Note:derivation of 3-31

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I. Substitue pressure modes for transverse velocity modes

A. mass equation

$$\frac{dU_\alpha^a}{ds} - \Psi_{\{\alpha\}\beta}[r]U_\beta^a - ia\kappa\Psi_{\alpha\beta}[r(1-\kappa cos(\phi))]P_\beta^a - \Psi_{[\alpha]\beta}[r(1-\kappa rcos(\phi))]\underline{\underline{V_\beta^a}} - \Psi_{(\alpha)\beta}[(1-\kappa rcos(\phi))]\underline{\underline{W_\beta^a}}$$
$$= \Psi_{\alpha\beta\gamma}[r(1-\kappa rcos(\phi))]\sum_{b=-\infty}^{+\infty}(-ib\kappa P_\beta^{a-b}P_\gamma^b - ib\kappa U_\beta^{a-b}U_\gamma^b - ib\kappa\underline{\underline{V_\beta^{a-b}V_\gamma^b}} - ib\kappa\underline{\underline{W_\beta^{a-b}W_\gamma^b}} - ia\kappa\frac{B}{2A}P_\beta^{a-b}P_\gamma^b) \tag{1}$$

Transform:

$$\frac{dU_\alpha^a}{ds} := \underline{\underline{I}}u_\alpha'^a$$

$$-\Psi_{\{\alpha\}\beta}[r]U_\beta^a := \underline{\underline{-\Psi_{\{\alpha\}\beta}[r]}}u_\beta^a \to \mathcal{G}$$

$$\begin{cases} -ia\kappa\Psi_{\alpha\beta}[r(1-\kappa cos(\phi))]P_\beta^a := \sum_{\beta=0}^{+\infty}\underline{\underline{-ia\kappa\Psi_{\alpha\beta}[r(1-\kappa cos(\phi))]}}p_\beta^a \to \mathcal{M}_1 \\ -\Psi_{[\alpha]\beta}[r(1-\kappa rcos(\phi))]\underline{\underline{V_\beta^a}} := \sum_{\beta=0}^{+\infty}\underline{\underline{-\Psi_{[\alpha]\delta}[r(1-\kappa rcos(\phi))]\mathbf{V}_{\delta\beta}^a}}p_\beta^a \to \mathcal{M}_2 + \Psi_{[\alpha]\delta}[r(1-krcos\phi)](N^{-1})(o(M_2^2))[1] \\ -\Psi_{(\alpha)\beta}[(1-\kappa rcos(\phi))]\underline{\underline{W_\beta^a}} := \sum_{\beta=0}^{+\infty}\underline{\underline{-\Psi_{(\alpha)\delta}[(1-\kappa rcos(\phi))]\mathbf{W}_{\delta\beta}^a}}p_\beta^a \to \mathcal{M}_3 + \Psi_{[\alpha]\delta}[r(1-krcos\phi)](N^{-1})(o(M_3^2)) \end{cases}$$

$$\Psi_{\alpha\beta\gamma}[r(1-\kappa rcos(\phi))]\sum_{b=-\infty}^{+\infty}(-ib\kappa P_\beta^{a-b}P_\gamma^b) := \sum_{b=-\infty}^{+\infty}-ib\kappa\sum_{\beta=0}^{+\infty}\sum_{\gamma=0}^{+\infty}\underline{\underline{\Psi_{\alpha\beta\gamma}[(1-\kappa rcos(\phi))]}}p_\beta^{a-b}p_\gamma^b \to \mathcal{B}_2$$

$$\Psi_{\alpha\beta\gamma}[r(1-\kappa rcos(\phi))]\sum_{b=-\infty}^{+\infty}(-ib\kappa U_\beta^{a-b}U_\gamma^b) := \sum_{b=-\infty}^{+\infty}-ib\kappa\sum_{\beta=0}^{+\infty}\sum_{\gamma=0}^{+\infty}\underline{\underline{\Psi_{\alpha\beta\gamma}[(1-\kappa rcos(\phi))]}}u_\beta^{a-b}u_\gamma^b \to \mathcal{A}_1$$

