Helicopter Lab

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1 Part 1 - Mathematical modeling

1.1 Problem 1

$$\sum T = J_p\ddot{p} = force \times arm = l_p K_f(V_f - V_b) = L_1 V_d \quad (1)$$

$$\sum T = J_e\ddot{e} = force \times arm = g(m_c l_c - 2m_p l_h)\cos e + K_f l_h(V_f + V_b)\cos p$$
$$= L_2 \cos e + L_3 V_s \cos p \quad (2)$$

$$\sum T = J_\lambda\ddot{\lambda} = force \times arm = -l_h K_f(V_f + V_b)\cos e \sin p$$
$$= L_4 V_s \cos e \sin p \quad (3)$$

Constants:

$$L_1 = l_p K_f \quad (4)$$
$$L_2 = g(m_c l_c - 2m_p l_h) \quad (5)$$
$$L_3 = K_f l_h \quad (6)$$
$$L_4 = -l_h K_f \quad (7)$$

1.2 Problem 2

$$J_p\ddot{p} = L_1 V_d = 0 \rightarrow V_d^* = V_d = 0 \quad (8)$$

$$J_e\ddot{e} = L_2 \cos e + L_3 V_s \cos p = 0 \rightarrow V_s^* = V_s = -L_2/L_3 \quad (9)$$

Transformation:

$$J_p\ddot{p} = L_1 V_d \quad (10)$$

$$J_p\ddot{\tilde{p}} = L_1(\tilde{V}_d + V_d^*) = L_1\tilde{V}_d \quad (11)$$

$$J_e\ddot{e} = L_2 \cos e + L_3 V_s \cos p \quad (12)$$

$$J_e\ddot{\tilde{e}} = L_2 \cos\tilde{e} + L_3(\tilde{V}_s + V_s^*)\cos\tilde{p} = L_2\cos\tilde{e} + L_3(\tilde{V}_s - L_2/L_3)\cos\tilde{p} \quad (13)$$

$$J_\lambda\ddot{\lambda} = L_4 V_s \cos e \sin p \quad (14)$$

$$J_\lambda \ddot{\tilde{\lambda}} = L_4(\tilde{V}_s + V_s^*)\cos\tilde{e}\sin\tilde{p} = L_4(\tilde{V}_s - L_2/L_3)\cos\tilde{e}\sin\tilde{p} \tag{15}$$

Linearization:

$$\begin{bmatrix} \dot{\tilde{p}} \\ \ddot{\tilde{p}} \\ \dot{\tilde{e}} \\ \ddot{\tilde{e}} \\ \dot{\tilde{\lambda}} \\ \ddot{\tilde{\lambda}} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -\frac{L_4 L_2}{J_\lambda L_3} & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{p} \\ \dot{\tilde{p}} \\ \tilde{e} \\ \dot{\tilde{e}} \\ \tilde{\lambda} \\ \dot{\tilde{\lambda}} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{L_1}{J_p} & 0 \\ 0 & 0 \\ 0 & \frac{L_3}{J_e} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{V}_d \\ \tilde{V}_s \end{bmatrix} \tag{16}$$

$$\ddot{\tilde{p}} = \frac{L_1}{J_p}\tilde{V}_d = K_1\tilde{V}_d \tag{17}$$

$$\ddot{\tilde{e}} = \frac{L_3}{J_e}\tilde{V}_s = K_2\tilde{V}_s \tag{18}$$

$$\ddot{\tilde{\lambda}} = -\frac{L_4 L_2}{J_\lambda L_3}\tilde{p} = K_3\tilde{p} \tag{19}$$

Constants:

$$K_1 = \frac{L_1}{J_p} = 0.3361 \tag{20}$$

$$K_2 = \frac{L_3}{J_e} = 0.1074 \tag{21}$$

$$K_3 = -\frac{L_4 L_2}{J_\lambda L_3} = -0.3361 \tag{22}$$

1.3 Problem 3

When we guesstimated the input we needed to control the helicopter, we tried to control it without any gain to see how it responded. We thought that the helicopter was to slow. We therefore added a gain from the output of the joystick to the voltage difference. We tried increasing the gains until we reached a gain we thought was appropriate. We ended up with gain of 3 on x and 5 on y.

The ideal formulas did not account for friction, drag and offset. The theoretical model did therefore not work as well as we had hoped. The helicopter was unstable and hard to control.

