Structure From Motion

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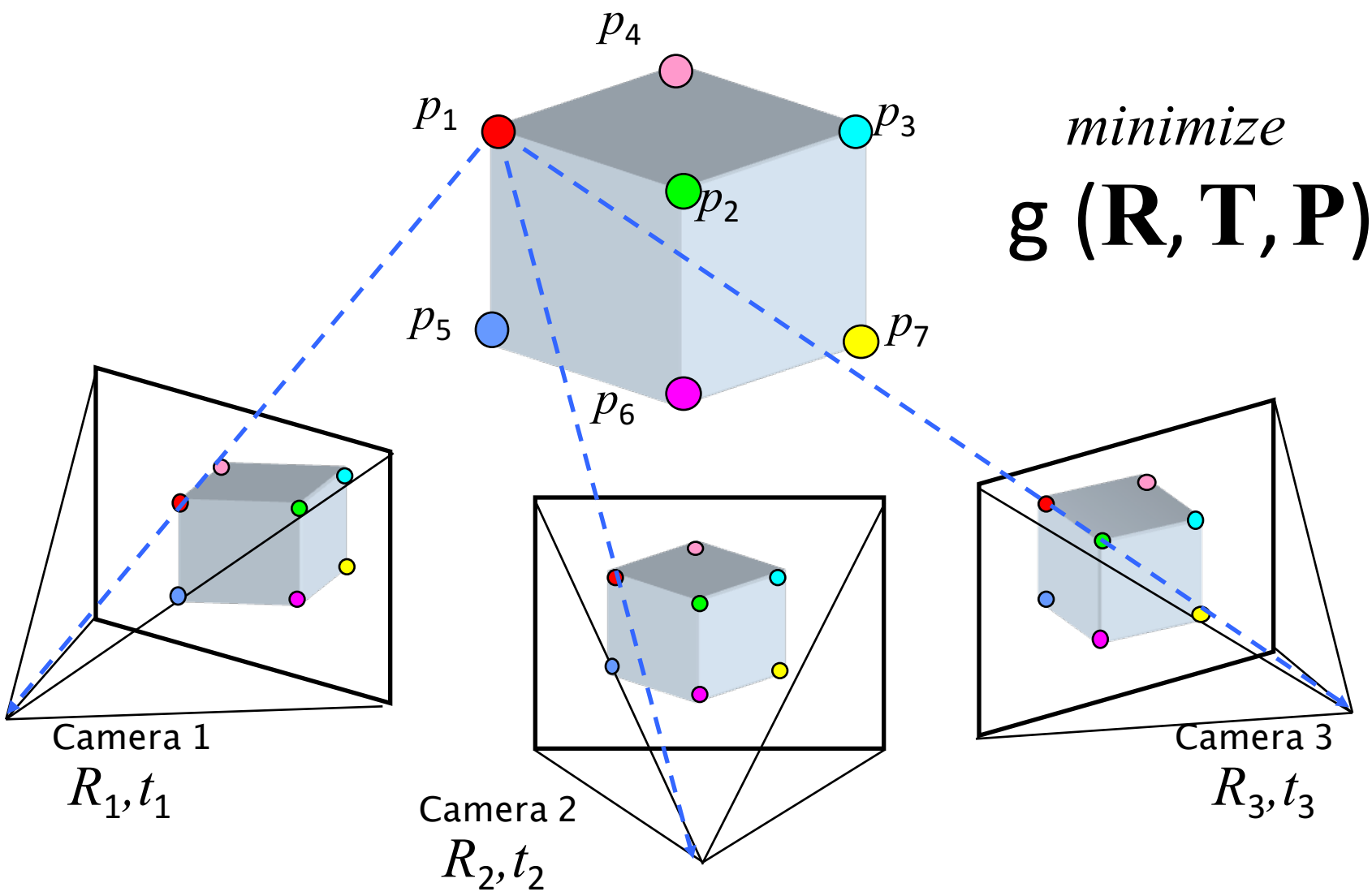
CSE 576

Several slides from Steve Seitz, Rick Szeliski, Martial Hebert, and Noha Snavely

Structure from motion

- aka "bundle adjustment" (texts: Zisserman; Faugeras)



SfM objective function

- Given point **x** and rotation and translation **R**, **t**

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \mathbf{R}\mathbf{x} + \mathbf{t} \qquad u' = \frac{f x'}{z'} \qquad v' = \frac{f y'}{z'} \qquad \begin{bmatrix} u' \\ v' \end{bmatrix} = \mathbf{P}(\mathbf{x}, \mathbf{R}, \mathbf{t})$$

- Minimize sum of squared reprojection errors:

$$g(\mathbf{X}, \mathbf{R}, \mathbf{T}) = \sum_{i=1}^{m} \sum_{j=1}^{n} w_{ij} \cdot \left\| \underbrace{\mathbf{P}(\mathbf{x}_i, \mathbf{R}_j, \mathbf{t}_j)}_{\text{predicted image location}} - \underbrace{\begin{bmatrix} u_{i,j} \\ v_{i,j} \end{bmatrix}}_{\text{observed image location}} \right\|^2$$

Solving structure from motion

- Minimizing g is difficult:
  - g is non-linear due to rotations, perspective division
  - lots of parameters: 3 for each 3D point, 6 for each camera
  - difficult to initialize
  - gauge ambiguity: error is invariant to a similarity transform (translation, rotation, uniform scale)

- Many techniques use non-linear least-squares optimization (*bundle adjustment*)
  - Levenberg-Marquardt is a popular algorithm
  - http://en.wikipedia.org/wiki/Levenberg-Marquardt_algorithm

- Good code online
  - Bundler: http://phototour.cs.washington.edu/bundler/
  - Multicore: http://grail.cs.washington.edu/projects/mcba/

Suppose we know 3D points and affine camera parameters …

then, we can compute the observed 2d positions of each point

$$\begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \\ \vdots \\ \mathbf{A}_m \end{bmatrix} \begin{bmatrix} \mathbf{X}_1 & \mathbf{X}_2 & \cdots & \mathbf{X}_n \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{x}}_{11} & \hat{\mathbf{x}}_{12} & \cdots & \hat{\mathbf{x}}_{1n} \\ \hat{\mathbf{x}}_{21} & \hat{\mathbf{x}}_{22} & \cdots & \hat{\mathbf{x}}_{2n} \\ & & \ddots & \\ \hat{\mathbf{x}}_{m1} & \hat{\mathbf{x}}_{m2} & \cdots & \hat{\mathbf{x}}_{mn} \end{bmatrix}$$

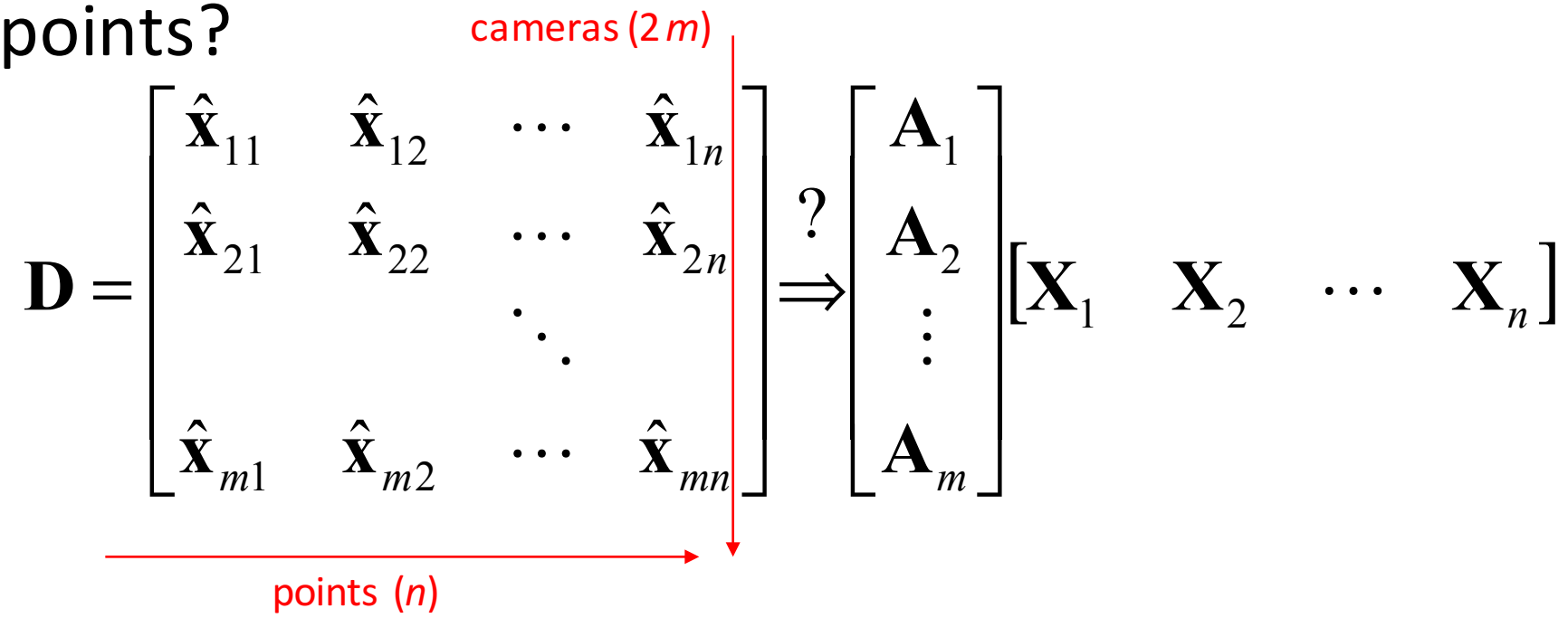
3D Points (3xn)

