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A user equilibrium, traffic assignment model of network route and parking lot choice, with search circuits and cruising flows

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ABSTRACT

The paper provides a novel network model of parking and route choice. Parking supply is represented by parking type, management strategy including the fare, capacity and occupancy rate of parking lot, and network location, in relation to access routes along the roadway network. Trip demand is segmented according to origin–destination pair, the disposal of private parking facilities and the individual preferences for parking quality of service. Each traveller is assumed to make a two stage choice of, first, network route on the basis of the expected cost of route and parking and, second, local diversion on the basis of a discrete choice model. Search circuits are explicitly considered on the basis of the success probability to get a slot at a given lot and of the transition probabilities between lots in case of failure.

The basic endogenous model variables are the route flows, the lot success probabilities and the transition probabilities between lots. These give rise to the cost of a travel route up to a target lot and to the expected cost of search and park from that lot to the destination. Traffic equilibrium is defined in a static setting. It is characterized by a mixed problem of variational inequality and fixed point. Equilibrium is shown to exist under mild conditions and a Method of Successive Averages is put forward to solve for it. Lastly, a planning instance is given to illustrate the effects of insufficient parking capacity on travel costs and network flows.

0. Introduction

0.1. Background

Every car trip requires to park the car at the destination place or close to it; it also depends on the parking conditions at the origin place. The parking conditions in terms of price and quality of service determine the trip-maker's decisions of travel mode, network route and parking mode, especially so in dense urban areas. Abstracting from location, a parking mode involves a parking type either on-street or off-street, operating conditions such as tariffs, limit duration or special rights of access notably so for residents. Let us call "parking lot" a set of parking slots with given location and parking mode. Regular activities such as home and work will entice the car user to hold a parking space of his own, be it by ownership,

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rental or garage subscription. The associated costs (Van Ommeren et al., 2011) may push some users to park elsewhere, or to travel at another time of day, or to make their trips by another mode of travel.

Furthermore, in every place the parking capacity is limited. Thus the users compete with one another to avail themselves of the parking slots. A user that cannot get a slot immediately in a given lot has to wait for a length of time that is difficult to predict, or to divert to another lot. Not only does the quest for an available parking slot take time to the user, but it also adds "cruising traffic" to the core, "through" roadway traffic.

These phenomena have become obvious in many cities throughout the world, due to urban development, mass individual motorization and the massive use of cars by their holders (Shoup, 2005). Parking management, from capacity planning to dynamic pricing, has become a key component in the urban mobility policy and the multimodal planning of transportation networks. Yet the decision-aiding toolbox of the transportation planners still lacks a simulation model to deal with parking plans and policies over a wide area – apart from the inclusion of parking conditions in mode choice modelling and the specific treatment of Park-and-Ride (P&R) facilities at the interface between mode choice and network assignment modelling (namely the Parkride macro in the Emme/3 package).

0.2. Previous work

Models of parking supply and demand may be classified into three streams. First, a branch of economic theory has focused on parking to emphasize the social need to invest in capacity and to price for it so as to limit congestion, and also the dead-weight loss of cruising traffic (Arnott and Inci, 2006). The associated theoretical models address the spatial features and the demand behaviours in a much abstract form.

Second, behavioural models for the discrete choice of a parking type and location emphasize the diversity of behavioural strategies and the linkage between parking modes and travel mode: while the first generation of behavioural models paid little if any consideration to supplied capacity and spatial configuration (e.g. Austin, 1973; Hensher and King, 2001), the next generation has addressed parking search and the associated cruising as processes of individual behaviour (Thompson and Richardson, 1998, pioneered by Polak and Axhausen, 1990), thereby leading to agent-based simulation (e.g. Benenson et al., 2008; Dieussaert et al., 2009; Martens et al., 2010; Waraich and Axhausen, 2012).

Third, parking choices are addressed in conjunction with route choice in the framework of traffic assignment to a network: models are either static (e.g. Gur and Beimborn, 1984; Li et al., 2007a) or dynamic (e.g. Bifulco, 1993; Lam et al., 2006; Li et al., 2007b). Recently, Gallo et al. (2011) provided the first macroscopic assignment model to deal with search traffic explicitly; they succeeded to model (i) public access to parking lots whatever the destination, (ii) the local lot search and (iii) the associated pedestrian path up to the destination. Yet, none of the macroscopic assignment models presented so far addressed the detailed physics of parking search, the eventuality of loops in search circuits and the resulting search time and cruising flow.

0.3. Objective and model features

The paper provides a user equilibrium, traffic assignment model of parking and route choice on a roadway transportation network including parking facilities. The model captures the following features of parking supply: by lot, the location, residual capacity available in the study period and management mode are taken as exogenous, while the lot occupation and terminal cost (by demand segment) are endogenous. The demand is modelled as a set of segments, each of which is characterized by its origin–destination pair, period flow, specific access rights to parking lots and a specific travel behaviour on the basis of individual preferences for path and lot quality of service and price.

It is hypothesized that every traveller makes a two-stage choice of, first, network route to a prior target lot on the basis of its expected overall cost (including expected parking cost) and, second, a sequence of local diversions up to parking success. At a given lot the user will succeed to park with a probability that depends on the prior capacity and the number of candidates during the study period. Upon failure, the user diverts to alternative lots according to a discrete choice model on the basis of transition costs and the expected cost of search and parking from the head lot. Thus, search loops may arise when the success probabilities of immediate parking are strictly less than one around a chain of lots. The cruising for parking flows thus result and contribute to the roadway flows and travel times. By demand segment, each parking lot is characterized by an ex-ante expected terminal cost which is endogenous and reduces to the cost of terminal pedestrian access to the destination if the lot has free capacity.

A traffic equilibrium is defined where the individual user selects only a route of minimum expected overall cost to himself. Traffic equilibrium is cast into a joint problem of variational inequality for route and target lot choice and fixed point for success probabilities as well as transition probabilities.

0.4. Contribution and approach

The model of looping is innovative in the field of macroscopic traffic assignment: it enables to capture search circuits as in the most recent agent-based simulation models, yet with simpler assumptions about user behaviour. Our macroscopic setting also captures some stochasticity in the interplay of parking capacity and demand flows, since it is assumed that the arrivals of candidate parkers as well as the delivery of the so-called prior capacity are progressive. This bears some resemblance to static traffic assignment to a transit network (Spiess and Florian, 1989).

The local stochasticity of a parking lot, combined with the stochastic model of lot diversion, yield a stochastic user cost – of which only the average value is included in the evaluation of a lot option by a user. By demand segment, the expected costs from target lots to destination are evaluated by solving a linear system of small dimension and whose matrix is invertible. It may be thought of as a sophisticated link travel time function, where the "link" refers to a parking lot and involves diversion circuits, while the "travel time" is composed of the circuit cost from that lot plus the parking cost at the final lot and the pedestrian access to the destination.

For simplicity, the setting is static, by assuming that the parking slots are made available in a continuous way due to either prior vacancy or the departure of their previous occupants. This assumption typically describes the morning peak hour in urban nuclei, when night occupants give place to day occupants motivated notably by work. In the authors' opinion, this makes the major issue of parking in urban transportation planning, since it determines the travel modes chosen in commuting trips.

Aside from the user cost of parking and the cruising flows, the model yields the areas of parking saturation along the roadway network: such prediction is of much interest to land-use planners.

0.5. Structure

The rest of the paper is organized in eight parts. Section 1 provides a detailed bibliographical review. The assumptions about supply and demand are introduced in Section 2 and Section 3, respectively. Section 4 brings about a structural analysis of the interaction between supply and demand: the elementary influences are articulated into a logical structure that enables one to identify the core model variables and to state traffic equilibrium as a system of conditions. Then, Section 5 is devoted to more formal though concise mathematical analysis: existence conditions for traffic equilibrium are discussed and a simple computation scheme is put forward. Next, Section 6 deals with a small numerical instance to demonstrate parking diversion from more to less demanded lots and the determination of success probabilities. A more realistic instance of parking planning in an urban district is presented in Section 7. Lastly, Section 8 concludes by pointing to potential developments and on-going work.

1. Bibliographical review

Over a 40 years time span, the scientific literature devoted to parking analysis has evolved from very scarce to relatively abundant. Three main streams may be distinguished according to their focus either on (i) economic analysis, (ii) parking behaviour or (iii) network traffic assignment.

Economic models of parking demand and supply have been developed to depict the user competition for parking spaces (Arnott et al., 1991; Arnott and Rowse, 1999, 2009; Verhoef et al., 1995; Arnott, 2006) and the phenomenon of cruising for parking (Anderson and De Palma, 2004; Arnott and Inci, 2006), in order to explore the scope for public policy of capacity planning and pricing and to demonstrate the potential effects of specific instruments and coordination schemes (Glazer and Niskanen, 1992; Arnott, 2006). These models address user behaviours and spatial features in a simplistic way so as to focus on their interrelationship and to provide policy guidance. Anderson and De Palma (2004) depict a monocentric city, where a central business district attracts workers and parking capacity is available along a series of radial positions. They showed that finely tuned pricing improves the social efficiency of the parking system. Arnott and Inci (2006) showed that cruising for parking is a pure deadweight loss and that the optimal parking policy is to levy a parking fee just sufficient to eliminate cruising, while the number of parking spaces must be planned so that the remaining congestion externality matches the fee. In their model, the spatial description is limited to a set of identical destination zones with parking capacity, each of which is surrounded by a road ring along which cruising takes place until a parking space is vacated.

