Example Q2 PS 2

1. Calculate the changes in the total entropy of the universe as a consequence of the following:

   a. A copper block of mass 1 kg, temperature 100 °C and specific heat capacity c equal to 0.4 J g^{-1} K^{-1} is placed in a lake of large thermal capacity at 10 °C.

   b. An equivalent block at 10 °C is dropped into the lake from a height of 10 m.

   c. Two such copper blocks, one at 0 °C and one at 100 °C, are joined together.

   You may assume that c is independent of temperature.

1. a)

100°C

$\boxed{1kg}$ $c = 0.4\ Jg^{-1}k^{-1}$

AAAAAAAh....

10°C

$H = U + pV$

$dH = TdS + Vdp$

$dU = TdS - pdV$

b)

100°C

$\boxed{1kg}$ $c = 0.4\ Jg^{-1}k^{-1}$

10m

10°c

c) 10°c — 100°c

cont'd:

a)

Calculate the changes in the total entropy of the universe as a consequence of the following:

a) A copper block of mass 1 kg, temperature 100 °C and specific heat capacity c equal to 0.4 J g^{-1} K^{-1} is placed in a lake of large thermal capacity at 10 °C.

$\Delta S = S_f - S_i = \int_i^f \frac{dQ_{rev}}{T}$ for any path, because S is function of state

lake is much bigger, so can be treated as a heat reservoir

entropy change has two components

$\Delta S_{Cu} = \int_{T_i}^{T_f} \frac{C dT}{T} = C \ln\frac{T_f}{T_i}$ negative, because heat release

heat released by Cu (cdT) is heat gained by lake, but lake temperature is unaffected

$$\Delta S_{lake} = -\frac{1}{T_f}\int_{T_i}^{T_f} C dT = C\frac{T_i - T_f}{T_f}$$

$C = cm$

b)

Calculate the changes in the total entropy of the universe as a consequence of the following:

b) An equivalent block at 10 °C is dropped into the lake from a height of 10 m.

height of 10 m.

no heat flow, but lake gains energy equivalent to initial gravitational potential energy of block

$$\Delta S = \frac{mgh}{T_f} = \frac{1\ \text{kg} \cdot 9.81\ \frac{\text{kg m}}{\text{s}} \cdot 10\ \text{m}}{283\ \text{K}} = 0.35\ \text{J/K}$$

Remember $dW = -Fdx = -\text{đ}Q$

c)

Calculate the changes in the total entropy of the universe as a consequence of the following:

c) Two such copper blocks, one at 0 °C and one at 100 °C, are joined together. You may assume that C is independent of temperature.

$$T_a = 0\ °\text{C} = 273\ \text{K}, \qquad T_b = 100\ °\text{C} = 373\ \text{K}$$

$$C(T_f - T_a) = -C(T_f - T_b)$$

$$T_f = \frac{T_a + T_b}{2} = 323\ \text{K}$$

$$\Delta S = \int_{T_a}^{T_f} \frac{CdT}{T} + \int_{T_b}^{T_f} \frac{CdT}{T} = C\left(\ln\frac{T_f}{T_a} + \ln\frac{T_f}{T_b}\right) = cm \ln\frac{T_f^2}{T_a \cdot T_b} = 9.702\ \text{J/K}$$