

MyWorld3: Equations and Explanations

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Introduction

1 History of World3

In 1968, a group of persons among heads of state and government, United Nation administrators, high-level politicians, scientists, economists, and business leaders from around the globe is formed, alarmed by the tendency mankind was taking. The international group created became known as "The Club Of Rome" after its first meeting took place in this location.

At the same time, Professor J.W. Forrester, a scientist expert in dynamic systems, is finishing its model on an industrial application of dynamic system using computer simulations. A few months after having proposed to design a model based on dynamic system which represent the world in order to observe the world-wide problem The Club Of Rome was worrying about, Forrester presents World2 to the organisation^[1].

Its approach seemed interesting to their eyes and so after having discussed about some components of the model which were approximately described and based on some strong assumptions, the new model World3 is created^[2]. Realised in cooperation with D.L. Meadows, it will be used in the "The Limits To Growth" to have a good based to explain the dynamic of different parameters. The Club Of Rome received criticisms from all around the world, because of their conclusions on the future of mankind which were alarming. After the model was upgraded and more data were collected, an update version of their book was published 30 years after the first one^[3].

2 This Study

This study aims at coding in Python the World3 model of 1972 and 2003 with its different scenario to observe its behavior. The objective is to do it in a way that everyone with basic knowledge in Python could modify it.^[4] This paper explains how this World3 model is implemented. It is based on previous explanations and implementations which have been realised^{[5] [6] [7] [8]}. The code is available [here](#)¹. It is under the licence CeCILL which is an adaptation of the GNU-GPL to the French laws. More information are available at [here](#)².

¹<https://github.com/Juji29/World3>

²<https://cecill.info/index.fr.html>

Equations of World3

3 Introduction

There is no real calibration of equations in order to make sure that values match with historical values. But the values have been inspected regarding historical tendency from 1900 to 1970. The choice of input have been made in order to obtain coherent curves with the model and historic data. The values given are valid for the scenario 1. To obtain the other scenario, you will have to change only the variables named in the part "Different scenario".

4 Initial conditions

Name	Type	Value	Dimension
IT	constant	1900	Year
FT	constant	2100	Year
TS	constant	0.5	Year
$PYEAR^{1972}$	constant	1975	Year
$PYEAR^{2003}$	constant	4000	Year

5 Units

Name	Type	Value	Dimension
OY	constant	1	Year
$UAGI$	constant	1	\$/({\text{Ha}\cdot\text{Year}})
UP	constant	1	Person
$GDPU$	constant	1	\$/({\text{Person}\cdot\text{Year}})

6 Population

6.1 Population dynamics

Constants

Name	Type	Value	Dimension
P_{1I}	initial value	650×10^6	Person
P_{2I}	initial value	700×10^6	Person
P_{3I}	initial value	190×10^6	Person
P_{4I}	initial value	60×10^6	Person

Equations

Total Population (POP) :

$$POP = P_1 + P_2 + P_3 + P_4 \quad (1)$$

Population aged from 0 to 14 years (\mathbf{P}_1) :

$$\frac{dP_1}{dt} = B - D_1 - MAT_1 \quad (2)$$

Deaths per year in population aged from 0 to 14 years (\mathbf{D}_1) :

$$D_1 = P_1 \times M_1 \quad (3)$$

Mortality rate in population aged from 0 to 14 years (\mathbf{M}_1) :

$\frac{LE}{OY}$	20	30	40	50	60	70	80
M_1	5.67×10^{-2}	3.66×10^{-2}	2.43×10^{-2}	1.55×10^{-2}	8.2×10^{-3}	2.3×10^{-3}	1×10^{-3}

(4)

Maturation rate of 14 years old people (\mathbf{MAT}_1) :

$$MAT_1 = \frac{P_1 \times (1 - M_1)}{15} \quad (5)$$

Population aged from 15 to 44 years (\mathbf{P}_2) :

$$\frac{dP_2}{dt} = MAT_1 - D_2 - MAT_2 \quad (6)$$

Deaths per year in population aged from 15 to 44 years (\mathbf{D}_2) :

$$D_2 = P_2 \times M_2 \quad (7)$$

Mortality rate in population aged from 15 to 44 years (\mathbf{M}_2) :

$\frac{LE}{OY}$	20	30	40	50	60	70	80
M_2	2.66×10^{-2}	1.71×10^{-2}	1.1×10^{-2}	6.5×10^{-3}	4×10^{-3}	1.6×10^{-3}	8×10^{-4}

(8)

Maturation rate of 44 years old people (\mathbf{MAT}_2) :

$$MAT_2 = \frac{P_2 \times (1 - M_2)}{30} \quad (9)$$

Population aged from 45 to 64 years (\mathbf{P}_3) :

$$\frac{dP_3}{dt} = MAT_2 - D_3 - MAT_3 \quad (10)$$

Deaths per year in population aged from 45 to 64 years (**D₃**) :

$$D_3 = P_3 \times M_3 \quad (11)$$

Mortality rate in population aged from 45 to 64 years (**M₃**) :

$\frac{LE}{OY}$	20	30	40	50	60	70	80
M_3	5.62×10^{-2}	3.73×10^{-2}	2.52×10^{-2}	1.71×10^{-2}	1.18×10^{-2}	8.3×10^{-3}	6×10^{-3}

(12)

Maturation rate of 64 years old people (**MAT₃**) :

$$MAT_3 = \frac{P_3 \times (1 - M_3)}{20} \quad (13)$$

Population older than age 65 (**P₄**) :

$$\frac{dP_4}{dt} = MAT_3 - D_4 \quad (14)$$

Deaths per year in population older than age 65 (**D₄**) :

$$D_4 = P_4 \times M_4 \quad (15)$$

Mortality rate in population older than age 65 (**M₄**) :

$\frac{LE}{OY}$	20	30	40	50	60	70	80
M_4	0.13	0.011	0.09	0.07	0.06	0.05	0.04

(16)

6.2 Death

Constants

Name	ID	Type	Value	Dimension
<i>LEN</i>		constant	28	Year
<i>HSID</i>		constant	20	Year
<i>EHSPCI</i>		initial value	0	\$/Year

Equations

Deaths per year (**D**) :

$$D = D_1 + D_2 + D_3 + D_4 \quad (17)$$

Crude Death Rate (**CDR**) :

$$CDR = \frac{1000 \times D}{POP} \quad (18)$$

Life Expectancy (**LE**) :

$$LE = LEN \times LMF \times LMHS \times LMP \times LMC \quad (19)$$

Lifetime Multiplier from Food (**LMF**) :

$\frac{FPC}{SFP\bar{C}}$	0	1	2	3	4	5
LMF^{1972}	0	1	1.2	1.3	1.35	1.4
LMF^{2003}	0	1	1.43	1.5	1.5	1.5

(20)

Health Services Allocations Per Capita (**HSAPC**) :

$\frac{SOPC}{GDP\bar{U}}$	0	250	500	750	1000	1250	1500	1750	2000
$HSAPC$	0	20	50	95	140	175	200	220	230

(21)

Effective Health Services Per Capita (**EHSPC**) :

$$\frac{dEHSPC}{dt} = \frac{HSAPC - EHSPC}{HSID} \quad (22)$$

Lifetime Multiplier from Health Services (**LMHS**) :

$$LMHS = \begin{cases} LMHS_2, & \text{if } TIME > 1940 \\ LMHS_1, & \text{else} \end{cases} \quad (23)$$

$\frac{EHSPC}{GDP\bar{U}}$	0	20	40	60	80	100
$LMHS_1$	1	1.1	1.4	1.6	1.7	1.8
$LMHS_2^{1972}$	1	1.4	1.6	1.8	1.95	2
$LMHS_2^{2003}$	1	1.5	1.9	2	2	2

(24)

Fraction of Population Urban (**FPU**) :