Furthermore, empirical economic research based on hedonic analysis yielded estimates for the values of, respectively, holding a private garage or holding a position in a waiting list to obtain an on-street parking permit near of one's home in Amsterdam (Van Ommeren et al., 2011).

The economic models can be considered as an abstract synthesis of the two other research streams that deal with individual behaviours, on one hand, and flows throughout space, on the other hand.

On the side of behavioural models, from the 1970s to the 2000s two generations of models addressed the issues of parking mode choice either on a standalone basis (Ergün, 1971; Austin, 1973; Axhausen, 1988; Hunt and Teply, 1993; Hess and Polak, 2004) or in relation to travel mode choice (Gillen, 1978; Gantvoort, 1984; Axhausen and Polak, 1991; Bradley et al., 1993; Hensher and King, 2001): first by multinomial or plain nested logit up to the 1990s, then by advanced discrete choice models with complex correlations between options (Muromachi, 2003) or distributed coefficients (Hess and Polak, 2004). Polak and Axhausen (1990) identified seven search strategies according to the type of target lot, its degree of availability and the eventual use of intermediate parking opportunities. The users that are certain to avail themselves of a given lot (private/garage/commercial area/off-street) drive to it directly, whereas other users consider intermediate parking opportunities and make individual trade-off between search effort, terminal walk and parking price. A hybrid strategy is to consider intermediate en-route opportunities while keeping a reliable option as recourse. Polak and Axhausen (1990) emphasized the need to model parking behaviour as a search process in two stages of first roadway access to a search sub-area and second parking search within that sub-area; each stage involves spatial features of its own. Thompson and

Richardson (1998) provided a modelling framework of behavioural process together with an application instance. Asakura (1996) designed a micro-simulation traffic model involving a two-stage behavioural process and applied it to simulate the effects of Parking Guidance Information systems. Muromachi (2003) applied complex choice structures (cross nested logit, generalized nested logit) to model the spatial topology of lots and routes in a particular case for which he obtained much statistical improvement over simpler structures insensitive to topology.

Kaplan and Bekhor (2011) put forward an advanced framework to model cruising for parking and the associated decisions as a behavioural process: their model is sensitive to individual characteristics. An initial choice of parking type between off-street and on-street leads to either the choice of an off-street facility or to on-street search (hence routing) for an available space, eventually returning to the choice of parking type. The binary choice of parking type is addressed by binary logit with respect to individual characteristics and to type attributes. The off-street selection of a parking facility is modelled by a multinomial logit, too. On-street parking facility and route choice is modelled as a biased random walk: at every junction the next link is selected from among viable alternatives (considering spatiotemporal thresholds at the trip level, notably on search duration and number of turns), with bias towards shortest paths.

From these contributions stems the third generation of behavioural research about parking, which involves agent-based models to deal with both behavioural processes and finely-described spatial features: the Parkagent model of Benenson et al. (2008), the Sustapark model of Dieussaert et al. (2009) and the Matsim-based parking model of Horni et al. (2011, 2012) and Waraich and Axhausen (2012). All of these involve a two stage process of individual behaviour, each with its own set of specific rules. The notion of search starting point is a key feature at the interface of main access and local search. Waraich and Axhausen (2012) designed a parking search algorithm as follows: the car user is faced with available parking spaces taken from a candidate set after some filtering on the basis of individual preferences: if the resulting set is empty then the candidate set is enlarged, otherwise a utility function is evaluated for each space and the user is assigned to the space of maximum utility.

The Parkagent model of Benenson et al. (2008) and Martens et al. (2010) is more sensitive to spatial features since the behavioural process is a sequence of (i) driving to search starting point, (ii) proximity driving up to destination with parking at any place depending on both its availability and the expected number of free places closer to destination, (iii) if unsuccessful and destination point is passed, then select the first space available not too far.

Complementarily, modern traffic data (including GPS tracks) have been used to depict behavioural patterns (Van Ommeren et al., 2012; Montini et al., 2012).

To sum up, behavioural and spatial features have been progressively recognised and modelled in an intricate relationship. While some consensus has been achieved about the sequence of phases in the search process, the decision model particular to each stage still makes a topic of behavioural research. Only the most sophisticated, agent-based behavioural models make cruising for parking explicit. Furthermore, despite the issues of capacity planning and pricing are emphasized in the quoted papers, to our knowledge no application to planning has yet been reported.

Whereas the behavioural models deal with parking demand in relation to supplied services, the other stream of parking research deals with parking supply throughout space in relation to demand, so as to depict the spatial features and variations of parking availability and usage. *This streams involves macroscopic models of traffic onto a roadway network* yielding access to parking facilities. In the 1970s and 1980s, a first generation of traffic models addressed the distribution of car trip flows from origin zones to parking lots yielding access to the destination zones either by transit (Florian and Los, 1979, 1980) or by walk (Goyal and Gomes, 1984; Gur and Beimborn, 1984). The second generation of traffic models has focused on the assignment of trip flows to network routes ended by a parking lot near of the destination. Nour Eldin et al. (1981) addressed parking links of given practical capacity as connector links to destination zones: the travel time function for such a link depicts the search delay as a base time plus a congested time inversely proportional to (1 – Flow/Capacity). Their model deals with a sequence of periods, each one in a static way with residual capacity taken from the previous period. Bifulco (1993) addressed stochastic user equilibrium in a dynamic (sequential) model with parking, again with parking facilities associated to destination zones as connector links. By period, his function of search time is derived from the assumption of a ring-shaped parking area along which vacant spaces are uniformly distributed; also parking fees and fines are represented. Li et al. (2007a) and Huang et al. (2005) modelled a multimodal network with capacity constraints on both road segments and parking facilities: in equilibrium the dual variable penalizing the capacity constraint is interpreted as a queuing cost which is added to the travel cost of the roadway path. These authors succeeded to model parking facilities with no prior relationship to a selected destination (i) by allowing for terminal access to a wider range of parking facilities close to the destination, (ii) by allowing each parking facility to be shared between one or several destinations. Subsequently, some dynamic assignment models have been designed to deal with parking. Li et al. (2007b) and Li et al. (2008) modelled the search delay by a penalty function between demand and capacity at a destination-related parking facility – essentially the same treatment as in previous static models but with dynamic variables of flow and residual capacity.

In these models, parking is essentially depicted by congestible facilities with ad-hoc time function or cost penalty; cruising for parking is not considered. A more recent model by Gallo et al. (2011) addresses cruising by distinguishing three trip phases of, first, main car path, second, cruising part and, third, pedestrian access. Each phase is dealt with in a dedicated network layer. The main car path takes place in the car network layer and leads to a proxy of the destination node yielding access to the cruising layer, where paths are available to reach parking facilities, which grant the access to the pedestrian layer. Each facility is modelled by a parking link, again with a travel time function that relates an ad-hoc search time to the ratio of flow to residual capacity. This model improves upon previous macroscopic models by the clear distinction of the trip phases, as in the most recent agent-based models.

Surprisingly enough, microscopic simulation models that have enjoyed vivid development over the recent years still do not deal with parking – to the best of our knowledge. The micro-simulation Parksim model of Young (1986a), Young (1986b) and Young et al. (1991) addresses a parking area so as to establish the traffic state in response to the area layout, without covering an urban network.

To synthesize, parking at the end of a car trip is an important issue, now well-known in economic theory which worries about cruising for parking as a pure deadweight loss. Agent-based models have been designed to simulate parking conditions and the process of space search; cruising loops may arise in the Parkagent model of Martens et al. (2010). So far, the macroscopic traffic assignment models that deal with parking address it as a local facility with ad-hoc wait time function, so that no cruising loop may arise.

2. The supply of parking and route services

On the supply side, transportation services for a motorist include a parking slot as well as a network route, in a joint fashion since the route provides access to the slot which makes its final endpoint.

2.1. Parking supply

Assume that the parking slots are grouped into lots denoted by $\ell \in L$. Each lot is located at a given place and connected to the roadway network. It is operated in a specific mode which imposes a parking fee, m_ℓ, and a time of transaction, t_ℓ. During a period of reference, the lot has "prior capacity" of κ_ℓ slots which are made available in a progressive way, assumedly from prior vacancy or the departure of previous occupants.

Assume also that Y_ℓ customers demand a slot in lot ℓ during the period: then the probability of immediate parking success is

$$\alpha_\ell = \min\left\{1, \frac{\kappa_\ell}{Y_\ell}\right\}, \text{ where in } \alpha_\ell \text{ is set to 1 if } Y_\ell = 0. \tag{2.1}$$

This probability is a key factor of parking search and associated circuits. Formula (2.1) is a first-round approximation specified so for the sake of simplicity. The essential feature is to capture the stochasticity of slot availability and to relate it to both prior capacity and the candidate flow.

It should be noted here that the candidate flow Y_ℓ is not restricted to the users of a particular destination; out of the previous macroscopic traffic assignment models dealing with parking, only those of Li et al. (2007a,b, 2008) and Gallo et al. (2011) also can accommodate several destinations in the same parking lot, which is quite natural in reality as in agent-based models.

