Note:derivation of 3-31

Email: jiaqi_wang@sjtu.edu.cn

A. Substitue pressure modes for transverse velocity modes

1. *mass equation*

$$
\begin{aligned}
&\frac{dU^a_\alpha}{ds} - \Psi_{\{\alpha\}\beta}[r]U^a_\beta - ia\kappa\Psi_{\alpha\beta}[r(1-\kappa cos(\phi))]P^a_\beta - \Psi_{[\alpha]\beta}[r(1-\kappa rcos(\phi))]\underline{\underline{\underline{V^a_\beta}}} - \Psi_{(\alpha)\beta}[(1-\kappa rcos(\phi))]\underline{\underline{\underline{W^a_\beta}}} \\
&= \Psi_{\alpha\beta\gamma}[r(1-\kappa rcos(\phi))]\sum_{b=-\infty}^{+\infty}(-ib\kappa P^{a-b}_\beta P^b_\gamma - ib\kappa U^{a-b}_\beta U^b_\gamma - ib\kappa \underline{\underline{\underline{V^{a-b}_\beta V^b_\gamma}}} - ib\kappa \underline{\underline{\underline{W^{a-b}_\beta W^b_\gamma}}} - ia\kappa\frac{B}{2A}P^{a-b}_\beta P^b_\gamma)
\end{aligned} \tag{1}
$$

Transform:

$$
\begin{aligned}
\frac{dU^a_\alpha}{ds} &:= \underline{\underline{I}}u'^a_\alpha \\
-\Psi_{\{\alpha\}\beta}[r]U^a_\beta &:= \underline{\underline{\underline{-\Psi_{\{\alpha\}\beta}[r]}}}u^a_\beta \to \mathcal{G} \\
&\begin{cases}
-ia\kappa\Psi_{\alpha\beta}[r(1-\kappa cos(\phi))]P^a_\beta := \sum_{\beta=0}^{+\infty}\underline{\underline{-ia\kappa\Psi_{\alpha\beta}[r(1-\kappa cos(\phi))]}}p^a_\beta \to \mathcal{M}_1 \\
-\Psi_{[\alpha]\beta}[r(1-\kappa rcos(\phi))]\underline{\underline{\underline{V^a_\beta}}} := \sum_{\beta=0}^{+\infty}\underline{\underline{-\Psi_{[\alpha]\delta}[r(1-\kappa rcos(\phi))]\mathbf{V}^a_{\delta\beta}}}p^a_\beta \to \mathcal{M}_2 \\
-\Psi_{(\alpha)\beta}[(1-\kappa rcos(\phi))]\underline{\underline{\underline{W^a_\beta}}} := \sum_{\beta=0}^{+\infty}\underline{\underline{-\Psi_{(\alpha)\delta}[(1-\kappa rcos(\phi))]\mathbf{W}^a_{\delta\beta}}}p^a_\beta \to \mathcal{M}_3
\end{cases} \\
\Psi_{\alpha\beta\gamma}[r(1-\kappa rcos(\phi))]\sum_{b=-\infty}^{+\infty}(-ib\kappa P^{a-b}_\beta P^b_\gamma) &:= \sum_{b=-\infty}^{+\infty}-ib\kappa\sum_{\beta=0}^{+\infty}\sum_{\gamma=0}^{+\infty}\underline{\underline{\Psi_{\alpha\beta\gamma}[(1-\kappa rcos(\phi))]}}p^{a-b}_\beta p^b_\gamma \to \mathcal{B}_2 \\
\Psi_{\alpha\beta\gamma}[r(1-\kappa rcos(\phi))]\sum_{b=-\infty}^{+\infty}(-ib\kappa U^{a-b}_\beta U^b_\gamma) &:= \sum_{b=-\infty}^{+\infty}-ib\kappa\sum_{\beta=0}^{+\infty}\sum_{\gamma=0}^{+\infty}\underline{\underline{\Psi_{\alpha\beta\gamma}[(1-\kappa rcos(\phi))]}}u^{a-b}_\beta u^b_\gamma \to \mathcal{A}_1 \\
\Psi_{\alpha\beta\gamma}[r(1-\kappa rcos(\phi))]\sum_{b=-\infty}^{+\infty}(-ib\kappa V^{a-b}_\beta V^b_\gamma) &:= \sum_{b=-\infty}^{+\infty}-ib\kappa\sum_{\beta=0}^{+\infty}\sum_{\gamma=0}^{+\infty}\underline{\underline{\sum_{\delta,\epsilon=0}^{\infty}\Psi_{\alpha\delta\epsilon}[r(1-\kappa rcos\phi)]\mathbf{V}^{a-b}_{\delta\beta}\mathbf{V}^b_{\epsilon\gamma}p^{a-b}_\beta p^b_\gamma}} \to \mathcal{B}_3 \\
\Psi_{\alpha\beta\gamma}[r(1-\kappa rcos(\phi))]\sum_{b=-\infty}^{+\infty}(-ib\kappa W^{a-b}_\beta W^b_\gamma) &:= \sum_{b=-\infty}^{+\infty}-ib\kappa\sum_{\beta=0}^{+\infty}\sum_{\gamma=0}^{+\infty}\underline{\underline{\sum_{\delta,\epsilon=0}^{\infty}\Psi_{\alpha\delta\epsilon}[r(1-\kappa rcos\phi)]\mathbf{W}^{a-b}_{\delta\beta}\mathbf{W}^b_{\epsilon\gamma}p^{a-b}_\beta p^b_\gamma}} \to \mathcal{B}_4 \\
\Psi_{\alpha\beta\gamma}[r(1-\kappa rcos(\phi))]\sum_{b=-\infty}^{+\infty}(-ia\kappa\frac{B}{2A}P^{a-b}_\beta P^b_\gamma) &:= \sum_{b=-\infty}^{+\infty}-ia\kappa\frac{B}{2A}\sum_{\beta=0}^{+\infty}\sum_{\gamma=0}^{+\infty}\underline{\underline{\Psi_{\alpha\beta\gamma}[(1-\kappa rcos(\phi))]}}p^{a-b}_\beta p^b_\gamma \to \mathcal{B}_1
\end{aligned} \tag{2}
$$

