1) $\vec{a} = 2t\vec{i} + \vec{j} + \vec{k}$ $\qquad$ $\vec{b} = (2t)^2\vec{j}$

$= \begin{pmatrix} 2t \\ 1 \\ 1 \end{pmatrix}$ $\qquad$ $= \begin{pmatrix} 0 \\ 4t^2 \\ 0 \end{pmatrix}$

- $\dfrac{da}{dt} = 2\vec{i}$ $\qquad$ • $\dfrac{db}{dt} = 8t\vec{j}$

- $a \times b = \begin{vmatrix} i & j & k \\ 2t & 1 & 1 \\ 0 & 4t^2 & 0 \end{vmatrix} = 8t^3\vec{k} - 4t^2\vec{i} = 4t^2(2t\vec{k} - \vec{i})$

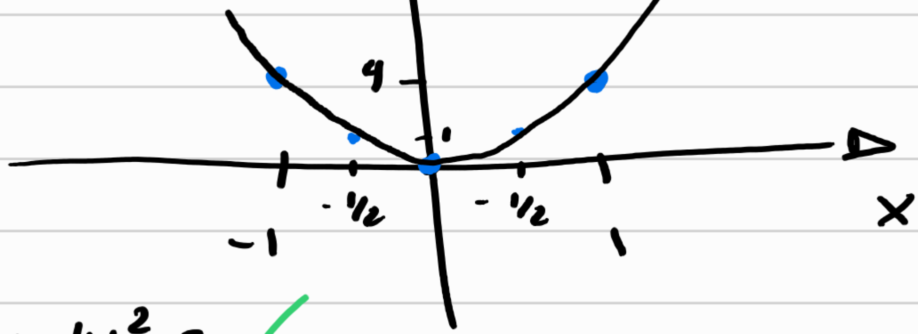
- $\dfrac{d}{dt}(a \times b) = 24t^2\vec{k} - 8t\vec{i} = 8t(3t\vec{k} - 1)$

- $a \times \dfrac{db}{dt} = \begin{vmatrix} i & j & k \\ 2t & 1 & 1 \\ 0 & 8t & 0 \end{vmatrix} = 16t^2\vec{k} - 8t\vec{i} = 8t(-\vec{i} + 2t\vec{k})$

- $\dfrac{da}{dt} \times b = \begin{vmatrix} i & j & k \\ 2 & 0 & 0 \\ 0 & 4t^2 & 0 \end{vmatrix} = 8t^2\vec{k}$

$$\Rightarrow \frac{d}{dt}(a \times b) = 24t^2\vec{k} - 8t\vec{i}$$
$$= (16t^2\vec{k} - 8t\vec{i}) + (8t^2\vec{k})$$
$$= a \times \frac{db}{dt} + \frac{da}{dt} \times b$$

2) a) $y = 4x^2$




$r(t) = t\vec{i} + 4t^2\vec{j}$ ✓

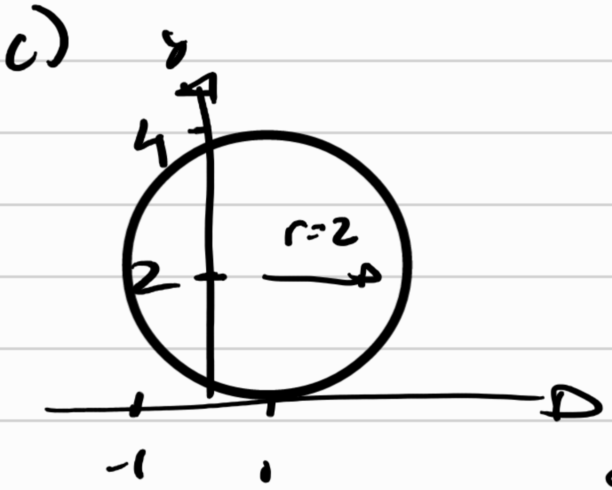
b) $x+y=0,\ 3x+6y-z=2$

$x=-y \Rightarrow 3y-z=2$

$z=3y-2$

$r(t) = -t\vec{i} + t\vec{j} + (3t-2)\vec{k}$ ✓

c)




