Algorithm 1 Conditional Inference: construct quantile-unbiased estimators for $\mu_Y(\hat{\theta})$

Require: $X = \left(X(\theta_1), \ldots, X(\theta_{|\Theta|})\right)'$ where $X \sim N(\mu, \Sigma)$ and $\Theta = \left\{\theta_1, \ldots, \theta_{|\Theta|}\right\}$ is a finite set, covariance matrix Σ, significance level α, number of draws B

$\tilde{\theta} \leftarrow \underset{\theta \in \Theta}{\operatorname{argmax}} X(\theta)$

$Z_{\tilde{\theta}} \leftarrow X - \left(\Sigma_{XY}(\cdot, \tilde{\theta}) / \Sigma_Y(\tilde{\theta})\right) Y(\tilde{\theta})$

$\mathcal{L}\left(\tilde{\theta}, Z_{\tilde{\theta}}\right) \leftarrow -\inf$

for θ in $|\Theta|$ **do**

  $\mathcal{L}\left(\theta, Z_{\tilde{\theta}}\right) \leftarrow \frac{\Sigma_Y(\tilde{\theta})\left(Z_{\tilde{\theta}}(\theta) - Z_{\tilde{\theta}}(\tilde{\theta})\right)}{\Sigma_{XY}(\tilde{\theta}) - \Sigma_{XY}(\tilde{\theta}, \theta)}$

  if $\Sigma_{XY}(\tilde{\theta}) > \Sigma_{XY}(\tilde{\theta}, \theta)$ and $\mathcal{L}\left(\theta, Z_{\tilde{\theta}}\right) > \mathcal{L}\left(\tilde{\theta}, Z_{\tilde{\theta}}\right)$ **then**

    $\mathcal{L}\left(\tilde{\theta}, Z_{\tilde{\theta}}\right) \leftarrow \mathcal{L}\left(\theta, Z_{\tilde{\theta}}\right)$

  end if

end for

$\mathcal{U}\left(\tilde{\theta}, Z_{\tilde{\theta}}\right) \leftarrow \inf$

for θ in $|\Theta|$ **do**

  $\mathcal{U}\left(\theta, Z_{\tilde{\theta}}\right) \leftarrow \frac{\Sigma_Y(\tilde{\theta})\left(Z_{\tilde{\theta}}(\theta) - Z_{\tilde{\theta}}(\tilde{\theta})\right)}{\Sigma_{XY}(\tilde{\theta}) - \Sigma_{XY}(\tilde{\theta}, \theta)}$

  if $\Sigma_{XY}(\tilde{\theta}) < \Sigma_{XY}(\tilde{\theta}, \theta)$ and $\mathcal{U}\left(\theta, Z_{\tilde{\theta}}\right) < \mathcal{U}\left(\tilde{\theta}, Z_{\tilde{\theta}}\right)$ **then**

    $\mathcal{U}\left(\tilde{\theta}, Z_{\tilde{\theta}}\right) \leftarrow \mathcal{U}\left(\theta, Z_{\tilde{\theta}}\right)$

  end if

end for

$F_{TN}\left(y; \mu_Y(\theta), \theta, z\right)$ the cdf of $\xi \mid \xi \in \mathcal{Y}(\tilde{\theta}, z)$ with

- ▶ $\xi \sim N\left(\mu_Y(\tilde{\theta}), \Sigma_Y(\tilde{\theta})\right)$
- ▶ $\mathcal{Y}(\tilde{\theta}, z) = \left\{y : z + \left(\Sigma_{XY}(\cdot, \tilde{\theta}) / \Sigma_Y(\tilde{\theta})\right) y \in \mathcal{X}(\tilde{\theta})\right\}$

Median:

  $\hat{\mu}_{\frac{1}{2}} \leftarrow F_{TN}^{-1}\left(Y(\hat{\theta}); \cdot, \tilde{\theta}, Z_{\tilde{\theta}}\right)(1/2)$

$1 - \alpha$ **Interval:** $CS_{ET} = \left[\hat{\mu}_{\alpha/2}, \hat{\mu}_{1-\alpha/2}\right]$

  $\hat{\mu}_{\alpha/2} \leftarrow F_{TN}^{-1}\left(Y(\hat{\theta}); \cdot, \tilde{\theta}, Z_{\tilde{\theta}}\right)(1 - \alpha/2)$

  $\hat{\mu}_{1-\alpha/2} \leftarrow F_{TN}^{-1}\left(Y(\hat{\theta}); \cdot, \tilde{\theta}, Z_{\tilde{\theta}}\right)(\alpha/2)$