2.2. Network routes

The roadway network is a set A of oriented links a, each one with travel time t_a and money cost m_a per trip to a user during the reference period. Any network route r is a sequence of links $\{a \in r\}$ with continuity of itinerary and the following route time and money cost, respectively:

$$t_r = \sum_{a \in r} t_a, \tag{2.2a}$$

$$m_r = \sum_{a \in r} m_a. \tag{2.2b}$$

Roadway traffic phenomena are modelled by link travel time functions with respect to the vector of link flows, $\mathbf{v}_A = [v_a: a \in A]$: in vector form to allow for complex dependencies,

$$\mathbf{t}_A = T_A(\mathbf{v}_A). \tag{2.3}$$

3. Demand

3.1. Segments of customers

Demand is analyzed as a set S of segments s of homogeneous customers, each one with a vehicle (presumably a car), an origin zone i_s and a destination zone j_s, a given activity at destination that motivates the trip from i to j and imposes a duration of parking, together with specific economic preferences that include tradeoffs between quality factors such as the time and money costs. The customers within a segment may differ from one another by the disaggregate activity location within the destination zone. Let us assume however that all of them take as valuable a subset L_s of parking lots, located within the destination zone or in the vicinity of it. The pedestrian time from lot ℓ to destination j_s is denoted as $t_{s\ell}$. Let also Q_s denote the trip flow of segment s.

3.2. Costs taken for certain

Assume that any customer of segment s evaluates a network route r in a deterministic fashion, synthesized into a single "generalized cost" of travel that depends on the route's money cost and travel time:

$$g_{sr} = G_s(m_r, t_r). \tag{3.1}$$

For instance, assuming a constant money value of a unit run time of β_s^R for a user in segment s, a simple form of generalized cost would be $g_{sr} = m_r + \beta_s^R t_r$.

Furthermore, if the customer gets a parking slot in lot ℓ then to him the generalized cost of parking amounts to the following:

$$c_{s\ell} = C_s(m_\ell, t_\ell, t_{s\ell}). \tag{3.2}$$

This abstract relationship enables one to account for segment's features such as activity duration (hence parking duration) and trade-off between time and money. For instance, assuming an activity duration of η_s and a unitary value of walking time of β_s^W for a user in segment s, a simple form of generalized parking cost would be $c_{s\ell} = m_\ell \eta_s + \beta_s^W (t_\ell + t_{s\ell})$.

Cost functions G_s and C_s are taken for certain in the sense that they are determined prior to the process of lot search, conditional on the lot that will come out of it.

3.3. Search process and the resulting flows

Each customer participates to the production of his own travel service, by looking for a convenient parking lot. Let us model the related search as the following process:

- The customer selects ex-ante a target lot, say ℓ_0, out of L_s.
- at a current lot ℓ there is at least one slot available with probability α_ℓ. If candidature is successful then the customer is satisfied and the search process ends up. Otherwise, the customer directs himself to any lot $n \in L_s$ with diversion probability $p_{\ell n}^s$ of transition (which will be specified at the end of the section). This step is repeated until success.

The notion of target lot is virtually identical to that of search starting point in the agent-based models of Martens et al. (2010) and Waraich and Axhausen (2012). In our model the target lot selection by the user is endogenous and stems from ex-ante travel information, whereas in the agent-based models it seems to be determined in two steps of, first, the identification of an origin–destination shortest path without parking and, second, the identification of the first node in that path to be closer to the destination point than a given threshold distance. Another difference in the hypothesized behavioural processes is that in the Parkagent model the user will accept an available slot only up to a given probability, whereas in our model it is assumed that any lot with immediate parking success is chosen for sure. However, the lot success probability (2.1) could be easily elaborated so as to include an acceptance rate from the user.

Denote as $\mathbf{p}_s = [p_{\ell n}^s : \ell, n \in L_s]$ the matrix of transition probabilities and let us analyze the search process by assuming an ex-ante demand of segment s for lots in L_s, i.e. a vector of trip flows by lot denoted as $\mathbf{q}_s = [q_{s\ell} : \ell \in L_s]$, and by focusing on the vector of candidate volume by lot, $\mathbf{y}_s = [y_{s\ell} : \ell \in L_s]$.

At any lot ℓ the number of candidates, $y_{s\ell}$, is made up of the ex-ante candidates, $q_{s\ell}$, plus the candidates diverted from unsuccessful requests, the ℓ-th component of $\mathbf{y}_s \mathbf{J}_{s\alpha} \mathbf{p}_s$, in which $\mathbf{J}_{s\alpha}$ is the diagonal matrix of term $1 - \alpha_\ell$ for $\ell \in L_s$. Thus, in vector form,

$$\mathbf{y}_s = \mathbf{q}_s + \mathbf{y}_s \mathbf{J}_{s\alpha} \mathbf{p}_s, \tag{3.3}$$

or equivalently, denoting by $\mathbf{I}_s$ the identity matrix on L_s,

$$\mathbf{y}_s (\mathbf{I}_s - \mathbf{J}_{s\alpha} \mathbf{p}_s) = \mathbf{q}_s. \tag{3.4}$$

In the Appendix, it is shown that matrix $\mathbf{I}_s - \mathbf{J}_{s\alpha} \mathbf{p}_s$ can be inverted; thus, denoting by $\mathbf{H}_{s\alpha p}$ its inverse matrix, it holds that

$$\mathbf{y}_s = \mathbf{q}_s \mathbf{H}_{s\alpha p}. \tag{3.5}$$

Denote by $\tau = (\ell, n)$ the transition from lot ℓ to lot n and by T_s the set of transitions allowed for segment s. The transition flows $x_{s\tau}$ stems from the search flow $y_{s\ell}$ restricted to the lot unsuccessful parking candidates that make a fraction $1 - \alpha_\ell$ of it and also from the transition probabilities $p_{\ell n}^s$, through $x_{s\tau} = y_{s\ell}.(1 - \alpha_\ell).p_{\ell n}^s$. In vector form, denoting by $\vec{\mathbf{p}}_s$ the matrix made up by juxtaposition of square blocks indexed by $\ell \in L_s$, each of which is null save for its ℓ -th row that is taken from $\mathbf{p}_s$:

$$\mathbf{x}_{sT} = \mathbf{y}_s \mathbf{J}_{s\alpha} \vec{\mathbf{p}}_s = \mathbf{q}_s \mathbf{H}_{s\alpha p} \mathbf{J}_{s\alpha} \vec{\mathbf{p}}_s. \tag{3.6}$$

3.4. Search cost and the expected costs by target lot

Let us now state the search cost from target lot to any final lot of first successful request with respect to the transition costs. Assume that to segment s a transition $\tau = (\ell, n)$ costs a time of $t_{s\tau}$ and a money expense of $m_{s\tau}$: the two types of costs are aggregated into a "generalized cost" denoted $c_{s\tau}$. For instance, it may be assumed that $c_{s\tau} = m_{s\tau} + \beta_s^C t_{s\tau}$ with β_s^C the money value of a unit cruising time.

Under the system conditions, the search cost of cruising for parking incurred by demand vector $\mathbf{q}_s$ depends on the transition expenses by user and the number of users that make a given transition. It amounts to the vector product of $\mathbf{x}_{sT}$ times the vector of generalized transition costs, $\mathbf{c}_{sT}$, so

$$\tilde{c}_s(\mathbf{q}_s, \alpha_L, \mathbf{p}_s, \mathbf{c}_{sT}) = \mathbf{x}_{sT}.\mathbf{c}_{sT} = \mathbf{q}_s \mathbf{H}_{s\alpha p}\ \mathbf{J}_{s\alpha} \vec{\mathbf{p}}_s.\mathbf{c}_{sT}.$$

A search that starts from lot ℓ corresponds to a particular demand vector $\delta_\ell^s = [1_{\{\ell=n\}} : n \in L_s]$, hence to a particular search cost as follows:

$$\tilde{c}_{s\ell} = \tilde{c}_s(\delta_\ell^s, \alpha_L, \mathbf{p}_s, \mathbf{c}_{sT}) = \delta_\ell^s \mathbf{H}_{s\alpha p} \mathbf{J}_{s\alpha} \vec{\mathbf{p}}_s.\mathbf{c}_{sT}. \tag{3.7}$$

Moreover, the "final" cost of parking also depends on demand vector $\mathbf{q}_s$ through the derived vector $\mathbf{y}_s$:

$$\hat{c}_s(\mathbf{q}_s, \alpha_L, \mathbf{p}_s, \mathbf{c}_{sL}) = \mathbf{y}_s diag(\alpha_L)\, [c_{s\ell} : \ell \in L_s] = \mathbf{q}_s \mathbf{H}_{s\alpha p} diag(\alpha_L) \mathbf{c}_{sL}. \tag{3.8}$$

Starting from lot ℓ, the expected cost of search and park is

$$\hat{g}_{s\ell} = \tilde{c}_{s\ell} + \hat{c}_s(\delta_\ell^s, \alpha_L, \mathbf{p}_s, \mathbf{c}_{sL}) = \delta_\ell^s \mathbf{H}_{s\alpha p}\, (\mathbf{J}_{s\alpha} \vec{\mathbf{p}}_s\, \mathbf{c}_{sT} + diag(\alpha_L)\, \mathbf{c}_{sL}). \tag{3.9}$$

The hypothesized behavior is that a user knows the *value* of the search and park cost, possibly with some inaccuracy as will be stated next. The latter formula is only an engineer formula to reproduce the value, of which the user knowledge can be interpreted to be acquired by successive trials and errors in the reiteration of his trips – which fits for frequent purposes.

3.5. Travel options and choice behaviour

Ex-post, the travel option of a customer includes a network route r to a target lot ℓ, that lot as ex-ante target and a sequence of transitions starting from ℓ and ending up at first success. The salient features are (r, ℓ) or r only since ℓ must be the final endpoint of r. The expected cost amounts to the travel cost along r, g_{sr}, plus the expected cost from ℓ, $\hat{g}_{s\ell}$:

$$\hat{g}_{sr} = g_{sr} + \hat{g}_{s\ell(r)}. \tag{3.10}$$

A two-stage choice behavior is assumed for the customer. First, a travel option of minimum ex-ante cost $\hat{g}_{sr}$ is chosen. Second, depending on the current occupancy state of parking lots, the user makes a local choice of transition to some next lot if he fails to get a slot at the current lot. Such local choices are repeated until parking success.

