Kinematic analysis of an Industrial Serial Robot with 6 DoF

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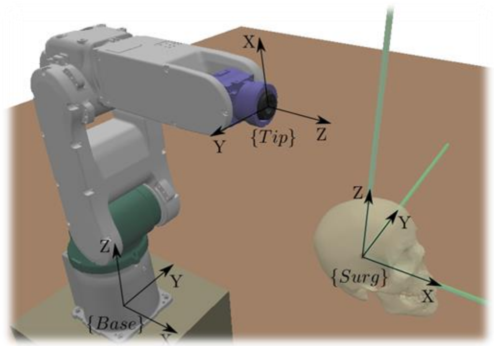
Yaskawa Motoman MH5



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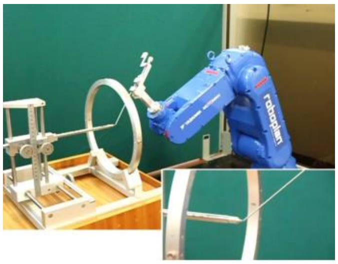
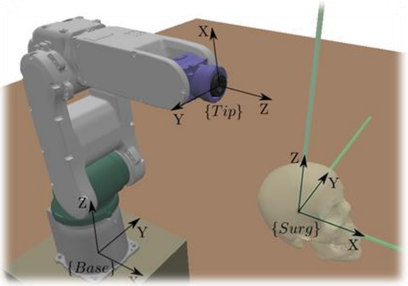
Task examples

Assisted-drilling.





What is specified is a postion and orientation for the reference frame on the Tip of the robot.

Inserting an electrode at a target position

Kinematics?

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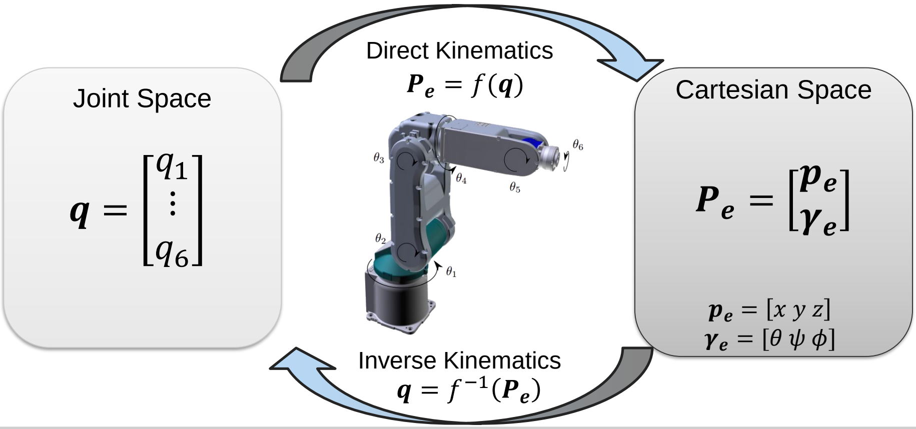
- Every robotic task can be split in a sequence of predefined motions.
- Kinematics studies the laws of body motion independent of the causes



Joint Space

$$\boldsymbol{q} = \begin{bmatrix} q_1 \\ \vdots \\ q_6 \end{bmatrix}$$

Direct Kinematics

$$\boldsymbol{P_e} = f(\boldsymbol{q})$$

Cartesian Space

$$\boldsymbol{P_e} = \begin{bmatrix} \boldsymbol{p_e} \\ \boldsymbol{\gamma_e} \end{bmatrix}$$

$$\boldsymbol{p_e} = [x\ y\ z]$$
$$\boldsymbol{\gamma_e} = [\theta\ \psi\ \phi]$$

Inverse Kinematics

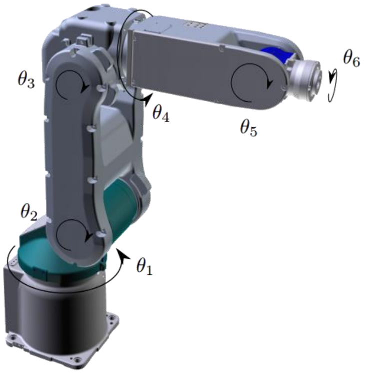
$$\boldsymbol{q} = f^{-1}(\boldsymbol{P_e})$$

