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Problem 4

Description:

The study of the butterfly effect. The Lorenz system is a system of ordinary differential equations first studied by Edward Lorenz. It is notable for having chaotic solutions for certain parameter values and initial conditions. In popular media the 'butterfly effect' stems from the real-world implications of the Lorenz system, i.e. that in any physical system, in the absence of perfect knowledge of the initial conditions (even the minuscule disturbance of the air due to a butterfly flapping its wings), our ability to predict its future course will always fail. The Lorenz system is given by a 3-dimensional nonlinear ODE:

$$\frac{dx}{dt} = \sigma(y - x) \tag{1}$$

$$\frac{dx}{dt} = x(\rho - z) - y \tag{2}$$

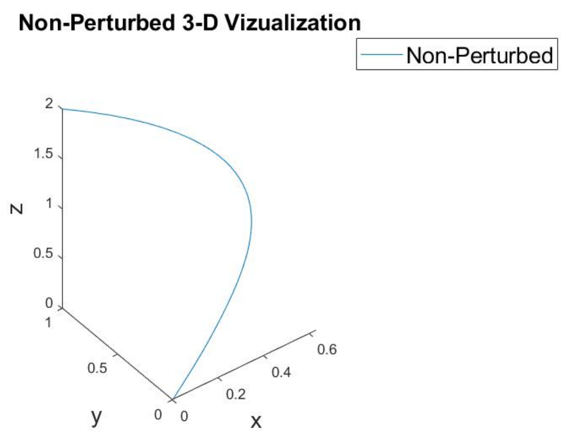
$$\frac{dx}{dt} = xy - \beta z \tag{3}$$

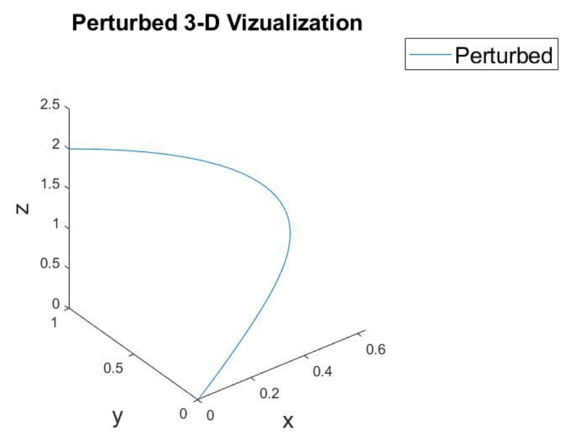
where σ, ρ, β are modeling parameters. You can now use the mathematical method you learnt in this class to quantitatively study the butterfly effect of the Lorenz system. Please read Section 5.9 and develop a numerical method to solve the Lorenz system numerically. With this numerical solver, you should finish the following tasks:

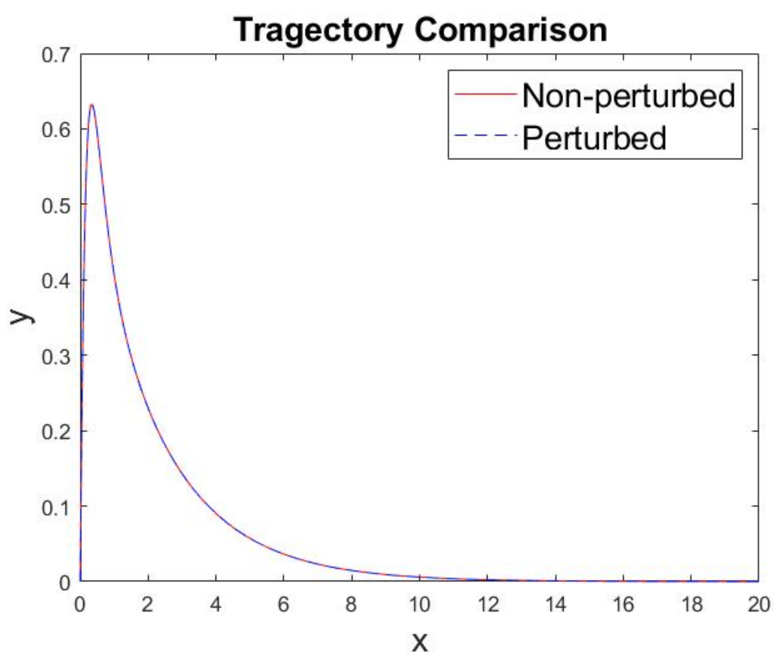
- (a) We first set $\sigma = 5, \beta = 3, \rho = 0.5$. Please choose an arbitrary initial condition $(x(0), y(0), z(0))$ and solve the Lorenz system numerically. With the same parameter setting and a slightly perturbed initial condition $(x(0) + \delta 1, y(0) + \delta 2, z(0) + \delta 3)$, where $\delta 1, \delta 2, \delta 3$ are constants with small absolute value such as $0.00001, 0.00003, 0.0002$, solve the Lorenz system numerically and compare the perturbed solution with the non-perturbed solution. Please use plots to show whether the system exhibits the butterfly effect. i.e. The perturbed solution deviates from the non-perturbed solution dramatically as time $t \to +\infty$.
- (b) Repeat what you have done for a new parameter setting $\sigma = 1, \beta = 2.5, \rho = 10$. Can you observe the butterfly effect?
- (c) Repeat what you have done for a new parameter setting $\sigma = 10, \beta = 8/3, \rho = 28$. Can you observe the butterfly effect?
- (d) Plot your numerical solution $[x(t), y(t), z(t)]$ in 3D with the parameter setting $\sigma = 10, \beta = 8/3, \rho = 28$. What does the trajectory look like?

Solution

- (a) The Perturbed 3-D Visualizations, Non-Perturbed 3-D Visualizations, and their trajectory comparison using parameters $\sigma = 5, \beta = 3, \rho = 0.5$ are shown below.

