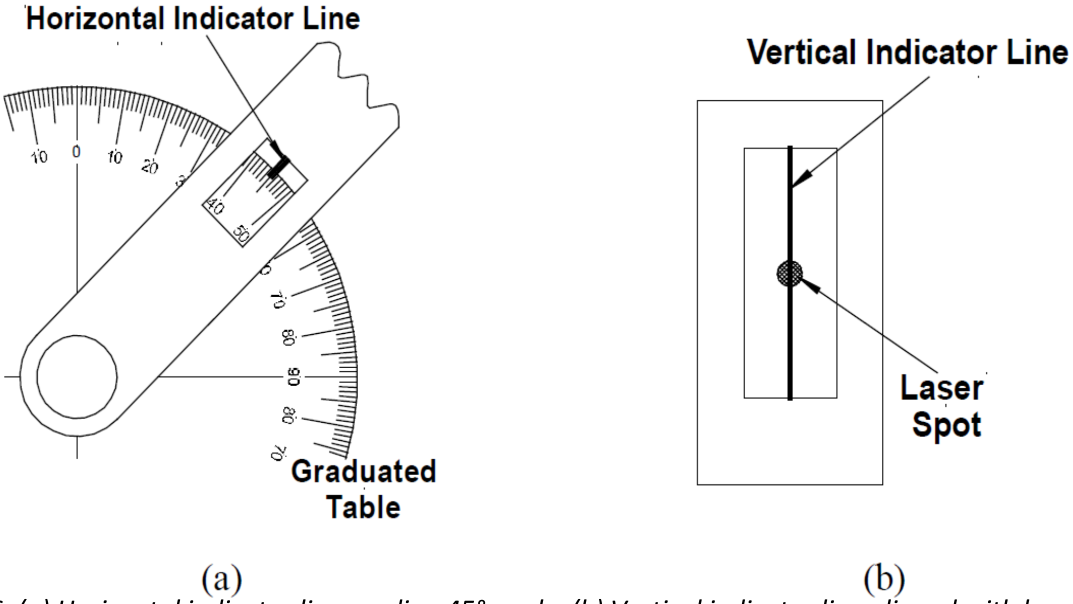

Figure 6: (a) Horizontal indicator line reading 45° angle, (b) Vertical indicator line aligned with laser spot.

4. The experimental system

The two key operating issues are the measurement of angle (i.e. angle of incidence and angle of refraction) and the measurement of optical power for the determination of reflectance and transmittance (i.e. incident, transmitted and reflected power). These will now be examined in turn.

4.1 Angular Measurements

All angles are measured relative to the normal at the flat optical interface on the half cylinder. In operation, the incidence angle of the laser beam onto the flat optical interface of the half cylinder may be varied simply by undoing the post locking screw (see Figure 5) and rotating the entire table assembly. To measure the incident angle, simply align the indicator arm with its vertical indicator line centred on the incident beam and determine the angle marked by the horizontal indicator line, see Figure 6. You should determine the angular width of the spot in the experiment and decide the uncertainty in determining the experimental angles using the graduated table.

For the transmitted (reflected) angle simply align the vertical indicator line with the transmitted (reflected) beam and the transmitted (reflected) angle is marked by the horizontal indicator line, as shown in Figure 6. The only other adjustment concerns the orientation of the polariser, which establishes the polarisation direction of the incident beam (discussed earlier). During your experiment, angle the prism as in Figure 7b so that the reflected and transmitted beams are projected backwards towards the wall of the lab rather than out into the room wherever possible.

4.2 Optical Power Measurements

A photodiode detector mounted on a rotating arm is provided for the measurement of optical power. The arm is mounted on the post beneath the rotating stage and rotates about the axis of the stage when the locking screw on its support collar is loosened slightly. Thus the detector can be positioned to measure transmitted power at any angle and reflected power at any angle except 10° on either side of the incident beam (since at angles below 10° the detector head blocks the incident beam). The detector should be connected to the photoreceiver unit so the detected optical power may be read directly from the panel meter display. To reduce measurement errors, adjust the angle/height to make sure the laser beam falls on the centre of the active area of the detector.

