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A General Analytical Algorithm for Collaborative Robot (cobot) with 6 Degree of Freedom (DOF)

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Abstract

The Forward and inverse kinematics of the robot is the connection of mechanical system motion and control system. There is no closed form solution for inverse kinematics of 6 *DOF* robots currently. The method of numerical iterative algorithm is used usually, but which is hard to ensure the requirement of real-time and accuracy. Considering the application and characteristic of cobot, we designed the mechanical structure and distributed the 6 *DOF* of it. This paper uses the analytical method to get the consequence of the forward and inverse kinematics of the robot. It ensures the most important characteristics of real-time and accuracy. It can also get the singularity of the robot easily. At last, we verified the method by simulation in *MATLAB*.

Key words: cobot, forward and inverse kinematics, analytical method, matrix theory

Introduction

Choosing appropriate kinematics algorithm what is fast and accuracy is the base of trajectory planning and control for the cobot. Iteration method is the frequently-used method. Iteration method is feasible in most cases, but it can't get all solutions. Geometric method just can be used to some special robot which structure is simple. Regnier proposed an algorithm at 1997 what bases on iteration method and distributed method [1].this method can calculate the inverse kinematics of many robots with 6 DOF (degree of freedom). But it need more time to get the solution. Jun proposed a method at 2009 what transforms the input speed of workspace to inverse kinematics [2].it can get the jacobian matrix quickly. Rolland proposed an optimization genetic algorithm at 2009 about parallel manipulator[3]. He transformed the non-linear equations solution to an optimization process separately.

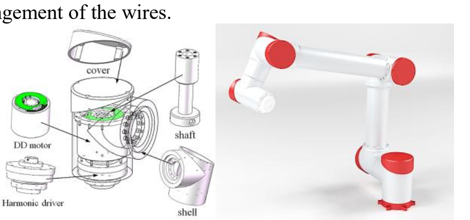
For the inverse kinematics, there are many new methods, such as artificial neural network[4], groebner basis and so on. These methods can achieve the target in theory[5]. But it need more time to calculate and can't meet with require of rapidity.

This cobot uses a general analytical algorithm to get the forward and inverse kinematics. It can ensure the requirement of real-time, accuracy and can be used to any other similar robot.

1. The structure and kinematic model of the robot.

Considering the application and characteristic of cobot, the distribution of the degrees of freedom (*DOF*), we designed the structure of the robot and then made lightweight design through topology optimization.

The Fig.1 shows the model of the robot. The robot mainly consists of DD motor, harmonic driver, feedback components (photoelectric encoder and magnetic encoder), and brake mechanism and so on. The robot adopts series structure. All the internal structure of the joint is hollow. This can ensure the arrangement of the wires.



(1) Single joint (2) Whole robot

Fig.1 The structure of the robot

In the aspect of *DOF* configuration, 6 *DOF* can reach any position of the work space. So, this robot uses 6 *DOF* to realize the flexible movement and enhance the ability of avoiding obstacles. All the 6 joints are rotational *DOF*, and adopt series structure.

As shown in the Fig.2, the kinematics model is built on the basis of the *D-H* theory. The reference coordinate system is set in the base joint. The Z_0 axis is vertical. The X_0 axis is perpendicular to Z_0 axis and follows outward direction of the paper. The direction of X_0 axis is determined by the right-hand screw rule. Another 6 coordinate systems O_i-$X_iY_iZ_i$(i=1~6) build at each rotary joint. The Z_i axis is heading in the forward direction of the motor. The directions of X_i and Y_i are determined by *D-H* theory.

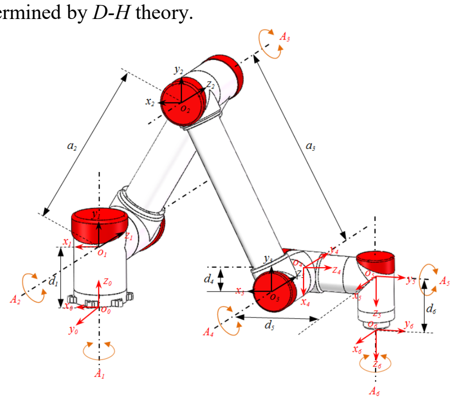


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Fig.2 The kinematics model

The Fig.2 shows the 6 axes A_i (i=1~6) of robot. The 7 frames O_i-$X_iY_iZ_i$(i=0~6) connect with each basic part.
θ_i , a_i , d_i , α_i represent the joint angle, link length, link offset and link twist respectively. The table 1 gives the *D-H* parameters of the robot.

