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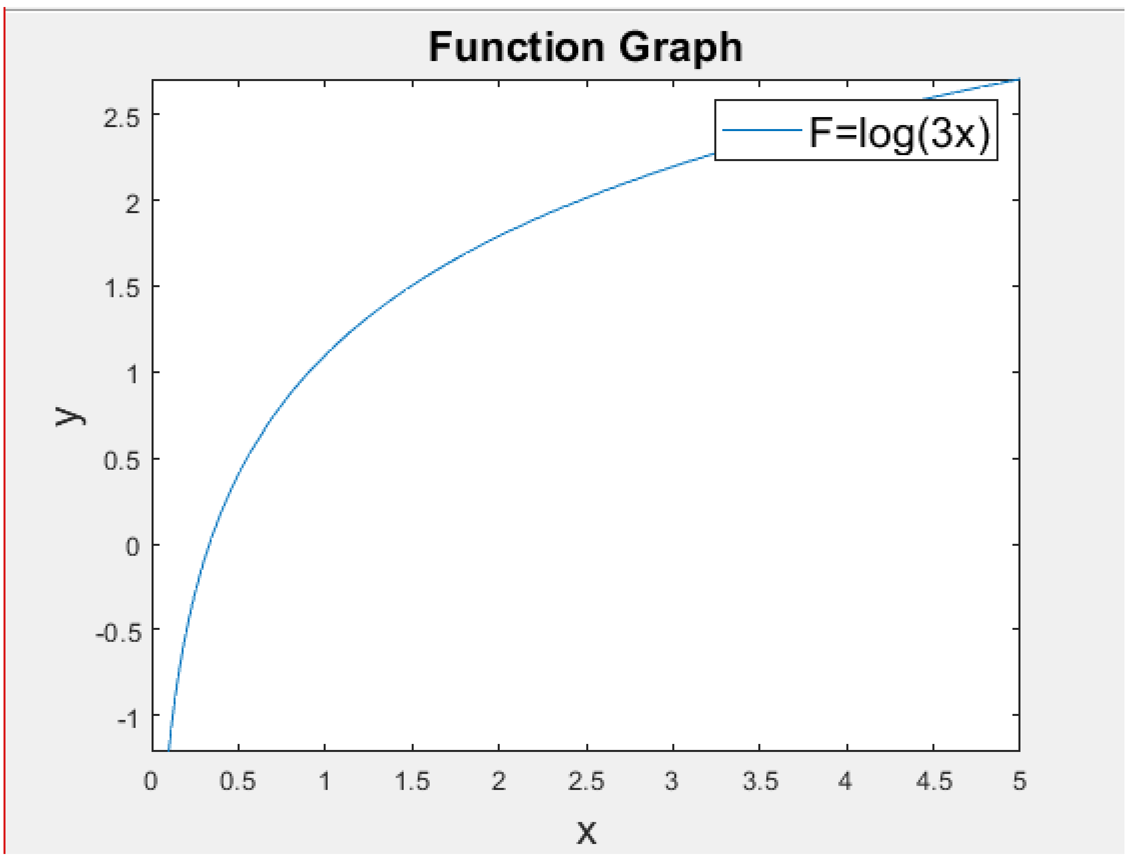
MATH 131 – Midterm 2 *Fall Semester 2021* **Duration: 48 hours**

1. (25 points) Consider a logarithmic function ***f*(*x*) = ln(3*x*)**. Please write Matlab code and finish the following tasks:

   (a) Derive the composite Simpson's 3/8-rule and then use it to calculate the definite integral $\int_{1}^{3} ln(3x)dx$ The accuracy is at least of the order $\mathbf{10^{-4}}$.

