TEACHING STATEMENT

JOE WEBSTER

A week before the term starts, I take a look at the list of names of students enrolled in my course. Every name represents a human being that is entitled to respect, understanding, and the best leadership I can provide through this segment of their mathematics education. Every name represents an individual with their own set of interests, strengths, weaknesses, and hardships. Every name represents a person that will invest a lot of money, attention, and time in my course. Thus it is my privilege and my obligation to design a course that makes a lasting impact on their understanding of and appreciation for mathematics, and to provide an inclusive and inspiring environment that makes this possible.

When setting up a course, I find highlights in the curriculum that are especially powerful, interesting, or surprising, introduce them as a "sales pitch" for the course in my first lecture, and develop the rest of the course around them. For example, I begin my Calculus III course with the following question: *Calculators are designed to add, multiply, and divide very efficiently, and this is essentially all they can do. But we rely on them for computing radical, exponential, logarithmic, and trigonometric values all the time. How can a calculator possibly know that* $\sin(1) \approx 0.841471$ *if it only knows how to add, multiply, and divide?* I then provide an answer, relying only on my students' prerequisite knowledge: Integrate the inequality $-1 \leq \sin(x) \leq 1$ from 0 to $x \geq 0$ to get $-x \leq \cos(x) - 1 \leq x$, then integrate again and multiply through by -1 to get $-x^2/2 \leq \sin(x) - x \leq x^2/2$. Carefully repeating these steps several times and evaluating at $x = 1$ reveals that

$$\sin(1) \approx 1 - \frac{1}{3!} + \frac{1}{5!} - \frac{1}{7!} + \frac{1}{9!} \approx 0.841471 \quad \text{with error } \leq \frac{1}{10!} \approx 2.76 \times 10^{-7}.$$

The example above quickly establishes important ideas. It shows that $\sin(1)$ can be approximated by a sum of simple rational numbers with easily controlled error, which begins to demystify what is going on "under the hood" of their calculator. It hints at the shocking and powerful idea that sine is really an "infinite degree polynomial", which raises another question: *What about other functions, like roots, exponentials, and logarithms?* I assure the students that we will soon reach the same revelation for all of these functions, but need to first carefully develop an understanding of infinite sums. This incentivizes our forthcoming study of sequences, series, and convergence, which begins in the second lecture and culminates two months later with Taylor's Theorem. In the final lectures, I try to fit the applications of Taylor's Theorem to the interests of my students. I dare the computer science students to write an efficient program that computes sine and cosine values to arbitrary precision using Taylor polynomials, and I discuss the practical importance of the small angle approximations $\sin(\theta) \approx \theta$ and $\cos(\theta) \approx 1 - \frac{\theta^2}{2}$ for the physics students. For the mathematics students, I use Taylor series to prove *Euler's Formula*, or $e^{i\theta} = \cos(\theta) + i\sin(\theta)$, and I make sure to comment that this formula is the reason I became a mathematician.

Sharing my enthusiasm for mathematics with students helps me recruit them into the UO Directed Reading Program (DRP), which I cofounded with three fellow graduate students in 2018. The program pairs undergraduates with graduate mentors that guide them through advanced topics for a term, and each student presents their topic to their peers at the end. Through this process, they have an opportunity to better understand what it means to be a graduate student and a teacher, and to study an interesting topic without the constant pressures of competition and exams. The DRP has been very popular and fulfilling, so I plan to create or participate in another such program at my next position.

Salesmanship and well-designed lectures are necessary for teaching mathematics, but may amount to little more than entertainment if they are not complemented by active learning. Depending on the level of the course, I mandate active learning in several ways. College algebra, introductory statistics, and trigonometry offer a wide variety of conceptual and short problems, so in these courses I typically use daily half-page quizzes or ask several clicker questions throughout each lecture. These usually constitute 5%-10% of the course grade and are an effective way to incentivize attendance, attention, and participation, without putting too much pressure on the students. I've used daily quizzes and clickers in Calculus courses as well, but I prefer to assign in-class worksheets for the lengthy computational problems. For a proof-intensive course such as Analysis, I like to use a semi-flipped method: Each lecture day is followed by a randomized group worksheet day. This helps the students get their hands on the material multiple times per week, learn how to explain mathematics to others (a huge part of proof writing), and build camaraderie with many classmates. It is also a fun opportunity to explore exotic examples, such the Fibonacci sequence, the Cantor set, and Thomae's function, which are ripe for further exploration through the DRP.

Outside lecture, my favorite part of teaching a course is writing homework assignments and discussing them with students in office hours. Writing my own assignments allows me to emphasize important and interesting concepts, incorporate visualization using Desmos (particularly for Calculus), and/or help students learn how to use LaTeX, while also protecting them from the temptation of cheating with well-known solutions from the internet. Homework is vital for solidifying the ideas of the course, so I typically make it worth 40% of the course grade. The assignments are often thorough and time consuming, which is necessary to prepare students for exams and future courses. That being said, it is not okay to demand a lot of work without providing enough availability and support. If possible, I always try to schedule my office hours so that every student can attend at least one each week, and I ask them to attend as often as they can. Office hours provide a safe space in which I can acknowledge that mathematics is challenging, convince students that they can and will overcome their struggles, and assure them that their individual success is my priority.

Mathematics is a challenging subject and a source of insecurity and anxiety for many students. The last thing I want is for students to think they are inadequate, and I follow several rules to assure them that they belong in my class. One of my first rules is to avoid using needless exclusive words like "easy", "obvious", or "trivial". This is because the moment I call something easy or obvious, anyone in the audience that disagrees might immediately feel bad about themselves. Every time I finish a complicated example on the board, I step back and say, "A lot just happened. Let's walk back through that to make sure I didn't make any mistakes." This shows that I make mistakes too (and sometimes I find one!), and that they shouldn't feel bad if it didn't make perfect sense to them right away. It also gives them a chance to finish copying, digesting, and annotating the example in their notes before the next big example appears. Finally, I make sure to respond to "silly" questions with a straight face and a patient explanation. If I indicate (even subtly) that I think their question is silly, this might embarrass them and discourage future questions. Keeping the students interacting and feeling welcome is essential to their success in the course and my success as a teacher. The more people we keep in the conversation about mathematics, the better off we will be.