An optimization model for power transformer maintenance

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Abstract. Power transformers are one of the main elements of a power grid, and their downtime impacts the entire network. Repairing their failures can be very costly, so sophisticated maintenance techniques are necessary. To attempt to solve this problem, we developed a mixed-integer nonlinear optimization model that, focusing on a single power transformer, both schedules this maintenance and also decides how much of the hourly demand it will satisfy. A high level of load on a power transformer increases its temperature, which increases its degradation, and so these two decisions have to be carefully balanced. We also consider that power transformers have several components that degrade differently.

Our model becomes very difficult to solve even in reasonably sized instances, so we also present an iterative refinement heuristic.

Keywords: Mixed-integer nonlinear optimization · Physics-Based Optimization · Scheduling · Production Planning

1 Introduction

In this article, we present a mixed-integer nonlinear program (MINLP) with a convex relaxation for power transformer (PT) maintenance. We focus on accurately modeling the degradation process of its components, especially on the degradation of the insulating paper of the copper windings, which is where most nonlinearities reside. We also present a simple heuristic to try to get better solutions for larger instances, given that the model's difficulty increases steeply with the number of periods.

This article is organized as follows: Section 1 frames the problem and provides a light background on power transformers and maintenance practices, besides presenting the related work. In Section 2, the main model is presented, where we explain the constraints and the simplifications we have made. Here, we also elaborate on the heuristic we developed. Section 3 presents the experimental setup and in Section 4 the results are shown. We conclude the paper in Section 5, where we provide an overview of the article and discuss future research.

1.1 Power transformers

A PT can be divided into subsystems, which are in turn composed of several components, each of which will be impacted differently under the same conditions [2]. Furthermore, the condition of one component may influence the deterioration of another but not that of a third. Our choice of components is based on [1] and [16]. It comprises the cooling system (CS), the oil preservation system (OPS), the oil (O), and the insulating paper of the copper windings (W), this last one being a critical component that will impose a hard limit on the transformer's remaining useful life (RUL, time until equipment failure under normal operation; see [19] for a literature review). The chosen components are essential to power transformer usage, interacting closely regarding maintenance and degradation, and were validated by experts in power transformer manufacturing.

1.2 Maintenance Practices

In maintaining power transformers, two maintenance strategies are commonly used: Time-based Maintenance (TbM), where maintenance is planned considering the time since the last maintenance action, and Condition Based Maintenance (CbM), where the asset is regularly tested and when some predefined threshold is reached, maintenance is scheduled. TBM is the most common practice, but CbM has already been implemented as well [1]. A component associated with the latter strategy is the Oil, which is regularly tested to understand when maintenance should be performed. Some examples of CbM applied to PTs can also be found in the literature (see [9, 13]). The most common decision is to combine TbM with CbM, depending on the considered components. However, both approaches make decisions using only locally available information, disregarding how the maintenance actions will impact the profit and cost over the long term. Considering that PTs are machines with a very long lifetime, it seems sensible and necessary to develop methods that take a more global view of this problem.

1.3 Related Work

This problem belongs to the class of production-maintenance scheduling problems with a single machine - see Geurtsen et al. [10] for a comprehensive literature review. Our work differs from the many variants we could find in the sense that it is the first to research this topic in the context of power transformers specifically (although optimization models for PT maintenance have been developed, see the next paragraph). Additionally, nearly every paper looks at linear variants of this problem (see [11] for a resource-production exception), mostly because physical considerations tend to be ignored in more abstract models.

The scheduling of maintenance actions in PTs has been studied extensively [1, 4, 9, 12, 17], but to the best of our knowledge, we were the first to attempt to incorporate the real-world degradation of the components into our model [8]. In other studies, the evolution of a PT's RUL is either modeled linearly or not modeled at all. Some works abstract the degradation process and equate it with a failure rate, modeled by a random distribution, most often a Weibull distribution [5]. This simplification ignores the inner workings of a power transformer, not taking advantage of the available engineering knowledge. Other works construct a health index, an artificial variable that assesses the condition of the power transformer as a whole based on the results of testing the machine [9].