2. *momentum equation I*

$$\frac{d}{ds}P^a_\alpha - ia\kappa\Psi_{\alpha\beta}[r(1-\kappa rcos\phi)]U^a_\beta - \int_0^{2\pi} hh'[\psi_\beta\psi_\alpha]_{r=h}d\theta P^a_\beta - \Psi_{\{\alpha\}\beta}[r]P^a_\beta$$
$$= \sum_{b=-\infty}^{+\infty} \underline{term\mathcal{D}1:}\Psi_{\{\alpha\}\beta\gamma}[r]U^{a-b}_\beta U^a_\gamma + \Psi_{[\alpha]\beta\gamma}[r(1-\kappa rcos\phi)]\underline{\underline{V^{a-b}_\beta}}U^a_\gamma + \Psi_{(\alpha)\beta\gamma}[(1-\kappa rcos\phi)]\underline{\underline{W^{a-b}_\beta}}U^a_\gamma$$
$$+\underline{term(D1+\mathcal{P}1):}i(a)\kappa\Psi_{\alpha\beta\gamma}[r(1-\kappa rcos\phi)]P^{a-b}_\beta U^b_\gamma$$
$$+\underline{term\mathcal{D}1:}-\frac{\partial}{\partial s}(\Psi_{\alpha\beta\gamma}[r])U^{a-b}_\beta U^b_\gamma - \Psi_{\alpha\beta\gamma}[r](\frac{dU^{a-b}_\beta}{ds}U^b_\gamma + \frac{dU^b_\gamma}{ds}U^{a-b}_\beta)$$
$$+\underline{term\mathcal{X}1:}\Psi_{\alpha\beta\gamma}[rcos\phi]U^{a-b}_\beta \underline{\underline{V^b_\gamma}} - \kappa\Psi_{\alpha\beta\gamma}[rsin\phi]U^{a-b}_\beta\underline{\underline{W^b_\gamma}} \tag{3}$$

