P6: Experiments with ultrasonic waves.

Aims

In this experiment, some aspects of wave theory are studied using ultrasonic waves. You will investigate some effects arising from acoustic waves propagating in a finite material with a thickness of the order of magnitude of the wavelength.

Safety

Take care not to spill water on electrical equipment or to create slip hazards. Always mop up any spills.

→ displacement

→ propagatio

1. Introduction

Ultrasonic waves in a fluid travel as longitudinal pressure waves in which the particle displacement is in the same direction as the propagation direction. The phase velocity is related to the density and the compressional elastic modulus of the fluid. $V_p \propto$ density $\propto$ elastic modulus

A solid medium can support shear stresses and thus ultrasonic waves in solids can have particle displacement components both in the direction of propagation and in a plane normal to the propagation direction. In an infinite homogeneous solid only two types of wave propagate; longitudinal and shear waves. The phase velocity of the former is usually several times larger than that of the latter. In this experiment, you will investigate effects arising from acoustic waves propagating in a finite material with a thickness of the order of magnitude of the wavelength.

Note that you can export data from the oscilloscope via a memory stick for analysis or to include as an illustration in your report.

?, same ?

2. Basic Wave Theory

2.1 General waves

Consider a wave propagating in the x-direction with an associated amplitude A. We can write this in its complex form as:

$$\Psi(x,t) = Ae^{i(\omega t - Kx)},$$

where ω is the angular frequency of the wave and the wave number $K = 2\pi/\lambda$, where λ is the wavelength. If we write this now as a wave propagating in three-dimensions then the wave number will become a wavevector that gives the direction of the motion of the wavefront in space. Also, the amplitude must become a vector as it represents some form of disturbance to an equilibrium state that will have amplitude and direction. Thus, our general wave is now written as:

$$\vec{\Psi}(r,t) = \vec{A}e^{i(\omega t - \vec{K}.\vec{r})},$$

where $\vec{K} = k_x\vec{i} + k_y\vec{j} + k_z\vec{k}$ is the wavevector and $r = x\vec{i} + y\vec{j} + z\vec{k}$ is the position vector. In a transverse wave, the direction of the amplitude will always be perpendicular to the wavevector whilst in a longitudinal wave the amplitude is parallel to the wavevector.

2.2 Acoustic Waves

There are two standard ways to represent an acoustic wave – either as a pressure wave or as a wave of particle velocities. We can write the pressure wave as:

$$\vec{p} = \vec{A}e^{i(\omega t - \vec{K}.\vec{r})},$$

and there is a particle velocity wave associated with this given by:

$$\vec{u} = \frac{\vec{A}}{\rho_0 c} e^{i(\omega t - \vec{K}.\vec{r})},$$

where ρ_0 is the equilibrium value of the mass density and c is the phase velocity of the wave. The factor, $\rho_0 c$, is the characteristic acoustic impedance of the material relating the amplitude of the pressure and velocity waves. The acoustic reflectivity at an interface between two materials depends on the impedance mismatch at the interface.

The velocity of acoustic waves in fluids and solids depends on the elastic moduli of the material (Young's modulus, bulk modulus, shear modulus). There is not one simple expression for this but they all have the form:

$$c^2 = \frac{f(\text{moduli})}{\rho_0},$$

where $f(\text{moduli})$ is the appropriate elastic modulus or a linear addition of them. Note also that it is always the case for the wave that $c = \omega/K$. So we can see that as a wave, which always has constant frequency, passes into a new material with different acoustic velocities then the wave number (magnitude of the wavevector) will change in the new material.

Figure 1 shows an acoustic wave in a liquid being transmitted at the boundary of a solid. There will also be a reflected wave that we are not considering here. The acoustic wave in the liquid must be a longitudinal wave as that is the only acoustic wave that can propagate through a liquid. The wave has the wavevector $\vec{K}_1$ and the amplitude of the wave pressure, $\vec{A}$, is parallel to the wavevector. However, as this wave is not striking the surface at normal incidence, there are components of the pressure that are both along the interface and perpendicular to the interface. These produce a longitudinal wave and a shear wave inside the material with wavevectors, $\vec{K}_2$ and $\vec{K}_3$.

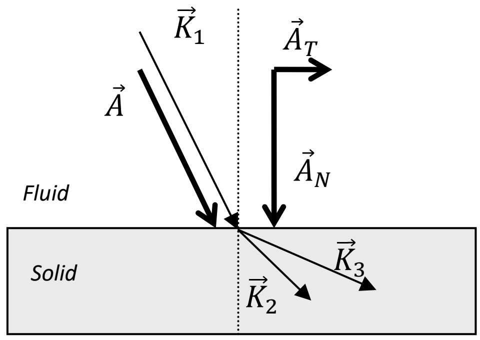


Figure 1. Schematic of acoustic transmission. The bold lines (→) show the pressure amplitude and its components compared to the interface.

As the wave velocity for the waves inside the solid is higher than that in the liquid the waves have been refracted away from the normal to the surface and the wave number has been reduced for both waves.

