This manuscript has been previously reviewed at another journal. This document only contains reviewer comments and rebuttal letters for versions considered at Communications Physics

Reviewers' comments:

Reviewer #1 (Remarks to the Author):

Report in the attached file.

Reviewer #2 (Remarks to the Author):

The paper proposes an experimental implementation of Bell states generation for elastic waves. The experimental sample is formed of three elastically coupled aluminum rods, connected by epoxy joints. Owing to the low-frequency range of operation, there is a single quasi-longitudinal guided elastic mode for the rod, that can propagate to either right (forward) or left (back). The elastic contrast between aluminum and epoxy is large, leading to the assumption that elastic energy remains mostly in the bars and the collective guided modes are limited to a number of 6 modes (3 aluminum rods each supporting 2 guided waves get slightly coupled by epoxy).
In the summary above, I have used on purpose elastic wave terminology and not Bell states as in the paper. The reason is that I feel the authors should better explain their assumptions and what is not straightforward from the point of view of elastic waves in solids. If they can do that, the paper should have more novelty and hence impact.
I have the following main remarks.
1. The frequency range of operation is limited to 50 kHz at maximum. I am missing the dispersion relation of the elastic guided waves in the free aluminium rods. Indeed, the guided spectrum of an elastic waveguide contains three branches starting from the zero frequency and an infinite number of bands as frequency increases. Why can we consider only the fundamental quasi-longitudinal dispersion branch here? [I have generated the dispersion relation for myself because I was unsure what the authors meant at lines 98-101: that statement is wrong or badly phrased and must be revised.]
2. Why matrix M33 has the form announced above equation (2) is unsubstantiated. Was epoxy really added exactly so as to obtain integer coefficients of the matrix or what is the idea behind that particular form? It looks like the authors consider a first-order finite difference approximation of some derivative between d.o.f. lying on each rod (by why?). Why does the coupling matrix has diagonal elements? (and what is the meaning of self-coupling for a mode of a rod?) Why is α^2 introduced bluntly and has the coefficient any reason to be real and positive from the start, whereas the final discussion stresses complex values for other parameters? Actually, coupling results from elastic waves in epoxy and is there a reason why some phase can not be possibly introduced between distant rods, just because of causality? As a remark, the eigenvalues of matrix M33 are 0, 1 and 3, but those values are given for its square root, is this an error?
3. Actually, obtaining the elastic guided modes of the composite of 3 rods plus epoxy should be very easy, e.g. with the finite element; why not show it as well? The analytic model to which the measurements are fitted against seems just too simple to be true at first sight. As a remark some numbers are missing in the paper in order to describe completely the elastic structure (elastic constants of aluminum for instance).
4. The main 'classical' property of Bell states appearing on the paper is said to be 'nonseparability'. What the word actually means in the context of elastic waves remains unclear to this reviewer. The analysis in the paper sets forward 6 degrees of freedom, which I understand to be 3*2 coupled L-waves of the rods. This is a simplification of an elastic wave field that exists throughout the sample (the 3 rods and the epoxy joints) and that is globally 'nonseparable' in the sense that if you change a boundary condition anywhere the global response is changed everywhere. Is 'nonseparability' a property of the discrete states used in the paper or is it just inherited from the elastic solid supporting the degrees of freedom?
I think all these comments must be addressed before the paper can be considered any further.

Reviewer 1 Review Attachment 1

Report on manuscript 19-0158-T:

The Sound of Bell States

The authors report the production of non-separable acoustic modes in elastic waveguides. They employed three waveguides made of aluminum rods coupled by epoxy fillings inserted in the gaps between the rods. The acoustic waves were excited by transducers placed at one end of the rods and detected by other transducers placed at the other end. The acoustic waves are initially described by a set of three coupled wave equations for three acoustic perturbations, one on each waveguide. Then, they factorize the set of three second order equations as a set of two first order equations in a similar procedure to the passage from the Klein-Gordon to the Dirac equation in relativistic Quantum Mechanics. The resulting Dirac-like equations involve a six-component wave function. The Hilbert space of those can be cast in the form of a tensor product between a spinor part associated with the propagation direction (backwards and forwards) in the waveguides and a three-level degree of freedom (DoF) associated with the eigenvectors of the coupling matrix. This three-level DoF is formally equivalent to the orbital angular momentum of a quantum particle. In this context non-separability is realized between the spinor and OAM degrees of freedom. These non-separable modes are the eigenmodes of the coupled system with well-defined resonance frequencies. By adjusting the excitation amplitudes and phases of the waveguides, the authors demonstrate the operation of non-separable modes at 33.25 kHz.

