World3

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Introduction

1 History of World3

In 1968, a group of persons among heads of state and government, United Nation administrators, high-level politicians, scientists, economists, and business leaders from around the globe is formed, alarmed by the tendency mankind was taking. The international group created became known as "The Club Of Rome" after its first meeting took place in this location.

At the same time, Professor J.W.Forrester, a scientist expert in dynamic systems, is finishing its model on an industrial application of dynamic system using computer simulations. A few months after having proposed to design a model based on dynamic system which represent the world in order to observe the world-wide problem The Club Of Rome was worrying about, Forrester presents World2 to the organisation[1].

Its approach seemed interesting to their eyes and so after having discussed about some components of the model which were approximately described and based on some strong assumptions, the new model World3 is created[2]. Realised in cooperation with D.L. Meadows, it will be used in the "The Limits To Growth" to have a good based to explain the dynamic of different parameters. The Club Of Rome received criticisms from all around the world, because of their conclusions on the future of mankind which were alarming. After the model was upgraded and more data were collected, an update version of their book was published 30 years after the first one[3].

2 This Study

This study aims at coding in Python the World3 model of 1972 and 2003 with its different scenario to observe its behavior. The objective is to do it in a way that everyone with basic knowledge in Python could modify it. This paper explains how this World3 model is implemented. It is based on previous explanations and implementations which have been realised[4] [5] [6] [7]. The code is available here[1].

[1]https://github.com/Juji29/World3

Equations of World3

3 Introduction

There is no real calibration of equations in order to make sure that values match with historical values. But the values have been inspected regarding historical tendency from 1900 to 1970. The choice of input have been made in order to obtain coherent curves with the model and historic data. The values given are valid for the scenario 1. To obtain the other scenario, you will have to change only the variables named in the part "Different scenario".

4 Initial conditions

Name	ID	Type	Value	Dimension
IT		constant	1900	Year
FT		constant	2100	Year
TS		constant	0.5	Year
$PYEAR^{1972}$		constant	1975	Year
$PYEAR^{2003}$		constant	4000	Year

5 Units

Name	ID	Type	Value	Dimension
OY		constant	1	Year
$UAGI$		constant	1	\$/(Ha·Year)
UP		constant	1	Person
$GDPU$		constant	1	\$/(Person·Year)

6 Population

6.1 Population dynamics

Constants

Name	ID	Type	Value	Dimension
P_{1I}		initial value	650×10^6	Person
P_{2I}		initial value	700×10^6	Person
P_{3I}		initial value	190×10^6	Person
P_{4I}		initial value	60×10^6	Person

Equations

Total Population (**POP**) :

$$POP = P_1 + P_2 + P_3 + P_4 \tag{1}$$

Population aged from 0 to 14 years ($\mathbf{P_1}$) :

$$\frac{dP_1}{dt} = B - D_1 - MAT_1 \tag{2}$$

Deaths per year in population aged from 0 to 14 years ($\mathbf{D_1}$) :

$$D_1 = P_1 \times M_1 \tag{3}$$

Mortality rate in population aged from 0 to 14 years ($\mathbf{M_1}$) :

$\frac{LE}{OY}$	20	30	40	50	60	70	80
M_1	5.67×10^{-2}	3.66×10^{-2}	2.43×10^{-2}	1.55×10^{-2}	8.2×10^{-3}	2.3×10^{-3}	1×10^{-3}

(4)

Maturation rate of 14 years old people ($\mathbf{MAT_1}$) :

$$MAT_1 = \frac{P_1 \times (1 - M_1)}{15} \tag{5}$$

Population aged from 15 to 44 years ($\mathbf{P_2}$) :

$$\frac{dP_2}{dt} = MAT_1 - D_2 - MAT_2 \tag{6}$$

Deaths per year in population aged from 15 to 44 years ($\mathbf{D_2}$) :

$$D_2 = P_2 \times M_2 \tag{7}$$

Mortality rate in population aged from 15 to 44 years ($\mathbf{M_2}$) :

$\frac{LE}{OY}$	20	30	40	50	60	70	80
M_2	2.66×10^{-2}	1.71×10^{-2}	1.1×10^{-2}	6.5×10^{-3}	4×10^{-3}	1.6×10^{-3}	8×10^{-4}

(8)

Maturation rate of 44 years old people ($\mathbf{MAT_2}$) :

$$MAT_2 = \frac{P_2 \times (1 - M_2)}{30} \tag{9}$$

Population aged from 45 to 64 years ($\mathbf{P_3}$) :

$$\frac{dP_3}{dt} = MAT_2 - D_3 - MAT_3 \tag{10}$$

Deaths per year in population aged from 45 to 64 years (**D_3**) :

$$D_3 = P_3 \times M_3 \tag{11}$$

Mortality rate in population aged from 45 to 64 years (**M_3**) :

$\frac{LE}{OY}$	20	30	40	50	60	70	80
M_3	5.62×10^{-2}	3.73×10^{-2}	2.52×10^{-2}	1.71×10^{-2}	1.18×10^{-2}	8.3×10^{-3}	6×10^{-3}

(12)

Maturation rate of 64 years old people (**MAT_3**) :

$$MAT_3 = \frac{P_3 \times (1 - M_3)}{20} \tag{13}$$

Population older than age 65 (**P_4**) :

$$\frac{dP_4}{dt} = MAT_3 - D_4 \tag{14}$$

Deaths per year in population older than age 65 (**D_4**) :

$$D_4 = P_4 \times M_4 \tag{15}$$

Mortality rate in population older than age 65 (**M_4**) :

$\frac{LE}{OY}$	20	30	40	50	60	70	80
M_4	0.13	0.011	0.09	0.07	0.06	0.05	0.04

(16)

6.2 Death

Constants

Name	ID	Type	Value	Dimension
LEN		constant	28	Year
$HSID$		constant	20	Year
$EHSPCI$		initial value	0	\$/Year

Equations

Deaths per year (**D**) :

$$D = D_1 + D_2 + D_3 + D_4 \tag{17}$$

Crude Death Rate (**CDR**) :

$$CDR = \frac{1\,000 \times D}{POP} \tag{18}$$

Life Expectancy (**LE**) :

$$LE = LEN \times LMF \times LMHS \times LMP \times LMC \tag{19}$$

Lifetime Multiplier from Food (**LMF**) :

$$\begin{array}{|c||c|c|c|c|c|c|} \hline \frac{FPC}{SFPC} & 0 & 1 & 2 & 3 & 4 & 5 \\ \hline LMF^{1972} & 0 & 1 & 1.2 & 1.3 & 1.35 & 1.4 \\ \hline LMF^{2003} & 0 & 1 & 1.43 & 1.5 & 1.5 & 1.5 \\ \hline \end{array} \tag{20}$$

Health Services Allocations Per Capita (**HSAPC**) :

$$\begin{array}{|c||c|c|c|c|c|c|c|c|c|} \hline \frac{SOPC}{GDPU} & 0 & 250 & 500 & 750 & 1\,000 & 1\,250 & 1\,500 & 1\,750 & 2\,000 \\ \hline HSAPC & 0 & 20 & 50 & 95 & 140 & 175 & 200 & 220 & 230 \\ \hline \end{array} \tag{21}$$

Effective Health Services Per Capita (**EHSPC**) :

$$\frac{dEHSPC}{dt} = \frac{HSAPC - EHSPC}{HSID} \tag{22}$$

Lifetime Multiplier from Health Services (**LMHS**) :

$$LMHS = \begin{cases} LMHS_2, \; if \; TIME > 1940 \\ LMHS_1, \qquad\quad else \end{cases} \tag{23}$$

$$\begin{array}{|c||c|c|c|c|c|c|} \hline \frac{EHSPC}{GDPU} & 0 & 20 & 40 & 60 & 80 & 100 \\ \hline LMHS_1 & 1 & 1.1 & 1.4 & 1.6 & 1.7 & 1.8 \\ \hline LMHS_2^{1972} & 1 & 1.4 & 1.6 & 1.8 & 1.95 & 2 \\ \hline LMHS_2^{2003} & 1 & 1.5 & 1.9 & 2 & 2 & 2 \\ \hline \end{array} \tag{24}$$