The full details of the transmission and reflection of waves at the boundary are found by considering the boundary conditions which are the requirement for continuity of pressure and continuity of the normal component of velocity at the interface. The full details of the proof are not important for this experiment. However, the effect of shear waves being produced by the incidence of longitudinal waves on the interface is worth noting. The equivalent does not happen for electromagnetic waves at an interface. This is studied in the second year unit Electromagnetism 1, where you will see that the two polarisations of light do not mix or generate each other at an optical interface.

For waves, the intensity – the energy transported per unit area per second – is proportional to the square of the wave amplitude. However, the voltages measured in this experiment at the transducers are proportional to the amplitude of the waves.

When working with a large range of wave amplitude, it is usual to consider the ratio of A_1 (measured amplitude) and A_0 (reference amplitude) and working in decibels allows quick conversion from an amplitude ratio to an intensity ratio. So, for the intensity transmissivity (transmission) the following definition is used to give a consistent definition of transmission in decibels:

$$T_{dB} = 10\log_{10}\left(\frac{I_1}{I_0}\right) = 10\log_{10}\left(\frac{{A_1}^2}{{A_0}^2}\right) = 20\log_{10}\left(\frac{A_1}{A_0}\right).$$

3. Part A: Familiarisation with Equipment

Figure 2 shows the arrangement of two ultrasonic transducers provided so that the transmission loss through solid plates may be measured as functions of angle of incidence and frequency. The transducers are based on the piezoelectric effect; an exciting voltage produces motion and hence radiation of sound from one transducer; incident pressure produces a potential and thus the other transducer acts as a sound receiver. The voltage measured is proportional to the amplitude of the wave.

15 cm

100mv

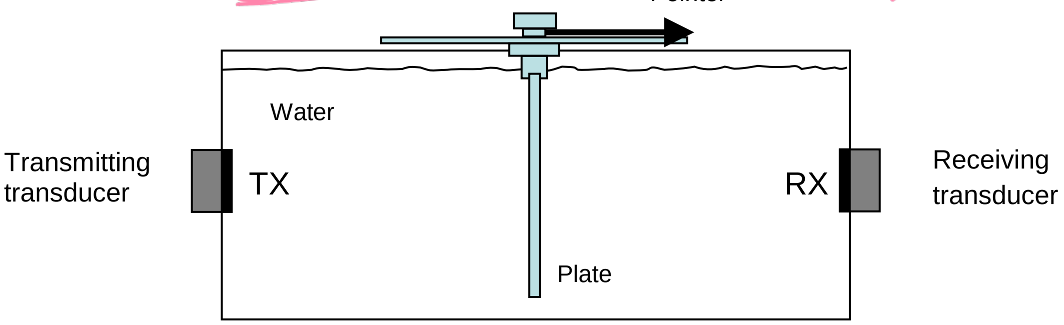


Figure 2. Schematic diagram of apparatus.

Figure 3 shows the setup of the equipment.

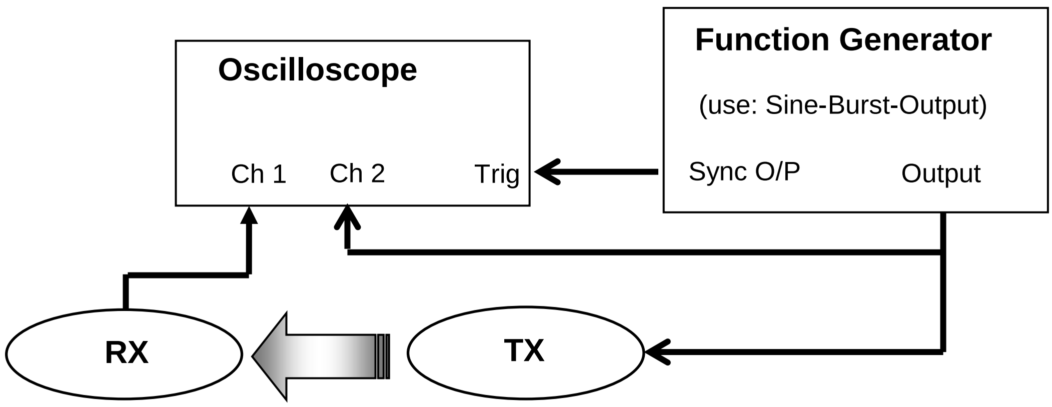


Figure 3. Details of pulse generation and detection system. TX = transmitting transducer, RX = receiving transducer.

The transmitting transducer (TX) is driven with "burst" pulses that are a set number of sine wave cycles. The received signal (RX) is displayed on the oscilloscope. Solid plates may be inserted in the water path between the two transducers and orientated for particular angles of incidence.

On the Agilent 33210A function generator:

To set the properties of the sine wave driving the transducers:

- Press "Sine" button – highlight "freq" on leftmost dark button below the display – then adjust frequency as desired. You can enter a frequency or decide which digit of the frequency you wish to adjust using the left/right buttons below the control knob.