TABLE 1
D-H PARAMETERS

	$\theta_i(rad)$	$a_2(m)$	$d_i(m)$	$\alpha_i(rad)$
joint 1	θ_1	0	d_1	$\pi/2$
joint 2	θ_2	a_2	0	0
joint 3	θ_3	a_3	0	0
joint 4	θ_4	0	d_4	$\pi/2$
joint 5	θ_5	0	d_5	$-\pi/2$
joint 6	θ_6	0	d_6	0

The kinematics research of the robot

The analysis of kinematic can solve the problem of mapping relation with the joint angle θ_i and the pose of end effector. This is the theoretical basis of motion control and one of the most important steps to realize close-loop control accurately[6].

A Forward Kinematics

The joint angle θ_i (i=1~6) is known, and then need to calculate the position and orientation of *TCP* (terminal center point).This process is described as forward kinematics. ${}^{i-1}{}_{i}T$ is the *D-H* transformation matrix and what transforms from the coordinate frame i to i-1. Every coordinate frame i can transform to the coordinate frame i-1 through 4 relative transformations orderly.

The *D-H* transformation matrix ${}^{i-1}{}_{i}T$ can be got as follows:

$$ {}^{i-1}_{i}T = T_{RZ-1}(\theta_i)\cdot T_Z(d_i)\cdot T_X(a_i)T_{RX}(\alpha_i) $$

$$ = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & 0 \\ s\theta_i & c\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\alpha_i & -s\alpha_i & 0 \\ 0 & s\alpha_i & c\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ = \begin{bmatrix} c\theta_i & -c\alpha_i s\theta_i & s\alpha_i s\theta_i & a_i c\theta_i \\ s\theta_i & c\alpha_i c\theta_i & -s\alpha_i c\theta_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.1} $$

The homogeneous transformation matrix of each joint can be got from (1.1) and table 1.

$$ {}^0_6T = {}^0T_1\,{}^1T_2\,{}^2T_3\,{}^3T_4\,{}^4T_5\,{}^5T_6 \tag{1.2} $$

${}^0T_6$ represents the position and orientation of TCP what is based on the base coordinate frame. It can be also written as this:

$$ {}^0T_6 = \begin{bmatrix} x_6 & y_6 & z_6 & p_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} {}^0R_6 & {}^0P_6 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \vec{n} & \vec{s} & \vec{a} & \vec{p} \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ = \begin{bmatrix} n_x & s_x & a_x & p_x \\ n_y & s_y & a_y & p_y \\ n_z & s_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.3} $$

n, s, a represent the coordinates of X_6 axis, Y_6 axis and Y_6 axis what are based on the base coordinate frame O_0-$X_0Y_0Z_0$. P represents the coordinate of *TCP* what is also based on the base coordinate frame O_0-$X_0Y_0Z_0$. So, ${}^0T_6$ can be got as this:

$$ {}^0_6T = {}^0T_1\,{}^1T_2\,{}^2T_3\,{}^3T_4\,{}^4T_5\,{}^5T_6 = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.4} $$

The position and orientation of TCP can be calculated when the initial θ_i (i=1~6), a_2, a_3, d_1, d_4, d_5, d_6 are given.

B Inverse Kinematics

The position and orientation of *TCP* (n_x, n_y... p_y, p_z) is known. Then we need to calculate the joint variate θ_i (i=1~6).This process is called inverse kinematics. In the field of engineering application, inverse kinematics is more important than forward kinematics. It's the base of motion planning and trajectory control. There are two sufficient conditions we should follow if we want to get the closed-form solution[7-8].

(1) There are 3 adjacent axes of the joints intersect at one point.
(2) There are 3 adjacent axes of the joints parallel to each other.