Camera Parameters (2mx3)

2D Image Points (2mxn)

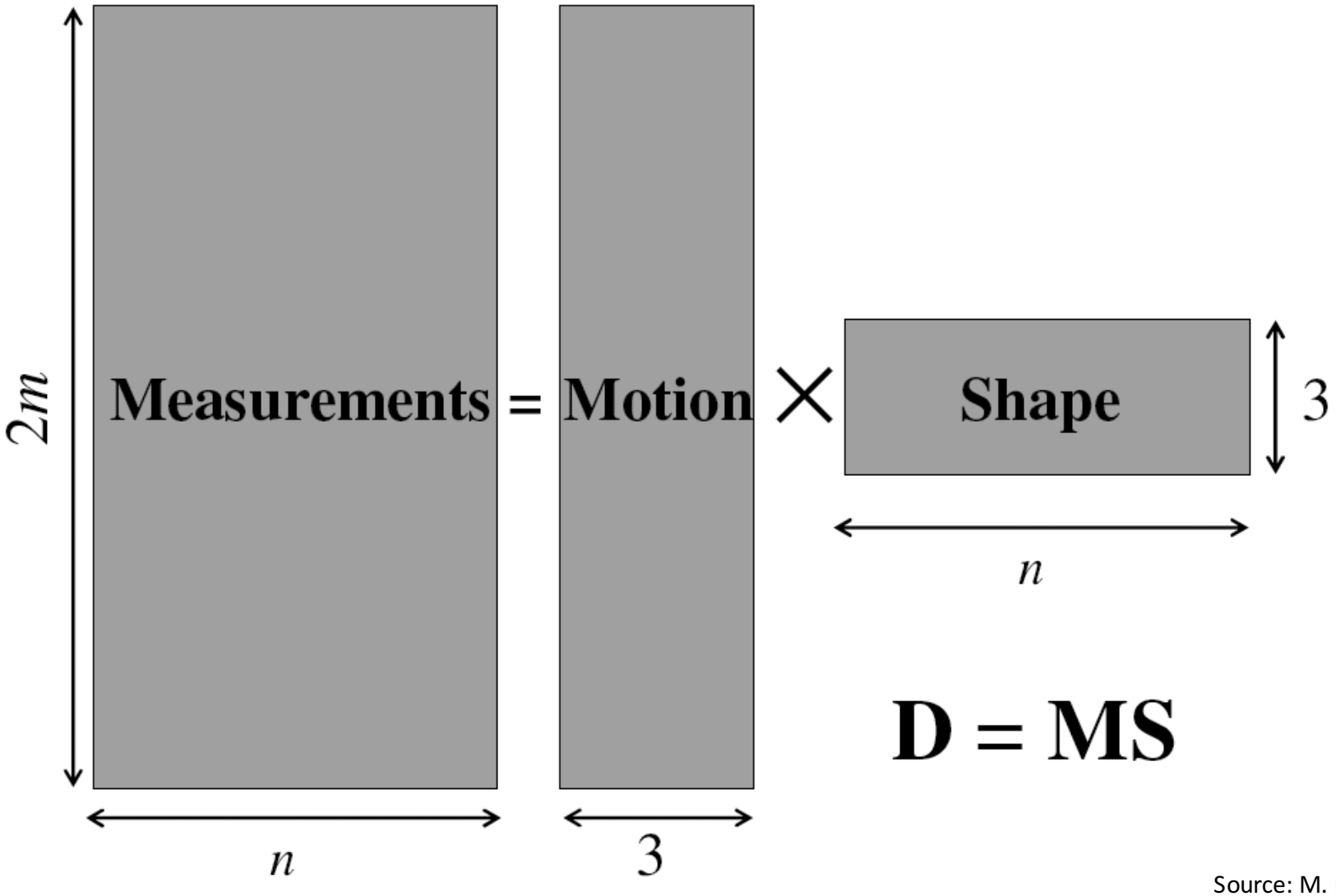
What if we instead observe corresponding 2d image points?

Can we recover the camera parameters and 3d points?



What rank is the matrix of 2D points?

Factorizing the measurement matrix

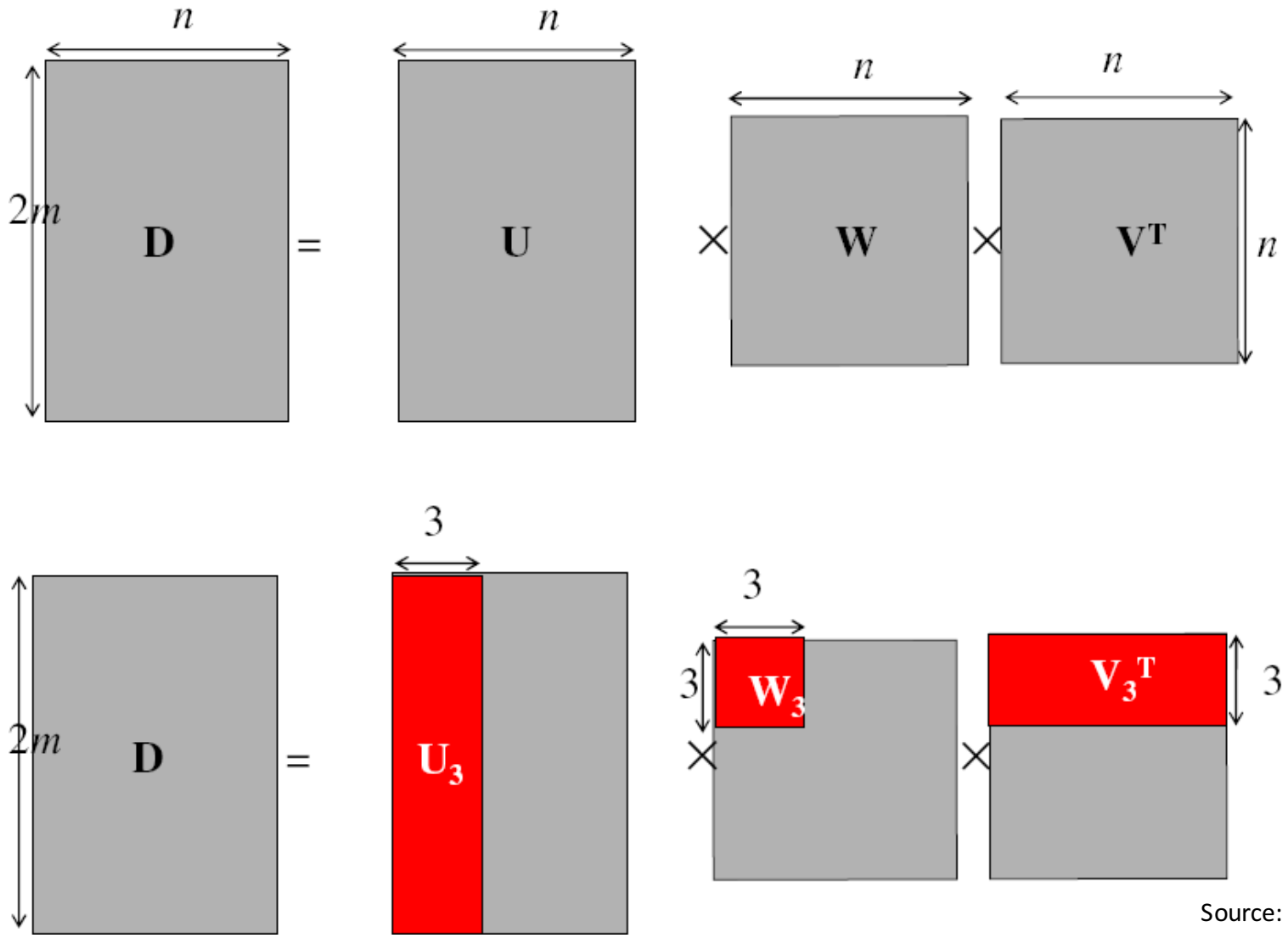


$$\mathbf{D} = \mathbf{MS}$$

Source: M. Hebert

Factorizing the measurement matrix

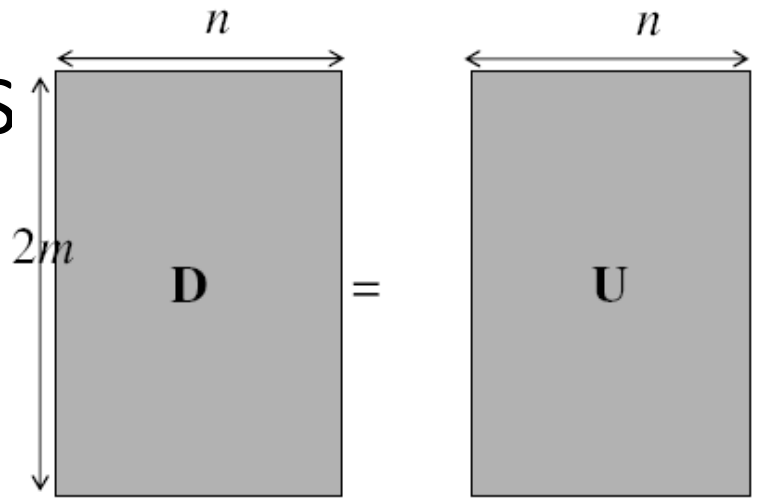
- Singular value decomposition of D:

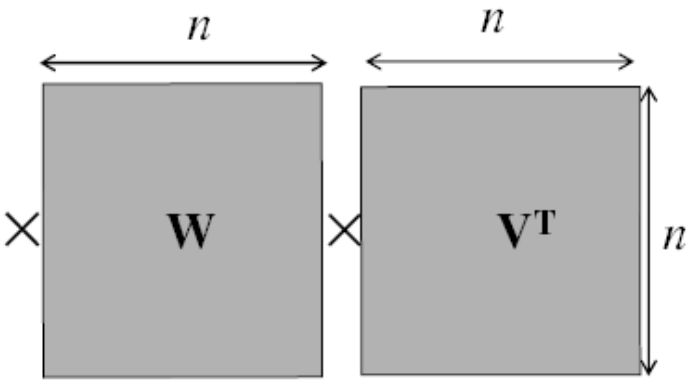


Source: M. Hebert

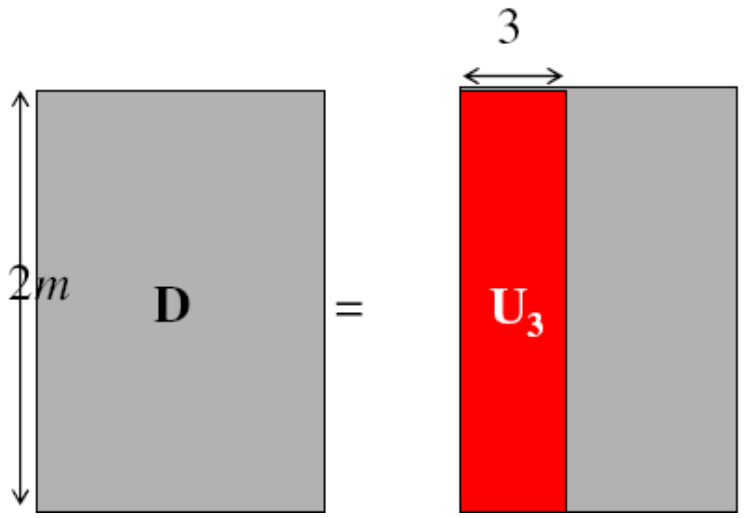
Factorizing the measurement matrix

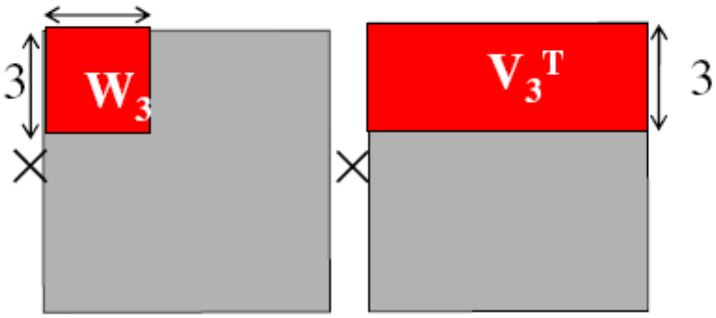
- S





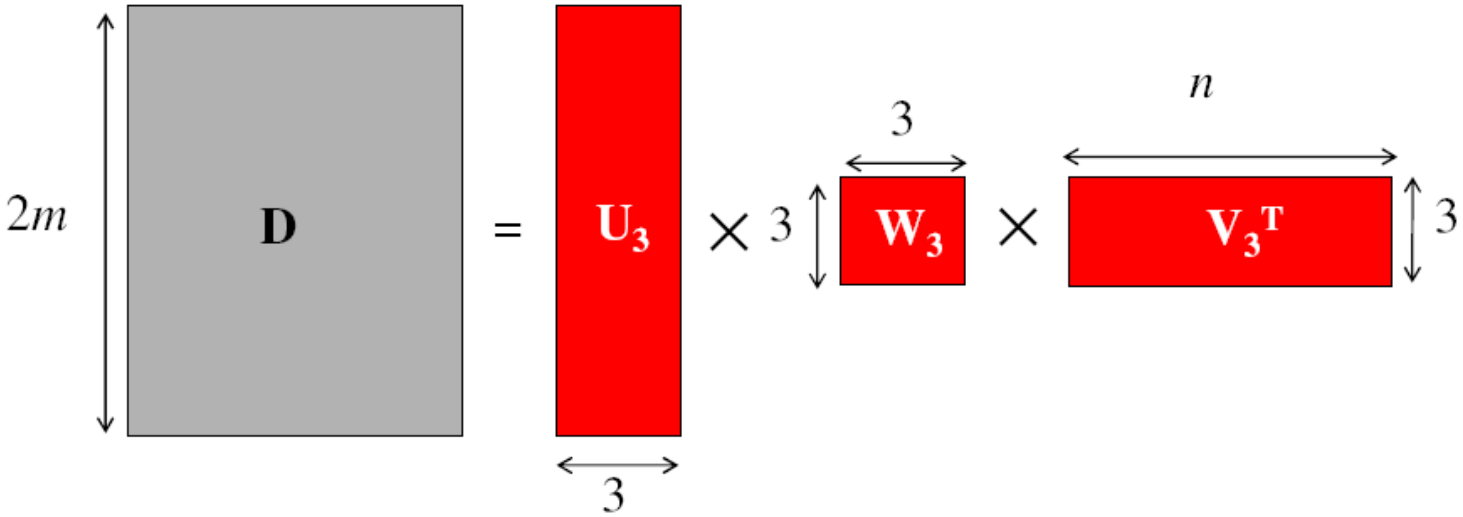
To reduce to rank 3, we just need to set all the singular values to 0 except for the first 3





Source: M. Hebert

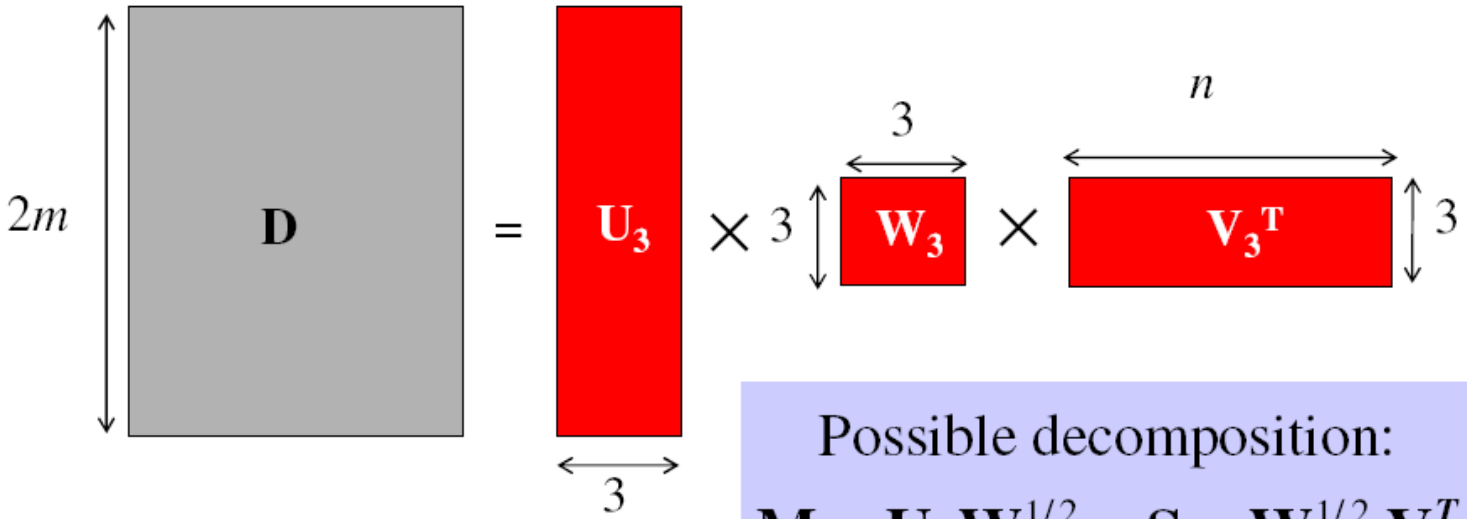
Factorizing the measurement matrix



Source: M. Hebert

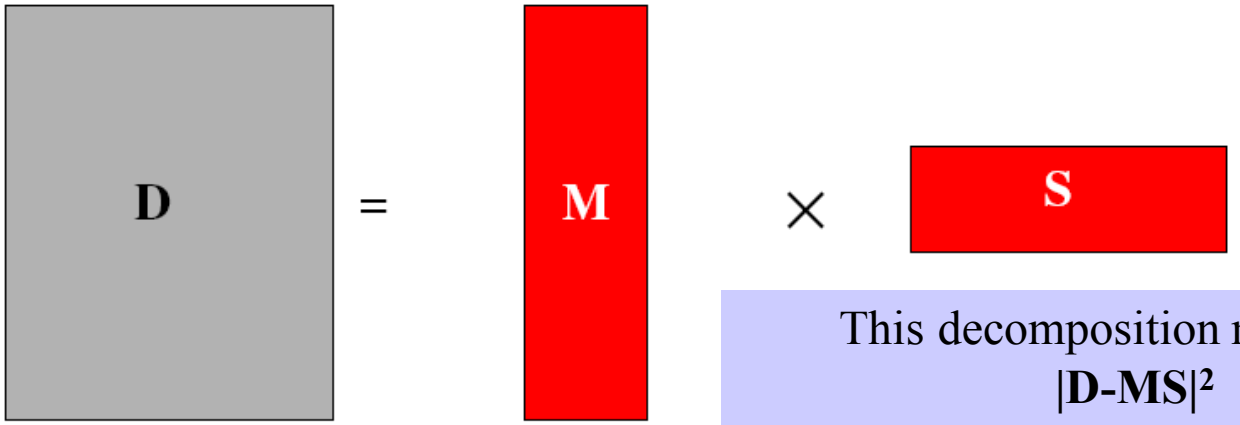
Factorizing the measurement matrix

- Obtaining a factorization from SVD:



Possible decomposition:

$$\mathbf{M} = \mathbf{U}_3 \mathbf{W}_3^{1/2} \qquad \mathbf{S} = \mathbf{W}_3^{1/2} \mathbf{V}_3^T$$



This decomposition minimizes $|\mathbf{D}-\mathbf{MS}|^2$

Source: M. Hebert

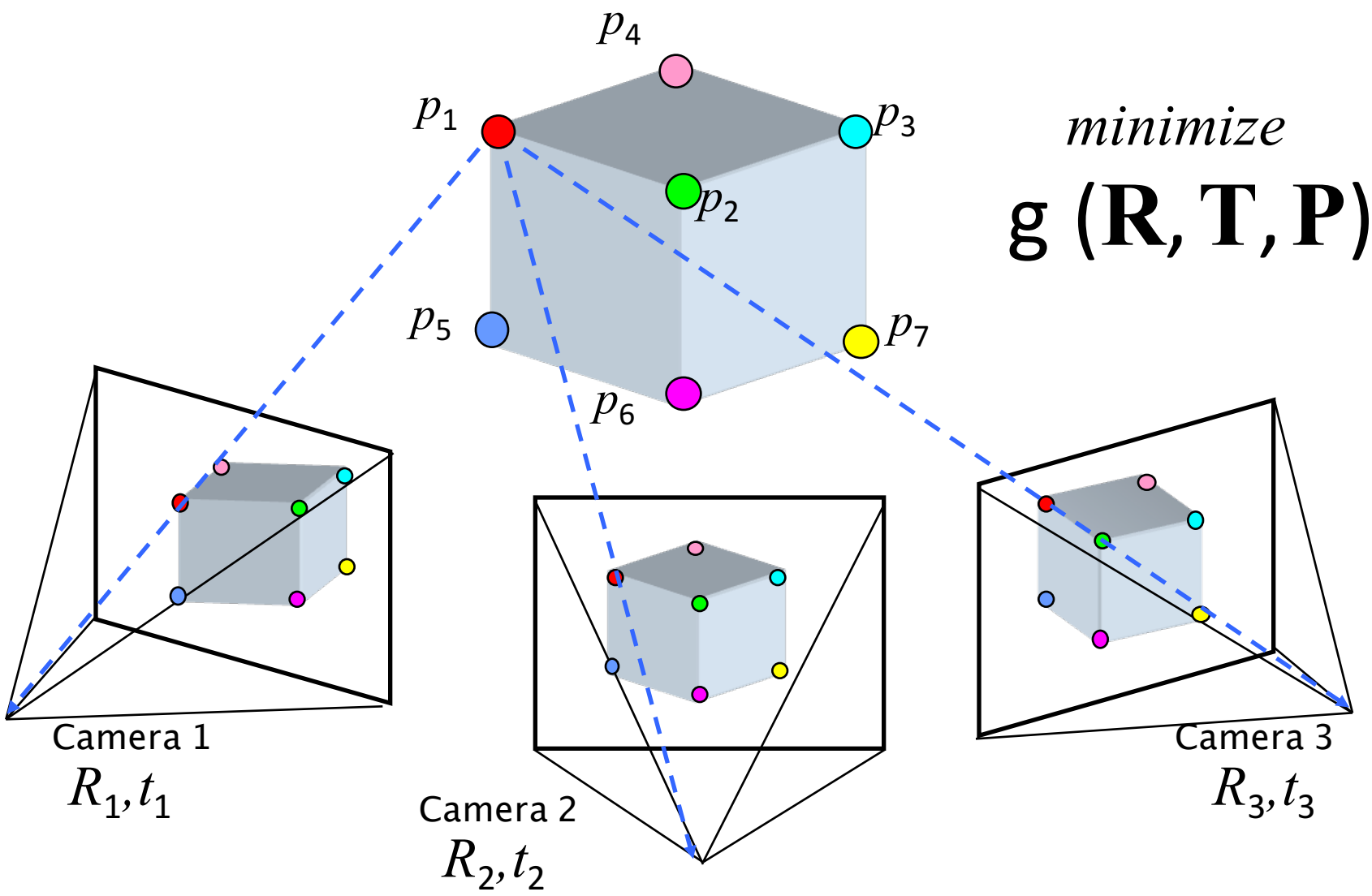
Algorithm summary

- Given: *m* images and *n* features $\mathbf{x}_{ij}$
- For each image *i, c*enter the feature coordinates
- Construct a $2m \times n$ measurement matrix **D**:
  - Column *j* contains the projection of point *j* in all views
  - Row *i* contains one coordinate of the projections of all the *n* points in image *i*
- Factorize **D**:
  - Compute SVD: $\mathbf{D} = \mathbf{U}\,\mathbf{W}\,\mathbf{V}^{\mathbf{T}}$
  - Create $\mathbf{U}_3$ by taking the first 3 columns of **U**
  - Create $\mathbf{V}_3$ by taking the first 3 columns of **V**
  - Create $\mathbf{W}_3$ by taking the upper left 3×3 block of **W**
- Create the motion and shape matrices:
  - $\mathbf{M} = \mathbf{U}_3\mathbf{W}_3^{½}$ and $\mathbf{S} = \mathbf{W}_3^{½}\,\mathbf{V}_3^{T}$ (**or** $\mathbf{M} = \mathbf{U}_3$ and $\mathbf{S} = \mathbf{W}_3\mathbf{V}_3^{T}$)

Source: M. Hebert

Structure from motion

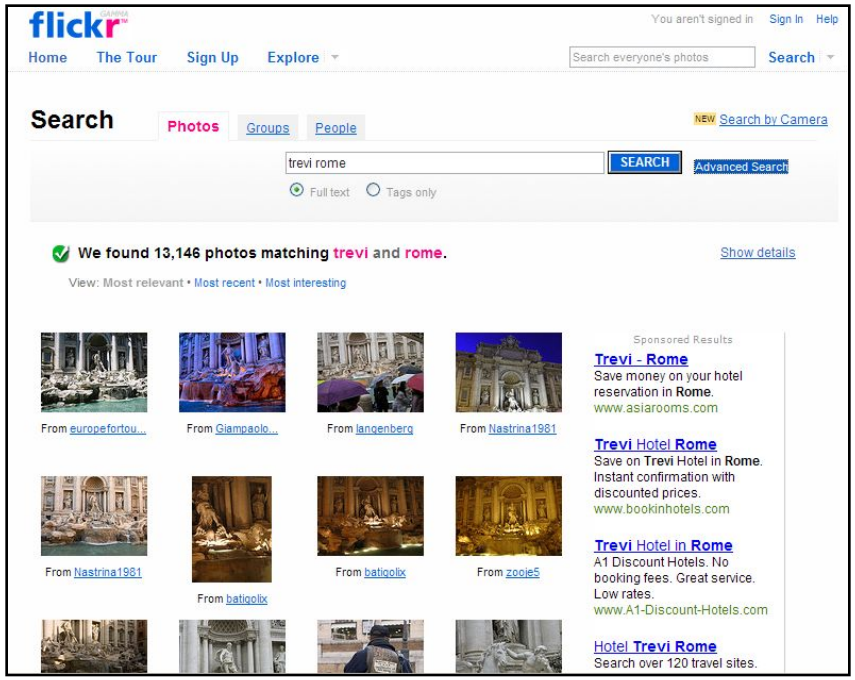
- aka "bundle adjustment" (texts: Zisserman; Faugeras)



minimize

$$\mathbf{g\,(R, T, P)}$$

Structure from motion



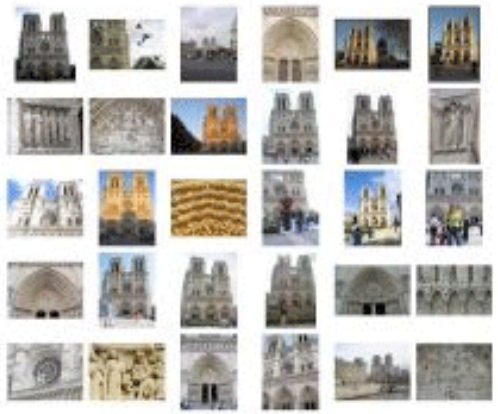
Images on the Internet

Computed 3D structure

Photo Tourism

Exploring photo collections in 3D

Microsoft



(a)

(b)

(c)

- Photo tourism video: http://www.youtube.com/watch?v=5Ji84zb2r8s
- Photosynth: http://photosynth.net/

