# Radar Mapping

## 1.1. Reminders

*Radar waves are usually mono-chromatic: 1 frequency ($f$)*

Wavelength: $\boxed{\lambda = \frac{c}{f}}$

Amplitude: $A$

Difference between 2 waves: $\boxed{10\log_{10}\frac{A_1}{A_2} = \Delta A\,(dB)}$

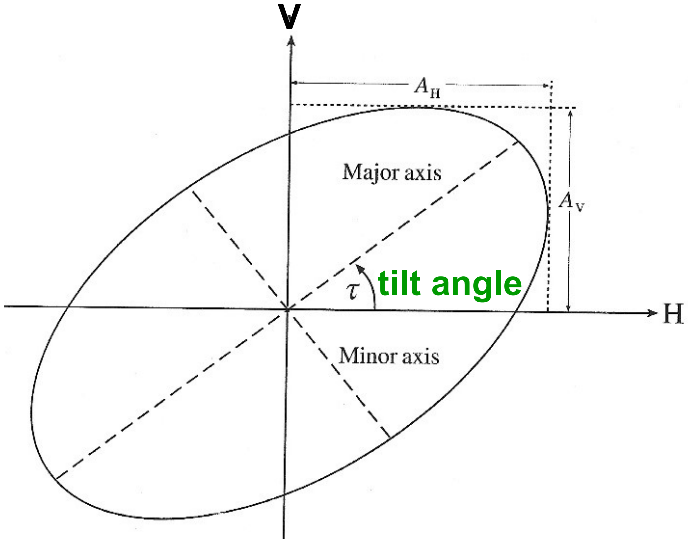
Time (space) difference between 2 waves:
relative phase $\delta$ (0º-360º)

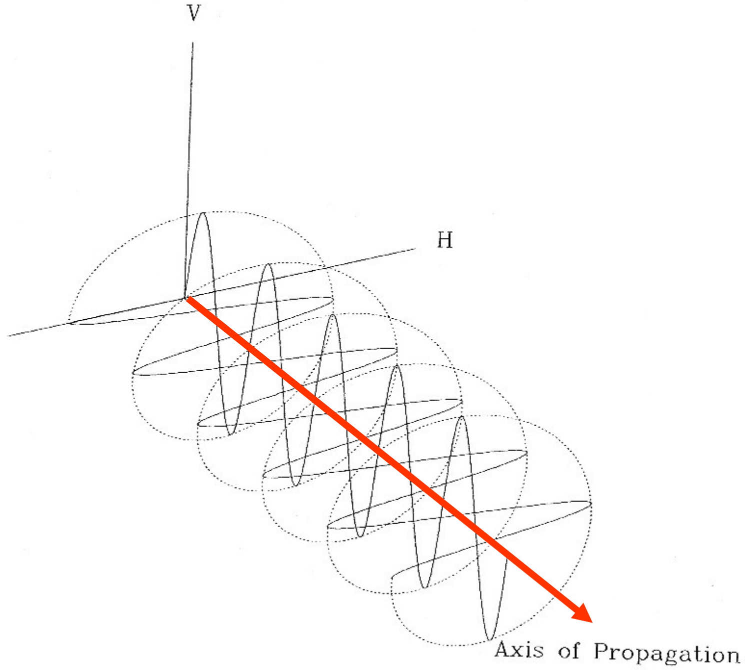
| *Frequency* | *Wavelength* | *IEEE band* |
|---|---|---|
| 1 – 2 GHz | 30 – 15 cm | L band |
| 2 – 4 GHz | 15 – 7.5 cm | S band |
| 4 – 8 GHz | 7.5 – 3.75 cm | C band |
| 8 – 12 GHz | 3.75 – 2.5 cm | X band |
| … | … | … |

*2nd-year experiment $P_{12}$: horn antenna, 9.4 GHz*
*10 mW power*

## 1.2. Polarisation, coherence and power

Electric field = sum of two orthogonal plane-polarised signals H and V




*In radar remote sensing:* ***H (Horizontal) and V (Vertical) polarisations*** *generally used.*

**Coherence** between 2 components: complex product (e.g. $E_V{}^* E_H$)

*Higher coherence is better to distinguish moving targets with small relative velocities*

**Power:** strength integrated over 1 period

*Higher power means we can receive better reflections from further away*

Different polarisations reveal different processes on the ground (e.g. water content, mineralogy)

**HH**

**VV**

**HV**

*ESA: fields in the Netherlands*

**R = HH**

**G = VV**

**B = HV**

Planetary radars mostly use only 1 polarisation (HH or VV) *(practical constraints …)*

** Note the sensitivity of radar to water content in the target volumes/areas*

## 1.3. Wave propagation

A vector electric field **E** in a lossy medium is described by the wave equation:

$$\boxed{\nabla^2 \mathbf{E} = \varepsilon\,\mu \frac{\partial \mathbf{E}^2}{\partial t^2} + \sigma\,\mu \frac{\partial \mathbf{E}}{\partial t}}$$

*$\sigma$ effective conductivity ($\Omega$/m)*
*$\mu$ permeability (H/m) – $\varepsilon$ permittivity (F/m)*

The solution is of the form

$$\boxed{\mathbf{E}(x,t) = A e^{i\omega t} \times \exp\left(-2\pi i\, f\, x \sqrt{\varepsilon\mu - i\frac{\sigma\mu}{2\pi f}}\right)}$$

Its components in the Horizontal (H) and Vertical (V) polarisation states are:

$$A = A_H\, e^{i\delta_H} + A_V\, e^{i\delta_V}$$

Of course, we still have: $\varepsilon\,\mu\, c^2 = 1$

The complex permittivity is: $\varepsilon - i\dfrac{\sigma}{2\pi f}$

The dimensionless, complex relative permittivity, or dielectric constant, is: $\boxed{\varepsilon_r = \dfrac{1}{\varepsilon_0}\left(\varepsilon - i\dfrac{\sigma}{2\pi f}\right)}$

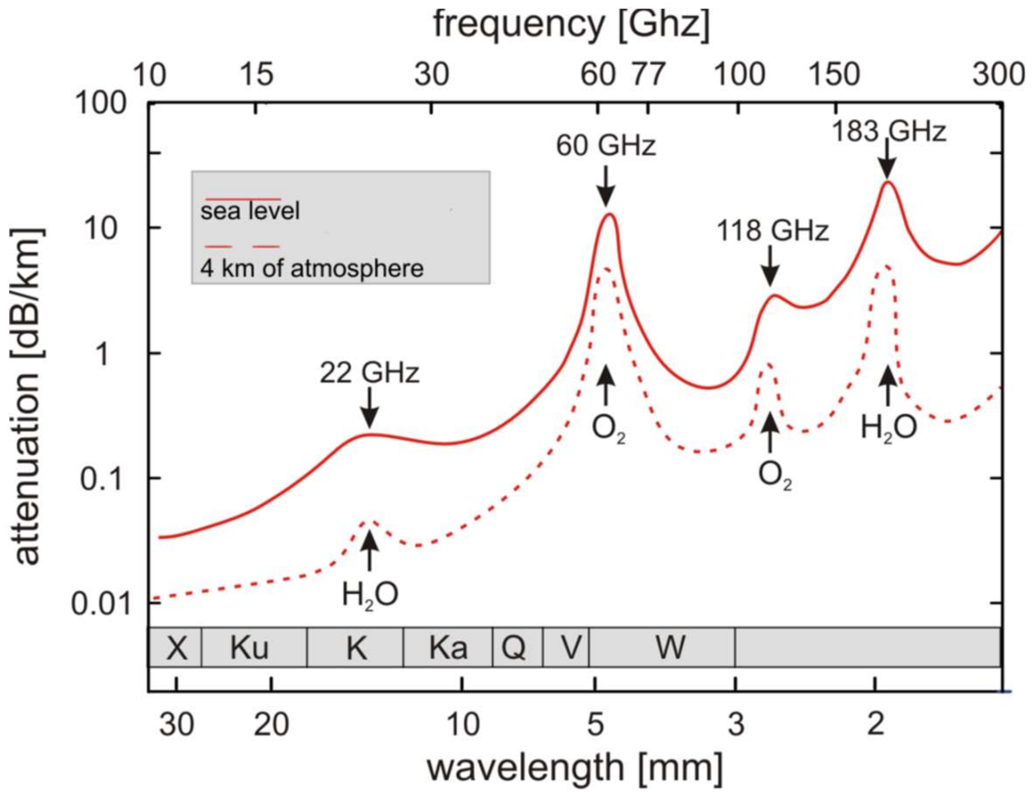
It is related to the loss tangent: $\boxed{\tan\delta = \dfrac{\sigma}{2\pi f\,\varepsilon}}$

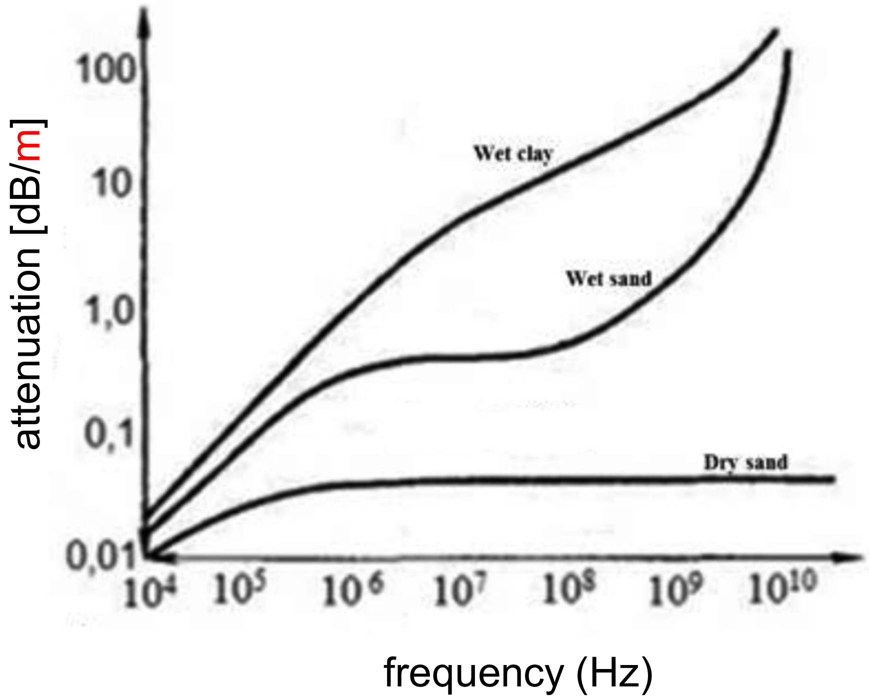
## Propagation through 1 medium

Loss in dB/m. Attenuation by *e* factor defines:

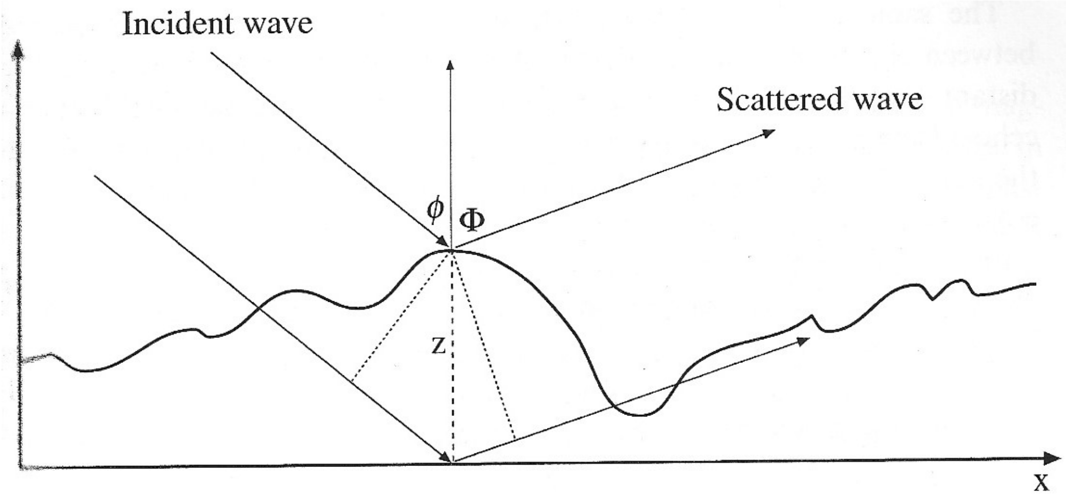
$$\text{penetration depth} = \frac{1}{2 \times 2\pi f \times \sqrt{\varepsilon\mu} \times \sqrt{\frac{1}{2}\left(\sqrt{1+\tan^2\delta}-1\right)}}$$

$$\text{skin depth} = 2 \times \text{penetration depth}$$



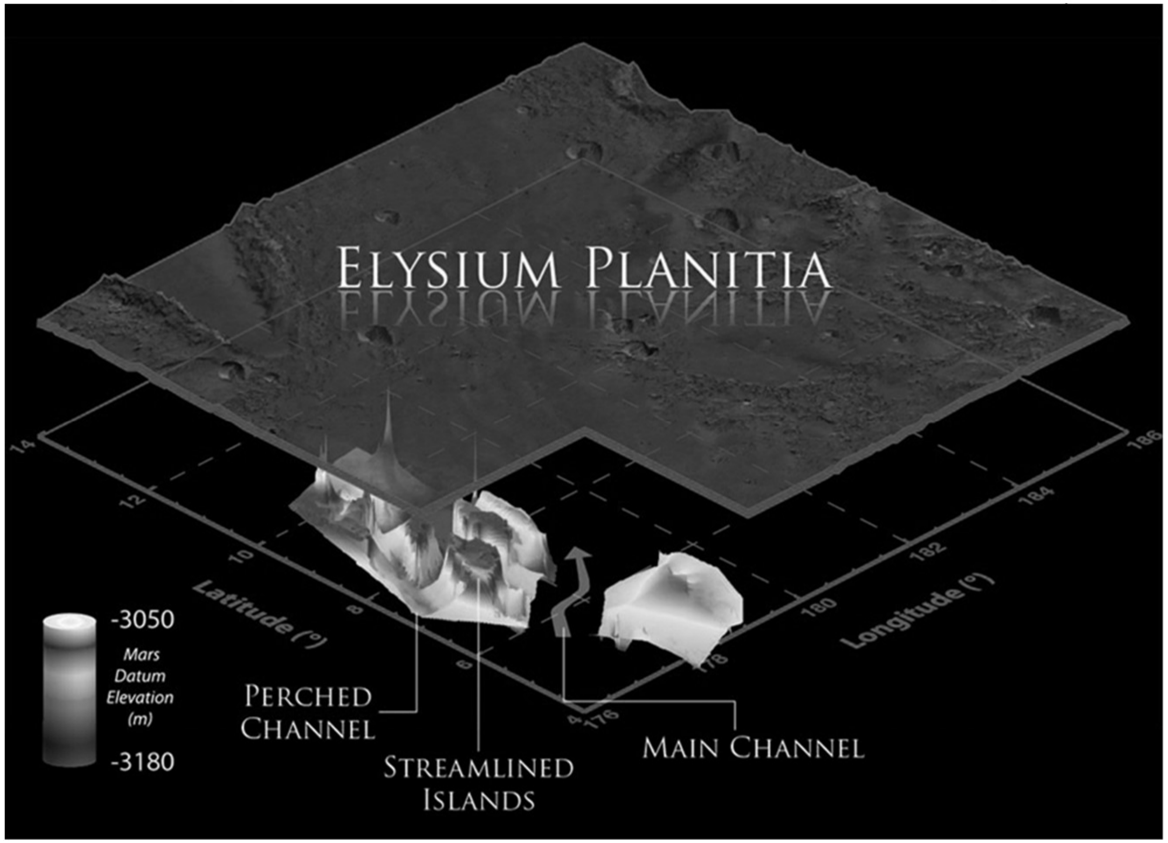



## Propagation through 2 media




Snell's law for transmitted wave:

$$\boxed{\theta = \sin^{-1}\left(\sin\phi\frac{\sqrt{\varepsilon_1}}{\sqrt{\varepsilon_2}}\right)}$$




<u>In some conditions</u>, radar images reveal details below the surface.

Example: Flood channels buried by lava flows (Mars)

*The surface has been elevated and scaled by 100 for clarity*

## 1.4. Radar equation

Radar reflectivity of a target is a function of

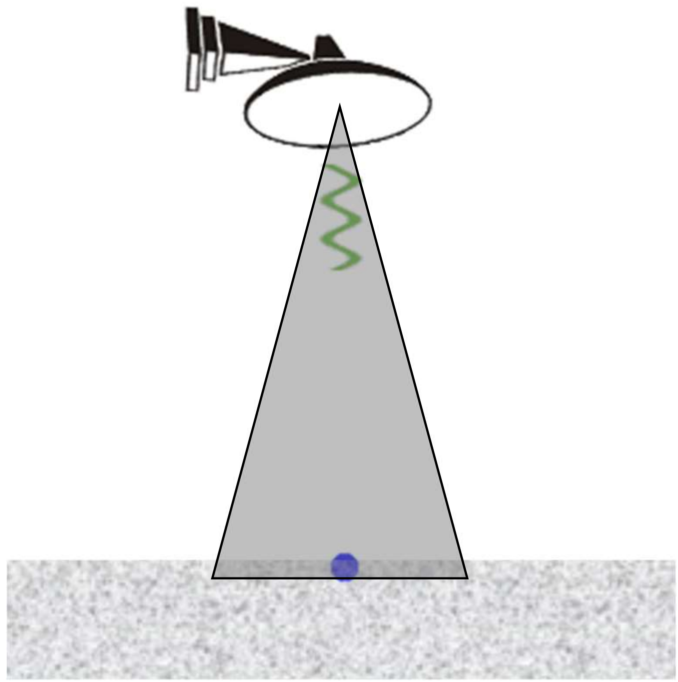
- power transmitted $P_t$ (in W/m$^2$)
- power received $P_r$ (in W/m$^2$)
- directive gain $G$ of antenna (dimensionless)
- illuminated area $A$ (in m$^2$)
- distance $R$ from radar to target (in m)

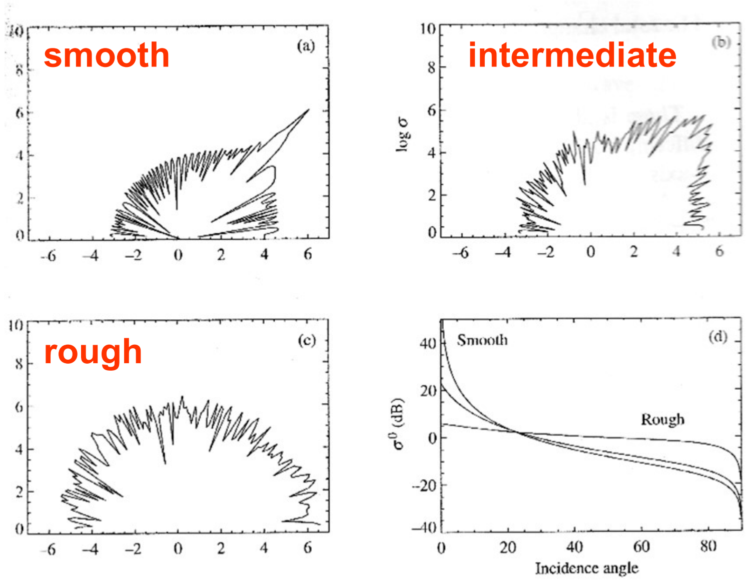
→ backscatter coefficient $\sigma_0$ (dimensionless)

Radar equation:

$$\boxed{P_r = P_t\,\sigma_0\,A\,\frac{G^2\lambda^2}{64\,R^4\,\pi^3}}$$

$\sigma_0$ exhibits high variations, e.g. for surfaces (hence need for dB scale)





# Radar Mapping

## 1. Radar scattering

1.1. Reminders
1.2. Polarisation, coherence and power
1.3. Wave propagation
1.4. Radar equation

## 2. Roughness and dielectric properties

2.1. Rough and smooth surfaces
2.2. Fourier and fractal analyses
2.3. Dielectric properties
2.4. Mixing of materials with different properties

## 3. Radar mapping – Synthetic Aperture Radar (SAR)

3.1. Data transmission and reception
3.2. Synthetic Aperture Radar

## 4. Scattering by rough surfaces

4.1. Integral equation
4.2. Simplifying scattering
4.3. Planetary Imaging: the Hagfors model

## 5. Application: Venus – Magellan SAR

5.1 Magellan radar mapping
5.2 Using the Hagfors model
5.3 Examples

# 2. Roughness and dielectric properties

## *2.1. Rough and smooth surfaces*

Real surfaces are not nicely smooth and flat.

Roughness, along with dielectric variations, will greatly influence radar scattering.

Surface $z = h(x,y)$, sampled at regular $\Delta x$

Roughness measured by:

(1) average height

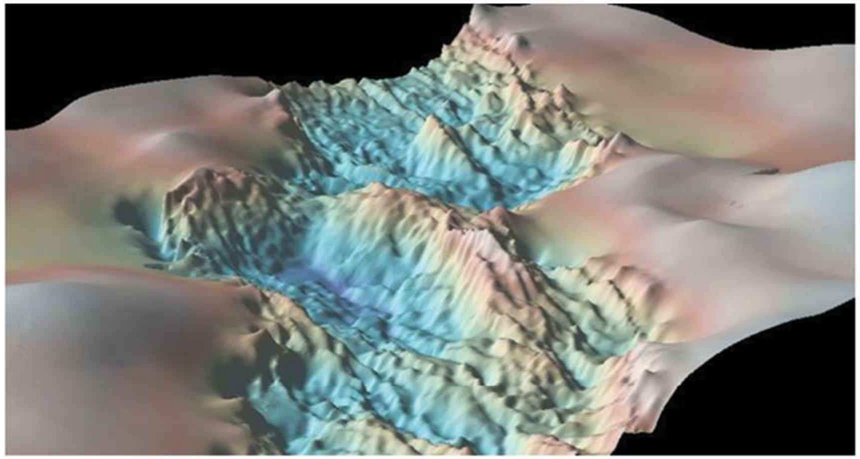
$$\boxed{\hat{h} = \frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} z_{i,j}}$$

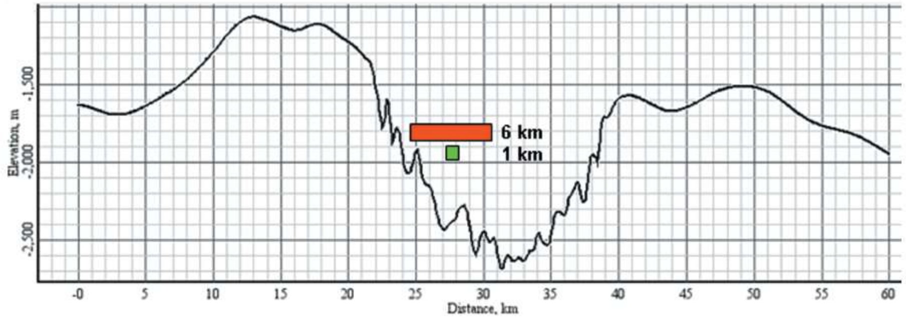
(2) rms roughness

$$\boxed{h_0 = \sqrt{\frac{1}{N^2 - 1} \sum_{i=1}^{N} \sum_{j=1}^{N} \left( z_{i,j} - \hat{h} \right)^2}}$$