Fraction of Population Urban (**FPU**) :

$$\begin{array}{|c||c|c|c|c|c|c|c|c|c|} \hline \frac{POP}{UP} & 0 & 2{\times}10^9 & 4{\times}10^9 & 6{\times}10^9 & 8{\times}10^9 & 1{\times}10^{10} & 1.2{\times}10^{10} & 1.4{\times}10^{10} & 1.6{\times}10^{10} \\ \hline FPU & 0 & 0.2 & 0.4 & 0.5 & 0.58 & 0.65 & 0.72 & 0.78 & 0.8 \\ \hline \end{array} \tag{25}$$

Crowding Multiplier from Industry (**CMI**) :

$$\begin{array}{|c||c|c|c|c|c|c|c|c|c|} \hline \frac{IOPC}{GDPU} & 0 & 200 & 400 & 600 & 800 & 1\,000 & 1\,200 & 1\,400 & 1\,600 \\ \hline CMI & 0.5 & 0.05 & -0.1 & -0.08 & -0.02 & 0.05 & 0.1 & 0.15 & 0.2 \\ \hline \end{array} \tag{26}$$

Lifetime Multiplier from Crowding (**LMC**) :

$$LMC = 1 - CMI \times FPU \tag{27}$$

Lifetime Multiplier from persistant Polution (**LMP**) :

$$\begin{array}{|c||c|c|c|c|c|c|c|c|c|c|c|} \hline PPOLX & 0 & 10 & 20 & 30 & 40 & 50 & 60 & 70 & 80 & 90 & 100 \\ \hline LMP & 1 & 0.99 & 0.97 & 0.95 & 0.9 & 0.85 & 0.75 & 0.65 & 0.55 & 0.4 & 0.2 \\ \hline \end{array} \tag{28}$$

6.3 Birth

Constants

Name	ID	Type	Value	Dimension
RLT		constant	30	Year
PET		constant	4000	Year
$MTFN$		constant	12	Dmnl
LPD		constant	20	Year
$AIOPCI$		initial value	43.3	\$/Year
$ZPGT$		constant	4000	Year
$DCFSN^{1972}$		constant	4	Dmnl
$DCFSN^{2003}$		constant	3.8	Dmnl
SAD		constant	20	Year
$IEAT$		constant	3	Year
$FCEST$		constant	4000	Year

Equations

Births per year (**B**) :

$$B = \begin{cases} D & , \; if \; TIME \geqslant PET \\ \frac{TF \times P_2}{2 \times RLT}, & else \end{cases} \tag{29}$$

Crude Birth Rate (**CBR**) :

$$CBR = \frac{1\,000 \times B}{POP} \tag{30}$$

Total Fertility (**TF**) :

$$TF = Min(MTF, MTF \times (1 - FCE) + DTF \times FCE) \tag{31}$$

Maximum Total Fertility (**MTF**) :

$$MTF = MTFN \times FM \tag{32}$$

Fertility Multiplier (**FM**) :

$\frac{LE}{OY}$	0	10	20	30	40	50	60	70	80
FM^{1972}	0	0.2	0.4	0.6	0.8	0.9	1	1.05	1.1
FM^{2003}	0	0.2	0.4	0.6	0.7	0.75	0.79	0.84	0.87

(33)

Desired Total Fertility (**DTF**) :

$$DTF = DCFS \times CMPLE \tag{34}$$

Compensatory Multiplier from Perceived Life Expectancy (**CMPLE**) :

$\frac{PLE}{OY}$	0	10	20	30	40	50	60	70	80
$CMPLE$	3	2.1	1.6	1.4	1.3	1.2	1.1	1.05	1

(35)

Perceived Life Expectancy (**PLE**) :

$$PLE = Delay3(LE, LPD) \tag{36}$$

Desired Completed Family Size (**DCFS**) :

$$DCFS = \begin{cases} 2 & , \ if \ TIME \geqslant ZPGT \\ DCFSN \times FRSN \times SFSN, & else \end{cases} \tag{37}$$

Social Family Size Norm (**SFSN**) :

$\frac{DIOPC}{GDPU}$	0	200	400	600	800
$SFSN^{1972}$	1.25	1	0.9	0.8	0.75
$SFSN^{2003}$	1.25	0.94	0.715	0.59	0.5

(38)

Delayed Industrial Output Per Capita (**DIOPC**) :

$$DIOPC = Delay3(IOPC, SAD) \tag{39}$$

Family Response to Social Norm (**FRSN**) :

FIE	-0.2	-0.1	0	0.1	0.2
$FRSN$	0.5	0.6	0.7	0.85	1

(40)

Family Income Expectation (**FIE**) :

$$FIE = \frac{IOPC - AIOPC}{AIOPC} \tag{41}$$

Average Industrial Output Per Capita (**AIOPC**) :

$$\frac{dAIOPC}{dt} = \frac{IOPC - AIOPC}{IEAT} \tag{42}$$

Need for Fertility Control (**NFC**) :

$$NFC = \frac{MTF}{DTF} - 1 \tag{43}$$

Fertility Control Effectiveness (**FCE**) :

$$FCE = \begin{cases} 1 & , \ if \ TIME \geqslant FCEST \\ \begin{array}{|c||c|c|c|c|c|c|c|} \hline \frac{FCFPC}{GDPU} & 0 & 0.5 & 1 & 1.5 & 2 & 2.5 & 3 \\ \hline FCE & 0.75 & 0.85 & 0.9 & 0.95 & 0.98 & 0.99 & 1 \\ \hline \end{array}, & else \end{cases} \tag{44}$$

Fertility Control Facilities Per Capita (**FCFPC**) :

$$FCFPC = Delay3(FCAPC, HSID) \tag{45}$$

Fertility Control Allocations Per Capita (**FCAPC**) :

$$FCAPC = FSAFC \times SOPC \tag{46}$$

Fraction of Services Allocated to Fertility Control (**FSAFC**) :

NFC	0	2	4	6	8	10
$FSAFC$	0	0.005	0.015	0.025	0.03	0.035

(47)

7 Capital

7.1 Industrial

Constants

Name	ID	Type	Value	Dimension
$ICOR_1$		constant	3	Year
$ICOR_2^{1972}$		constant	3	Year
ICI		initial value	2.1×10^{11}	\$
$ALIC_1$		constant	14	Year
$ALIC_2$		constant	14	Year
IET		constant	4000	Year
$FIOAC_1$		constant	0.43	Dmnl
$FIOAC_2$		constant	0.43	Dmnl
$IOPCD$		constant	400	\$/(Person·Year)

Equations

Industrial Output Per Capita (**IOPC**) :

$$IOPC = \frac{IO}{POP} \tag{48}$$

Industrial Output (**IO**) :

$$IO = \frac{IC \times (1 - FCAOR) \times CUF}{ICOR} \tag{49}$$

Industrial Capital-Output Ratio (**ICOR**) :

$$ICOR = \begin{cases} ICOR_2, & if\ TIME \geqslant PYEAR \\ ICOR_1, & else \end{cases} \tag{50}$$

Industrial Capital (**IC**) :

$$\frac{dIC}{dt} = ICIR - ICDR \tag{51}$$

Industrial Capital Depreciation Rate (**ICDR**) :

$$ICDR = \frac{IC}{ALIC} \tag{52}$$

Average Lifetime of Industrial Capital (**ALIC**) :

$$ALIC = \begin{cases} ALIC_2, & if\ TIME \geqslant PYEAR \\ ALIC_1, & else \end{cases} \tag{53}$$