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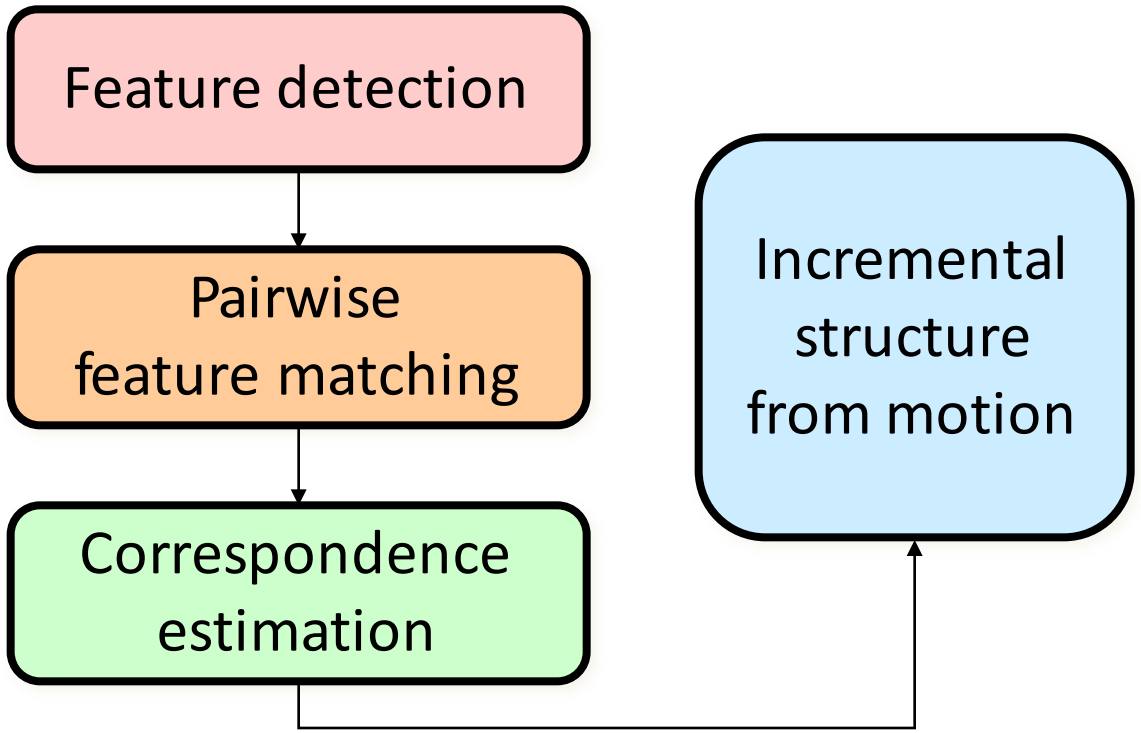
From formicheit

From Lisa...

From Rick's_Picks

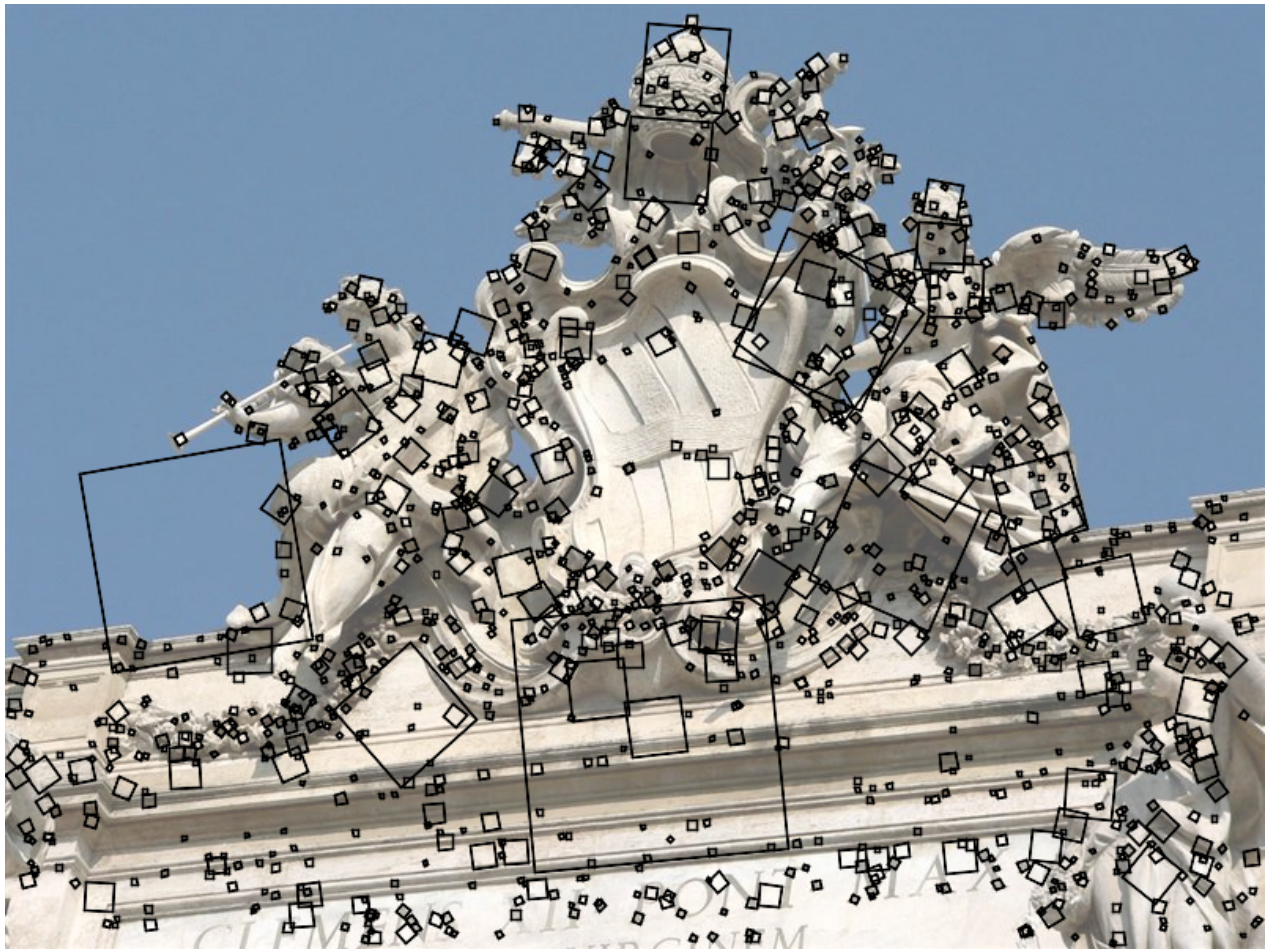
Scene reconstruction

- Automatically estimate
  - position, orientation, and focal length of cameras
  - 3D positions of feature points



Feature detection

Detect features using SIFT [Lowe, IJCV 2004]



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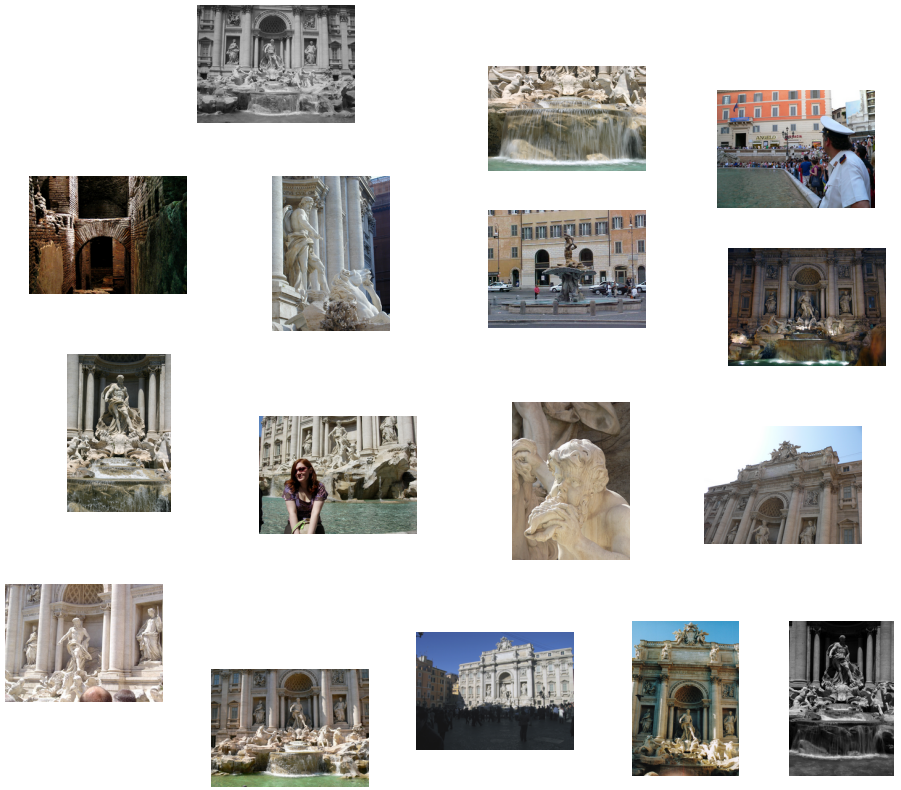

