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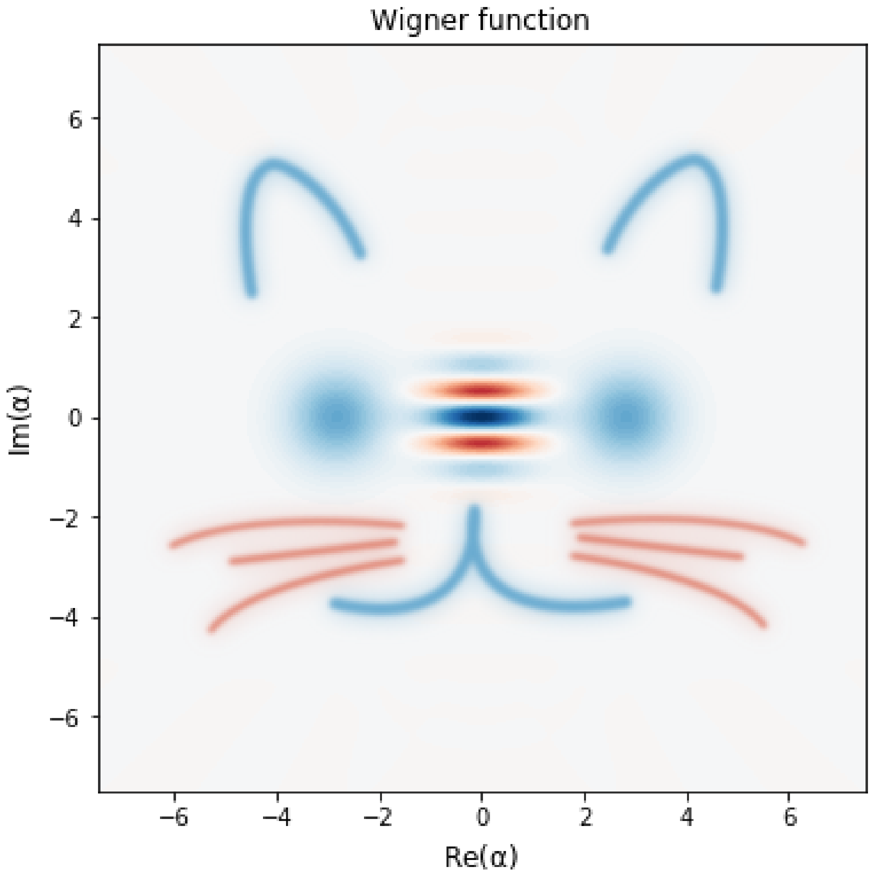

Cat State Preparation By Quantum Optimal Control with Krotov's Method

How to prepare your cat

Master's thesis in Quantum Computing

JOHAN WINTHER

Department of Microtechnology and Nanoscience
CHALMERS UNIVERSITY OF TECHNOLOGY
Göteborg, Sweden 2019

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Abstract

Quantum computing has gained a lot of interest in recent years and commercial products are just now entering the market. However one of the main challenges in realising a quantum computer is noise and one key technology to remedy this is quantum error correction (QEC). One way of performing QEC is to store the quantum information of a qubit, while it's idle, as special basis states in a resonator. To do this one needs to apply encoding and decoding pulses to the coupled qubit-resonator system to do a state transfer. These pulses are hard or perhaps impossible to solve for analytically, which means they need to be numerically obtained by simulation.

This thesis studies the prospects of using Krotov's method (gradient ascent) to numerically optimize encoding pulses for state transfer. The Python package Krotov, a package for quantum optimal control using Krotov's method, is used to perform state transfers —0¿ to —1¿ and —0¿ to —2¿ of an anharmonic resonator. It is shown that, assuming a maximum drive amplitude and no dissipation, the method can realise a —0¿ to —1¿ transfer with fidelity F ¿ 0.99999 with a total pulse length of only 10.75 ns. For the —0¿ to —2¿ transfer a total pulse length of 30 ns is needed to reach the same fidelity. Thus it is concluded that state transfers using Krotov for non-coupled systems is viable.

However for coupled systems there is great difficulty in using Krotov, as the package assumes a time-independent control Hamiltonian. A potential workaround for this problem is also presented.

Keywords: quantum computing, quantum optimal control, cat state, cat code

Sammanfattning

Preface

Acknowledgements

Nomenclature

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Contents

1 Introduction

Quantum computing is starting to appear in commercial products [1], however, despite a lot of progress [2], there are still a lot of challenges to be solved before large scale quantum computers become commonplace. Unlike classical computers, where the transistors' on and off state is dictated by the stream of many electrons, quantum computers rely on single or very few number of particles which make the quantum states very delicate [3].

One of the key technologies to keep these states in quantum computing is quantum error correction (QEC) [3], which is a way to retain the quantum information in a qubit by introducing redundancy into the physical system. Leghtas, Kirchmair, Vlastakis, *et al.* [4], Mirrahimi, Leghtas, Albert, *et al.* [5] propose and Ofek, Petrenko, Heeres, *et al.* [6] demonstrate a method to encode the quantum information in a clever basis in quantum harmonic resonators.

To further clarify, in this scheme the quantum information of a qubit is carefully encoded and decoded into a resonator by applying optimized microwave pulses to the qubit and resonator system which will realise state transfers in both these systems. This thesis will focus on how to numerically optimize encoding and decoding pulses using Krotov's method [7], a gradient ascent based optimization algorithm available publicly as a ready-to-use Python package [8].

1.1 Purpose

The purpose of this thesis is to numerically optimize microwave pulses to transfer the quantum information in a qubit to a resonator. This will be done using the Krotov Python package [8].

1.2 Limitations of the thesis

In this thesis, these limitations will :

- No Liouvillians
-

2 Theory

In this chapter the theoretic concepts will be explained.

2.1 Superconducting resonators

Superconducting resonators are used in quantum computing both as the basis for qubits and as readout and control components. Although ideal resonators have equally spaced energy levels, in reality they are more or less anharmonic and the general Hamiltonian for a quantum anharmonic resonator is

$$\hat{\mathcal{H}} = \omega \hat{a}^\dagger \hat{a} - \frac{\kappa}{2}\left(\hat{a}^\dagger \hat{a}\right)^2 \tag{2.1}$$

where ω is the resonance frequency, κ is the anharmonic (self-Kerr) term and $\hat{a}$ is the destruction operator which removes an excitation from the resonator.

The anharmonicity can be vizualised, see fig. 2.1, by plotting the eigenenergies of eq. (2.1) as a function of κ with $\omega = 1$. A larger anharmonicity gives larger energy spacing for higher excitation states.

2.2 Jaynes-Cummings Model

2.3 Visualization of quantum states

2.3.1 Bloch sphere

Briefly explain the Bloch sphere representation and some examples

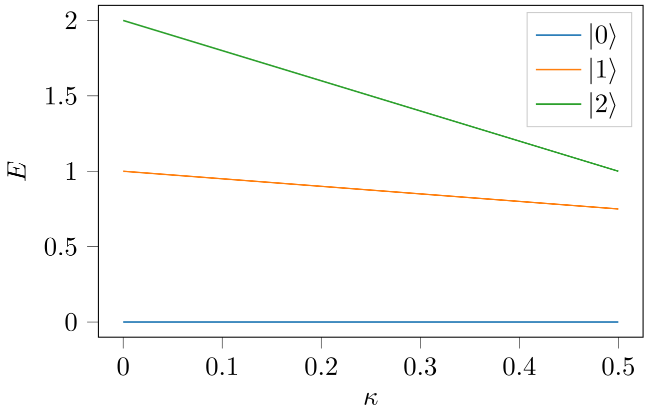


Figure 2.1: The energy levels of a resonator for anharmonicity $\kappa \in [0, 1]$.