$$\Psi_{\alpha\beta\gamma}[r(1-\kappa rcos(\phi))]\sum_{b=-\infty}^{+\infty}(-ib\kappa V_\beta^{a-b}V_\gamma^b) := \sum_{b=-\infty}^{+\infty}-ib\kappa\sum_{\beta=0}^{+\infty}\sum_{\gamma=0}^{+\infty}\underline{\underline{\sum_{\delta,\epsilon=0}^{\infty}\Psi_{\alpha\delta\epsilon}[r(1-\kappa rcos\phi)]\mathbf{V}_{\delta\beta}^{a-b}\mathbf{V}_{\epsilon\gamma}^b}}p_\beta^{a-b}p_\gamma^b \to \mathcal{B}_3$$

$$\Psi_{\alpha\beta\gamma}[r(1-\kappa rcos(\phi))]\sum_{b=-\infty}^{+\infty}(-ib\kappa W_\beta^{a-b}W_\gamma^b) := \sum_{b=-\infty}^{+\infty}-ib\kappa\sum_{\beta=0}^{+\infty}\sum_{\gamma=0}^{+\infty}\underline{\underline{\sum_{\delta,\epsilon=0}^{\infty}\Psi_{\alpha\delta\epsilon}[r(1-\kappa rcos\phi)]\mathbf{W}_{\delta\beta}^{a-b}\mathbf{W}_{\epsilon\gamma}^b}}p_\beta^{a-b}p_\gamma^b \to \mathcal{B}_4$$

$$\Psi_{\alpha\beta\gamma}[r(1-\kappa rcos(\phi))]\sum_{b=-\infty}^{+\infty}(-ia\kappa\frac{B}{2A}P_\beta^{a-b}P_\gamma^b) := \sum_{b=-\infty}^{+\infty}-ia\kappa\frac{B}{2A}\sum_{\beta=0}^{+\infty}\sum_{\gamma=0}^{+\infty}\underline{\underline{\Psi_{\alpha\beta\gamma}[(1-\kappa rcos(\phi))]}}p_\beta^{a-b}p_\gamma^b \to \mathcal{B}_1 \tag{2}$$

1. momentum equation I

$$
\begin{aligned}
&\frac{d}{ds}P^a_\alpha - ia\kappa\Psi_{\alpha\beta}[r(1-\kappa rcos\phi)]U^a_\beta - \int_0^{2\pi} hh'[\psi_\beta\psi_\alpha]_{r=h}d\theta P^a_\beta - \Psi_{\{\alpha\}\beta}[r]P^a_\beta \\
&= \sum_{b=-\infty}^{+\infty} \underline{term\mathcal{D}1:}\Psi_{\{\alpha\}\beta\gamma}[r]U^{a-b}_\beta U^a_\gamma + \Psi_{[\alpha]\beta\gamma}[r(1-\kappa rcos\phi)]\underline{\underline{V^{a-b}_\beta}}U^a_\gamma + \Psi_{(\alpha)\beta\gamma}[(1-\kappa rcos\phi)]\underline{\underline{W^{a-b}_\beta}}U^a_\gamma \\
&\quad + \underline{term(D1+\mathcal{P}1):}i(a)\kappa\Psi_{\alpha\beta\gamma}[r(1-\kappa rcos\phi)]P^{a-b}_\beta U^b_\gamma \\
&\quad + \underline{term\mathcal{D}1:} - \frac{\partial}{\partial s}(\Psi_{\alpha\beta\gamma}[r])U^{a-b}_\beta U^b_\gamma - \Psi_{\alpha\beta\gamma}[r](\frac{dU^{a-b}_\beta}{ds}U^b_\gamma + \frac{dU^b_\gamma}{ds}U^{a-b}_\beta) \\
&\quad + \underline{term\mathcal{X}1:}\Psi_{\alpha\beta\gamma}[rcos\phi]U^{a-b}_\beta\underline{\underline{V^b_\gamma}} - \kappa\Psi_{\alpha\beta\gamma}[rsin\phi]U^{a-b}_\beta\underline{\underline{W^b_\gamma}}
\end{aligned}
\tag{3}
$$