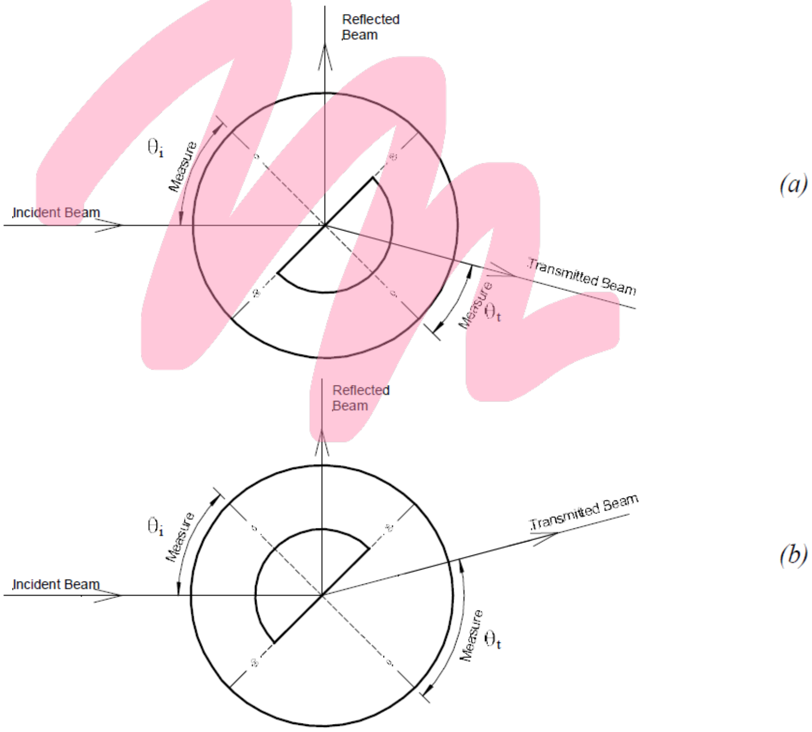

Figure 7: Plan view of incident and transmitted angle measurements for (a) Lo-Hi and (b) Hi-Lo refractive index interface.

5. Experiments

Throughout your experiment you will need to think carefully about sources of errors. Any values derived should have an associated error.

5.1 Snell's Law

Firstly, you should ensure that the system has been set up and aligned as described in Section 3. Since Snell's law is independent of the polarisation state of the incident light, you should remove the polariser from the system for the following experiments.

5.1.1 Snell's Law - low to high index

With the beam from the laser incident directly on the flat optical interface, measure the transmitted angle as a function of incidence angle at 10° intervals (as described in 4.1) and plot your results.

5.1.2 Snell's Law - high to low index

Repeat with the beam incident on the flat optical interface through the curved surface of the half cylinder. Estimate the value of the critical angle, which should be readily observed as you rotate the stage. As you approach the critical angle, view the transmitted beam on a white card/screen and rotate the stage until the transmitted beam is just extinguished. The incident angle at this point is then the critical angle.

Practical Note - As you approach the critical angle, the transmitted laser beam will spread out making it more difficult to measure its centre position. This error should be taken into account when determining the angle of transmission.

5.1.3 Analysis of the Snell's Law measurements

From your measurement of the critical angle in 5.1.2, propose an initial estimate of refractive index of the transparent half cylinder.

Use all your data on angles on incidence and transmission with Snell's law (Equation 2) to find a more precise value for the refractive index of the prism along with your uncertainty in that value. Note: You can plot all the Snell's law data on one straight line using: $n_A \sin\theta_A = n_G \sin\theta_G$.