To set the properties of the sine-wave burst:

- Press "Burst" – highlight "# cycles" on the second dark button under display – set desired number of cycles.

SET EQUIPMENT UP

Set up the equipment as follows:

i. Connect the waveform generator output into the scope instead of transducer TX with the following settings: frequency 1.50 MHz and peak-to-peak amplitude 10V (the maximum peak-to-peak voltage). Pulse length 100 cycles and burst period 1.5 ms.

ii. Observe the scope display and vary the settings of signal generator and scope to become familiar with their capabilities.

iii. Return the scope and waveform generator to the above settings. Connect transducers TX and RX as in Figures 2 and 3.

iv. Observe the scope display of receiver, RX. You should see an initial electrical pick-up from the voltage pulse supplied to TX, followed by an acoustic signal received about 100µs later, which has travelled through the water path. This is followed (at max gain for the scope) by about four visible multiple echoes from propagation up and down the tank.

v. Calculate the velocity of the pulse in the water using the time values of the multiple pulses.

$v = \delta / \lambda =$

vi. Change the pulse spacing (using the burst period button in the Burst display) and note that if it is too short the signals from previous transmissions interfere with the present transmission and overlap occurs.

vii. Adjust the pulse length by varying the number of cycles. Observe that the maximum useful pulse length is about 80 µs.

4. Part B: Simulation of Semi-infinite interface

4.1 Theory: Two Semi-Infinite Media

At an interface between two semi-infinite media, the boundary conditions lead to conversions of wave types and also to the possibility of what are termed surface waves. Figure 4 shows a plane longitudinal wave in a fluid (in this case water) incident on a plane boundary between a fluid and a solid. There is, in general, a reflected longitudinal wave, a transmitted longitudinal wave and a transmitted shear wave.

!

There is a 'Snell's Law' relation between the angle of incidence and the angles of refraction:

$$\frac{V_W}{\sin(\theta_i)} = \frac{V_{L_p}}{\sin\left(\theta_{L_p}\right)} = \frac{V_{S_p}}{\sin\left(\theta_{S_p}\right)}, \tag{1}$$

where V_W, V_{L_p} and V_{S_p} are the velocity in water, and the longitudinal and shear velocities in the solid (plate material) respectively.

40°

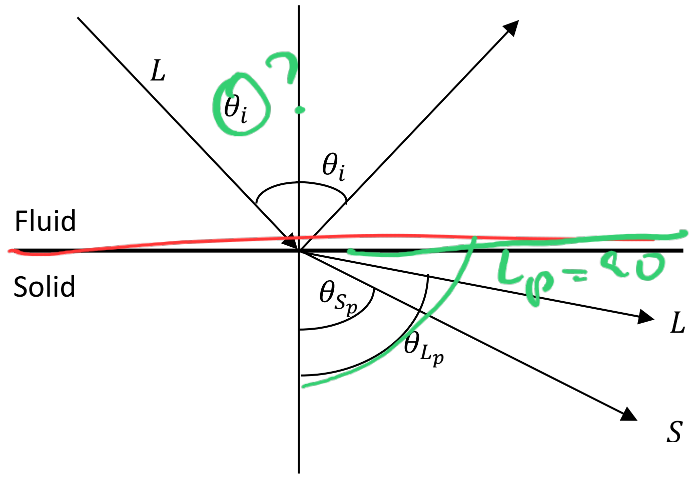


Figure 4. Reflection and transmission at a fluid/solid interface

Since $V_W < V_{L_p}$, there exists a critical angle of incidence for the longitudinal wave. For angles beyond this critical angle there is no transmitted longitudinal wave but rather an inhomogeneous shear wave which travels along the interface, its associated particle displacement amplitude decaying rapidly with increasing distance from the interface. If $V_{S_p} > V_W$ then there will also be an associated critical angle for the shear wave.

Figures 5 and 6 show the reflection coefficient (expressed in decibels [1]) for a water/brass and a water/Perspex interface as a function of the incident angle of a plane wave in water. The reflection coefficient, R, is the ratio of the reflected amplitude to the incident amplitude. These reflection and transmission coefficients at the interface between two semi-infinite media are independent of frequency as we are assuming velocities that are independent of frequency.

BEFORE TODO LABS

You can see that the calculated reflection coefficients approach 0dB (a value of 1) at some angles. What will the transmission coefficient be at these angles?