Transform:

$$\frac{d}{ds}P^a_\alpha := \underline{\underline{I}}p'^a_\alpha$$
$$-ia\kappa\Psi_{\alpha\beta}[r(1-\kappa rcos\phi)]U^a_\beta := \underline{\underline{-ia\kappa\Psi_{\alpha\beta}[r(1-\kappa cos(\phi))]}}u^a_\beta \to \mathcal{N}$$
$$-\int_0^{2\pi} hh'[\psi_\beta\psi_\alpha]_{r=h}d\theta P^a_\beta - \Psi_{\{\alpha\}\beta}[r]P^a_\beta := \underline{\underline{-\int_0^{2\pi} hh'[\psi_\beta\psi_\alpha]_{r=h}d\theta}}p^a_\beta - \underline{\underline{\Psi_{\{\alpha\}\beta}[r]}}p^a_\beta = -\underline{\underline{\Psi_{\{\alpha\}\beta}[r]}}p^a_\beta \to \mathcal{H}$$
$$\Psi_{\{\alpha\}\beta\gamma}[r]\sum_{b=-\infty}^{+\infty} U^{a-b}_\beta U^a_\gamma := \sum_{b=-\infty}^{+\infty}\sum_{\beta=0}^{+\infty}\sum_{\gamma=0}^{+\infty}\underline{\underline{\Psi_{\{\alpha\}\beta\gamma}[r]}}u^{a-b}_\beta u^b_\gamma \to \mathcal{D}$$
$$\Psi_{[\alpha]\beta\gamma}[r(1-\kappa rcos\phi)]\sum_{b=-\infty}^{+\infty}\underline{\underline{V^{a-b}_\beta}}U^a_\gamma := \sum_{b=-\infty}^{+\infty}\sum_{\beta=0}^{+\infty}\sum_{\gamma=0}^{+\infty}\underline{\underline{\sum_{\delta,\epsilon=0}^{\infty}\Psi_{[\alpha]\delta\epsilon}[r(1-\kappa rcos\phi)]\mathbf{V}^{a-b}_{\delta\beta}\mathbf{I}^b_{\epsilon\gamma}}}p^{a-b}_\beta u^b_\gamma \to \mathcal{C}_4$$
$$\Psi_{(\alpha)\beta\gamma}[(1-\kappa rcos\phi)]\sum_{b=-\infty}^{+\infty}\underline{\underline{W^{a-b}_\beta}}U^a_\gamma := \sum_{b=-\infty}^{+\infty}\sum_{\beta=0}^{+\infty}\sum_{\gamma=0}^{+\infty}\underline{\underline{\sum_{\delta,\epsilon=0}^{\infty}\Psi_{(\alpha)\delta\epsilon}[(1-\kappa rcos\phi)]\mathbf{W}^{a-b}_{\delta\beta}\mathbf{I}^b_{\epsilon\gamma}}}p^{a-b}_\beta u^b_\gamma \to \mathcal{C}_5$$
$$ia\kappa\Psi_{\alpha\beta\gamma}[r(1-\kappa rcos\phi)]\sum_{b=-\infty}^{+\infty}P^{a-b}_\beta U^b_\gamma := ia\kappa\sum_{b=-\infty}^{+\infty}\sum_{\beta=0}^{+\infty}\sum_{\gamma=0}^{+\infty}\underline{\underline{\Psi_{\alpha\beta\gamma}[r(1-\kappa rcos\phi)]}}p^{a-b}_\beta u^b_\gamma \to \mathcal{C}_3$$
$$-\frac{\partial}{\partial s}(\Psi_{\alpha\beta\gamma}[r])\sum_{b=-\infty}^{+\infty}U^{a-b}_\beta U^b_\gamma := \sum_{b=-\infty}^{+\infty}\sum_{\beta=0}^{+\infty}\sum_{\gamma=0}^{+\infty}\underline{\underline{-\frac{\partial}{\partial s}(\Psi_{\alpha\beta\gamma}[r])}}u^{a-b}_\beta u^b_\gamma \to \mathcal{D}$$
$$-\Psi_{\alpha\beta\gamma}[r]\sum_{b=-\infty}^{+\infty}(\frac{dU^{a-b}_\beta}{ds}U^b_\gamma + \frac{dU^b_\gamma}{ds}U^{a-b}_\beta) := \Psi_{\alpha\beta\gamma}[r]\sum_{b=-\infty}^{+\infty}([(Mp)^{a-b}_\beta + (Gu)^{a-b}_\beta]U^b_\gamma + [(Mp)^b_\gamma + (Gu)^b_\gamma]U^{a-b}_\beta)$$
$$= \sum_{b=-\infty}^{+\infty}(\Psi_{\alpha\beta\gamma}[r][\mathbf{G},\mathbf{I}] + \Psi_{\alpha\beta\gamma}[r][\mathbf{I},\mathbf{G}])u^{a-b}_\beta u^b_\gamma \to \mathcal{D}_{2,3}$$
$$+ \sum_{b=-\infty}^{+\infty}(\Psi_{\alpha\beta\gamma}[r][\mathbf{M},\mathbf{I}] + \Psi_{\alpha\beta\gamma}[r][\mathbf{I},\mathbf{M}])u^{a-b}_\beta p^b_\gamma \to \mathcal{C}_{1,2}$$
$$\Psi_{\alpha\beta\gamma}[rcos\phi]U^{a-b}_\beta\underline{\underline{V^b_\gamma}} - \kappa\Psi_{\alpha\beta\gamma}[rsin\phi]U^{a-b}_\beta\underline{\underline{W^b_\gamma}} := \{\sum_{b=-\infty}^{+\infty}\Psi_{\alpha\beta\gamma}[rcos\phi][\mathbf{I},\mathbf{V}] - \kappa\Psi_{\alpha\beta\gamma}[rsin\phi][\mathbf{I},\mathbf{W}]\}u^{a-b}_\gamma p^b_\beta \to \mathcal{C}_{6,7} \tag{4}$$