1.4 Problem 4

$$V_s = 3.7 \tag{23}$$

$$V_d = -0.055 \tag{24}$$

$$g(m_c l_c - 2m_p l_h) + K_f l_h V_s = 0 \tag{25}$$

$$K_f = -\frac{g(m_c l_c - 2m_p l_h)}{l_h V_s} = 0.1935907... \approx 0.1936 \tag{26}$$

In order to keep the helicopter horizontally stable, we had to set V_s and V_d. Based on the value of V_s we found the motor force constant K_f.

2 Part 2 - Monovariable control

2.1 Problem 1

$$\tilde{V_d} = K_{pp}(\tilde{p_c} - \tilde{p}) - K_{pd}\dot{\tilde{p}} \tag{27}$$

$$\ddot{\tilde{p}} = K_1\tilde{V_d} = K_1(K_{pp}(\tilde{p}c - \tilde{p}) - K_{pd}\dot{\tilde{p}}) \tag{28}$$

$$\ddot{\tilde{p}} + K_1K_{pd}\dot{\tilde{p}} - K_1(K_{pp}(\tilde{p}c - \tilde{p})) = 0 \tag{29}$$

$$\tilde{p}s^2 + K_1K_{pd}\tilde{p}s - K_1(K_{pp}(p_c - \tilde{p})) = 0 \tag{30}$$

$$\frac{p}{p_c}(s) = \frac{K_1K_{pp}}{s^2 + K_1K_{pd}s + K_1K_{pp}} \tag{31}$$

$$s = -\frac{K_1K_{pd} \pm \sqrt{(K_1K_{pd})^2 - 4K_1K_{pp}}}{2} \tag{32}$$

Critical damped system:

$$(K_1K_{pd})^2 - 4K_1K_{pp} = 0 \tag{33}$$

$$K_{pp} = \frac{1}{4}K_1(K_{pd})^2 \tag{34}$$

As long as equation (34) is true, the system will be critical damped.

We chose K_{pd} to be 7, and from equation (34) we then got K_{pp} to be 4.12. The reason we chose K_{pd} to be 7 is because with a larger K_{pd} the system got unstable in form of oscillations and it took to long to get to the referance value. With a smaller K_{pd} the system was non responsive, and too slow.

We could achieve the same result by studying the transfer function.

$$H(s) = K\frac{\omega_0^2}{s^2 + 2\gamma\omega_0 s + \omega_0^2} \tag{35}$$

And adjusting our parameters so that $\gamma = 1$.

With the PD controller, the helicopter is easier to control than it was when only feed forward was used.

2.2 Problem 2

$$\tilde{p_c} = K_{rp}(\dot{\tilde{\lambda}}_c - \dot{\tilde{\lambda}}) \tag{36}$$

$$\ddot{\tilde{\lambda}} = K_3\tilde{p} \tag{37}$$

$$\frac{\ddot{\lambda}}{K_3} = \tilde{p} \tag{38}$$

Since $\tilde{p_c} = \tilde{p}$, we can set (33) = (35)

$$\frac{s\dot{\tilde{\lambda}}}{K_s} = K_{rp}(\dot{\tilde{\lambda}}_c - \dot{\tilde{\lambda}}) \tag{39}$$

$$\dot{\tilde{\lambda}}(\frac{s}{K_3} + K_r p) = K_{rp}\dot{\tilde{\lambda}}_c \tag{40}$$

$$\frac{\dot{\tilde{\lambda}}}{\tilde{\lambda}_c} = \frac{K_{rp}}{\frac{s}{K_3} + K_{rp}} = \frac{K_{rp}K_3}{s + K_{rp}K_3} \tag{41}$$

We chose K_{rp} to be -1.6 because with a lower K_{rp} it oscillated and was too aggressive and with a higher K_{rp} it was too slow.