Other works assume that the PTs are under a constant load, ignoring that they can work under different loads, which has a large impact on their condition and profitability. Additionally, some of these papers focus on maintaining the PT in an optimal state [20], but this is not a realistic assumption.

Given that most models for PT maintenance tend to be linear, they can be solved to optimality in most reasonably sized instances, making heuristics not especially useful. An exception is Jahromi et al.'s work [5] which presents a large MIP for maintaining multiple grid transformers that must satisfy the client's demand. The authors develop a two-stage framework that divides the model into two parts, the first of which schedules maintenance on a long-term time horizon, and afterward, with that solution, operational considerations are taken. In this article, we present a heuristic with similar reasoning, but considerably different, given that it iteratively refines the maintenance decisions (rather than doing it one time), and operational decisions are always present.

2 Model

The objective of the model is to maximize the profit: revenue from electricity sales with maintenance costs subtracted. Electricity demand must not be exceeded, though it does not need to be satisfied, in conformity with the day-ahead trading of electricity present in many countries with a liberalized market [18].

Higher electricity sales impact the degradation of all components, each requiring maintenance. It also increases the transformer's temperature, which is reduced by the oil and the CS and has a strong effect on the degradation of the winding component. Moisture is also an important aspect of winding degradation and is a function of the condition of the OPS.

As for notation, variables will be denoted by lowercase letters (e.g., r) and parameters by capital letters (e.g., R), using different fonts. Tables 1 and 2 detail the parameters and the variables used, respectively.

Some of these parameters and variables will require additional superscripts, which detail the component that they reference. Parameter C, representing the cost associated with component maintenance, for example, will have different values for each component. As such, we will say C^{W}, C^{O}, $\mathsf{C}^{\mathsf{OPS}}$ and C^{CS} to refer to the maintenance cost of the Winding, Oil, OPS, and CS components respectively. Other parameters will use the same notation. Variables and some parameters will also require a subscript, detailing the time they are referring to. For example, the RUL of the Winding component at time $t-1$ is denoted by r_{t-1}^{W}, while the maintenance of the Oil component at time t is m_t^{O}.

Parameter	Parameter Meaning	Parameter Range
A	Temperature's scaling factor	$\mathbb{R}^+$
B	Temperature's growth factor	$\mathbb{R}^+$
C^k	Cost of maintaining component k	$\mathbb{R}^+$
D^k	Natural wear of component k	$[0, 1] \times \{$Oil, OPS, CS$\}$
E_t	Electricity demand at period t	[0,2]
H	Maximum Permissible temperature	$\mathbb{R}^+$
Ht^k	Temp. reduction of component k	$\mathbb{R}^+$
$\mathcal{K}$	Components to be maintained	$\{$Oil, OPS, CS,Winding$\}$
L	Effect of load in component wear	$\mathbb{R}^+$
M	Big constant to dominate constraint	$\mathbb{R}^+$
P_t	Electricity price at period t	$\mathbb{R}^+$
Q	Maximum permissible load	$\mathbb{R}^+$
R^k	Maximum RUL of component k	$\mathbb{R}^+$
T	Number of periods	$\mathbb{N}$
$\mathcal{T}$	Set of periods ($= \{1, 2, \ldots, \mathsf{T}\}$)	$\mathbb{N}$
V	Moisture Dependent Degradation	$\mathbb{R}^+$ See [3, Table in page 21]

Table 1: Description of the parameters used in the model.