Solution 1A:



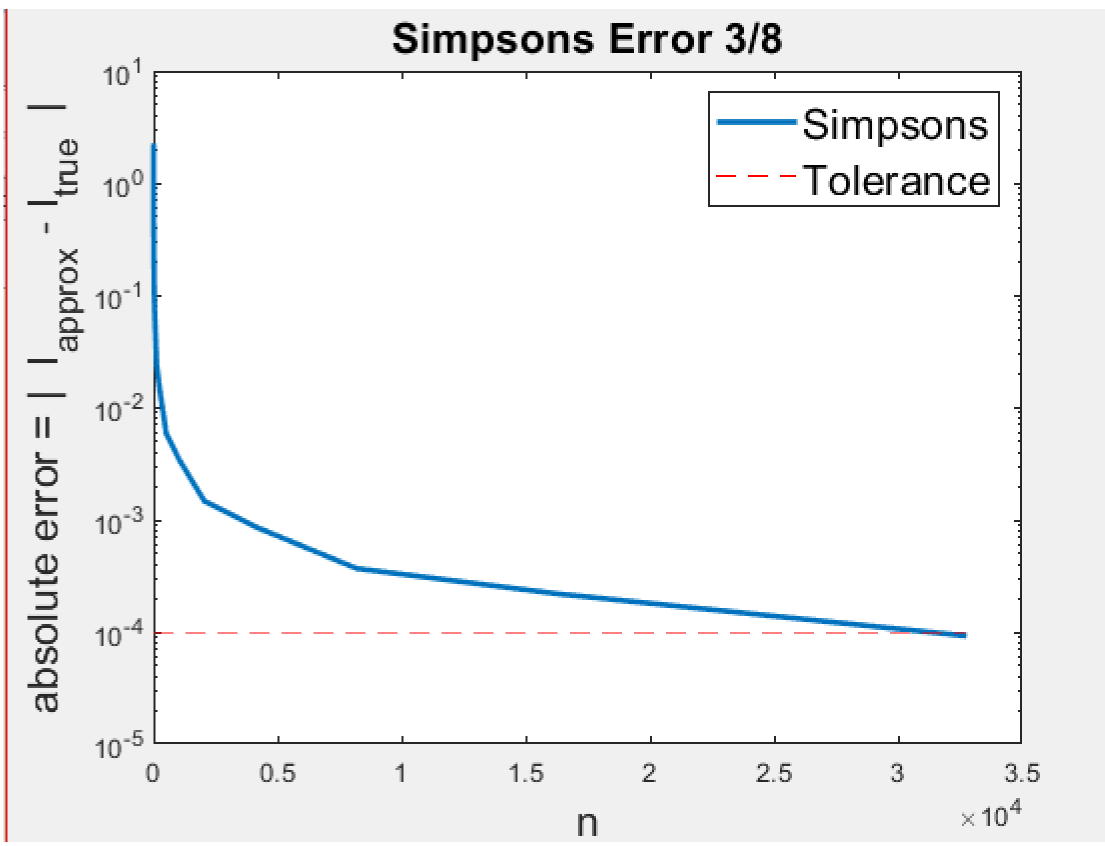
Figure(1)

After updating the composite simpson's rule into its 3/8ths counterpart and running it where: F(x)= log(3x), a=1,b=3, and with an accuracy of at least 10^{-4} we get the answer **3.4930.** This result should be more accurate than the regular ⅓ simpsons due to the way the algorithm is structured.

(b) Plot the numerical error of the composite Simpson's 3/8-rule with respect to N (the total number of the subinterval). (Hint: You should use the Matlab syntax semilogy to plot the error in the log scale. The integration by parts formula can be used to get the exact result for the definite integral $\int_{1}^{3} ln(3x)dx$)

Solution 1B:

We begin by deriving the **true value=(5*log(3)-2)** and subtracting this value with our results from Part A in order to get the error. We then plot our n value as well as our error to get the following graph where **y=absolute error and x= n.**



Figure(2)

Analyzing the graph we can see that the algorithm converges fairly quickly in,of course, a logarithmic curve.

