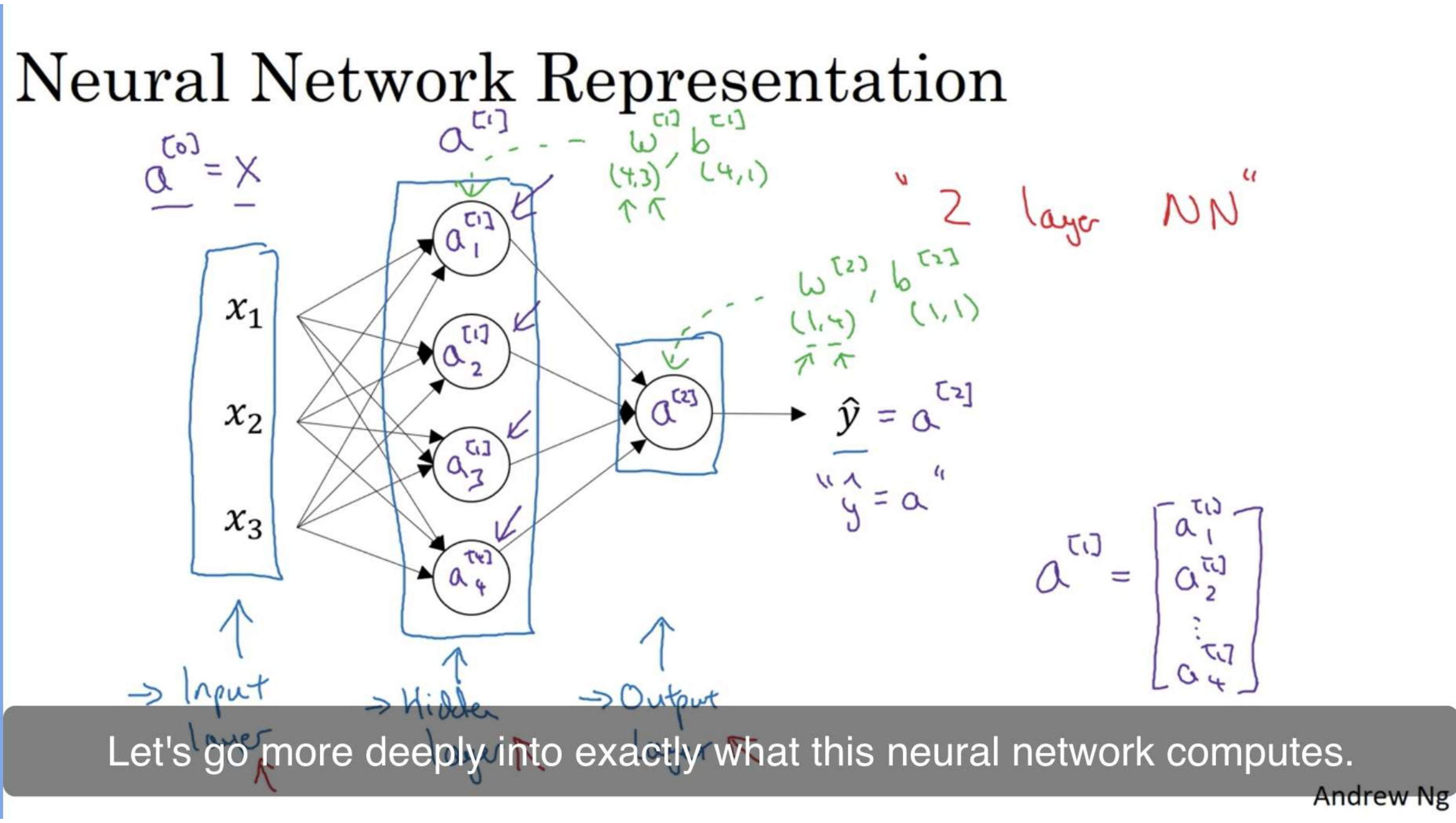