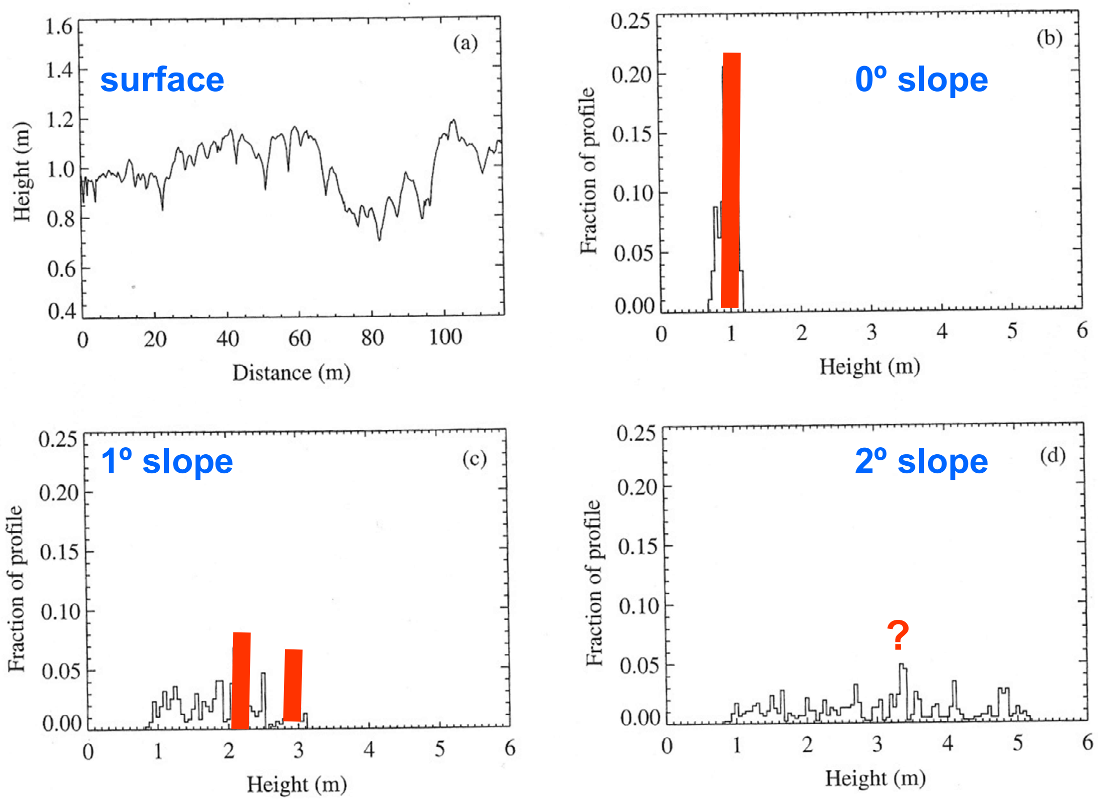
(3) rms tilt

$$\boxed{s_0 = \frac{\sqrt{\left\langle \left( z(x) - z(x + \Delta x) \right)^2 \right\rangle}}{\Delta x}}$$





This will vary with the footprint ($\Delta x$) → role of wavelength / frequency …
Important to know the local slope




Typical height distributions in nature:

– Gaussian

$$p(z) = \frac{e^{\frac{-(z-\hat{h})^2}{2h_0^2}}}{h_0\sqrt{2\pi}}$$

– exponential …

– Rayleigh …

## 2.2. Fourier and fractal analyses

Fourier transform

$$\boxed{F(f) = \int_{-\infty}^{+\infty} z(x)\, e^{-i 2\pi f x}\, dx}$$

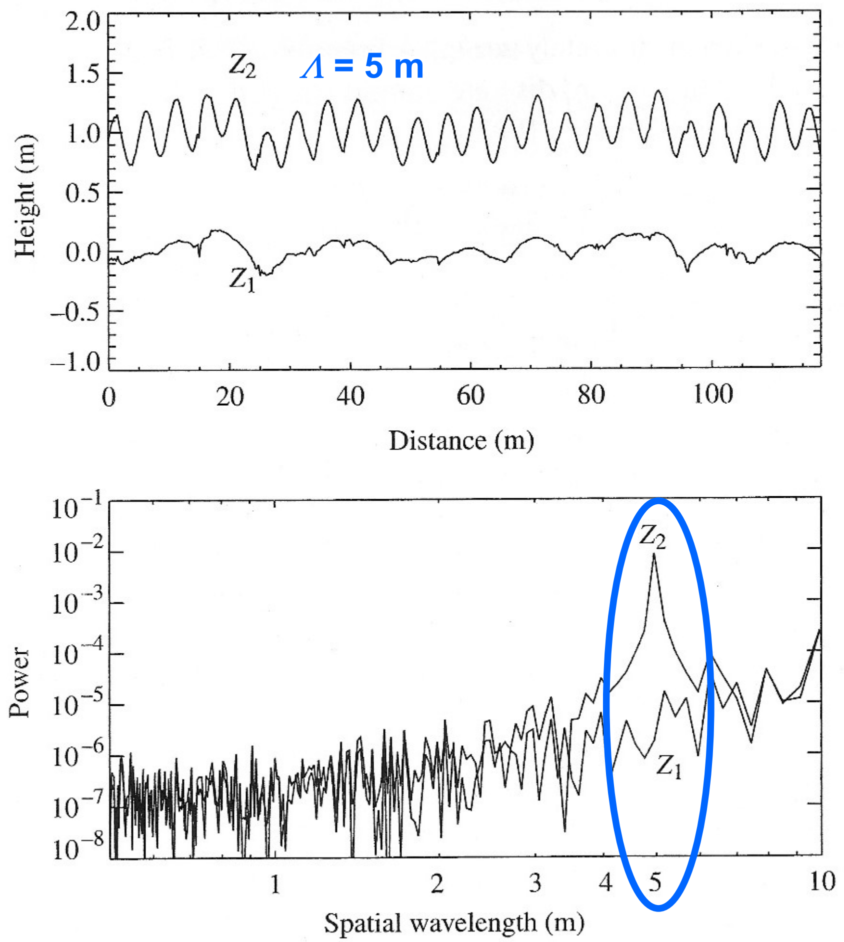
Concept of *spatial wavelength Λ*

Terrain profiles are real-valued functions:

$$F(-f) = F(f)^*$$

Power spectral density W(f)

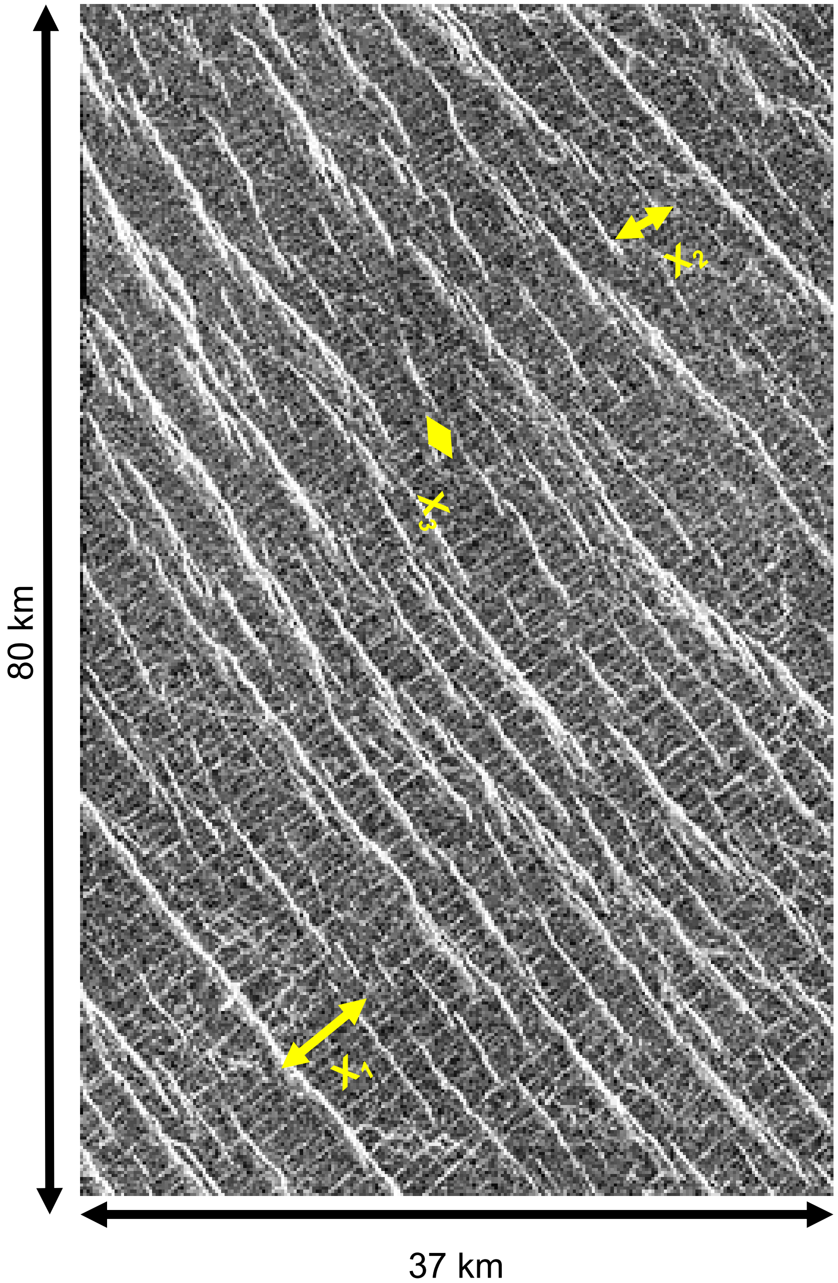
$$W(f) = \frac{1}{L}\left(\mathrm{Re}(F(f))^2 + \mathrm{Im}(F(f))^2\right)$$




*Sand ripples with spacing X will show much higher power at spatial frequency 1/X*

*Tessera terrains on Venus: crisscrossing patterns ⇒ series of spatial frequencies 1/$X_i$*

*Variations and frequency peaks show creation processes (tectonics, local dome growth or impact)*




*Tessera terrains on Venus:*

*crisscrossing patterns ⇒ series of spatial frequencies 1/$X_i$*

*Variations and frequency peaks show creation processes (tectonics, local dome growth or impact)*

*Tesserae (“tiles” in Latin) cover ca. 8% of Venus’s surface.*

## A quick introduction to fractals

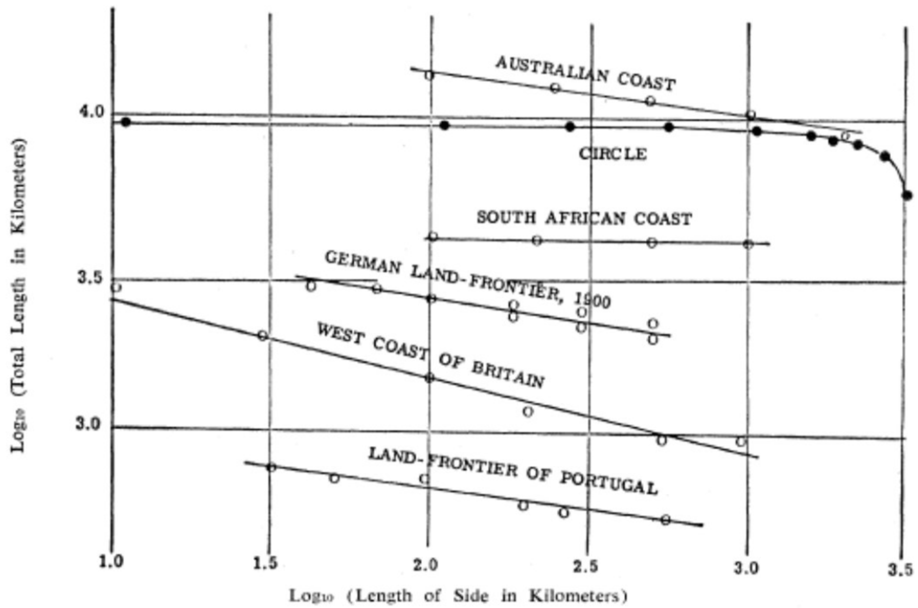
Research started by B.B. Mandelbrot (Science, 1967)

*More at: http://www.math.yale.edu/mandelbrot/*

Unit = 200 km
Length = 2400 km

Unit = 100 km
Length = 2800 km

Unit = 50 km
Length = 3400 km

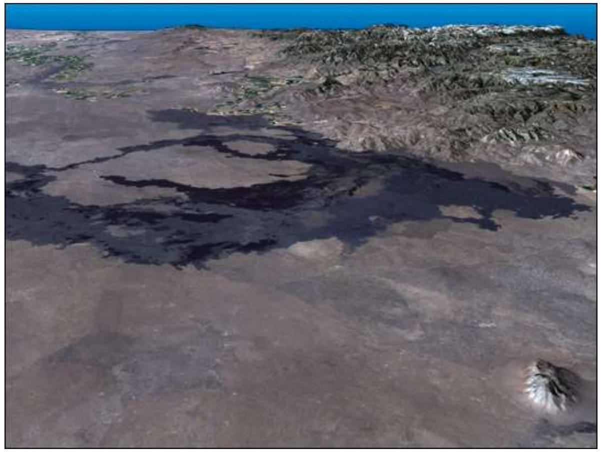



The size of complex lengths/surfaces will increase as the measuring unit decreases
(for the circle, it tends to a limit)

This can be plotted as a logarithmic function, indicative of the fractal dimension (slope = D–1).