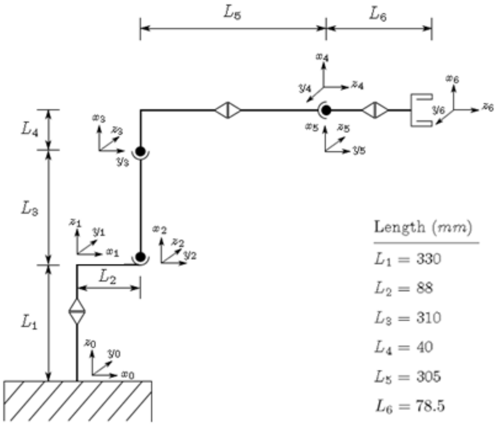
Required background knowledge

- Denavit-Hartenberg Convention
- General Transformation matrixes ${}^{i-1}\mathbf{T}_i$
- Roll-Pitch-Yaw convention and RPY Matrix
- Inverse kinematics of a two-link arm – geometric solution
- Inverse kinematics via algebraic solution

Robotic Structure and reference frames

- Yaskawa Motoman MH5 robotic structure can be split into a consecutive series of links connected by joints.





Length (mm)
$L_1 = 330$
$L_2 = 88$
$L_3 = 310$
$L_4 = 40$
$L_5 = 305$
$L_6 = 78.5$

Fig. 1 – Yaskawa Motoman MH5, 6 degrees of freedom (DoF) serial robotic manipulator.

Fig. 2 – Yaskawa Motoman MH5, kinematic structure (frames attached to each joint).

Denavit-Hartenberg Convention

- Most common convention to characterise the robotic structure.

1. Start by assigning frames to each joint along the manipulator arm (Fig. 2).
2. The relation between $link_{(i)}$ and $link_{(i+1)}$ is represented by four parameters:
   - α_{i-1}, angle of rotation around $\boldsymbol{X_i}$ axis;
   - a_{i-1}, distance of translation along $\boldsymbol{X_i}$ axis;
   - θ_i, angle of rotation around $\boldsymbol{Z_i}$ axis, after the rotation around $\boldsymbol{X_i}$ axis;
   - d_i, distance of translation along $\boldsymbol{Z_i}$ axis, after the rotation around $\boldsymbol{X_i}$ axis.
3. The homogenous transformation ${}^{\boldsymbol{i-1}}\boldsymbol{T}_{\boldsymbol{i}}$, from frame {i-1} to {i} is defined as the result of:
   - ${}^{i-1}T_i = Rot_X(\alpha_{i-1}) * Trans_X(a_{i-1}) * Trans_Z(d_i) * Rot_Z(\theta_i)$

Denavit-Hartenberg parameters

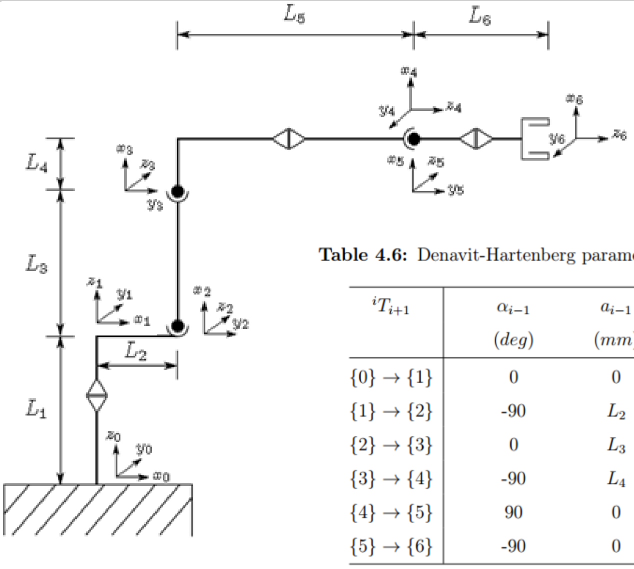


Table 4.6: Denavit-Hartenberg parameters for the Motoman MH5 arm.