2.3.2 Density matrices and Hinton diagrams

Explain the Hinton diagram and density matrix

2.3.3 Wigner function

Explain why the wigner function is good for visualization.

2.4 Bosonic codes

Explain what bosonic codes are

2.4.1 Cat codes

Explain what the cat code basis is and why it can be used for QEC.

2.5 Quantum control

Quantum control is the process of controlling a quantum system by controlling the amplitude of a set of control operators in time [9]. Such a system can be described [9] by a Hamiltonian of the following form

$$\hat{\mathcal{H}}(t) = \underbrace{\hat{\mathcal{H}}_d}_{\text{Drift}} + \underbrace{u_0(t)\hat{\mathcal{H}}_0 + \ldots + u_N(t)\hat{\mathcal{H}}_N}_{\text{Control}} . \tag{2.2}$$

The controls are usually electromagnetic pulses changing in time and thus will be referred to as "pulse shapes" in this thesis [9].

There are two main questions in quantum control: one of *controllability* and one of *optimal control* where the first deals with the *existence* of solutions given the Hamiltonian and the second with the *optimized* solutions for the pulse shapes $\{u_i(t)\}$ [10]. The optimal solutions are generally not analytically solvable and thus the pulse shapes need to be discretised in time and numerically optimized using algorithms. The algorithm used for this thesis will be presented in the Method chapter.

$\mathrm{Re}(\Omega)$ and $\mathrm{Im}(\Omega)$ drive rotations around the x-axis and y-axis respectively (in the Bloch sphere).

3 Method

In this chapter Krotov's method for quantum optimal control will be briefly introduced and an implementation as a Python package. Then the numerical experiments will be explained.

3.1 Krotov's Method for quantum optimal control

Krotov's method fundamentally relies on the variational principle to minimize a functional

$$J\left[\left\{\left|\phi_k^{(i)}(t)\right\rangle\right\},\left\{\epsilon_l^{(i)}(t)\right\}\right]$$

where the constraints are included as Lagrange multipliers [8]. A detailed explanation of this functional when the method is applied to quantum systems can be found in [7].

3.2 Krotov: the Python package

A Python implementation of the Krotov package is available at `https://krotov.readthedocs.io/en/latest/`. It provides simple functions and objects to

3.3 Optimization Experiments

In this chapter the numerical optimization experiments are presented and motivated.
Optimization will realise state transfers
Rotating frame
4 gigasamples/s
Amplitude constraint (with pi pulse calibration)
Guess pulses half amplitude (actually blackman pulses)
convergence criteria fidelity F change between iterations falls below a certain critera ΔF

3.3.1 $|0\rangle \rightarrow |1\rangle$ state transfer

Pulse shapes were optimized with varying lengths from 4.25 ns to 30 ns with convergence criteria $F > 0.99999$ or $\Delta F < 10^{-7}$. Step size $\lambda = \frac{1}{\frac{1}{2}A_m}$

3.3.2 $|0\rangle \rightarrow |2\rangle$ state transfer

Pulse shapes were optimized with varying lengths from 22 ns to 30 ns with convergence criteria $F > 0.99999$ or $\Delta F < 10^{-9}$. Step size $\lambda = \frac{1}{2A_m}$

4 Results

The results of the numerical experiments will be presented in this chapter. For both optimization targets the optimized pulses, their properties and the final evolved state will be presented.

4.1 $|0\rangle \rightarrow |1\rangle$ state transfer

In fig. 4.1 the fidelity during all optimization runs are plotted. For pulse lengths longer than 15 ns the fidelity starts at values close to the goal ($F > 0.9$) and the number of iterations is relatively low (less than 85 iterations). In contrast, pulses shorter than 15 ns start at lower fidelities while the number of iterations are roughly one order of magnitude larger with no clear pattern.

To give a more detailed picture, the starting fidelity and optimized fidelity is plotted over pulse length in fig. 4.2. The optimizations where the fidelity goal was not reached, pulse lengths equal to and below 10.50 ns, are marked with stars.

Further analysis is done on pulses with lengths 4.25 ns, 6.0 ns, 8.0 ns, 10.0 ns, 20.0 ns and 30.0 ns. The optimized pulse shapes $\mathrm{Re}(\Omega)$ and $\mathrm{Im}(\Omega)$ are plotted in fig. 4.3 together with the guess pulses. Pulses longer than 20 ns require only fine adjustments to the Blackman guess pulse while shorter pulses have an imaginary part which is maximized for the whole duration of the pulse.

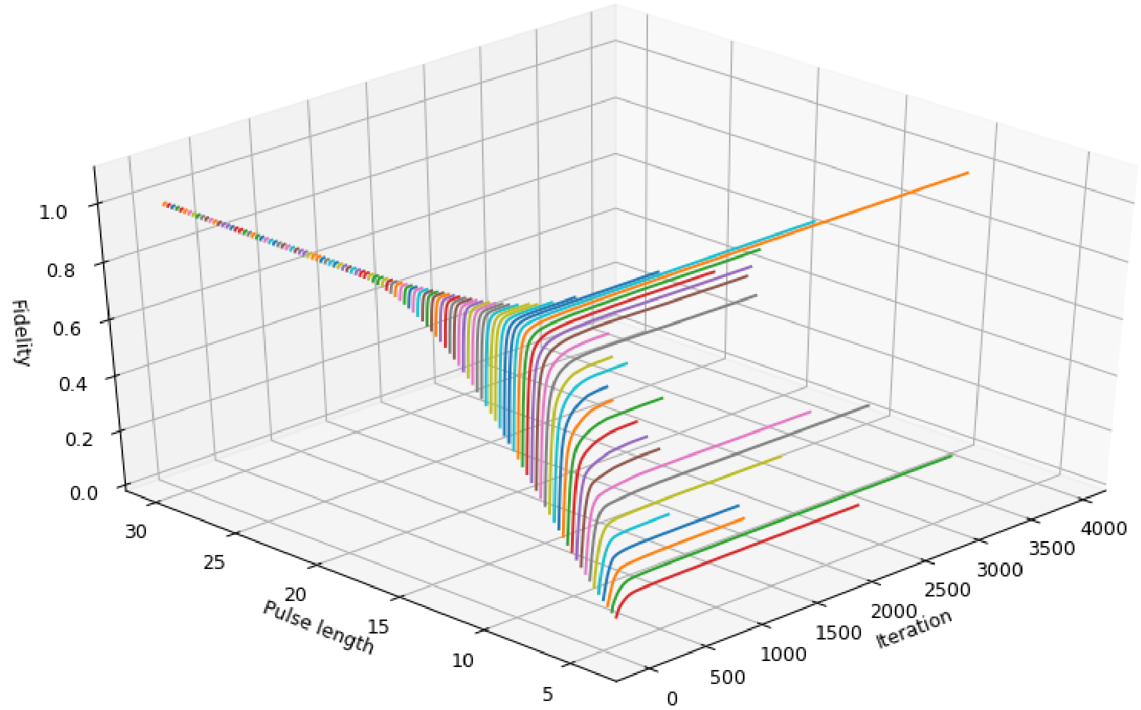


Figure 4.1: Fidelity during optimizations for every pulse length (ns). The different colors help distinguish the lines.