Transform:

$$
\begin{aligned}
&\frac{d}{ds}P^a_\alpha := \underline{\underline{I}}p'^a_\alpha \\
&-ia\kappa\Psi_{\alpha\beta}[r(1-\kappa rcos\phi)]U^a_\beta := \underline{\underline{-ia\kappa\Psi_{\alpha\beta}[r(1-\kappa cos(\phi))]}}u^a_\beta \rightarrow \mathcal{N} \\
&-\int_0^{2\pi} hh'[\psi_\beta\psi_\alpha]_{r=h}d\theta P^a_\beta - \Psi_{\{\alpha\}\beta}[r]P^a_\beta := \underline{\underline{-\int_0^{2\pi} hh'[\psi_\beta\psi_\alpha]_{r=h}d\theta}}p^a_\beta - \underline{\underline{\Psi_{\{\alpha\}\beta}[r]}}p^a_\beta = -\underline{\underline{\Psi_{\{\alpha\}\beta}[r]}}p^a_\beta \rightarrow \mathcal{H} \\
&\Psi_{\{\alpha\}\beta\gamma}[r]\sum_{b=-\infty}^{+\infty} U^{a-b}_\beta U^a_\gamma := \sum_{b=-\infty}^{+\infty}\sum_{\beta=0}^{+\infty}\sum_{\gamma=0}^{+\infty}\underline{\underline{\Psi_{\{\alpha\}\beta\gamma}[r]}}u^{a-b}_\beta u^b_\gamma \rightarrow \mathcal{D} \\
&\Psi_{[\alpha]\beta\gamma}[r(1-\kappa rcos\phi)]\sum_{b=-\infty}^{+\infty}\underline{\underline{V^{a-b}_\beta}}U^a_\gamma := \sum_{b=-\infty}^{+\infty}\sum_{\beta=0}^{+\infty}\sum_{\gamma=0}^{+\infty}\underline{\underline{\sum_{\delta,\epsilon=0}^{\infty}\Psi_{[\alpha]\delta\epsilon}[r(1-\kappa rcos\phi)]\mathbf{V}^{a-b}_{\delta\beta}\mathbf{I}^b_{\epsilon\gamma}}}p^{a-b}_\beta u^b_\gamma \rightarrow \mathcal{C}_4 \\
&\Psi_{(\alpha)\beta\gamma}[(1-\kappa rcos\phi)]\sum_{b=-\infty}^{+\infty}\underline{\underline{W^{a-b}_\beta}}U^a_\gamma := \sum_{b=-\infty}^{+\infty}\sum_{\beta=0}^{+\infty}\sum_{\gamma=0}^{+\infty}\underline{\underline{\sum_{\delta,\epsilon=0}^{\infty}\Psi_{(\alpha)\delta\epsilon}[(1-\kappa rcos\phi)]\mathbf{W}^{a-b}_{\delta\beta}\mathbf{I}^b_{\epsilon\gamma}}}p^{a-b}_\beta u^b_\gamma \rightarrow \mathcal{C}_5 \\
&ia\kappa\Psi_{\alpha\beta\gamma}[r(1-\kappa rcos\phi)]\sum_{b=-\infty}^{+\infty}P^{a-b}_\beta U^b_\gamma := ia\kappa\sum_{b=-\infty}^{+\infty}\sum_{\beta=0}^{+\infty}\sum_{\gamma=0}^{+\infty}\underline{\underline{\Psi_{\alpha\beta\gamma}[r(1-\kappa rcos\phi)]}}p^{a-b}_\beta u^b_\gamma \rightarrow \mathcal{C}_3 \\
&-\frac{\partial}{\partial s}(\Psi_{\alpha\beta\gamma}[r])\sum_{b=-\infty}^{+\infty}U^{a-b}_\beta U^b_\gamma := \sum_{b=-\infty}^{+\infty}\sum_{\beta=0}^{+\infty}\sum_{\gamma=0}^{+\infty}\underline{\underline{-\frac{\partial}{\partial s}(\Psi_{\alpha\beta\gamma}[r])}}u^{a-b}_\beta u^b_\gamma \rightarrow \mathcal{D} \\
&-\Psi_{\alpha\beta\gamma}[r]\sum_{b=-\infty}^{+\infty}(\frac{dU^{a-b}_\beta}{ds}U^b_\gamma + \frac{dU^b_\gamma}{ds}U^{a-b}_\beta) := \Psi_{\alpha\beta\gamma}[r]\sum_{b=-\infty}^{+\infty}([(Mp)^{a-b}_\beta + (Gu)^{a-b}_\beta]U^b_\gamma + [(Mp)^b_\gamma + (Gu)^b_\gamma]U^{a-b}_\beta) \\
&\qquad = \sum_{b=-\infty}^{+\infty}(\Psi_{\alpha\beta\gamma}[r][\mathbf{G},\mathbf{I}] + \Psi_{\alpha\beta\gamma}[r][\mathbf{I},\mathbf{G}])u^{a-b}_\beta u^b_\gamma \rightarrow \mathcal{D}_{2,3} \\
&\qquad + \sum_{b=-\infty}^{+\infty}(\Psi_{\alpha\beta\gamma}[r][\mathbf{M},\mathbf{I}] + \Psi_{\alpha\beta\gamma}[r][\mathbf{I},\mathbf{M}])u^{a-b}_\beta p^b_\gamma \rightarrow \mathcal{C}_{1,2} \\
&\Psi_{\alpha\beta\gamma}[rcos\phi]U^{a-b}_\beta\underline{\underline{V^b_\gamma}} - \kappa\Psi_{\alpha\beta\gamma}[rsin\phi]U^{a-b}_\beta\underline{\underline{W^b_\gamma}} := \{\sum_{b=-\infty}^{+\infty}\Psi_{\alpha\beta\gamma}[rcos\phi][\mathbf{I},\mathbf{V}] - \kappa\Psi_{\alpha\beta\gamma}[rsin\phi][\mathbf{I},\mathbf{W}]\}u^{a-b}_\gamma p^b_\beta \rightarrow \mathcal{C}_{6,7}
\end{aligned}
\tag{4}
$$