$\frac{POP}{UP}$	0	2×10^9	4×10^9	6×10^9	8×10^9	1×10^{10}	1.2×10^{10}	1.4×10^{10}	1.6×10^{10}
FPU	0	0.2	0.4	0.5	0.58	0.65	0.72	0.78	0.8

(25)

Crowding Multiplier from Industry (**CMI**) :

$\frac{IOPC}{GDP\bar{U}}$	0	200	400	600	800	1000	1200	1400	1600
CMI	0.5	0.05	-0.1	-0.08	-0.02	0.05	0.1	0.15	0.2

(26)

Lifetime Multiplier from Crowding (**LMC**) :

$$LMC = 1 - CMI \times FPU \quad (27)$$

Lifetime Multiplier from persistant Polution (**LMP**) :

$PPOLX$	0	10	20	30	40	50	60	70	80	90	100
LMP	1	0.99	0.97	0.95	0.9	0.85	0.75	0.65	0.55	0.4	0.2

(28)

6.3 Birth

Constants

Name	Type	Value	Dimension
RLT	constant	30	Year
PET	constant	4000	Year
$MTFN$	constant	12	Dmnl
LPD	constant	20	Year
$AIOPCI$	initial value	43.3	\$/Year
$ZPGT$	constant	4000	Year
$DCFSN^{1972}$	constant	4	Dmnl
$DCFSN^{2003}$	constant	3.8	Dmnl
SAD	constant	20	Year
$IEAT$	constant	3	Year
$FCEST$	constant	4000	Year

Equations

Births per year (**B**) :

$$B = \begin{cases} \frac{D}{2 \times RLT}, & \text{if } TIME \geq PET \\ \frac{TF \times P_2}{2 \times RLT}, & \text{else} \end{cases} \quad (29)$$

Crude Birth Rate (**CBR**) :

$$CBR = \frac{1000 \times B}{POP} \quad (30)$$

Total Fertility (**TF**) :

$$TF = \text{Min}(MTF, MTF \times (1 - FCE) + DTF \times FCE) \quad (31)$$

Maximum Total Fertility (**MTF**) :

$$MTF = MTFN \times FM \quad (32)$$

Fertility Multiplier (**FM**) :

$\frac{LE}{OY}$	0	10	20	30	40	50	60	70	80
FM^{1972}	0	0.2	0.4	0.6	0.8	0.9	1	1.05	1.1
FM^{2003}	0	0.2	0.4	0.6	0.7	0.75	0.79	0.84	0.87

Desired Total Fertility (**DTF**) :

$$DTF = DCFS \times CMPLE \quad (34)$$

Compensatory Multiplier from Perceived Life Expectancy (**CMPLE**) :

$\frac{PLE}{OY}$	0	10	20	30	40	50	60	70	80
$CMPLE$	3	2.1	1.6	1.4	1.3	1.2	1.1	1.05	1

Perceived Life Expectancy (**PLE**) :

$$PLE = \text{Delay3}(LE, LPD) \quad (36)$$

Desired Completed Family Size (**DCFS**) :

$$DCFS = \begin{cases} 2, & \text{if } TIME \geq ZPGT \\ DCFSN \times FRSN \times SFSN, & \text{else} \end{cases} \quad (37)$$

Social Family Size Norm (**SFSN**) :

$\frac{DIOPC}{GDPU}$	0	200	400	600	800
$SFSN^{1972}$	1.25	1	0.9	0.8	0.75
$SFSN^{2003}$	1.25	0.94	0.715	0.59	0.5

(38)

Delayed Industrial Output Per Capita (**DIOPC**) :

$$DIOPC = Delay3(IOPC, SAD) \quad (39)$$

Family Response to Social Norm (**FRSN**) :

FIE	-0.2	-0.1	0	0.1	0.2
$FRSN$	0.5	0.6	0.7	0.85	1

(40)

Family Income Expectation (**FIE**) :

$$FIE = \frac{IOPC - AIOPC}{AIOPC} \quad (41)$$

Average Industrial Output Per Capita (**AIOPC**) :

$$\frac{dAIOPC}{dt} = \frac{IOPC - AIOPC}{IEAT} \quad (42)$$

Need for Fertility Control (**NFC**) :

$$NFC = \frac{MTF}{DTF} - 1 \quad (43)$$

Fertility Control Effectiveness (**FCE**) :

$$FCE = \begin{cases} 1, & \text{if } TIME \geq FCEST \\ \begin{array}{|c|c|c|c|c|c|c|c|} \hline \frac{FCFPC}{GDPU} & 0 & 0.5 & 1 & 1.5 & 2 & 2.5 & 3 \\ \hline FCE & 0.75 & 0.85 & 0.9 & 0.95 & 0.98 & 0.99 & 1 \\ \hline \end{array}, & \text{else} \end{cases} \quad (44)$$

Fertility Control Facilities Per Capita (**FCFPC**) :

$$FCFPC = Delay3(FCAPC, HSID) \quad (45)$$

Fertility Control Allocations Per Capita (**FCAPC**) :

$$FCAPC = FSAFC \times SOPC \quad (46)$$

Fraction of Services Allocated to Fertility Control (**FSAFC**) :

NFC	0	2	4	6	8	10
$FSAFC$	0	0.005	0.015	0.025	0.03	0.035

(47)

7 Capital

7.1 Industrial

Constants

Name	Type	Value	Dimension
$ICOR_1$	constant	3	Year
$ICOR_2^{1972}$	constant	3	Year
ICI	initial value	2.1×10^{11}	\$
$ALIC_1$	constant	14	Year
$ALIC_2$	constant	14	Year
IET	constant	4000	Year
$FIOAC_1$	constant	0.43	Dmnl
$FIOAC_2$	constant	0.43	Dmnl
$IOPCD$	constant	400	\$/(\text{Person}\cdot\text{Year})

Equations

Industrial Output Per Capita (**IOPC**) :

$$IOPC = \frac{IO}{POP} \quad (48)$$

Industrial Output (**IO**) :

$$IO = \frac{IC \times (1 - FCAOR) \times CUF}{ICOR} \quad (49)$$

Industrial Capital-Output Ratio (**ICOR**) :

$$ICOR = \begin{cases} ICOR_2, & \text{if } TIME \geq PYEAR \\ ICOR_1, & \text{else} \end{cases} \quad (50)$$

Industrial Capital (**IC**) :

$$\frac{dIC}{dt} = ICIR - ICDR \quad (51)$$

Industrial Capital Depreciation Rate (**ICDR**) :

$$ICDR = \frac{IC}{ALIC} \quad (52)$$

Average Lifetime of Industrial Capital (**ALIC**) :

$$ALIC = \begin{cases} ALIC_2, & \text{if } TIME \geq PYEAR \\ ALIC_1, & \text{else} \end{cases} \quad (53)$$

Industrial Capital Investment Rate (**ICIR**) :

$$ICIR = IO \times FIAOI \quad (54)$$

Fraction of Industrial Output Allocated to Industry (**FIOAI**) :

$$FIOAI = 1 - FIAOA - FIOAS - FIOAC \quad (55)$$

Fraction of Industrial Output Allocated to Consumption (**FIOAC**) :

$$FIOAC = \begin{cases} FIOACV, & \text{if } TIME \geq IET \\ FIOACC, & \text{else} \end{cases} \quad (56)$$

Fraction of Industrial Output Allocated to Consumption Constant (**FIOACC**) :

$$FIOACC = \begin{cases} FIOAC_2, & \text{if } TIME \geq PYEAR \\ FIOAC_1, & \text{else} \end{cases} \quad (57)$$

Fraction of Industrial Output Allocated to Consumption Variable (**FIOACV**) :

<i>IOPC</i>	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
<i>IOPCD</i>	0.3	0.32	0.34	0.36	0.38	0.43	0.73	0.77	0.81	0.82	0.83