Industrial Capital Investment Rate (**ICIR**) :

$$ICIR = IO \times FIAOI \tag{54}$$

Fraction of Industrial Output Allocated to Industry (**FIOAI**) :

$$FIOAI = 1 - FIAOA - FIOAS - FIOAC \tag{55}$$

Fraction of Industrial Output Allocated to Consumption (**FIOAC**) :

$$FIOAC = \begin{cases} FIOACV, \ if\ TIME \geqslant IET \\ FIOACC, \qquad else \end{cases} \tag{56}$$

Fraction of Industrial Output Allocated to Consumption Constant (**FIOACC**) :

$$FIOACC = \begin{cases} FIOAC_2, \ if\ TIME \geqslant PYEAR \\ FIOAC_1, \qquad else \end{cases} \tag{57}$$

Fraction of Industrial Output Allocated to Consumption Variable (**FIOACV**) :

$IOPC$	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
$IOPCD$	0.3	0.32	0.34	0.36	0.38	0.43	0.73	0.77	0.81	0.82	0.83

(58)

In addition for the version 2003:
$\mathbf{ICOR_2^{2003}}$:

$$ICOR_2^{2003} = ICOR_{2T} \times COYM \times COPM \tag{59}$$

7.2 Services

Constants

Name	**ID**	**Type**	**Value**	**Dimension**
SCI		initial value	$1.44{\times}10^{11}$	\$
$ALSC_1$		constant	20	Year
$ALSC_2$		constant	20	Year
$SCOR_1$		constant	1	Year
$SCOR_2$		constant	1	Year

Equations

Indicated Service Output Per Capita (**ISOPC**) :

$$ISOPC = \begin{cases} ISOPC_2, \ if\ TIME \geqslant PYEAR \\ ISOPC_1, \qquad else \end{cases} \tag{60}$$

$\frac{IOPC}{GDPU}$	0	200	400	600	800	1 000	1 200	1 400	1 600
$ISOPC_1$	40	300	640	1 000	1 220	1 450	1 650	1 800	2 000
$ISOPC_2$	40	300	640	1 000	1 220	1 450	1 650	1 800	2 000

(61)

Fraction of Industrial Output Allocated to Services (**FIOAS**) :

$$FIOAS = \begin{cases} FIOAS_2, \ if\ TIME \geqslant PYEAR \\ FIAOS_1, \qquad else \end{cases} \tag{62}$$

$\frac{SOPC}{ISOPC}$	0	0.5	1	1.5	2
$FIOAS_1$	0.3	0.2	0.1	0.05	0
$FIOAS_2$	0.3	0.2	0.1	0.05	0

(63)

Service Capital Investment Rate (**SCIR**) :

$$SCIR = IO \times FIOAS \tag{64}$$

Service Capital (**SC**) :

$$\frac{dSC}{dt} = SCIR - SCDR \tag{65}$$

Service Capital Depreciation Rate (**SCDR**) :

$$SCDR = \frac{SC}{ALSC} \tag{66}$$

Average Lifetime of Service Capital (**ALSC**) :

$$ALSC = \begin{cases} ALSC_2, & if\ TIME \geqslant PYEAR \\ ALSC_1, & else \end{cases} \tag{67}$$

Service Output (**SO**) :

$$SO = \frac{SC \times CUF}{SCOR} \tag{68}$$

Service Output Per Capita (**SOPC**) :

$$SOPC = \frac{SO}{POP} \tag{69}$$

Service Capital-Output Rate (**SCOR**) :

$$SCOR = \begin{cases} SCOR_2, & if\ TIME \geqslant PYEAR \\ SCOR_1, & else \end{cases} \tag{70}$$

7.3 Job

Constants

Name	ID	Type	Value	Dimension
$LFPF$		constant	0.75	Dmnl
$LUFDT$		constant	2	Year
$LUFDI$		initial value	1	Dmnl

Equations

Job (**J**) :

$$J = PJIS + PJAS + PJSS \tag{71}$$

Potential Jobs in Industrial Sector (**PJIS**) :

$$PJIS = IC \times JPICU \tag{72}$$

Jobs Per Industrial Capital Unit (**JPICU**) :

$\frac{IOPC}{GDPU}$	50	200	350	500	650	800
$JPICU$	3.7×10^{-4}	1.8×10^{-4}	1.2×10^{-4}	9×10^{-5}	7×10^{-5}	6×10^{-5}

(73)

Potential Jobs in Service Sector (**PJSS**) :

$$PJSS = SC \times JPSCU \tag{74}$$

Jobs Per Service Capital Unit (**JPSCU**) :

$\frac{SOPC}{GDPU}$	50	200	350	500	650	800
$JPSCU$	1.1×10^{-3}	6×10^{-4}	3.5×10^{-4}	2×10^{-4}	1.5×10^{-4}	1.5×10^{-4}

(75)

Potential Jobs in Agricultural Sector (**PJAS**) :

$$PJAS = JPH \times AL \tag{76}$$

Jobs Per Hectare (**JPH**) :

$\frac{AIPH}{UAGI}$	2	6	10	14	18	22	26	30
JPH	2	0.5	0.4	0.3	0.27	0.24	0.2	0.2

(77)

Labor Force (**LF**) :

$$LF = (P_2 + P_3) \times LFPF \tag{78}$$

Labor Utilization Fraction (**LUF**) :

$$LUF = \frac{J}{LF} \tag{79}$$

Labor Utilization Fraction Delayed (**LUFD**) :

$$\frac{dLUFD}{dt} = \frac{LF - LUFD}{LUFDT} \tag{80}$$

Capital Utilization Fraction (**CUF**) :

$LUFD$	1	3	5	7	9	11
CUF	1	0.9	0.7	0.3	0.1	0.1

(81)

8 Agriculture

8.1 Loop 1

Constants

Name	ID	Type	Value	Dimension
$PALT$		constant	3.2×10^{9}	Year
ALI		initial value	9×10^{8}	Year
$PALI$		initial value	2.3×10^{9}	\$
LFH		constant	0.7	Dmnl
PL		constant	0.1	Dmnl

Equations

Land Fraction Cultivated (**LFC**) :

$$LFC = \frac{AL}{PALT} \tag{82}$$

Arable Land (**AL**) :

$$\frac{dAL}{dt} = LDR - LER - LRUI \tag{83}$$

Potentially Arable Land (**PAL**) :

$$\frac{dPAL}{dt} = -LDR \tag{84}$$

Food (**F**) :

$$F = LY \times AL \times LFH \times (1 - PL) \tag{85}$$

Fraction Per Capita (**FPC**) :

$$FPC = \frac{F}{POP} \tag{86}$$

Indicated Food Per Capita (**IFPC**) :

$$IFPC = \begin{cases} IFPC_2, \ if \ TIME \geqslant PYEAR \\ IFPC_1, \qquad else \end{cases} \tag{87}$$

$\frac{IOPC}{GDPU}$	0	200	400	600	800	1 000	1 200	1 400	1 600
$IFPC_1$	230	480	690	850	970	1 070	1 150	1 210	1 250
$IFPC_2$	230	480	690	850	970	1 070	1 150	1 210	1 250

(88)

Total Agricultural Investment (**TAI**) :

$$TAI = \frac{IO}{FIOAA} \tag{89}$$

Fraction of Industrial Output Allocated to Agriculture (**FIOAA**) :

$$FIOAA = \begin{cases} FIOAA_2, \ if \ TIME \geqslant PYEAR \\ FIOAA_1, \qquad else \end{cases} \tag{90}$$

$\frac{FPC}{IFPC}$	0	0.5	1	1.5	2	2.5
$FIOAA_1$	0.4	0.2	0.1	0.025	0	0
$FIOAA_2$	0.4	0.2	0.1	0.025	0	0

(91)

Land Development Rate (**LDR**) :

$$LDR = \frac{TAI \times FIALD}{DCPH} \tag{92}$$

Development Cost Per Hectare (**DCPH**) :