${}^{i}T_{i+1}$	α_{i-1}	a_{i-1}	θ_i	d_i
	(deg)	(mm)	(deg)	(mm)
$\{0\} \to \{1\}$	0	0	θ_1	L_1
$\{1\} \to \{2\}$	-90	L_2	$\theta_2 - 90$	0
$\{2\} \to \{3\}$	0	L_3	θ_3	0
$\{3\} \to \{4\}$	-90	L_4	θ_4	L_5
$\{4\} \to \{5\}$	90	0	θ_5	0
$\{5\} \to \{6\}$	-90	0	θ_6	L_6

Denavit-Hartenberg parameters

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$\{4\} \to \{5\}$	90	0	θ_5	0
$\{5\} \to \{6\}$	-90	0	θ_6	L_6

Denavit-Hartenberg Matrix

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$$^{i-1}\mathbf{T}_i = Rotation_X(\alpha_{i-1}) \cdot Translation_X(a_{i-1}) \cdot Translation_Z(d_i) \cdot Rotation_Z(\theta_i)$$

$$^{i-1}\mathbf{T}_i = \begin{bmatrix} \cos(\theta_i) & -\sin(\theta_i) & 0 & a_{i-1} \\ \sin(\theta_i)\cos(\alpha_{i-1}) & \cos(\theta_i)\cos(\alpha_{i-1}) & -\sin(\alpha_{i-1}) & -\sin(\alpha_{i-1})d_i \\ \sin(\theta_i)\sin(\alpha_{i-1}) & \cos(\theta_i)\sin(\alpha_{i-1}) & \cos(\alpha_{i-1}) & \cos(\alpha_{i-1})d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4.27)$$

Do it yourself: compute the individual matrices

Direct Kinematics

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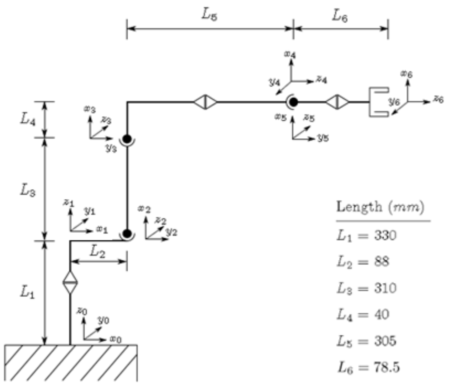
- Having defined the homogeneous transformation matrices along the robotic manipulator arm (Denavit-Hartenberg notation), the transformation from the robot base (frame) to the robot-tip (frame 6) is given by:

$$\boxed{{}^0T_6 = {}^0T_1\ {}^1T_2\ {}^2T_3\ {}^3T_4\ {}^4T_5\ {}^5T_6}$$

Do it yourself: compute ${}^0T_6$



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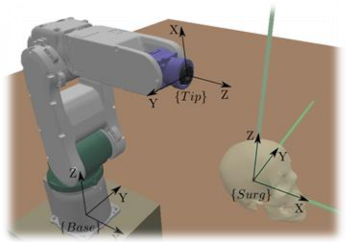
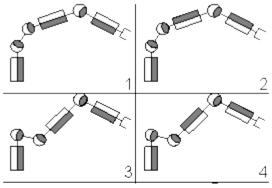


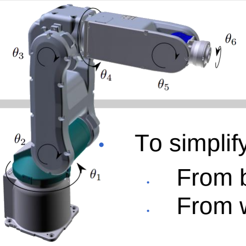
Fig. 2 – Yaskawa Motoman MH5, kinematic structure (frames attached to each joint).

Inverse Kinematics Problem

- Unlike Direct Kinematics (a solution always exists and it's unique), **Inverse Kinematics may generate:**
  - Several possible solutions
  - Possible and unique solutions (near dexterous workspace limit)
  - Undetermined solutions (in dexterous workspace)
  - Impossible solutions (outside dexterous workspace)



- A non-redundant 6-DoF manipulator like Yaskawa Motoman MH5 can reach a given position and orientation in space with **4 different postures**.
- A **cost-function** is usually defined to choose the best fitting solution from the 4 different poses.