(58)

In addition for the version 2003:

ICOR₂²⁰⁰³ :

$$ICOR_2^{2003} = ICOR_{2T} \times COYM \times COPM \quad (59)$$

7.2 Services

Constants

Name	Type	Value	Dimension
<i>SCI</i>	initial value	1.44×10^{11}	\$
<i>ALSC</i> ₁	constant	20	Year
<i>ALSC</i> ₂	constant	20	Year
<i>SCOR</i> ₁	constant	1	Year
<i>SCOR</i> ₂	constant	1	Year

Equations

Indicated Service Output Per Capita (**ISOPC**) :

$$ISOPC = \begin{cases} ISOPC_2, & \text{if } TIME \geq PYEAR \\ ISOPC_1, & \text{else} \end{cases} \quad (60)$$

<i>IOPC</i> <i>GDP</i> _U	0	200	400	600	800	1 000	1 200	1 400	1 600
<i>ISOPC</i> ₁	40	300	640	1 000	1 220	1 450	1 650	1 800	2 000
<i>ISOPC</i> ₂	40	300	640	1 000	1 220	1 450	1 650	1 800	2 000

(61)

Fraction of Industrial Output Allocated to Services (**FIOAS**) :

$$FIOAS = \begin{cases} FIOAS_2, & \text{if } TIME \geq PYEAR \\ FIAOS_1, & \text{else} \end{cases} \quad (62)$$

$\frac{SOPC}{ISOPC}$	0	0.5	1	1.5	2	(63)
$FIOAS_1$	0.3	0.2	0.1	0.05	0	
$FIOAS_2$	0.3	0.2	0.1	0.05	0	

Service Capital Investment Rate (**SCIR**) :

$$SCIR = IO \times FIOAS \quad (64)$$

Service Capital (**SC**) :

$$\frac{dSC}{dt} = SCIR - SCDR \quad (65)$$

Service Capital Depreciation Rate (**SCDR**) :

$$SCDR = \frac{SC}{ALSC} \quad (66)$$

Average Lifetime of Service Capital (**ALSC**) :

$$ALSC = \begin{cases} ALSC_2, & \text{if } TIME \geq PYEAR \\ ALSC_1, & \text{else} \end{cases} \quad (67)$$

Service Output (**SO**) :

$$SO = \frac{SC \times CUF}{SCOR} \quad (68)$$

Service Output Per Capita (**SOPC**) :

$$SOPC = \frac{SO}{POP} \quad (69)$$

Service Capital-Output Rate (**SCOR**) :

$$SCOR = \begin{cases} SCOR_2, & \text{if } TIME \geq PYEAR \\ SCOR_1, & \text{else} \end{cases} \quad (70)$$

7.3 Job

Constants

Name	Type	Value	Dimension
$LFPF$	constant	0.75	Dmnl
$LUFDT$	constant	2	Year
$LUFDI$	initial value	1	Dmnl

Equations

Job (**J**) :

$$J = PJIS + PJAS + PJSS \quad (71)$$

Potential Jobs in Industrial Sector (**PJIS**) :

$$PJIS = IC \times JPICU \quad (72)$$

Jobs Per Industrial Capital Unit (**JPICU**) :

$\frac{IOPC}{GDP_U}$	50	200	350	500	650	800
$JPICU$	3.7×10^{-4}	1.8×10^{-4}	1.2×10^{-4}	9×10^{-5}	7×10^{-5}	6×10^{-5}

(73)

Potential Jobs in Service Sector (**PJSS**) :

$$PJSS = SC \times JPSCU \quad (74)$$

Jobs Per Service Capital Unit (**JPSCU**) :

$\frac{SOPC}{GDP_U}$	50	200	350	500	650	800
$JPSCU$	1.1×10^{-3}	6×10^{-4}	3.5×10^{-4}	2×10^{-4}	1.5×10^{-4}	1.5×10^{-4}

(75)

Potential Jobs in Agricultural Sector (**PJAS**) :

$$PJAS = JPH \times AL \quad (76)$$

Jobs Per Hectare (**JPH**) :

$\frac{AIPH}{UAGI}$	2	6	10	14	18	22	26	30
JPH	2	0.5	0.4	0.3	0.27	0.24	0.2	0.2

(77)

Labor Force (**LF**) :

$$LF = (P_2 + P_3) \times LFPF \quad (78)$$

Labor Utilization Fraction (**LUF**) :

$$LUF = \frac{J}{LF} \quad (79)$$

Labor Utilization Fraction Delayed (**LUF_{FD}**) :

$$\frac{dLUFD}{dt} = \frac{LF - LUFD}{LUFDT} \quad (80)$$

Capital Utilization Fraction (**CUF**) :

$LUFD$	1	3	5	7	9	11
CUF	1	0.9	0.7	0.3	0.1	0.1

(81)

8 Agriculture

8.1 Loop 1

Constants

Name	Type	Value	Dimension
$PALT$	constant	3.2×10^9	Year
ALI	initial value	9×10^8	Year
$PALI$	initial value	2.3×10^9	\$
LFH	constant	0.7	Dmnl
PL	constant	0.1	Dmnl

Equations

Land Fraction Cultivated (**LFC**) :

$$LFC = \frac{AL}{PALT} \quad (82)$$

Arable Land (**AL**) :

$$\frac{dAL}{dt} = LDR - LER - LRUI \quad (83)$$

Potentially Arable Land (**PAL**) :

$$\frac{dPAL}{dt} = -LDR \quad (84)$$

Food (**F**) :

$$F = LY \times AL \times LFH \times (1 - PL) \quad (85)$$

Fraction Per Capita (**FPC**) :

$$FPC = \frac{F}{POP} \quad (86)$$

Indicated Food Per Capita (**IFPC**) :

$$IFPC = \begin{cases} IFPC_2, & \text{if } TIME \geq PYEAR \\ IFPC_1, & \text{else} \end{cases} \quad (87)$$

$\frac{IOPC}{GDPU}$	0	200	400	600	800	1 000	1 200	1 400	1 600
$IFPC_1$	230	480	690	850	970	1 070	1 150	1 210	1 250
$IFPC_2$	230	480	690	850	970	1 070	1 150	1 210	1 250

Total Agricultural Investment (**TAI**) :

$$TAI = \frac{IO}{FIOAA} \quad (88)$$

Fraction of Industrial Output Allocated to Agriculture (**FIOAA**) :

$$FIOAA = \begin{cases} FIOAA_2, & \text{if } TIME \geq PYEAR \\ FIOAA_1, & \text{else} \end{cases} \quad (90)$$

$\frac{FPC}{IFPC}$	0	0.5	1	1.5	2	2.5
$FIOAA_1$	0.4	0.2	0.1	0.025	0	0
$FIOAA_2$	0.4	0.2	0.1	0.025	0	0

Land Development Rate (**LDR**) :

$$LDR = \frac{TAI \times FIALD}{DCPH} \quad (92)$$

Development Cost Per Hectare (**DCPH**) :

$\frac{PAL}{PALT}$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$DCPH$	100 000	7 400	5 200	3 500	2 400	1 500	750	300	150	75	50

8.2 Loop 2

Constants

Name	Type	Value	Dimension
AII	initial value	5×10^9	\$/Year
$ALAI_1$	constant	2	Year
$ALAI_2$	constant	2	Year
LYF_1	constant	1	Dmnl
LYF_2^{1972}	constant	1	Dmnl
IO_{70}	constant	7.9×10^{11}	\$/Year
SD	constant	0.07	$Year^{-1}$

Equations

Current Agricultural Inputs (**CAI**) :

$$CAI = TAI \times (1 - FIALD) \quad (94)$$

Agricultural Inputs (**AI**) :

$$\frac{dAI}{dt} = \frac{CAI - AI}{ALAI} \quad (95)$$

Average Lifetime of Agricultural Inputs (**ALAI**) :

$$ALAI = \begin{cases} ALAI_2, & \text{if } TIME \geq PYEAR \\ ALAI_1, & \text{else} \end{cases} \quad (96)$$