$x^2+y^2=r^2$

$x^2+y^2=4$

$x^2=4-y^2$

$x=\sqrt{4-y^2}$

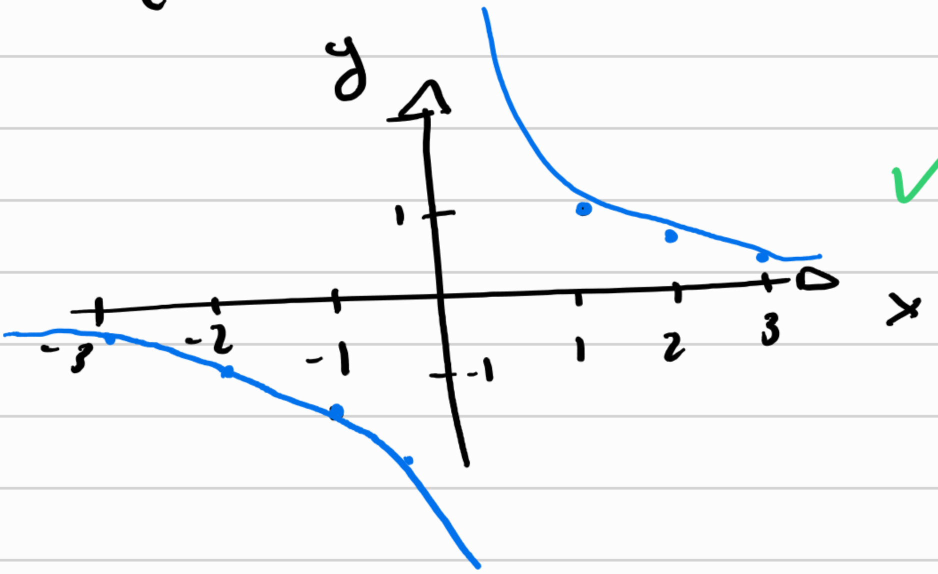
$\cancel{r(t)=(\sqrt{4-t})\vec{i}+t\vec{j}}$

circle with center $(a,b)$:

$r(t) = (a+r\cos(t))\vec{i} + (b+r\sin(t))\vec{j}$

3) $r(t) = e^{t}\vec{i} + e^{-t}\vec{j}$

$e^{-t} = \frac{1}{e^{t}}$ $\qquad t = \frac{1}{y}$ ?




4) a) $\hat{T} = \dfrac{r'(t)}{\|r'(t)\|}$

$r(t) = t\vec{i} + t^2\vec{j}$

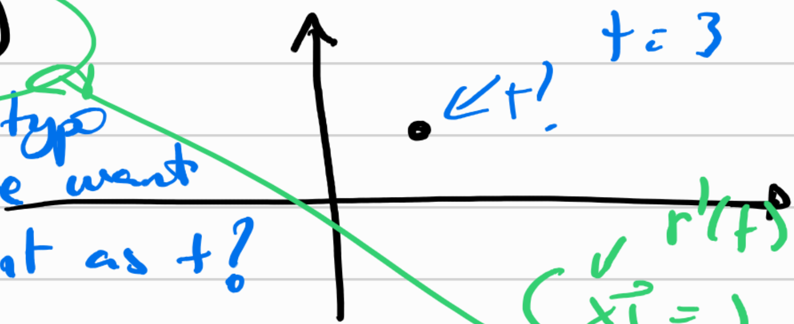
$r'(t) = \vec{i} + 2t\,\vec{j}$

$\hat{T} = \dfrac{\vec{i} + 2t\vec{j}}{\|\vec{i} + 2t\vec{j}\|}$

$\| = \sqrt{1+4t^2}$

$t = 3$

$P = (1,1,0)$ ← is that a typo or does he want a 3D point as $t$?




$r'(t) = x\vec{i} + y\vec{j} + z\vec{k}$

$\left\{ \begin{array}{l} x\vec{i} = 1 \\ y\vec{j} = 1 \\ z\vec{k} = 0 \end{array} \right. \Rightarrow t = 1$

$\hat{T} = \dfrac{1+2}{\|1+2\|} = 1$

Post comprehension corrections:

$t=1 \rightarrow \hat{T} = \dfrac{\vec{i}+2t\vec{j}}{\sqrt{(\cancel{1+2t})^2}} = \dfrac{\vec{i}+2\vec{j}}{\sqrt{\cancel{3}5}} = \dfrac{\vec{i}+2\vec{j}}{\sqrt{5}} \Big\} \checkmark \dfrac{(1,2,0)}{\sqrt{5}}$

