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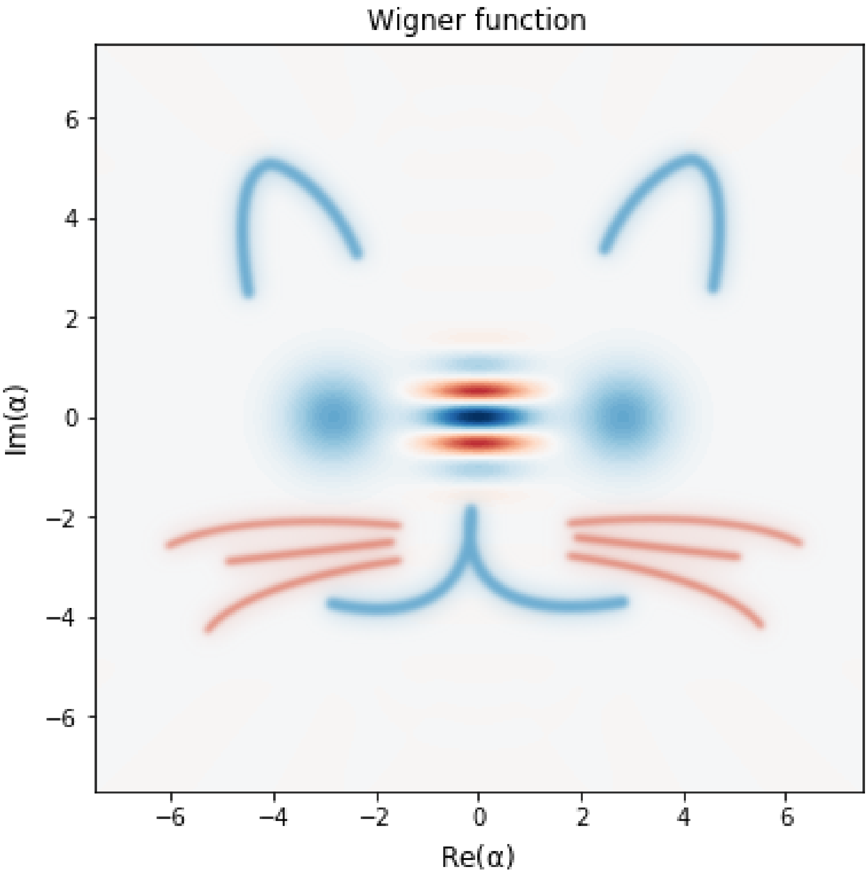

Cat State Preparation By Quantum Optimal Control with Krotov's Method

How to prepare your cat

Master's thesis in Quantum Computing

JOHAN WINTHER

Department of Microtechnology and Nanoscience
CHALMERS UNIVERSITY OF TECHNOLOGY
Göteborg, Sweden 2019

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Abstract

Keywords: quantum computing, quantum optimal control, cat state, cat code

Sammanfattning

Preface

Acknowledgements

Nomenclature

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Contents

1 Introduction

Quantum computing is starting to appear in commercial products [**santos˙ibm˙2016**], however, despite a lot of progress [**preskill˙quantum˙2018**], there are still a lot of challenges to be solved before large scale quantum computers become commonplace. Main problem is noise and keeping quantum states. Unlike classical computers which has natural error correction due to the number of particles (electrons), quantum computers rely on single or very few number of particles which make the states very delicate [**gottesman˙introduction˙2009**].

One of the key technologies for quantum computing is quantum error correction (QEC) [**gottesman˙introduction˙2009**] which is a way to retain the quantum information in a qubit by introducing redundancy into the physical system. **leghtas˙hardware-efficient˙2013** [**leghtas˙hardware-efficient˙2013**], **mirrahimi˙dynamically˙2014** [**mirrahimi˙dyn** propose and **ofek˙extending˙2016** [**ofek˙extending˙2016**] demonstrate a method to encode the quantum information in a clever basis in quantum harmonic resonators. To further clarify, in this scheme the quantum information of a qubit is carefully encoded and decoded into a resonator by applying optimised microwave pulses to the qubit and resonator system which will realise state transfers in both these systems. This thesis will focus on how to numerically optimise encoding and decoding pulses using Krotov's method [**reich˙monotonically˙2012**], a gradient ascent based optimisation algorithm available publicly as a ready-to-use Python package [**goerz˙krotov:˙2019**].

1.1 Purpose

The purpose of this thesis is to numerically optimise microwave pulses to transfer the quantum information in a qubit to a resonator. This will be done using the Krotov Python package [**goerz˙krotov:˙2019**].