12

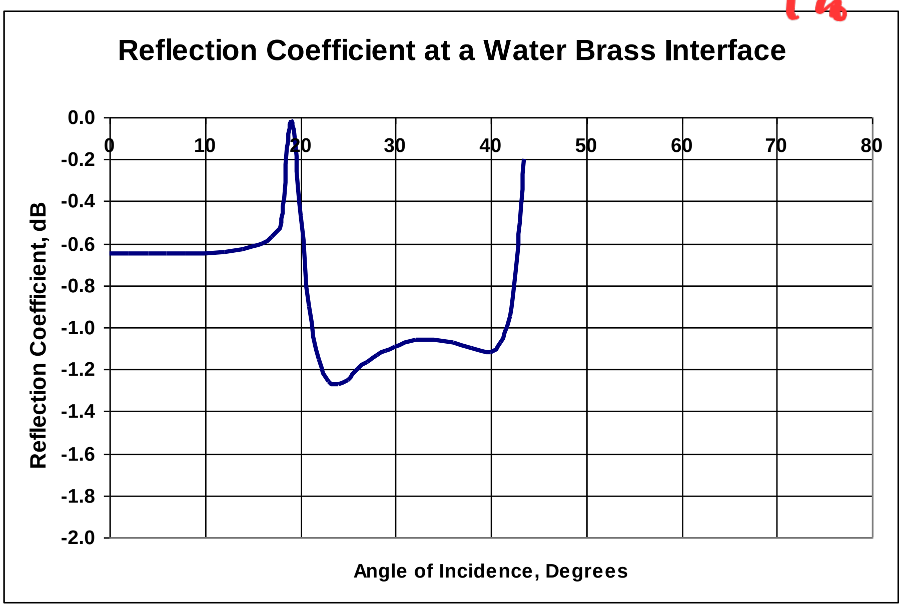


Figure 5. Reflection coefficient for acoustic plane waves at a water/brass interface

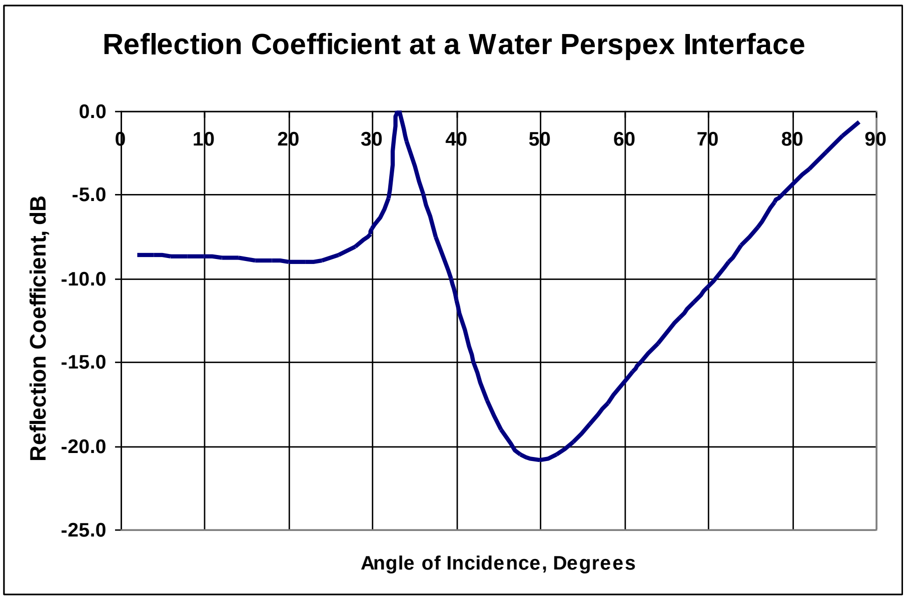


Figure 6. Reflection coefficient for acoustic plane waves at a water/Perspex interface

Intro section

4.2 Measurements Using Very Short Pulses

This part of the experiment simulates transmission at an interface between two semi-infinite media. If the duration of the transmitted pulse is short enough (e.g. 1-2 cycles of sine wave), the reflections from the two faces of the plate can be time separated and hence interference effects do not occur between the wave of successive pulses.

Set the function generator to 2.15 MHz with 1 sine cycle.

Draw set up

Brass

a) Insert the thick (8.04 mm) brass plate and orientate it at normal incidence. You can set for normal incidence using the pointer – but also observe the effect on the set of reflections as you move close to and through normal incidence to check the correct zero position.

b) Sketch the multiple reflections produced within the plate. Use the time spacing of these multiples to obtain the velocity of longitudinal waves in brass.

c) Calculate the critical angle for longitudinal waves at a brass/water interface using Equation 1 and your knowledge of the meaning of critical angle. (Take the water velocity as 1490 ms^{-1}.)

d) Rotate the thick brass plate to an angle beyond this critical angle. Observe how the pulse from the longitudinal wave disappears as you move through the critical angle.

e) Sketch the received signal beyond the critical angle. Its appearance arises due to a shear wave traversing the plate. You are going to obtain the shear wave velocity in the plate by noting the arrival time of this signal in comparison with the arrival time in the absence of a plate. This is called the replacement technique; it is illustrated in Figure 7 and yields:

$$V_S = V_W \left\{ \left(\frac{\Delta t V_W}{d} + \cos\theta_i \right)^2 + \sin^2\theta_i \right\}^{-\frac{1}{2}}, \tag{2}$$

where Δt is positive if $V_S < V_W$ and negative otherwise. The method for obtaining this equation is shown in Appendix A. Greater delay times are obtained at higher angles. Pay attention to the direction of the time shift when you insert the plate.