$\frac{PAL}{PALT}$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$DCPH$	100 000	7 400	5 200	3 500	2 400	1 500	750	300	150	75	50

(93)

8.2 Loop 2

Constants

Name	ID	Type	Value	Dimension
AII		initial value	5×10^9	\$/Year
$ALAI_1$		constant	2	Year
$ALAI_2$		constant	2	Year
LYF_1		constant	1	Dmnl
LYF_2^{1972}		constant	1	Dmnl
IO_{70}		constant	7.9×10^{11}	\$/Year
SD		constant	0.07	Year^{-1}

Equations

Current Agricultural Inputs (**CAI**) :

$$CAI = TAI \times (1 - FIALD) \tag{94}$$

Agricultural Inputs (**AI**) :

$$\frac{dAI}{dt} = \frac{CAI - AI}{ALAI} \tag{95}$$

Average Lifetime of Agricultural Inputs (**ALAI**) :

$$ALAI = \begin{cases} ALAI_2, & if\ TIME \geqslant PYEAR \\ ALAI_1, & else \end{cases} \tag{96}$$

Agricultural Inputs Per Hectare (**AIPH**) :

$$AIPH = \frac{AI \times (1 - FALM)}{AL} \tag{97}$$

Land Yield Multiplier from Capital (**LYMC**) :

$\frac{AIPH}{UAGI}$	0	40	80	120	160	200	240	280	320	360	400	440	480
$LYMC^{1972}$	1	3	3.8	4.4	4.9	5.4	5.7	6	6.3	6.6	6.9	7.2	7.4
$LYMC^{2003}$	1	3	4.5	5	5.3	5.6	5.9	6.1	6.35	6.6	6.9	7.2	7.4

$\frac{AIPH}{UAGI}$	520	560	600	640	680	720	760	800	840	880	920	960	1 000
$LYMC^{1972}$	7.6	7.8	8	8.2	8.4	8.6	8.8	9	9.2	9.4	9.6	9.8	10
$LYMC^{2003}$	7.6	7.8	8	8.2	8.4	8.6	8.8	9	9.2	9.4	9.6	9.8	10

(98)

Land Yield (**LY**) :

$$LY = LYF \times LFERT \times LYMC \times LYMAP \tag{99}$$

Land Yield Factor (**LYF**) :

$$LYF = \begin{cases} LYF_2, & if\ TIME \geqslant PYEAR \\ LYF_1, & else \end{cases} \tag{100}$$

Land Yield Multiplier from Air Pollution (**LYMAP**) :

$$LYMAP = \begin{cases} LYMAP_2, \ if\ TIME \geqslant PYEAR \\ LYMAP_1, \qquad\qquad else \end{cases} \tag{101}$$

$\frac{IO}{IO_{70}}$	0	10	20	30
$LYMAP_1$	1	1	0.7	0.4
$LYMAP_2^{1972}$	1	1	0.7	0.4
$LYMAP_2^{2003}$	1	1	0.98	0.95

(102)

Fraction of Inputs Allocated to Land Development (**FIALD**) :

$\frac{MPLD}{MPAI}$	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2
$FIALD$	0	0.05	0.15	0.3	0.5	0.7	0.85	0.95	1

(103)

Marginal Productivity of Land Development (**MPLD**) :

$$MPLD = \frac{LY}{DCPH \times SD} \tag{104}$$

Marginal Productivity of Agricultural Inputs (**MPAI**) :

$$MPAI = \frac{ALAI \times LY \times MLYMC}{LYMC} \tag{105}$$

Marginal Land Yield Multiplier from Capital (**MLYMC**) :

$\frac{AIPH}{UAGI}$	0	40	80	120	160	200	240	280
$FIALD$	0.075	0.03	0.015	0.011	0.009	0.007	0.006	0.005

$\frac{AIPH}{UAGI}$	320	360	400	440	480	520	560	600
$FIALD$	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005

(106)

In addition for the version 2003:

Name	ID	Type	Value	Dimension
TDD		constant	20	Year

LYF$_2^{2003}$:

$$LYF_2^{2003} = Delay3(LYTD, TDD) \tag{107}$$

8.3 Loop 3

Constants

Name	ID	Type	Value	Dimension
$ALLN^{1972}$		constant	6 000	Year
$ALLN^{2003}$		constant	1 000	Year
$UILDT$		constant	10	Year
$UILI$		initial value	8.2×10^6	Ha
$LLMYTM$		constant	4000	Year

Equations

Average Life of Land (**ALL**) :

$$ALL = ALLN \times LLMY \tag{108}$$

Land Life Multiplier from Yield (**LLMY**) :

$$LLMY = \begin{cases} 0.95^{\frac{TIME-LLMYTM}{OY}} LLMY_1 + (1 - 0.95^{\frac{TIME-LLMYTM}{OY}})LLMY_2, & if\ TIME \geqslant LLMYTM \\ LLMY_1, & else \end{cases} \tag{109}$$

$\frac{LY}{ILF}$	0	1	2	3	4	5	6	7	8	9
$LLMY_1$	1.2	1	0.63	0.36	0.16	0.055	0.04	0.025	0.015	0.01
$LLMY_2^{1972}$	1.2	1	0.63	0.36	0.16	0.055	0.04	0.025	0.015	0.01
$LLMY_2^{2003}$	1.2	1	0.63	0.36	0.29	0.26	0.24	0.22	0.21	0.2

(110)

Land Erosion Rate (**LER**) :

$$LER = \frac{AL}{ALL} \tag{111}$$

Urban-Industrial Land Per Capita (**UILPC**) :

$\frac{IOPC}{GDPU}$	0	200	400	600	800	1 000	1 200	1 400	1 600
$UILPC$	0.005	0.008	0.015	0.025	0.04	0.055	0.07	0.08	0.09

(112)

Urban-Industrial Land Required (**UILR**) :

$$UILR = UILPC \times POP \tag{113}$$

Land Removal from Urban-Industrial use (**LRUI**) :

$$LRUI = \frac{Max(0,\ UILR - UIL)}{UILDT} \tag{114}$$

Urban-Industrial Land (**UIL**) :

$$\frac{dUIL}{dt} = LRUI \tag{115}$$

8.4 Loop 4

Constants

Name	ID	Type	Value	Dimension
$LFERTI$		constant	600	Veg eq kg/(Year·Ha)

Equations

Land Fertility (**LFERT**) :

$$\frac{dLFERT}{dt} = LFR - LFD \tag{116}$$

Land Fertility Degradation Rate (**LFDR**) :

$PPOLX$	0	10	20	30
$LFDR$	0	0.1	0.3	0.5

(117)

Land Fertility Degradation (**LFD**) :

$$LFD = LFERT \times LFDR \tag{118}$$

8.5 Loop 5

Constants

Name	ID	Type	Value	Dimension
ILF		constant	600	Veg eq kg/(Year·Ha)
$SFPC$		constant	230	Veg eq kg/(Year·Ha)
$PFRI$		initial value	1	Dmnl
$FSDP$		constant	2	Year

Equations

Land Fertility Regeneration (**LFR**) :

$$LFR = \frac{ILF - LFERT}{LFRT} \tag{119}$$

Land Fertility Regeneration Time (**LFRT**) :

$FALM$	0	0.02	0.04	0.06	0.08	0.1
$LFRT$	20	13	8	4	2	1.2

(120)

Fraction of inputs Allocated to Land Maintenance (**FALM**) :

PFR	0	1	2	3	4
$FALM$	0	0.04	0.07	0.09	0.1

(121)

Food Ratio (**FR**) :

$$FR = \frac{FPC}{SFPC} \tag{122}$$

Perceived Food Ratio (**PFR**) :

$$\frac{dPFR}{dt} = \frac{FR - PFR}{FSDP} \tag{123}$$

In addition for the version 2003:

Name	ID	Type	Value	Dimension
DFR		constant	2	Dmnl

Land Yield Technology Development (**LYTD**) :

$$\frac{dLYTD}{dt} = LYTDR \tag{124}$$

Land Yield Technology Development Rate (**LYTDR**) :

$$LYTDR = \begin{cases} LYTD \times LYCM, & if\ TIME \geqslant PYEAR \\ 0 & , \qquad else \end{cases} \tag{125}$$

Land Yield Change Multiplier (**LYCM**) :

$DFR - FR$	0	1
$LYCM$	0	0

(126)

Capital Output Yield Multiplier (**COYM**) :

LYF	1	1.2	1.4	1.6	1.8	2
$COYM$	1	1.05	1.12	1.25	1.35	1.5

(127)

9 Resources

Constants

Name	ID	Type	Value	Dimension
NRI		initial value	10^{12}	Resource Units
$NRUF_1$		constant	1	Dmnl
$NRUF_2^{1972}$		constant	1	Dmnl
$FACORTM$		constant	4000	Year

Equations

Non-renewable Resources (**NR**) :

$$\frac{dNR}{dt} = -NRUR \tag{128}$$

Non-renewable Resources Usage Rate (**NRUR**) :

$$NRUR = POP \times PCRUM \times NRUF \tag{129}$$

Non-renewable Resources Usage Factor (**NRUF**) :

$$NRUF = \begin{cases} NRUF_2, & if\ TIME \geqslant PYEAR \\ NRUF_1, & else \end{cases} \tag{130}$$

Per Capita Resources Usage Multiplier (**PCRUM**) :

$\frac{IOPC}{GDPU}$	0	200	400	600	800	1 000	1 200	1 400	1 600
$PCRUM$	0	0.85	2.6	3.4	3.8	4.1	4.4	4.7	5

(131)

Non-renewable Resources Fraction Remaining (**NRFR**) :

$$NRFR = \frac{NR}{NRI} \tag{132}$$

Fertility Control Allocated to Obtaining Resources (**FCAOR**) :

$$FCAOR = \begin{cases} FCAOR_2, \ if \ TIME \geqslant FCAORTM \\ FCAOR_1, \qquad\qquad else \end{cases} \tag{133}$$

$NRFR$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$FCAOR_1$	1	0.9	0.7	0.5	0.2	0.1	0.05	0.05	0.05	0.05	0.05
$FCAOR_2^{1972}$	1	0.9	0.7	0.5	0.2	0.1	0.05	0.05	0.05	0.05	0.05
$FCAOR_2^{2003}$	1	0.2	0.1	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05

(134)

In addition for the version 2003:

Name	**ID**	**Type**	**Value**	**Dimension**
$DNRUR$		constant	4 000	Year

$\mathbf{NRUF_2^{2003}}$:

$$NRUF_2^{2003} = Delay3(NRTD, TDD) \tag{135}$$

Non-renewable Resources Technology Development (**NRTD**) :

$$\frac{dNRTD}{dt} = NRATE \tag{136}$$

Non-renewable Resource Technology Improvement Rate (**NRATE**) :

$$NRATE = \begin{cases} NRTD \times NRCM, \ if \ TIME \geqslant PYEAR \\ \quad 0 \qquad\qquad , \qquad\qquad else \end{cases} \tag{137}$$

Non-renewable Resources Change Multiplier (**NRCM**) :

$1 - \frac{NRUR}{DNRUR}$	-1	0
$NRCM$	0	0

(138)

Industrial Capital-Output Ratio Technology Transmission (**ICOR2T**) :

$NRUF$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$ICOR2T$	3.75	3.6	3.47	3.36	3.25	3.16	3.1	3.06	3.02	3.01	3

(139)

10 Pollution

Constants

Name	ID	Type	Value	Dimension
$PPGF_1$		constant	1	Dmnl
$PPGF_2^{1972}$		constant	1	Dmnl
$FRPM$		constant	0.02	Dmnl
$IMEF$		constant	0.1	Dmnl
$IMTI$		constant	10	Pollution Unit/Resource Unit
$FIPM$		constant	0.001	Dmnl
$AMTI$		constant	1	Pollution Unit/$
$PPTD$		constant	20	Year
$PPOLI$		initial value	2.5×10^7	Pollution Unit
$PPOL_{70}$		constant	1.36×10^8	Pollution Unit
AHL_{70}		constant	1.5	Year

Equations

Persistent Pollution Generation Rate (**PPGR**) :

$$PPGR = (PPGIO + PPGAO) \times PPGF \tag{140}$$

Persistent Pollution Generation Factor (**PPGF**) :

$$PPGF = \begin{cases} PPGF_2, & if\ TIME \geqslant PYEAR \\ PPGF_1, & else \end{cases} \tag{141}$$

Persistent Pollution Generated by Industrial Output (**PPGIO**) :

$$PPGIO = PCRUM \times POP \times FRPM \times IMEF \times IMTI \tag{142}$$

Persistent Pollution Generated by Agricultural Output (**PPGAO**) :

$$PPGAO = AIPH \times AL \times FIPM \times AMTI \tag{143}$$

Persistent Pollution Appearance Rate (**PPAPR**) :

$$PPAPR = Delay3(PPGR, PPTD) \tag{144}$$

Persistent Pollution (**PPOL**) :

$$\frac{dPPOL}{dt} = PPAPR - PPASR \tag{145}$$

Index of Persistent Pollution (**PPOLX**) :

$$PPOLX = \frac{PPOL}{PPOL_{70}} \tag{146}$$

Persistent Pollution Assimilation Rate (**PPASR**) :

$$PPASR = \frac{PPOL}{1.4 \times AHL} \tag{147}$$

Assimilation Half-Life Multiplier (**AHLM**) :

$PPOLX$	1	251	501	751	1 001
$AHLM$	1	11	21	31	41

(148)

Assimilation Half-Life (**AHL**) :

$$AHL = AHL_{70} \times AHLM \tag{149}$$

In addition for the version 2003:

Name	ID	Type	Value	Dimension
$DPOLX$		constant	1.2	Dmnl

PPGF2^{2003} :

$$PPGF_2^{2003} = Delay3(PTD, TDD) \tag{150}$$

Pollution Technology Development (**PTD**) :

$$\frac{dPTD}{dt} = PTDR \tag{151}$$

Pollution Technology Development Rate (**PTDR**) :

$$PTDR = \begin{cases} PTD \times POLGFM, & if\ TIME \geqslant PYEAR \\ 0 & , \quad else \end{cases} \tag{152}$$

Pollution Control Technology Change Multiplier (**POLGFM**) :

$1 - \frac{PPOLX}{DPOLX}$	-1	0
$POLGFM$	0	0

(153)

Capital Output Pollution Multiplier (**COPM**) :

$PPGF$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$COPM$	1.25	1.2	1.15	1.11	1.08	1.05	1.03	1.02	1.01	1	1

(154)

11 Indexes

11.1 Distribution of outputs between the different sector

Constants

Name	ID	Type	Value	Dimension
POF		constant	0.22	\$/Veg eq kg

Equations

Fraction of Output in Agriculture (**FOA**) :

$$FOA = \frac{POF \times F}{POF \times F + SO + IO} \tag{155}$$

Fraction Of Output in Industry (**FOI**) :

$$FOI = \frac{IO}{POF \times F + SO + IO} \tag{156}$$

Fraction Of Output in Service (**FOS**) :

$$FOS = \frac{S}{POF \times F + SO + IO} \tag{157}$$

11.2 Industrial outputs indexes

Equations

Resource usage Intensity (**RESINT**) :

$$RESINT = \frac{NRUR}{IO} \tag{158}$$

Pollution Intensity Indicator (**PLINID**) :

$$PLINID = \frac{PPGIO \times PPGF}{IO} \tag{159}$$

Consumed Industrial Output (**CIO**) :

$$CIO = IO \times FIOAC \tag{160}$$

Consumed Industrial Output Per Capita (**CIOPC**) :

$$CIOPC = \frac{CIO}{POP} \tag{161}$$

11.3 Human Welfare Index

Constants

Name	ID	Type	Value	Dimension
$RHGDP$		constant	9 508	\$/(Year·Person)
$RLGDP$		constant	24	\$/(Year·Person)