$$
\begin{aligned}
&-ia\kappa\Psi_{\alpha\beta}[r(1-\kappa rcos\phi)]V^a_\beta \\
&+\int_0^{2\pi}[\psi_\alpha\psi_\beta r(1-\kappa rcos\phi)]_0^h d\theta P^a_\beta - \Psi_{[\alpha]\beta}[r(1-\kappa rcos\phi)]P^a_\beta - \Psi_{\alpha\beta}[1-2\kappa rcos\phi]P^a_\beta \\
&= \underline{term\mathcal{D}2:}\Psi_{\{\alpha\}\beta\gamma}[r]U^{a-b}_\beta V^a_\gamma + \Psi_{[\alpha]\beta\gamma}[r(1-\kappa rcos\phi)]V^{a-b}_\beta V^a_\gamma + \Psi_{(\alpha)\beta\gamma}[(1-\kappa rcos\phi)]W^{a-b}_\beta V^a_\gamma \\
&+\underline{term(D2+\mathcal{P}2):}i(a)\kappa\Psi_{\alpha\beta\gamma}[r(1-\kappa rcos\phi)]P^{a-b}_\beta V^b_\gamma \\
&+\underline{term\mathcal{D}2:}-\frac{\partial}{\partial s}(\Psi_{\alpha\beta\gamma}[r])U^{a-b}_\beta V^b_\gamma - \Psi_{\alpha\beta\gamma}[r](\frac{dU^{a-b}_\beta}{ds}V^b_\gamma + \frac{dV^b_\gamma}{ds}U^{a-b}_\beta) \\
&+\underline{term\mathcal{X}2:}\Psi_{\alpha\beta\gamma}[1-\kappa rcos\phi]W^{a-b}_\beta W^b_\gamma - \kappa\Psi_{\alpha\beta\gamma}[rcos\phi]U^{a-b}_\beta U^b_\gamma
\end{aligned} \tag{5}
$$

With:

$$
\begin{aligned}
&\frac{d}{ds}V^a_\alpha = \{\int_0^{2\pi}\frac{h'^2}{1-\kappa hcos\phi}[r\psi_\beta\psi_\alpha]_0^h d\theta \\
&+\int_0^{2\pi}[r(1-\kappa rcos\phi)\psi_\beta\psi_\alpha]_0^h d\theta - \Psi_{[\alpha]\beta}[r(1-\kappa rcos\phi)] - \Psi_{\alpha\beta}[(1-\kappa rcos\phi)]\}U^a_\beta \\
&-G^a_{\alpha\beta}V^a_\beta
\end{aligned} \tag{6}
$$