A discrete choice model of behaviour is assumed for local transition choice: local options at ℓ are the transitions starting at current lot and ending at $n \in L_s$, $\tau = (\ell, n)$, with travel disutility of, say, $d_{s\tau} = c_{s\tau} + \hat{g}_{sn} + \varepsilon_{sn}$, in which ε_{sn} is a random variable. The joint distribution of $[\varepsilon_{sn}: n \in L_s]$, together with the transition generalized costs and the lot expected costs, determine the choice probability by transition:

$$p_{\ell n}^s = \Pr\{d_{sn} \leqslant d_{sm} : \forall m \in L_s\}. \tag{3.11}$$

For simplicity, a multinomial logit model can be used for local diversion choices: assuming independent, identically distributed Gumbel variables ε_{sn} with parameter θ, then

$$p_{\ell n}^s = \frac{\exp(-\theta[c_{s(\ell,n)} + \hat{g}_{sn}])}{\sum_{m \in L_s \setminus \ell} \exp(-\theta[c_{s(\ell,m)} + \hat{g}_{sm}])}, \quad \forall n \in L_s \setminus \ell.$$

The abstract formula (3.11) allows for much elaboration of the local choice process, which may be tailored to the segment characteristics and preferences, as well as to the spatial configuration of parking lots – e.g. by restricting local diversion to the *immediately* neighbouring lots or by biasing the probabilities to account for turn geometry. Only memory effects seem to be out of the reach of our formulation: however such effects are somewhat mimicked by the notion of "expected cost" by target lot which is hypothesized to be known by the user.

To sum up, the travel behaviour is rational at both stages on the basis of costs which the customer seeks to minimize. At the network level the cost of a travel option of main route and target lot to be chosen ex-ante is an expected cost evaluated in an average, deterministic way (a stochastic part could be added to it to model disaggregate travel conditions at the origin point). At the local level the cost is modelled in a stochastic way – to facilitate the determination of equilibrium and also to address the disaggregate location of trip endpoints in the destination zone.

4. The interaction of supply and demand

The model developed so far consists in a set of variables that are involved in a set of relationships – more precisely, of dependencies. Demand-side variables depend on supply-side variables, and conversely demand variables are factors of supply variables. In this section, our purpose is to gather the dependencies and to lay the emphasis on the overall logical structure.

We shall state each dependency under abstract form, such as $\mathbf{x}_I = \mathbf{X}_I(Z_J \ldots)$ in which $\mathbf{x}$ is the dependent variable, I is an index set taken from among routes (R), links (A), lots (L), segments (S) or transitions (T), $\mathbf{Z}_J$ is a factor and $\mathbf{X}_I$ is a mathematical function in abridged notation for an influence that has been stated formerly in a detailed way.

The section is organized in four parts: the three first ones are devoted to gather and re-state the dependencies that pertain to demand (Section 5.1), traffic (Section 5.2) and costs (Section 5.3), respectively. Then the logical structure is depicted in an influence diagram (Section 5.4).

4.1. Demand functions

At the network level, the assignment of customers to target lots of minimum cost can be stated as follows: Find vector $\mathbf{f}_{SR} = [f_{sr}: r \in R_s, s \in S]$ and dual variables $[\mu_s: s \in S]$ such that

$$f_{sr} \geq 0 \quad \forall r \in R_s, \quad \forall s \in S \tag{4.1a}$$

$$\sum_{r \in R_s} f_{sr} = Q_s \quad \forall s \in S \tag{4.1b}$$

$$\hat{g}_{sr} - \mu_s \geq 0 \quad \forall r \in R_s, \quad \forall s \in S \tag{4.1c}$$

$$f_{sr}(\hat{g}_{sr} - \mu_s) = 0 \quad \forall r \in R_s, \quad \forall s \in S \tag{4.1d}$$

At solution, μ_s is equal to the minimum cost among the route options of customer segment s. System (4.1) can be abstracted into a multi-valued mapping as follows:

$$\mathbf{f}_{SR} \in F_{SR}(\mathbf{Q}_S, \hat{\mathbf{g}}_{SR}). \tag{4.2}$$

Lot demand $\mathbf{q}_{SL}$ stems from route flows in a straightforward way: $q_{s\ell} = \sum_{r \in \ell} f_{sr}$, in which $\{r \in \ell\}$ indicates that lot ℓ is located at the final endpoint of route r. In abstract form,

$$\mathbf{q}_{SL} = Q_{SL}(\mathbf{f}_{SR}). \tag{4.3}$$

Concerning local routing behaviour for the search of an available parking slot, by segment the matrix of transition probabilities, $\mathbf{p}_{ST}$, depends on that of transition costs, $\mathbf{c}_{ST} = [c_{s\tau}: \tau \in T_s, s \in S]$ and on the lot expected costs, $\hat{\mathbf{g}}_{SL} = [\hat{g}_{s\ell} : \ell \in L_s, s \in S]$:

$$\mathbf{p}_{ST} = P_{ST}(\mathbf{c}_{ST}, \hat{\mathbf{g}}_{SL}). \tag{4.4}$$

4.2. Traffic functions

By segment s and parking lot ℓ, the search flow $y_{s\ell}$ depends on the flow inputs of all target lots of that segment, $[q_{sn}: n \in L_s]$, the success probabilities α_L and the transition probabilities $\mathbf{p}_{sL}$: thus

$$\mathbf{y}_{SL} = Y_{SL}(\mathbf{q}_{SL}, \alpha_L, \mathbf{p}_{ST}). \tag{4.5}$$

By segment s and transition $\tau = (\ell, n)$ between lots in L_s, the transition customer volume $x_{s\tau}$ stems from (3.6). Thus, in vector form,

$$\mathbf{x}_{ST} = X_{ST}(\mathbf{y}_{SL}, \alpha_L, \mathbf{p}_{ST}). \tag{4.6}$$

By demand segment and network link, the route volume due to network routes is derived from the route flows of all segments and network routes: $f_{sa} = \sum_{r \ni a} f_{sr}$, so

$$\mathbf{f}_{SA} = F_{SA}(\mathbf{f}_{SR}). \tag{4.7}$$

At the link level, the flow volume v_{sa} stems from route volumes and also from the network effect of the transition flows. Assuming that a given proportion $\gamma_{as\tau}$ of $x_{s\tau}$ is assigned to link a, then

$$\mathbf{v}_{SA} = V_{SA}(\mathbf{f}_{SA}, \mathbf{x}_{ST}, \Gamma_{AST}). \tag{4.8}$$

For simplicity, any transition could be assigned deterministically to one network route, yielding only values 0 or 1 for $\gamma_{as\tau}$.

By lot, the probability of parking success, α_ℓ, depends on the lot demand $Y_\ell = \sum_{s \in S} y_{s\ell}$, so

$$\alpha_L = A_L(\mathbf{y}_{SL}, \kappa_L). \tag{4.9}$$

Link travel times are related to link flows on the basis of travel time functions:

$$\mathbf{t}_{SA} = T_{SA}(\mathbf{v}_{SA}). \tag{4.10}$$

In turn, route travel times stem from links' ones by serial composition, $t_{sr} = \sum_{a \in r} t_{sa}$, so

$$\mathbf{t}_{SR} = T_{SR}(\mathbf{t}_{SA}). \tag{4.11}$$

4.3. Cost functions

To a customer of segment s, the money cost of travelling once along link a depends on exogenous fares and also on the link travel time (for energy expenses etc): so

$$\mathbf{m}_{SA} = M_{SA}(\mathbf{t}_{SA}). \tag{4.12}$$

The money expenses are composed by route:

$$\mathbf{m}_{SR} = M_{SR}(\mathbf{m}_{SA}). \tag{4.13}$$

By route, the generalized cost of travel results from time and money expenses:

$$\mathbf{g}_{SR} = G_{SR}(\mathbf{t}_{SR}, \mathbf{m}_{SR}). \tag{4.14}$$

The costs of parking at lots, $\mathbf{c}_{SL}$, are assumed exogenous and may vary by segment.

The transition costs depend on the travel times, money expenses, and the linkage $\Gamma_{AST} = [\gamma_{as\tau} : a \in A, s \in S, \tau \in T_s]$ between transitions and network links, on the basis of:

$$\mathbf{c}_{ST} = C_{ST}(\mathbf{t}_{SA}, \mathbf{m}_{SA}, \Gamma_{AST}). \tag{4.15}$$

The lot expected costs of search and park comply to

$$\hat{\mathbf{g}}_{SL} = \hat{G}_{SL}(\alpha_L, \mathbf{p}_{ST}, \mathbf{c}_{ST}, \mathbf{c}_{SL}). \tag{4.16}$$

Lastly, the route expected cost includes the travel time to targeted lot plus the search and park cost from that lot:

$$\hat{\mathbf{g}}_{SR} = \widehat{G}_{SR}(\mathbf{g}_{SR}, \hat{\mathbf{g}}_{SL}). \tag{4.17}$$

4.4. Logical structure

Eqs. (4.2)–(4.17) make up an interconnected system of dependencies between the state variables in the model. An overview is provided in Fig. 1 in order to depict the logical structure and to trace the various influences. The diagram enables us to identify sub-systems as follows:

- A route demand model in which the route flow vector is determined and yields link flows as well as lot target flows.
- A local demand model for transition probabilities.