## *Analyses of spatial wavelengths are heavily dependent on scale of analysis (footprint)*

To quantify the variations of height/slope statistics with the horizontal measurement scale, one uses **fractals**, a.k.a. self-affine or fractional Brownian statistics.



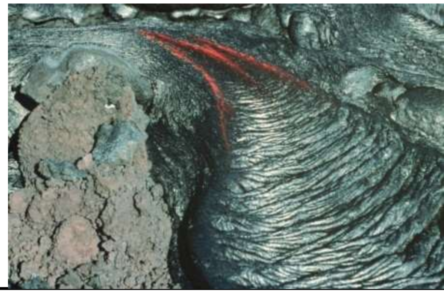
$$\boxed{rms \text{ height } h_0 = C_h \left( \frac{L}{L_0} \right)^H}$$

Scaling constant $C_H$ = rms height when $L = L_0$ (unit scale length)
$H$ = Hurst exponent ($H \in [0, 1]$)

Fractal dimension:

$$\boxed{D = \begin{cases} 2 - H \text{ for profile} \\ 3 - H \text{ for surface} \end{cases}}$$

Useful to quantify lava flows, sedimentary processes, mountain ranges, etc.

| Smooth pahoehoe | Rough pahoehoe | A'a |
|---|---|---|
|  |  |  |
| H = 0.63<br>$C_H$ = 0.05 | H = 0.48<br>$C_H$=0.16 | H = 0.26<br>$C_H$ = 0.24 |

*Fractals (and multifractals) are a rich and developing field of Physics.*
*Check it out in the Library (63+ books, including 16 e-books)*

## 2.3. Dielectric properties

Minerals: Re($\varepsilon$) $\in$ [2; 150]

Silicates: 4 – 10

Iron-bearing: 25 – 81

Sulfates – sulfides: 5 – 14

Halides – carbonates: 4 – 11

Dry rock powders: Re($\varepsilon$) related to bulk density $\rho$ (g/cm$^3$)

Im($\varepsilon$) more difficult to measure

$$\boxed{\mathrm{Re}(\varepsilon) = 1.96^{\rho}}$$

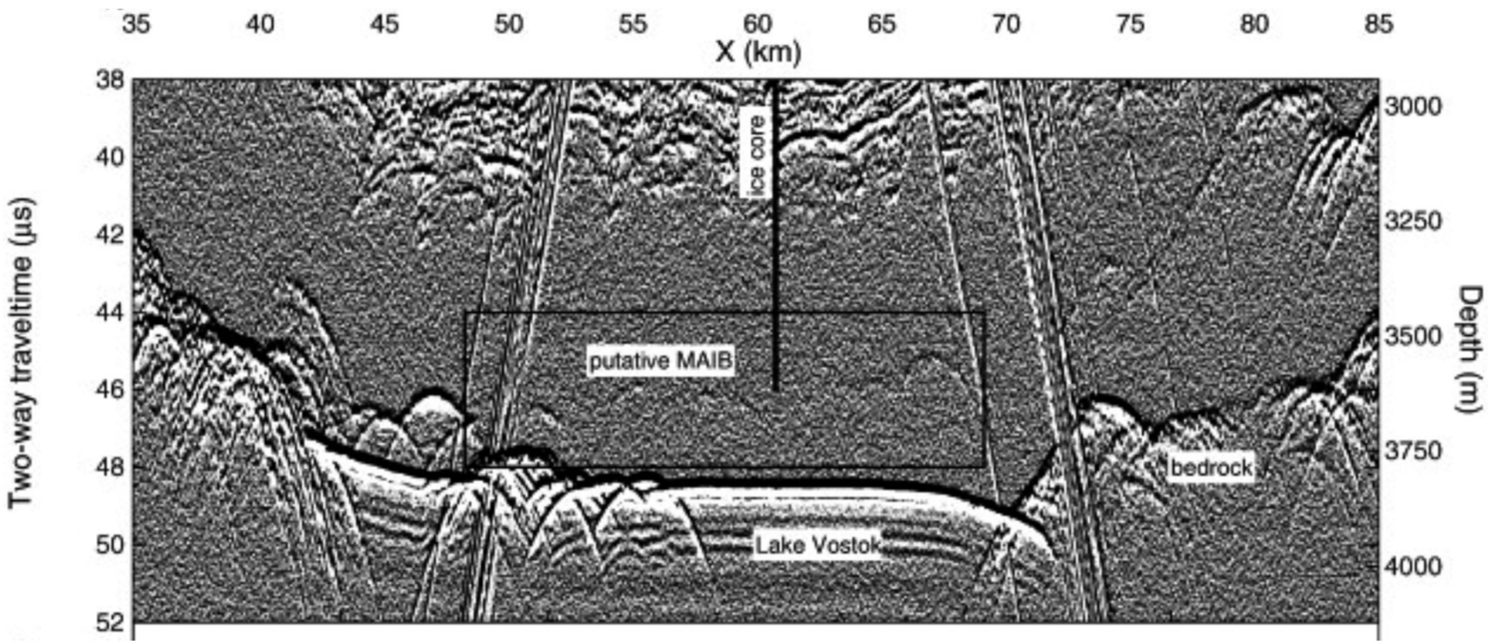
Water: depends on salinity, temperature and frequency. Debye formula (complicated)

*Includes relaxation time $\tau$: rate at which polar molecules align themselves with electric field*

For seawater on Earth, at radar frequencies: $\varepsilon \cong 80 \Rightarrow$ signal penetration negligible

Freshwater ice (pure): $\varepsilon \cong 3.15$
Sea ice: more complex (need to account for mixing of different components)
Loss tangent $\tan\delta$ declines with decreasing temperature (important for icy planets with potential oceans)

$$skin\ depth = \frac{1}{2\,\pi\,f\ \times \sqrt{\epsilon\,\mu}\ \times \sqrt{\frac{1}{2}\left(\sqrt{1+\tan^2\delta}-1\right)}}$$



## 2.4. Mixing of materials with different properties

*Full details in Ulaby et al. (Microwave Remote Sensing, vol. 2, 1986), 686.78 ULA*
*More in Campbell (Radar Remote Sensing of Planetary Surfaces, 2002), 523.164 CAM*

Several formulas:

- Polder-Van Santen-de Loor (PVL) formula for background matrix and ellipsoidal inclusions (e.g. spheres, needles, plates);

- Maxwell formula for matrix with spherical inclusions further than their diameters;

- Lichtenecker formula for random mixtures of two components with volume fractions $V_1$ and $V_2$ ($\varepsilon_1$ and $\varepsilon_2$ not too different):

$$\boxed{\log \varepsilon = V_1 \log \varepsilon_1 + V_2 \log \varepsilon_2}$$

- **Lorentz-Lorenz formula for *n*-component mixture**

$$\boxed{\frac{\varepsilon - 1}{\varepsilon + 2} = \sum_{k=1}^{n} V_k \frac{\varepsilon_k - 1}{\varepsilon_k + 2}}$$

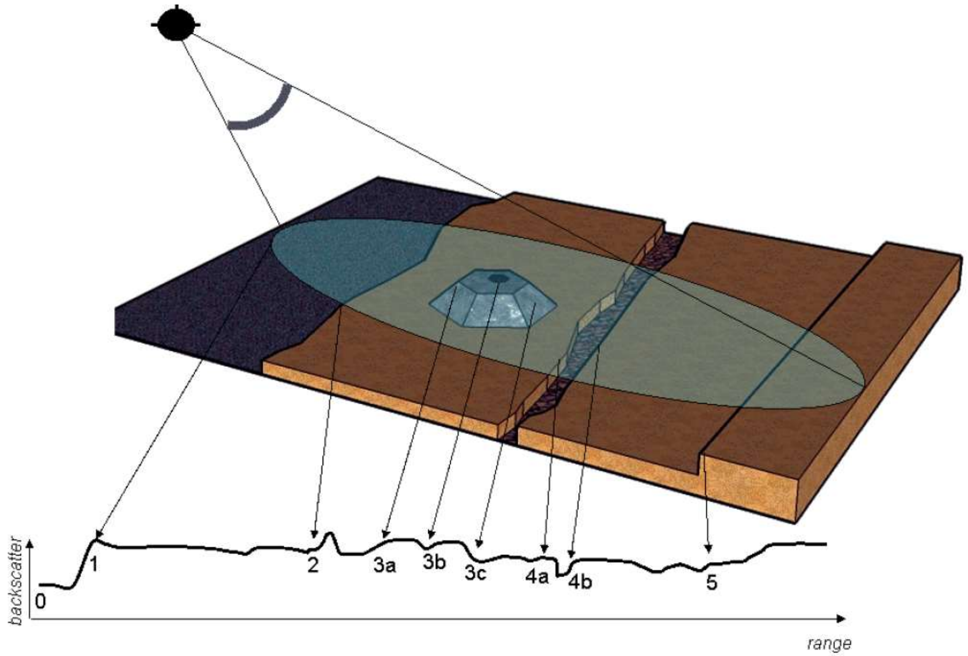
A4!

# Radar Mapping

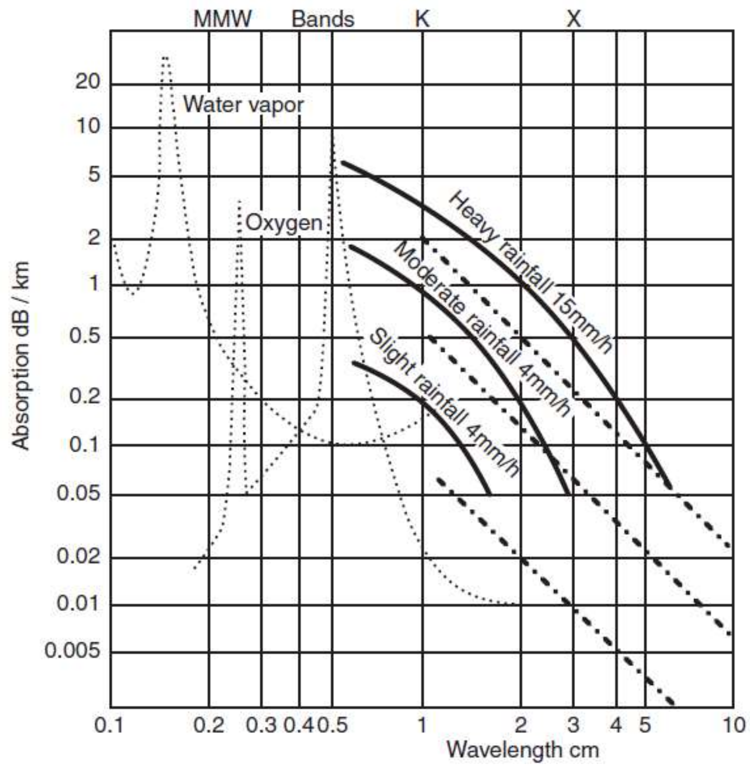
## 3. Radar mapping – Synthetic Aperture Radar (SAR)

### *3.1. Data transmission and reception*

Signal attenuated through atmosphere, by surface scattering and through atmosphere again. Time variation of signal shows returns as a function of time, within footprint