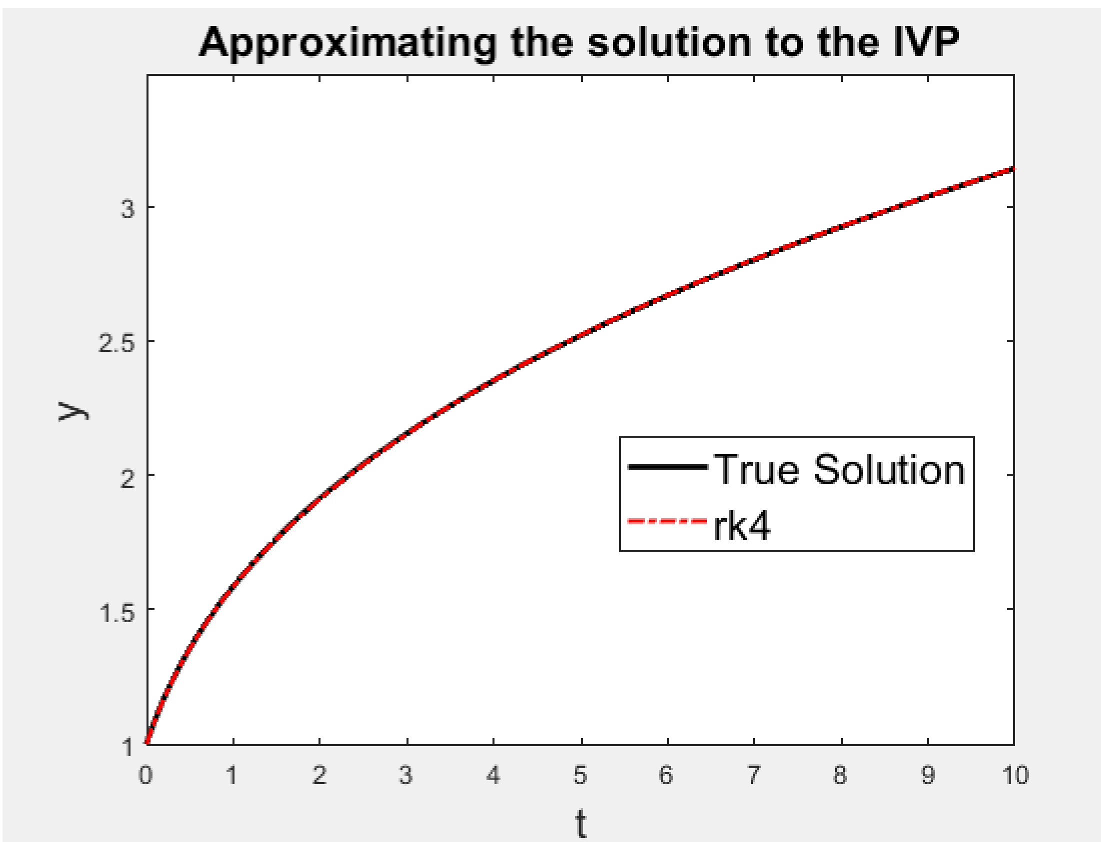
2. (25 points) For the one-dimensional differential equation:

$$\frac{dy}{dx} = \frac{1}{y^2} \quad (1)$$

please write Matlab code and finish the following tasks:

(a) Use the 4-th order Runge-Kutta method to solve differential equation (1) numerically for $t \in [0, 10]$ with the initial condition $y(0) = 1$. The step size $h = 10^{-3}$. Plot your solution.

Solution 2A:



Figure(3)