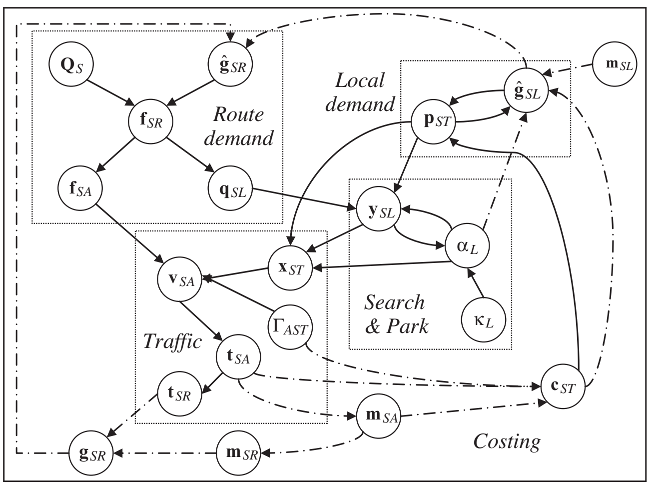

Fig. 1. Influences in the routing and parking model (dashed line for Costing model).

- A parking supply and demand model (Search and Park), yielding search flows and success probabilities on the basis of target flows, transition probabilities and lot capacities.
- A traffic model for the determination of transition flows, link flows, link times and route times.
- A costing model to yield link and route money expenses, link/route/lot costs and lot/route expected costs.

The route demand, traffic and costing models are simple in that their outputs exert no feedback on themselves or the inputs in a straightforward way. But the local demand model and the search-and-park model exhibit such straightforward feedback, which makes them harder to solve. Fig. 2 depicts the sub-models as connected by the logical flow.

5. Mathematical analysis

Let us now turn to the two major issues of, first, equilibrium i.e. the determination of a system state consistent with all influences and, second, a computation scheme to solve for equilibrium.

5.1. Equilibrium

Most of the variables in the model are endogenous: the exceptions are the segment volumes, the lot capacities and the tariffs by lot or link. Among the endogenous variables, let us select the triple $\mathbf{z} = (\mathbf{f}_{SR}, \mathbf{p}_{ST}, \alpha_L)$ as the "basic" state variable. It can be checked on the influence diagram that, on cutting every arrow which points to any of the three components, then everything else is determined.

Definition: Routing and parking equilibrium. State $\mathbf{z} = (\mathbf{f}_{SR}, \mathbf{p}_{ST}, \alpha_L)$ is an equilibrium if and only if it satisfies jointly the conditions (4.2)–(4.17).

In fact, system (4.2)–(4.17) amounts to a Fixed Point Problem in $\mathbf{z}$: find $\mathbf{z}^*$ such that (4.2), (4.4) and (4.9) hold true when all other endogenous variables are based on $\mathbf{z}^*$.

It is classical to address a route choice model of traffic assignment to a network on the basis of the route flow vector (e.g. Sheffi, 1985). So it would be tempting to take the same approach to our routing and parking model, by trying to solve in an integrated way for $(\mathbf{p}_{ST}, \mathbf{c}_{ST})$ on the one hand, and for $(\alpha_L, \mathbf{y}_{SL})$ on the other hand. However this would be awkward since the first sub-problem would require much effort, whereas the second problem may have no solution – when a subset L_s provides little opportunity of lot diversion while the lot target flows are in excess of the local parking capacity.

By replacing system (4.1) of deterministic network routing with analogous conditions of stochastic network routing, then the mapping F_{SR} in (4.2) would be a continuous function and the system (4.2)–(4.17) would make a continuous function of $\mathbf{z}$. As the values in $\mathbf{p}_{ST}$ and α_L are probabilities, while the route flows are non negative and bounded by the segment volume, the admissible set is compact. Let us assume that the system is feasible, i.e. that the overall parking capacity is greater than the overall trip volume and that the subsets L_s enable for sufficiently wide dispersion of local demand. Then the admissible set is compact and nonempty so that, if the fixed point function is continuous, there must be a solution of routing and parking equilibrium.

This proof can be extended to deterministic routing by taking it as the limiting case of stochastic routing. The uniqueness of an R&P equilibrium is still an open issue – at least concerning the success probabilities.

System feasibility can be asserted by solving an auxiliary problem of "feasible distribution" on a specific network, in which one source node is associated to every demand segment s, one node is associated to every parking lot ℓ, a last node makes a sink and there are two kinds of links: first, from every demand node s to every parking node of a lot in L_s, there is an oriented link of infinite capacity; second, from every parking node ℓ to the sink node, there is an oriented link of capacity κ_ℓ. The feasible distribution problem consists in finding a flow on that network in consistency with both the inflows and the link capacities. It can be solved using a specific algorithm which is relatively simple (Rockafellar, 1984). An obvious requirement is that the total capacity of the lots must be larger than the total inflows demanded by the segments. However this condition

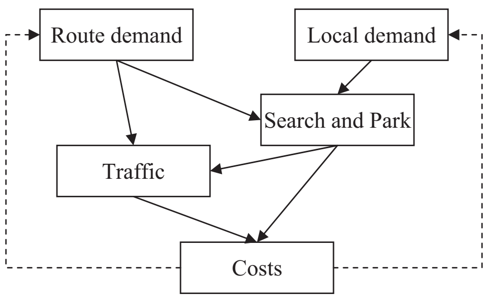

Fig. 2. Sub-models and logical flow.

is not sufficient in general for system feasibility since there may be local shortages of capacity that could not be cleared by the diversion of demand to neighboring lots.

5.2. Computation scheme

The R&P equilibrium can be searched for by solving a mixed problem of variational inequality (on route flows) and fixed point (on success probabilities and transition probabilities). As a first, simple approach, let us address the variational inequality by a Method of Successive Averages, i.e. convex combination of current state $\mathbf{f}_{SR}^{(k)}$ with an auxiliary state $\tilde{\mathbf{f}}_{SR}^{(k)}$ that solves (4.1) with respect to the current state of costs, into a new state $\mathbf{f}_{SR}^{(k+1)} = \mathbf{f}_{SR}^{(k)} + \zeta_k(\tilde{\mathbf{f}}_{SR}^{(k)} - \mathbf{f}_{SR}^{(k)})$ where ζ_k is a decreasing sequence of numbers in]0,1[save for $\zeta_0 = 1$.

On the probability side, the new state can be obtained by convex combination also but using a fixed coefficient, say ω_α for success probabilities and ω_p for transition probabilities.

Here is an abridged flowchart:

- *Initialization*. Let k: = 0, $\alpha_\ell := 1\ \forall \ell \in L, p^s_{\ell\tau} := 1/|L_s| \forall \ell, n \in L_s$ and $\forall s \in S$, $\mathbf{f}_{SR}^{(0)} := \mathbf{0}$.
- *Route and lot costing*. Based on $\mathbf{z}^{(k)}$, evaluate link costs, route costs, transition costs, lot expected costs and route expected costs, $\hat{\mathbf{g}}_{SR}^{(k)}$.
- *Auxiliary state*. Assign demand volumes to routes of minimum expected cost, yielding $\tilde{\mathbf{f}}_{SR}^{(k)}$. From these and the current probabilities, derive the $\tilde{\mathbf{q}}_{SL}^{(k)}$ and $\tilde{\mathbf{y}}_{SL}^{(k)}$. Then derive $\tilde{\alpha}_L^{(k)}$ on the basis of (4.9) and $\tilde{\mathbf{p}}_{ST}^{(k)}$ on the basis of the current transition costs and expected lot costs.
- *Convex combination*. Let $\mathbf{f}_{SR}^{(k+1)} := \mathbf{f}_{SR}^{(k)} + \zeta_k(\tilde{\mathbf{f}}_{SR}^{(k)} - \mathbf{f}_{SR}^{(k)}), \alpha_L^{(k+1)} := \alpha_L^{(k)} + \omega_\alpha(\tilde{\alpha}_L^{(k)} - \alpha_L^{(k)})$ and $\mathbf{p}_{ST}^{(k+1)} := \mathbf{p}_{ST}^{(k)} + \omega_p(\tilde{\mathbf{p}}_{ST}^{(k)} - \mathbf{p}_{ST}^{(k)})$.
- *Convergence test*. If distance between $\mathbf{z}^{(k)}$ and $\mathbf{z}^{(k+1)}$ is small enough then stop, else increment k and go to Costing step.

As a distance criterion, a sum of functions by component in $\mathbf{z}$ is appropriate: for instance a duality gap on $\mathbf{f}_{SR}$ and squared distance on each probability vector, with formulae as follows:

$$DG(\mathbf{f}_{SR}) = \sum_{s,r} \hat{\mathbf{g}}_{sr}^{(k)}(\mathbf{f}_{sr}^{(k)} - \tilde{\mathbf{f}}_{sr}^{(k)}),$$

$$D_\alpha^2 = \|\tilde{\alpha}_L^{(k)} - \alpha_L^{(k)}\|^2 \text{ and } D_p^2 = \|\tilde{\mathbf{p}}_{ST}^{(k)} - \mathbf{p}_{ST}^{(k)}\|^2.$$

6. Toy instance

6.1. Case design

As a classroom instance, let us consider a toy network with four roadway junctions (nodes 0–3) providing access to three parking lots (nodes P1–P3) by directional links (numbered from 1 to 7 on Fig. 3). Node 0 is taken as an origin node whereas a destination node D is linked to each of the three parking lots by a pedestrian link. The lot capacities are of 350, 850 and 1300 veh, respectively, and the walking distances to destination of .2, .3 and .4 km, respectively.