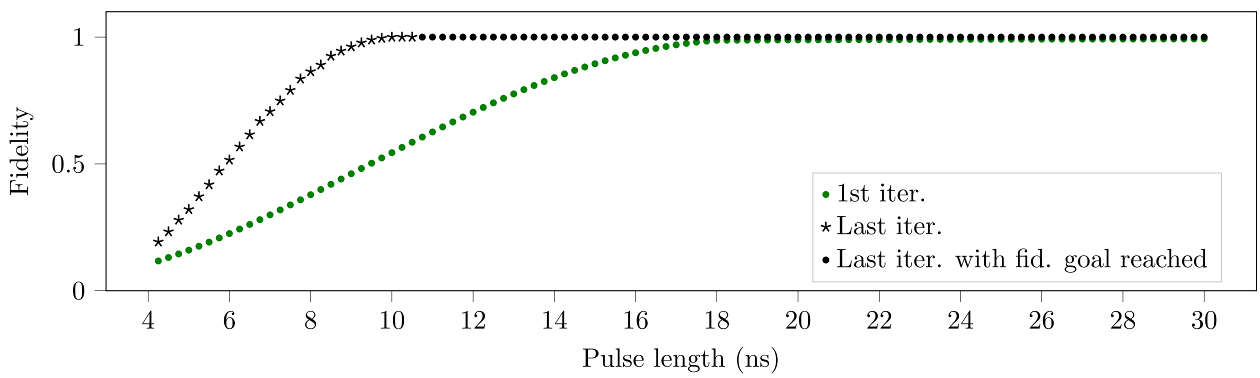
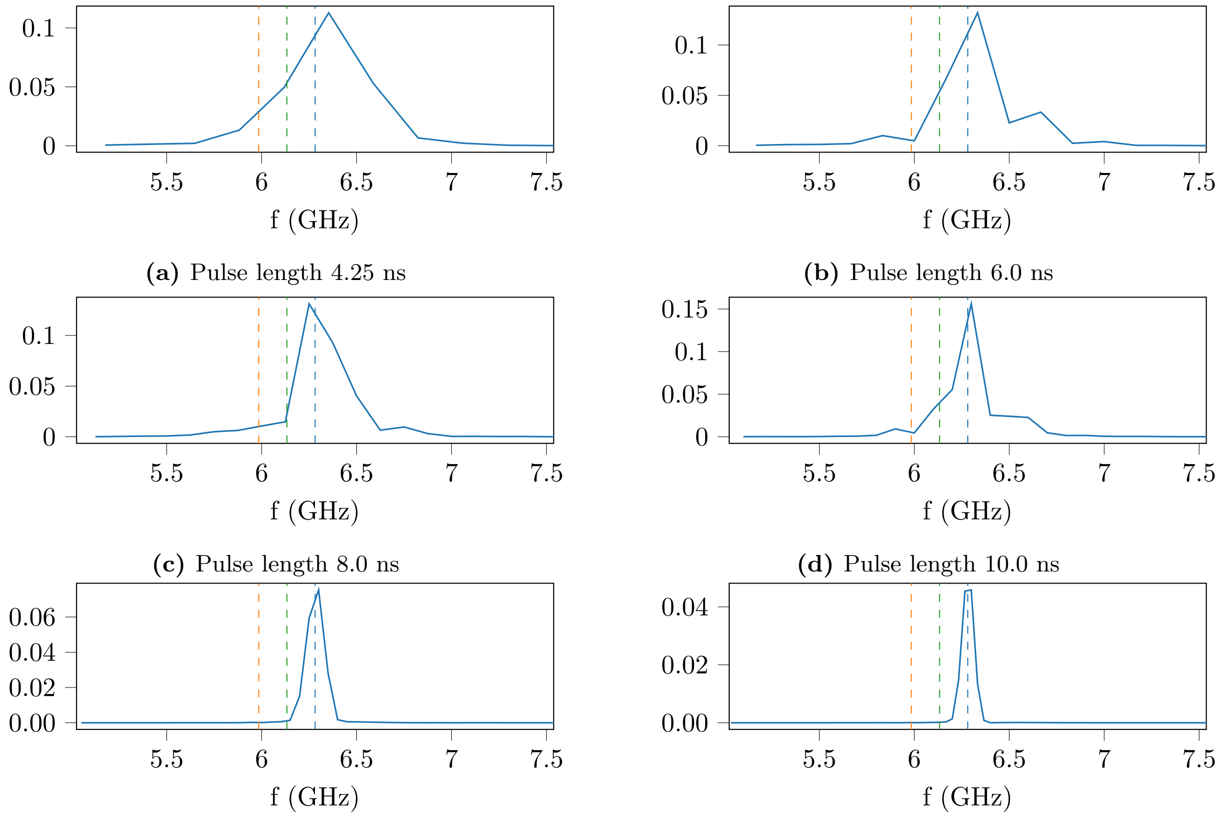


Figure 4.2: Fidelity of first and last iteration of every pulse length. The stable region above

The spectrum of $\Omega(t)$ in the lab frame ($\Omega(t)\mathrm{e}^{i\omega_{01}t}$) is shown in fig. 4.4. For all pulse lengths there is a peak centered roughly at ω_{01} and the width of the peak becomes narrower for longer pulses. For the (maximum) pulse length 30 ns there is almost no support at ω_{02} and ω_{12}.



(a) Pulse length 4.25 ns

(b) Pulse length 6.0 ns

(c) Pulse length 8.0 ns

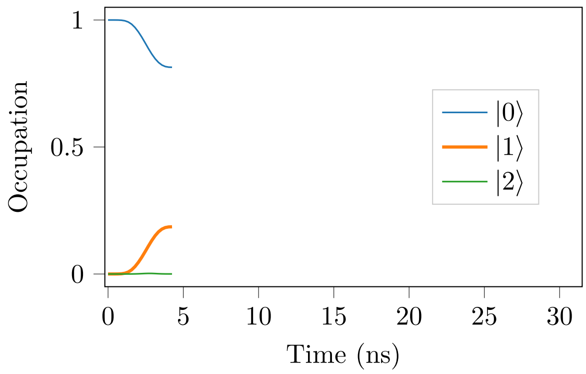
(d) Pulse length 10.0 ns

(e) Pulse length 20.0 ns

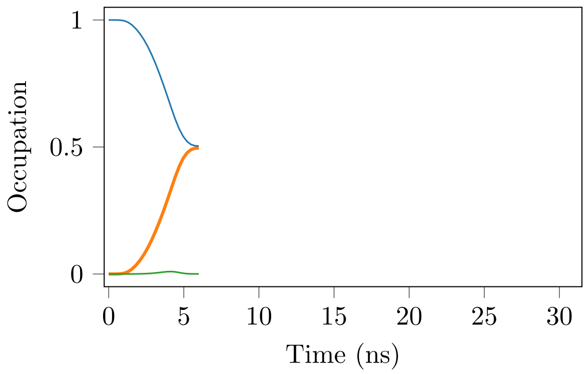
(f) Pulse length 30.0 ns

Figure 4.4: Pulse spectrum of the complex pulses in fig. 4.3. The vertical lines indicate (from left to right) ω_{02}, ω_{12}, ω_{01}.