Variable	Variable Meaning	Variable Type
h_t	Temperature at period t, dependent on load	Continuous
m_t^k	Maintenance of component k at period t	Binary
q_t	Transformer load at period t	Continuous
r_t^k	RUL of component k at period t	Continuous
x_t	OPS RUL above 1/3 at period t	Binary
y_t	OPS RUL above 2/3 at period t	Binary

Table 2: Description of the variables used in the model.

$$\underset{q,m}{\text{maximize}} \quad \sum_{t\in\mathcal{T}}(\mathsf{P}_t \cdot q_t - \sum_{k\in\mathcal{K}} \mathsf{C}^k \cdot m_t^k) \tag{1a}$$

subject to

$$q_t \leq \mathsf{E}_t, \forall t \in \mathcal{T}, \tag{1b}$$

$$r_0^k = \mathsf{R}^k, \forall k \in \mathcal{K}, \tag{1c}$$

$$r_t^{\mathsf{O}} \leq r_{t-1}^{\mathsf{O}} \cdot \mathsf{D}^{\mathsf{O}} - \frac{h_t}{\mathsf{H}} + \mathsf{M}_1 \cdot m_t^{\mathsf{O}}, \forall t \in \mathcal{T}, \tag{1d}$$

$$r_t^{\mathsf{CS}} \leq r_{t-1}^{\mathsf{CS}} \cdot \mathsf{D}^{\mathsf{CS}} - \mathsf{L} \cdot \frac{q_t}{\mathsf{Q}} + \mathsf{M}_2 \cdot m_t^{\mathsf{CS}}, \forall t \in \mathcal{T}, \tag{1e}$$

$$r_t^{\mathsf{OPS}} \leq r_{t-1}^{\mathsf{OPS}} \cdot \mathsf{D}^{\mathsf{OPS}} - \mathsf{L} \cdot \frac{q_t}{\mathsf{Q}} + \mathsf{M}_3 \cdot m_t^{\mathsf{OPS}}, \forall t \in \mathcal{T}, \tag{1f}$$

$$r_t^{\mathsf{W}} \leq r_{t-1}^{\mathsf{W}} - 2^{\frac{h_t-98}{6}} + \mathsf{M}_4 \cdot m_t^{\mathsf{W}}, \forall t \in \mathcal{T}, \tag{1g}$$

$$r_t^{\mathsf{W}} \leq r_{t-1}^{\mathsf{W}} - \mathsf{V}_y \cdot 2^{\frac{h_t-98}{6}} + \mathsf{M}_4 \cdot m_t^{\mathsf{W}} + \mathsf{M}_5 \cdot y_t, \forall t \in \mathcal{T}, \tag{1h}$$

$$r_t^{\mathsf{W}} \leq r_{t-1}^{\mathsf{W}} - \mathsf{V}_x \cdot 2^{\frac{h_t-98}{6}} + \mathsf{M}_4 \cdot m_t^{\mathsf{W}} + \mathsf{M}_5 \cdot x_t, \forall t \in \mathcal{T}, \tag{1i}$$

$$q_t \leq \mathsf{Q} \cdot \frac{r_t^k}{\mathsf{R}^k}, \forall k \in \mathcal{K}, t \in \mathcal{T}, \tag{1j}$$

$$m_t^{\mathsf{OPS}} \leq m_t^{\mathsf{O}}, \forall t \in \mathcal{T}, \tag{1k}$$

$$m_t^{\mathsf{W}} \leq m_t^k, \forall k \in \mathcal{K}, t \in \mathcal{T}, \tag{1l}$$

$$y_t \leq x_t, \forall t \in \mathcal{T}, \tag{1m}$$

$$3\frac{r_t^{\mathsf{OPS}}}{\mathsf{R}^{\mathsf{OPS}}} \geq y_t + x_t, \forall t \in \mathcal{T}, \tag{1n}$$

$$h_t \geq \mathsf{A} \cdot e^{\mathsf{B}\cdot q_t} - \mathsf{Ht}^{\mathsf{CS}} \cdot \frac{r_t^{\mathsf{CS}}}{\mathsf{R}^{\mathsf{CS}}} - \mathsf{Ht}^{\mathsf{O}} \cdot \frac{r_t^{\mathsf{O}}}{\mathsf{R}^{\mathsf{O}}}, \forall t \in \mathcal{T}. \tag{1o}$$

Objective 1a is the maximization of the sum of the profit over every time period. The revenue is generated from electricity sales, with the load at period t represented by the continuous variable q_t, and the price at period t by P_t. The cost is calculated with a binary variable m_t^k, which is equal to 1 if component k is maintained at time period t, and 0 otherwise. This binary variable is then multiplied by the cost of maintaining component k. In our work, we assume that the maintenance of a component restores it to its optimal state, and can thus be seen as its replacement.