B. momentum equation II

$$
\begin{aligned}
&-ia\kappa\Psi_{\alpha\beta}[r(1-\kappa r cos\phi)]V^a_\beta \\
&+\int_0^{2\pi}[\psi_\alpha\psi_\beta r(1-\kappa r cos\phi)]_0^h d\theta P^a_\beta - \Psi_{[\alpha]\beta}[r(1-\kappa r cos\phi)]P^a_\beta - \Psi_{\alpha\beta}[1-2\kappa r cos\phi]P^a_\beta \\
&= \underline{term\mathcal{D}2:}\Psi_{\{\alpha\}\beta\gamma}[r]U^{a-b}_\beta V^a_\gamma + \Psi_{[\alpha]\beta\gamma}[r(1-\kappa r cos\phi)]V^{a-b}_\beta V^a_\gamma + \Psi_{(\alpha)\beta\gamma}[(1-\kappa r cos\phi)]W^{a-b}_\beta V^a_\gamma \\
&+\underline{term(D2+\mathcal{P}2):}i(a)\kappa\Psi_{\alpha\beta\gamma}[r(1-\kappa r cos\phi)]P^{a-b}_\beta V^b_\gamma \\
&+\underline{term\mathcal{D}2:}-\frac{\partial}{\partial s}(\Psi_{\alpha\beta\gamma}[r])U^{a-b}_\beta V^b_\gamma - \Psi_{\alpha\beta\gamma}[r](\frac{dU^{a-b}_\beta}{ds}V^b_\gamma + \frac{dV^b_\gamma}{ds}U^{a-b}_\beta) \\
&+\underline{term\mathcal{X}2:}\Psi_{\alpha\beta\gamma}[1-\kappa r cos\phi]W^{a-b}_\beta W^b_\gamma - \kappa\Psi_{\alpha\beta\gamma}[r cos\phi]U^{a-b}_\beta U^b_\gamma
\end{aligned}
\tag{5}
$$