Transform:

$$
\begin{aligned}
&-ia\kappa\Psi_{\alpha\beta}[r(1-\kappa r cos\phi)]V^a_\beta := -N^a_{\alpha\beta}V^a_\beta \\
&\{\int_0^{2\pi}[\psi_\alpha\psi_\beta r(1-\kappa r cos\phi)]_0^h d\theta - \Psi_{[\alpha]\beta}[r(1-\kappa r cos\phi)] - \Psi_{\alpha\beta}[1-2\kappa r cos\phi]\}P^a_\beta \\
&\Psi_{\{\alpha\}\beta\gamma}[r]U^{a-b}_\beta V^a_\gamma := \sum_{b=-\infty}^{+\infty}\sum_{\beta=0}^{+\infty}\sum_{\gamma=0}^{+\infty}\underline{\underline{\Psi_{\{\alpha\}\delta\epsilon}[r]\mathbf{I}^{a-b}_{\delta\beta}\mathbf{V}^b_{\epsilon\gamma}}}u^{a-b}_\beta p^b_\gamma = \sum_{b=-\infty}^{+\infty}\Psi_{\{\alpha\}\beta\gamma}[r][\mathbf{I},\mathbf{V}]u^{a-b}_\beta p^b_\gamma \to \varepsilon_{1.4} \\
&\Psi_{[\alpha]\beta\gamma}[r(1-\kappa r cos\phi)]V^{a-b}_\beta V^a_\gamma := \sum_{b=-\infty}^{+\infty}\sum_{\beta=0}^{+\infty}\sum_{\gamma=0}^{+\infty}\underline{\underline{\Psi_{[\alpha]\delta\epsilon}[r(1-\kappa r cos\phi)]\mathbf{V}^{a-b}_{\delta\beta}\mathbf{V}^b_{\epsilon\gamma}}}p^{a-b}_\beta p^b_\gamma \to \mathcal{B}_{5.3} \\
&\Psi_{(\alpha)\beta\gamma}[(1-\kappa r cos\phi)]W^{a-b}_\beta V^a_\gamma := \sum_{b=-\infty}^{+\infty}\sum_{\beta=0}^{+\infty}\sum_{\gamma=0}^{+\infty}\underline{\underline{\Psi_{(\alpha)\delta\epsilon}[1-\kappa r cos\phi]\mathbf{W}^{a-b}_{\delta\beta}\mathbf{V}^b_{\epsilon\gamma}}}p^{a-b}_\beta p^b_\gamma \to \mathcal{B}_{5.4} \\
&i(a)\kappa\Psi_{\alpha\beta\gamma}[r(1-\kappa r cos\phi)]P^{a-b}_\beta V^b_\gamma := \sum_{b=-\infty}^{+\infty}\sum_{\beta=0}^{+\infty}\sum_{\gamma=0}^{+\infty}\underline{\underline{ia\kappa\Psi_{\alpha\delta\epsilon}[r(1-\kappa r cos\phi)]\mathbf{I}^{a-b}_{\delta\beta}\mathbf{V}^b_{\epsilon\gamma}}}p^{a-b}_\beta p^b_\gamma \to \mathcal{B}_{5.2} \\
&-\frac{\partial}{\partial s}(\Psi_{\alpha\beta\gamma}[r])U^{a-b}_\beta V^b_\gamma := \sum_{b=-\infty}^{+\infty}\sum_{\beta=0}^{+\infty}\sum_{\gamma=0}^{+\infty}\underline{\underline{-\frac{\partial}{\partial s}(\Psi_{\alpha\delta\epsilon}[r])\mathbf{I}^{a-b}_{\delta\beta}\mathbf{V}^b_{\epsilon\gamma}}}u^{a-b}_\beta p^b_\gamma \to \varepsilon_{1.1} \\