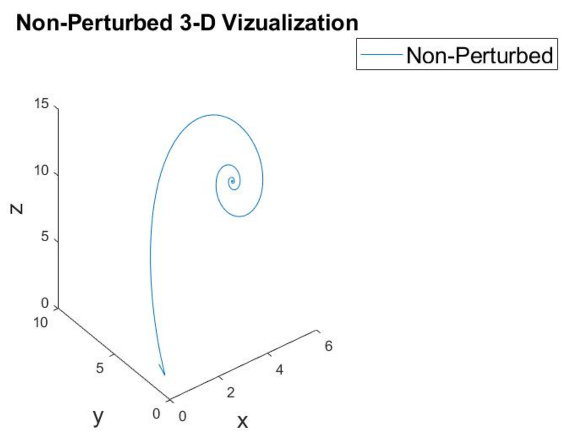


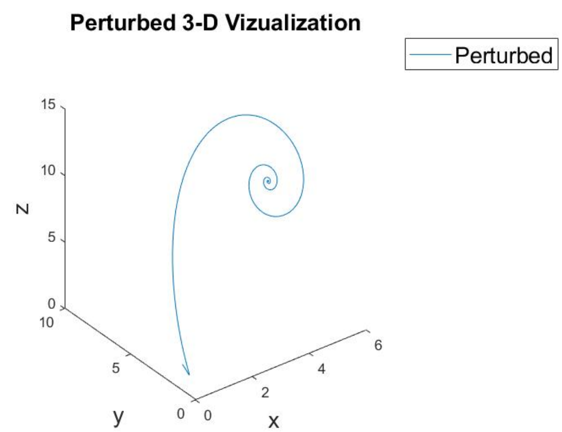


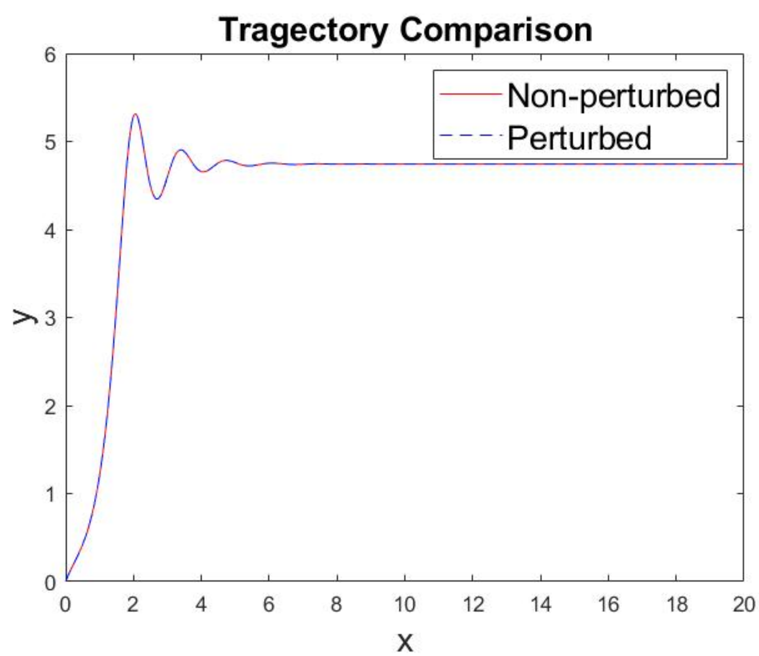


We do not see the butterfly effect in both the Perturbed and Non-perturbed Lorenz models. The fact that the trajectories are identical reinforce this claim.

- (b) The Perturbed 3-D Visualizations, Non-Perturbed 3-D Visualizations, and their trajectory comparison using parameters $\sigma = 1, \beta = 2.5, \rho = 10$ are shown below.

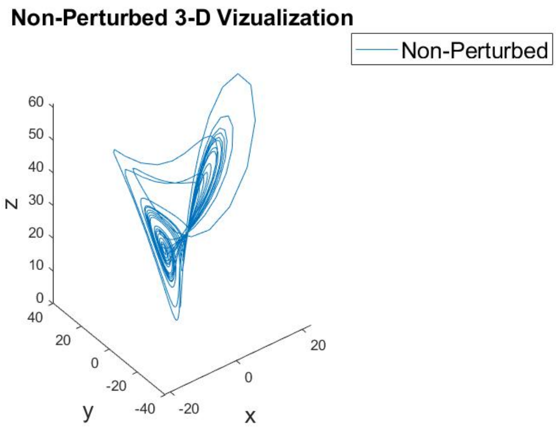


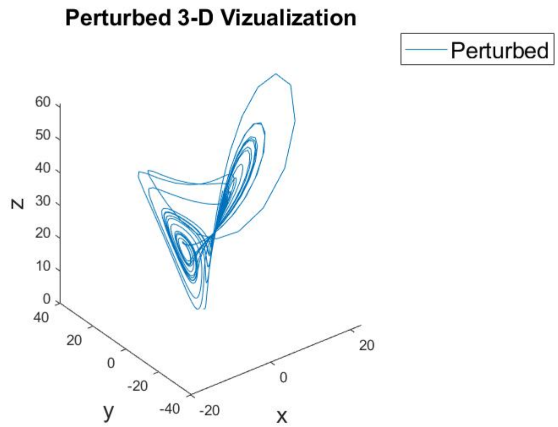


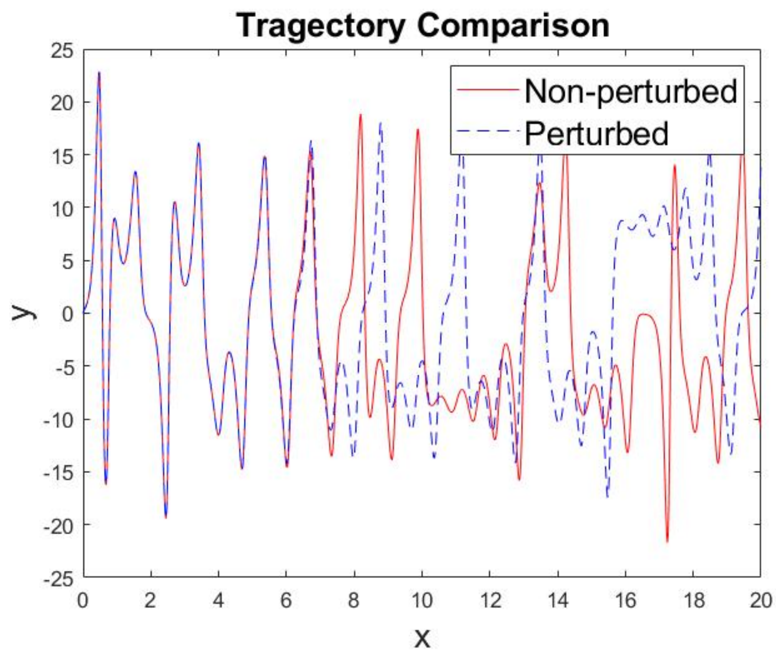


We do not see the butterfly effect in both the Perturbed and Non-perturbed Lorenz models. Similarly to problem 4a, the fact that the trajectories are identical reinforce this claim.