Agricultural Inputs Per Hectare (**AIPH**) :

$$AIPH = \frac{AI \times (1 - FALM)}{AL} \quad (97)$$

Land Yield Multiplier from Capital (**LYMC**) :

$\frac{AIPH}{UAGI}$	0	40	80	120	160	200	240	280	320	360	400	440	480
$LYMC^{1972}$	1	3	3.8	4.4	4.9	5.4	5.7	6	6.3	6.6	6.9	7.2	7.4
$LYMC^{2003}$	1	3	4.5	5	5.3	5.6	5.9	6.1	6.35	6.6	6.9	7.2	7.4
$\frac{AIPH}{UAGI}$	520	560	600	640	680	720	760	800	840	880	920	960	1000
$LYMC^{1972}$	7.6	7.8	8	8.2	8.4	8.6	8.8	9	9.2	9.4	9.6	9.8	10
$LYMC^{2003}$	7.6	7.8	8	8.2	8.4	8.6	8.8	9	9.2	9.4	9.6	9.8	10

(98)

Land Yield (**LY**) :

$$LY = LYF \times LFERT \times LYMC \times LYMAP \quad (99)$$

Land Yield Factor (**LYF**) :

$$LYF = \begin{cases} LYF_2, & \text{if } TIME \geq PYEAR \\ LYF_1, & \text{else} \end{cases} \quad (100)$$

Land Yield Multiplier from Air Pollution (**LYMAP**) :

$$LYMAP = \begin{cases} LYMAP_2, & \text{if } TIME \geq PYEAR \\ LYMAP_1, & \text{else} \end{cases} \quad (101)$$

$\frac{IO}{IO_{70}}$	0	10	20	30	
$LYMAP_1$	1	1	0.7	0.4	
$LYMAP_2^{1972}$	1	1	0.7	0.4	
$LYMAP_2^{2003}$	1	1	0.98	0.95	

(102)

Fraction of Inputs Allocated to Land Development (**FIALD**) :

$\frac{MPLD}{MPAI}$	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2
FIALD	0	0.05	0.15	0.3	0.5	0.7	0.85	0.95	1

(103)

Marginal Productivity of Land Development (**MPLD**) :

$$MPLD = \frac{LY}{DCPH \times SD} \quad (104)$$

Marginal Productivity of Agricultural Inputs (**MPAI**) :

$$MPAI = \frac{ALAI \times LY \times MLYMC}{LYMC} \quad (105)$$

Marginal Land Yield Multiplier from Capital (**MLYMC**) :

$\frac{AIPH}{UAGI}$	0	40	80	120	160	200	240	280	
FIALD	0.075	0.03	0.015	0.011	0.009	0.007	0.006	0.005	
$\frac{AIPH}{UAGI}$	320	360	400	440	480	520	560	600	
FIALD	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	

(106)

In addition for the version 2003:

Name	Type	Value	Dimension
<i>TDD</i>	constant	20	Year

LYF₂²⁰⁰³ :

$$LYF_2^{2003} = Delay3(LYTD, TDD) \quad (107)$$

8.3 Loop 3

Constants

Name	Type	Value	Dimension
<i>ALLN</i> ¹⁹⁷²	constant	6 000	Year
<i>ALLN</i> ²⁰⁰³	constant	1 000	Year
<i>UILDT</i>	constant	10	Year
<i>UILI</i>	initial value	8.2×10^6	Ha
<i>LLMYTM</i>	constant	4000	Year

Equations

Average Life of Land (**ALL**) :

$$ALL = ALLN \times LLMY \quad (108)$$

Land Life Multiplier from Yield (**LLMY**) :

$$LLMY = \begin{cases} 0.95^{\frac{TIME-LLMYTM}{OY}} LLMY_1 + (1 - 0.95^{\frac{TIME-LLMYTM}{OY}}) LLMY_2, & \text{if } TIME \geq LLMYTM \\ LLMY_1 & \text{else} \end{cases}, \quad (109)$$

$\frac{LY}{ILF}$	0	1	2	3	4	5	6	7	8	9
$LLMY_1$	1.2	1	0.63	0.36	0.16	0.055	0.04	0.025	0.015	0.01
$LLMY_2^{1972}$	1.2	1	0.63	0.36	0.16	0.055	0.04	0.025	0.015	0.01
$LLMY_2^{2003}$	1.2	1	0.63	0.36	0.29	0.26	0.24	0.22	0.21	0.2

Land Erosion Rate (**LER**) :

$$LER = \frac{AL}{ALL} \quad (111)$$

Urban-Industrial Land Per Capita (**UILPC**) :

$\frac{IOPC}{GDPU}$	0	200	400	600	800	1 000	1 200	1 400	1 600
$UILPC$	0.005	0.008	0.015	0.025	0.04	0.055	0.07	0.08	0.09

Urban-Industrial Land Required (**UILR**) :

$$UILR = UILPC \times POP \quad (113)$$

Land Removal from Urban-Industrial use (**LRUI**) :

$$LRUI = \frac{\text{Max}(0, UILR - UIL)}{UILDT} \quad (114)$$

Urban-Industrial Land (**UIL**) :

$$\frac{dUIL}{dt} = LRUI \quad (115)$$

8.4 Loop 4

Constants

Name	Type	Value	Dimension
$LFERTI$	constant	600	Veg eq kg/(Year·Ha)

Equations

Land Fertility (**LFERT**) :

$$\frac{dLFERT}{dt} = LFR - LFD \quad (116)$$

Land Fertility Degradation Rate (**LFDR**) :

<i>PPOLX</i>	0	10	20	30
<i>LFDR</i>	0	0.1	0.3	0.5

(117)

Land Fertility Degradation (**LFD**) :

$$LFD = LFERT \times LFDR \quad (118)$$

8.5 Loop 5

Constants

Name	Type	Value	Dimension
<i>ILF</i>	constant	600	Veg eq kg/(Year·Ha)
<i>SFPC</i>	constant	230	Veg eq kg/(Year·Ha)
<i>PFRI</i>	initial value	1	Dmnl
<i>FSDP</i>	constant	2	Year

Equations

Land Fertility Regeneration (**LFR**) :

$$LFR = \frac{ILF - LFERT}{LFRT} \quad (119)$$

Land Fertility Regeneration Time (**LFRT**) :

<i>FALM</i>	0	0.02	0.04	0.06	0.08	0.1
<i>LFRT</i>	20	13	8	4	2	1.2

(120)

Fraction of inputs Allocated to Land Maintenance (**FALM**) :

<i>PFR</i>	0	1	2	3	4
<i>FALM</i>	0	0.04	0.07	0.09	0.1

(121)

Food Ratio (**FR**) :

$$FR = \frac{FPC}{SFPC} \quad (122)$$

Perceived Food Ratio (**PFR**) :

$$\frac{dPFR}{dt} = \frac{FR - PFR}{FSDP} \quad (123)$$

In addition for the version 2003:

Name	Type	Value	Dimension
DFR	constant	2	Dmnl

Land Yield Technology Development (**LYTD**) :

$$\frac{dLYTD}{dt} = LYTDR \quad (124)$$

Land Yield Technology Development Rate (**LYTDR**) :

$$LYTDR = \begin{cases} LYTD \times LYCM, & \text{if } TIME \geq PYEAR \\ 0 & \text{else} \end{cases} \quad (125)$$

Land Yield Change Multiplier (**LYCM**) :

$DFR - FR$	0	1
$LYCM$	0	0

Capital Output Yield Multiplier (**COYM**) :