5.2 Fresnel Equations - Reflectance and Transmittance

Read: tips

When taking power measurements always ensure that the laser beam is incident on the centre of the active area of the photodiode. The maximum power reading should then be noted as the active area of the detector is manually scanned across the laser beam. This will ensure that discrepancies in the power readings due to beam position on the active area will be minimised. As the photodiode can experience varied ambient light conditions as it is rotated, it is imperative to take an ambient light reading for each experimental reading taken. **It is not a constant offset.** This correction is easily obtained by blocking the laser beam and noting the detector power reading with no laser light incident on the detector. This value should then be subtracted from your experimental reading to yield the true laser power reading.

5.2.1 Determination of the Correction Factors for the Calculation of Reflectance and Transmission from Power Measurements

We wish to measure the transmittance and reflectance of a single interface. However, the geometry of the half cylinder means that for most measurements of transmitted and reflected power, we must consider the reflection loss from the second interface. This unwanted source of loss **must** be taken into account in the determination of the single interface transmittance and reflectance from the measured powers.

To determine the correction factors, you need to measure the incident power and the reflected powers for normal incidence directly onto the flat face of the half cylinder (see Appendix A for detailed derivations).

Replace the polariser on the optical rail as shown in Figure 5.

Carry out the following procedure:

- Position the photoreceiver to intersect and measure the incident beam, making sure that the entire cross-section of the incident beam falls within the detector area. You will need to turn the receiver in its mount for this.
- Measure the power reading on the detector for the polariser positions of $P_i(0°)$ and $P_i(90°)$, corresponding to axes of polarisation to be **O**ut and **I**n the plane of incidence. **These values should be roughly equal to obtain good data in the rest of the experiment.** If they are not then please alert your demonstrator or a member of technical staff. You will use these for the next section of the experiment.
- Now, adjust the angle of the polariser to obtain maximum light transmission (keep the polariser in this position for the remaining measurements in this section). Note both the angle and the power reading.
- Record this power reading and then close the laser shutter and take an ambient light reading from the detector. This ambient value should be subtracted from your measured power with the shutter open to yield the maximum incident laser power, P_i^M.
- When you have completed the measurement of P_i^M, rotate the table assembly so that the beam has an incidence angle of about 10°. Rotate the detector into its normal position and measure the reflected power, again taking an ambient light reading to yield P_r^M.

For a 10° angle of incidence, there is negligible difference in the reflected power from that at normal incidence, this value can be used to determine the normal incidence reflection coefficient, $R_n = P_r^M / P_i^M$ (see Appendix A). Compare your measured value of R_n with that calculated from Equation 7 (using your value for the half cylinder refractive index calculated in 5.1.3) and comment on the validity of your assumptions.

FOLLOWING NOT PAST

The correction factor $(1 - R_n)$ should now be used appropriately in the following sections to correct the measured values of reflected and transmitted power to enable the reflectance and/or the transmittance of any single interface to be obtained (see Appendix A).

5.2.2 Reflectivity – low to high index

The Fresnel reflection and transmission coefficients are strongly polarisation dependent. You must measure R_O and R_I so you can see and model this effect. The time efficient way to do this is to move the table and detector to the correct angles and then make a power measurement at the two polarisations (polariser at 0° and at 90°). This will work well so long as the polariser is carefully set to the correct positions during the experiment.

With the beam incident directly on the flat optical interface (i.e. Lo-Hi index interface), measure the reflected power, P_r, as a function of incidence angle at 5° intervals. Remember to also take an ambient light reading (i.e. block the laser beam from hitting the prism) at each angle. This value should be subtracted from each measured P_r value to account for any ambient light variations. The reflectance, R, can then be simply obtained from Equation A1, using your values of $P_i(0°)$ and $P_i(90°)$ measured earlier for P_i as appropriate.

In this experiment, Brewster's angle should be readily observed for the horizontal polarisation of light as you rotate the stage. An accurate estimate of Brewster's angle (θ_B) may be obtained by viewing the reflected beam on a white card and rotating the table assembly to observe the minimum intensity. You will find that as the reflected horizontally polarised light falls in intensity, it will most convenient to set the detector in the correct position using vertically polarised light.