Equations

Human Welfare Index (**HWI**) :

$$HWI = \frac{LEI + EI + GDPI}{3} \tag{162}$$

Life Expectancy Index (**LEI**) :

$\frac{LE}{OY}$	25	35	45	55	65	75	85
LEI	0	0.16	0.33	0.5	0.67	0.84	1

(163)

Education Index (**EI**) :

$\frac{GDPPC}{GDPU}$	0	1 000	2 000	3 000	4 000	5 000	6 000	7 000
EI	0	0.81	0.88	0.92	0.95	0.98	0.99	1

(164)

Gross Domestic Product Index (**GDPI**) :

$$GDPI = \frac{log_{10}(\frac{GDPPC}{RLGDP})}{log_{10}(\frac{RHGDP}{RLGDP})} \tag{165}$$

Gross Domestic Product Per Capita (**GDPPC**) :

$\frac{IOPC}{GDPU}$	0	200	400	600	800	1 000
$GDPPC$	120	600	1 200	1 800	2 500	3 200

(166)

11.4 Human Ecological Footprint

Constants

Name	ID	Type	Value	Dimension
HUP		constant	4	Ha/(Pollution Unit/Year)
TL		constant	1.91	GHa
$HGHA$		constant	10^9	GHa/Ha

Equations

Human Ecological Footprint (**HEF**) :

$$HEF = \frac{ALGGHA + ULGHA + ALGHA}{TL} \tag{167}$$

Absorption Land in Gigahectare (**ALGHA**) :

$$ALGHA = \frac{PPGR \times HUP}{GHAH} \tag{168}$$

Arable Land in Gigahectare (**ALGGHA**) :

$$ALGGHA = \frac{AL}{GHAH} \tag{169}$$

Urban Land in Gigahectare (**ULGHA**) :

$$ULGHA = \frac{UIL}{GHAH} \tag{170}$$

12 Explanations on function $Delay3$

$Delay3$:

$$\begin{gathered}
Y = Delay3(VAL, DT) \\
\Leftrightarrow \\
DL = \frac{DT}{3} \\
RT1 = I1_{t-1}/DL \\
I1_t = I1_{t-1} + (VAL_t - RT1) \times TS \ avec \ I1_0 = DL \times VAL_0 \\
RT2 = I2_{t-1}/DL \\
I2_t = I2_{t-1} + (RT1 - RT2) \times TS \ avec \ I2_0 = DL \times VAL_0 \\
I3_t = I3_{t-1} + (RT2 - \frac{I3_{t-1}}{DL}) \times TS \ avec \ I3_0 = DL \times VAL_0 \\
Y_t = I3_t/DL
\end{gathered} \tag{171}$$

13 Different scenario

In "Limits to Growth: The 30-Year Update", different scenario are given starting from the standard run described above. Here are the changes to make in order to obtain the different scenario:

- Scenario 2: This scenario starts with the same assumptions as Scenario 1.

$$NRI = 2{\times}10^{-9}$$

$$FCAORTM = 2002$$

$NRFR$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$FCAOR_2^{2003}$	1	0.1	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05

- Scenario 3: This scenario starts with the same assumptions as Scenario 2.

$$PYEAR = 2002$$

$1 - \frac{PPOLX}{DPOLX}$	-1	0
$POLGFM$	-0.04	0

- Scenario 4: This scenario starts with the same assumptions as Scenario 3.

$DFR - FR$	0	1
$LYCM$	0	0.04

- Scenario 5: This scenario starts with the same assumptions as Scenario 4.

$$LLMYTM = 2002$$

- Scenario 6: This scenario starts with the same assumptions as Scenario 5.

$1 - \frac{NRUR}{DNRUR}$	-1	0
$NRCM$	-0.04	0

- Scenario 7: This scenario starts with the same assumptions as Scenario 2.

$$FCEST = 2002$$

$$ZPGT = 2002$$

- Scenario 8: This scenario starts with the same assumptions as Scenario 7.

$$IOPCD = 350$$

$$ALAI_2 = 2.5$$

$$ALIC_2 = 18$$

$$ALSC_2 = 25$$

$$IET = 2002$$

$$PYEAR = 2002$$

- Scenario 9: This scenario starts with the same assumptions as Scenario 8.

$$LLMYTM = 2002$$

$1 - \frac{PPOLX}{DPOLX}$	-1	0
$POLGFM$	-0.04	0

$DFR - FR$	0	1
$LYCM$	0	0.04

$1 - \frac{NRUR}{DNRUR}$	-1	0
$NRCM$	-0.04	0

- Scenario 10: This scenario starts with the same assumptions as Scenario 9.

$$FCAORTM = 1982$$

$$FCEST = 1982$$
$$IET = 1982$$
$$LLMYTM = 1982$$
$$PYEAR = 1982$$
$$ZPGT = 1982$$

- Scenario 11: This scenario starts with the same assumptions as Scenario 10.

$$FCAORTM = 2012$$

$$FCEST = 2012$$
$$IET = 2012$$
$$LLMYTM = 2012$$
$$PYEAR = 2012$$
$$ZPGT = 2012$$

Implementation

In this part, it will be explain how the code given in introduction works and how you can modify it in order to experiment other scenario.
The code is composed of 4 files:

- world3_dynamic.py which concerns the resolution of the graph
- world3_graph.py which concerns the creation of the graph
- world3_plot.py which concerns posting of the results
- world3_run.py which concerns the execution of the code

14 World3_dynamic.py

First, a general class Node is created in order to have a base for all nodes. Each node will have for attributes: a name, a value, an associated function, predecessors, successors, a rank and a graph to which it is linked.

Then, four types of node are defined: NodeStock, NodeFlow, NodeDelay3 and NodeConstant.

- A NodeStock is a node which is initialized with a value and at each step of time, its variation is calculated. So it will be integrated after all the other nodes have been computed.
- A NodeFlow is a node which is calculated with the value of of NodeStock, NodeConstant or other NodeFlows. Its value is calculated and not its variation.
- A NodeDelay3 is a node which has the same characteristics as a NodeFlow except the fact that it takes into account the value it had at the step n-1 and n-2. So the value of the step n is smoothed with its previous value. In order to do that, it has for attributes the constant which is the delta of time, the node used to obtain the value of the current step and finally I1, I2 and I3 to compute the smoothed value. The way they are computed is given in the section 12.
- A NodeConstant is a node which is constant during all the execution of the program. There are two types of NodeConstant: constant and tables. Constants have a single value where tables are used to define piece-wise linear functions. They are used instead of equation for some nodes. Their implementation is made with a list of couple (x, y) where x is the input and y the output.