LYF	1	1.2	1.4	1.6	1.8	2
$COYM$	1	1.05	1.12	1.25	1.35	1.5

9 Resources

Constants

Name	Type	Value	Dimension
NRI	initial value	10^{12}	Resource Units
$NRUF_1$	constant	1	Dmnl
$NRUF_2^{1972}$	constant	1	Dmnl
$FACORTM$	constant	4000	Year

Equations

Non-renewable Resources (**NR**) :

$$\frac{dNR}{dt} = -NRUR \quad (128)$$

Non-renewable Resources Usage Rate (**NRUR**) :

$$NRUR = POP \times PCRUM \times NRUF \quad (129)$$

Non-renewable Resources Usage Factor (**NRUF**) :

$$NRUF = \begin{cases} NRUF_2, & \text{if } TIME \geq PYEAR \\ NRUF_1, & \text{else} \end{cases} \quad (130)$$

Per Capita Resources Usage Multiplier (**PCRUM**) :

$\frac{IOPC}{GDPU}$	0	200	400	600	800	1 000	1 200	1 400	1 600	
PCRUM	0	0.85	2.6	3.4	3.8	4.1	4.4	4.7	5	

(131)

Non-renewable Resources Fraction Remaining (**NRFR**) :

$$NRFR = \frac{NR}{NRI} \quad (132)$$

Fertility Control Allocated to Obtaining Resources (**FCAOR**) :

$$FCAOR = \begin{cases} FCAOR_2, & \text{if } TIME \geq FCAORTM \\ FCAOR_1, & \text{else} \end{cases} \quad (133)$$

NRFR	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
FCAOR₁	1	0.9	0.7	0.5	0.2	0.1	0.05	0.05	0.05	0.05	0.05
FCAOR₂¹⁹⁷²	1	0.9	0.7	0.5	0.2	0.1	0.05	0.05	0.05	0.05	0.05
FCAOR₂²⁰⁰³	1	0.2	0.1	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05

(134)

In addition for the version 2003:

Name	Type	Value	Dimension
<i>DNRUR</i>	constant	4 000	Year

NRUF₂²⁰⁰³ :

$$NRUF_2^{2003} = Delay3(NRTD, TDD) \quad (135)$$

Non-renewable Resources Technology Development (**NRTD**) :

$$\frac{dNRTD}{dt} = NRATE \quad (136)$$

Non-renewable Resource Technology Improvement Rate (**NRATE**) :

$$NRATE = \begin{cases} NRTD \times NRCM, & \text{if } TIME \geq PYEAR \\ 0 & \text{else} \end{cases} \quad (137)$$

Non-renewable Resources Change Multiplier (**NRCM**) :

$1 - \frac{NRUF}{DNRUR}$	-1	0
NRCM	0	0

(138)

Industrial Capital-Output Ratio Technology Transmission (**ICOR2T**) :

NRUF	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
ICOR2T	3.75	3.6	3.47	3.36	3.25	3.16	3.1	3.06	3.02	3.01	3

(139)

10 Pollution

Constants

Name	Type	Value	Dimension
$PPGF_1$	constant	1	Dmnl
$PPGF_2^{1972}$	constant	1	Dmnl
$FRPM$	constant	0.02	Dmnl
$IMEF$	constant	0.1	Dmnl
$IMTI$	constant	10	Pollution Unit/Resource Unit
$FIPM$	constant	0.001	Dmnl
$AMTI$	constant	1	Pollution Unit/\$
$PPTD$	constant	20	Year
$PPOLI$	initial value	2.5×10^7	Pollution Unit
$PPOL_{70}$	constant	1.36×10^8	Pollution Unit
AHL_{70}	constant	1.5	Year

Equations

Persistent Pollution Generation Rate (**PPGR**) :

$$PPGR = (PPGIO + PPGAO) \times PPGF \quad (140)$$

Persistent Pollution Generation Factor (**PPGF**) :

$$PPGF = \begin{cases} PPGF_2, & \text{if } TIME \geq PYEAR \\ PPGF_1, & \text{else} \end{cases} \quad (141)$$

Persistent Pollution Generated by Industrial Output (**PPGIO**) :

$$PPGIO = PCRUM \times POP \times FRPM \times IMEF \times IMTI \quad (142)$$

Persistent Pollution Generated by Agricultural Output (**PPGAO**) :

$$PPGAO = AIPH \times AL \times FIPM \times AMTI \quad (143)$$

Persistent Pollution Appearance Rate (**PPAPR**) :

$$PPAPR = Delay3(PPGR, PPTD) \quad (144)$$

Persistent Pollution (**PPOL**) :

$$\frac{dPPOL}{dt} = PPAPR - PPASR \quad (145)$$

Index of Persistent Pollution (**PPOLX**) :

$$PPOLX = \frac{PPOL}{PPOL_{70}} \quad (146)$$

Persistent Pollution Assimilation Rate (**PPASR**) :

$$PPASR = \frac{PPOL}{1.4 \times AHL} \quad (147)$$

Assimilation Half-Life Multiplier (**AHLM**) :

<i>PPOLX</i>	1	251	501	751	1 001
<i>AHLM</i>	1	11	21	31	41

(148)

Assimilation Half-Life (**AHL**) :

$$AHL = AHL_{70} \times AHLM \quad (149)$$

In addition for the version 2003:

Name	Type	Value	Dimension
<i>DPOLEX</i>	constant	1.2	Dmnl

PPGF2²⁰⁰³ :

$$PPGF_2^{2003} = Delay3(PTD, TDD) \quad (150)$$

Pollution Technology Development (**PTD**) :

$$\frac{dPTD}{dt} = PTDR \quad (151)$$

Pollution Technology Development Rate (**PTDR**) :

$$PTDR = \begin{cases} PTD \times POLGFM, & \text{if } TIME \geq PYEAR \\ 0 & \text{else} \end{cases} \quad (152)$$

Pollution Control Technology Change Multiplier (**POLGFM**) :

$1 - \frac{PPOLX}{DPOLEX}$	-1	0
<i>POLGFM</i>	0	0

(153)

Capital Output Pollution Multiplier (**COPM**) :

<i>PPGF</i>	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
<i>COPM</i>	1.25	1.2	1.15	1.11	1.08	1.05	1.03	1.02	1.01	1	1

(154)

11 Indexes

11.1 Distribution of outputs between the different sector Constants

Name	Type	Value	Dimension
<i>POF</i>	constant	0.22	\$/Veg eq kg

Equations

Fraction of Output in Agriculture (**FOA**) :

$$FOA = \frac{POF \times F}{POF \times F + SO + IO} \quad (155)$$

Fraction Of Output in Industry (**FOI**) :

$$FOI = \frac{IO}{POF \times F + SO + IO} \quad (156)$$

Fraction Of Output in Service (**FOS**) :

$$FOS = \frac{S}{POF \times F + SO + IO} \quad (157)$$

11.2 Industrial outputs indexes

Equations

Resource usage Intensity (**RESINT**) :

$$RESINT = \frac{NRUR}{IO} \quad (158)$$

Pollution Intensity Indicator (**PLINID**) :

$$PLINID = \frac{PPGIO \times PPGF}{IO} \quad (159)$$

Consumed Industrial Output (**CIO**) :

$$CIO = IO \times FIOAC \quad (160)$$

Consumed Industrial Output Per Capita (**CIOPC**) :

$$CIOPC = \frac{CIO}{POP} \quad (161)$$

11.3 Human Welfare Index

Constants

Name	Type	Value	Dimension
<i>RHGDP</i>	constant	9 508	\$/(Year·Person)
<i>RLGDP</i>	constant	24	\$/(Year·Person)

Equations

Human Welfare Index (**HWI**) :

$$HWI = \frac{LEI + EI + GDPI}{3} \quad (162)$$

Life Expectancy Index (**LEI**) :

$\frac{LE}{OY}$	25	35	45	55	65	75	85
LEI	0	0.16	0.33	0.5	0.67	0.84	1

(163)