3 Part 3 - Multivariable control

3.1 Problem 1

$$\dot{x} = Ax + Bu \tag{42}$$

with

$$x = \begin{bmatrix} \tilde{p} \\ \dot{\tilde{p}} \\ \dot{\tilde{e}} \end{bmatrix} \tag{43}$$

and

$$u = \begin{bmatrix} \tilde{V_s} \\ \tilde{V_d} \end{bmatrix} \tag{44}$$

we get

$$\begin{bmatrix} \dot{\tilde{p}} \\ \ddot{\tilde{p}} \\ \ddot{\tilde{e}} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{p} \\ \dot{\tilde{p}} \\ \dot{\tilde{e}} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & K_1 \\ K_2 & 0 \end{bmatrix} \begin{bmatrix} \tilde{V_s} \\ \tilde{V_d} \end{bmatrix} \tag{45}$$

3.2 Problem 2

We checked that the matrix was controllable with the function $ctrb(A, B)$ in MatLab. We got a matrix of rank 3 as expected which has full rank and therefore the matrix is controllable.

We used Brayson's rule to find the two matrices. (Skriv kort om regelen**)

$$Q = \begin{bmatrix} \frac{1}{(\frac{\pi}{4})^2} & 0 & 0 \\ 0 & \frac{1}{(\frac{200\pi}{180})^2} & 0 \\ 0 & 0 & \frac{1}{(\frac{35\pi}{180})^2} \end{bmatrix} \tag{46}$$

$$R = \begin{bmatrix} \frac{1}{9^2} & 0 \\ 0 & \frac{1}{9^2} \end{bmatrix} \tag{47}$$

We found K by using the MatLab function $lqr(A, B, Q, R)$

$$K = \begin{bmatrix} 0 & 0 & 14.7332 \\ 11.4592 & 8.6507 & 0 \end{bmatrix} \tag{48}$$

$$P = \begin{bmatrix} 0 & 14.7332 \\ 11.4592 & 0 \end{bmatrix} \tag{49}$$

3.3 Problem 3

When we add the two new states, we get the new matrices:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \tag{50}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & K_1 \\ K_2 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{51}$$

$$Q = \begin{bmatrix} \frac{1}{(\frac{\pi}{4})^2} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{(\frac{200\pi}{180})^2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{(\frac{35\pi}{180})^2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{(\frac{\pi}{4})^2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{(\frac{200\pi}{180})^2} \end{bmatrix} \tag{52}$$

$$K = \begin{bmatrix} 0 & 0 & 16.2808 & 0 & 2.5783 \\ 19.6487 & 11.1160 & 0 & 11.4592 & 0 \end{bmatrix} \tag{53}$$

$$P = \begin{bmatrix} 0 & 16.2808 \\ 19.6487 & 0 \end{bmatrix} \tag{54}$$

With integral effect, the helicopter stays at the elevation we set. **

4 Part 4 - Multivariable control

4.1 Problem 1

$$\begin{bmatrix} \dot{\tilde{p}} \\ \ddot{\tilde{p}} \\ \dot{\tilde{e}} \\ \ddot{\tilde{e}} \\ \dot{\tilde{\lambda}} \\ \ddot{\tilde{\lambda}} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -K_3 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{p} \\ \dot{\tilde{p}} \\ \tilde{e} \\ \dot{\tilde{e}} \\ \tilde{\lambda} \\ \dot{\tilde{\lambda}} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & K_1 \\ 0 & 0 \\ K_2 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{V_s} \\ \tilde{V_d} \end{bmatrix} \tag{55}$$

$$y = \begin{bmatrix} \tilde{p} \\ \tilde{e} \\ \tilde{\lambda} \end{bmatrix} = Cx = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \tilde{p} \\ \dot{\tilde{p}} \\ \tilde{e} \\ \dot{\tilde{e}} \\ \tilde{\lambda} \\ \dot{\tilde{\lambda}} \end{bmatrix} \tag{56}$$

4.2 Problem 2

We checked the observability of the system with the function $obsv(A, C)$ in MatLab. We got a matrix of rank 6 as expected which has full rank and therefore the matrix is observable.
Then we could create a linear observer for the system. We did this by choosing eigenvalues with negative real parts with the same radius and angel between them. At first we had a radius ten times the largest system pole, but this turned out to be too small. In the end we chose r = 200 and an total angel of 80 degrees. It was important to find poles which was not too close to the imagenary axis (- angel too big), because this made the system oscillate. A too large radius made the system too sensitive to noice. With these new poles we used the Matlab function place(A',C',poles') to find the matrix L.