- (c) The Perturbed 3-D Visualizations, Non-Perturbed 3-D Visualizations, and their trajectory comparison using parameters $\sigma = 10, \beta = 8/3, \rho = 28$ are shown below.

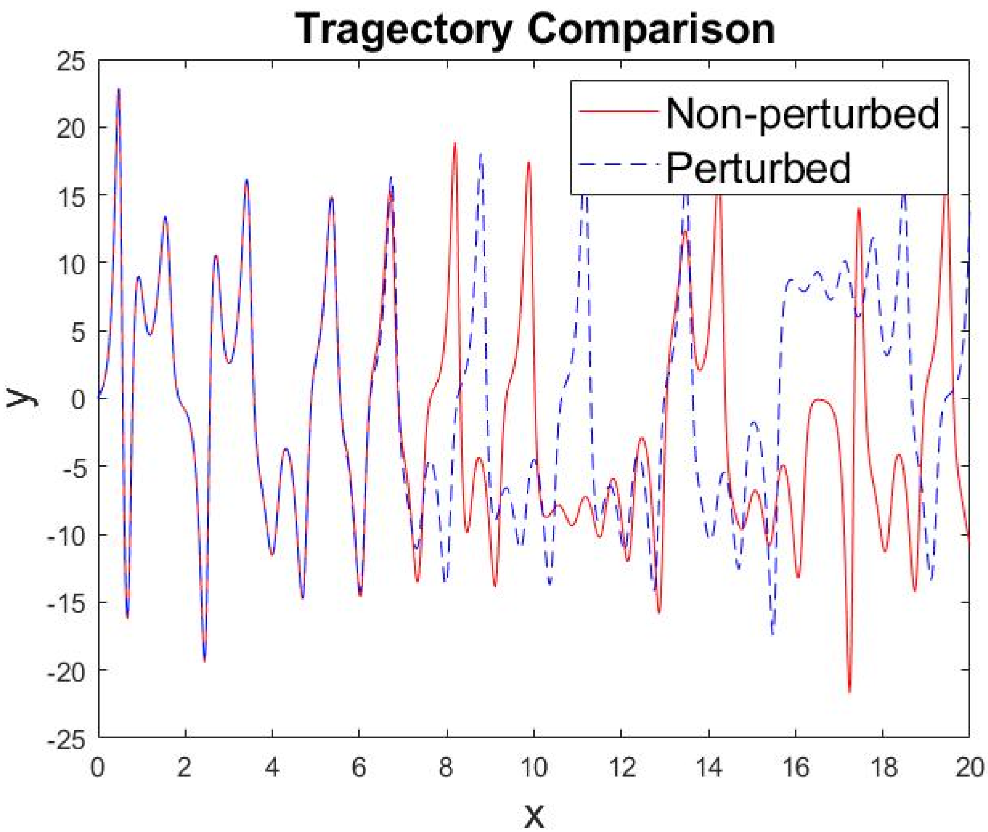






We can see the butterfly effect in both the Perturbed and Non-Perturbed 3-D visualizations using parameters $\sigma = 10, \beta = 8/3$, and $\rho = 28$.

- (d) The trajectory comparison for the Perturbed and Non-Perturbed Lorenz models using parameters $\sigma = 10, \beta = 8/3$, and $\rho = 28$ can be seen below. The trajectory for both models appears identical until $x = 7$. From then on the trajectories begin to differ, this further proves that we observe the butterfly effect using the given parameters.


Tragectory Comparison
Non-perturbed
Perturbed
25
20
15
10
5
0
-5
-10
-15
-20
-25
0
2
4
6
8
10
12
14
16
18
20
y
x

solution