Education Index (**EI**) :

$\frac{GDPPC}{GDPU}$	0	1 000	2 000	3 000	4 000	5 000	6 000	7 000
EI	0	0.81	0.88	0.92	0.95	0.98	0.99	1

(164)

Gross Domestic Product Index (**GDPI**) :

$$GDPI = \frac{\log_{10}(\frac{GDPPC}{RLGDP})}{\log_{10}(\frac{RHGDP}{RLGDP})} \quad (165)$$

Gross Domestic Product Per Capita (**GDPPC**) :

$\frac{IOPC}{GDPU}$	0	200	400	600	800	1 000
$GDPPC$	120	600	1 200	1 800	2 500	3 200

(166)

11.4 Human Ecological Footprint

Constants

Name	Type	Value	Dimension
HUP	constant	4	Ha/(Pollution Unit/Year)
TL	constant	1.91	GHa
$HGHA$	constant	10^9	GHa/Ha

Equations

Human Ecological Footprint (**HEF**) :

$$HEF = \frac{ALGGHA + ULGHA + ALGHA}{TL} \quad (167)$$

Absorption Land in Gigahectare (**ALGHA**) :

$$ALGHA = \frac{PPGR \times HUP}{GHAH} \quad (168)$$

Arable Land in Gigahectare (**ALGGHA**) :

$$ALGGHA = \frac{AL}{GHAH} \quad (169)$$

Urban Land in Gigahectare (**ULGHA**) :

$$ULGHA = \frac{UIL}{GHAH} \quad (170)$$

12 Explanations on function *Delay3*

Delay3 :

$$\begin{aligned}
 Y &= \text{Delay3}(VAL, DT) \\
 &\Leftrightarrow \\
 DL &= \frac{DT}{3} \\
 RT1 &= I1_{t-1}/DL \\
 I1_t &= I1_{t-1} + (VAL_t - RT1) \times TS \text{ avec } I1_0 = DL \times VAL_0 \\
 RT2 &= I2_{t-1}/DL \\
 I2_t &= I2_{t-1} + (RT1 - RT2) \times TS \text{ avec } I2_0 = DL \times VAL_0 \\
 I3_t &= I3_{t-1} + (RT2 - \frac{I3_{t-1}}{DL}) \times TS \text{ avec } I3_0 = DL \times VAL_0 \\
 Y_t &= I3_t/DL
 \end{aligned} \tag{171}$$

13 Different scenario

In "Limits to Growth: The 30-Year Update", different scenario are given starting from the standard run described above. Here are the changes to make in order to obtain the different scenario:

- Scenario 2: This scenario starts with the same assumptions as Scenario 1.

$$NRI = 2 \times 10^{-9}$$

$$FCAORTM = 2002$$

$NRFR$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$FCAOR_2^{2003}$	1	0.1	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05

- Scenario 3: This scenario starts with the same assumptions as Scenario 2.

$$PYEAR = 2002$$

$1 - \frac{PPOLX}{DPOLX}$	-1	0
$POLGFM$	-0.04	0

- Scenario 4: This scenario starts with the same assumptions as Scenario 3.

$DFR - FR$	0	1
$LYCM$	0	0.04

- Scenario 5: This scenario starts with the same assumptions as Scenario 4.

$$LLMYTM = 2002$$

- Scenario 6: This scenario starts with the same assumptions as Scenario 5.

$1 - \frac{NRUR}{DNRUR}$	-1	0
$NRCM$	-0.04	0

- Scenario 7: This scenario starts with the same assumptions as Scenario 2.

$$FCEST = 2002$$

$$ZPGT = 2002$$

- Scenario 8: This scenario starts with the same assumptions as Scenario 7.

$$IOPCD = 350$$

$$ALAI_2 = 2.5$$

$$ALIC_2 = 18$$

$$ALSC_2 = 25$$

$$IET = 2002$$

$$PYEAR = 2002$$

- Scenario 9: This scenario starts with the same assumptions as Scenario 8.

$$LLMYTM = 2002$$

$1 - \frac{PPOLX}{DPOLX}$	-1	0
$POLGFM$	-0.04	0
$DFR - FR$	0	1
$LYCM$	0	0.04
$1 - \frac{NRUR}{DNRUR}$	-1	0
$NRCM$	-0.04	0

- Scenario 10: This scenario starts with the same assumptions as Scenario 9.

$$FCAORTM = 1982$$

$$FCEST = 1982$$

$$IET = 1982$$

$$LLMYTM = 1982$$

$$PYEAR = 1982$$

$$ZPGT = 1982$$

- Scenario 11: This scenario starts with the same assumptions as Scenario 10.

$$FCAORTM = 2012$$

$$FCEST = 2012$$

$$IET = 2012$$

$$LLMYTM = 2012$$

$$PYEAR = 2012$$

$$ZPGT = 2012$$

Implementation

In this part, it will be explain how the code given in introduction works and how you can modify it in order to experiment other scenario.

The code is composed of 4 files:

- `world3_dynamic.py` which concerns the resolution of the graph
- `world3_graph.py` which concerns the creation of the graph
- `world3_plot.py` which concerns posting of the results
- `world3_run.py` which concerns the execution of the code

14 World3_dynamic.py

First, a general class `Node` is created in order to have a base for all nodes. Each node will have for attributes: a name, a value, an associated function, predecessors, successors, a rank and a graph to which it is linked.

Then, four types of node are defined: `NodeStock`, `NodeFlow`, `NodeDelay3` and `NodeConstant`.

- A `NodeStock` is a node which is initialized with a value and at each step of time, its variation is calculated. So it will be integrated after all the other nodes have been computed.
- A `NodeFlow` is a node which is calculated with the value of `NodeStock`, `NodeConstant` or other `NodeFlows`. Its value is calculated and not its variation.
- A `NodeDelay3` is a node which has the same characteristics as a `NodeFlow` except the fact that it takes into account the value it had at the step $n-1$ and $n-2$. So the value of the step n is smoothed with its previous value. In order to do that, it has for attributes the constant which is the delta of time, the node used to obtain the value of the current step and finally $I1$, $I2$ and $I3$ to compute the smoothed value. The way they are computed is given in the section 12.
- A `NodeConstant` is a node which is constant during all the execution of the program. There are two types of `NodeConstant`: constant and tables. Constants have a single value where tables are used to define piece-wise linear functions. They are used instead of equation for some nodes. Their implementation is made with a list of couple (x, y) where x is the input and y the output.

The class `Hypergraph` represents the entire graph which is composed of different nodes described above. It takes as parameters the version of `World3` desired and eventually nodes already created. Its attributes are a dictionary of its nodes(`nodes`), its version(`version`), the number maximum of rank(`nrank`), a list of nodes ordered by their rank(`nodesrank`) and a list of its `NodeStocks`(`stocks`). Several methods are implemented so each of them are explained below:

- `add_node(self, node)`: add node to the graph.
- `add_nodes(self, nodes)`: add all nodes given in the list nodes to the graph.
- `add_edge(self, f, x_target, x_s)`: create an edge between `x_target` and `x_s` described by the function `f`.
- `eval(self, ts)`: compute the value of all nodes in the `nodesrank`.
- `run(self, it, ft, ts)`: start the execution of the code by computing values for all time steps.
- `sub_graph_vertex(self, cond)`: return the list of name of nodes respecting the condition `cond` as well as a list of the rank of their predecessors and another with the rank of their successors
- `set_rank(self)`: sort a list of nodes by their rank(`nodesrank`) and compute `nrank`

15 World3_graph.py

All the graph given in the chapter Equations of World3 is implemented here using classes of `World3_dynamic.py`. If you want to create a new scenario, I suggest you to give a number to this scenario and to modify `World3_graph.py` as it is done for the existing ones. The tables are used with the following function:

Algorithm 1 Function f_{tab}

Require: $x \in \mathbb{Z}$

```

if  $tab[0][0] \geq x$  then
    return  $tab[0][1]$ 
else if  $tab[-1][0] \leq x$  then
    return  $tab[-1][1]$ 
else
     $i \leftarrow 0$ 
    while  $i < \text{length}(tab)$  do
        if  $tab[i][0] \geq x \geq tab[i + 1][0]$  then
             $coef \leftarrow \frac{tab[i + 1][1] - tab[i][1]}{tab[i + 1][0] - tab[i][0]}$ 
            return  $tab[i][1] + coef \times (x - tab[i][0])$ 
        else
             $i \leftarrow i + 1$ 
        end if
    end while
end if

```

16 World3_plot.py

All nodes have also an historic which can be showed at the end of the execution of the program. `World3_plot.py` contains functions which allow to observe evolution of variables in the way it is done in "The Limits to Growth". All backgrounds are taken from this book and from "The Limits to Growth: The 30-Year Update". They are available from scenario 1 to 10.