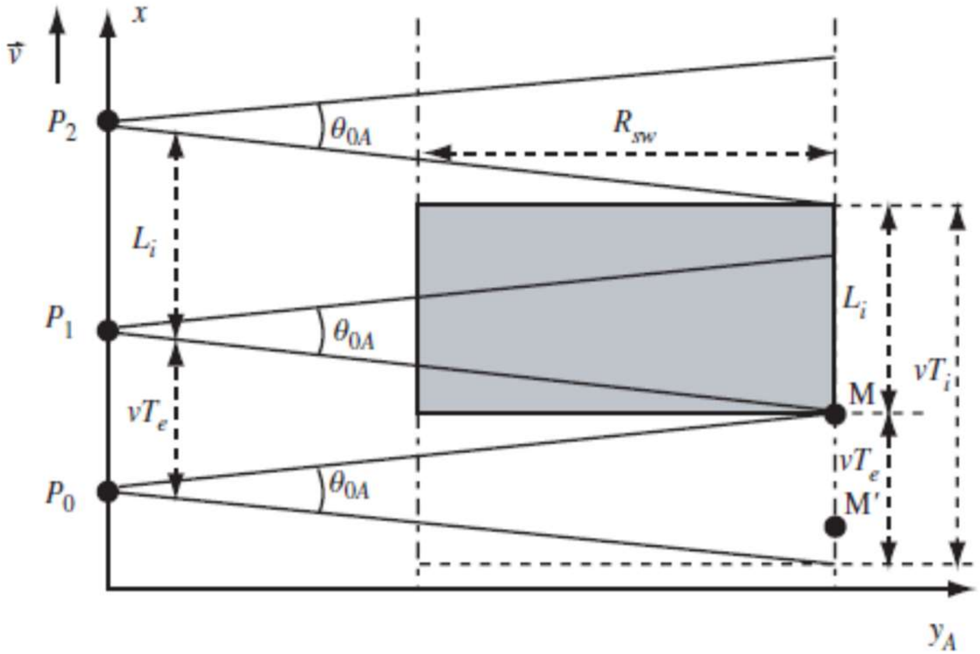



*Typical scattering return*




*Attenuation through Earth's atmosphere*

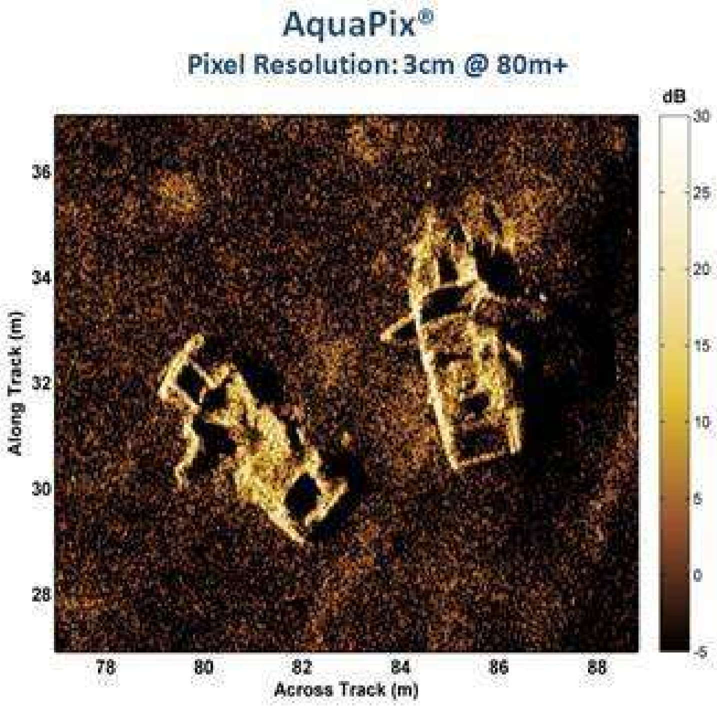
## 3.2. Synthetic Aperture Radar

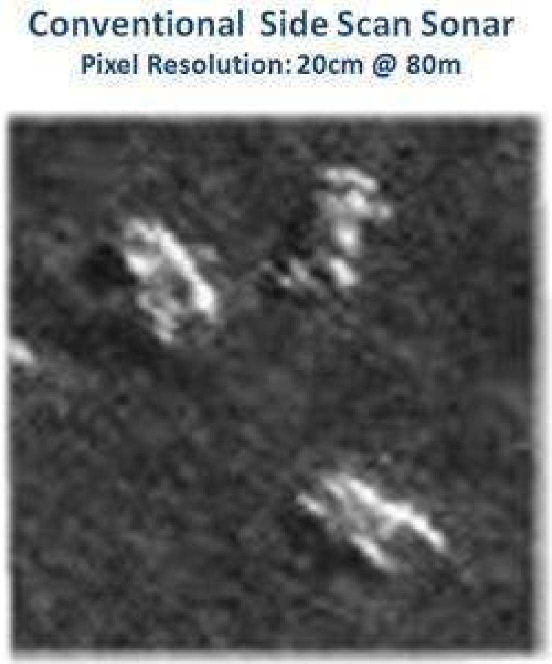



Synthesises longer antenna by looking at the same ground cell from several locations on the flight path.

*For Magellan, 3.7-m synthetic antenna equivalent to 672(!)-m real antenna.*

## SAR data processing




*The same principle works with sonar too …*

Relates lines of constant Doppler/range to specific ground cells.

*Computationally intensive.*
*<u>Very complex</u> (worth entire books)*

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- 1.1. Reminders
- 1.2. Polarisation, coherence and power
- 1.3. Wave propagation
- 1.4. Radar equation

## 2. Roughness and dielectric properties

- 2.1. Rough and smooth surfaces
- 2.2. Fourier and fractal analyses
- 2.3. Dielectric properties
- 2.4. Mixing of materials with different properties

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- 4.1. Integral equation
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# 4. Scattering by rough surfaces

## *4.1. Integral equation*

The wave equation (conservation of electric field) can be solved for every point in scattering area



To perform the integration, it is clear that some form of the autocorrelation coefficient must be assumed. The works of *Hayre and Moore* [1961], *Daniels* [1961], and the experimental results of *Evans* [1963] show that the exponential form gives the best result when the angle of incidence is not very large, say, less than 35°. Hence, letting $r = \exp\,[-[(S\varphi)^2 + (t \csc\alpha)^2]^{1/2}/a]$, where $a$ is the horizontal correlation distance, we obtain

$$E(p) = \kappa_1 \,\mathrm{Re} \Big\{ \iiiint \Big[ A_0$$

$$\cdot \exp\,[-ipt \csc\alpha + iqS\varphi - k^2\sigma^2B^2(1 - e^{-d/a})]$$

$$- C_0\sigma^2\Big[\Big(\frac{t^2\csc^2\alpha}{ad^3} - \frac{1}{ad} + \frac{t^2\csc^2\alpha}{a^2d^2}\Big)$$

$$\cdot e^{-d/a} + \frac{t^2\csc^2\alpha}{a^2d^2}e^{-2d/a}\Big]$$

$$\cdot \exp\,[-ipt\csc\alpha + iqS\varphi - k^2\sigma^2B^2(1 - e^{-d/a})]\Big]$$

$$\cdot S_0^2 \csc^2\alpha \, dS\, d\gamma\, dt\, d\varphi\Big\} \qquad (22)$$

where $d^2 = S^2\varphi^2 + t^2\csc^2\alpha$.

If we make the following change of variables

$$S\varphi = d\cos\theta'$$

$$t\csc\alpha = d\sin\theta'$$

equation 22 becomes

$$E(p) = \kappa_1$$

$$\cdot \mathrm{Re}\Big\{\iiiint A_0 \exp\,[i\psi d - \kappa(1 - e^{-d/a})]$$

$$\cdot S_0 \csc\alpha\, d\, dd\, d\theta'\, dS\, d\gamma - \iiiint C_0\sigma^2$$

$$\cdot\Big[\Big(-\frac{1}{ad}\cos^2\theta' + \frac{1}{a^2}\sin^2\theta'\Big)e^{-d/a}$$

$$+ \kappa\frac{\sin^2\theta'}{a^2}e^{-2d/a}\Big] \exp\,[i\psi d - \kappa(1 - e^{-d/a})]$$

$$\cdot S_0\csc\alpha\, d\, dd\, d\theta'\, dS\, d\gamma\Big\} \qquad (23)$$

where $\psi = -p\sin\theta' + q\cos\theta'$.

Upon expanding $\exp\,(\kappa e^{-d/a})$ into a series in $\kappa e^{-d/a}$, we see that integration with respect to $d$ becomes a trivial matter. The integration with respect to $\theta'$ can be performed by means of standard contour integration technique by the following change of variables onto a circle.

$$\cos\theta' = (z + z^{-1})/2$$

$$\sin\theta' = (z - z^{-1})/2$$

$$d\theta' = dz/iz$$

The advantage of evaluating the integral this way is that no approximation needs to be made for the autocorrelation function as in the works of Hayre and Davies; also, the terms (which involve partial derivatives of the random function $Z(x, y)$ in (8) and appear as the second integral in (23)) that have been neglected in the works of *Winter* [1962] and others are evaluable in the same manner. (Details of the evaluations are given in the appendix.)

The final result of the integration for $E(p)$ is

$$E(p) = \mathrm{Re}\, K\Big[\Big(\frac{a}{\lambda}\Big)^2 \frac{\alpha(\sin^2\theta\cos^2\phi + \cos^2\theta)}{\sin\alpha} \exp\,\{-k^2\sigma^2(\cos\theta + \cos\alpha)^2\}$$

$$\cdot\sum_{n=1}^{\infty}\frac{[k^2\sigma^2(\cos\theta + \cos\alpha)^2]^n}{(n-1)!\,[n^2 + a^2(p^2+q^2)]^{3/2}} + \Big(\frac{\sigma}{\lambda}\Big)^2\frac{\alpha\sin^2\theta}{\sin\alpha}\exp\,\{-k^2\sigma^2(\cos\theta + \cos\alpha)^2\}$$

$$\cdot\sum_{n=0}^{\infty}\frac{[k^2\sigma^2(\cos\theta+\cos\alpha)^2]^n}{n!}\Big\{-\frac{n}{a^2(p+iq)^2} + \frac{1}{[(n+1)^2 + a^2(p^2+q^2)]^{1/2}}$$

$$\cdot\Big[\frac{1}{2} + \frac{1}{4}\Big(\frac{-(n+1) + [(n+1)^2 + a^2(p^2+q^2)]^{1/2}}{a(p-iq)}\Big)^2$$

$$+ \frac{1}{4}\Big(\frac{a(p-iq)}{-(n+1) + [(n+1)^2 + a^2(p^2+q^2)]^{1/2}}\Big)^2\Big] + \frac{1}{2[(n+1)^2 + a^2(p^2+q^2)]^{3/2}}$$



$$\cdot\Big[-(n+1) + \frac{1}{2}\Big(\frac{-(n+1) + [(n+1)^2 + a^2(p^2+q^2)]^{1/2}}{a(p-iq)}\Big)^2$$

$$\cdot\Big((n+1) + 2[(n+1)^2 + a^2(p^2+q^2)]^{1/2}\Big)$$

$$+ \frac{1}{2}\Big(\frac{a(p-iq)}{-(n+1) + [(n+1)^2 + a^2(p^2+q^2)]^{1/2}}\Big)^2\Big((n+1) - 2[(n+1)^2 + a^2(p^2+q^2)]^{1/2}\Big)\Big]\Big\}$$

$$+ \Big(\frac{\sigma}{\lambda}\Big)^2\frac{\alpha(2\pi)^2(\cos\theta+\cos\alpha)^2\sin^2\theta}{\sin\alpha}\exp\,\{-k^2\sigma^2(\cos\theta+\cos\alpha)^2\}\sum_{n=0}^{\infty}\frac{[k^2\sigma^2(\cos\theta+\cos\alpha)^2]^n}{n!}$$

$$\cdot\Big\{\frac{1}{a^2(p+iq)^2} + \frac{1}{2[(n+2)^2 + a^2(p^2+q^2)]^{3/2}}$$

$$\cdot\Big[-(n+2) + \frac{1}{2}\Big(\frac{-(n+2) + [(n+2)^2 + a^2(p^2+q^2)]^{1/2}}{a(p-iq)}\Big)^2$$

$$\cdot\Big((n+2) + 2[(n+2)^2 + a^2(p^2+q^2)]^{1/2}\Big)$$

$$+ \frac{1}{2}\Big(\frac{a(p-iq)}{-(n+2) + [(n+2)^2 + a^2(p^2+q^2)]^{1/2}}\Big)^2\Big((n+2) - 2[(n+2)^2 + a^2(p^2+q^2)]^{1/2}\Big)\Big]\Big\}\Big] \qquad (24)$$

$$K = 2\eta H_0^2(c\tau S_0\pi)/R_0^2$$

$$p = k(\sin\alpha - \sin\theta\sin\phi)$$

$$q = k\sin\theta\cos\phi$$

Under the special conditions of backscatter (i.e., the transmitter and the receiver in the same location) (24) reduces to

$$E(p) = K\Big[\Big(\frac{a}{\lambda}\Big)^2\frac{\alpha\cos^2\alpha}{\sin\alpha}\exp\,[-4k^2\sigma^2\cos^2\alpha]\sum_{n=1}^{\infty}\frac{(4k^2\sigma^2\cos^2\alpha)^n}{(n-1)!\,(n^2 + a^2p^2)^{3/2}} + \Big(\frac{\sigma}{\lambda}\Big)^2\alpha\sin\alpha$$