In my opinion, realization of classical entanglement in elastic waves can lead to interesting quantum information inspired experiments in Acoustics. The manuscript is well written and the whole presentation is good. However, there are a few points to be addressed before acceptance:

1- In the discussion the authors make an inaccurate statement: “for the case of laser light the coupling between the degrees of freedom is weak and, hence, realizing all the possible relationships for the nonseparable superpositions is not possible.” Q-plates are a quite flexible and efficient tool for preparation of non-separable spin-OAM states of a laser beam thanks to the strong spin-orbit coupling achieved in liquid crystals. There are also the so-called S-plates for preparation of radially polarized beams, that are also non-separable spin-OAM states. With these devices complemented by birefringent wave plates, any spin-OAM state can be easily realized. I suggest the authors remove this kind of assessment in the manuscript.

2- The authors talk about Bell states, but they do not evaluate any entanglement witness such as the parameter *S* that figures in the CHSH inequality or the concurrence. I think that the numerical evaluation of such parameters would give more strength to their claims.

3- The authors should improve the motivation of the field by mentioning some applications of classical entanglement in quantum simulations, as in J. Opt. Soc. Am. B **30**, 3210 (2013).

REVIEWERS' COMMENTS

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In my opinion, realization of classical entanglement in elastic waves can lead to interesting quantum information inspired experiments in Acoustics. The manuscript is well written and the whole presentation is good.

Response: We appreciate the reviewer’s remark

However, there are a few points to be addressed before acceptance:

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flexible and efficient tool for preparation of non-separable spin-OAM states of a laser beam thanks to the strong spin-orbit coupling achieved in liquid crystals. There are also the so-called S-plates for preparation of radially polarized beams, that are also non-separable spin-OAM states. With these devices complemented by birefringent wave plates, any spin-OAM state can be easily realized. I suggest the authors remove this kind of assessment in the manuscript.

Response: The above mentioned statement has now been removed in the revised manuscript.

2- The authors talk about Bell states, but they do not evaluate any entanglement witness such as the parameter *S* that figures in the CHSH inequality or the concurrence. I think that the numerical evaluation of such parameters would give more strength to their claims.

Response: In the Supplementary Information, we have discussed the notion of entropy of 'entanglement', the classical nature of nonseparability. From equation (A7) of the Supplementary Information we see that if the coefficients of the Bell state are same, then we obtain the maximally "entangled" state. In the revised manuscript, we have calculated the entropy of 'entanglement' and have added the following:

"In particular, for the three illustrated ratios, we find the following values of $A^{(i)} \approx 0.00496e^{i\frac{7\pi}{16}}$, $A^{(ii)} \approx 0.00627e^{i\frac{7\pi}{8}}$, and $A^{(iii)} \approx 0.01640e^{i\frac{29\pi}{32}}$ with an estimated experimental uncertainty of $\frac{1}{62}\pi$ in $\arg(A)$, and $B^{(i)} \approx 0.00377$, $B^{(ii)} \approx 0.00751$, and $B^{(iii)} \approx 0.01433$ (see Methods for details). We calculate the entropy of 'entanglement', $S(\rho_{OAM})$, for the states labeled (i), (ii) and (iii), and we find $S(\rho_{OAM})^{(i)} = \frac{15}{16}ln2$, $S(\rho_{OAM})^{(ii)} = \frac{31}{32}ln2$, and $S(\rho_{OAM})^{(iii)} = \frac{63}{64}ln2$ (see Supplementary Information)."