The class Hypergraph represents the entire graph which is composed of different nodes described above. It takes as parameters the version of World3 desired and eventually nodes already created. Its attributes are a dictionary of its nodes(nodes), its version(version), the number maximum of rank(nbrank), a list of nodes ordered by their rank(nodesrank) and a list of its NodeStocks(stocks). Several methods are implemented so each of them are explained below:

- add_node(self, node): add node to the graph.
- add_nodes(self, nodes): add all nodes given in the list nodes to the graph.
- add_edge(self, f, x_target, x_s): create an edge between x_target and x_s described by the function f.
- eval(self, ts): compute the value of all nodes in the nodesrank.
- run(self, it, ft, ts): start the execution of the code by computing values for all time steps.
- sub_graph_vertex(self, cond): return the list of name of nodes respecting the condition cond as well as a list of the rank of their predecessors and another with the rank of their successors
- set_rank(self): sort a list of nodes by their rank(nodesrank) and compute nbrank

15 World3_graph.py

All the graph given in the chapter Equations of World3 is implemented here using classes of World3_dynamic.py. If you want to create a new scenario, I suggest you to give a number to this scenario and to modify World3_graph.py as it is done for the existing ones. The tables are used with the following function:

Algorithm 1 Function f_{tab}

Require: $x \in \mathbb{Z}$
  if $tab[0][0] \geq x$ **then**
    return $tab[0][1]$
  else if $tab[-1][0] \leq x$ **then**
    return $tab[-1][1]$
  else
    $i \leftarrow 0$
    while $i < length(tab)$ **do**
      if $tab[i][0] \geq x \geq tab[i+1][0]$ **then**
        $coef \leftarrow \frac{tab[i+1][1]-tab[i][1]}{tab[i+1][0]-tab[i][0]}$
        return $tab[i][1] + coef \times (x - tab[i][0])$
      else
        $i \leftarrow i + 1$
      end if
    end while
  end if

16 World3_plot.py

All nodes have also an historic which can be showed at the end of the execution of the program. World3_plot.py contains functions which allow to observe evolution of variables in the way it is done in "The Limits to Growth". All backgrounds are taken from this book and from "The Limits to Growth: The 30-Year Update". They are available from scenario 1 to 10.

17 World3_run.py

This file is the one to execute. You can choose the version of World3, the initial time, the final one, time step and the scenario you desire to execute. Each run is by default printed with in background the result obtained in "The Limits to Growth".

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Acronyms and Appendix[2]

AHL Assimilation Half-Life

AHL_{70} Assimilation Half-Life in 1970

AHLM Assimilation Half-Life Multiplier

AI Agricultural Inputs

AII Initial Agricultural Inputs

AIOPC Average Industrial Output Per Capita

AIOPCI Initial Average Industrial Output Per Capita

AIPH Agricultural Inputs Per Hectare

AL Arable Land

ALAI Average Lifetime of Agricultural Inputs

ALGGHA Arable Land in Gigahectare

ALGHA Absorption Land in Gigahectare

ALI Initial Arable Land

ALIC Average Lifetime of Industrial Capital

ALL Average Life of Land

ALLN Average Life of Land Normal

ALSC Average Lifetime of Service Capital

AMTI Agricultural Materials Toxicity Index

B Births per year

CAI Current Agricultural Inputs

CBR Crude Birth Rate

CDR Crude Death Rate

CIO Consumed Industrial Output

[2]Extracted from "Understanding Global Systems Today — A Calibration of the World3-03 Model between 1995 and 2012" [8]

CIOPC Consumed Industrial Output Per Capita

CMI Crowding Multiplier from Industry

CMPLE Compensatory Multiplier from Perceived Life Expectancy

COPM Capital Output Pollution Multiplier

COYM Capital Output Yield Multiplier

CUF Capital Utilization Fraction

D Deaths per year

$\mathbf{D_1}$ Deaths per year in population aged from 0 to 14 years

$\mathbf{D_2}$ Deaths per year in population aged from 15 to 44 years

$\mathbf{D_3}$ Deaths per year in population aged from 45 to 64 years

$\mathbf{D_4}$ Deaths per year in population older than age 65

DCFS Desired Completed Family Size

DCFSN Desired Completed Family Size Normal

DCPH Development Cost Per Hectare

DFR Desired Food Ratio

DIOPC Delayed Industrial Output Per Capita

DNRUR Delayed Non-renewable Resources Usage Rate

DPOLX Desired Index of persistent Pollution

DTF Desired Total Fertility

EHSPC Effective Health Services Per Capita

EHSPCI Initial Effective Health Services Per Capita

EI Education Index

F Food

FALM Fraction of inputs Allocated to Land Maintenance

FCAOR Fertility Control Allocated to Obtaining Resources

FCAORTM Fertility Control Allocated to Obtaining Resources switch Time

FCAPC Fertility Control Allocations Per Capita

FCE Fertility Control Effectiveness

FCEST Fertility Control Effectiveness Set Time

FCFPC Fertility Control Facilities Per Capita

FIALD Fraction of Inputs Allocated to Land Development

FIE Family Income Expectation

FIOAA Fraction of Industrial Output Allocated to Agriculture

FIOAC Fraction of Industrial Output Allocated to Consumption

FIOACC Fraction of Industrial Output Allocated to Consumption Constant

FIOACV Fraction of Industrial Output Allocated to Consumption Variable

FIOAI Fraction of Industrial Output Allocated to Industry

FIOAS Fraction of Industrial Output Allocated to Services

FIPM Fraction of Inputs as Persistent Materials

FM Fertility Multiplier

FOA Fraction of Output in Agriculture

FOI Fraction Of Output in Industry

FOS Fraction Of Output in Service

FPC Fraction Per Capita

FPU Fraction of Population Urban

FR Food Ratio

FRPM Fraction of Resources as Persistent Materials

FRSN Family Response to Social Norm

FSAFC Fraction of Services Allocated to Fertility Control

FSDP Food Shortage Perception Delay

FT Final Time

GDPI Gross Domestic Product Index

GDPPC Gross Domestic Product Per Capita

GDPU Gross Domestic Product Unit

GHAH Gigahectare per Hectare

HEF Human Ecological Footprint

HSAPC Health Services Allocations Per Capita

HSID Health Services Impact Delay

HUP Hectare per Unit of Pollution

HWI Human Welfare Index

IC	Industrial Capital
ICDR	Industrial Capital Depreciation Rate
ICI	Initial Industrial Capital
ICIR	Industrial Capital Investment Rate
ICOR	Industrial Capital-Output Ratio
ICOR2T	Industrial Capital-Output Ratio Technology Transmission
IET	Industrial Equilibrium Time
IEAT	Income Expectation Averaging Time
IFPC	Indicated Food Per Capita
ILF	Inherent Land Fertility
IMEF	Industrial Materials Emission Factor
IMTI	Industrial Materials Toxicity Index
IO	Industrial Output
$\mathbf{IO_{70}}$	Industrial Output in 1970
IOPC	Industrial Output Per Capita
IOPCD	Industrial Output Per Capita Desired
ISOPC	Indicated Service Output Per Capita
IT	Initial Time
J	Job
JPH	Jobs Per Hectare
JPICU	Jobs Per Industrial Capital Unit
JPSCU	Jobs Per Service Capital Unit
LDR	Land Development Rate
LE	Life Expectancy
LEI	Life Expectancy Index
LER	Land Erosion Rate
LEN	Life Expectancy Normal
LF	Labor Force
LFC	Land Fraction Cultivated
LFD	Land Fertility Degradation