With:

$$
\begin{aligned}
&\frac{d}{ds}V^a_\alpha = \{\int_0^{2\pi}\frac{h'^2}{1-\kappa h cos\phi}[r\psi_\beta\psi_\alpha]_0^h d\theta \\
&+\int_0^{2\pi}[r(1-\kappa r cos\phi)\psi_\beta\psi_\alpha]_0^h d\theta - \Psi_{[\alpha]\beta}[r(1-\kappa r cos\phi)] - \Psi_{\alpha\beta}[(1-\kappa r cos\phi)]\}U^a_\beta \\
&-G^a_{\alpha\beta}V^a_\beta
\end{aligned}
\tag{6}
$$

Transform:

$$-ia\kappa\Psi_{\alpha\beta}[r(1-\kappa rcos\phi)]V^a_\beta := -N^a_{\alpha\beta}V^a_\beta$$

$$\{\int_0^{2\pi}[\psi_\alpha\psi_\beta r(1-\kappa rcos\phi)]_0^h d\theta - \Psi_{[\alpha]\beta}[r(1-\kappa rcos\phi)] - \Psi_{\alpha\beta}[1-2\kappa rcos\phi]\}P^a_\beta$$

$$\Psi_{\{\alpha\}\beta\gamma}[r]U^{a-b}_\beta V^a_\gamma := \sum_{b=-\infty}^{+\infty}\sum_{\beta=0}^{+\infty}\sum_{\gamma=0}^{+\infty}\underline{\underline{\Psi_{\{\alpha\}\delta\epsilon}[r]\mathbf{I}^{a-b}_{\delta\beta}\mathbf{V}^b_{\epsilon\gamma}}}u^{a-b}_\beta p^b_\gamma = \sum_{b=-\infty}^{+\infty}\Psi_{\{\alpha\}\beta\gamma}[r][\mathbf{I},\mathbf{V}]u^{a-b}_\beta p^b_\gamma \to \varepsilon_{1.4}$$

$$\Psi_{[\alpha]\beta\gamma}[r(1-\kappa rcos\phi)]V^{a-b}_\beta V^a_\gamma := \sum_{b=-\infty}^{+\infty}\sum_{\beta=0}^{+\infty}\sum_{\gamma=0}^{+\infty}\underline{\underline{\Psi_{[\alpha]\delta\epsilon}[r(1-\kappa rcos\phi)]\mathbf{V}^{a-b}_{\delta\beta}\mathbf{V}^b_{\epsilon\gamma}}}p^{a-b}_\beta p^b_\gamma \to \mathcal{B}_{5.3}$$

$$\Psi_{(\alpha)\beta\gamma}[(1-\kappa rcos\phi)]W^{a-b}_\beta V^a_\gamma := \sum_{b=-\infty}^{+\infty}\sum_{\beta=0}^{+\infty}\sum_{\gamma=0}^{+\infty}\underline{\underline{\Psi_{(\alpha)\delta\epsilon}[1-\kappa rcos\phi]\mathbf{W}^{a-b}_{\delta\beta}\mathbf{V}^b_{\epsilon\gamma}}}p^{a-b}_\beta p^b_\gamma \to \mathcal{B}_{5.4}$$

$$i(a)\kappa\Psi_{\alpha\beta\gamma}[r(1-\kappa rcos\phi)]P^{a-b}_\beta V^b_\gamma := \sum_{b=-\infty}^{+\infty}\sum_{\beta=0}^{+\infty}\sum_{\gamma=0}^{+\infty}\underline{\underline{ia\kappa\Psi_{\alpha\delta\epsilon}[r(1-\kappa rcos\phi)]\mathbf{I}^{a-b}_{\delta\beta}\mathbf{V}^b_{\epsilon\gamma}}}p^{a-b}_\beta p^b_\gamma \to \mathcal{B}_{5.2}$$

$$-\frac{\partial}{\partial s}(\Psi_{\alpha\beta\gamma}[r])U^{a-b}_\beta V^b_\gamma := \sum_{b=-\infty}^{+\infty}\sum_{\beta=0}^{+\infty}\sum_{\gamma=0}^{+\infty}\underline{\underline{-\frac{\partial}{\partial s}(\Psi_{\alpha\delta\epsilon}[r])\mathbf{I}^{a-b}_{\delta\beta}\mathbf{V}^b_{\epsilon\gamma}}}u^{a-b}_\beta p^b_\gamma \to \varepsilon_{1.1}$$