This robot meets the second condition. So, we can get the closed-form solution.

1、*Calculate joint angle* q_1

Multiplying each side of (1.4) by ${}_0T_1^{-1}$.

$$ {}^0T_1^{-1} \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = {}^1T_2\,{}^2T_3\,{}^3T_4\,{}^4T_5\,{}^5T_6 \tag{1.5} $$

The left side of (1.5) is:

$$ \begin{bmatrix} n_xC_1 + n_yS_1 & o_xC_1 + o_yS_1 & a_xC_1 + a_yS_1 & p_xC_1 + p_yS_1 \\ n_z & o_z & a_z & p_z - d_1 \\ n_xS_1 - n_yC_1 & o_xS_1 - o_yC_1 & a_xS_1 - a_yC_1 & p_xS_1 - p_yC_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.6} $$

The right side of (1.5) is:

$$ \begin{bmatrix} C_{234}C_5C_6 - S_{234}S_6 & -S_{234}C_6 - C_{234}C_5S_6 & -C_{234}S_5 \\ C_{234}S_6 + S_{234}C_5C_6 & C_{234}C_6 - S_{234}C_5S_6 & -S_{234}S_5 \\ C_6S_5 & -S_5S_6 & C_5 \\ 0 & 0 & 0 \end{bmatrix} $$

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$$\begin{matrix} d_5S_{234}+a_3C_{23}+a_2C_2+d_6S_5C_{234} \\ d_5C_{234}+a_3S_{23}+a_2S_2-d_6S_5S_{234} \\ d_4+d_6C_5 \\ 1 \end{matrix}\Bigg] \quad (1.7)$$

There is one linear equation about q_1 can be got, comparing (1.6) and (1.7) in Row 3/Column 3 and 4.

$$\begin{cases} a_xS_1-a_yC_1=C_5 \\ p_xS_1-p_yC_1=d_4+d_6C_5 \end{cases} \quad (1.8)$$

It can be derived out from (1.8),

$$(p_x-d_6a_x)S_1+(d_6a_y-p_y)C_1=d_4 \quad (1.9)$$

Using trigonometric substitution:

$$\begin{cases} \rho\sin\phi=d_6a_y-p_y \\ \rho\cos\phi=p_x-d_6a_x \end{cases} \quad (1.10)$$

$$\phi=a\tan2(d_6a_y-p_y,p_x-d_6a_x) \quad (1.11)$$

Putting (1.10) and (1.11) into (1.9), we can get: $\sin(\phi+q_1)=\frac{d_4}{\rho}$

So, $\phi+q_1=a\tan2(\frac{d_4}{\rho},\pm\sqrt{1-\frac{d_4^{\ 2}}{\rho^2}})$

Combining with (1.10), we can get 2 solutions of q_1.

$$q_1=a\tan2(d_4,\pm\sqrt{(d_6a_y-p_y)^2+(p_x-d_6a_x)^2-d_4^{\ 2}}) \\ -a\tan2(d_6a_y-p_y,p_x-d_6a_x) \quad (1.12)$$

2、*Calculate joint angle q_5*

There is one linear equation about q_5 can be got, comparing (1.6) and (1.7) in Row 3/Column 1 and 2.

$$\begin{cases} n_xS_1-n_yC_1=C_6S_5 \\ o_xS_1-o_yC_1=-S_5S_6 \end{cases} \quad (1.13)$$

Squaring the two equations in (1.13), the variate q_6 can be eliminated.

$$S_5=\pm\sqrt{(n_xS_1-n_yC_1)^2+(o_xS_1-o_yC_1)^2} \quad (1.14)$$

Because $a_xS_1-a_yC_1=C_5$ in equation (1.8), so

$$q_5=a\tan2(\pm\sqrt{(n_xS_1-n_yC_1)^2+(o_xS_1-o_yC_1)^2},a_xS_1-a_yC_1) \quad (1.15)$$

There are two solutions of q_5 from the symbol '±' in (1.15).

3、*Calculate joint angle q_6*

As we can see in (1.13), the robot has singular value if S_5=0(q_5=0or q_5=π).So, we can get q_6 just when $S_5 \neq 0$.