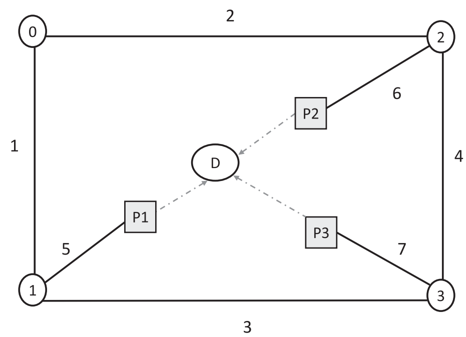

Fig. 3. Toy network.

Table 1
Characteristics of roadway links.

Link	From node	To node	Length (km)	Free flow time (min)
1	0	1	1.2	1.44
2	0	2	1	1.20
3	1	3	0.8	0.96
4	2	3	0.5	0.80
5	1	P1	0.1	0.20
6	2	P2	0.15	0.30
7	3	P3	0.1	0.20

Table 1 provides the rest of supply settings: link length and free-flow travel time t_{ff}. By traffic direction, every link has a BPR travel time function with flowing capacity of $k = 1000$ veh/h, factor of 1.1 and exponent of 5 on the ratio of flow x to capacity k: $t(x) = t_{ff}.\left[1 + 1.1\left(\frac{x}{k}\right)^5\right]$.

On the demand side there is a single origin–destination pair from 0 to D but two segments of commuters versus non-commuters. Each segment has access to every parking lot. The walking speed is set to 3.6 km/h for every user. Search time has a penalty coefficient of 1.38 as compared to base car time, while the penalty factor of walk time amounts to 1.2 for commuters and 1.5 for non-commuters. Diversion between the parking lots is modelled by a multinomial logit model with parameter $\theta = .1$/min for both segments.

6.2. Parametric analysis

To demonstrate the model, the demand flow is varied from 0 to 2250 car trips in a 1 h period: the latter value amounts to 90% of the total parking capacity. By assumption it is equally shared between the two segments.

The case is designed so that under free flow conditions, lot 1 is preferred with physical time of 1.6 + 0 + 3.3 = 4.9 min along the run + search + walk sequence, yielding generalized time of 1.6 + 0 + 5.5 = 7.1 min to a commuter and of 1.6 + 0 + 4 = 5.6 min to a non-commuter. Lot 2 has physical time of 1.5 + 0 + 5 = 6.5 min, yielding generalized time of 9.8 min to a commuter and of 7.5 min to a non-commuter. Lot 3 has physical time of 2.2 + 0 + 6.7 = 9.9 min, yielding generalized time of 10.2 min to a commuter and of 13.2 min to a non-commuter.

As the demand flow is increased, lot 1 is filled progressively until its saturation at $Q = \kappa_1 = 350$ veh. Then lot 2 gets its share of demand, both directly as an attractive option for the non-commuter segment and indirectly as a diversion option for the other segment. As for lot 3, it is used as diversion lot from lot 1 beyond its saturation and also from lot 2 when the first two lots are jointly saturated, which happens beyond $Q = \kappa_1 + \kappa_2 = 1200$ veh.

Fig. 4 depicts the variation of the lot success probability according to demand inflow Q (right scale) together with lot parking load (left scale), whereas Fig. 5 shows the candidate volume by lot.

The overall cruising traffic, evaluated as the sum over the roadway links of the cruising flow times the link length, is depicted in Fig. 6. Its share of the total car traffic along the toy network is null for Q less than κ_1; then it increases more than linearly with Q. The cruising flows exert effects on the car link flows and in turn on the user costs – through the car path time from origin to target lot and even more through the search process (Fig. 7).

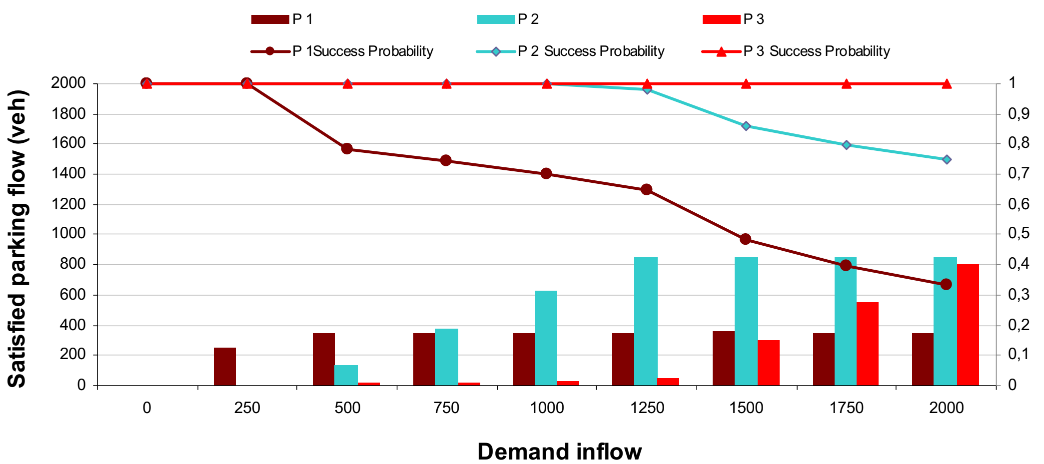

Fig. 4. Lot flow (left scale) and probability of success (right scale) w.r.t. demand inflow.

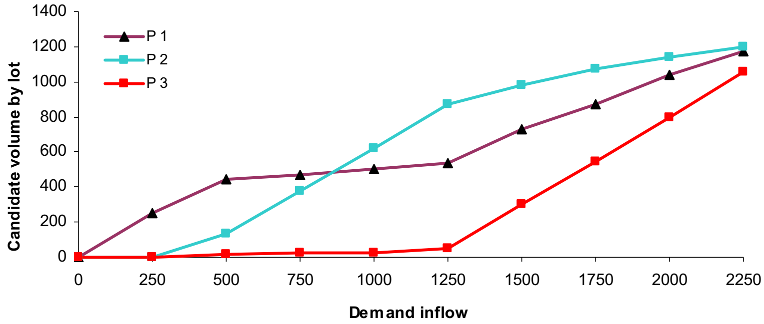

Fig. 5. Lot candidate flow w.r.t. demand inflow.

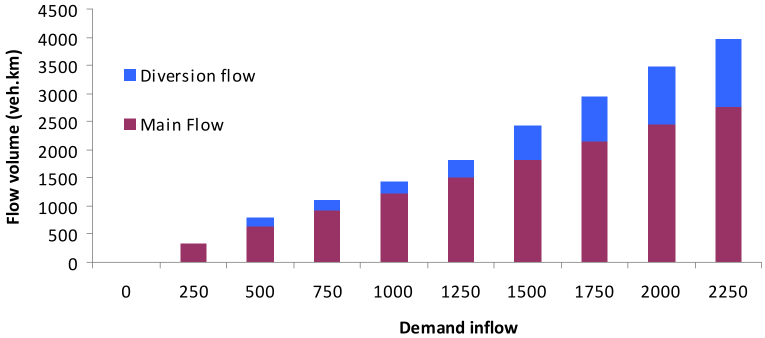

Fig. 6. Cruising traffic in addition to the base car traffic w.r.t. demand inflow.

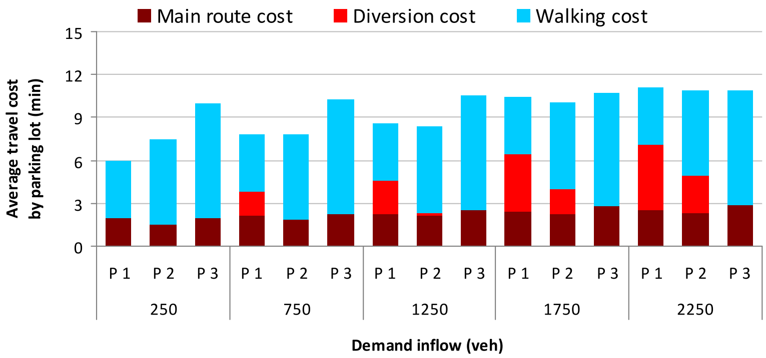

Fig. 7. Travel cost by target lot option to a non-commuter user.

7. Application to a planning study

7.1. Case presentation

Let us now report on an operational application to the "Cité Descartes" Area (CDA), which is a 1.2 km^2 area in the eastern fringe of the Grand Paris urbanized area at about 25 km from the city centre. As of 2012, the CDA accommodates about 5800 inhabitants, together with 6000 jobs and 15,000 students. It is endowed with a railway station served by the major regional train line (RER A) which traverses the Paris area along a west-east axis.

On the supply side, the road network is modelled by 259 nodes and 290 links, plus 473 parking access links and 243 nodes for parking lots (Fig. 8). There are 124 destination centroids – one by building set including that for the railway station.

The parking capacity amounts to 3320 slots on-street, 3100 private off-street and 590 in the three park-and-ride lots. No parking fee is levied in the area as of 2012, which is likely to change in the coming years since two new metro lines of the Grand Paris Express scheme will get connected at the railway station between 2020 and 2030.

