~~Past exercise:~~

PH20107 MATHEMATICAL METHODS (1)

LECTURE 4 12^{th} OCTOBER 2022

GIVEN

$$\vec F = q(\dot{\vec r}\times\vec B) \quad \text{(LORENTZ FORCE)}$$

EXPLAIN WHY THE ACCELERATION OF THE PARTICLE IS ALWAYS $\perp$ TO $\vec B$

BY INTEGRATING THE EQUATION OF MOTION, SHOW THAT

$$\dot{\vec r} = \frac{q}{M}(\vec r\times\vec B) + \vec C$$

mass of particle

constant vector

HENCE, SHOW THAT

$$\ddot{\vec r} + \frac{q^2}{M^2}\left(B^2\vec r - (\vec r\cdot\vec B)\vec B\right) = \frac{q}{M}(\vec C\times\vec B).$$

$$\ddot{\vec r} = \frac{q}{M}(\dot{\vec r}\times\vec B)$$

$$\int \ddot{\vec r}\,dt = \int \frac{q}{M}(\dot{\vec r}\times\vec B)\,dt$$

const B field

$$\dot{\vec r} + \vec C_1 = \frac{q}{M}(\vec r\times\vec B) + \vec C_2$$

$$= \frac{q}{M}(\vec r\times\vec B) + C \quad (\text{where } \vec C = \vec C_2 - \vec C_1)$$

$\therefore$ I can integrate vectors

Sub for $\dot{\vec r}$ into original eq° to get

$$\ddot{\vec r} = \frac{q}{M}(\dot{\vec r}\times\vec B)$$

$\vec{r} \quad \frac{}{\mu}$

becomes $\ddot{\vec{r}} = \frac{q}{\mu}\left(\frac{q}{\mu}\left(\vec{r}\times\vec{B}\right)+\vec{c}\right)\times\vec{B})$

$$\cancel{= \frac{q^2}{\mu^2}\left(\vec{r}\times\vec{B}\right) + \left(\vec{c}\times\vec{B}\right)}$$

$$= \frac{q^2}{\mu^2}\left(\vec{r}\times\vec{B}\right)\times\vec{B} + \frac{q}{\mu}\left(\vec{c}\times\vec{B}\right)$$

From FB

$$\left(\vec{e}\times\vec{f}\right)\times\vec{g} = \left(\vec{e}\cdot\vec{g}\right)\vec{f} - \left(\vec{f}\cdot\vec{g}\right)\vec{e}$$

let $\vec{r}$ be $\vec{e}$, let $\vec{B}$ be $\vec{f}$ &

let $\vec{B}$ be $\vec{g}$

So $\left(\vec{r}\times\vec{B}\right)\times\vec{B} = \left(\vec{r}\cdot\vec{B}\right)\vec{B} - \left(\vec{B}\cdot\vec{B}\right)\vec{r}$

$$= \left(\vec{r}\cdot\vec{B}\right)\vec{B} - B^2\vec{r}$$

Thus $\ddot{r} \quad \frac{q^2}{} \left(\left(\vec{r}\cdot\vec{B}\right)\vec{B} - \vec{B}\,\vec{r}\right)$

$$\ddot{r} = \frac{q}{m}\left(\left(\vec{r}\cdot\vec{B}\right)\vec{B} - B^2\vec{r}\right)$$

$$+\frac{q}{m}\left(\vec{c}\times\vec{B}\right)$$

$$\ddot{\vec{r}} - \frac{q^2}{m}\left(\left(\vec{r}\cdot\vec{B}\right)\vec{B} - B^2 r\right) = \frac{q}{m}\left(\vec{c}\times\vec{B}\right)$$

$$\ddot{\vec{r}} + \frac{q^2}{m^2}\left(B^2\vec{r} - \left(\vec{r}\cdot\vec{B}\right)\vec{B}\right) = \frac{q}{m}\left(\vec{c}\times\vec{B}\right)$$

QED

What is a vector field?

Consider a vector quantity

$$\vec{a}\left(\vec{r}\right)$$

or

$$\vec{a}(x, y, z)$$

or

$$\vec{a}\left(\vec{r}\right) = a_1\left(\vec{r}\right)\vec{i} + a_2\left(\vec{r}\right)\vec{j}$$

$a_x(\ldots)\hat{i} + a_y(\ldots)\hat{j} + \ldots$

$a_x(x,y,z)\hat{i} + a_y(x,y,z)\hat{j} + \dots$

a is a vector field

A vector field is a physical quantity that that has a

- magnitude
- direction

at every point in space.

Examples: Electric field ($\vec{E}$)
Magnetic field ($\vec{B}$)
wind velocity

Also we can have a physical quantity that has a magnitude at every point in a space. eg temperature

Differentiation of vector & scalar fields

Scalar field ϕ

$$\frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial y}, \frac{\partial \phi}{\partial z}$$

vector field $\vec{a}$

$$\frac{\partial \vec{a}}{\partial x}, \frac{\partial \vec{a}}{\partial y}, \frac{\partial \vec{a}}{\partial z}$$

where $\frac{\partial \vec{a}}{\partial x} = \frac{\partial a_x}{\partial x}\vec{i} + \frac{\partial a_y}{\partial x}\vec{j} + \frac{\partial a_z}{\partial x}\vec{k}$

$$\frac{\partial \vec{a}}{\partial y} = \frac{\partial a_x}{\partial y}\vec{i} + \frac{\partial a_y}{\partial y}\vec{j} + \frac{\partial}{\partial y} a_z \vec{k}$$

$$\frac{\partial \vec{a}}{\partial z} = \partial \ldots$$