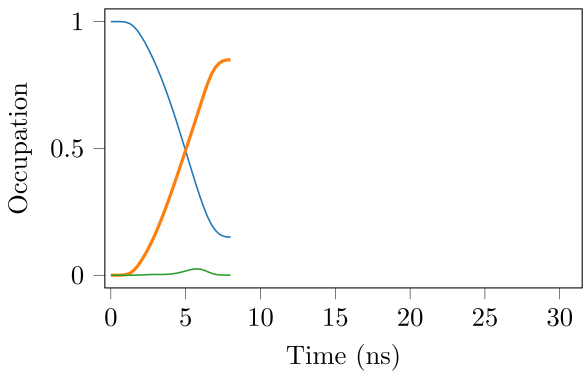
The time evolution of the system under the optimized pulses are visualized by plotting the occupation of the states over time, fig. 4.5, the projection of the state on the Bloch sphere over time, fig. 4.6, and a Hinton diagram of the evolved final state, fig. 4.7.



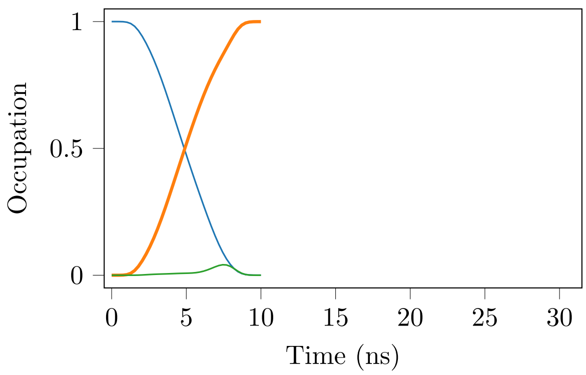
(a) Pulse length 4.25 ns



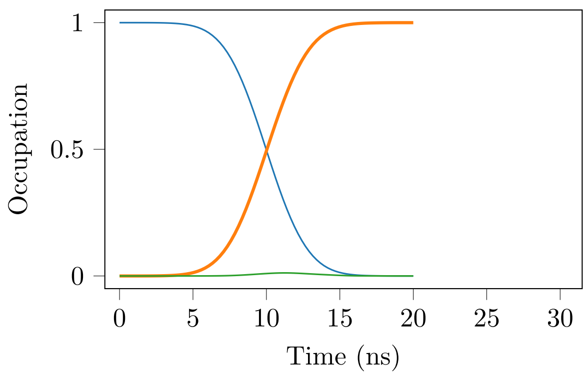
(b) Pulse length 6.0 ns



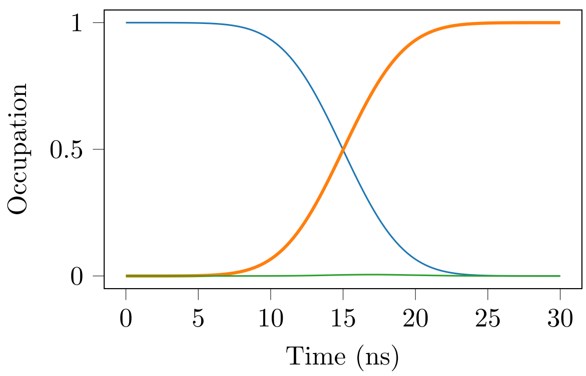
(c) Pulse length 8.0 ns



(d) Pulse length 10.0 ns



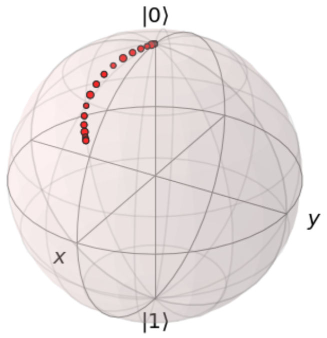
(e) Pulse length 20.0 ns



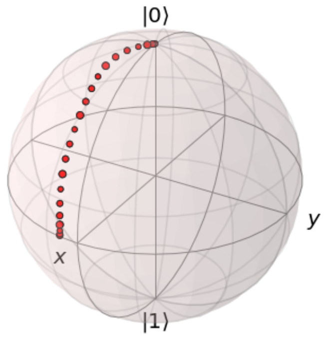
(f) Pulse length 30.0 ns

Figure 4.5: Energy level occupation over time for different lengths of optimized pulses.

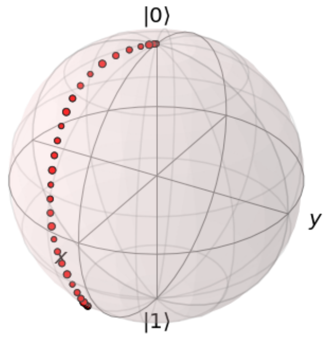
For short pulse lengths there is not enough time for the transfer from $|0\rangle$ to $|1\rangle$, but for a pulse length of roughly 10.0 ns the goal is reached. Figure 4.5 **(b)** shows a little rise in occupation of $|2\rangle$ around 7.5 ns.



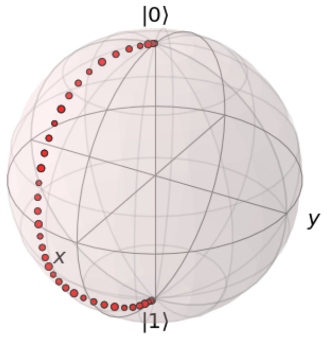
(a) Pulse length 4.25 ns



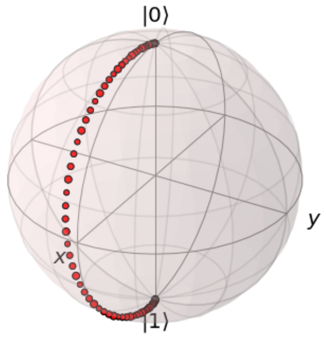
(b) Pulse length 6.0 ns



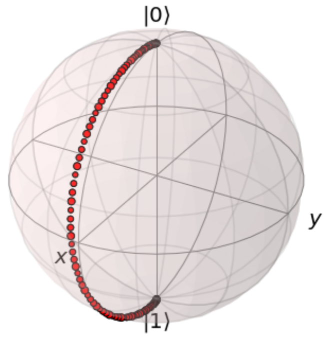
(c) Pulse length 8.0 ns



(d) Pulse length 10.0 ns



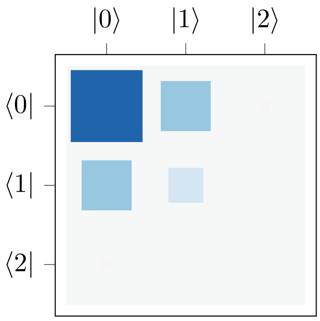
(e) Pulse length 20.0 ns



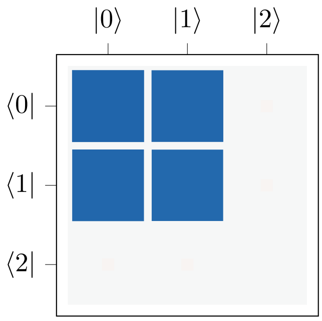
(f) Pulse length 30.0 ns

Figure 4.6: Time dynamics on the Bloch sphere for different lengths of optimized pulses.

