

Q Question

A one-dimensional solid contains atoms separated by the lattice constant $a = 2 \text{ \AA}$. The lowest two energy bands are given (in eV) by $\epsilon(k) = A + B \cos(ka)$ and $\epsilon(k) = C + D \cos(ka)$ with $A = 2 \text{ eV}$, $B = 2 \text{ eV}$, $C = 9 \text{ eV}$ and $D = -4 \text{ eV}$.

(a) Assume the solid contains one monovalent atom per unit cell.

- Sketch the energy bands for wave vectors in the Brillouin zone, and clearly indicate the occupied portion of the energy bands.
- If the material is a metal, calculate the group velocity (in m/s) of the electrons at the Fermi energy.

• If the material is an insulator, calculate (in Hz) the lowest frequency of light incident on the material that will be absorbed.

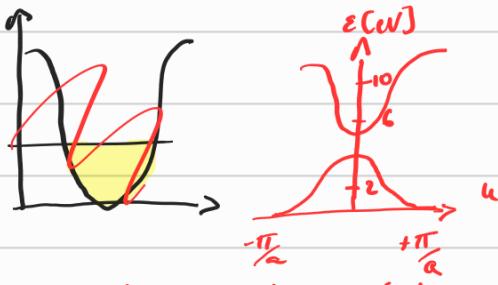
• If the material is a semiconductor, calculate the effective mass (as a multiple of the free electron mass m_e) of the conduction electrons.

(b) Repeat part (i) and (ii), this time for the case of the solid containing one divalent atom per unit cell.

[8]

$\epsilon = 1.60 \times 10^{-19} \text{ C}$, $c = 3.00 \times 10^8 \text{ m/s}$, $\hbar = 1.05 \times 10^{-34} \text{ Js}$, $m_e = 9.11 \times 10^{-31} \text{ kg}$.

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Filled band holds $2e/\text{unit cell}$: 1 monovalent atom
half-filled band

$$\text{group velocity} = \frac{1}{\hbar} \frac{d\epsilon}{dk}$$

At $k_F, k_F \approx \frac{\pi}{2a}$

$$v = \pm 6.0 \times 10^5 \text{ m/s}$$

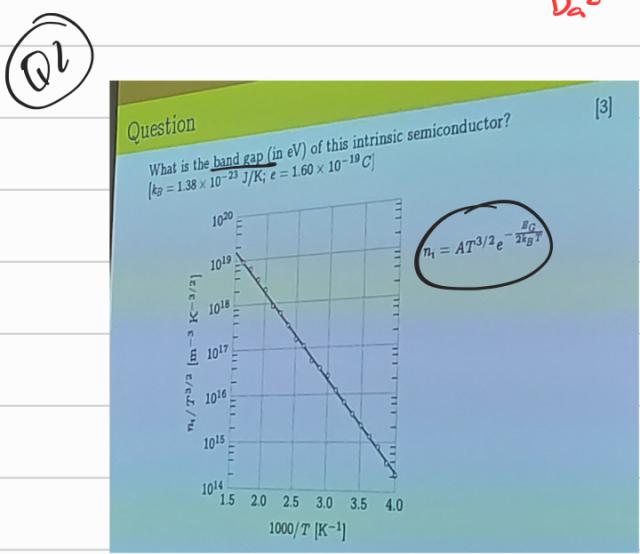
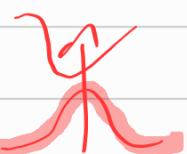
b) Answer (b)

A filled band holds 2 electrons/unit cell. 1 divalent atom \Rightarrow filled band.
1 eV gap between highest occupied and lowest unoccupied state: semiconductor

$$m^* = \frac{k^2}{(\frac{d^2\epsilon}{dk^2})} = -\Delta \epsilon \cos ka \quad \text{for upper band}$$

Semiconductor at bottom conductive band

$$k_F \text{ lower band}, m^* = \frac{-k^2}{D \epsilon_m} = \frac{-k^2}{D a^2 m_e} \quad m_e = 0.47 m_e$$



a) band gap

Answer

For an intrinsic semiconductor, $n_i = AT^{3/2} e^{-\frac{E_G}{2k_B T}}$. So

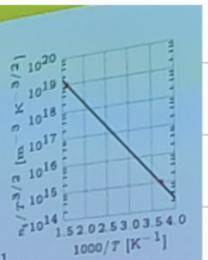
$$\ln(n_i/T^{3/2}) = \ln A - \frac{E_G}{2k_B T}$$