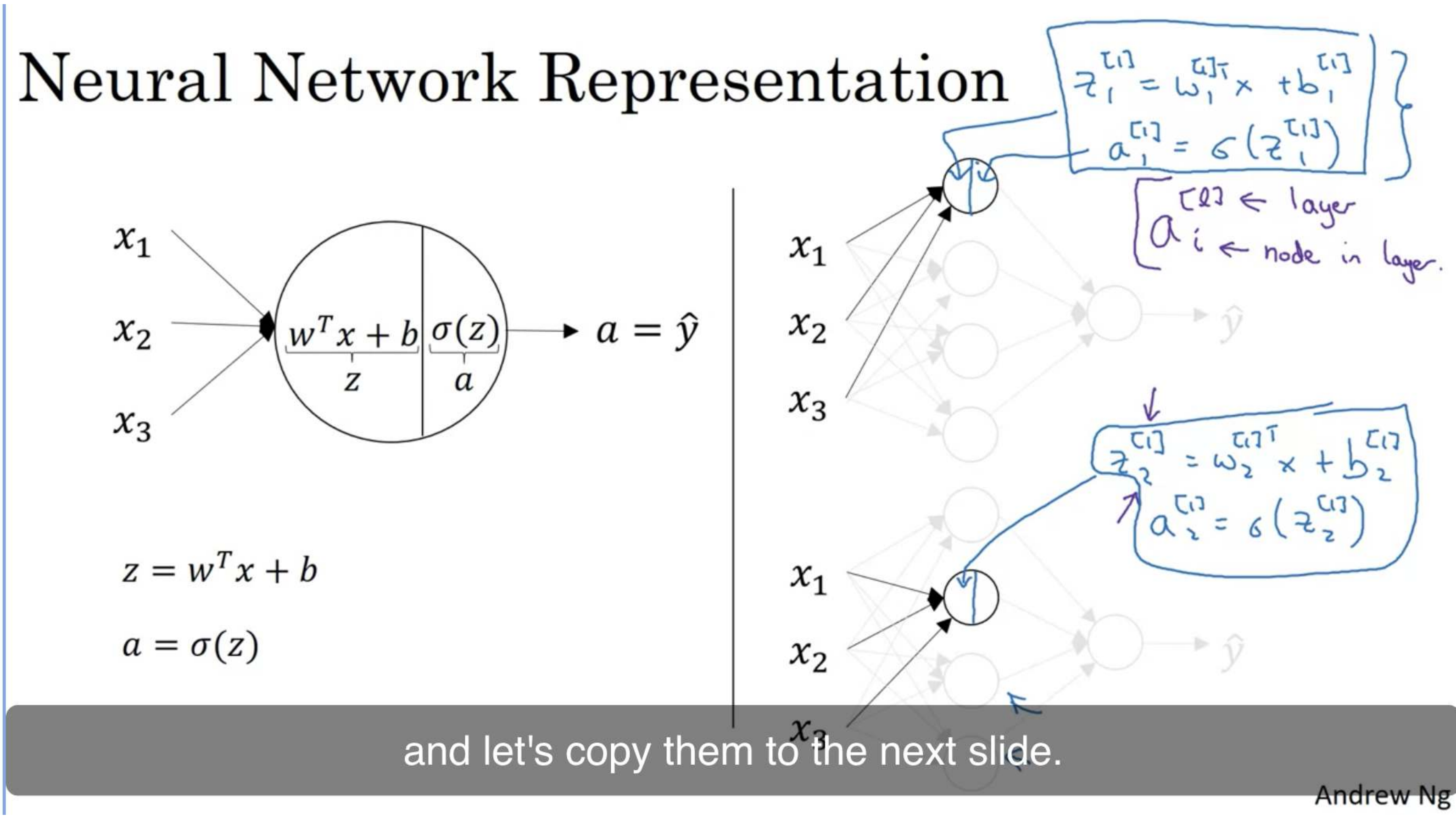