$$\cdot\exp\,[-4k^2\sigma^2\cos^2\alpha]\sum_{n=1}^{\infty}\frac{(4k^2\sigma^2\cos^2\alpha)^n}{n!}\Big\{-\frac{n}{a^2p^2} + \frac{1}{[(n+1)^2 + a^2p^2]^{1/2}}$$

$$\cdot\Big[\frac{1}{2} + \frac{1}{4}\Big(\frac{(n+1) - [(n+1)^2 + a^2p^2]^{1/2}}{ap}\Big)^2 + \frac{1}{4}\Big(\frac{ap}{(n+1) - [(n+1)^2 + a^2p^2]^{1/2}}\Big)^2\Big]$$

$$+ \frac{1}{2[(n+1)^2 + a^2p^2]^{3/2}}\Big[-(n+1) + \frac{1}{2}\Big(\frac{(n+1) - [(n+1)^2 + a^2p^2]^{1/2}}{ap}\Big)^2$$

$$\cdot\Big((n+1) + 2[(n+1)^2 + a^2p^2]^{1/2}\Big) + \frac{1}{2}\Big(\frac{ap}{(n+1) - [(n+1)^2 + a^2p^2]^{1/2}}\Big)^2$$

$$\cdot\Big((n+1) - 2[(n+1)^2 + a^2p^2]^{1/2}\Big)\Big]\Big\}$$

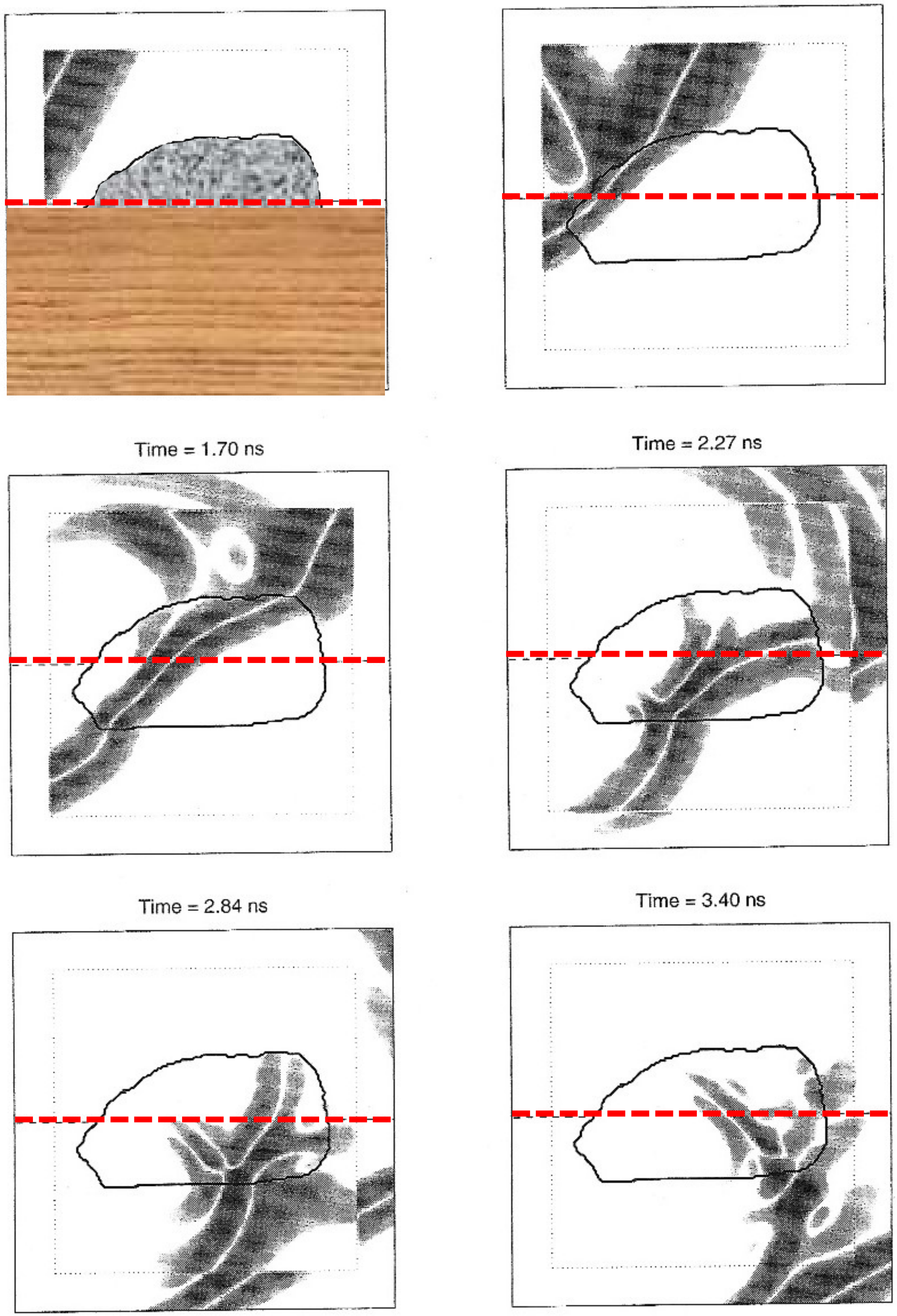
$$+ \Big(\frac{\sigma}{\lambda}\Big)^4 4(2\pi)^2\alpha\sin\alpha\cos^2\alpha\exp\,[-4k^2\sigma^2\cos^2\alpha]\sum_{n=0}^{\infty}\frac{(4k^2\sigma^2\cos^2\alpha)^n}{n!}$$

$$\cdot\Big\{\frac{1}{a^2p^2} + \frac{1}{2[(n+2)^2 + a^2p^2]^{3/2}}\Big[-(n+2) + \frac{1}{2}\Big(\frac{(n+2) - [(n+2) + a^2p^2]^{1/2}}{ap}\Big)^2$$

$$\cdot\Big((n+2) + 2[(n+2)^2 + a^2p^2]^{1/2}\Big)$$

$$+ \frac{1}{2}\Big(\frac{ap}{(n+2) - [(n+2)^2 + a^2p^2]^{1/2}}\Big)^2\Big((n+2) - 2[(n+2)^2 + a^2p^2]^{1/2}\Big)\Big]\Big\}\Big] \qquad (25)$$

***Extract** from Fung, 1964*




The integral equation is not suitable for planetary-scale radar images.

It can however be solved for small objects, using different approaches.

*Examples: finite-element models, COMSOL or ANSYS packages*

*Simulation of scattering on a half-buried rock (from Campbell, 2002)*

## 4.2. Simplifying scattering

Radar scattering: impossible to solve analytically (local topography and dielectric variations)
One uses approximations, e.g. scattering by gently undulated surfaces

correlation length $L_c$, rms height $h_0$ $\quad L_C > \lambda \quad \frac{L_C^2}{h_0} > 3\lambda$

For a square scattering area of side $L$:

$$E_s = E_i \times \left( \frac{-ik \exp(ikr_0)}{4\pi r_0} \right) \int_{-L/2}^{+L/2} \int_{-L/2}^{+L/2} K\left( \frac{dz}{dx}, \frac{dz}{dy}, \varepsilon_r, \mu \right) e^{-ik\left(\hat{k}_i - \hat{k}_s\right)} dx\, dy$$

**Kirchhoff phase function** **local slopes** **phase shift** **unit vectors**

3 approaches for calculating $K$ by simplifying its expression:

1. physical optics (integrating over distribution of surface slopes)
2. ray optics (surfaces represented as tilted facets)
3. geometric optics (surface is gently curved dielectric interface)

All lead to very similar results.

For a Gaussian surface, for example:

$$\sigma(\phi) = \frac{\rho_0^2}{\left(\frac{4h_0^2}{L_C^2}\right)^2 \cos^4\phi} \exp\left( \frac{-\tan^2\phi}{\left(\frac{4h_0^2}{L_C^2}\right)^2} \right)$$

## 4.3. The Hagfors model

Often used for gently undulating surfaces (e.g. Moon, Venus).
Assumes Gaussian distribution of heights.

Backscatter coefficient becomes:

$$\boxed{\sigma(\phi) = \frac{C\rho_0}{2}\left(\cos^4\phi + C\sin^2\phi\right)^{-3/2}}$$

Parameter *C* generally taken as inverse square of rms slope
Parameter $\rho_0$: apparent Fresnel normal reflectivity (used to derived emissivity)

Other simplified models use the fractal nature of the surface (e.g. Shephard & Campbell, 1999):

$$\boxed{\sigma(\phi) = 16\pi^3 \,\mathrm{Re}\left[\int_{\hat{r}=0}^{+\infty} \exp\left(-4\pi^2 s_\lambda^2 \hat{r}^{2H}\cos^2\phi\right)\hat{r}\, J_0(4\pi\hat{r})\sin\phi\, d\hat{r}\right]}$$

$$\hat{r} = \frac{r}{\lambda}$$

*s: rms slope divided by* $\lambda$
$J_0$*: zero-order Bessel function*
*H: Hurst exponent*

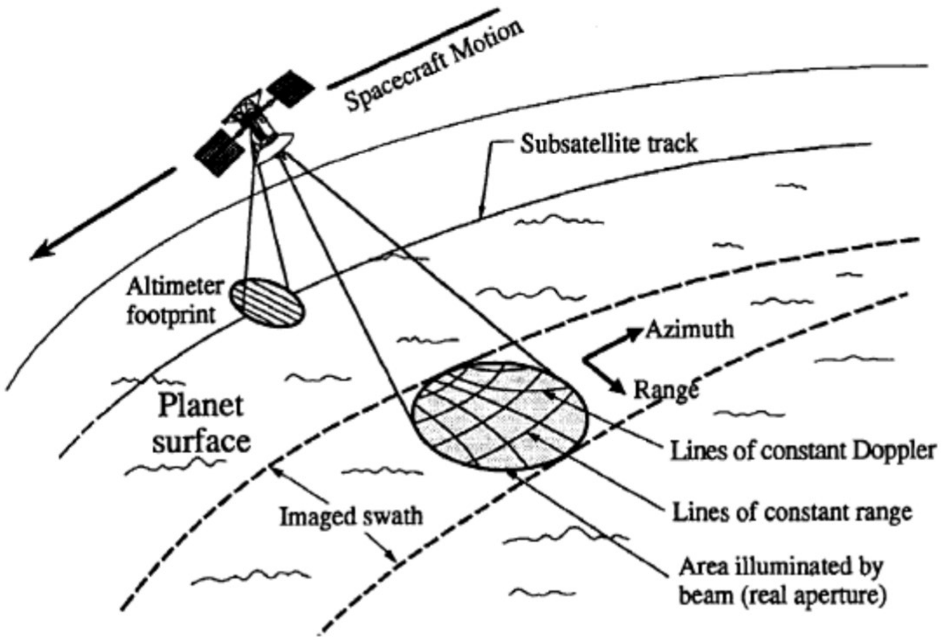
Composite models look at the expected contributions from different roughness scales

Best model is still matter of <u>intense</u> debate (especially if accounting for polarisation effects).
Depends on planetary surface under investigation, and on time available for analyses.