Inverse Kinematics

- To simplify the problem we split the robotic arm structure in 2 parts:
  - From base {B} (or frame 0 in Fig. 2) to wrist {W} (or frame 4 in Fig. 2);
  - From wrist {W} (frame 4) to robot tip (or frame 6 in Fig. 2)
- A geometric approach is used to determine the lower arm joint values $(\theta_1, \theta_2, \theta_3)$, and an algebraic approach is used to determine the upper arm joint values $(\theta_4, \theta_5, \theta_6)$ → ${}^0\boldsymbol{T}_3$ and ${}^3\boldsymbol{T}_6$ are going to be useful matrixes

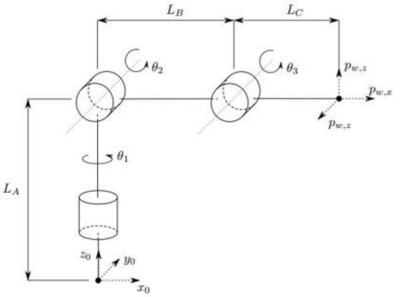


Figure 4.2: Anthropormorphic arm.

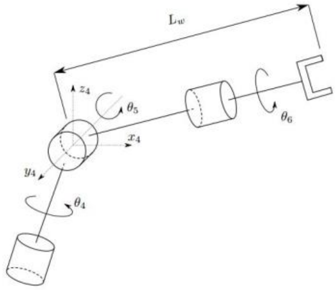


Figure 4.4: Spherical wrist.

Inverse Kinematics

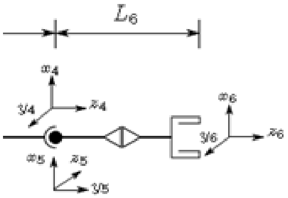
Knowing the desired robot tip position and orientation in the robot base frame {B}, ${}^{B}\boldsymbol{P}_{\boldsymbol{e}} = ({}^{B}p_{e,x}, {}^{B}p_{e,y}, {}^{B}p_{e,z}, {}^{B}\gamma_{e,\theta}, {}^{B}\gamma_{e,\psi}, {}^{B}\gamma_{e,\phi})$ we start by **computing the wrist {W} position**.

$\hookrightarrow$ *elements of* ${}^{0}\boldsymbol{T}_{6}$ *are knwown*

1. Compute the RPY Matrix from $\boldsymbol{\gamma}_e = ({}^{B}\gamma_{e,\theta}, {}^{B}\gamma_{e,\psi}, {}^{B}\gamma_{e,\phi}) \rightarrow {}^{B}\hat{\mathbf{z}}_{RPY}$=?

2. The position of the wrist {W} (${}^{B}p_w$) in {B} can be calculated by:

$$ {}^{B}\boldsymbol{p}_{w} = {}^{B}\boldsymbol{p}_{e} - L_6 * {}^{B}\hat{\mathbf{z}}_{RPY} $$



Being:

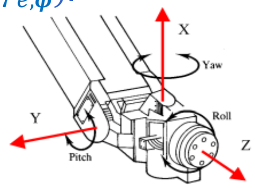
${}^{B}\boldsymbol{p}_{e} = ({}^{B}p_{e,x}, {}^{B}p_{e,y}, {}^{B}p_{e,z})$ – the position specified for the robot tip (3x1 vector)

- L_6, the distance from the robot tip to joint 5 (frame 4 or 5 in Fig.2).
- ${}^{B}\hat{\mathbf{z}}_{RPY}$, is the z-axis of the RPY matrix.

frame {B}≡frame{0}

Euler Angles : Roll-Pitch-Yaw (RPY)

1. Compute the RPY Matrix from γ=(${}^{B}\gamma_{e,\boldsymbol{\theta}}, {}^{B}\gamma_{e,\boldsymbol{\psi}}, {}^{B}\gamma_{e,\boldsymbol{\phi}}$).