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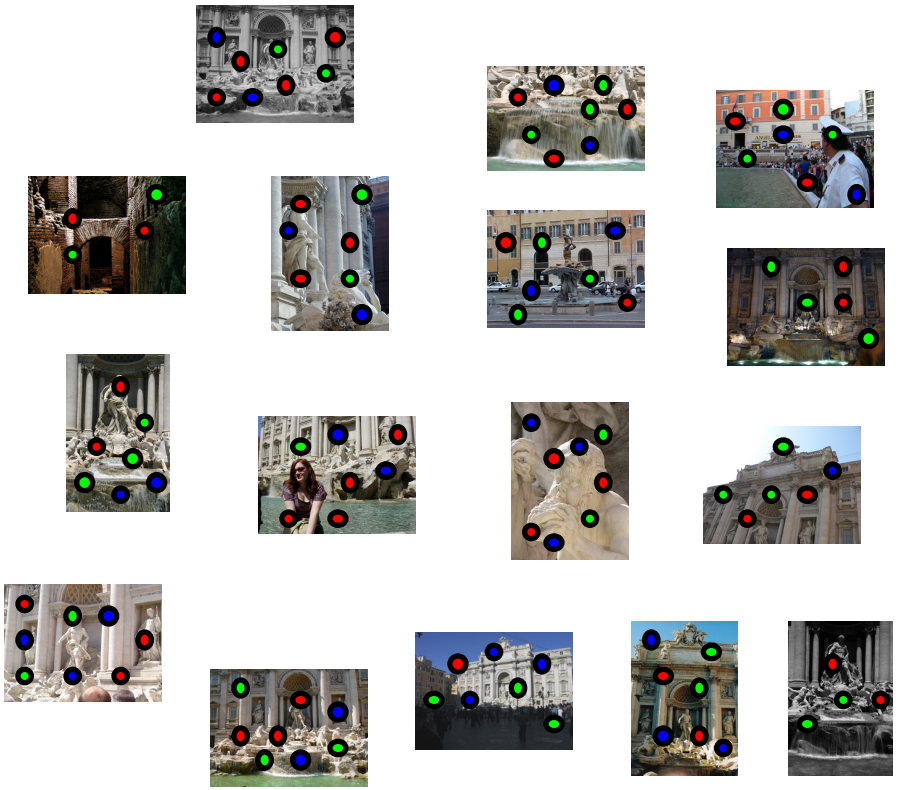


Correspondence estimation

- Link up pairwise matches to form connected components of matches across several images

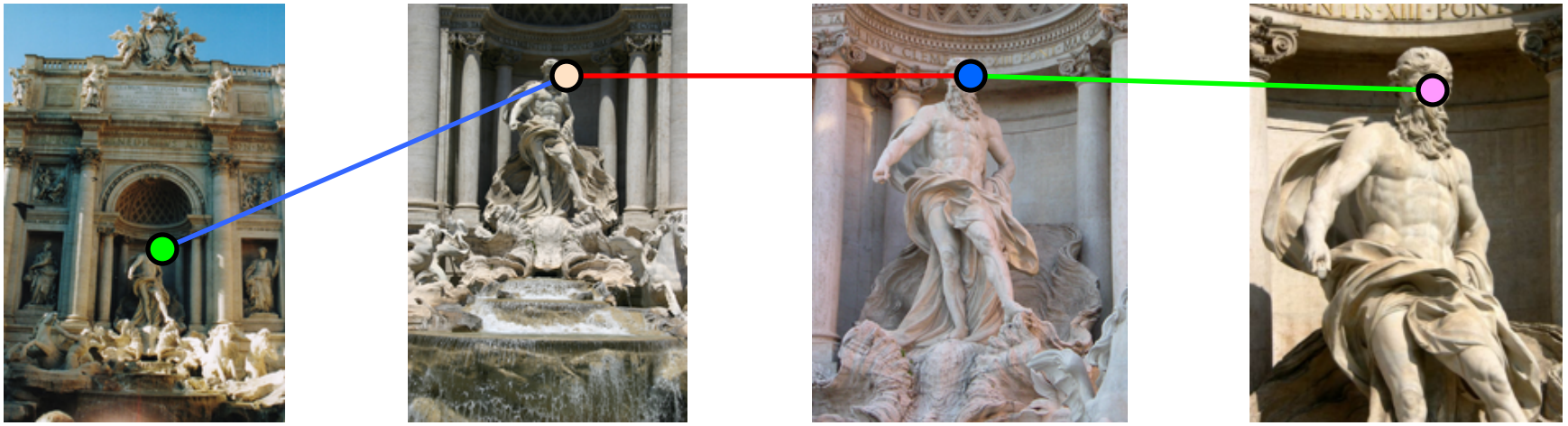
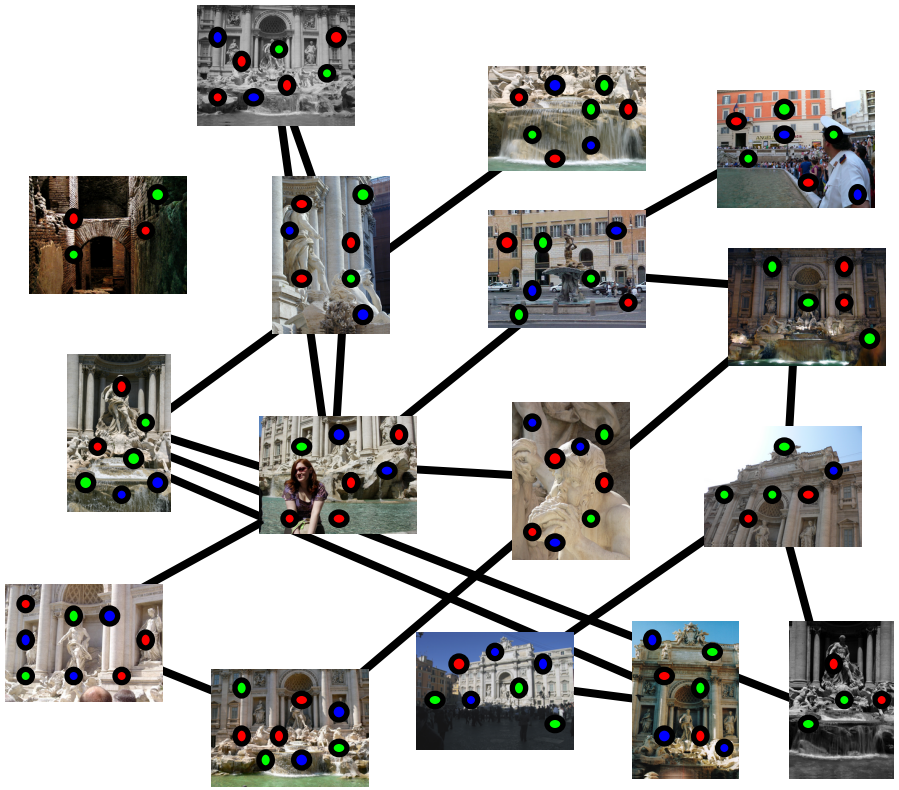


Image 1 Image 2 Image 3 Image 4

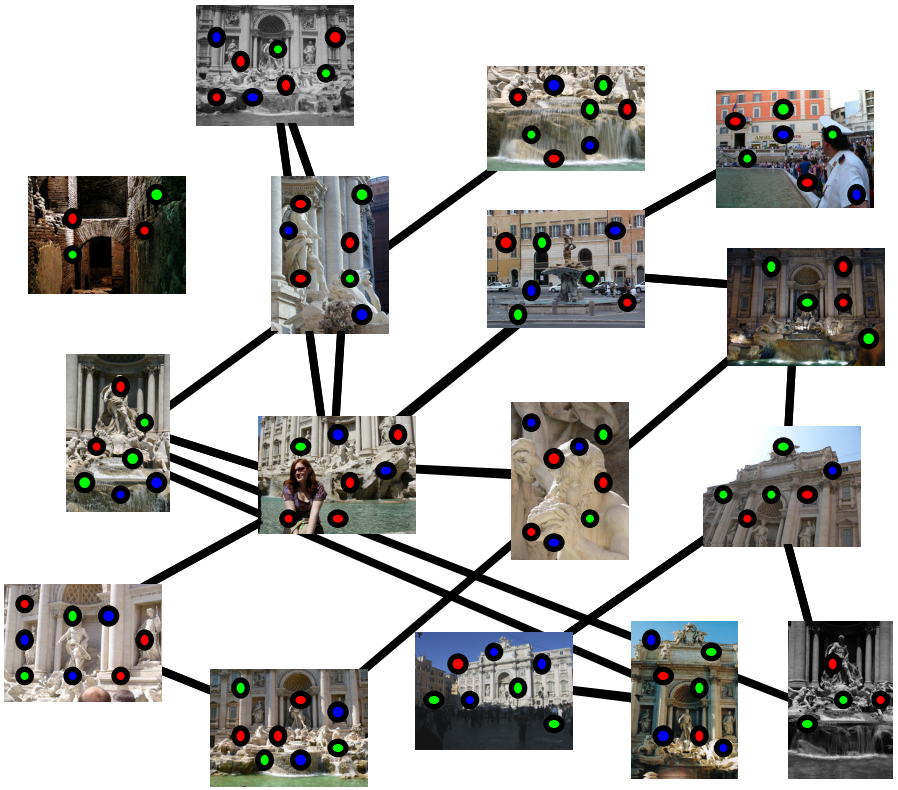
Feature matching

Match features between each pair of images

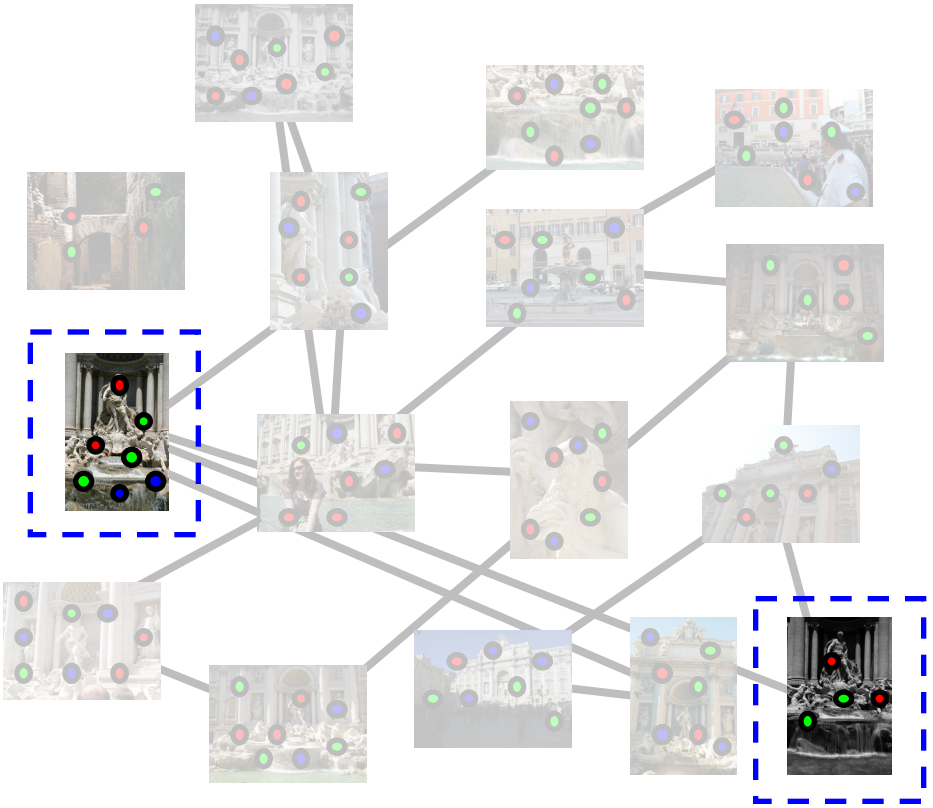


Feature matching

Refine matching using RANSAC [Fischler & Bolles 1987]

