1 Theory

In this chapter the theoretical concepts will be explained. However it is assumed that the reader has a basic understanding of quantum mechanics.

1.1 Superconducting resonators

Superconducting resonators are used in quantum computing both as the basis for qubits and as readout and control components. Although ideal resonators have equally spaced energy levels, in reality they are more or less anharmonic and the general Hamiltonian for a quantum anharmonic resonator is

$$\hat{H} = \omega \hat{a}^\dagger \hat{a} + \frac{\kappa}{2}\left(\hat{a}^\dagger \hat{a}\right)^2 \tag{1.1}$$

where ω is the resonance frequency, κ is the anharmonic (self-Kerr) term and $\hat{a}$ is the destruction operator which removes an excitation from the resonator. Note that this is still an approximation as higher order terms have been neglected.

The anharmonicity can be vizualised, see fig. 1.1, by plotting the eigenenergies of eq. (1.1) as a function of κ. A larger (negative) anharmonicity makes the energy spacing smaller for higher excitation states. This anharmonicity is what permits a resonator to act as a qubit, as it is possible to adress only the first two states $|0\rangle$ and $|1\rangle$. Throughout this thesis, a qubit will be used to refer to an anharmonic resonator even though it has more than two energy levels. Further, the energy spacing between states $|i\rangle$ and $|j\rangle$ will be referred to as ω_{ij} and the "resonance frequency of the qubit" as ω_{01}.

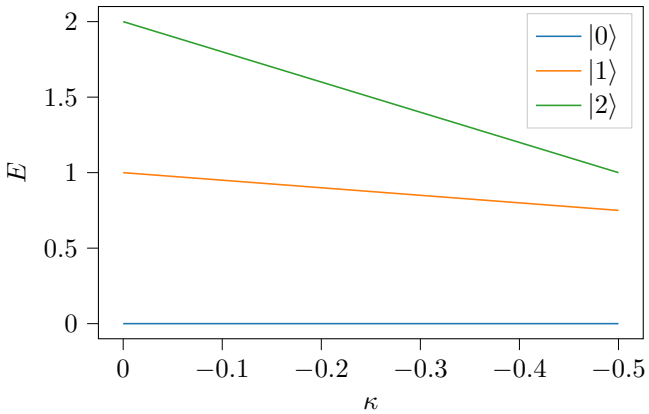

Figure 1.1: The energy levels of a three-level resonator for anharmonicity $\kappa \in [-0.5, 0]$ and $\omega = 1$. Note that the x-axis is reversed.

1.2 Coupled Qubit Cavity System

The Hamiltonian for a coupled qubit and resonator is chosen as

$$\hat{H}(t) = \underbrace{\omega_r \hat{a}^\dagger \hat{a} + \frac{\kappa_r}{2}\left(\hat{a}^\dagger \hat{a}\right)^2}_{\text{Resonator}} + \underbrace{\omega_{01} \hat{b}^\dagger \hat{b} + \frac{\kappa_q}{2}\left(\hat{b}^\dagger \hat{b}\right)^2}_{\text{Qubit}} + \underbrace{g\left(\hat{a}^\dagger \hat{b} + \hat{a}\hat{b}^\dagger\right)}_{\text{Coupling}} \tag{1.2}$$

where $\hat{b}$ is the destruction operator for the qubit. There is now a coupling term with the coupling strength g.

1.3 Visualization of quantum states

In order to understand the results presented later in this thesis, some visualization techniques of quantum states are shown and explained.

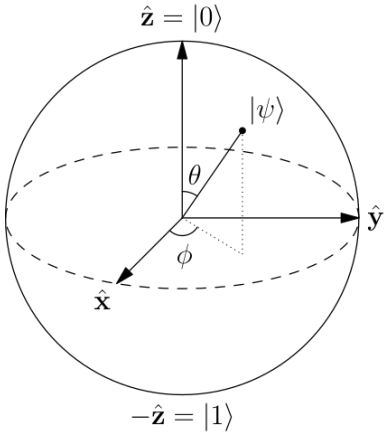

Figure 1.2: Representation of an abritrary pure quantum state on the Bloch sphere.
Source: [1] (CC BY-SA 3.0).

1.3.1 Bloch sphere

The pure state of a qubit can be visualized on the surface of a unit sphere with the following parametrization

$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + \mathrm{e}^{i\phi}\sin\left(\frac{\theta}{2}\right)|1\rangle \tag{1.3}$$

where θ and ϕ are angles which are shown in fig. 1.2. The ground state $|0\rangle$ is located on the "north pole" and the excited state $|1\rangle$ on the "south pole".

1.3.2 Density matrices and Hinton diagrams

1.3.3 Wigner function

1.4 Bosonic codes

1.4.1 Cat codes

1.5 Quantum optimal control

Quantum control is the process of controlling a quantum system by controlling the amplitude of a set of control operators in time [2]. Such a system can be described [2] by a Hamiltonian of the following form

$$\hat{H}(t) = \underbrace{\hat{H}_d}_{\text{Drift}} + \underbrace{u_0(t)\hat{H}_0 + \ldots + u_N(t)\hat{H}_N}_{\text{Control}}\,. \tag{1.4}$$

The controls are usually electromagnetic pulses changing in time and thus will be referred to as "pulse shapes" [2] in this thesis .

There are two main questions in quantum control: one of *controllability* and one of *optimal control*. The first deals with the *existence* of solutions given a Hamiltonian and the second with the *optimized* solutions for the pulse shapes $\{u_i(t)\}$ [3]. The optimal solutions are generally not analytically solvable and thus the pulse shapes need to be discretized in time and numerically optimized using algorithms. The algorithm used for this thesis is called Krotov's method and will be presented in the Method chapter.

1.5.1 Unitary transformation

A unitary transformation can significantly simplify systems that are hard to simulate. This idea will be conceptually presented here and then implemented in the Method chapter. The unitary transformation that is

used is a special case of transformations called the *interaction picture* where the Hamiltonian is split up into a time-independent and time-dependent part

$$\hat{H}(t) = \hat{H}_A + \hat{H}_B(t). \tag{1.5}$$

By choosing the unitary operator $\hat{U} = \mathrm{e}^{i\hat{H}_A t}$ the unitary transformation takes us into the interaction picture

$$\begin{aligned}
\hat{H} \rightarrow\ & \hat{U}\Big[\hat{H}_A + \hat{H}_B(t)\Big]\hat{U}^\dagger + i\frac{\mathrm{d}\hat{U}}{\mathrm{d}t}\hat{U}^\dagger = \\
=\ & \hat{U}\hat{H}_A\hat{U}^\dagger + \hat{U}\hat{H}_B\hat{U}^\dagger + i\Big(i\hat{H}_A t\Big)\mathrm{e}^{i\hat{H}_A t}\mathrm{e}^{-i\hat{H}_A t} = \\
=\ & \hat{H}_A + \hat{U}\hat{H}_B\hat{U}^\dagger - \hat{H}_A = \hat{U}\hat{H}_B\hat{U}^\dagger
\end{aligned}$$