Remember:

- **Euler convention used (Roll-Pitch-Yaw): accumulation of three rotations:**
  1. $Rot_{\boldsymbol{\theta}}$ around **X**; (Yaw)
  2. $Rot_{\boldsymbol{\psi}}$ around **Y;** (Pitch)
  3. $Rot_{\boldsymbol{\phi}}$ around **Z;** (Roll)

Pre-multiplying: $RPY_{3\times3} = R_{\boldsymbol{\phi}}\, R_{\boldsymbol{\psi}}\, R_{\boldsymbol{\theta}}$

$$RPY(\gamma) = Rotation_Z(\phi) \quad Rotation_Y(\psi) \quad Rotation_X(\theta)$$

$$RPY(\gamma) = \begin{bmatrix} c(\phi)c(\psi) & -s(\phi)c(\theta)+c(\phi)s(\psi)s(\theta) & s(\phi)s(\theta)+c(\phi)s(\psi)c(\theta) \\ s(\phi)c(\psi) & c(\phi)c(\theta)+s(\phi)s(\psi)s(\theta) & -c(\phi)s(\theta)+s(\phi)s(\psi)c(\theta) \\ -s(\psi) & c(\psi)s(\theta) & c(\psi)c(\theta) \end{bmatrix} \tag{4.16}$$

${}^{B}\hat{\mathbf{z}}_{\boldsymbol{RPY}}$

Note that:
the RPY(γ) ≡ ${}^{0}\boldsymbol{R}_{6}$
Thus we know the numeric values of the elements in this matrix!

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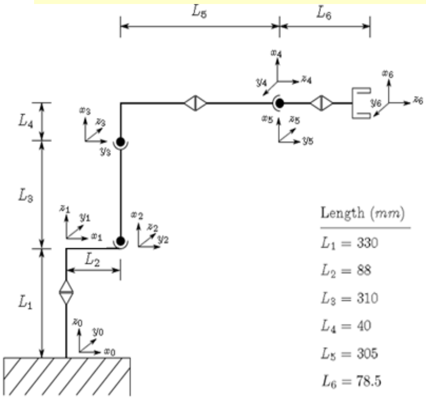
[2] **section 4.2** The Roll-Pitch-Yaw matrix (4.16) is written in a compressed form to fit the printing area. Therefore, c() stands for cos() and s() stands for sin().

Positioning the wrist

- Joints responsible for placing the wrist at a specific position are θ_1, θ_2 and θ_3.
- The position ${}^{B}\boldsymbol{p}_w$ (with respect to the base) is known

$(\theta_1, \theta_2, \theta_3)$ = ?



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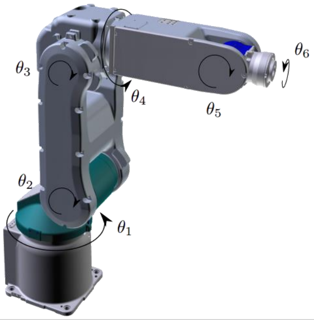
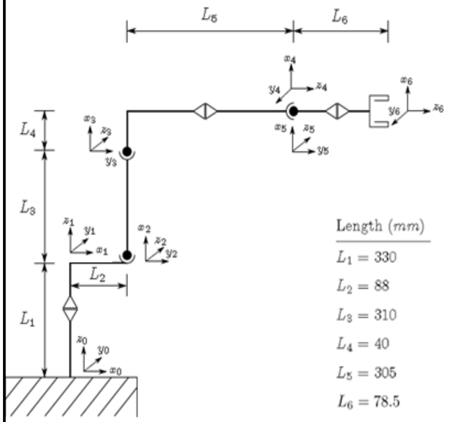


Inverse Kinematics: θ_1

A geometric approach:

3. Now, since the wrist position ${}^{B}\boldsymbol{p}_w$ (with respect to the base) is known, we can compute the first joint angle (θ_1):

$$\theta_1 = \begin{cases} \text{arctan}_2(p_{w,y}, p_{w,x}) & \text{if } p_{w,y} \neq 0 \\ 0 & \text{if } p_{w,y} = 0 \end{cases}$$



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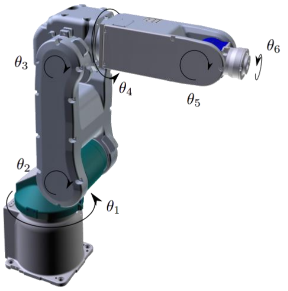
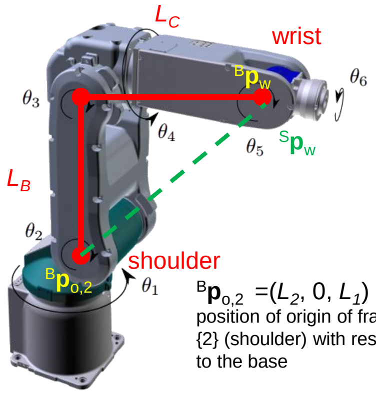