Constraints 1c-1i are modeling the initial condition, and its evolution, for each of the components of the power transformer. A big-M term in Constraints 1d-1i allows modeling the reduction in RUL (when the corresponding maintenance variable, m, is 0) and the replacement of the corresponding component (when $m = 1$). The winding component has three different on-off constraints (1g-1i) due to the impact of moisture, which scales the degradation by V_y and V_x. These constraints have their own big-M term, controlled by variables x and y. If the level of moisture in the transformer is high, then we will have $x = 0$, activating Constraint 1i and speeding up the degradation of the winding component by the factor V_x. If $x = 1$, then moisture is either at a low or medium level, and thus Constraint 1i is not binding. Constraints 1m-1n make variables x and y model moisture, according to their definition in Table 2. Constraint 1n will force variables y and x to 0 as the OPS becomes more degraded, and Constraint 1m makes it so y becomes 0 before x.

With constraint 1j we are conservatively limiting the load of the PT by the condition of its most damaged component, as a way to prevent unexpected failure. In 1b, we are enforcing an upper bound on the load. In our experiments, the demand is based on the consumption data found in [15].

Constraints 1k and 1l are due to real-world limitations on power transformer maintenance, where some components need to be maintained when others are. For example, replacing the OPS requires the replacement of the oil as well.

Constraint 1o models the evolution of the PT temperature as a function of its load, as a simplification of the equations presented in [3], which is reduced by the oil and CS components, based on their relative condition. Note that there is an exponential term here, that will be present in an exponent in constraints 1g- 1i, conveying the very nonlinear degradation when the load raises above a certain threshold.

2.1 Simplifications

The number of constraints and variables in this model increases linearly with the number of periods, but it is considerable for the largest instances in our set, with 83250 constraints (28731 nonlinear), and 44961 variables (1800 binary). However, even for average-sized instances, the model can be difficult to solve: the time required to find an optimal solution increases rapidly, as will be seen in Section 4. To aid in the tractability of the problem, we conceded some simplifications.

We assume that maintenance actions can only be taken on the first period of each year, in order to reduce the number of binary variables. The same is true for the indicator variables x and y.

Additionally, we use a representative day for each year. That is, every year has 365 identical days, each with 24 periods, which are used to compute the degradation of the components throughout the year. The RUL of the components is also measured in 24h periods. The aim is again to reduce the size of the model, this time the number of continuous variables.

2.2 Iterative Refinement Heuristic

Without the aforementioned simplifications, the model would very quickly become intractable. To further aid in the solving process and to hopefully be able to lift some of these simplifications in the future, we created an iterative refinement heuristic, which should reduce the computational burden due to the (binary) maintenance variables.

The objective of this heuristic is to decrease the impact of the binary variables in the solution time, by solving very coarse models when it comes to maintenance decisions and then refining the possibilities around the maintenance actions of the previous iteration. This idea is somewhat similar to what is presented by Jahromi et al. in [5], where a two-stage model first schedules maintenance on the long-term and then passes the decision to a medium-term scheduler. However, our heuristic does not distinguish between long and medium-term scheduling, always keeping a view of the global problem, and usually requires several iterations.