f) Measure the delay time at a few angles of incidence and hence calculate the shear velocity for acoustic waves in brass. ??

change material

Perspex

g) Now obtain the longitudinal velocity in Perspex (use the thick 5.86 mm sheet). The contrast in the acoustic impedance of water and Perspex is much smaller than for a brass/water interface, hence multiple internal reflections will not be seen. Thus, you must use the replacement technique at normal incidence to find the velocity.

h) Calculate the critical angle for longitudinal waves at the water/Perspex interface.

i) Rotate the thick Perspex sheet to an angle beyond this critical angle and thus obtain the shear wave velocity using Equation 2.

Note: There is no critical angle for shear waves at a water/Perspex interface.

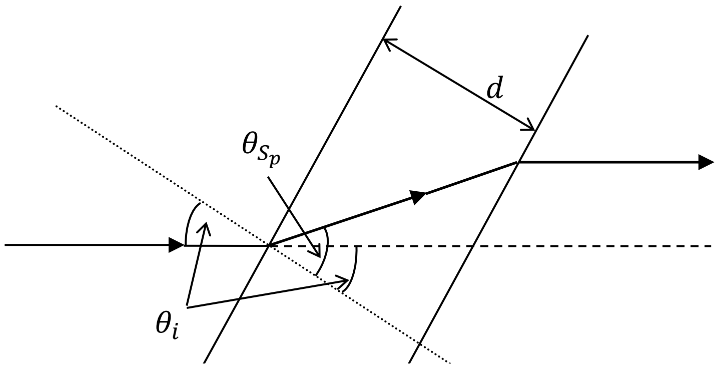


Figure 7. Replacement technique. This diagram shows the basis for calculating the shear wave velocity in the plate given the travel time of a short pulse with and without the plate (Δt). The method can also be used to calculate longitudinal and shear velocity in Perspex.

5. Part C: Transmission at normal incidence for a finite thickness plate

5.1 Theory: Shear Waves

In reality no solid is of infinite thickness. The reflection or transmission coefficients at the interface between a fluid and a *finite* thickness solid placed in the fluid is a complicated function of the frequency, plate thickness and angle of incidence [2].

At normal incidence, shear waves are not generated and the transmission through the plate is as a pure longitudinal wave. The transmission coefficient (T, the ratio of transmitted to incident acoustic intensity) for any thickness d at normal incidence is approximately given by:

$$T \approx \frac{1}{1 + \frac{1}{4}\left(\frac{\rho V_{L_p}}{\rho_W V_W}\right)^2 \sin^2\left(K_{L_p} d\right)}, \tag{3}$$

when sin | → min
sin ≈ 0 → max

where ρV_{L_p} is the acoustic impedance of the solid plate where ρ is the density of the plate, $\rho_W V_W$ is the acoustic impedance of the medium (in this case water), $\frac{\rho V_{L_p}}{\rho_W V_W} \gg 1$ and $K_{L_p} = 2\pi \frac{f}{V_{L_p}}$ where f is the frequency of the wave.

From this we can see that for maximum transmission, $T_{\max}$, we need $\sin^2\left(K_{L_p} d\right) = 0$ and then the transmission is ~1. This occurs when $\sin\left(K_{L_p} d\right) = 0$ which holds when $K_{L_p} d = n\pi$ where n is an integer.

BEFORE LABS TODO

Show that this requires $d = \frac{n\lambda}{2}$ or $f = n\frac{V_{L_p}}{2d}$.

$u_{Lp} = 2\pi \, n \, U_{Lp} / 2d \, / \, U_{Lp}$

$u_{Lp} = \pi n$

This shows the resonance condition for standing waves in the solid plate. The optical equivalent of this is used in the Fabry-Pérot interferometer in the Zeeman effect experiment.

Minimum transmission will occur when $\sin^2\left(K_{L_p} d\right) = 1$ giving the condition $d = (2n+1)\frac{\lambda}{4}$. Hence, we find the minimum transmission using:

$$T_{\min} = \frac{1}{1+\left(\frac{\rho V_{L_p}}{2\rho_W V_W}\right)^2}. \quad (4)$$

From here we can work out the modulat depth: see how & why our comp graph is wrong + look loss

5.2 Measurements

This part of the experiment simulates continuous wave signals in a finite thickness plate. By making the pulse duration longer than the two way travel time between the plate faces, the signal transmission through the plate as a result of interference in the plate can be measured.

To measure the effect of the interference in the plate without effects from other resonances in the experiment, **measure the amplitude of the 7th cycle of the transmitted pulse**.

- Use the thin (1.77 mm) brass plate
- Turn the pulse length to 75 cycles at 1.00 MHz.

details set-up

By following the instructions below, you will investigate the transmission loss through the plate (the ratio of received signal intensity with and without the plate) at normal incidence as a function of frequency and compare with the expression given above in Equation 3.