$1^2+(2t)^2 = 1+4t^2$

b) $r(t) = 3\cos(t)\,\vec{i} + 3\sin(t)\,\vec{j} + 4t\,\vec{k}$

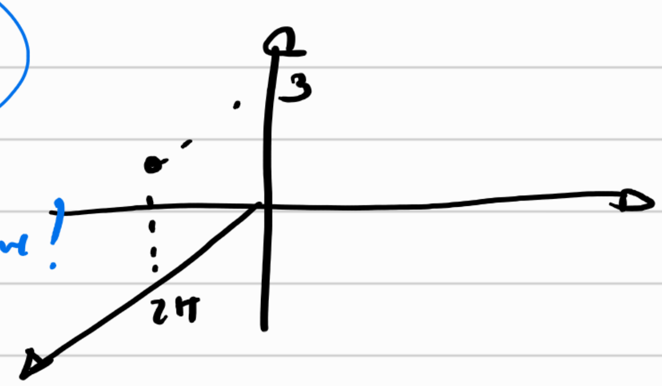
$r'(t) = -3\sin(t)\,\vec{i} + 3\cos(t)\,\vec{j} + 4\vec{k}$

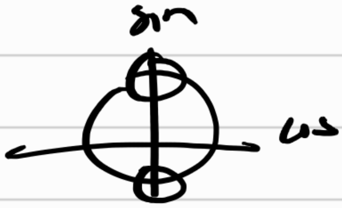
$\hat{T} = -3\sin(t)\,\vec{i} + 3\cos(t)\,\vec{j} + 4\vec{k}$

$$T = \frac{-3\sin(t)\,\vec{i} + 3\cos(t)\,\vec{j} + 4\vec{k}}{\|-3\sin(t)\,\vec{i} + 3\cos(t)\,\vec{j} + 4\vec{k}\|}$$

$P(0, 3, 2\pi)$

separate points of same!




After a) correction:

$$P(0,3,2\pi) \Rightarrow \begin{cases} 3\cos(t) = 0 & \Rightarrow t = \{\pi/2,\ 3\pi/2\} \\ 3\sin(t) = 3 & \Rightarrow t = \{\pi/2,\ 3\pi/2\} \\ 4t = 2\pi & \Rightarrow t = \frac{\pi}{2} \end{cases}$$

$\Rightarrow t = \pi/2$

$$\hat{T} = \frac{-3\sin(t)\,\vec{i} + 3\cos(t)\,\vec{j} + 4\vec{k}}{\sqrt{(-3\sin(t))^2 + (3\cos(t))^2 + (4)^2}}$$

$$= \frac{-3\sin(t)\,\vec{i} + 3\cos(t)\,\vec{j} + 4\vec{k}}{\sqrt{9\sin^2 + 9\cos^2 + 16}}$$

$$= \frac{-3\sin(t)\,\vec{i} + 3\cos(t)\,\vec{j} + 4\vec{k}}{\sqrt{9+16} \Rightarrow 5}$$

$$= \frac{(-3,\ 3,\ 4)}{5}$$

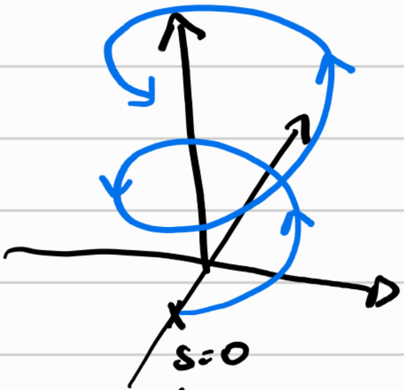
5) $r(t) = \cos(2\pi t)\vec{i} + \sin(2\pi t)\vec{j} + (2\pi t)\vec{k}$

a) $\frac{dr}{dt} = -2\pi\sin(2\pi t)\vec{i} + 2\pi\cos(2\pi t)\vec{j}$
$+ 2\pi\vec{k}$ ✓

$\frac{d^2r}{dt^2} = -4\pi^2\cos(2\pi t)\vec{i} - 4\pi^2\cos(2\pi t)\vec{j}$ ✓

$\left|\frac{dr}{dt}\right| = \sqrt{4\pi^2 \cancel{\sin^2} + 4\pi^2\cancel{\cos^2} + 4\pi^2}$

$= 2\pi\sqrt{2}$ ✓

b) 