# Radar Mapping

# 5. Application: Venus – Magellan SAR

*5.1. Magellan radar mapping*



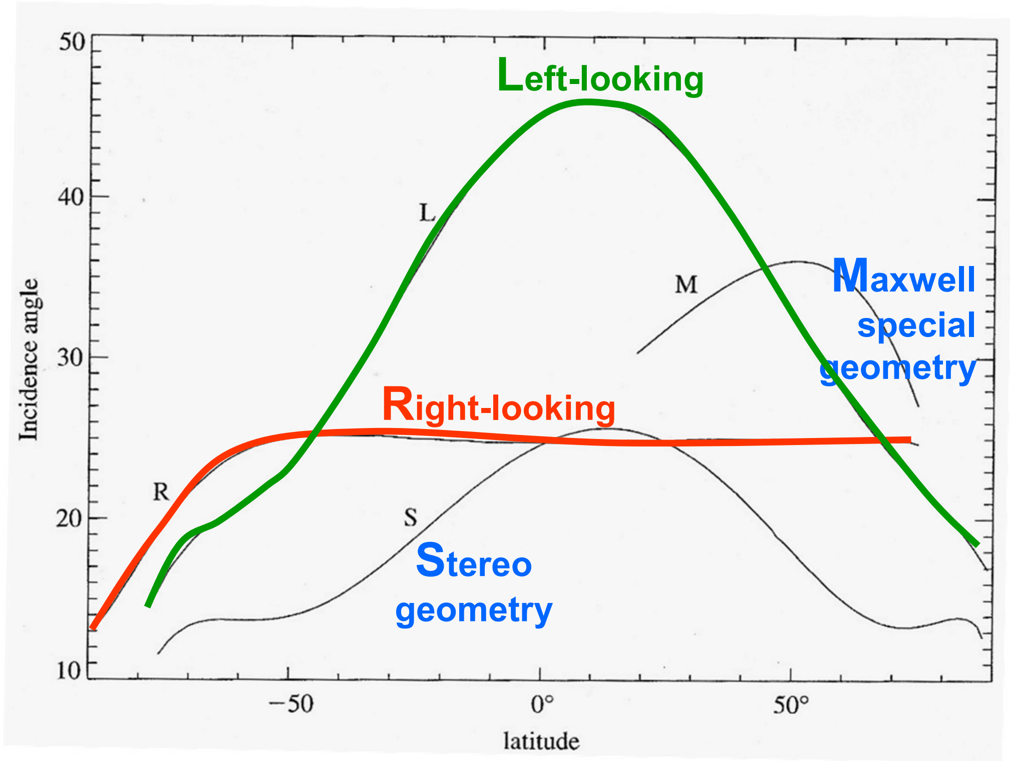




**Table 1.** Initial orbit parameters.

| Parameter | Design orbit | Actual orbit |
|---|---|---|
| Periapsis altitude | 275.0 km | 294 km |
| Periapsis latitude | 10.0°N | 9.9°N |
| Altitude at pole | 2145 km | 2225 km |
| Orbit period | 3.186 hours | 3.26 hours |
| Inclination | 85.30° | 85.5° |

**Table 5.** Radiometer surface footprint resolution versus spacecraft altitude.

| Spacecraft height (km) | Resolution | |
|---|---|---|
| | Along track (km) | Across track (km) |
| 290 | 16 | 24 |
| 500 | 25 | 33 |
| 1000 | 44 | 50 |
| 2100 | 83 | 87 |

*From NASA JPL Publication 89-41, 1989*

## 5.2. Using the Hagfors model

Image swaths: north-south strips ca. 20 km wide. Varying incidence angles (~25°-45°)
Primary mission data: left-looking, HH polarisation
Second mapping cycle: right-looking, HH polarisation (a few VV tracks toward the end)

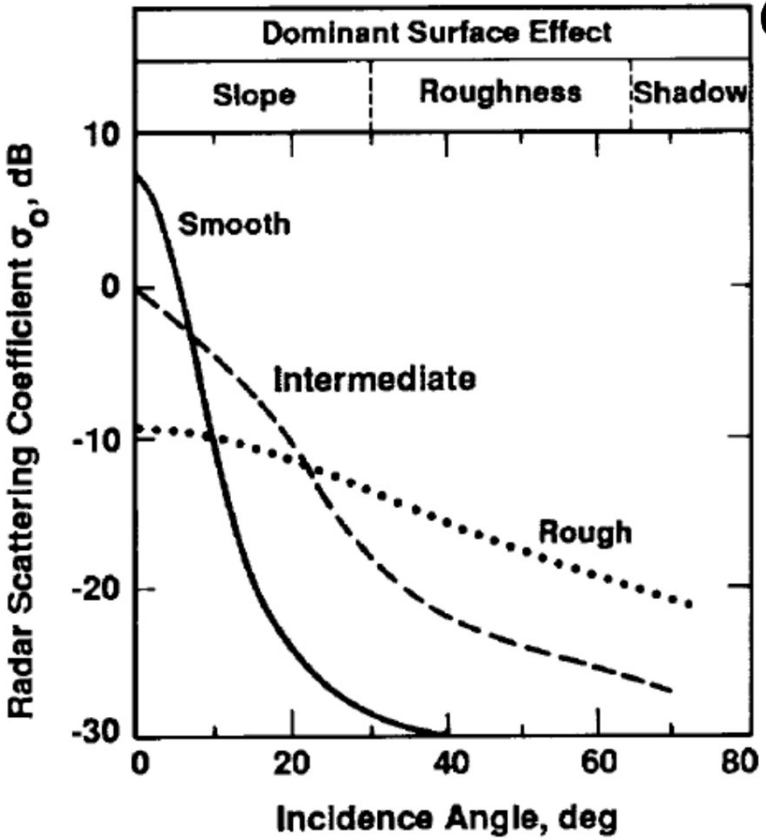
Backscatter values scaled using the empirical model of Muhleman (1964).
Each *DN* (grey level) is related to $\sigma(\phi)$ by:

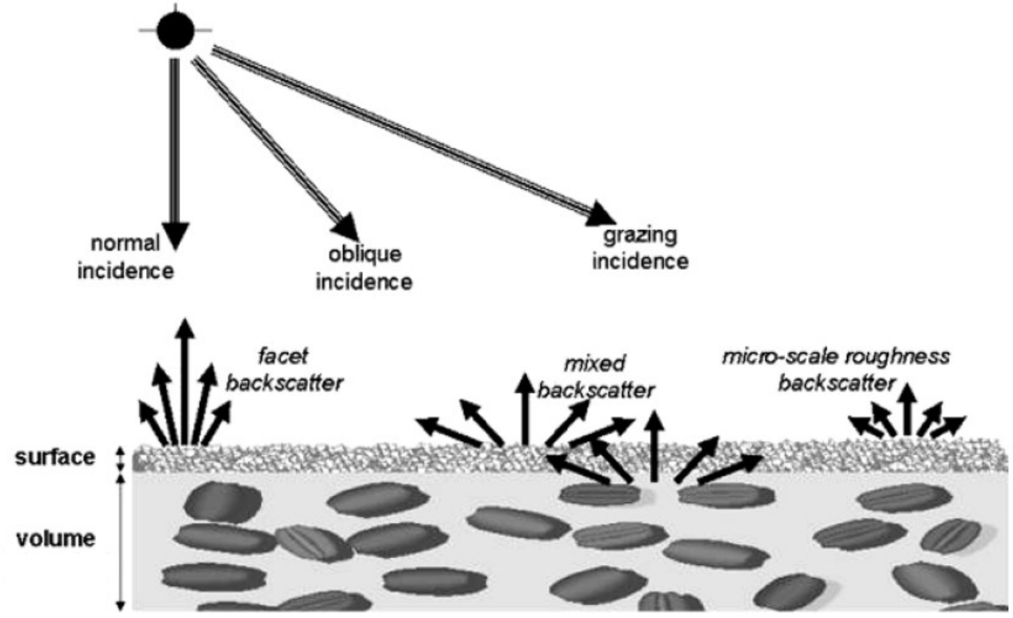
$$\boxed{\sigma(\phi) = 10^{\frac{-20+\frac{DN-1}{5}}{10}} \times \frac{0.0118\cos(\phi+0.5^\circ)}{\left[\sin(\phi+0.5^\circ)+0.111\cos(\phi+0.5^\circ)\right]^3}}$$

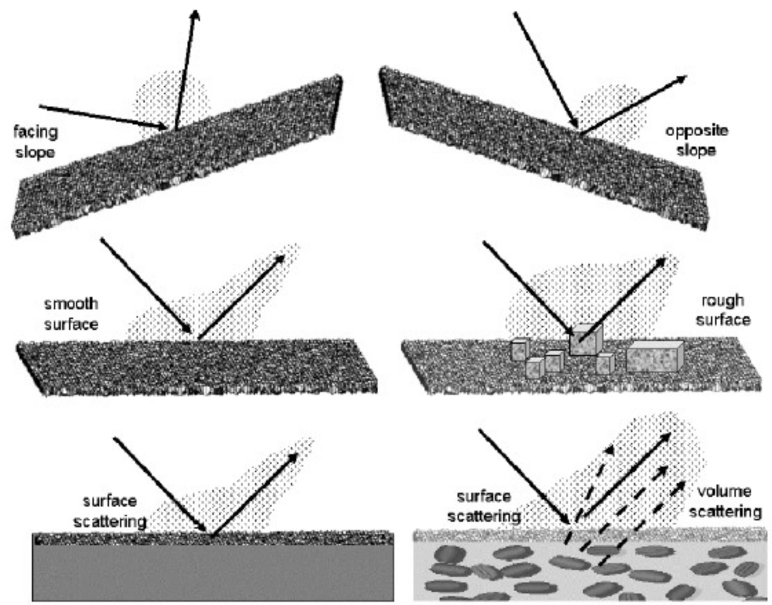
0.5° shift of $\phi$ added "inadvertently" during data processing

The Muhleman model incorporates several approximations of the Hagfors model.
It allows compressing the data to an 8-bit format (*DN* = 0–255)

we can infer how the surface should look like



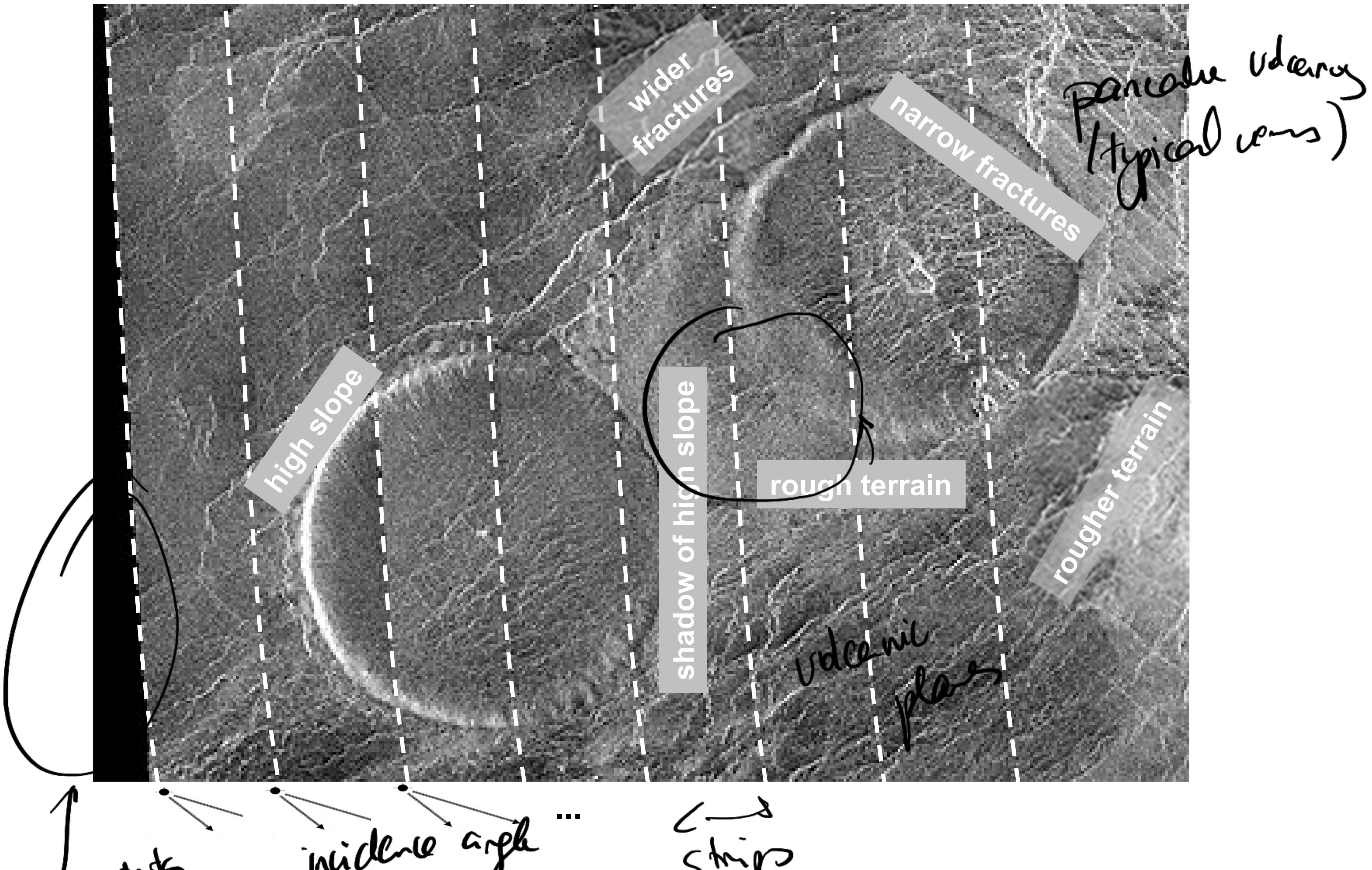




Knowing the backscatter $\sigma$ and the incidence angle $\phi$, it is possible to infer from the Hagfors law the main effects for this wavelength $\lambda$.