Manually scan the frequency between 1.0 and 3.0 MHz using the knob on the waveform generator. Observe the general features of the frequency dependence of the transmission of the plate. You will measure the transmission of the plate across these resonances.

- Remove the brass plate.

← 0.85

The efficiency of the transducers is frequency dependent so to find the signal amplitude through water alone; you must first measure the received amplitude as a function of frequency with no plate present. It is probably easier to do this first using a series of frequencies – at say 0.2 MHz spacing - to find the transducer response, rather than moving the plate in and out of the water at each frequency during a transmission experiment, which is likely to change the angle of incidence.

The transducer response is a smooth function so you can make a fit to it and hence calibrate for all frequencies. Its maximum efficiency is at 2.15 MHz, which is why this frequency was used earlier. If you subsequently investigate frequencies not on your initial grid, you can calculate the transmitted amplitude through water using your calibration curve or by interpolating between measured points.

1) • Measure the signal amplitude through water alone against frequency
2) • Replace the thin brass plate
3) • Measure transmission through plate

Take measurements between 1.00 and 3.00 MHz in steps of 0.1 MHz, adding more measurements, say at 0.05 MHz separation, around interesting frequencies.

- Plot the transmission coefficient as a function of frequency. Remember to plot your results in decibels as $20\log_{10}\left(\frac{V_{RX}}{V_{TX}}\right)$.

Explain the result in terms of the physical parameters of the brass plate.

& why?? ← TODO

add details waves & plate + PUT UNCERTNS

6. Part D: Lamb Waves in Finite Thickness Plate

6.1 Theory: Non-normal Incidence

As the angle of incidence moves away from normal both longitudinal and shear waves are generated at each interface of the plate with the water as indicated in Figure 8. Reflections at bottom and top surface produce a set of waves and a complicated interference takes place within the plate. This forms a consistent and ongoing self-supporting result, which is a Lamb wave [3,4] which propagates in the plate parallel to the plate faces.

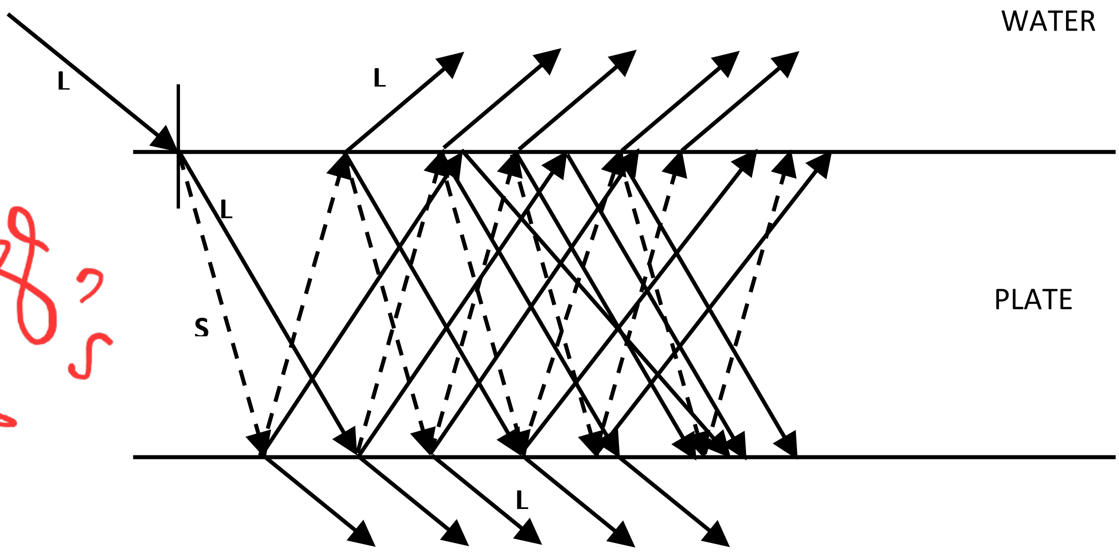


Figure 8. An indication of the effect of a longitudinal wave in water incident at an oblique angle on a solid plate.

The velocity of the Lamb wave is a strong function of the angle of incidence of the exciting wave in the water, and the product of the frequency and plate thickness. Additionally the elastic moduli of the plate material influence the form of the dependence.
For certain angles of incidence, constructive interference of the waves inside the plate leads to a resonance-like situation and the transmission through the plate is a maximum.
The vibration pattern of the plate surface allows two classes to be defined namely symmetric (s) and antisymmetric (a) modes, as depicted in Figure 9.

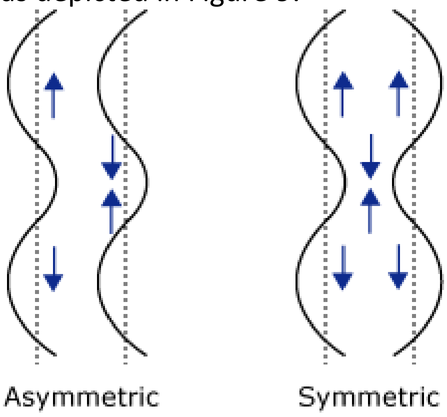


Figure 9 Vibration patterns of plate surfaces due to Lamb wave propagation (A) symmetric (B) antisymmetric. (Image from www.ndt-ed.org.)