17 World3_run.py

This file is the one to execute. You can choose the version of World3, the initial time, the final one, time step and the scenario you desire to execute. Each run is by default printed with in background the result obtained in "The Limits to Growth".

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Acronyms and Appendix³

AHL	Assimilation Half-Life
AHL₇₀	Assimilation Half-Life in 1970
AHLM	Assimilation Half-Life Multiplier
AI	Agricultural Inputs
AII	Initial Agricultural Inputs
AIOPC	Average Industrial Output Per Capita
AIOPCI	Initial Average Industrial Output Per Capita
AIPH	Agricultural Inputs Per Hectare
AL	Arable Land
ALAI	Average Lifetime of Agricultural Inputs
ALGGHA	Arable Land in Gigahectare
ALGHA	Absorption Land in Gigahectare
ALI	Initial Arable Land
ALIC	Average Lifetime of Industrial Capital
ALL	Average Life of Land
ALLN	Average Life of Land Normal
ALSC	Average Lifetime of Service Capital
AMTI	Agricultural Materials Toxicity Index
B	Births per year
CAI	Current Agricultural Inputs
CBR	Crude Birth Rate
CDR	Crude Death Rate
CIO	Consumed Industrial Output

³Extracted from "Understanding Global Systems Today — A Calibration of the World3-03 Model between 1995 and 2012" [9]

CIOPC	Consumed Industrial Output Per Capita
CMI	Crowding Multiplier from Industry
CMPLE	Compensatory Multiplier from Perceived Life Expectancy
COPM	Capital Output Pollution Multiplier
COYM	Capital Output Yield Multiplier
CUF	Capital Utilization Fraction
D	Deaths per year
D₁	Deaths per year in population aged from 0 to 14 years
D₂	Deaths per year in population aged from 15 to 44 years
D₃	Deaths per year in population aged from 45 to 64 years
D₄	Deaths per year in population older than age 65
DCFS	Desired Completed Family Size
DCFSN	Desired Completed Family Size Normal
DCPH	Development Cost Per Hectare
DFR	Desired Food Ratio
DIOPC	Delayed Industrial Output Per Capita
DNRUR	Delayed Non-renewable Resources Usage Rate
DPOLX	Desired Index of persistent Pollution
DTF	Desired Total Fertility
EHSPC	Effective Health Services Per Capita
EHSPCI	Initial Effective Health Services Per Capita
EI	Education Index
F	Food
FALM	Fraction of inputs Allocated to Land Maintenance
FCAOR	Fertility Control Allocated to Obtaining Resources
FCAORTM	Fertility Control Allocated to Obtaining Resources switch Time
FCAPC	Fertility Control Allocations Per Capita
FCE	Fertility Control Effectiveness
FCEST	Fertility Control Effectiveness Set Time
FCFPC	Fertility Control Facilities Per Capita

FIALD	Fraction of Inputs Allocated to Land Development
FIE	Family Income Expectation
FIOAA	Fraction of Industrial Output Allocated to Agriculture
FIOAC	Fraction of Industrial Output Allocated to Consumption
FIOACC	Fraction of Industrial Output Allocated to Consumption Constant
FIOACV	Fraction of Industrial Output Allocated to Consumption Variable
FIOAI	Fraction of Industrial Output Allocated to Industry
FIOAS	Fraction of Industrial Output Allocated to Services
FIPM	Fraction of Inputs as Persistent Materials
FM	Fertility Multiplier
FOA	Fraction of Output in Agriculture
FOI	Fraction Of Output in Industry
FOS	Fraction Of Output in Service
FPC	Fraction Per Capita
FPU	Fraction of Population Urban
FR	Food Ratio
FRPM	Fraction of Resources as Persistent Materials
FRSN	Family Response to Social Norm
FSAFC	Fraction of Services Allocated to Fertility Control
FSDP	Food Shortage Perception Delay
FT	Final Time
GDPI	Gross Domestic Product Index
GDPPC	Gross Domestic Product Per Capita
GDPU	Gross Domestic Product Unit
GHAH	Gigahectare per Hectare
HEF	Human Ecological Footprint
HSAPC	Health Services Allocations Per Capita
HSID	Health Services Impact Delay
HUP	Hectare per Unit of Pollution
HWI	Human Welfare Index

IC	Industrial Capital
ICDR	Industrial Capital Depreciation Rate
ICI	Initial Industrial Capital
ICIR	Industrial Capital Investment Rate
ICOR	Industrial Capital-Output Ratio
ICOR2T	Industrial Capital-Output Ratio Technology Transmission
IET	Industrial Equilibrium Time
IEAT	Income Expectation Averaging Time
IFPC	Indicated Food Per Capita
ILF	Inherent Land Fertility
IMEF	Industrial Materials Emission Factor
IMTI	Industrial Materials Toxicity Index
IO	Industrial Output
IO₇₀	Industrial Output in 1970
IOPC	Industrial Output Per Capita
IOPCD	Industrial Output Per Capita Desired
ISOPC	Indicated Service Output Per Capita
IT	Initial Time
J	Job
JPH	Jobs Per Hectare
JPICU	Jobs Per Industrial Capital Unit
JPSCU	Jobs Per Service Capital Unit
LDR	Land Development Rate
LE	Life Expectancy
LEI	Life Expectancy Index
LER	Land Erosion Rate
LEN	Life Expectancy Normal
LF	Labor Force
LFC	Land Fraction Cultivated
LFD	Land Fertility Degradation

LFDR	Land Fertility Degradation Rate
LFERT	Land Fertility
LFERTI	Initial Land Fertility
LFH	Land Fraction Harvest
LFPF	Labor Force Participation Fraction
LFR	Land Fertility Regeneration
LFRT	Land Fertility Regeneration Time
LLMY	Land Life Multiplier from Yield
LLMYTM	Land Life Multiplier from Yield switch Time
LMC	Lifetime Multiplier from Crowding
LMF	Lifetime Multiplier from Food
LMHS	Lifetime Multiplier from Health Services
LMP	Lifetime Multiplier from persistant Polution
LPD	Lifetime Perception Delay
LRUI	Land Removal from Urban-Industrial use
LUF	Labor Utilization Fraction
LUFD	Labor Utilization Fraction Delayed
LUFDI	Initial Labor Utilization Fraction Delayed
LUFDT	Labor Utilization Fraction Delay Time
LY	Land Yield
LYCM	Land Yield Change Multiplier
LYF	Land Yield Factor
LYMAP	Land Yield Multiplier from Air Pollution
LYMC	Land Yield Multiplier from Capital
LYTD	Land Yield Technology Development
LYTDR	Land Yield Technology Development Rate
M₁	Mortality rate in population aged from 0 to 14 years
M₂	Mortality rate in population aged from 15 to 44 years
M₃	Mortality rate in population aged from 45 to 64 years
M₄	Mortality rate in population older than age 65