Neural Network Representation

Let's go more deeply into exactly what this neural network computes.

Andrew Ng



Neural Network Representation

$z = w^T x + b$

$a = \sigma(z)$

$z_1^{[1]} = w_1^{[1]T} x + b_1^{[1]}$

$a_1^{[1]} = \sigma(z_1^{[1]})$

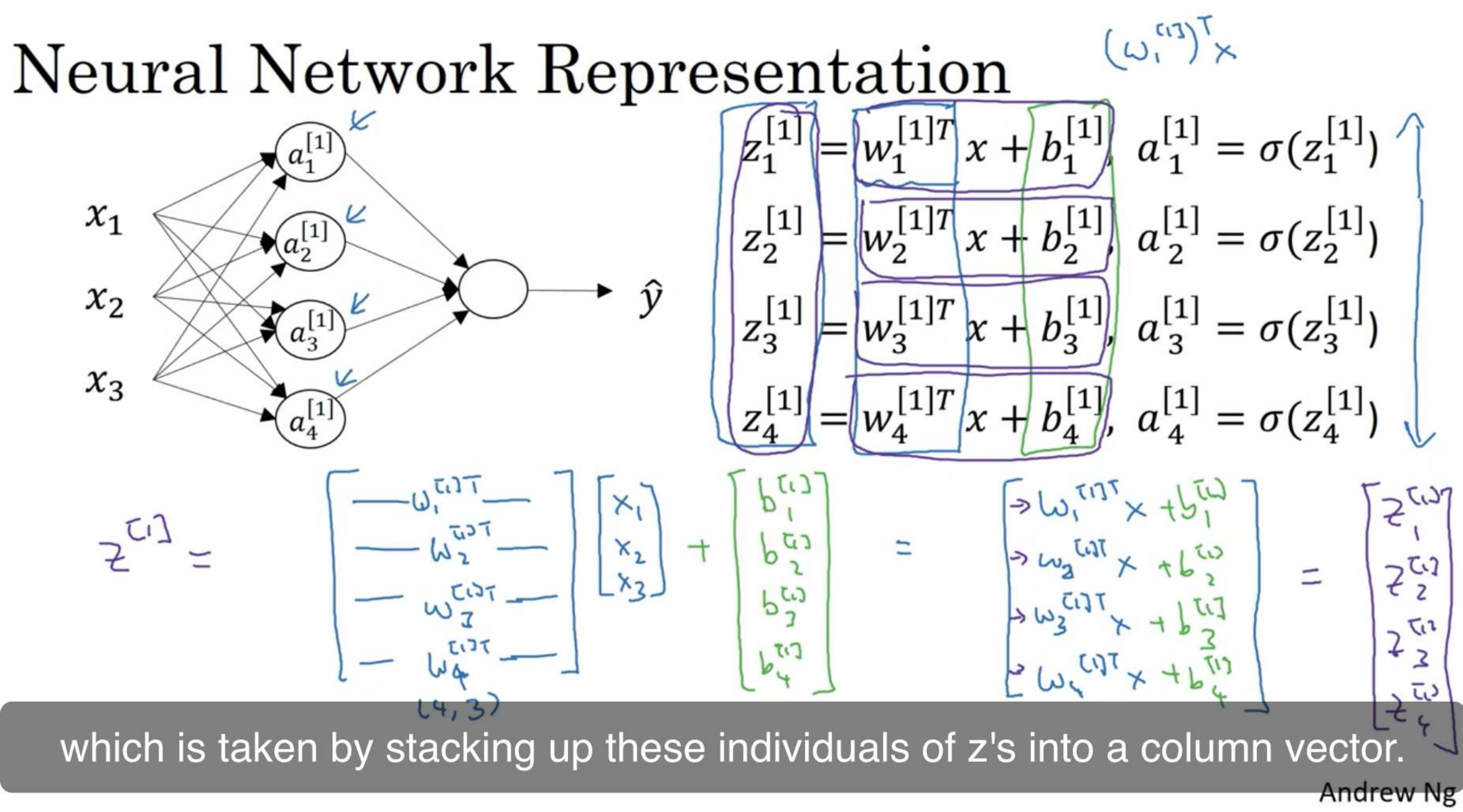
$a_i^{[l]}$ ← layer
← node in layer.

$z_2^{[1]} = w_2^{[1]T} x + b_2^{[1]}$

$a_2^{[1]} = \sigma(z_2^{[1]})$

and let's copy them to the next slide.