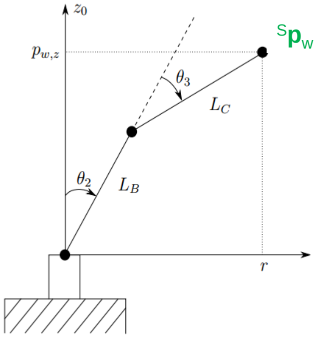
Inverse Kinematics: θ_2 and θ_3

A geometric approach:

4. We can simplify the problem of computing θ_2 and θ_3 to a planar 2-link robot.



${}^B\mathbf{p}_{o,2} = (L_2, 0, L_1)$
position of origin of frame {2} (shoulder) with respect to the base



nd 3.

${}^S\mathbf{p}_w$ - position of the wrist with respect to the shoulder:

${}^S\mathbf{p}_w = {}^B\mathbf{p}_w - {}^B\mathbf{p}_{o,2} = (p_{w,x}, p_{w,y}, p_{w,z}) = ({}^Bp_{w,x} - L_2, {}^Bp_{w,y}, {}^Bp_{w,z} - L_1)$

Inverse Kinematics: θ_2 and θ_3

4 (cont). We can simplify the problem of computing θ_2 and θ_3 to a planar 2-link robot.

Considering $r = \sqrt{p_{w,x}{}^2 + p_{w,y}{}^2}$, we easily prove that,

${}^S\mathbf{p}_w = (p_{w,x}, p_{w,y}, p_{w,z})$

$$\begin{cases} r = L_B \sin\theta_2 + L_C \sin(\theta_2 + \theta_3) \\ p_{w,z} = L_B \cos\theta_2 + L_C \cos(\theta_2 + \theta_3) \end{cases}$$

Which (as we've seen) can be solved using:

- an algebraic approach
- a geometric approach

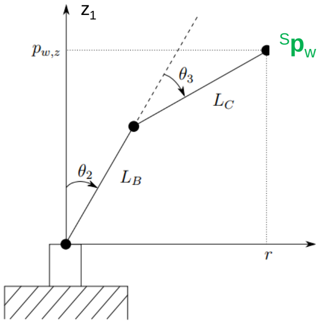


Figure 4.3: Plane projection formed by links 2 and 3

Inverse Kinematics: θ_2 and θ_3

4 (cont). We can simplify the problem of computing θ_2 and θ_3 to a planar 2-link robot.

GEOMETRIC APPROACH:

Cosine theorem: Computation of θ_3

$$\underbrace{{p_{w,x}}^2 + {p_{w,y}}^2 + {p_{w,z}}^2}_{r'^2} = {L_B}^2 + {L_C}^2 + 2\,L_B\,L_C\cos\theta_3$$

$$\cos\theta_3 = \frac{{p_{w,x}}^2 + {p_{w,y}}^2 + {p_{w,z}}^2 - {L_B}^2 - {L_C}^2}{2\,L_B\,L_C}$$

$$\sin\theta_3 = \pm\sqrt{1 - {\cos\theta_3}^2}$$

Different arm postures (1)

$$\theta_3 = \arctan_2(\sin\theta_3, \cos\theta_3)$$

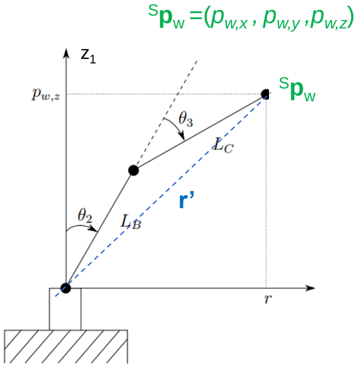


Figure 4.3: Plane projection formed by links 2 and 3

Inverse Kinematics: θ_2 and θ_3

4 (cont). We can simplify the problem of computing θ_2 and θ_3 to a planar 2-link robot.