5.2.3 Reflectivity – high to low index

Orientate the prism so that the beam is incident on the flat <u>through</u> the curved surface (i.e. Hi-Lo index interface). You will probably find that in this configuration the laser spot has diverged slightly. Hence, to ensure that the laser beam still falls within the photodiode active area move the detector head along its horizontal mounting post towards the table assembly if this is required to measure the whole laser spot. In addition, make any detector height adjustments required to centralise the laser beam on the active area.

As detailed in Appendix A, in order to obtain an accurate value for the reflectance at the flat surface of the half cylinder (Equation A7) for the Hi-Lo index case, the correction factors measured in 5.2.1 must be taken into account. Thus you should measure the reflected power from the flat interface, P_r^I, as a function of incidence angle (remember to take ambient light readings in to account as well). Calculate the reflectance, R, using Equation A7, using your values of $P_i(0°)$ and $P_i(90°)$ measured earlier for P_i as appropriate.

Note any interesting features as you take your measurements.

5.3 Analysis of results

Determine the Brewster angles using the data collected in 5.2.2. From the Brewster angles, calculate a value for the refractive index of the half cylinder (using Equation 4). Comment on its agreement with that obtained previously from the critical angle.

Plot your results for reflectance as a function of incidence angle for both polarisations of light for both sets of data (similar to Figure 4). Use the Fresnel Equations (Equation 5) to make plots of the theoretical reflectance, R_o and R_I, for the air/glass and glass/air interface. In order to plot the Fresnel Equations you will need to pick a suitable set of incidence angles, calculate the ideal transmittance angles using your calculated refractive index and Snell's Law and then use these values to calculate R_o and R_I. To save time, use cell references to store the incident and transmitted refractive indices such that you can switch them round when calculating both the Low-to-High and High-to-Low values for the Fresnel Equations.

Note: you should plot your measured values as points and calculated values from theory as a line <u>on the same graph</u>.

From the above measurements, you should now have the experimental/theoretical graphs of the Fresnel relationships for reflectance for each of the four possible combinations of polarisation state and incidence direction.

Compare experimental results and theoretical prediction. Comment on how well the theory agrees with your measured relationships. If necessary, you can adjust the value of refractive index used in your calculation of theory to get a better match to the data.

At normal incidence (use the value obtained at 10° incidence), how well do your reflectance results from Sections 5.2.1 and 5.2.2, for the low to high index interface, agree with the values calculated from Equation 7, using the refractive index value estimated above?

You have now measured the refractive index using Snell's law, the critical angle, the Brewster angle and the Fresnel equations for reflectivity. Decide which measurement gave the most precise value for the refractive index of the prism and explain your reasoning.

END

Appendix A

The geometry of the half cylinder is such that for most measurement situations the light encounters more than one interface, incurring additional, unwanted reflection losses. These unwanted sources of loss must be taken into account in the determination of the transmittance and reflectance of any single interface from the direct measurements of transmitted and reflected powers.

Let us first consider the determination of the transmittance, T, and reflectance, R, of an interface for propagation in the direction of Low to High (Lo-Hi) index as depicted in Figure A1.

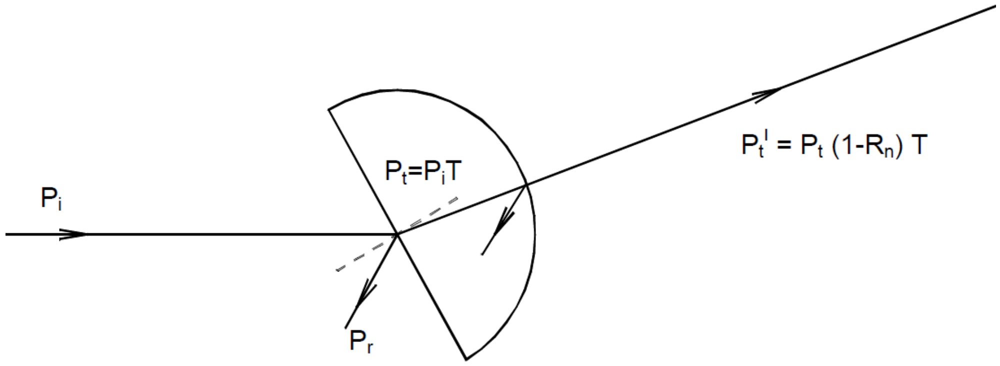

Figure A1: Determination of reflectance and transmittance of a Lo-Hi optical interface.