3- The authors should improve the motivation of the field by mentioning some applications of classical entanglement in quantum simulations, as in J. Opt. Soc. Am. B 30**, 3210 (2013).**

Response: We appreciate the reviewer's suggestion. As the reviewer suggested, the above-mentioned reference and other related references have now been cited in the revised manuscript in the Introduction section, by adding:

“The classical entanglement of laser beams have found applications in quantum information science[16–18].”

Reviewer #2 (Remarks to the Author):

The paper proposes an experimental implementation of Bell states generation for elastic waves. The experimental sample is formed of three elastically coupled aluminum rods, connected by epoxy joints. Owing to the low-frequency range of operation, there is a single quasi-longitudinal guided elastic mode for the rod, that can propagate to either right (forward) or left (back). The elastic contrast between aluminum and epoxy is large, leading to the assumption that elastic energy remains mostly in the bars and the collective guided modes are limited to a number of 6 modes (3 aluminum rods each supporting 2 guided waves get slightly coupled by epoxy).

In the summary above, I have used on purpose elastic wave terminology and not Bell states as in the paper. The reason is that I feel the authors should better explain their assumptions and what is not straightforward from the point of view of elastic waves in solids. If they can do that, the paper should have more novelty and hence impact.

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Response: For clarity, we have removed lines 98-101 in the revised manuscript. We have also added an experimental result of the free standing single aluminum rod in the Supplementary Information and have added the effect of ultrasonic couplant in the main body of the manuscript by adding the following:

"The experimental setup is devised to explore only longitudinal modes in the system (see Methods and Supplementary Information)."

and

"Furthermore, the use of honey couplant in conjunction with the longitudinal wave transducers suppress all non-longitudinal modes (torsional, transversal, etc.) of the aluminum waveguides, as shown in Supplementary Fig. A1 in the Supplementary Information for $\boldsymbol{N} = \mathbf{1}$ free standing aluminum rod."

2. Why matrix M33 has the form announced above equation (2) is unsubstantiated. Was epoxy really added exactly so as to obtain integer coefficients of the matrix or what is the idea behind that particular form? It looks like the authors consider a first-order finite difference approximation of some derivative between d.o.f. lying on each rod (by why?). Why does the coupling matrix has diagonal elements? (and what is the meaning of self-coupling for a mode of a rod?) Why is α^2 introduced bluntly and has the coefficient any reason to be real and positive from the start, whereas the final discussion stresses complex values for other parameters? Actually, coupling results from elastic waves in epoxy and is there a reason why some phase can not be possibly introduced between distant rods, just because of causality? As a remark, the eigenvalues of matrix M33 are 0, 1 and 3, but those values are given for its square root, is this an error?

Response: In the Supplementary Information, we have added the derivation of the coupling matrix $M_{3\times3}$. We have also corrected the eigen values in the main body of the manuscript. In addition, from the Supplementary Information we see that $\alpha^2 = k_c/m$, where k_c describes the stiffness of the springs that couples the chains, and hence has to be real and positive.

3. Actually, obtaining the elastic guided modes of the composite of 3 rods plus epoxy should be very easy, e.g. with the finite element; why not show it as well? The analytic model to which the measurements are fitted against seems just too simple to be true at first sight. As a remark some numbers are missing in the paper in order to describe completely the elastic structure (elastic constants of aluminum for instance).

Response: In the Supplementary Information, we have developed numerical model for the coupled three-chain mass-spring waveguides. In the limit of long wavelength, the equations of motion for the coupled three-chain mass-spring waveguides take the form similar to equation (1) of the manuscript. By choosing $k_c = 33.5 \times 10^6$ N/m, which is the stiffness of the springs that couples the chains, we can numerically calculate the dispersion relation that also fits the experimental band structure for three coupled rods with cutoff frequencies 14.08 kHz and 24.43 kHz.

The elastic modulus of aluminum rod is added in the revised manuscript.