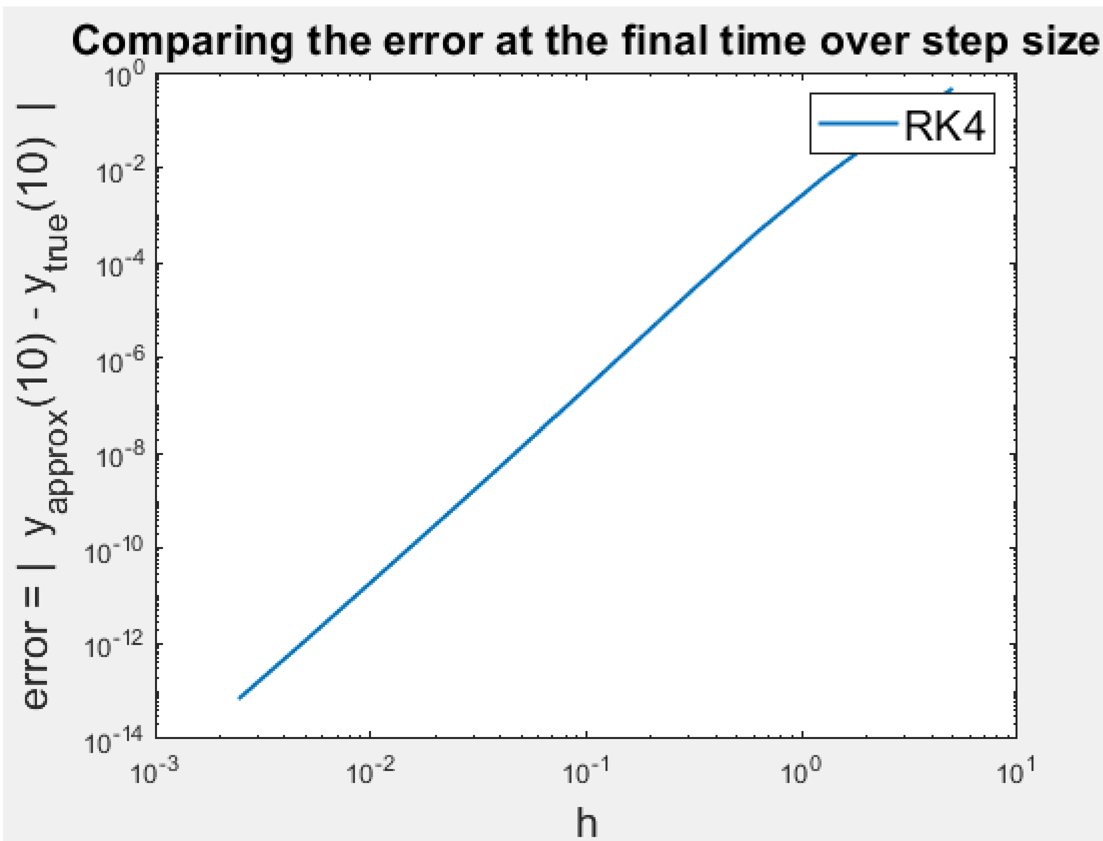
Analyzing the graph while having the parameters above we can see that the 4-th order Runge-Kutta Method gets very close, almost identical, to the true solution. In other words, the RK4 Method does a great job at approximation.

(b) Solve Equation (1) exactly using the separation of variables method. With this analytical solution, plot the endpoint approximation error $|y(10)-y_a(10)|$ with respect to the step size h. Here $y(t)$ is the exact solution and $y_a(t)$ is the approximated solution you got with the numerical scheme. (Hints: You should use the semilogy to plot the error in the log scale).

Solution 2B:

Equation (1) with initial conditions y(0)=1 admits an analytical solution where we use the separation of variables method:

$$y(t) = \sqrt[3]{3t + 1}$$



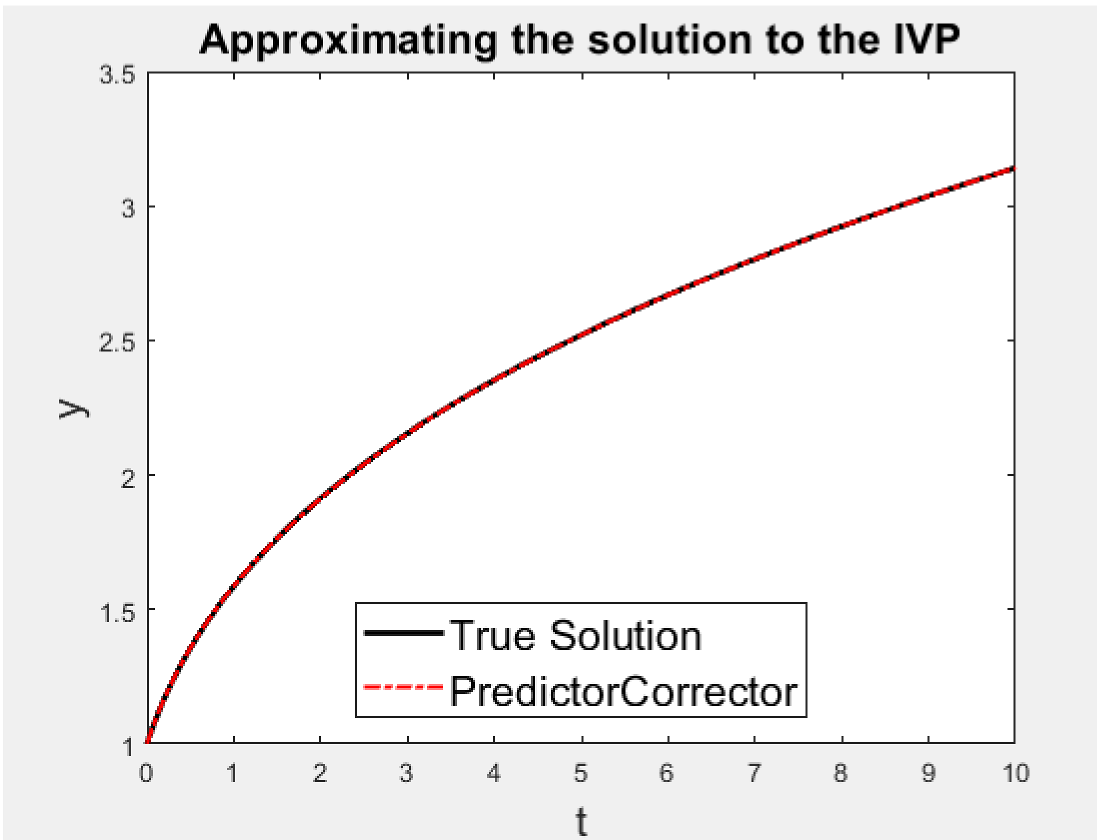
Figure(4)

The graph above shows the errors at the last time over a decreasing h for RK3. We can see that the slope is fairly steep which indicates a faster convergence time.

3. (Bonus problem 25 points) Please use the 4-th order Runge-Kutta method as the predictor and the Adams-Moulton 4-step implicit method as the corrector to solve Equation (1) numerically using the predictor-corrector method. After that, please compare the error $|y(10) - y_a(10)|$ with the one obtained by Euler's method in the same figure and comment on the convergence speed of these two methods.

Solution 3:

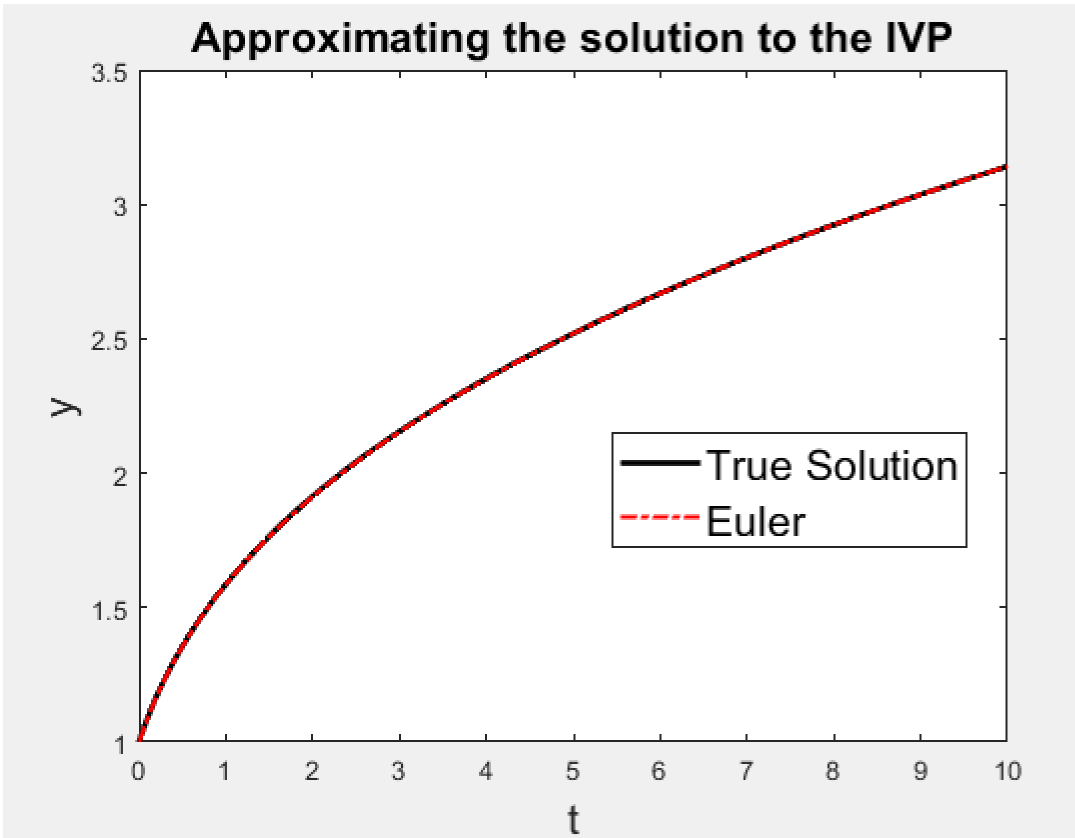
Graphing the **Predictor Method Approximation** we get:



Figure(5)

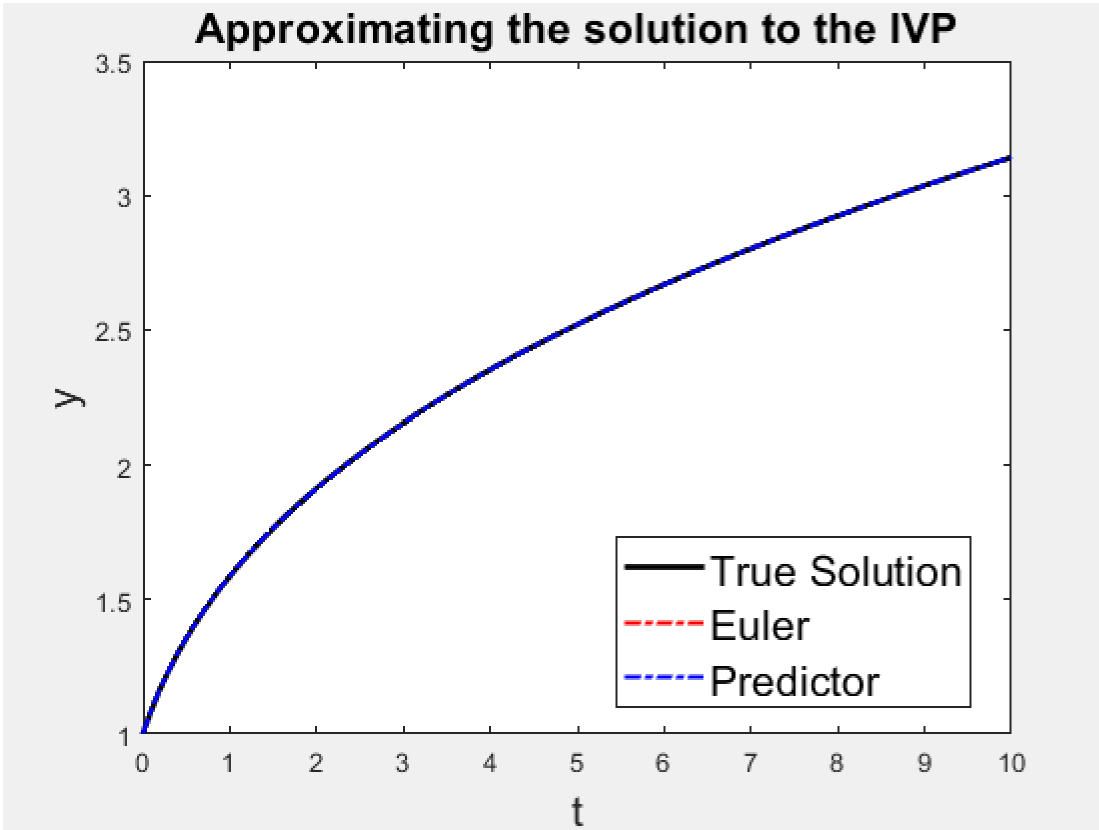
Here we can see that the Method does a great job at approximating the true solution.