1.2 Limitations of the thesis

In this thesis, these limitations will :

- No Liouvillians
-

2 Theory

In this chapter the theoretic concepts will be explained.

2.1 Superconducting resonators

Superconducting resonators are used in quantum computing both as the basis for qubits and as readout and control components []. Although ideal resonators have equally spaced energy levels, in reality they are more or less anharmonic and the general Hamiltonian for a quantum anharmonic resonator is

$$\hat{\mathcal{H}} = \omega \hat{a}^\dagger \hat{a} - \frac{\kappa}{2}\left(\hat{a}^\dagger \hat{a}\right)^2 \tag{2.1}$$

where ω is the resonance frequency, κ is the anharmonic (self-Kerr) term and $\hat{a}$ is the destruction operator which removes an excitation from the resonator.

The anharmonicity can be visualised, see fig. 2.1, by plotting the eigenenergies of eq. (2.1) as a function of κ with $\omega = 1$. A larger anharmonicity gives larger energy spacing for higher excitation states.

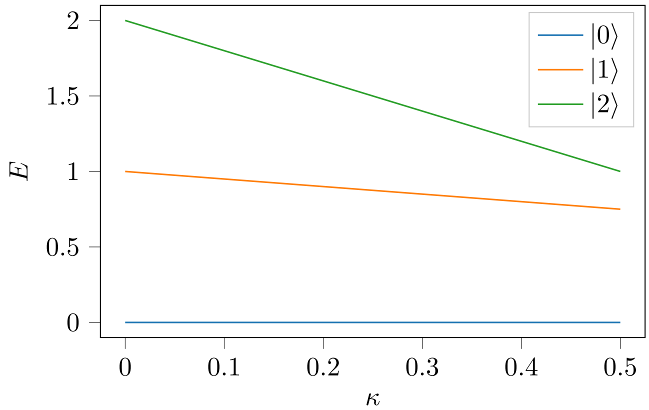

Figure 2.1: The energy levels of a resonator for anharmonicity $\kappa \in [0,1]$.

2.2 Jaynes-Cummings Model

2.3 Bosonic codes

2.3.1 Cat codes

2.4 Quantum control

Quantum control is the process of controlling a quantum system by controlling the amplitude of a set of control operators in time [**fisher˙optimal˙2010**]. Such a system can be described [**fisher˙optimal˙2010**] by a Hamiltonian of the following form

$$\hat{\mathcal{H}}(t) = \underbrace{\hat{\mathcal{H}}_d}_{\text{Drift}} + \underbrace{u_0(t)\hat{\mathcal{H}}_0 + \ldots + u_N(t)\hat{\mathcal{H}}_N}_{\text{Control}}. \tag{2.2}$$

The controls are usually electromagnetic pulses changing in time and thus will be referred to as "pulse shapes" in this thesis [**fisher˙optimal˙2010**].

There are two main questions in quantum control: one of *controllability* and one of *optimal control* where the first deals with the *existence* of solutions given the Hamiltonian and the second with the *optimised* solutions for the pulse shapes $\{u_i(t)\}$ [**dalessandro˙introduction˙2007**]. The optimal solutions are generally not analytically solvable and thus the pulse shapes need to be discretised in time and numerically optimised using algorithms. The algorithm used for this thesis will be presented in the Method chapter.

3 Method

In this chapter Krotov's method will be briefly introduced along with an implementation in Python. Then the numerical experiments will be presented.

3.1 Krotov's Method for quantum optimal control

Krotov's method fundamentally relies on the variational principle to minimise a functional $J\left[\left\{\left|\phi_k^{(i)}(t)\right\rangle\right\}, \left\{\epsilon_l^{(i)}(t)\right\}\right]$ where the constraints are included as Lagrange multipliers [**goerz˙krotov:˙2019**]. A detailed explanation of this functional when the method is applied to quantum systems can be found in [**reich˙monotonically˙2012**].

3.2 Krotov: the Python package

A Python implementation of the Krotov package is available at `https://krotov.readthedocs.io/en/latest/`. It provides simple functions and objects to Test

3.3 Optimisation Experiments

In this chapter the numerical optimisation experiments are presented and motivated.