On the demand side, three user classes of commuters (workers and students attracted by the CDA), visitors and train customers have been distinguished among the car users. Destination places are detailed finely whereas five roadway junctions

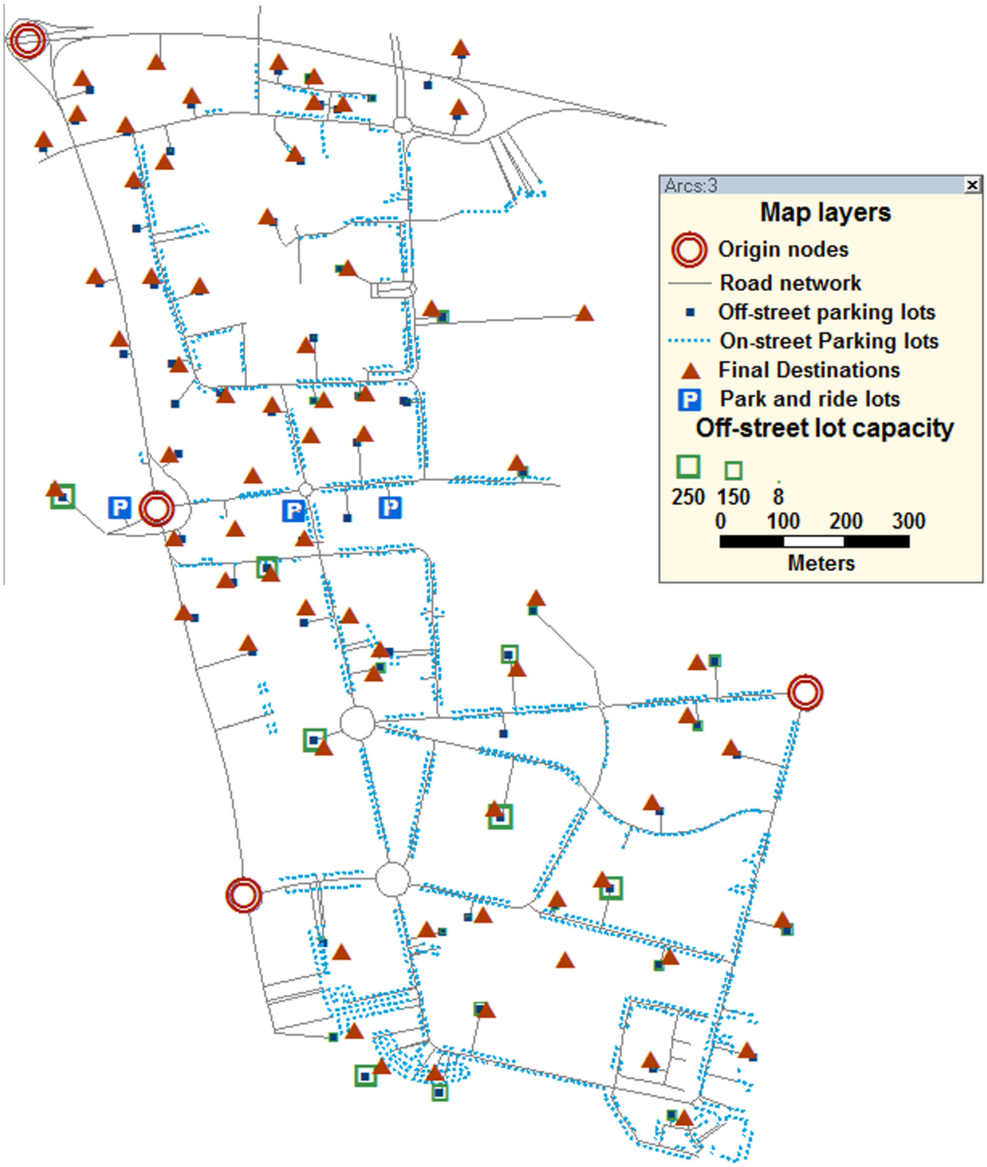

Fig. 8. The Cité Descartes area with its roadway network and parking lots.

are taken as origin centroids to model the access to the local area from the external world. Overall, 124 segments depict the local demand: their total flow amounts to 5106 veh at the morning peak hour, among which 4388 commuters, 79 visitors and 639 train riders.

For each demand segment, the lot subset L_s includes every parking lot for which the segment holds a right-of-access and which is located at a distance less than 900 meters from the destination point. Assumedly, every private lot is accessible only to their own segment. As in the toy instance, lot diversion is modelled by multinomial logit with parameter θ =.1/min; walk speed is set to 3.6 km/h; search time is penalized by a 1.38 factor relatively to base car time. The penalty factor for walk time is set to 1.2 for visitors and to 1.65 for workers and train customers.

A preliminary analysis of supply and demand was performed to evaluate the night occupant cars that remain in the CDA all day long, to be subtracted from the local parking capacity to yield the residual local capacities for the morning peak hour assignment.

7.2. Simulation results

The map of the lot probabilities of immediate parking success reveals the location of parking saturation throughout the study area (Fig. 9). A large saturated area emerges in the southern part due to three focal zones: first, the railway station (letter R on Fig. 9), second, the eastern buildings of the university campus (letter U) where the parking capacity is scarce relatively to the large numbers of students and employees, third, the western part of the economic activity zone (letter A). Within the saturated area, a lower probability of parking success indicates a higher ratio of demand to supply hence a sharper local shortage of parking capacity. Yet some parking facilities have positive residual capacity in that area: notably so in the western university block (letter W) since it cannot be accessed by the excess parkers of the activity zone.

Overall, this map is quite consistent with our own daily observations of the CDA, wherein our laboratory is located (letter L). The area activity has steadily increased over the past 10 years and so has the occupancy of on-street parking capacity, which has reached saturation along most of the southern roadway segments.

The cruising flows that come out of the simulation are depicted in Fig. 10. Although they increase significantly the roadway traffic around the focal zones of capacity saturation, yet their level is still far from reaching the link flowing capacities. No traffic signals have yet been implemented in the southern part of the CDA; the junctions have only priority regulation (including roundabouts) and there is no significant junction delay at the morning peak. Such is not the case at the evening peak when, once or twice a month, it happens that queues of cars extend from the closest motorway interchange (very close to zone A) up to the railway station (letter R).

So parking capacity makes the main stake of roadway planning in the CDA. The user cost of parking search amounts from 1 to 3 min on average, which is relatively important (Fig. 11). The comparison between the user classes demonstrates significant differences that are related to their destination places with respect to the parking lots with large capacity. Moreover the terminal walking time of train customers is relatively high, too. This is partly consistent with field data from a survey of parkers which we conducted in 2010 (Boujnah et al., 2013), in which train customers reported an average search delay of about 3 min, while the other types of parkers declared negligible delay.

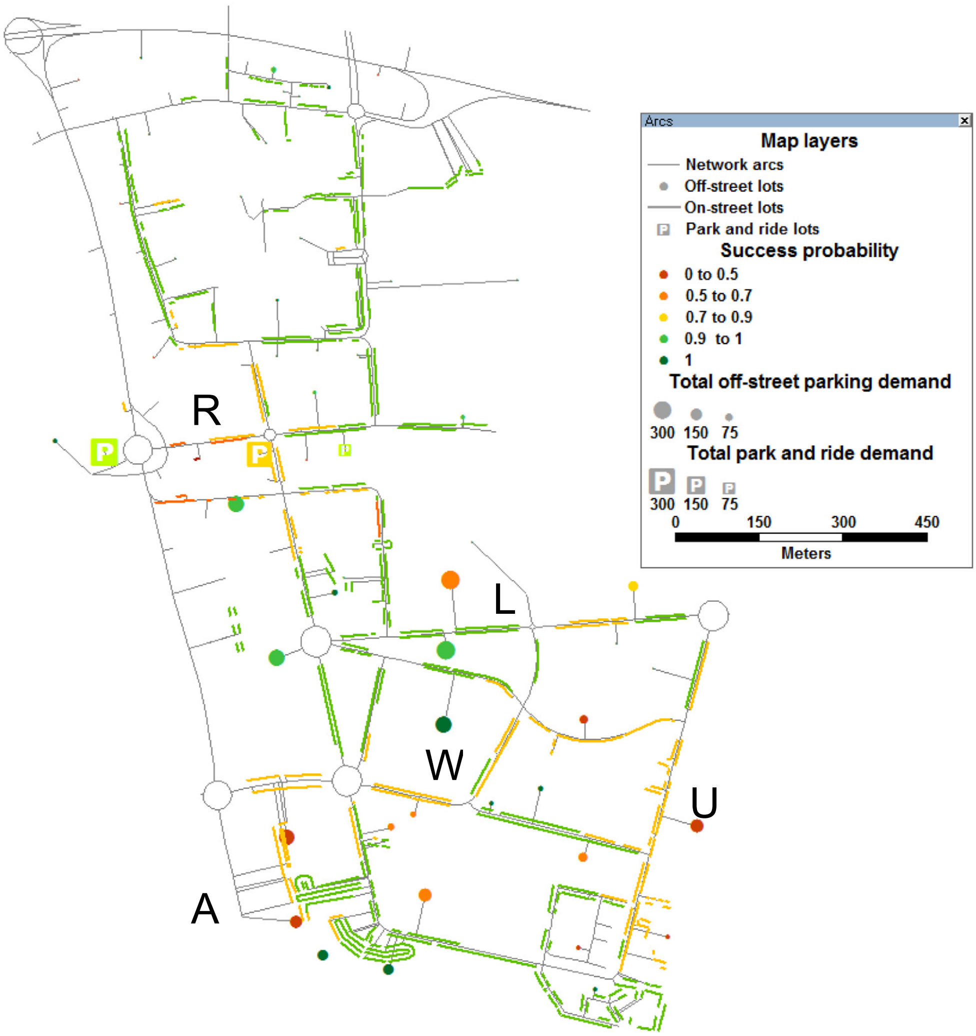

Fig. 9. Lot success probabilities and parking saturation at the morning peak hour.