Estimate two points on the straight line relationship, e.g.

$$\ln(n_i/T^{3/2}) = \begin{cases} \ln 10^{19} & \text{for } 1000/T = 1.6 \text{ K}^{-1} \\ \ln 10^{15} & \text{for } 1000/T = 3.7 \text{ K}^{-1} \end{cases}$$

Then

$$\ln 10^{19} = \ln A - \frac{E_G}{2k_B} \frac{1.6}{1000}$$



$$\ln 10^{15} = \ln A - \frac{E_G}{2k_B} \frac{3.7}{1000}$$

$$\text{subtract } \ln 10^4 = \frac{E_G}{2000k_B} (3.7 - 1.6) = \frac{2.1E_G}{2000k_B}.$$

$$E_I = \frac{2000k_B \ln 10^4}{m} = 0.76 \text{ eV}$$

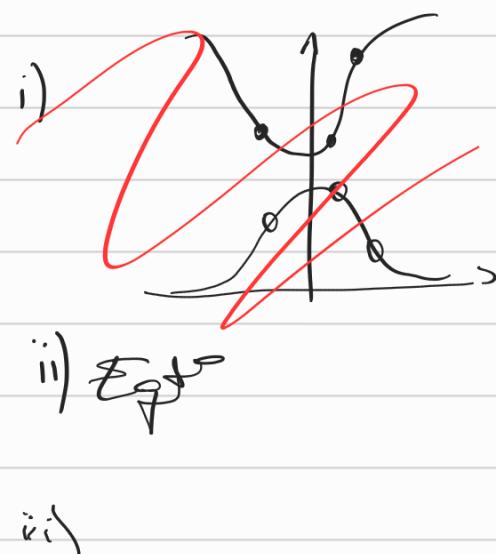
(Q3)

Question

A semiconductor has $E_G = 0.76 \text{ eV}$, $m_e^*/m = 0.02$, $m_h^*/m = 0.40$ and a relative permittivity $\epsilon_r = 20.0$.

It is doped with acceptor impurities at a concentration level of $n_A = 2.0 \times 10^{15}/\text{cm}^3$.

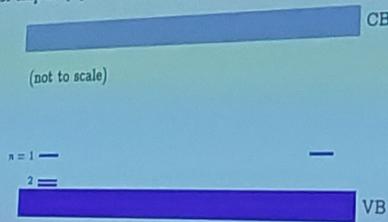
- (i) Indicate on a sketch the location of the impurity levels relative to the valence and conduction bands of the semiconductor. [2]
- (ii) Using the hydrogenic model of dopants, estimate the ionisation energy of the dopants. [2]
- (iii) Estimate the number density of charge carriers in the extrinsic semiconductor at room temperature. [1]



Answer (i)

- (i) Indicate on a sketch the location of the impurity levels relative to the valence and conduction bands of the semiconductor. [2]

An acceptor impurity provides holes.



$$n_A \approx 2.0 \times 10^{15}/\text{cm}^3$$

Answer (ii)

The semiconductor [$E_G = 0.76 \text{ eV}$, $m_e^*/m = 0.02$, $m_h^*/m = 0.40$] has a relative permittivity $\epsilon_r = 20.0$.

It is now doped with acceptor impurities at a concentration level of $n_A = 2.0 \times 10^{15}/\text{cm}^3$.

- (ii) Using the hydrogenic model of dopants, estimate the ionisation energy of the dopants. [2]

$$\text{From notes, } \epsilon_1^A = -\frac{1}{\epsilon_r^2} \left(\frac{m_h^*}{m} \right) \times \frac{13.6}{n^2} \text{ eV.}$$

$$\text{Ionisation energy} = -\epsilon_1^A = \frac{1}{20^2} \times 0.4 \times 13.6 = 0.0136 \text{ eV} = 13.6 \text{ meV}$$