4. The main 'classical' property of Bell states appearing on the paper is said to be 'nonseparability'. What the word actually means in the context of elastic waves remains unclear to this reviewer. The analysis in the paper sets forward 6 degrees of freedom, which I understand to be 3*2 coupled L-waves of the rods. This is a simplification of an elastic wave field that exists throughout the sample (the 3 rods and the epoxy joints) and that is globally 'nonseparable' in the sense that if you change a boundary condition anywhere the global response is changed everywhere. Is 'nonseparability' a property of the discrete states used in the paper or is it just inherited from the elastic solid supporting the degrees of freedom?

Response: In the Supplementary Information, we have discussed the notion of entropy of ‘entanglement’, the classical nature of nonseparability. In our acoustic waveguides, nonseparable states have been obtained by considering superpositions of quasi-standing longitudinal modes at two different but coupled pseudospin bands, which is however different than classical mixed state or classical nonseparable mixture of longitudinal and torsional/shear modes. In addition, this nonseparable states are also different than the problems that fall into a class that are nonseparable classically, where the nonseparability stems from media corners and crack edges[1].

1. Miklowitz, J. Wavefront Fields in the Scattering of Elastic Waves by Surface-Breaking and Sub-Surface Cracks. in *Review of Progress in Quantitative Nondestructive Evaluation: Volume 2A* (eds. Thompson, D. O. & Chimenti, D. E.) 413–440 (Springer US, 1983). doi:10.1007/978-1-4613-3706-5_25

I think all these comments must be addressed before the paper can be considered any further.

Finally, we would like to thank the reviewers for their constructive comments that helped us to improve our presentation.

REVIEWERS' COMMENTS:

Reviewer #1 (Remarks to the Author):

The authors have properly answered the points raised by the referees and improved the manuscript presentation accordingly. In my opinion, the notion of classical nonseparability can inspire new acoustic experiments and devices. I recommend publication of the manuscript in its present form.

Reviewer #2 (Remarks to the Author):

In my opinion the authors have only provided a very minimal revision. As a result, the improvement to the manuscript is only marginal. The results are presumably correct and might be publishable, but I still feel they could have been presented better.
In my second comment, I was requesting an explanation of the particular form of matrix M33. The answer is in short that a chain-and-springs discrete model is used, not the more accurate continuum mechanics or elastic wave model that I would have expected. The real and positive value for α^2 is a consequence of the same assumption and would not hold for a more accurate elastic wave model (there are actually no springs in the experimental sample, but epoxy plates).
I was not convinced by the answer to my fourth comment.

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Fourth comment of the second reviewer:

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Response to fourth comment: In the current manuscript, nonseparability of elastic waves have been achieved by considering superpositions of quasi-standing longitudinal modes at two different but coupled pseudospin bands i.e. the two non-trivial orbital angular momentum (OAM) eigen modes: Mode 1 0 -1 and Mode 1 -2 1. We have to, however, keep in mind that any excitation amplitude and/or frequency does not produce such nonseparable states of elastic waves. Therefore, in Fig. 2 of the manuscript the resonance frequencies correspond essentially to separable states. To achieve nonseparability, an appropriate excitation amplitude and frequency has to be chosen. Selecting a frequency below the second cut off frequency in Fig. 2 will not allow us to achieve such nonseparability. The isofrequency state of 33.25 kHz enabled us to excite such nonseparable state in the experiment. Moreover, from the calculation of entropy of ‘entanglement’, the classical nature of nonseparability, we are able to compare our experimental “entangled” or nonseparable state to that of maximal “entangled” state. It is however true that once the nonseparability is achieved, such nonseparability is ‘global’ in the sense that any change in the boundary condition change the global response everywhere. Finally, to clarify the notion of nonseparability in relation to either the discrete or the continuum elastic field, in the revised manuscript we have added the following in the Results Section:

“As mentioned before, the experimental realization requires a mechanical system which elastic wave behavior is effectively described by equation (1). We, therefore, develop a model of the experimental system by starting with a discrete mass spring model which in the long wavelength limit approaches the continuum system (see Supplementary Note 2).”