Helps toward interpretation

## 5.3. Main results



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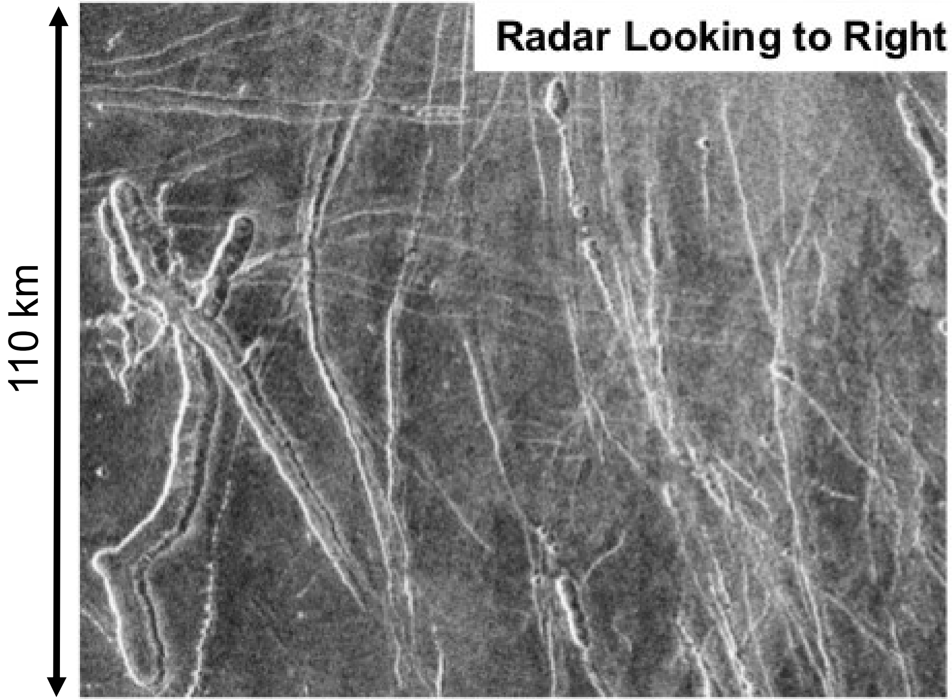
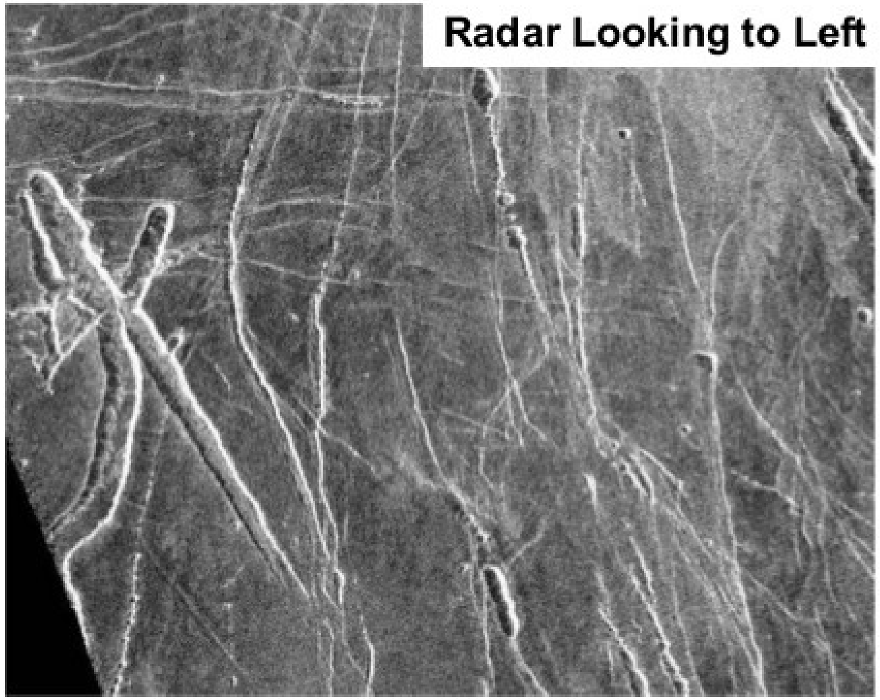
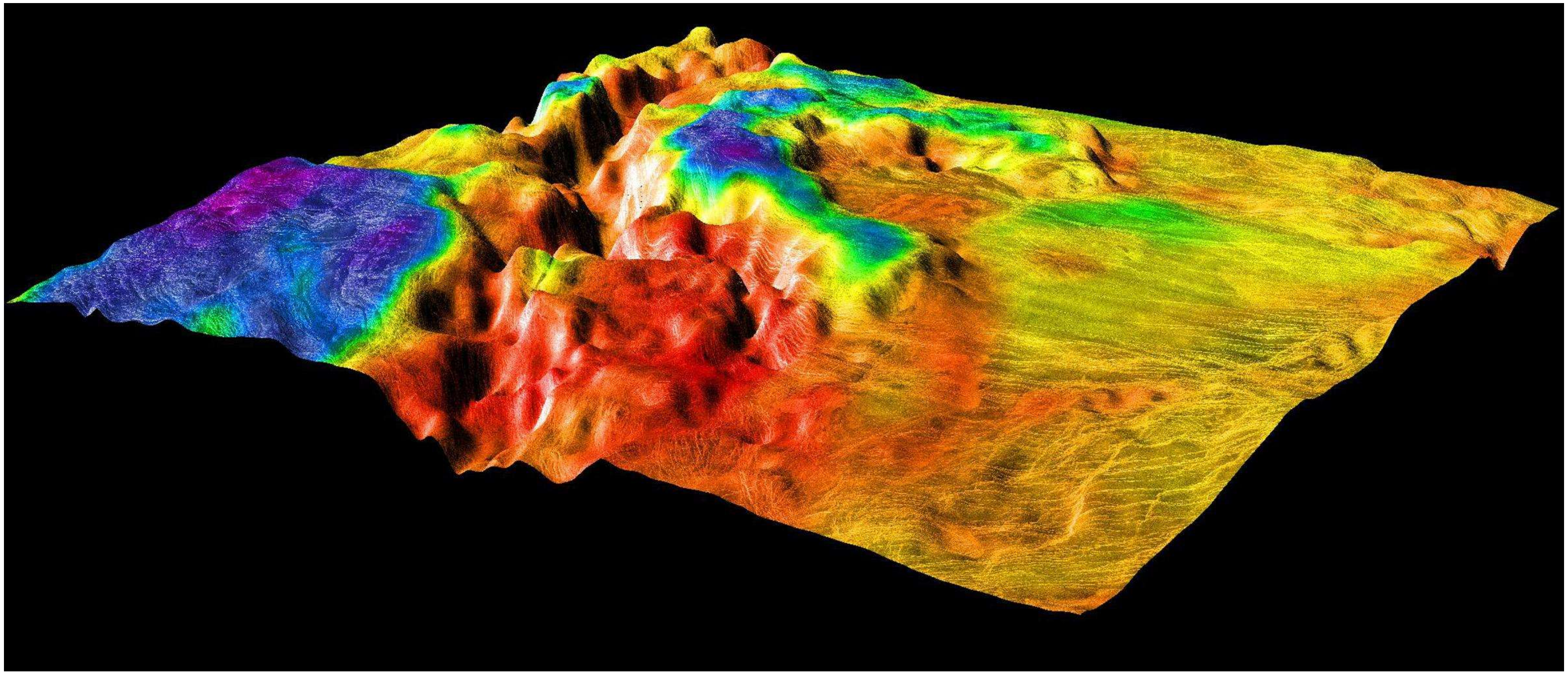
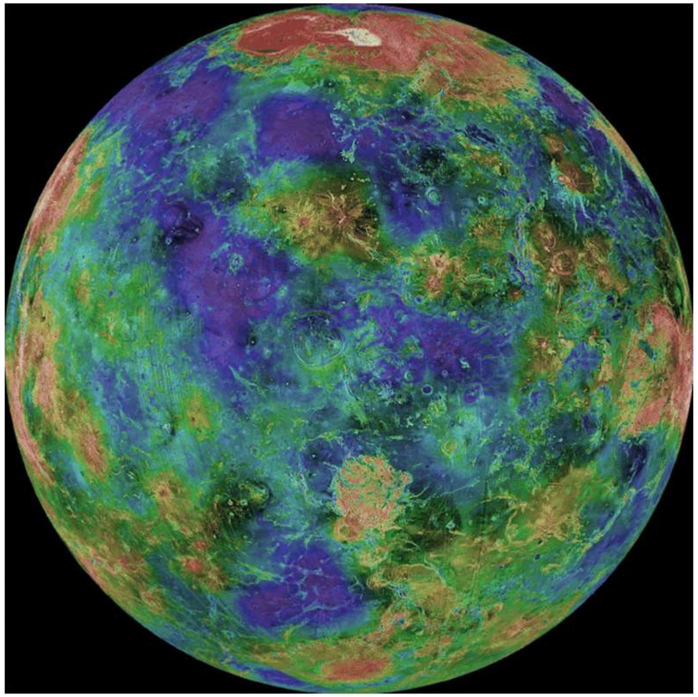
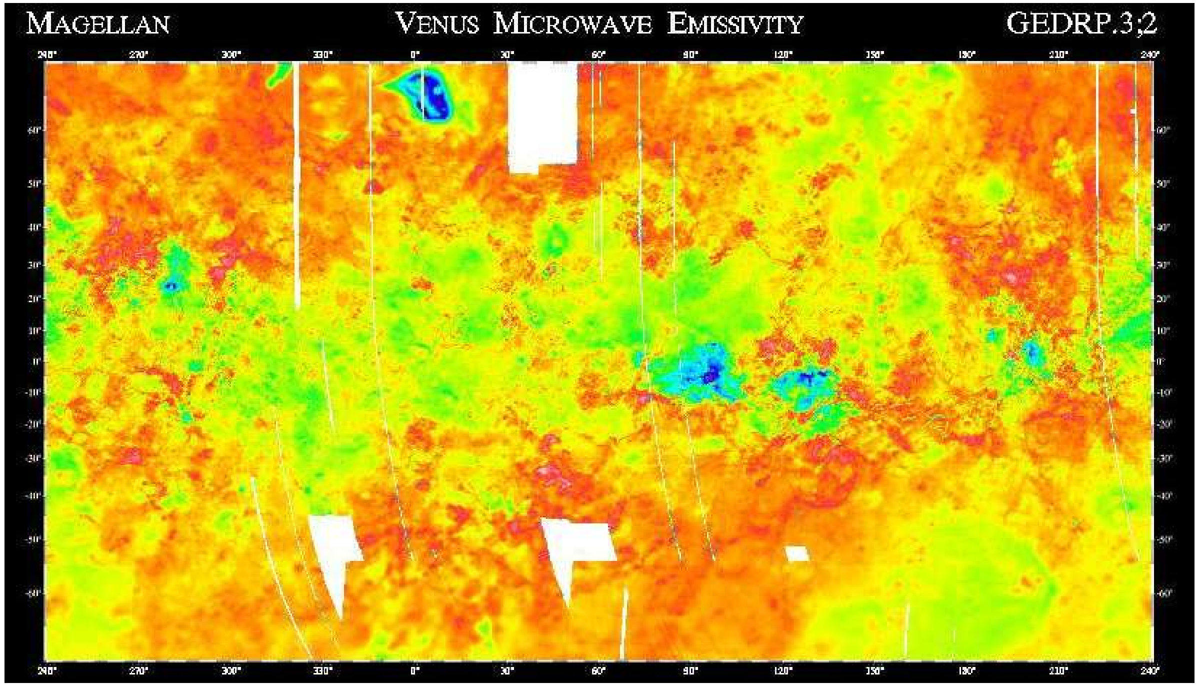


## 5.3. Main results






Two mapping cycles gave complete map of entire planet (Earth-size) at 75-m resolution:

- SAR imagery
- Topography
- Brightness temperature – Emissivity

# Radar Mapping