The relation between the angle of incidence of the wave in water and the product of plate thickness and frequency ($f.d$) required to excite a Lamb wave, is shown in Figure 10 for brass. The plate measured in this experiment has an $f.d$ product of 2.3MHz mm.

When the angle of incidence, the frequency and plate thickness are such as to excite a Lamb mode [3], the transmission coefficient of the plate approaches unity. This may be seen by studying Figure 11 which shows transmission loss (coefficient) expressed in decibels versus angle of incidence for brass plates in water.

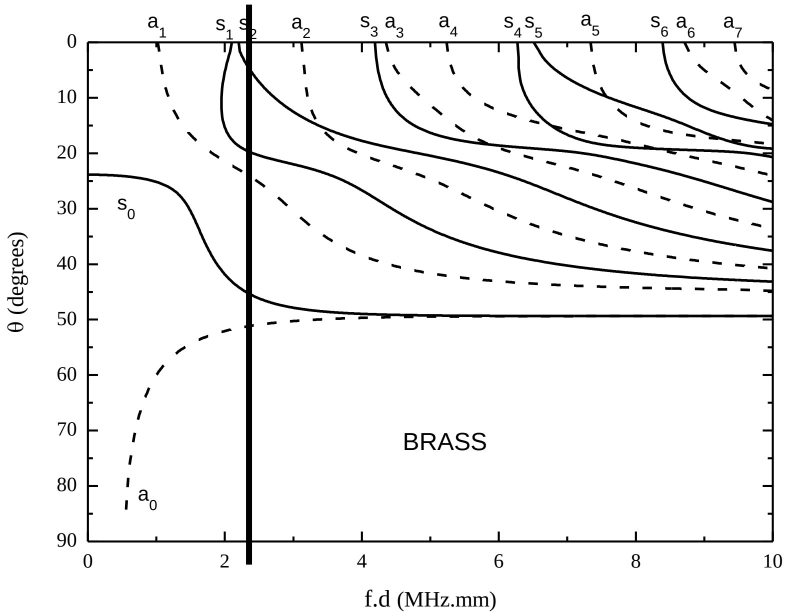
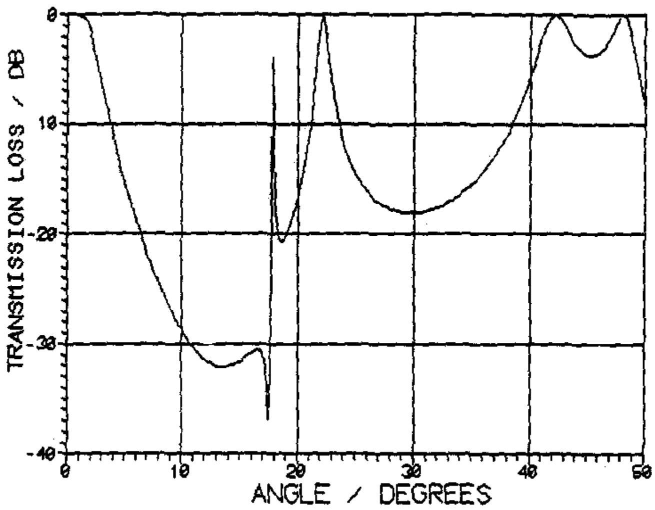


Figure 10. Condition for Lamb waves in terms of angle of incidence θ and the $f.d$ product in brass. a_i and s_i indicate antisymmetric and symmetric Lamb modes. The bold vertical line indicates an $f.d$ product of 2.3 MHz mm, relevant to the brass plate used in measurement. (Plot by Simon Dodd).



• ∞ plate
• diff f.d

*Figure 11. Transmission loss (in decibels, dB) versus angle of incidence θ for an $f.d$ product of 1.3MHz*1.77mm (2.3MHz.mm)*

6.2 Measurements

This part of the experiment simulates continuous wave signals in a finite thickness plate. By making the pulse duration many times longer than the two way travel time between the plate faces a steady state amplitude region will become evident as the multiply reflected waves superpose. Measurements in such a region give results consistent with CW theory.

Check that the pulse length is 125 cycles

a) Measure the angular dependence of the transmission loss, for angles of incidence from 0° to 60° at 3° intervals, for the thin (1.77mm) brass plate at its half wavelength frequency (1.32MHz). Increase the number of measurements around sharp features in the data. Plot your results in decibels.

Note: it is perfectly acceptable to try to locate maxima and minima by "tuning" to that angle by hand as well as by measuring points on the grid.

b) Compare the angles of maximum transmission (T~1, or 0 dB loss) to see if they coincide with the angles for Lamb mode excitation as given in Figures 10 and 11. Comment on your results.

do not join points in report for these graphs!