We can get equation (1.16) from (1.13).

$$\begin{cases} \frac{n_xS_1-n_yC_1}{S_5}=C_6 \\ -\frac{o_xS_1-o_yC_1}{S_5}=S_6 \end{cases} \quad (1.16)$$

Then, $q_6=a\tan2(-\frac{o_xS_1-o_yC_1}{S_5},\frac{n_xS_1-n_yC_1}{S_5})$ (1.17)

4、*Calculate joint angle q_{234}*

There is one linear equation about $q_2+q_3+q_4$ can be got, comparing (1.6) and (1.7) in Row 1 and 2/Column 3.

$$\begin{cases} a_xC_1+a_yS_1=-C_{234}S_5 \\ a_z=-S_{234}S_5 \end{cases} \quad (1.18)$$

As the same in q_6, the robot has singular value if S_5=0(q_5=0or q_5=π).So, we can get $q_2+q_3+q_4$ just when $S_5 \neq 0$.

We can get equation (1.19) from (1.18).

$$\begin{cases} -\frac{a_xC_1+a_yS_1}{S_5}=C_{234} \\ -\frac{a_z}{S_5}=S_{234} \end{cases} \quad (1.19)$$

Then,

$$q_{234}=q_2+q_3+q_4=a\tan2(-\frac{a_z}{S_5},-\frac{a_xC_1+a_yS_1}{S_5}) \quad (1.20)$$

5、*Calculate joint angle q_2*

There is one linear equation about q_2 can be got, comparing (1.6) and (1.7) in Row 1 and 2/Column 4.

$$\begin{cases} p_xC_1+p_yS_1=d_5S_{234}+a_3C_{23}+a_2C_2-d_6S_5C_{234} \\ p_z-d_1=-d_5C_{234}+a_3S_{23}+a_2S_2-d_6S_5S_{234} \end{cases} \quad (1.21)$$

Squaring the two equations in (1.13), the variate q_{23} can be eliminated.

$$Then, 2a_2AC_2+2a_2BS_2=A^2+B^2+a_2^{\ 2}-a_3^{\ 2} \quad (1.22)$$

We can assume that

$A=p_xC_1+p_yS_1-d_5S_{234}+d_6S_5C_{234}$,

$B=p_z-d_1+d_5C_{234}+d_6S_5S_{234}$ in the (1.22) , and order

$$\begin{cases} \eta\sin\varphi=A \\ \eta\cos\varphi=B \end{cases} \quad (1.23)$$

Then, $\eta=\sqrt{A^2+B^2}$ and $\varphi=a\tan2(A,B)$.

We can get (1.24) when putting (1.23) into (1.22).

$$2a_2\eta\sin\varphi C_2+2a_2\eta\cos\varphi S_2=A^2+B^2+a_2^{\ 2}-a_3^{\ 2} \quad (1.24)$$

And then, $\sin(\varphi+q_2)=\frac{A^2+B^2+a_2^{\ 2}-a_3^{\ 2}}{2a_2\eta}$

$\cos(\varphi+q_2)=\pm\sqrt{1-(\frac{A^2+B^2+a_2^{\ 2}-a_3^{\ 2}}{2a_2\eta})^2}$.

So,

$$\varphi+q_2=a\tan2(\frac{A^2+B^2+a_2^{\ 2}-a_3^{\ 2}}{2a_2\eta},\pm\sqrt{1-(\frac{A^2+B^2+a_2^{\ 2}-a_3^{\ 2}}{2a_2\eta})^2})$$

we can get 2 solutions of q_2.

$$q_2=a\tan2(A^2+B^2+a_2^{\ 2}-a_3^{\ 2}, \\ \pm\sqrt{(4a_2^{\ 2}-1)(A^2+B^2)-a_2^{\ 2}+a_3^{\ 2}}))-a\tan2(A,B) \quad (1.25)$$

6、*Calculate joint angle q_3*

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We can get the solution of q_{23} after q_2 was calculated.