$$-\Psi_{\alpha\beta\gamma}[r]\sum_{b=-\infty}^{+\infty}(\frac{dU^{a-b}_\beta}{ds}V^b_\gamma + \frac{dV^b_\gamma}{ds}U^{a-b}_\beta)$$

$$:= \Psi_{\alpha\beta\gamma}[r]\sum_{b=-\infty}^{+\infty}([(\mathbf{M}p)^{a-b}_\beta + (\mathbf{G}u)^{a-b}_\beta]V^b_\gamma$$

$$+\{\int_0^{2\pi}\frac{h'^2}{1-\kappa hcos\phi}[r\psi_\beta\psi_\alpha]_0^h d\theta$$

$$+\int_0^{2\pi}[r(1-\kappa rcos\phi)\psi_\beta\psi_\alpha]_0^h d\theta - \Psi_{[\alpha]\beta}[r(1-\kappa rcos\phi)] - \Psi_{\alpha\beta}[(1-\kappa rcos\phi)]\}U^a_\beta U^{a-b}_\beta$$

$$-G^a_{\alpha\beta}V^a_\beta U^{a-b}_\beta)$$

$$= \sum_{b=-\infty}^{+\infty}(\Psi_{\alpha\beta\gamma}[r][\mathbf{G},\mathbf{V}]])u^{a-b}_\beta p^b_\gamma \to \varepsilon_{1.2}$$

$$+\sum_{b=-\infty}^{+\infty}(\Psi_{\alpha\beta\gamma}[r][\mathbf{M},\mathbf{V}]])p^{a-b}_\beta p^b_\gamma \to \mathcal{B}_{5.1}$$

$$+\sum_{b=-\infty}^{+\infty}\Psi_{\alpha\beta\gamma}[r]\{\int_0^{2\pi}\frac{h'^2}{1-\kappa hcos\phi}[r\psi_\beta\psi_\alpha]_0^h d\theta$$

$$+\int_0^{2\pi}[r(1-\kappa rcos\phi)\psi_\beta\psi_\alpha]_0^h d\theta - \Psi_{[\alpha]\beta}[r(1-\kappa rcos\phi)] - \Psi_{\alpha\beta}[(1-\kappa rcos\phi)]\}u^{a-b}_\beta u^b_\gamma \to \mathcal{A}_{2.1,2.2,2.4,2.3}$$

$$-\sum_{b=-\infty}^{+\infty}\Psi_{\alpha\beta\gamma}[r][\mathbf{I},\mathbf{GV}])u^{a-b}_\beta p^b_\gamma \to \varepsilon_{1.3}$$

$$\Psi_{\alpha\beta\gamma}[1-\kappa rcos\phi]W^{a-b}_\beta W^b_\gamma := \sum_{b=-\infty}^{+\infty}\sum_{\beta=0}^{+\infty}\sum_{\gamma=0}^{+\infty}\underline{\underline{\Psi_{\alpha\delta\epsilon}[1-\kappa rcos\phi]\mathbf{W}^{a-b}_{\delta\beta}\mathbf{W}^b_{\epsilon\gamma}}}p^{a-b}_\beta p^b_\gamma \to \mathcal{B}_{5.5}$$

$$-\kappa\Psi_{\alpha\beta\gamma}[rcos\phi]U^{a-b}_\beta U^b_\gamma := \sum_{b=-\infty}^{+\infty}\sum_{\beta=0}^{+\infty}\sum_{\gamma=0}^{+\infty}\underline{\underline{-\kappa\Psi_{\alpha\beta\gamma}[rcos\phi]}}u^{a-b}_\beta u^b_\gamma \to \mathcal{A}_{2.5} \tag{7}$$