~~$s(t) = \sum_{i=0}^{n}(r(t) - r(t+\delta t))$~~

$\underbrace{\frac{ds}{dt} = 2\sqrt{2}\pi}$ $\Rightarrow s = 2\sqrt{2}\pi t$

$\frac{ds}{dt} = \left|\frac{dr}{dt}\right|$

c) $s = 2\sqrt{2}\pi t \Rightarrow t = \frac{s}{2\sqrt{2}\pi}$
$r(t) = \cos(2\pi t)\vec{i} + \sin(2\pi t)\vec{j} + (2\pi t)\vec{k}$

$r(s) = \cos(s/\sqrt{2})\vec{i} + \sin(s/\sqrt{2})\vec{j} + (s/\sqrt{2})\vec{k}$ ✓

d) $\hat{T}(t) = \frac{\frac{dr}{dt}}{\left|\frac{dr}{dt}\right|} = \frac{-\cancel{2\pi}\sin(2\pi t)\vec{i} + \cancel{2\pi}\cos(2\pi t)\vec{j} + \cancel{2}\pi\vec{k}}{2\pi\sqrt{2}}$

$\sqrt{2}\,(\sin(2\pi t))$

$$= \sqrt{2}\begin{pmatrix} \sin(2\pi t) \\ \cos(2\pi t) \\ 1 \end{pmatrix}$$ ✓

$$\hat{T}(s) = \frac{\frac{dr(s)}{ds}}{\left|\frac{dr(s)}{ds}\right|}$$

$$r(s) = \cos\left(s/\sqrt{2}\right)\vec{i} + \sin\left(s/\sqrt{2}\right)\vec{j} + \left(s/\sqrt{2}\right)\vec{k}$$

$$\frac{dr(s)}{ds} = \frac{-\sin\left(s/\sqrt{2}\right)\vec{i} + \cos\left(s/\sqrt{2}\right)\vec{j} + 1\vec{k}}{\sqrt{2}}$$

$$\left|\frac{dr(s)}{ds}\right| = \sqrt{\frac{\cancel{\sin^2 + \cos^2} + 1}{2}} = 1$$

$$\hat{T}(s) = \frac{-\sin\left(s/\sqrt{2}\right)\vec{i} + \cos\left(s/\sqrt{2}\right)\vec{j} + 1\vec{k}}{\sqrt{2}}$$ ✓

—needed to look at correct° 4 this

6) $F = m(\ddot{x}\vec{i} + \ddot{y}\vec{j} + \ddot{z}\vec{k})$, $t = 0$

$\ddot{x} = 0$, $\ddot{y} = qE/m$, $\ddot{z} = 0$

$x = 0$ $\quad y = 0$ $\quad z = 0$

$\dot{x} = v_0$ $\quad \dot{y} = 0$ $\quad \dot{z} = 0$

$\ddot{x} = \dot{v}_0$ $\quad \ddot{y} = 0$ $\quad \ddot{z} = 0$

What?

$r(t) = ?$

$r(t) = V_0 t \vec{i} + 0\vec{j} + 0\vec{k}$

$= t \begin{pmatrix} V_0 \\ 0 \\ 0 \end{pmatrix} \in$ set set ?

y, z α const
but then
$y \neq 0$ or $\dot{y} \neq 0$
$z \neq 0$ $\dot{z} \neq 0$

$F = ma^2 \Rightarrow F = m \frac{d^2 r}{dt^2}$

$(\ddot{x}\vec{i} + \ddot{y}\vec{j} + \ddot{z}\vec{k}) = \frac{d^2 r}{dt^2}$

$\dot{x}\vec{i} + \dot{y}\vec{j} + \dot{z}\vec{k} = \frac{dr}{dt}$

$x\vec{i} + y\vec{j} + z\vec{k} = r$

$x = 0 \quad y = 0 \quad z = 0 \Rightarrow r = 0$

opt° 2

$\ddot{x} = 0, \ \ddot{y} = qE/m, \ \ddot{z} = 0$

$\dot{x} = V_0, \ x = V_0 t + \cancel{c} = V_0 t$

$\ddot{y} = qE/m \qquad \dot{y} = \frac{qEt}{m} + \cancel{c}^0 \qquad y = \frac{qEt^2}{2m} + \cancel{ct}^0 + \cancel{E}^0$

$\ddot{z} = 0, \ \dot{z} = 0 \quad z = 0$

$\bar{r}(t) = V_0 t \vec{i} + \frac{qEt^2}{2m}\vec{j}$