$$\begin{cases} p_x C_1 + p_y S_1 - d_5 S_{234} - a_2 C_2 + d_6 S_5 C_{234} = a_3 C_{23} \\ p_z - d_1 + d_5 C_{234} - a_2 S_2 + d_6 S_5 S_{234} = a_3 S_{23} \end{cases} \tag{1.26}$$

So,

$$q_{23} = q_2 + q_3 = a\tan 2\left(\frac{p_z - d_1 + d_5 C_{234} - a_2 S_2 + d_6 S_5 S_{234}}{a_3}, \frac{p_x C_1 + p_y S_1 - d_5 S_{234} - a_2 C_2 + d_6 S_5 C_{234}}{a_3}\right) \tag{1.27}$$

Thereby, the remaining joints $q_3 = q_{23} - q_2$, $q_4 = q_{234} - q_{23}$.

At this moment, all the joints (q_1~q_6) of inverse kinematics were calculated.

The Simulation with Matlab

A Simulation of Forward Kinematics

The length of link a_2=-0.5, a_3=-0.4 and the setover of link d_1=0.1, d_4=0.1, d_5=0.1, d_6=0.08 is known. Putting these initial values into the equation (1.4).The terminal position and orientation can be got as follows，when angle $\theta_1 = \theta_4 = \theta_6 = 0$, $\theta_2 = \theta_3 = \theta_5 = \pi/2$ of the robot was given.

$$ {}^0_6T = \begin{bmatrix} 0 & 0 & 1 & 0.48 \\ -1 & 0 & 0 & -0.1 \\ 0 & -1 & 0 & -0.3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

The software Matlab Robotics Toolbox was used to make forward kinematics simulation. The terminal position and orientation can be got as Fig.3.

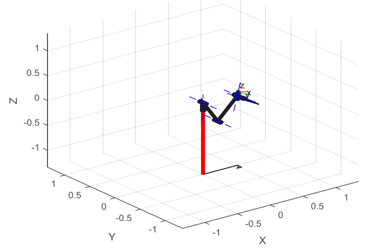

Fig.3 terminal position and orientation of the robot

B Simulation of Inverse Kinematics

In order to prove the correctness of the algorithm, we put the result we calculated from chapter 3.1as the known parameter. Then, the terminal position and orientation was known and used it to calculate the 8 solutions of joint angle θ_i (i=1~6). We used the software Matlab to make numerical and matrix operations. The table 2 gives the 8 solutions of inverse kinematics. We can see from the table 2.The NO 1 is one solution of the 8 solutions.it shows that $\theta_1 = \theta_4 = \theta_6 = 0$, $\theta_2 = \theta_3 = \theta_5 = \pi/2$ and correspond to the chapter 3.1.so,the algorithm is correct.

TABLE 2
EIGHT SOLUTIONS OF INVERSE KINEMATICS

No	joint	q(rad)	joint	q(rad)
1	1	0	2	1.5708
	3	1.5708	4	0
	5	1.5708	6	0
2	1	0	2	-0.2213
	3	-0.3805	4	-2.5398
	5	1.5708	6	0
3	1	0	2	-1.4665
	3	0.3213	4	1.055
	5	-1.5708	6	3.1416
4	1	0	2	0.1795
	3	-0.6111	4	0.232
	5	-1.5708	6	3.1416
5	1	2.6516	2	2.9621
	3	-4.7531	4	2.9096
	5	1.0808	6	-3.1416
6	1	2.6516	2	4.6081
	3	-5.8432	4	2.0866
	5	1.0808	6	-3.1416
7	1	2.6516	2	2.9468
	3	-4.7385	4	2.9471
	5	-1.0808	6	-3.1416
8	1	2.6516	2	4.3487
	3	-5.6938	4	2.3237
	5	-1.0808	6	-3.1416

Conclusion

This paper proposed an analytical algorithm to get the consequence of the forward and inverse kinematics of the robot with 6 DOF. It ensured the most important characteristics of real-time and accuracy of the robot, and also can get the singularity of the robot easily. The detailed derivation process given by this paper can be used in any similar robot.at the last of this paper, we given the simulation of the forward and inverse kinematics. The algorithm was efficient proved by the simulation of matlab.

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