Algorithm 1: Iterative Refinement.

```
Data: Initial maintenance possibilities, time/iteration limits
while time, iterations do not exceed limit do
    build model with current maintenance possibilities;
    if available, use previous best solution as starting point;
    optimize model with given time limit;
    if model is infeasible then
        add possibly at midpoint of every consecutive maintenance possibility;
    else
        for possible maintenance variables do
            if maintenance was scheduled then
                add maintenance possibility at the midpoint between previous
                  and current one;
                add maintenance possibility at the midpoint between current
                  and next one;
    if no new maintenance possibility was added then
        break;
return best solution;
```

The initial data consists of a set of initial maintenance possibilities (one every $\mathsf{T}/4$ years by default), and an iteration and time limit. If these last two parameters are surpassed, the algorithm is stopped (line 1) and it returns the best solution found (line 14). Using the currently available maintenance possibilities, it builds a model and optimizes it (lines 2 and 4), without exceeding the remaining computational budget and using the best solution found, if it exists (line 3). Any solution from an earlier iteration is feasible for further iterations. Given the maintenance possibilities at a given iteration, the model may be infeasible (which can happen even if the transformer does not produce anything, as there is load-independent degradation) (line 5). If this is the case, we double the maintenance possibilities (line 6) unless all time indices are already in use, in which case we return a null solution (lines 12, 13 and 14). If we are still in the loop, we iterate through each maintenance possibility currently available, and if maintenance was scheduled (the corresponding binary variable equals 1), then two maintenance possibilities are added, in the midpoint between the current one and the previous, and in the midpoint between the current and the next one (lines $8-11$), all rounded to the lowest year. If all of these maintenance possibilities were already present in the current iteration, then we break and return the best solution.

Given high enough limits on time, iterations, and maintenance possibilities, this algorithm converges to a solution, as it will eventually be unable to add new maintenance options.

Figure 1 below exemplifies three iterations of this heuristic. Starting with maintenance possibilities every $\mathsf{T}/4$ years, in this case, at $0, 5, 10, 15, 20$, the optimized model schedules maintenance for year 10 (marked with a red triangle), and so we add possibilities $7, 12$, at the midpoints between the scheduled maintenance actions of the previous iteration, and the ones that precede/succeed it (rounded to the lowest year). On the second iteration, this model is optimized and schedules maintenance on years $7, 12$. Note that additional maintenance may result in higher available capacity, and hence in higher profit due to the possibility of selling more electricity. At the end of the second iteration, the possibilities $6, 8, 11, 13$ are added (midpoints between 5 and 7, 7 and 10, 10 and 12, and 12 and 15). On the third iteration, solving the model gives optimal maintenance in years 6 and 11. No maintenance possibilities are added, so the model stops in this iteration.

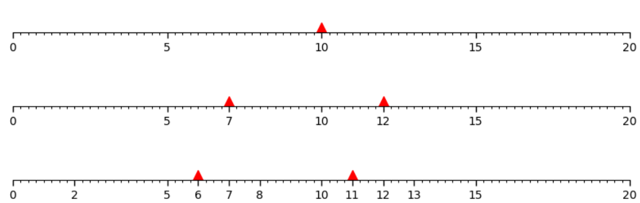

Fig. 1: Example of the iterative refinement heuristic.

This algorithm is not exact, as it may happen that the optimal maintenance configuration is not reachable by the algorithm. Consider an instance where the only profitable maintenance action consists of scheduling maintenance on year x, but year x is not a part of the initial coarse maintenance decisions. So, the optimal solution would not schedule any maintenance actions, meaning that maintenance possibilities will never be added, since maintenance is never scheduled.

3 Benchmark Instances Used

In this section, we will detail the experimental setup used to validate the model and the metrics used. We present results for the two different methods, the original model and the heuristic, for a varying number of years in the planning horizon, from 1 to 20, along with longer time periods, namely 50 and 100 years, all with a 2 hour time limit.

We run these experiments for 10 instances that differ in their parameters, which were randomly sampled from what we understand to be sensible intervals. After having the results, we average over the instances. For the two longest time frames, we also show the evolution of the incumbents of the two methods for a single instance. We chose the instance whose results were closest to the average.

When presenting the results, the objective value is unitless, and we will use the relative gap: $\frac{|\text{primal bound} - \text{dual bound}|}{\min(|\text{primal bound}|, |\text{dual bound}|)}$. The relative gap of the heuristic in the two largest sets of instances (50 and 100 years, marked with an asterisk in Table 4) is calculated in relation to the original model's dual bound. For smaller instances, the heuristic's gap is in relation to the result of the original model; in the above formula, the dual bound should be replaced by the primal of the original model. The heuristics' gap is calculated using the average of the results, rather than being calculated individually, and then averaged.