LFDR Land Fertility Degradation Rate

LFERT Land Fertility

LFERTI Initial Land Fertility

LFH Land Fraction Harvest

LFPF Labor Force Participation Fraction

LFR Land Fertility Regeneration

LFRT Land Fertility Regeneration Time

LLMY Land Life Multiplier from Yield

LLMYTM Land Life Multiplier from Yield switch Time

LMC Lifetime Multiplier from Crowding

LMF Lifetime Multiplier from Food

LMHS Lifetime Multiplier from Health Services

LMP Lifetime Multiplier from persistant Polution

LPD Lifetime Perception Delay

LRUI Land Removal from Urban-Industrial use

LUF Labor Utilization Fraction

LUFD Labor Utilization Fraction Delayed

LUFDI Initial Labor Utilization Fraction Delayed

LUFDT Labor Utilization Fraction Delay Time

LY Land Yield

LYCM Land Yield Change Multiplier

LYF Land Yield Factor

LYMAP Land Yield Multiplier from Air Pollution

LYMC Land Yield Multiplier from Capital

LYTD Land Yield Technology Development

LYTDR Land Yield Technology Development Rate

$\mathbf{M_1}$ Mortality rate in population aged from 0 to 14 years

$\mathbf{M_2}$ Mortality rate in population aged from 15 to 44 years

$\mathbf{M_3}$ Mortality rate in population aged from 45 to 64 years

$\mathbf{M_4}$ Mortality rate in population older than age 65

MAT_1 Maturation rate of 14 years old people

MAT_2 Maturation rate of 44 years old people

MAT_3 Maturation rate of 64 years old people

MLYMC Marginal Land Yield Multiplier from Capital

MPAI Marginal Productivity of Agricultural Inputs

MPLD Marginal Productivity of Land Development

MTF Maximum Total Fertility

MTFN Maximum Total Fertility Normal

NFC Need for Fertility Control

NR Non-renewable Resources

NRATE Non-renewable Resource Technology Improvement Rate

NRCM Non-renewable Resources Change Multiplier

NRFR Non-renewable Resources Fraction Remaining

NRI Initial Non-renewable Resources

NRTD Non-renewable Resources Technology Development

NRTDR Non-renewable Resources Technology Development Rate

NRUF Non-renewable Resources Usage Factor

NRUR Non-renewable Resources Usage Rate

OY One Year

P_1 Population aged from 0 to 14 years

P_{1I} Initial population aged from 0 to 14 years at the year 1900

P_2 Population aged from 15 to 44 years

P_{2I} Initial population aged from 15 to 44 years at the year 1900

P_3 Population aged from 45 to 64 years

P_{3I} Initial population aged from 45 to 64 years at the year 1900

P_4 Population older than age 65

P_{4I} Initial population older than age 65 at the year 1900

PAL Potentially Arable Land

PALI Initial Potentially Arable Land

PALT Potentially Arable Land Total

PCRUM Per Capita Resources Usage Multiplier

PET Population Equilibrium Time

PFR Perceived Food Ratio

PFRI Initial Perceived Food Ratio

PJAS Potential Jobs in Agricultural Sector

PJIS Potential Jobs in Industrial Sector

PJSS Potential Jobs in Service Sector

PL Processing Loss

PLE Perceived Life Expectancy

PLINID Pollution Intensity Indicator

POF Price Of Food

POLGFM Pollution Control Technology Change Multiplier

POP Total Population

PPAPR Persistent Pollution Appearance Rate

PPASR Persistent Pollution Assimilation Rate

PPGAO Persistent Pollution Generated by Agricultural Output

PPGF Persistent Pollution Generation Factor

PPGIO Persistent Pollution Generated by Industrial Output

PPGR Persistent Pollution Generation Rate

PPOL Persistent Pollution

$\mathbf{PPOL_{70}}$ Persistent Pollution in 1970

PPOLI Initial Persistent Pollution

PPOLX Index of Persistent Pollution

PPTD Persistent Pollution Transmission Delay

PTD Pollution Technology Development

PTDR Pollution Technology Development Rate

PYEAR New policy Year

RESINT Resource usage Intensity

RHGDP Real Highest Gross Domestic Product

RLGDP Real Lowest Gross Domestic Product

RLT Reproduction Lifetime

SAD Social Adjustment Delay

SC Service Capital

SCDR Service Capital Depreciation Rate

SCI Initial Service Capital

SCIR Service Capital Investment Rate

SCOR Service Capital-Output Rate

SD Social Discount

SFPC Subsistence Food Per Capita

SFSN Social Family Size Norm

SO Service Output

SOPC Service Output Per Capita

TAI Total Agricultural Investment

TDD Technology Development Delay

TF Total Fertility

TL Total Land

TS Time Step

UAGI Unit of Agricultural Inputs per Hectare

UIL Urban-Industrial Land

UILDT Urban-Industrial Land Development Time

ULGHA Urban Land in Gigahectare

UILI Initial Urban-Industrial Land

UILPC Urban-Industrial Land Per Capita

UILR Urban-Industrial Land Required

UP Unit of Person

ZPGT Zero Population Growth Time

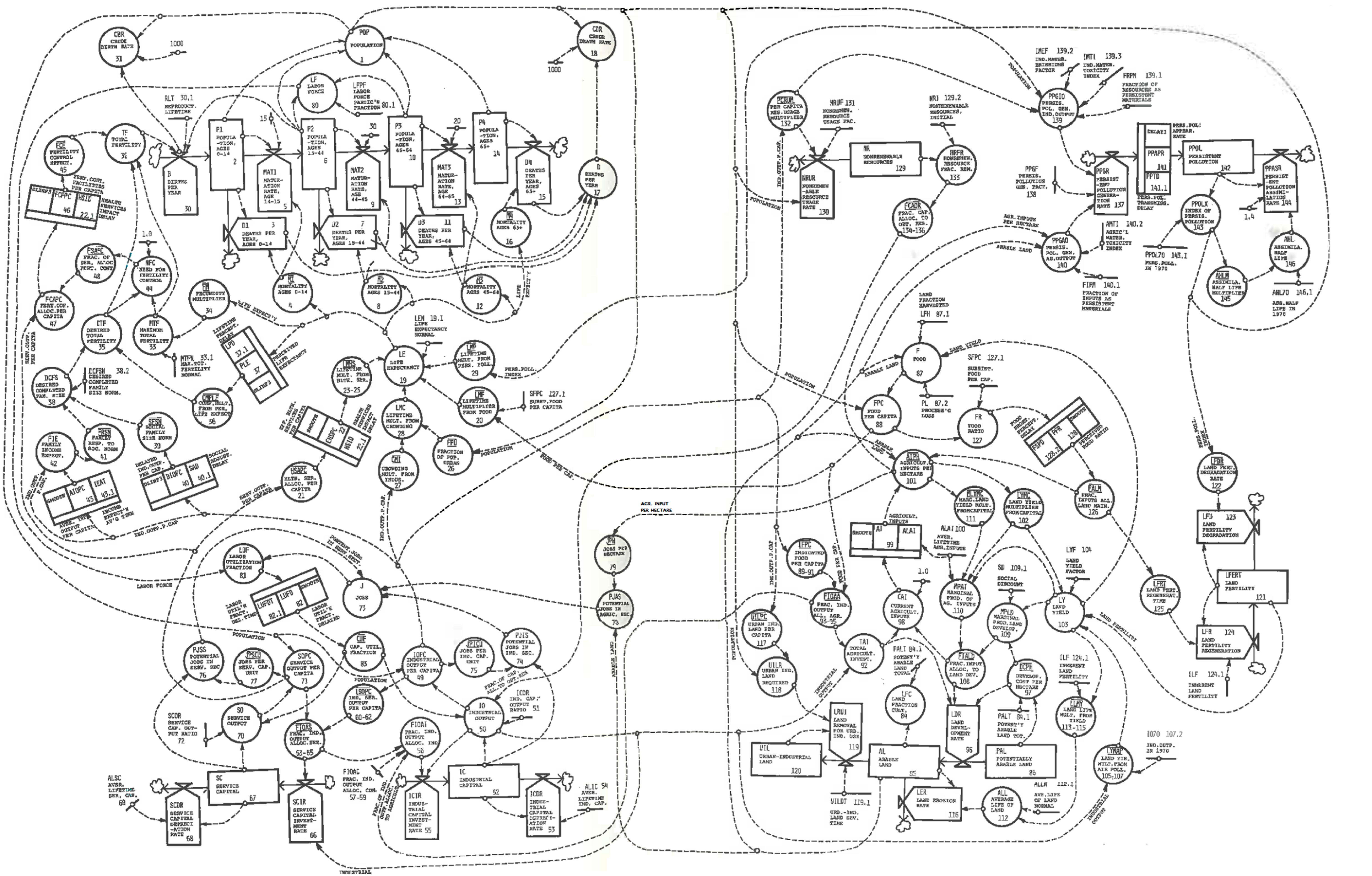

Figure 1: DYNAMO diagram of the 1972 World3 model