&-\Psi_{\alpha\beta\gamma}[r]\sum_{b=-\infty}^{+\infty}(\frac{dU^{a-b}_\beta}{ds}V^b_\gamma + \frac{dV^b_\gamma}{ds}U^{a-b}_\beta) \\
&:= \Psi_{\alpha\beta\gamma}[r]\sum_{b=-\infty}^{+\infty}([(\mathbf{M}p)^{a-b}_\beta + (\mathbf{G}u)^{a-b}_\beta]V^b_\gamma \\
&+\{\int_0^{2\pi}\frac{{h'}^2}{1-\kappa h cos\phi}[r\psi_\beta\psi_\alpha]_0^h d\theta \\
&+\int_0^{2\pi}[r(1-\kappa r cos\phi)\psi_\beta\psi_\alpha]_0^h d\theta - \Psi_{[\alpha]\beta}[r(1-\kappa r cos\phi)] - \Psi_{\alpha\beta}[(1-\kappa r cos\phi)]\}U^a_\beta U^{a-b}_\beta \\
&-G^a_{\alpha\beta}V^a_\beta U^{a-b}_\beta) \\
&= \sum_{b=-\infty}^{+\infty}(\Psi_{\alpha\beta\gamma}[r][\mathbf{G},\mathbf{V}]])u^{a-b}_\beta p^b_\gamma \to \varepsilon_{1.2} \\
&+\sum_{b=-\infty}^{+\infty}(\Psi_{\alpha\beta\gamma}[r][\mathbf{M},\mathbf{V}]])p^{a-b}_\beta p^b_\gamma \to \mathcal{B}_{5.1} \\
&+\sum_{b=-\infty}^{+\infty}\Psi_{\alpha\beta\gamma}[r]\{\int_0^{2\pi}\frac{{h'}^2}{1-\kappa h cos\phi}[r\psi_\beta\psi_\alpha]_0^h d\theta \\
&+\int_0^{2\pi}[r(1-\kappa r cos\phi)\psi_\beta\psi_\alpha]_0^h d\theta - \Psi_{[\alpha]\beta}[r(1-\kappa r cos\phi)] - \Psi_{\alpha\beta}[(1-\kappa r cos\phi)]\}u^{a-b}_\beta u^b_\gamma \to \mathcal{A}_{2.1,2.2,2.4,2.3} \\
&-\sum_{b=-\infty}^{+\infty}\Psi_{\alpha\beta\gamma}[r][\mathbf{I},\mathbf{G}\mathbf{V}])u^{a-b}_\beta p^b_\gamma \to \varepsilon_{1.3} \\
&\Psi_{\alpha\beta\gamma}[1-\kappa r cos\phi]W^{a-b}_\beta W^b_\gamma := \sum_{b=-\infty}^{+\infty}\sum_{\beta=0}^{+\infty}\sum_{\gamma=0}^{+\infty}\underline{\underline{\Psi_{\alpha\delta\epsilon}[1-\kappa r cos\phi]\mathbf{W}^{a-b}_{\delta\beta}\mathbf{W}^b_{\epsilon\gamma}}}p^{a-b}_\beta p^b_\gamma \to \mathcal{B}_{5.5} \\
&-\kappa\Psi_{\alpha\beta\gamma}[r cos\phi]U^{a-b}_\beta U^b_\gamma := \sum_{b=-\infty}^{+\infty}\sum_{\beta=0}^{+\infty}\sum_{\gamma=0}^{+\infty}\underline{\underline{-\kappa\Psi_{\alpha\beta\gamma}[r cos\phi]}}u^{a-b}_\beta u^b_\gamma \to \mathcal{A}_{2.5}
\end{aligned} \tag{7}
$$