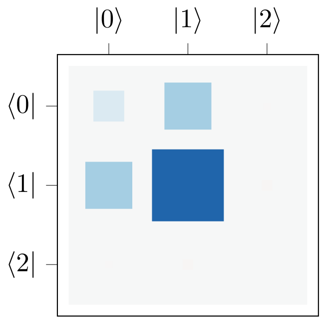
Looking at fig. 4.6 we see that the transfer occurs along the y-axis for longer pulses.



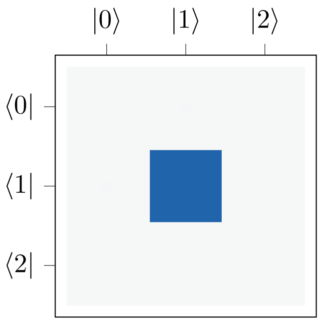
(a) Pulse length 4.25 ns



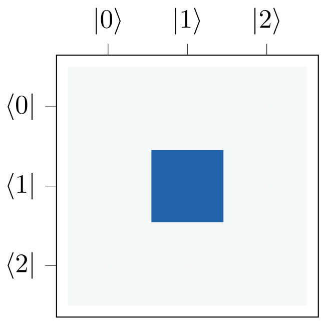
(b) Pulse length 6.0 ns



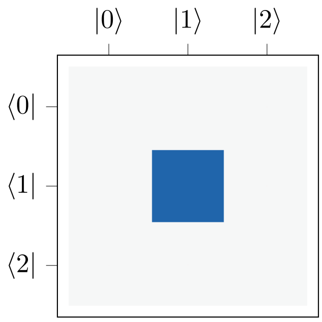
(c) Pulse length 8.0 ns



(d) Pulse length 10.0 ns



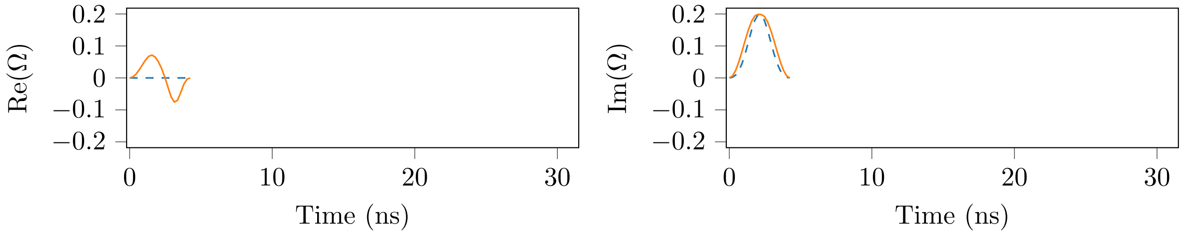
(e) Pulse length 20.0 ns



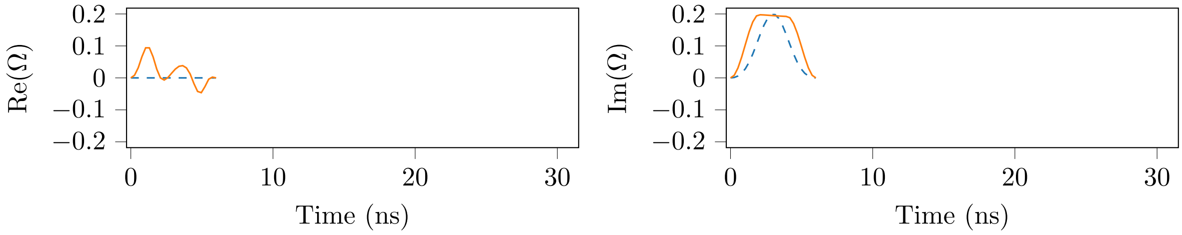
(f) Pulse length 30.0 ns

Figure 4.7: Hinton diagram of $|\psi(T)\rangle\langle\psi(T)|$

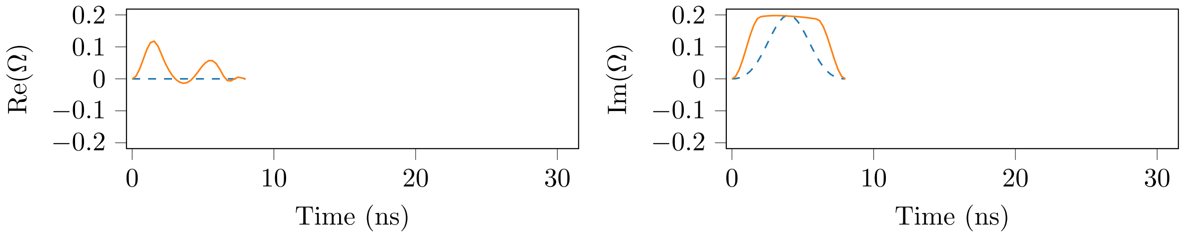
The Hinton diagram in fig. 4.7 provides a visualization of the density matrix $|\psi(T)\rangle\langle\psi(T)|$. The final density matrix shows how the much of the state is in the