The experiments were run on two 6-Core Intel Core i7 with 64 GB running at 3.20GHz. The models were run the models with the SCIP solver, version 8.0.0 [6]. For the implementation, we used its Python interface, PySCIPOpt [14] with Python version 3.7.9. The code can be found on Github [7].

4 Results and Analysis

Using the original model, most instances were solved to optimality within the given time limit. As for the iterative refinement heuristic, it did quite well for large instances, providing good incumbents, better than the original model, needing only a few iterations. It was also able to reach optimality in small to medium-sized instances.

From years $1-14$, the model optimally solves all instances. Afterward, some instances could not be solved to optimality and the last two instances have a very large gap. Only here (for 50 and 100 years) has the model not been able to solve any of the 10 instances for each year, which can be seen by the average time spent being the same as the time limit.

Years	Incumbent	Time(s)	Gap(%)
1	37.769	0.2	0.0
2	73.241	0.7	0.0
3	104.711	1.2	0.0
4	131.909	1.9	0.0
5	155.746	9.3	0.0
6	176.672	14.4	0.0
7	194.737	20.9	0.0
8	211.864	30.2	0.0
9	228.736	43.7	0.0
10	245.755	61.7	0.0
11	263.884	84.7	0.0
12	282.787	121.9	0.0
13	299.335	224.9	0.0
14	317.607	290.8	0.0
15	334.53	1108.9	0.2
16	350.923	2078.7	0.1
17	366.029	3325.7	0.8
18	382.292	5382.4	2.5
19	399.014	4819.0	3.2
20	416.57	5954.9	5.4
50	806.774	7200.1	56.2
100	1113.605	7200.1	159.8

Table 3: Results for the original model.

Years	Incumbent	Time(s)	Gap(%)	Iterations
1	37.769	0.2	0.0	1.0
2	73.241	0.5	0.0	1.0
3	104.711	0.8	0.0	1.0
4	131.909	3.5	0.0	2.0
5	155.746	9.5	0.0	2.0
6	176.672	20.5	0.0	2.3
7	194.737	24.3	0.0	1.8
8	211.788	47.3	0.04	2.3
9	228.736	53.8	0.0	2.2
10	244.681	83.3	0.44	2.5
11	263.773	114.3	0.04	3.2
12	282.541	203.5	0.09	3.7
13	299.261	298.1	0.02	4.3
14	317.563	289.6	0.01	4.0
15	333.371	525.0	0.35	3.8
16	349.75	1337.0	0.34	3.5
17	365.247	1000.7	0.21	3.6
18	387.126	2133.1	0.15	3.9
19	398.457	3292.3	0.14	4.1
20	414.689	4902.7	0.5	4.1
50	857.162	7201.0	47.02*	3.0
100	1227.91	7200.7	135.61*	2.7

Table 4: Results for iterative refinement.

The heuristic got an optimal solution for years $1-7$ and 9. Outside of these years, it arrived at a gap, in comparison to the original model's incumbent, of less than 1% for the remaining years (and surpassing the original model in years $50, 100$). In medium-sized instances, the heuristic actually takes longer, most likely due to maintenance actions not having a large impact, leading to time being wasted in additional iterations. From years 14 to 20, the heuristic starts to get progressively faster in comparison to the original model. For years 50 and 100, it even gets a significantly better incumbent than the original model, even though it is still far away from a provably optimal solution, with a gap of 47.02% and 135.61% in relation to the original model's dual bound, for years 50 and 100 respectively.