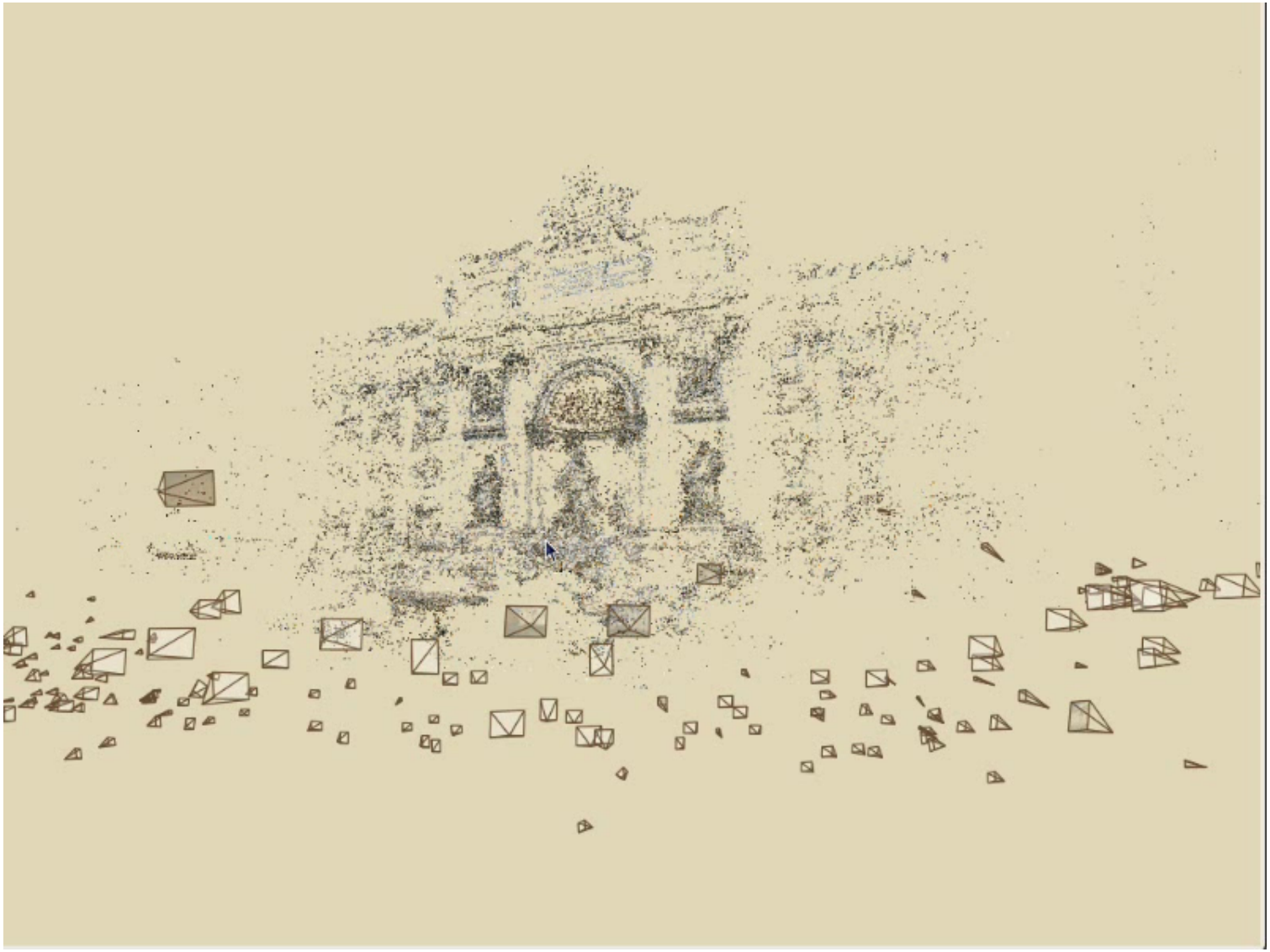


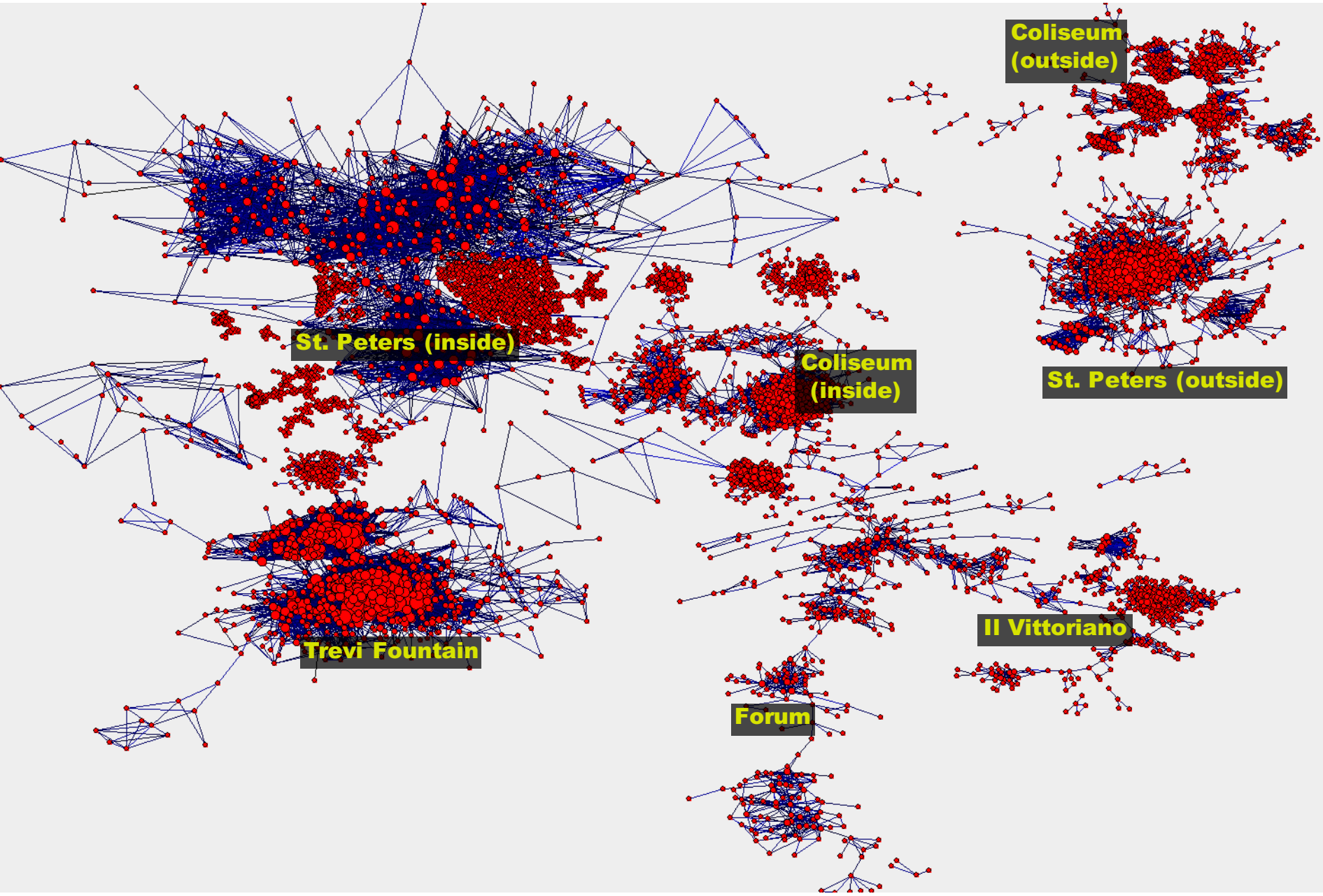
Incremental structure from motion



Incremental structure from motion

Photo Explorer

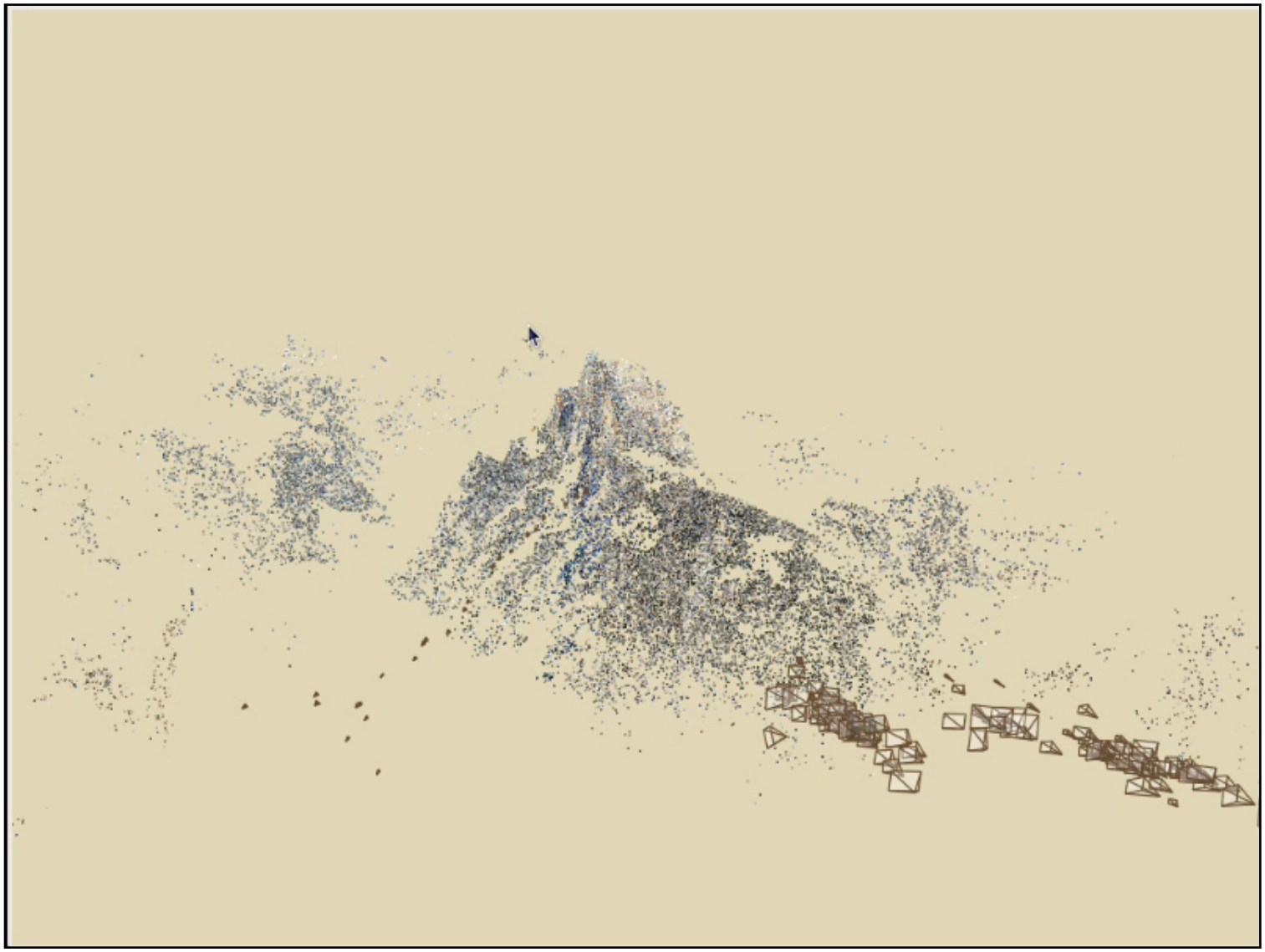



Coliseum (outside)
St. Peters (inside)
Coliseum (inside)
St. Peters (outside)
Il Vittoriano
Trevi Fountain
Forum

Navigation: Prague Old Town Square

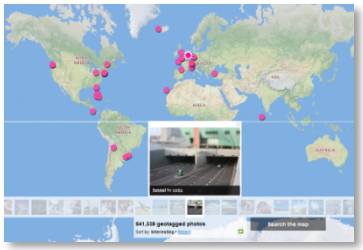
Hierarchical annotations

Locking the camera (stabilization)



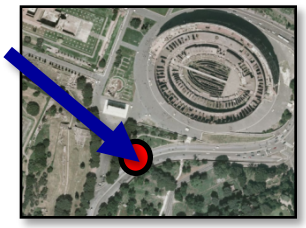
Applications

Community photo collections



- "Wikipedia for photos" – visual record of world through community of photographers