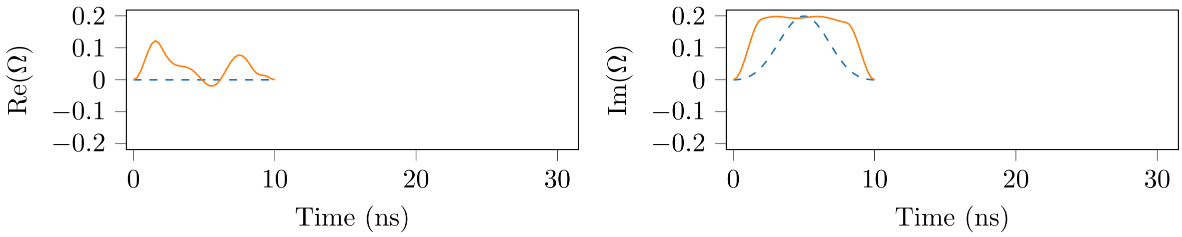
(a) Pulse length 4.25 ns



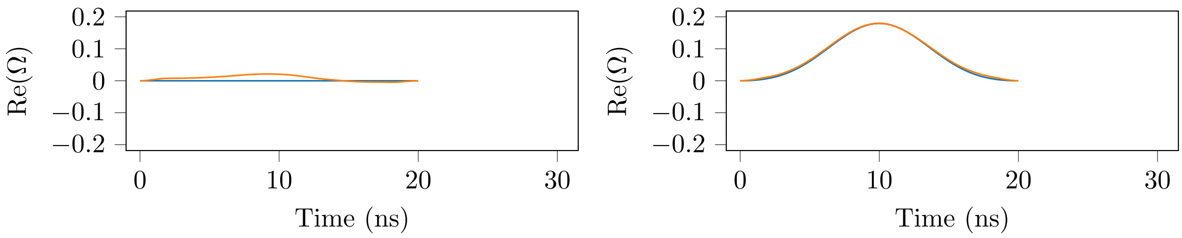
(b) Pulse length 6.0 ns



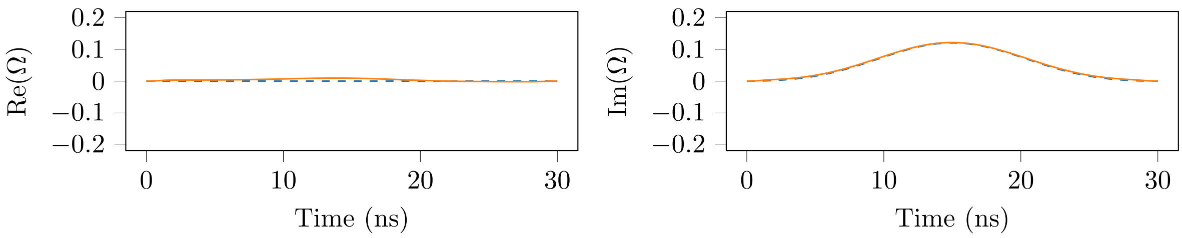
(c) Pulse length 8.0 ns



(d) Pulse length 10.0 ns



(e) Pulse length 20.0 ns



(f) Pulse length 30.0 ns

Figure 4.3: Optimised pulse shapes and guess pulses for pulse lengths **(a)** 4.25 ns, **(b)** 6.0 ns, **(c)** 8.0 ns, **(d)** 10.0 ns, **(e)** 20.0 ns, and **(f)** 30.0 ns. Short pulses (<10 ns) change substantially from the starting Blackman shape while long pulses (>20 ns) only require fine adjustments.

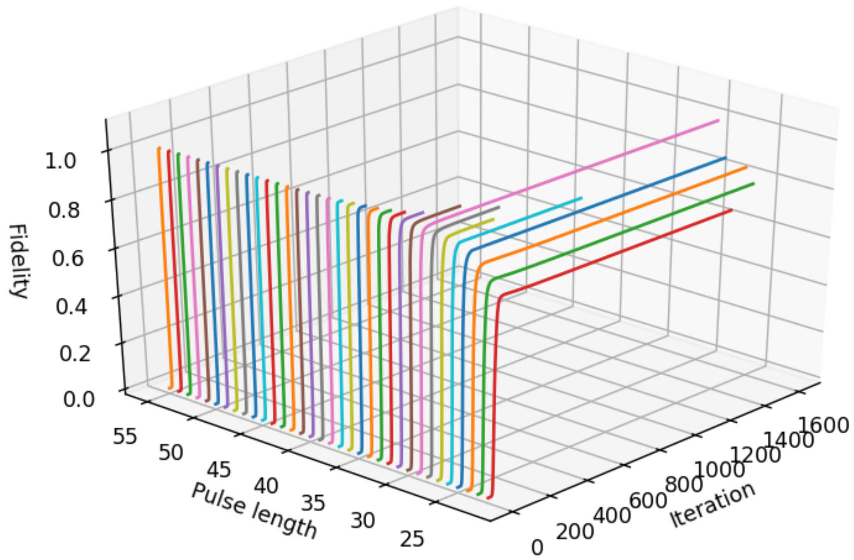


Figure 4.8: Fidelity during optimizations for every pulse length (ns).

4.2 $|0\rangle \rightarrow |2\rangle$ state transfer

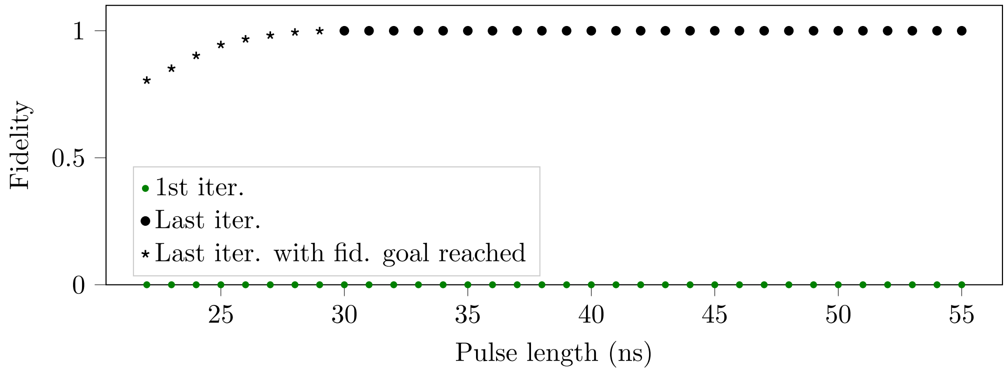


Figure 4.9: Fidelity of first and last iteration of every pulse length.

5 Discussion

6 Conclusion

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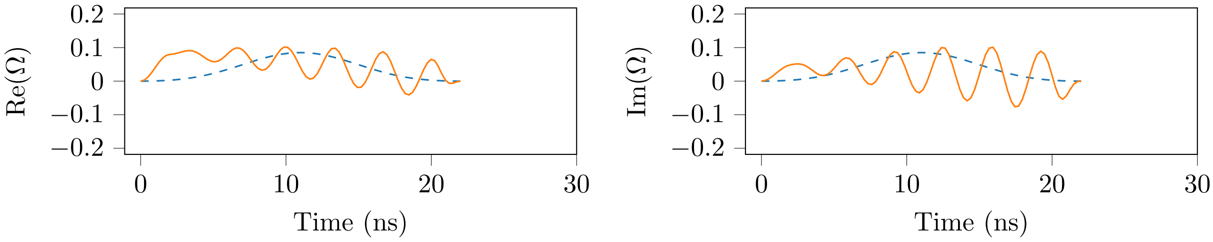
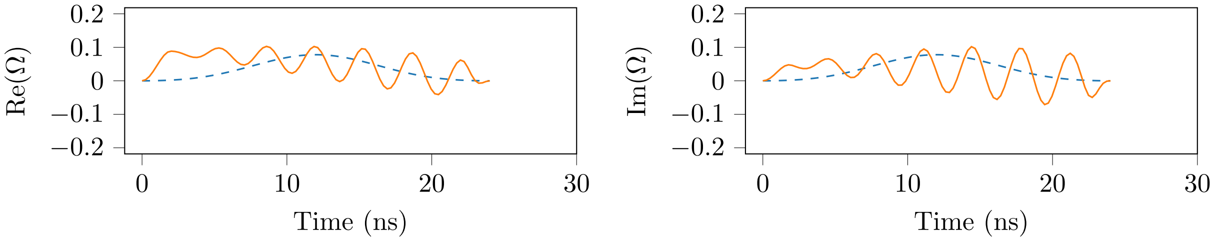
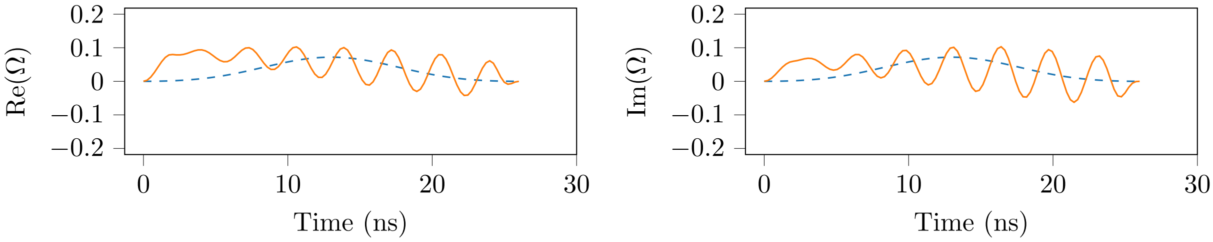