END

References

[1] **Revision: Decibels** [http://en.wikipedia.org/wiki/Decibels] When working with intensities and a large variation of field amplitude, it is usual to consider the ratio of the squares of A_1 (measured amplitude) and A_0 (reference amplitude). For waves, the power is proportional to the square of amplitude, and for the two decibel formulations to give the same result the following definition is used: $R_{dB} = 10\log_{10}\left(\frac{{A_1}^2}{{A_0}^2}\right) = 20\log_{10}\left(\frac{A_1}{A_0}\right)$.

[2] Theory for this is given in *Fundamentals of Acoustics*, Fourth Edition, Kinsler, Frey, Coppens and Sanders. (Wiley, New York, 2000) Section 6.3 p152. The solid plate can be considered to be equivalent to a liquid when only longitudinal waves are being generated.

[3] *Surface Waves in Acoustics*, Warren, WP.Mason and R.N.Thurston, Physical Acoustics – Principles and Methods, Vol. X, p17 . (Academic Press, New York: London. 1973)

[4] Named after Sir Horace Lamb FRS (27 November 1849 – 4 December 1934). He was a British applied mathematician and author of several influential texts on classical physics, among them Hydrodynamics (1879) and Dynamical Theory of Sound (1910). He first analysed Lamb waves. [http://en.wikipedia.org/wiki/Horace_Lamb]

Appendix A: Path lengths and time difference for replacement technique.

The velocity of the shear wave is found using the replacement technique by considering the path length and velocity of the wave going through the plate compared to through the water. With reference to the figure below, the path of the wave that is a shear wave in the plate is TABR and the path of the wave through the water is TACR'. The distances to the receiving transducer BR and CR' are the same. So, the difference in path length is between paths AB and AC.

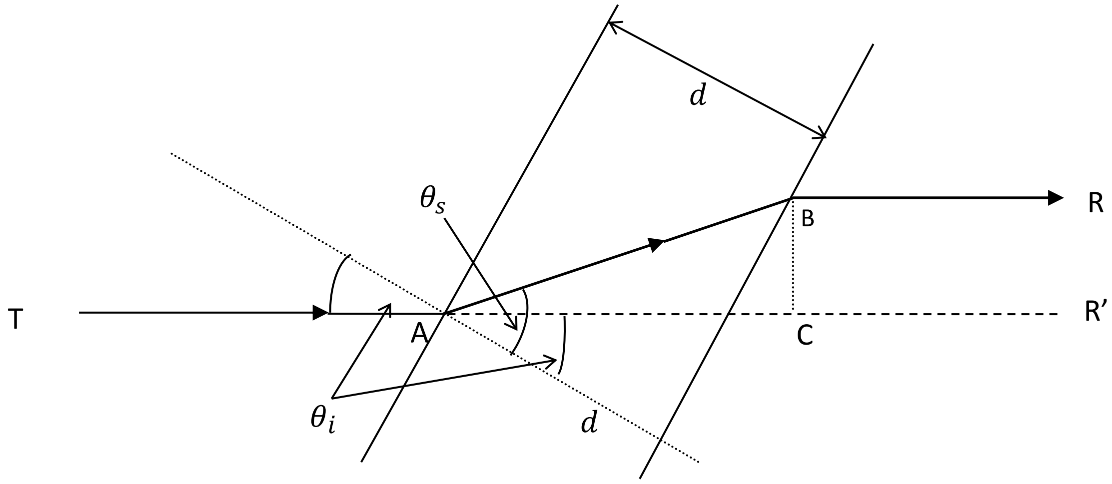


Figure A1: Replacement technique. This diagram shows the path lengths that must be considered to calculate the velocity using the replacement technique.

The measured difference in arrival time for the sound is Δt for waves with a velocity in water of V_w and a shear velocity in the plate of V_s is given by:

$$\Delta t = \frac{AB}{V_s} - \frac{AC}{V_W}.$$

where,

$$AC = AB\cos(\theta_s - \theta_i) \text{ and } d = AB\cos(\theta_s).$$

Thus,

$$\Delta t = \frac{d}{\cos(\theta_s)}\left[\frac{1}{V_s} - \frac{\cos(\theta_s - \theta_i)}{V_W}\right]. \tag{A1}$$

To solve this for V_s in terms of the measurable parameters, θ_i and Δt, we must eliminate the unknown internal angle θ_s, which depends on V_s. To do this we use some standard trigonometric relationships and Snell's Law:

$$\cos(\theta_s - \theta_i) = \cos(\theta_s)\cos(\theta_i) + \sin(\theta_s)\sin(\theta_i)$$

$$\cos(\theta) = \sqrt{1 - \sin^2(\theta)}$$

$$\sin(\theta_s) = \sin(\theta_i)\frac{V_s}{V_W}$$

Substituting these relationships into Equation A1 and solving for V_s results in Equation 2.