Andrew Ng



Neural Network Representation

$(w_1^{[1]})^T x$

$$z_1^{[1]} = w_1^{[1]T}x + b_1^{[1]}, \quad a_1^{[1]} = \sigma(z_1^{[1]})$$

$$z_2^{[1]} = w_2^{[1]T}x + b_2^{[1]}, \quad a_2^{[1]} = \sigma(z_2^{[1]})$$

$$z_3^{[1]} = w_3^{[1]T}x + b_3^{[1]}, \quad a_3^{[1]} = \sigma(z_3^{[1]})$$

$$z_4^{[1]} = w_4^{[1]T}x + b_4^{[1]}, \quad a_4^{[1]} = \sigma(z_4^{[1]})$$

$$z^{[1]} = \underset{(4,3)}{\begin{bmatrix} — w_1^{[1]T} — \\ — w_2^{[1]T} — \\ — w_3^{[1]T} — \\ — w_4^{[1]T} — \end{bmatrix}} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} b_1^{[1]} \\ b_2^{[1]} \\ b_3^{[1]} \\ b_4^{[1]} \end{bmatrix} = \begin{bmatrix} \rightarrow w_1^{[1]T}x + b_1^{[1]} \\ \rightarrow w_2^{[1]T}x + b_2^{[1]} \\ \rightarrow w_3^{[1]T}x + b_3^{[1]} \\ \rightarrow w_4^{[1]T}x + b_4^{[1]} \end{bmatrix} = \begin{bmatrix} z_1^{[1]} \\ z_2^{[1]} \\ z_3^{[1]} \\ z_4^{[1]} \end{bmatrix}$$

which is taken by stacking up these individuals of z's into a column vector.

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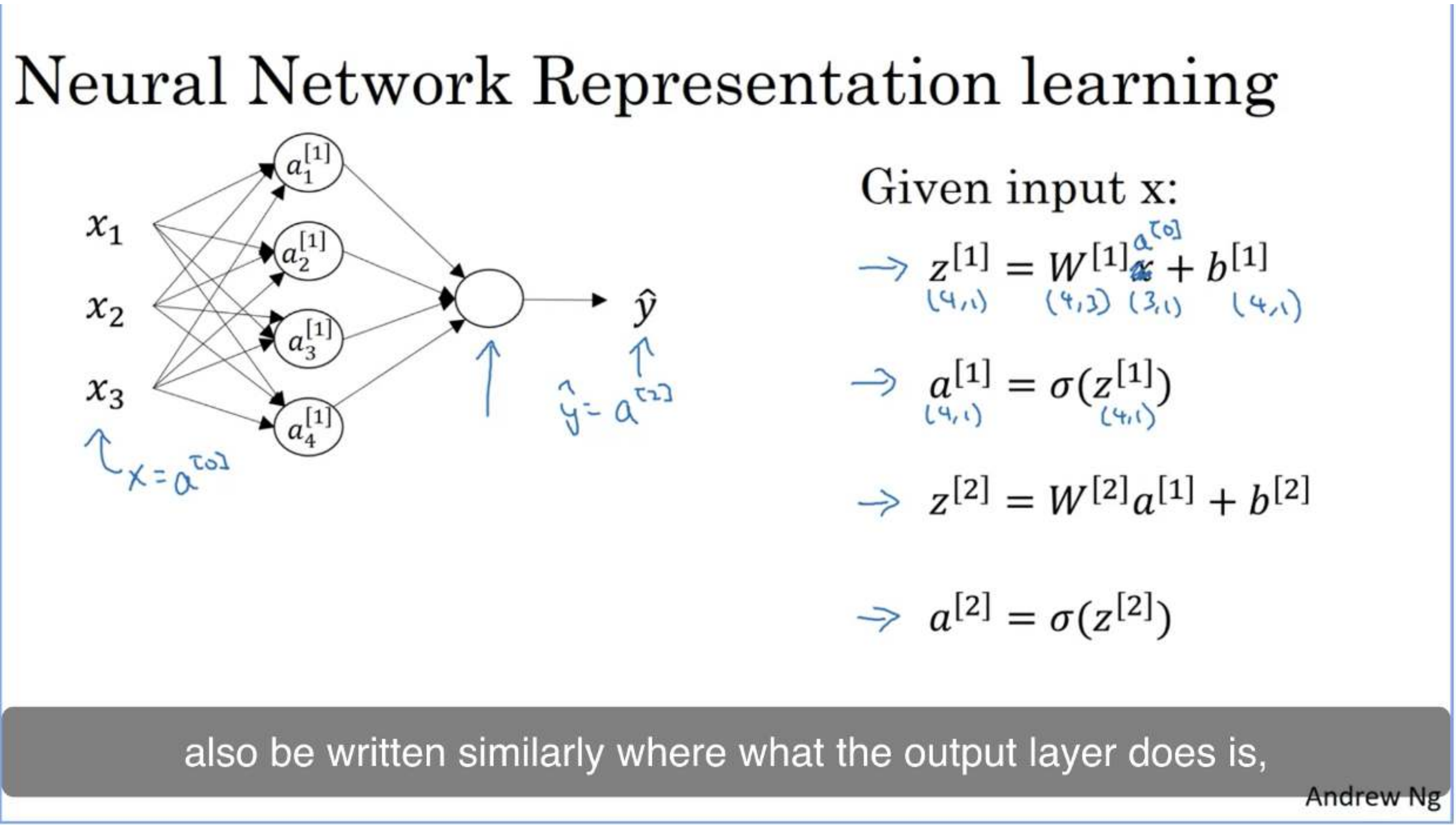