MAT₁	Maturation rate of 14 years old people
MAT₂	Maturation rate of 44 years old people
MAT₃	Maturation rate of 64 years old people
MLYMC	Marginal Land Yield Multiplier from Capital
MPAI	Marginal Productivity of Agricultural Inputs
MPLD	Marginal Productivity of Land Development
MTF	Maximum Total Fertility
MTFN	Maximum Total Fertility Normal
NFC	Need for Fertility Control
NR	Non-renewable Resources
NRATE	Non-renewable Resource Technology Improvement Rate
NRCM	Non-renewable Resources Change Multiplier
NRFR	Non-renewable Resources Fraction Remaining
NRI	Initial Non-renewable Resources
NRTD	Non-renewable Resources Technology Development
NRTDR	Non-renewable Resources Technology Development Rate
NRUF	Non-renewable Resources Usage Factor
NRUR	Non-renewable Resources Usage Rate
OY	One Year
P₁	Population aged from 0 to 14 years
P_{1I}	Initial population aged from 0 to 14 years at the year 1900
P₂	Population aged from 15 to 44 years
P_{2I}	Initial population aged from 15 to 44 years at the year 1900
P₃	Population aged from 45 to 64 years
P_{3I}	Initial population aged from 45 to 64 years at the year 1900
P₄	Population older than age 65
P_{4I}	Initial population older than age 65 at the year 1900
PAL	Potentially Arable Land
PALI	Initial Potentially Arable Land
PALT	Potentially Arable Land Total

PCRUM	Per Capita Resources Usage Multiplier
PET	Population Equilibrium Time
PFR	Perceived Food Ratio
PFRI	Initial Perceived Food Ratio
PJAS	Potential Jobs in Agricultural Sector
PJIS	Potential Jobs in Industrial Sector
PJSS	Potential Jobs in Service Sector
PL	Processing Loss
PLE	Perceived Life Expectancy
PLINID	Pollution Intensity Indicator
POF	Price Of Food
POLGFM	Pollution Control Technology Change Multiplier
POP	Total Population
PPAPR	Persistent Pollution Appearance Rate
PPASR	Persistent Pollution Assimilation Rate
PPGAO	Persistent Pollution Generated by Agricultural Output
PPGF	Persistent Pollution Generation Factor
PPGIO	Persistent Pollution Generated by Industrial Output
PPGR	Persistent Pollution Generation Rate
PPOL	Persistent Pollution
PPOL₇₀	Persistent Pollution in 1970
PPOLI	Initial Persistent Pollution
PPOLX	Index of Persistent Pollution
PPTD	Persistent Pollution Transmission Delay
PTD	Pollution Technology Development
PTDR	Pollution Technology Development Rate
PYEAR	New policy Year
RESINT	Resource usage Intensity
RHGDP	Real Highest Gross Domestic Product
RLGDP	Real Lowest Gross Domestic Product

RLT	Reproduction Lifetime
SAD	Social Adjustment Delay
SC	Service Capital
SCDR	Service Capital Depreciation Rate
SCI	Initial Service Capital
SCIR	Service Capital Investment Rate
SCOR	Service Capital-Output Rate
SD	Social Discount
SFPC	Subsistence Food Per Capita
SFSN	Social Family Size Norm
SO	Service Output
SOPC	Service Output Per Capita
TAI	Total Agricultural Investment
TDD	Technology Development Delay
TF	Total Fertility
TL	Total Land
TS	Time Step
UAGI	Unit of Agricultural Inputs per Hectare
UIL	Urban-Industrial Land
UILDT	Urban-Industrial Land Development Time
ULGHA	Urban Land in Gigahectare
UILI	Initial Urban-Industrial Land
UILPC	Urban-Industrial Land Per Capita
UILR	Urban-Industrial Land Required
UP	Unit of Person
ZPGT	Zero Population Growth Time

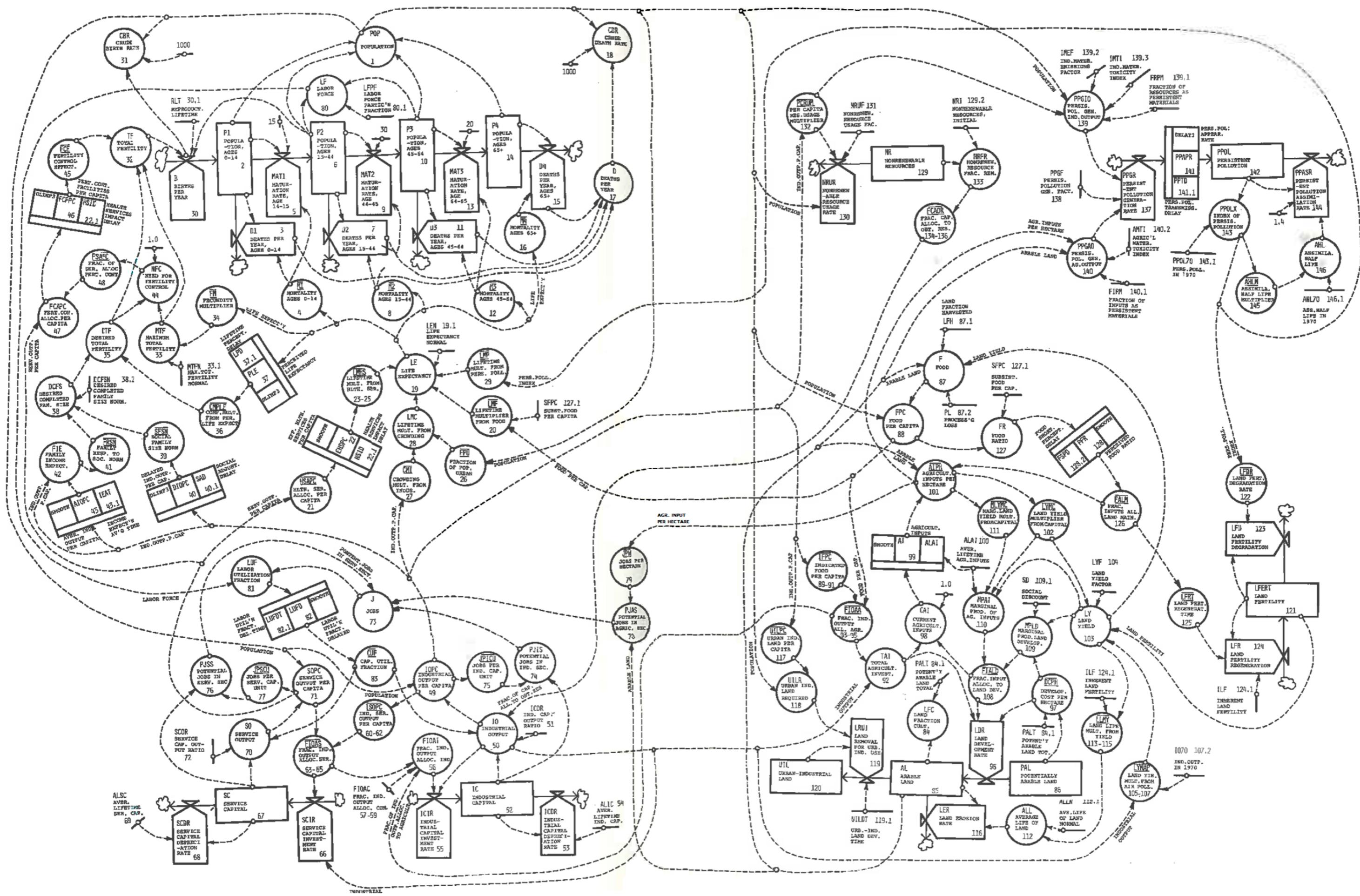


Figure 1: DYNAMO diagram of the 1972 World3 model