We can measure the incident power, P_i, the reflected power, P_r, and the transmitted power,P_t^I. In this case the reflectance, R, is simply given by:

$$R = \frac{P_r}{P_i}. \tag{A1}$$

However, to determine the transmittance, T, from the measured powers we must take account of the effect of the reflection at the exit interface. Figure A1 shows the measurement set up and traces the evolution of power through the half cylinder. It is readily seen from such an analysis of the power evolution in Figure A1 that the transmitted power, P_t^I, is related to the incident power by:

$$P_t^I = P_i(1 - R_n)T, \tag{A2}$$

so the transmittance is thus:

$$T = \frac{P_t^I}{P_i(1 - R_n)}, \tag{A3}$$

where R_n is the reflectance at normal incidence.

R_n is determined from Equation A1 with P_r taken as the reflected power from a Lo-Hi interface at normal incidence. In practice, the measurement of P_r is made at an angle of incidence of 10° to accommodate the detector. However, the difference in reflectance between normal incidence and 10° is negligible and the error incurred is insignificant.

Figure A2 shows the measurement set up for determining the transmittance and reflectance of an interface for propagation in the direction of High to Low (Hi-Lo) index and traces the evolution of power in the half cylinder.

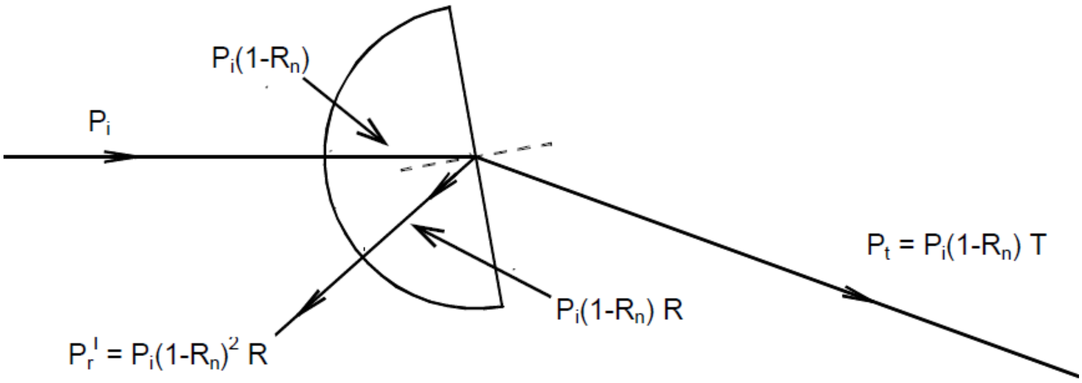

Figure A2: Determination of reflectance and transmittance of a Hi-Lo optical interface.

In this case, we measure P_i, P_t and P_r^I and it is readily seen from the power evolution in Figure A2 that P_t and P_r^I are related to P_i by:

$$P_t = P_i(1 - R_n)T, \tag{A4}$$

and

$$P_r^I = P_i(1 - R_n)^2 R\,, \tag{A5}$$

where R is the angle dependent reflectance at the Hi to Lo index interface.

This means that the transmittance and reflectance of the interface may be obtained from the measured values of P_t and P_r^I using:

$$T = \frac{P_t}{P_i(1 - R_n)}, \tag{A6}$$

and

$$R = \frac{P_r^I}{P_i(1 - R_n)^2}. \tag{A7}$$

Acknowledgements

This script is an amended version of the “Reflection and Refraction Educator Kit” Student Manual by OptoSci Ltd, which was provided with the experiment.