4. momentum equation III

$$
\begin{aligned}
&-ia\kappa\Psi_{\alpha\beta}[r(1-\kappa rcos\phi)]W^a_\beta \\
&-\Psi_{(\alpha)\beta}[(1-\kappa rcos\phi)]P^a_\beta - \kappa\Psi_{\alpha\beta}[rsin\phi]P^a_\beta \\
= \underline{term\mathcal{D}2:}&\Psi_{\{\alpha\}\beta\gamma}[r]U^{a-b}_\beta W^a_\gamma + \Psi_{[\alpha]\beta\gamma}[r(1-\kappa rcos\phi)]V^{a-b}_\beta W^a_\gamma + \Psi_{(\alpha)\beta\gamma}[(1-\kappa rcos\phi)]W^{a-b}_\beta W^a_\gamma \\
&+\underline{term(D2+\mathcal{P}2):}i(a)\kappa\Psi_{\alpha\beta\gamma}[r(1-\kappa rcos\phi)]P^{a-b}_\beta W^b_\gamma \\
&+\underline{term\mathcal{D}2:}-\frac{\partial}{\partial s}(\Psi_{\alpha\beta\gamma}[r]U^{a-b}_\beta V^b_\gamma - \Psi_{\alpha\beta\gamma}[r](\frac{dU^{a-b}_\beta}{ds}W^b_\gamma + \frac{dW^b_\gamma}{ds}U^{a-b}_\beta) \\
&+\underline{term\mathcal{X}2:}\kappa\Psi_{\alpha\beta\gamma}[rsin\phi]U^{a-b}_\beta U^b_\gamma - \Psi_{\alpha\beta\gamma}[1-\kappa rcos\phi]W^{a-b}_\beta V^b_\gamma
\end{aligned} \tag{8}
$$

With:

$$
\underline{\underline{\frac{d}{ds}W^a_\alpha = \int_0^{2\pi}\frac{dh(s)}{ds}[\psi_\beta r\psi_\alpha]^h_0 d\theta W^a_\beta + \Psi_{\{\alpha\}\beta}[r]W^a_\beta - \Psi_{(\alpha)\beta}[1-\kappa rcos\phi]U^a_\beta = -\Psi_{\alpha\{\beta\}}[r]W^a_\beta - \Psi_{(\alpha)\beta}[1-\kappa rcos\phi]U^a_\beta}} \tag{9}
$$

Transform:

$$
\begin{aligned}
-ia\kappa\Psi_{\alpha\beta}[r(1-\kappa rcos\phi)]W^a_\beta &:= -N^a_{\alpha\beta}W^a_\beta \\
&-\Psi_{(\alpha)\beta}[(1-\kappa rcos\phi)]p^a_\beta - \kappa\Psi_{\alpha\beta}[rsin\phi]p^a_\beta \\
\Psi_{\{\alpha\}\beta\gamma}[r]U^{a-b}_\beta W^a_\gamma &:= \sum_{b=-\infty}^{+\infty}\sum_{\beta=0}^{+\infty}\sum_{\gamma=0}^{+\infty}\underline{\underline{\Psi_{\{\alpha\}\delta\epsilon}[r]\mathbf{I}^{a-b}_{\delta\beta}\mathbf{W}^b_{\epsilon\gamma}}}u^{a-b}_\beta p^b_\gamma = \sum_{b=-\infty}^{+\infty}\Psi_{\{\alpha\}\beta\gamma}[r][\mathbf{I},\mathbf{W}]u^{a-b}_\beta p^b_\gamma \to \varepsilon_{2.4} \\
\Psi_{[\alpha]\beta\gamma}[r(1-\kappa rcos\phi)]V^{a-b}_\beta W^a_\gamma &:= \sum_{b=-\infty}^{+\infty}\sum_{\beta=0}^{+\infty}\sum_{\gamma=0}^{+\infty}\underline{\underline{\Psi_{[\alpha]\delta\epsilon}[r(1-\kappa rcos\phi)]\mathbf{V}^{a-b}_{\delta\beta}\mathbf{W}^b_{\epsilon\gamma}}}p^{a-b}_\beta p^b_\gamma \to \mathcal{B}_{6.3} \\
\Psi_{(\alpha)\beta\gamma}[(1-\kappa rcos\phi)]W^{a-b}_\beta W^a_\gamma &:= \sum_{b=-\infty}^{+\infty}\sum_{\beta=0}^{+\infty}\sum_{\gamma=0}^{+\infty}\underline{\underline{\Psi_{(\alpha)\delta\epsilon}[1-\kappa rcos\phi]\mathbf{W}^{a-b}_{\delta\beta}\mathbf{W}^b_{\epsilon\gamma}}}p^{a-b}_\beta p^b_\gamma \to \mathcal{B}_{6.4} \\
i(a)\kappa\Psi_{\alpha\beta\gamma}[r(1-\kappa rcos\phi)]P^{a-b}_\beta W^b_\gamma &:= \sum_{b=-\infty}^{+\infty}\sum_{\beta=0}^{+\infty}\sum_{\gamma=0}^{+\infty}\underline{\underline{ia\kappa\Psi_{\alpha\delta\epsilon}[r(1-\kappa rcos\phi)]\mathbf{I}^{a-b}_{\delta\beta}\mathbf{W}^b_{\epsilon\gamma}}}p^{a-b}_\beta p^b_\gamma \to \mathcal{B}_{6.2} \\
-\frac{\partial}{\partial s}(\Psi_{\alpha\beta\gamma}[r])U^{a-b}_\beta W^b_\gamma &:= \sum_{b=-\infty}^{+\infty}\sum_{\beta=0}^{+\infty}\sum_{\gamma=0}^{+\infty}\underline{\underline{-\frac{\partial}{\partial s}(\Psi_{\alpha\delta\epsilon}[r])\mathbf{I}^{a-b}_{\delta\beta}\mathbf{W}^b_{\epsilon\gamma}}}u^{a-b}_\beta p^b_\gamma \to \varepsilon_{2.1} \\
&-\Psi_{\alpha\beta\gamma}[r]\sum_{b=-\infty}^{+\infty}(\frac{dU^{a-b}_\beta}{ds}W^b_\gamma + \frac{dW^b_\gamma}{ds}U^{a-b}_\beta) \\
&:= \Psi_{\alpha\beta\gamma}[r]\sum_{b=-\infty}^{+\infty}([(\mathbf{M}p)^{a-b}_\beta + (\mathbf{G}u)^{a-b}_\beta]W^b_\gamma + \{\mathbf{H}W^a_\beta - \Psi_{(\alpha)\beta}[1-\kappa rcos\phi]U^a_\beta\}U^{a-b}_\beta \\
&= \sum_{b=-\infty}^{+\infty}(\Psi_{\alpha\beta\gamma}[r][\mathbf{G},\mathbf{W}]])u^{a-b}_\beta p^b_\gamma \to \varepsilon_{2.2} \\
&\sum_{b=-\infty}^{+\infty}(\Psi_{\alpha\beta\gamma}[r][\mathbf{M},\mathbf{W}]])p^{a-b}_\beta p^b_\gamma \to \mathcal{B}_{6.1} \\
&\sum_{b=-\infty}^{+\infty}\Psi_{\alpha\beta\gamma}[r][I,\mathbf{HW}]u^{a-b}_\beta p^b_\gamma \to \varepsilon_{2.3} \\
&\sum_{b=-\infty}^{+\infty}-\Psi_{(\alpha)\beta}[1-\kappa rcos\phi]u^a_\beta u^{a-b}_\beta \to \mathcal{A}_{3.1} \\
\kappa\Psi_{\alpha\beta\gamma}[rsin\phi]U^{a-b}_\beta U^b_\gamma &:= \sum_{b=-\infty}^{+\infty}\sum_{\beta=0}^{+\infty}\sum_{\gamma=0}^{+\infty}\underline{\underline{\kappa\Psi_{\alpha\beta\gamma}[rsin\phi]}}u^{a-b}_\beta u^b_\gamma \to \mathcal{A}_{3.2} \\
-\Psi_{\alpha\beta\gamma}[1-\kappa rcos\phi]W^{a-b}_\beta V^b_\gamma &:= \sum_{b=-\infty}^{+\infty}\sum_{\beta=0}^{+\infty}\sum_{\gamma=0}^{+\infty}\underline{\underline{-\Psi_{\alpha\delta\epsilon}[1-\kappa rcos\phi]\mathbf{W}^{a-b}_{\delta\beta}\mathbf{V}^b_{\epsilon\gamma}}}p^{a-b}_\beta p^b_\gamma \to \mathcal{B}_{6.5}
\end{aligned}
\tag{10}
$$