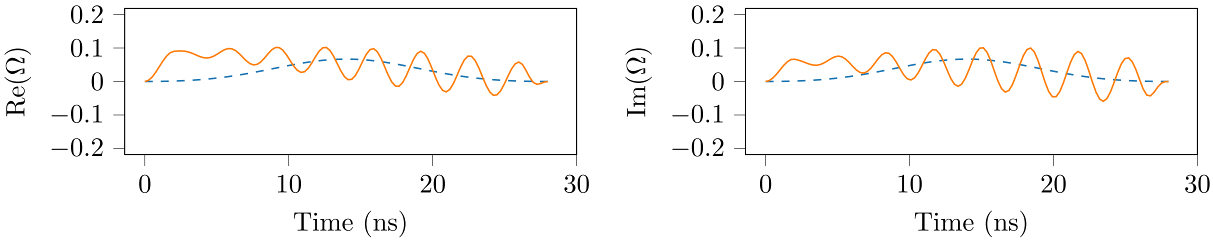
(a) Pulse length 22.0 ns



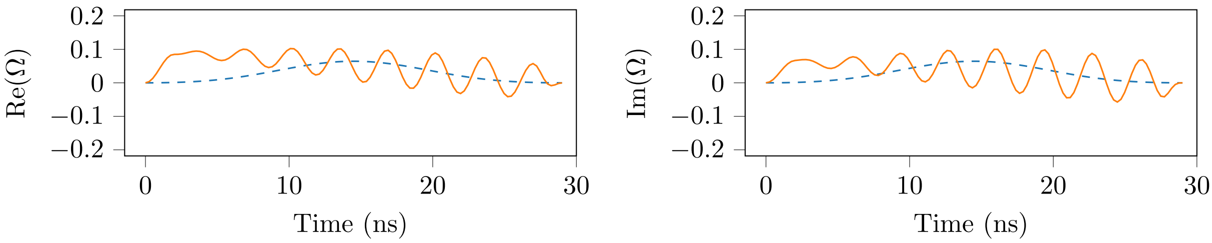
(b) Pulse length 24.0 ns



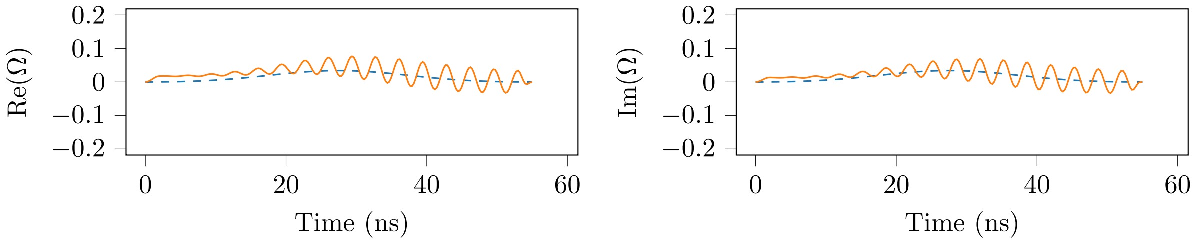
(c) Pulse length 26.0 ns



(d) Pulse length 28.0 ns

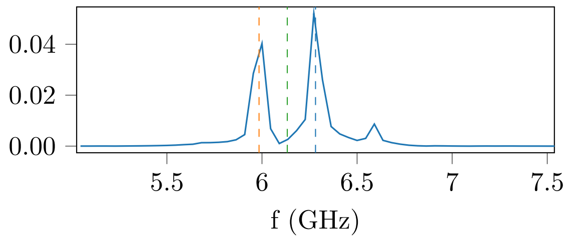


(e) Pulse length 29.0 ns

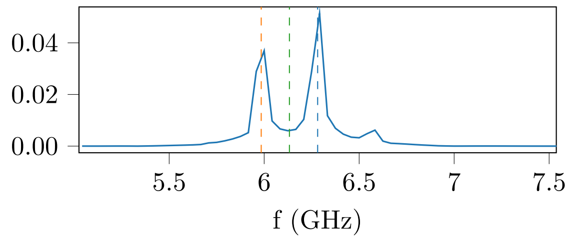


(f) Pulse length 55.0 ns

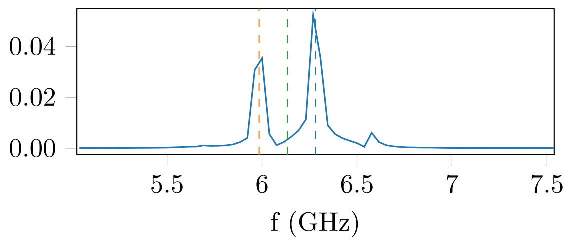
Figure 4.10: Pulse shapes.