C. momentum equation III

$$-ia\kappa\Psi_{\alpha\beta}[r(1-\kappa rcos\phi)]W^a_\beta$$

$$-\Psi_{(\alpha)\beta}[(1-\kappa rcos\phi)]P^a_\beta - \kappa\Psi_{\alpha\beta}[rsin\phi]P^a_\beta$$

$$= \underline{term\mathcal{D}2:}\Psi_{\{\alpha\}\beta\gamma}[r]U^{a-b}_\beta W^a_\gamma + \Psi_{[\alpha]\beta\gamma}[r(1-\kappa rcos\phi)]V^{a-b}_\beta W^a_\gamma + \Psi_{(\alpha)\beta\gamma}[(1-\kappa rcos\phi)]W^{a-b}_\beta W^a_\gamma$$

$$+\underline{term(D2+\mathcal{P}2):}i(a)\kappa\Psi_{\alpha\beta\gamma}[r(1-\kappa rcos\phi)]P^{a-b}_\beta W^b_\gamma$$

$$+\underline{term\mathcal{D}2:} - \frac{\partial}{\partial s}(\Psi_{\alpha\beta\gamma}[r]U^{a-b}_\beta V^b_\gamma - \Psi_{\alpha\beta\gamma}[r](\frac{dU^{a-b}_\beta}{ds}W^b_\gamma + \frac{dW^b_\gamma}{ds}U^{a-b}_\beta)$$

$$+\underline{term\mathcal{X}2:}\kappa\Psi_{\alpha\beta\gamma}[rsin\phi]U^{a-b}_\beta U^b_\gamma - \Psi_{\alpha\beta\gamma}[1-\kappa rcos\phi]W^{a-b}_\beta V^b_\gamma \tag{8}$$

With:

$$\frac{d}{ds}W^a_\alpha = \int_0^{2\pi}\frac{dh(s)}{ds}[\psi_\beta r\psi_\alpha]^h_0 d\theta W^a_\beta + \Psi_{\{\alpha\}\beta}[r]W^a_\beta - \Psi_{(\alpha)\beta}[1-\kappa rcos\phi]U^a_\beta = -\Psi_{\alpha\{\beta\}}[r]W^a_\beta - \Psi_{(\alpha)\beta}[1-\kappa rcos\phi]U^a_\beta \tag{9}$$

Transform:

$$
\begin{aligned}
&-ia\kappa\Psi_{\alpha\beta}[r(1-\kappa rcos\phi)]W^a_\beta := -N^a_{\alpha\beta}W^a_\beta \\
&-\Psi_{(\alpha)\beta}[(1-\kappa rcos\phi)]p^a_\beta - \kappa\Psi_{\alpha\beta}[rsin\phi]p^a_\beta \\
\Psi_{\{\alpha\}\beta\gamma}[r]U^{a-b}_\beta W^a_\gamma &:= \sum_{b=-\infty}^{+\infty}\sum_{\beta=0}^{+\infty}\sum_{\gamma=0}^{+\infty}\underline{\underline{\Psi_{\{\alpha\}\delta\epsilon}[r]\mathbf{I}^{a-b}_{\delta\beta}\mathbf{W}^b_{\epsilon\gamma}}}u^{a-b}_\beta p^b_\gamma = \sum_{b=-\infty}^{+\infty}\Psi_{\{\alpha\}\beta\gamma}[r][\mathbf{I},\mathbf{W}]u^{a-b}_\beta p^b_\gamma \to \varepsilon_{2.4} \\
\Psi_{[\alpha]\beta\gamma}[r(1-\kappa rcos\phi)]V^{a-b}_\beta W^a_\gamma &:= \sum_{b=-\infty}^{+\infty}\sum_{\beta=0}^{+\infty}\sum_{\gamma=0}^{+\infty}\underline{\underline{\Psi_{[\alpha]\delta\epsilon}[r(1-\kappa rcos\phi)]\mathbf{V}^{a-b}_{\delta\beta}\mathbf{W}^b_{\epsilon\gamma}}}p^{a-b}_\beta p^b_\gamma \to \mathcal{B}_{6.3} \\
\Psi_{(\alpha)\beta\gamma}[(1-\kappa rcos\phi)]W^{a-b}_\beta W^a_\gamma &:= \sum_{b=-\infty}^{+\infty}\sum_{\beta=0}^{+\infty}\sum_{\gamma=0}^{+\infty}\underline{\underline{\Psi_{(\alpha)\delta\epsilon}[1-\kappa rcos\phi]\mathbf{W}^{a-b}_{\delta\beta}\mathbf{W}^b_{\epsilon\gamma}}}p^{a-b}_\beta p^b_\gamma \to \mathcal{B}_{6.4} \\
i(a)\kappa\Psi_{\alpha\beta\gamma}[r(1-\kappa rcos\phi)]P^{a-b}_\beta W^b_\gamma &:= \sum_{b=-\infty}^{+\infty}\sum_{\beta=0}^{+\infty}\sum_{\gamma=0}^{+\infty}\underline{\underline{ia\kappa\Psi_{\alpha\delta\epsilon}[r(1-\kappa rcos\phi)]\mathbf{I}^{a-b}_{\delta\beta}\mathbf{W}^b_{\epsilon\gamma}}}p^{a-b}_\beta p^b_\gamma \to \mathcal{B}_{6.2} \\
-\frac{\partial}{\partial s}(\Psi_{\alpha\beta\gamma}[r])U^{a-b}_\beta W^b_\gamma &:= \sum_{b=-\infty}^{+\infty}\sum_{\beta=0}^{+\infty}\sum_{\gamma=0}^{+\infty}\underline{\underline{-\frac{\partial}{\partial s}(\Psi_{\alpha\delta\epsilon}[r])\mathbf{I}^{a-b}_{\delta\beta}\mathbf{W}^b_{\epsilon\gamma}}}u^{a-b}_\beta p^b_\gamma \to \varepsilon_{2.1} \\
&-\Psi_{\alpha\beta\gamma}[r]\sum_{b=-\infty}^{+\infty}(\frac{dU^{a-b}_\beta}{ds}W^b_\gamma + \frac{dW^b_\gamma}{ds}U^{a-b}_\beta) \\
&:= \Psi_{\alpha\beta\gamma}[r]\sum_{b=-\infty}^{+\infty}([(\mathbf{M}p)^{a-b}_\beta + (\mathbf{G}u)^{a-b}_\beta]W^b_\gamma + \{\mathbf{H}W^a_\beta - \Psi_{(\alpha)\beta}[1-\kappa rcos\phi]U^a_\beta\}U^{a-b}_\beta \\
&= \sum_{b=-\infty}^{+\infty}(\Psi_{\alpha\beta\gamma}[r][\mathbf{G},\mathbf{W}]])u^{a-b}_\beta p^b_\gamma \to \varepsilon_{2.2} \\
&\sum_{b=-\infty}^{+\infty}(\Psi_{\alpha\beta\gamma}[r][\mathbf{M},\mathbf{W}]])p^{a-b}_\beta p^b_\gamma \to \mathcal{B}_{6.1} \\
&\sum_{b=-\infty}^{+\infty}\Psi_{\alpha\beta\gamma}[r][\mathbf{I},\mathbf{HW}]u^{a-b}_\beta p^b_\gamma \to \varepsilon_{2.3} \\
&\sum_{b=-\infty}^{+\infty}-\Psi_{(\alpha)\beta}[1-\kappa rcos\phi]u^a_\beta u^{a-b}_\beta \to \mathcal{A}_{3.1} \\
\kappa\Psi_{\alpha\beta\gamma}[rsin\phi]U^{a-b}_\beta U^b_\gamma &:= \sum_{b=-\infty}^{+\infty}\sum_{\beta=0}^{+\infty}\sum_{\gamma=0}^{+\infty}\underline{\underline{\kappa\Psi_{\alpha\beta\gamma}[rsin\phi]}}u^{a-b}_\beta u^b_\gamma \to \mathcal{A}_{3.2} \\
-\Psi_{\alpha\beta\gamma}[1-\kappa rcos\phi]W^{a-b}_\beta V^b_\gamma &:= \sum_{b=-\infty}^{+\infty}\sum_{\beta=0}^{+\infty}\sum_{\gamma=0}^{+\infty}\underline{\underline{-\Psi_{\alpha\delta\epsilon}[1-\kappa rcos\phi]\mathbf{W}^{a-b}_{\delta\beta}\mathbf{V}^b_{\epsilon\gamma}}}p^{a-b}_\beta p^b_\gamma \to \mathcal{B}_{6.5}
\end{aligned} \tag{10}
$$

[1] Here, may be a little question of transform with P, think about N^-1, transform it as matrix we could solve it.