Neural Network Representation learning

Given input x:

$$\rightarrow \underset{(4,1)}{z^{[1]}} = \underset{(4,3)}{W^{[1]}} \underset{(3,1)}{\overset{a^{[0]}}{\cancel{x}}} + \underset{(4,1)}{b^{[1]}}$$

$$\rightarrow \underset{(4,1)}{a^{[1]}} = \sigma(\underset{(4,1)}{z^{[1]}})$$

$$\rightarrow z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$

$$\rightarrow a^{[2]} = \sigma(z^{[2]})$$

also be written similarly where what the output layer does is,

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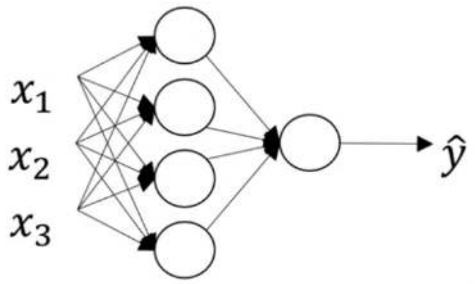
Vectorizing across multiple examples



$$\left\{\begin{aligned} z^{[1]} &= W^{[1]}x + b^{[1]} \\ a^{[1]} &= \sigma(z^{[1]}) \\ z^{[2]} &= W^{[2]}a^{[1]} + b^{[2]} \\ a^{[2]} &= \sigma(z^{[2]}) \end{aligned}\right\} \Leftarrow$$