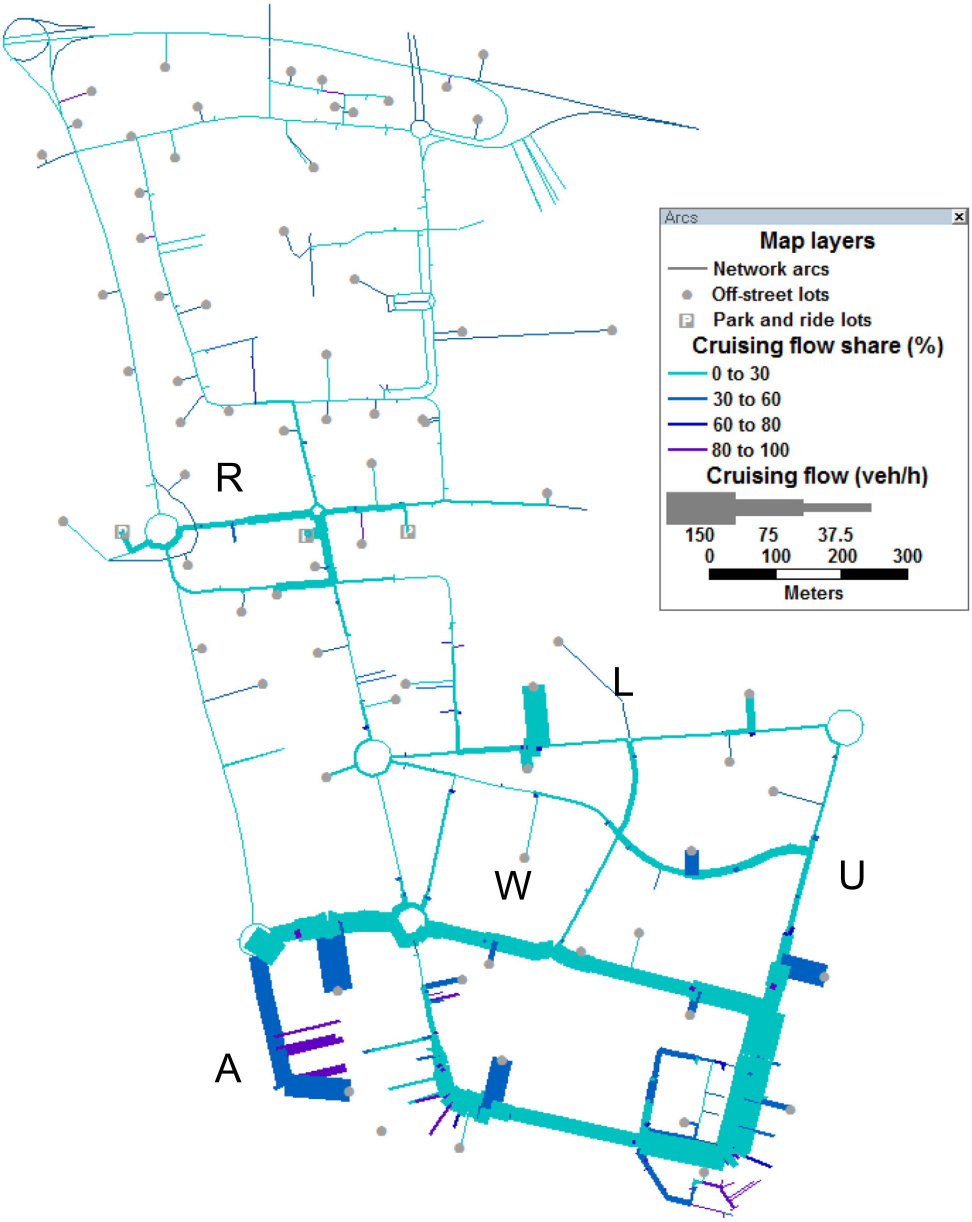

Fig. 10. Cruising flows at the morning peak hour.

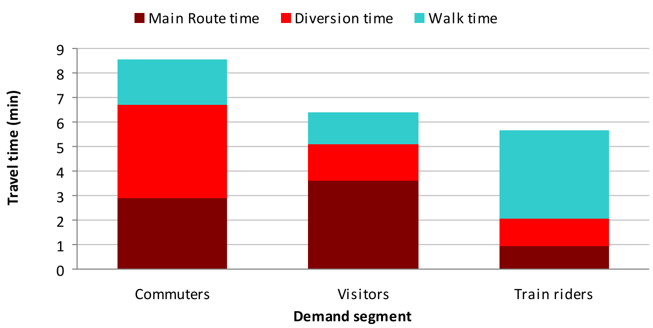

Fig. 11. User costs at the morning peak hour.

The conjunction of widespread parking saturation, on one hand, with relatively high average search time, on the other hand, make it obvious that the overall public parking capacity is insufficient. This calls for additional capacity investment, or alternatively for an improved usage of the most important private lots (cf. the W place on the CDA maps) – maybe so by some sharing scheme. So the model application provides clear insight for field analysis and urban planning.

About the model computation, let us just mention that one hundred of iterations of the MSA are sufficient to achieve a very good level of convergence. The computation time is very small since the model software is coded in C++ and applied to a medium-size case of assignment which involves only some hundreds of elements – zones, links, lots.

8. Conclusion

A model of parking supply and demand along a roadway network has been provided to simulate the occupancy of parking lots and its influence on user paths from origin to destination. Search loops may arise from the saturation of parking lots in a particular area, thus giving rise to cruising flows that add to, and interfere with, the core, "through" roadway traffic. To the individual user, the search for a parking lot with available residual capacity adds to the parking cost and in turn to the trip cost.

The mathematical framework is macroscopic, primarily so to address capacity issues and loading phenomena together with the traffic equilibrium of demand and supply. The demand is modelled as a set of segments that can be refined and tailored to any application requirement. The supply of parking is modelled as a set of lots, each one with exogenous capacity and tariff and endogenous load and candidate flows.

The user costs incurred due to lot saturation stem from diversion opportunities, not from an ad-hoc penalty function. Neighbouring parking lots are put in weak interaction by the demand flows, in a spatial process of diffusion. Thus our parking model is a mesoscopic physical model as it captures the relationship between the lots in terms of spatial features and of time costs to the users, whereas a microscopic model would identify each parking slot and at least detail the layout and internal paths inside each lot.

The lack of physical detail inside a lot makes a first direction of further research: the resulting slot search time should be added to the candidate user cost (at least in the absence of dynamic traffic information guidance about slot availability at the lot entrance) as well as to the lot transaction time of a user that is assigned to the lot. The order of magnitude of such lot internal search time will range from some seconds for an on-street lot to several tenths of seconds for a large off-street parking.

More generally, the parking time by car trip is likely to range from zero (for a private slot) to a handful of minutes: during an individual search the system state is likely to remain about stable, which makes our static treatment a reasonable one. Of course the dynamic extension of our model makes a second direction for further research: a multi-period treatment as in Nour Eldin et al. (1981) and Gallo et al. (2011) seems appropriate on the supply side and also on the demand side to address the choice of departure time. A dynamic extension would be most relevant to relate the access and egress of parking at the individual level in order to capture the tour features and to understand the travel mode choice better. The return issue is particularly important as concerns the terminal, walking sub-path: its costs to the user involve the walking distance twice (before and after the activity giving rise to the trip) and maybe also a memory effort to access a slot that has been assigned randomly.

There are many other directions of further research about the demand and supply of parking, among which:

- The behavioural process in the parking search, about lot departure, waiting strategy, local diversion, local turn strategy in relation to the current state of local traffic signals…
- The interplay of parking conditions with the choice of destination, particularly so for non-compulsory purposes such as shopping or personal services.
- The inclusion of urban freight deliveries in parking demand.
- The interplay of cruising flows (presumably with lower speed) with the rest of traffic, as well as that of savage parking with roadway capacity.

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Appendix. Invertibility of matrix $\mathbf{I}_s - \mathbf{J}_{s\alpha}\mathbf{p}_s$

Recall that $\mathbf{p}_s$ is a matrix of transition probabilities while $\mathbf{J}_{s\alpha}$ is the diagonal matrix of failure probabilities, $\bar{\alpha}_\ell = 1 - \alpha_\ell$ for $\ell \in L_s$. Assume that $\alpha_\ell > 0$ i.e. that every lot has positive capacity. Then $\bar{\alpha}_\ell < 1$. Consider the series of matrices, $\mathbf{M}^n = \sum_{i=0}^{n} (\mathbf{J}_{s\alpha}\mathbf{p}_s)^i$.

As matrix $\mathbf{J}_{s\alpha}$ is diagonal with coefficients in [0,1[and $\mathbf{p}_s$ is a probability matrix, the product $\mathbf{J}_{s\alpha}\mathbf{p}_s$ has modulus strictly less than one, hence $\mathbf{M}^n$ converges to a given matrix $\mathbf{M}^*$ as n tends to infinity. It holds that

$$(\mathbf{I}_s - \mathbf{J}_{s\alpha}\mathbf{p}_s)\mathbf{M}^n = \sum_{i=0}^{n}(\mathbf{J}_{s\alpha}\mathbf{p}_s)^i - \sum_{i=0}^{n}(\mathbf{J}_{s\alpha}\mathbf{p}_s)^{i+1} = \mathbf{I}_s - (\mathbf{J}_{s\alpha}\mathbf{p}_s)^{n+1},$$

so that

$$(\mathbf{I}_s - \mathbf{J}_{s\alpha}\mathbf{p}_s)\mathbf{M}* = \mathbf{I}_s$$

which shows that $\mathbf{M}^*$ is the inverse matrix of $\mathbf{I}_s - \mathbf{J}_{s\alpha}\mathbf{p}_s$.

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