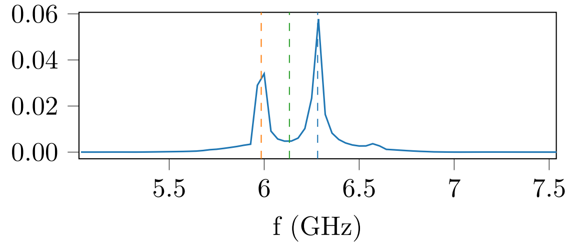
(a) Pulse length 22.0 ns



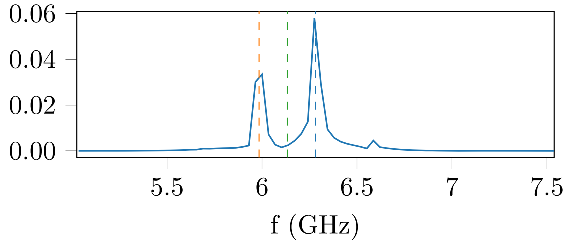
(b) Pulse length 24.0 ns



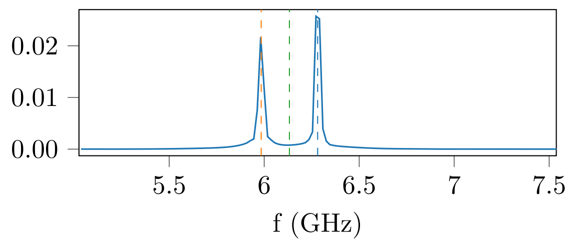
(c) Pulse length 26.0 ns



(d) Pulse length 28.0 ns

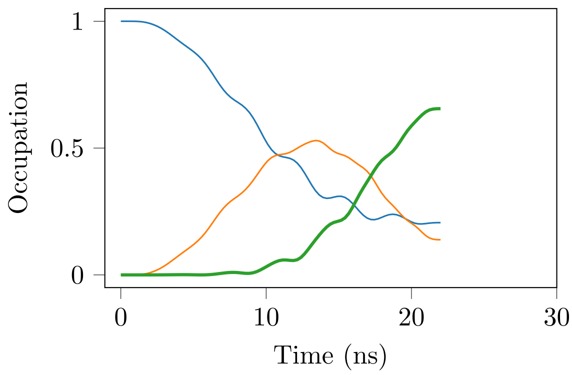


(e) Pulse length 29.0 ns

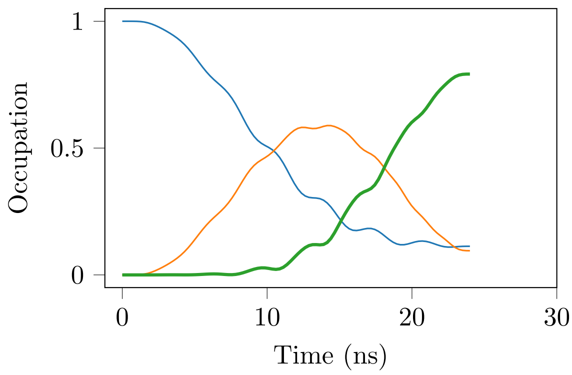


(f) Pulse length 55.0 ns

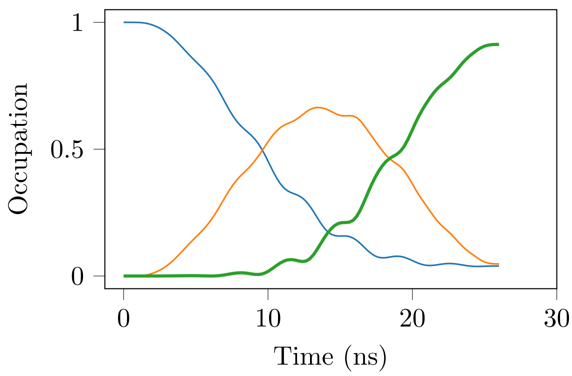
Figure 4.11: Pulse spectrum



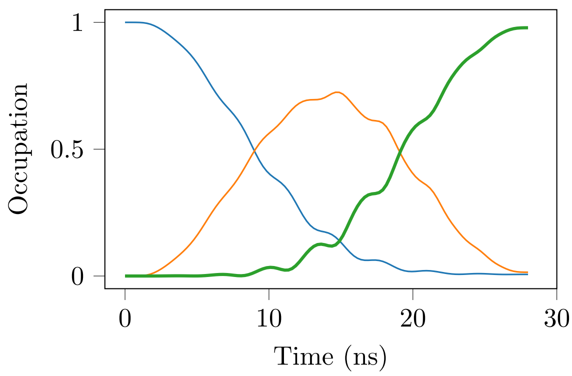
(a) Pulse length 22.0 ns



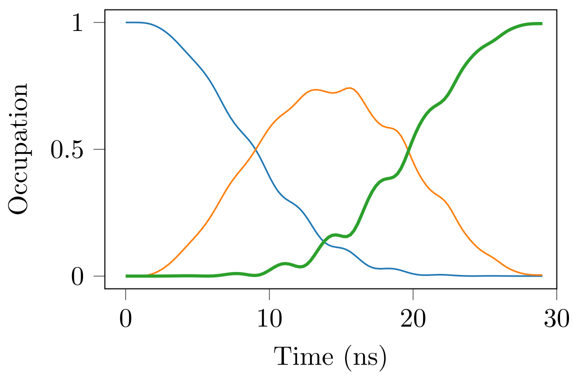
(b) Pulse length 24.0 ns



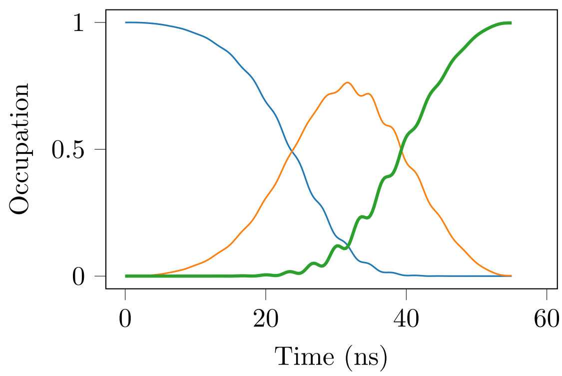
(c) Pulse length 26.0 ns



(d) Pulse length 28.0 ns

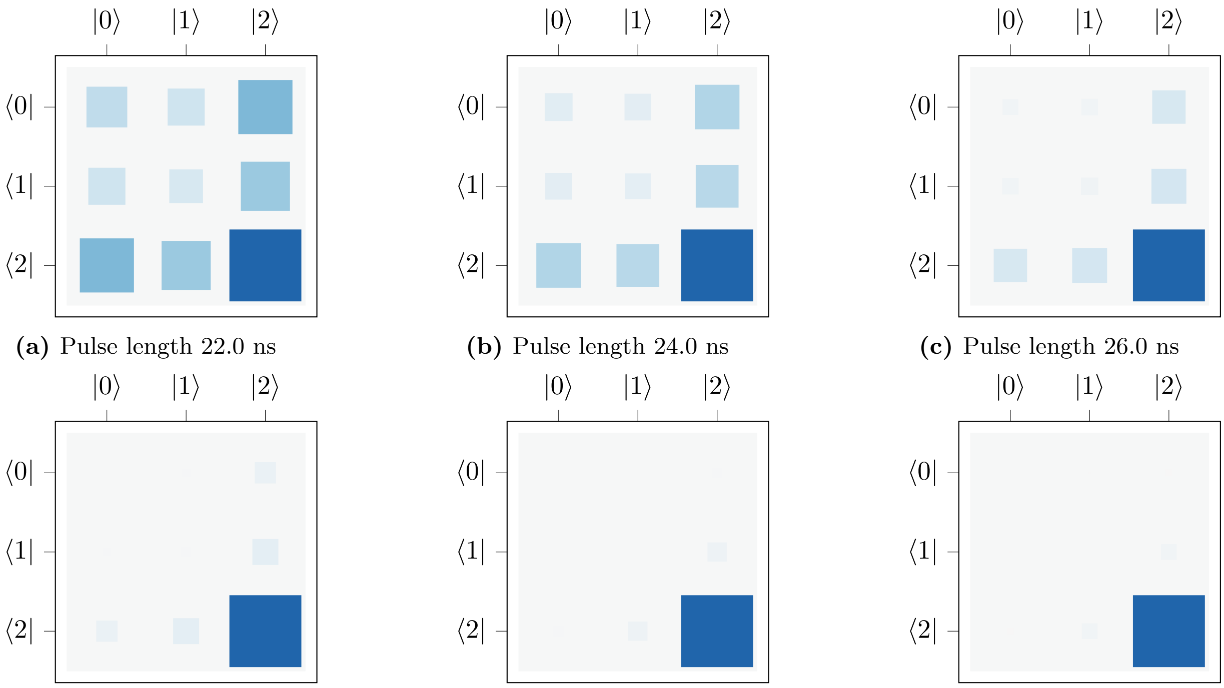


(e) Pulse length 29.0 ns



(f) Pulse length 55.0 ns

Figure 4.12: Energy level occupation over time for different lengths of optimized pulses.



(a) Pulse length 22.0 ns

(b) Pulse length 24.0 ns

(c) Pulse length 26.0 ns

(d) Pulse length 28.0 ns

(e) Pulse length 29.0 ns

(f) Pulse length 55.0 ns

Figure 4.13: Hinton diagram of $|\psi(T)\rangle\langle\psi(T)|$