4.3 Problem 3

$$y = \begin{bmatrix} \tilde{e} \\ \tilde{\lambda} \end{bmatrix} \rightarrow C = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{57}$$

The observability matrix in this case has rank six and the system is observable.

$$y = \begin{bmatrix} \tilde{p} \\ \tilde{e} \end{bmatrix} \rightarrow C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \tag{58}$$

Now the observability matrix has rank four and the system is not observable.

This observer does not work well event though it's controllable, because the $\ddot{\tilde{\lambda}}$ is dependent of the pitch. The pitch is not measuered or estimated so we have no control over it as we can see in figure 2. Therefore the $\ddot{\tilde{\lambda}}$ is not estimated as well as in problem 2. We can therefore conclude that some observers are poor if they are dependent of another variable that are not measured/estimated.

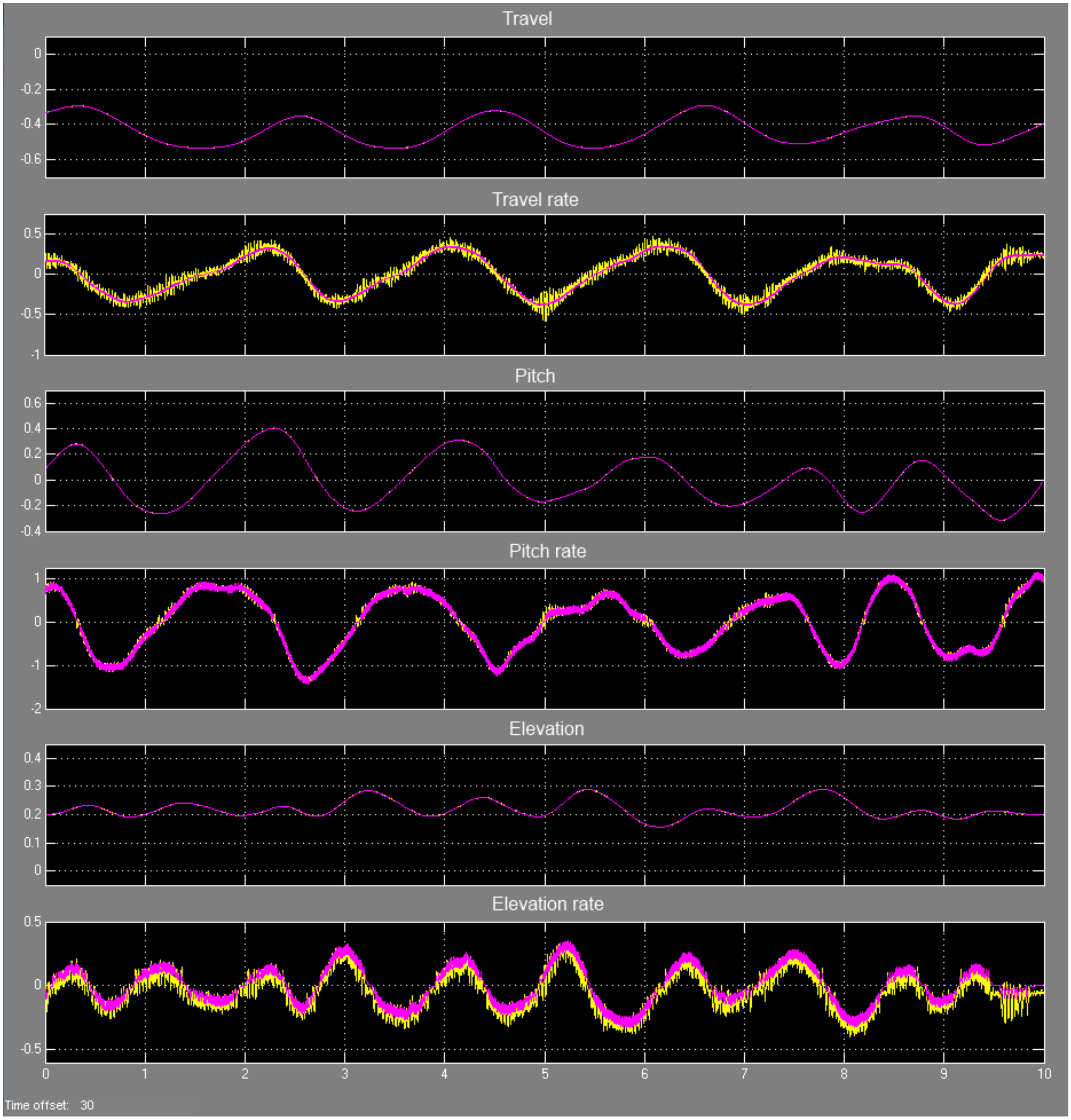

Figure 1: Estimated (yellow) vs measured (pink) with pitch

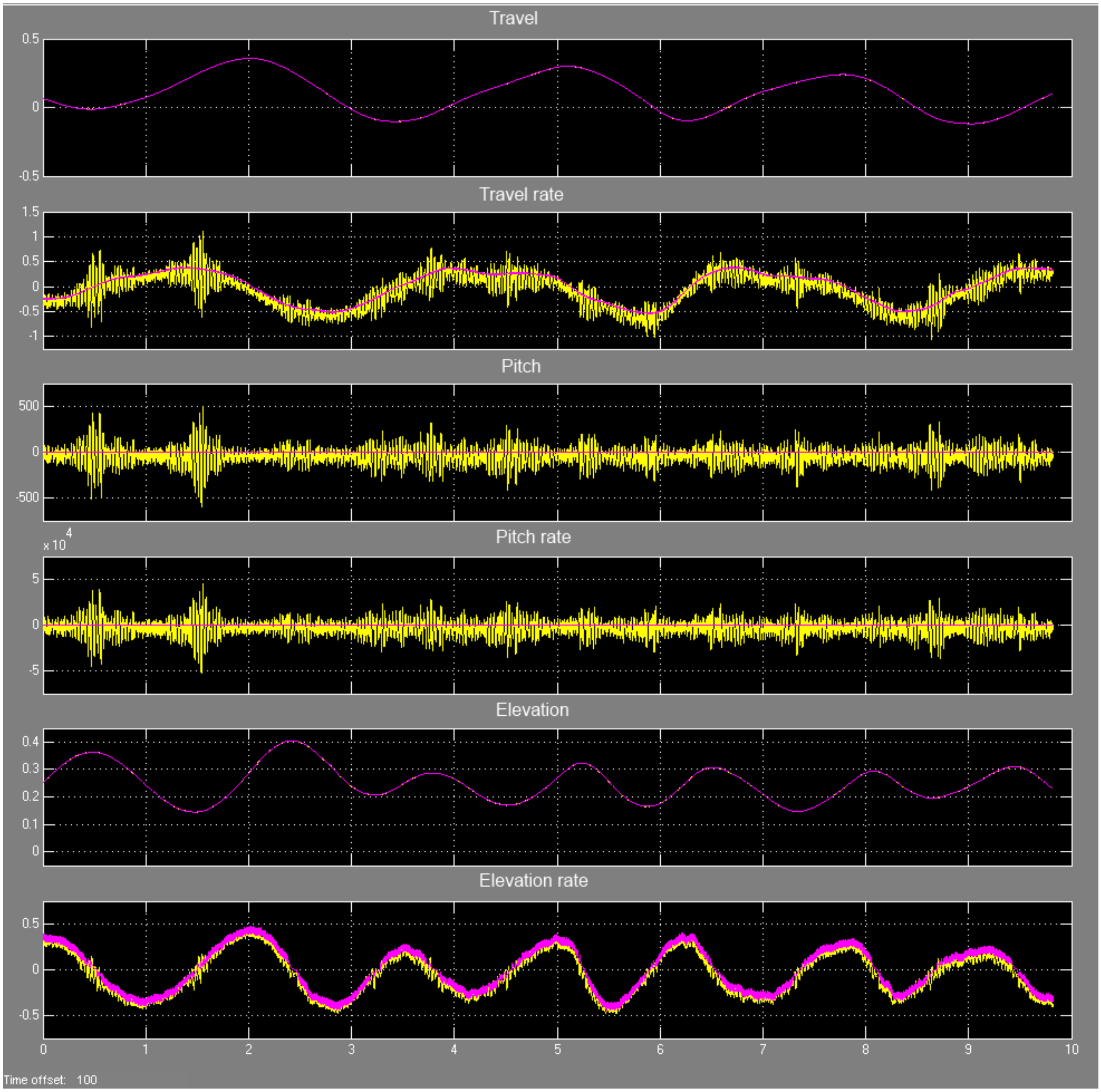

Figure 2: Estimated (yellow) vs measured (pink) without pitch