Additionally, graphing the **Euler method** we get similar results.



Figure(6)

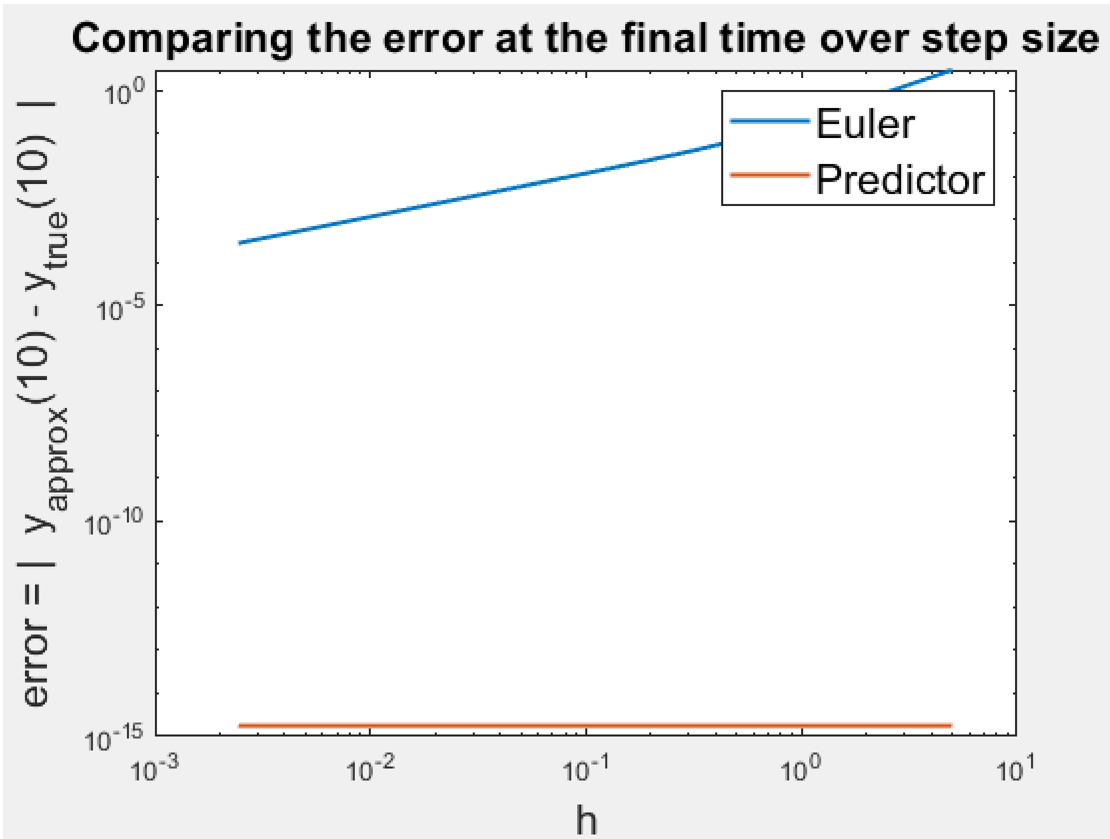
Graphing **Both methods** with the true solution we get the following results.



Figure(7)

From this graph we can identify that both methods work very well to the naked eye.

Finally, when comparing the error in relation to Predictor Vs Euler in the same figure we get...



...An incomplete graph after all that. Unfortunately I wasn't able to get the graph to work properly with my Predictor Method. Regardless, using the theory that I know I can determine the following. The Predictor method will converge faster than Euler's Method. This is thanks to the fact that the method uses RK4 as a predictor, and we know that RK4 is faster than Eulers. Additionally, it will be significantly more accurate than Euler's method due to the complex nature of the Adams-Moulton 4-step method.