  - *Geograph British Isles* http://www.geograph.org.uk/

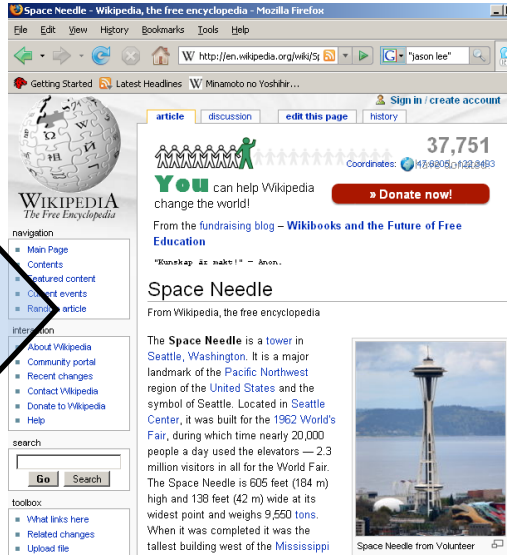
- Users can tag and comment on photos, link to other content
  - *World-wide telescope*



- "Where should I take a photo?" http://photocitygame.com/

Community photo collections

- Leveraging large databases of photos, large number of users
  - Annotations / augmented reality

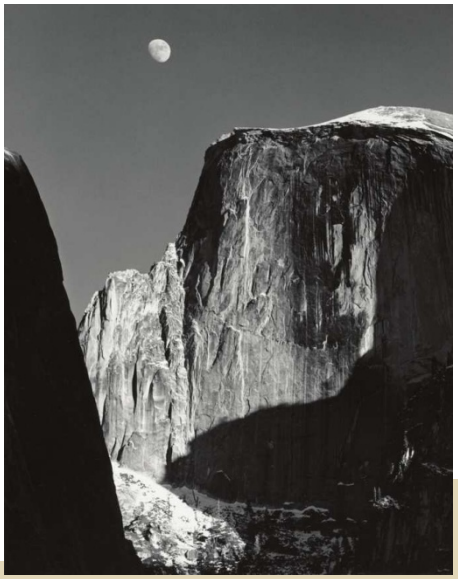


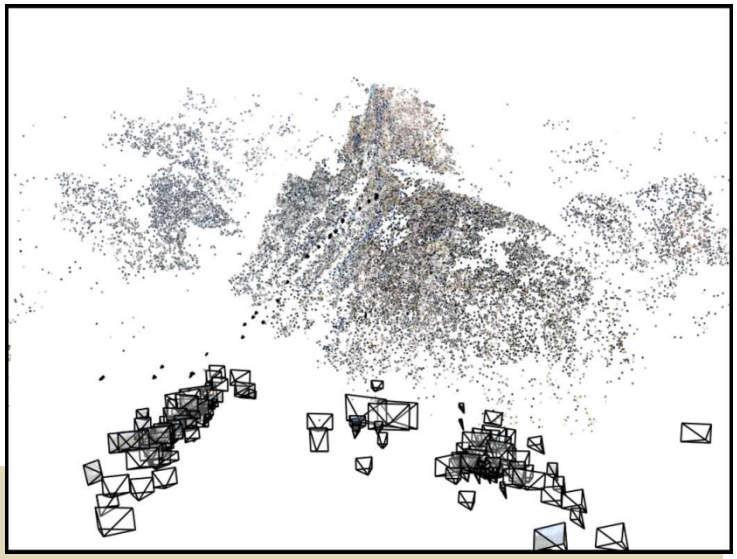
Virtual tour guide scenario

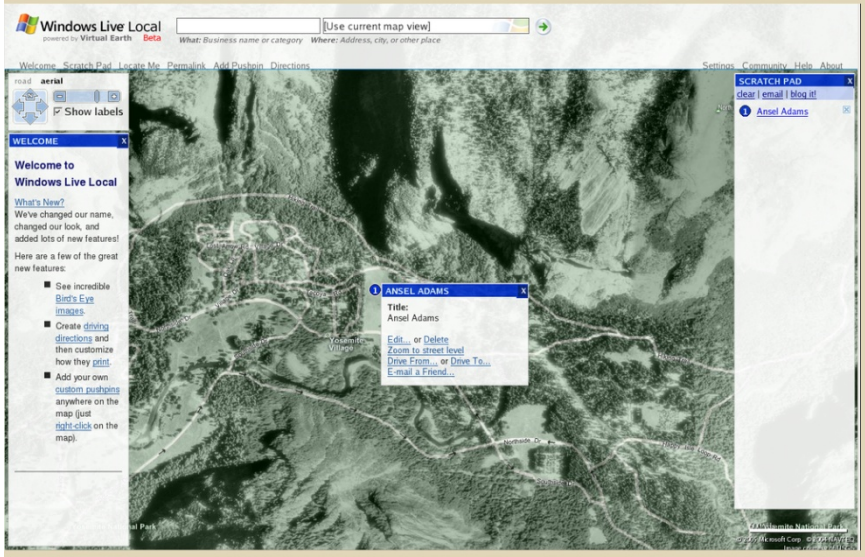
St. Peter's Basilica

- Built: 1506-1626
- http://en.wikipedia.org/wiki/St._Peter's_Basilica

Rephotography







Topographic data courtesy USGS