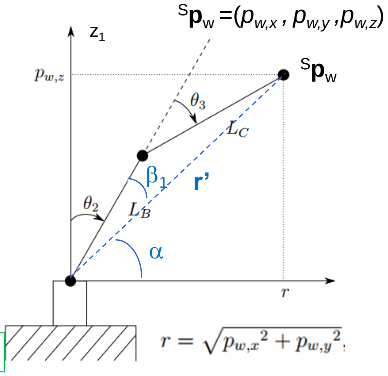
GEOMETRIC APPROACH:

Cosine theorem: Computation of θ_2

$$\theta_2 = 90^o - \alpha - \beta_1$$

$$\begin{cases} \alpha = \arctan(p_{w,z}, r) \\ L_c^2 = L_B^2 + r'^2 - 2L_B r' \cos(\beta_1) \end{cases}$$

$$\boxed{\theta_2 = 90^o - \arctan(p_{w,z}, r) - \operatorname{acos}((L_B^2 + r'^2 - L_c^2)/(2L_B r'))}$$



$r = \sqrt{{p_{w,x}}^2 + {p_{w,y}}^2}$

Figure 4.3: Plane projection formed by links 2 and 3.

Inverse Kinematics: θ_4, θ_5 and θ_6

- Having solved the inverse kinematics for joints $(\theta_1, \theta_2, \theta_3)$, we can compute the numeric values of the elements in ${}^{0}\boldsymbol{R}_3$. Being ${}^{0}\boldsymbol{R}_6$, the RPY matrix computed from γ_e (which is known), we can solve the equation for ${}^{3}\boldsymbol{R}_6$ (upper arm or spherical wrist):

$$ {}^{3}\boldsymbol{R}_6 = {}^{3}\boldsymbol{R}_0 \, {}^{0}\boldsymbol{R}_6 = \boldsymbol{inv}({}^{0}\boldsymbol{R}_3) \, {}^{0}\boldsymbol{R}_6 = {}^{0}\boldsymbol{R}_3{}^{T} \, {}^{0}\boldsymbol{R}_6 $$

$$ {}^{3}\boldsymbol{R}_6(\theta_4, \theta_5, \theta_6) = $$

$$ {}^{3}\boldsymbol{R}_6 = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} $$

- Using the wrist spherical properties, θ_5:

Different arm postures (2)

$$ \theta_5 = \arctan_2(\pm\sqrt{1 - (r_{23})^2}, r_{23}) $$

- Then, depending on θ_5, we get different values for θ_4 and θ_6:

$$ \begin{cases} \theta_4 = 0 \quad \wedge \quad \theta_6 = \arctan_2(r_{12}, r_{32}) & \text{if } \theta_5 = 0 \\ \theta_4 = 0 \quad \wedge \quad \theta_6 = -\arctan_2(-r_{12}, -r_{32}) & \text{if } \theta_5 = \pi \end{cases} $$

Singularity

$$ \begin{cases} \theta_4 = \arctan_2(r_{33}, -r_{13}) \\ \theta_6 = \arctan_2(-r_{22}, -r_{21}). \end{cases} $$

Do it yourself!

Conclusion

- We have studied how to solve the problems of the direct and inverse kinematics of the industrial 6 DoF manipulator Yaskawa Motoman MH5
- Direct kinematics was obtained using the Denavit-Hartenberg convention and the general transformation matrixes
- The solutions of the inverse kinematics were obtained combining two approaches:
  - Geometrical approach for the lower arm joint values $(\theta_1, \theta_2, \theta_3)$
  - Algebraic approach for the upper arm joint values $(\theta_4, \theta_5, \theta_6)$

Bibliography:

(2013) C. Faria, E. Bicho, M. Rito, L. Louro, S. Monteiro, W. Erlhagen. "*Robotic Assisted Deep Brain Stimulation Neurosurgery: first steps on system development*". In Proceedings of the **10th** IASTED International Conference on Biomedical Engineering (BioMed 2013), February 13-15, 2013, Innsbruck, Austria. DOI: 10.2316/P.2013.791-033.

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