There are two cases in which the heuristic takes longer to converge to a solution than it does on a smaller model. This happens in years 14 and 13 (289.6s vs 298.1s) and in years 17 and 16 (1000.7 vs 1337.0). This can happen because a different set of initial maintenance possibilities may lead to very different subsequent iterations. For example, with the initial maintenance possibilities in years 13 and 14, maintenance on year 7 might be the best possible action in one case, but on year 3 for the other, thus leading to different maintenance possibilities in the next iteration. It is possible that a larger instance converges faster on average (the number of iterations for 14 years is smaller than for 13 years).

Below we show the evolution of the incumbents of the original model and iterative refinement, concerning a specific instance, as described in Section 3.

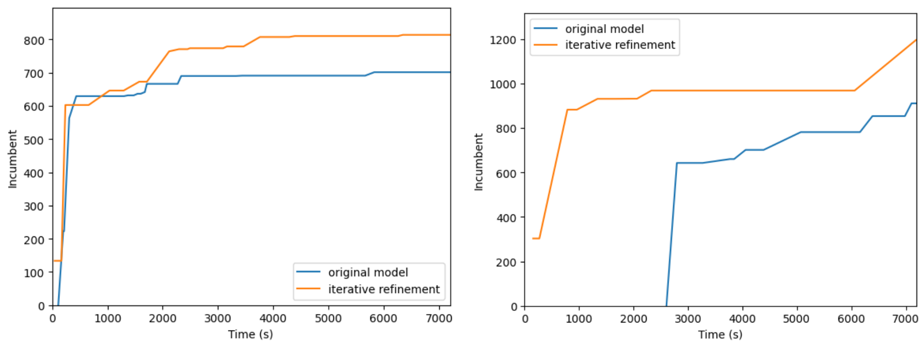

Fig. 2: Best solution as a function of CPU time for the Model and Heuristic for 50 years (left) and 100 years (right).

In both of these cases, the heuristic outperformed the original model by a wide margin. In the 50 years case, it gets an incumbent of about 760 after 1700s, and reaches 814 by the end, while the original model can only get to 700 after 2h. The difference is even larger in the 100 years scenario, where the heuristic gets an incumbent of 1196 against the 910 of the original model. In this instance, we exclude from the graphic the initial solution of the original model, obtained after 163s with objective -2854, for the sake of presentation.

The heuristic had 2 iterations in the first case, where the second iteration started at around 400s. The effects of the new iteration can be seen, as the heuristic plateaus around that time, before surpassing the original model again. In the 100 years case, the heuristic could only use 1 iteration within the given time limit. It seems that for very large models, heavily restricting the maintenance possibilities can still produce very good results.

Based on these results, the heuristic appears to be preferable for very large instances, and should also be considered in medium to large instances where results need to be obtained quickly and optimality is not a concern.

5 Conclusion and Future Work

This article presented a MINLP for scheduling maintenance and operational decisions in a power transformer. Given the model's difficulty, we developed a heuristic that focuses on the difficulty presented by the number of binary variables, starting with a subset of them and iteratively adding more.

The model is able to solve multiple medium-sized instances to optimality, but on larger instances, the difficulty grows rapidly. The heuristic has trouble reaching optimality but can obtain good solutions quickly, and in larger instances, it gets better results that the original model for the given time limit. These results leave us optimistic that we can decrease the granularity of small and medium-sized instances, and still obtain good solutions.

Regarding future work, we intend to study a related problem, where there are multiple power transformers instead of just one. In this problem, the electricity demand must be satisfied, and the operational decision is how to best allocate this demand to each of the machines at a given time period. This is a much more difficult problem, but amenable to a Dantzig-Wolfe reformulation. Unfortunately, the pricing problem will be equivalent to the problem described here. In order to successfully apply column generation, we need better heuristics.

In this model, fixing the production variables makes the problem significantly easier (solved in a few minutes in large instances). However, in a few milliseconds, we are able to obtain a solution that is near-optimal in most instances, by scheduling maintenance at the last possible period. While this strategy may lead to a suboptimal solution (for a fixed production), being able to find a good solution so quickly suggests using a sampling-based heuristic, where we continuously fix the production variables and quickly obtain the corresponding objective, choosing the best sample in the end.

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