Optimisation will realise state transfers

4 gigasamples/s

Amplitude constraint (with pi pulse calibration)

Guess pulses half amplitude (actually blackman pulses)

convergence criteria fidelity F change between iterations falls below a certain critera ΔF

3.3.1 $|0\rangle \rightarrow |1\rangle$ state transfer

Pulse shapes were optimised with varying lengths from 4.25 ns to 30 ns with convergence criteria $F > 0.99999$ or $\Delta F < 10^{-7}$. Step size $\lambda = \frac{1}{\frac{1}{2}A_m}$

3.3.2 $|0\rangle \rightarrow |2\rangle$ state transfer

Pulse shapes were optimised with varying lengths from 22 ns to 30 ns with convergence criteria $F > 0.99999$ or $\Delta F < 10^{-9}$. Step size $\lambda = \frac{1}{2A_m}$

4 Results

4.1 State transfer optimisation

Graph of final fidelities

The optimisation converges for pulse lengths 10 ns? and above while dropping

5 Discussion

6 Conclusion

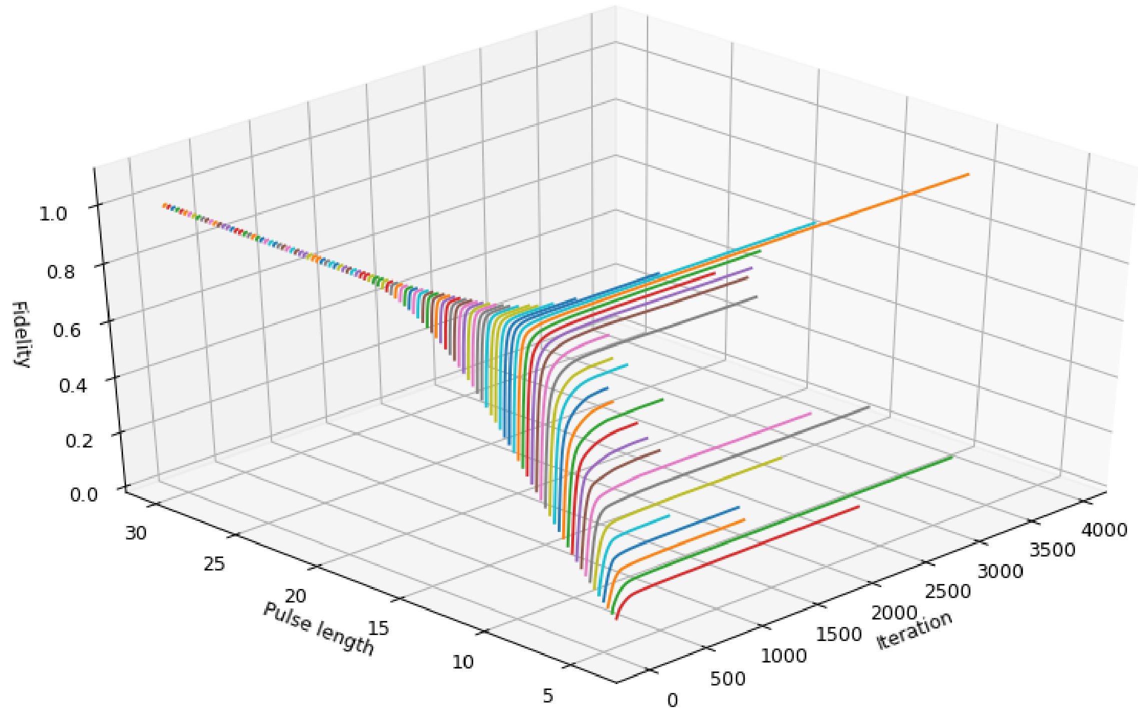

Figure 4.1: Caption

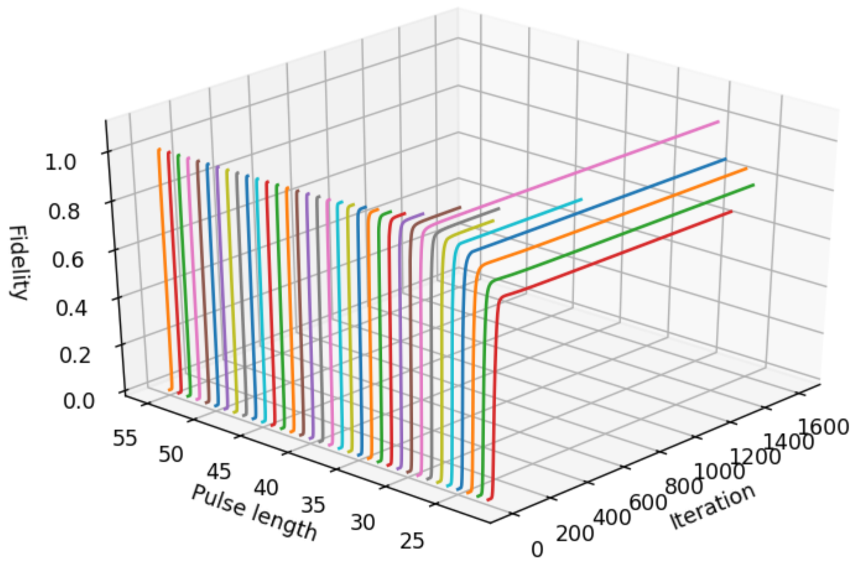

Figure 4.2: Caption

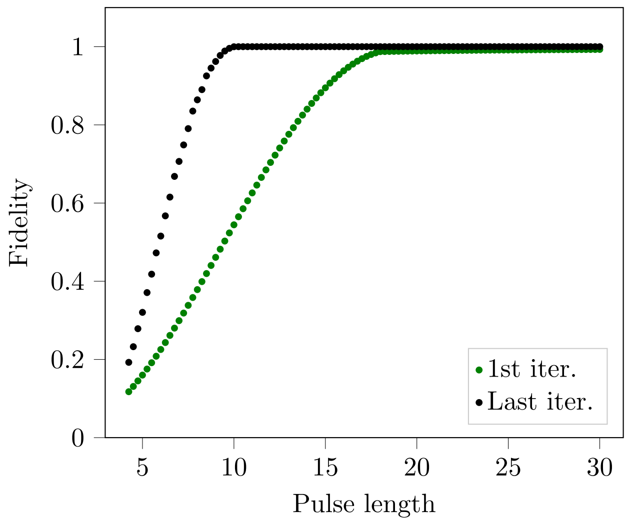

Figure 4.3: Text

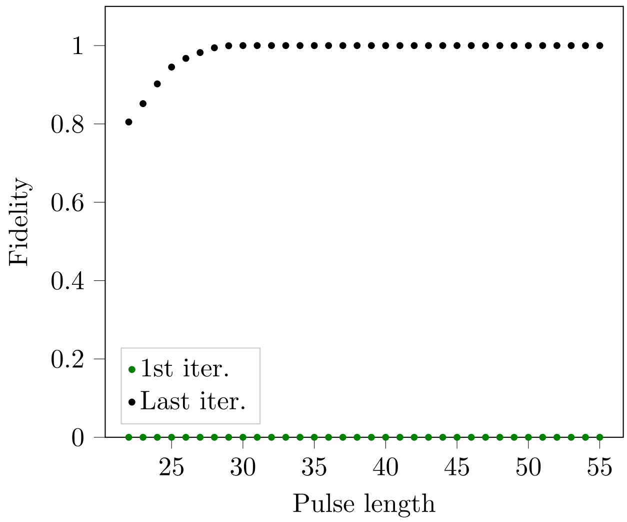

Figure 4.4: Text

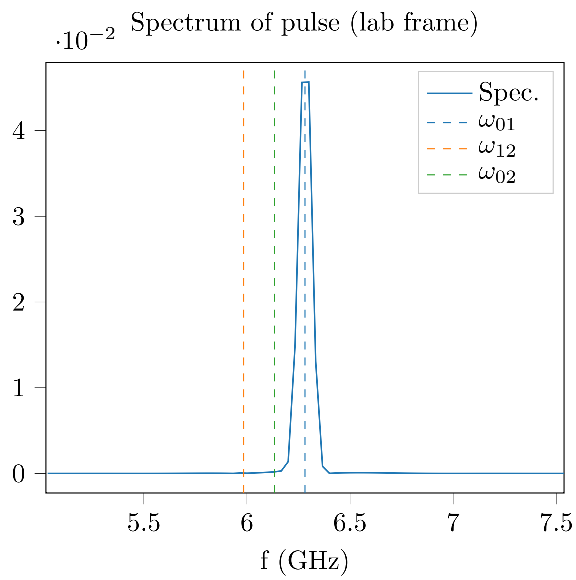

Figure 4.5: Text

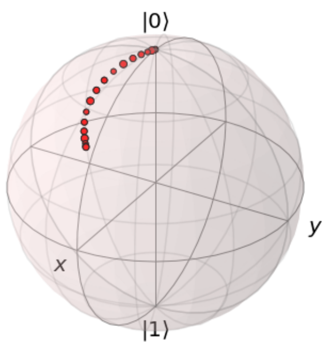

Figure 4.6: Caption