$$X \longrightarrow a^{[2]} = \hat{y}$$

$$x^{(1)} \longrightarrow a^{[2](1)} = \hat{y}^{(1)}$$

$$x^{(2)} \longrightarrow a^{[2](2)} \quad \hat{y}^{(2)}$$

$$\vdots$$

$$x^{(m)} \longrightarrow a^{[2](m)} \quad \hat{y}^{(m)}$$

$a^{[2](i)}$ ↖ example i

↖ layer 2

for $i = 1$ to m,

$$z^{[1](i)} = W^{[1]} x^{(i)} + b^{[1]}$$

$$a^{[1](i)} = \sigma(z^{[1](i)})$$

$$z^{[2](i)} = W^{[2]} a^{[1](i)} + b^{[2]}$$

$$a^{[2](i)} = \sigma(z^{[2](i)})$$

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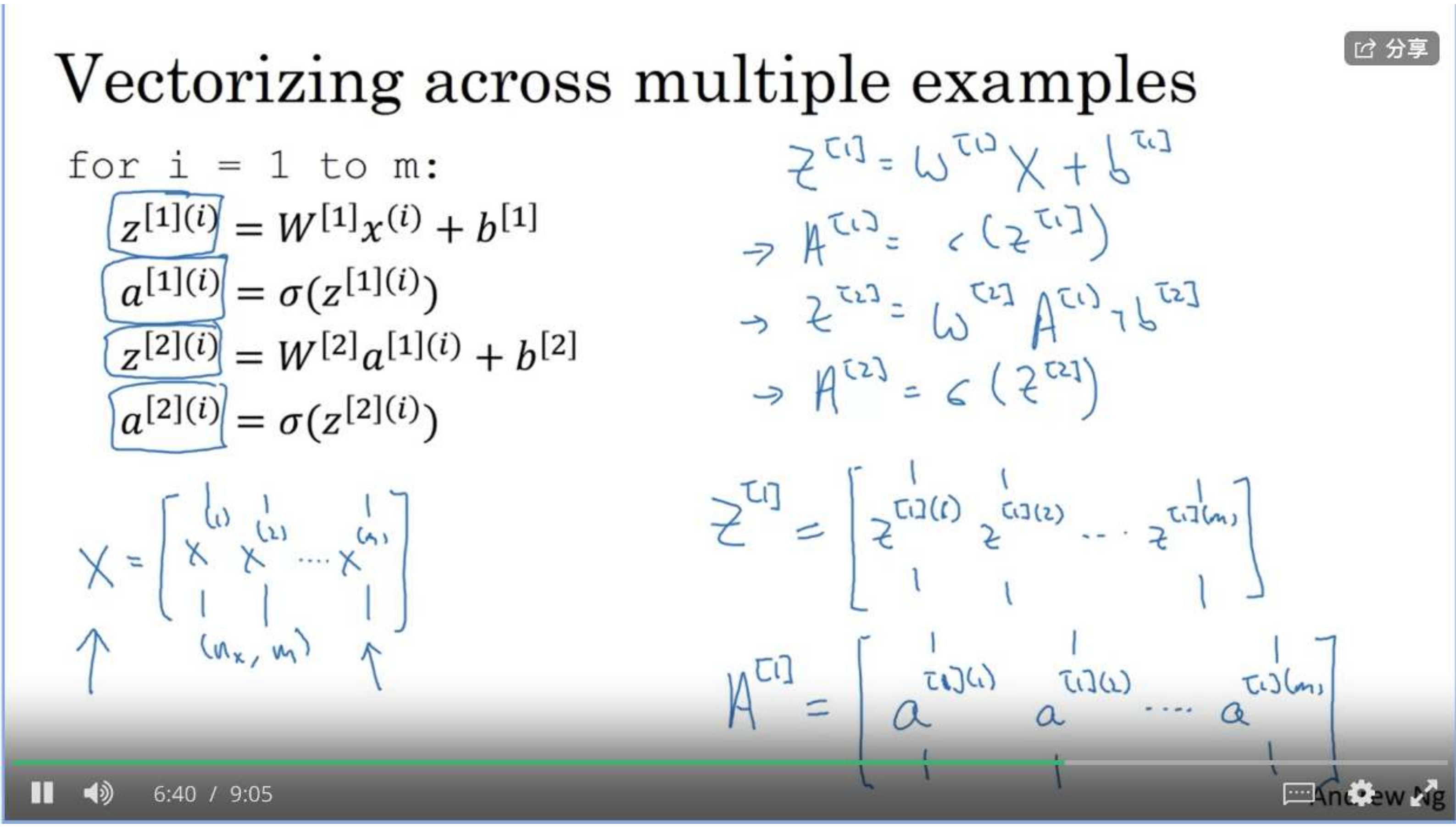


Vectorizing across multiple examples

```
for i = 1 to m:
    $\boxed{z^{[1](i)}} = W^{[1]}x^{(i)} + b^{[1]}$
    $\boxed{a^{[1](i)}} = \sigma(z^{[1](i)})$
    $\boxed{z^{[2](i)}} = W^{[2]}a^{[1](i)} + b^{[2]}$
    $\boxed{a^{[2](i)}} = \sigma(z^{[2](i)})$
```

$$Z^{[1]} = W^{[1]} X + b^{[1]}$$

$$\rightarrow A^{[1]} = \sigma(Z^{[1]})$$

$$\rightarrow Z^{[2]} = W^{[2]} A^{[1]} + b^{[2]}$$

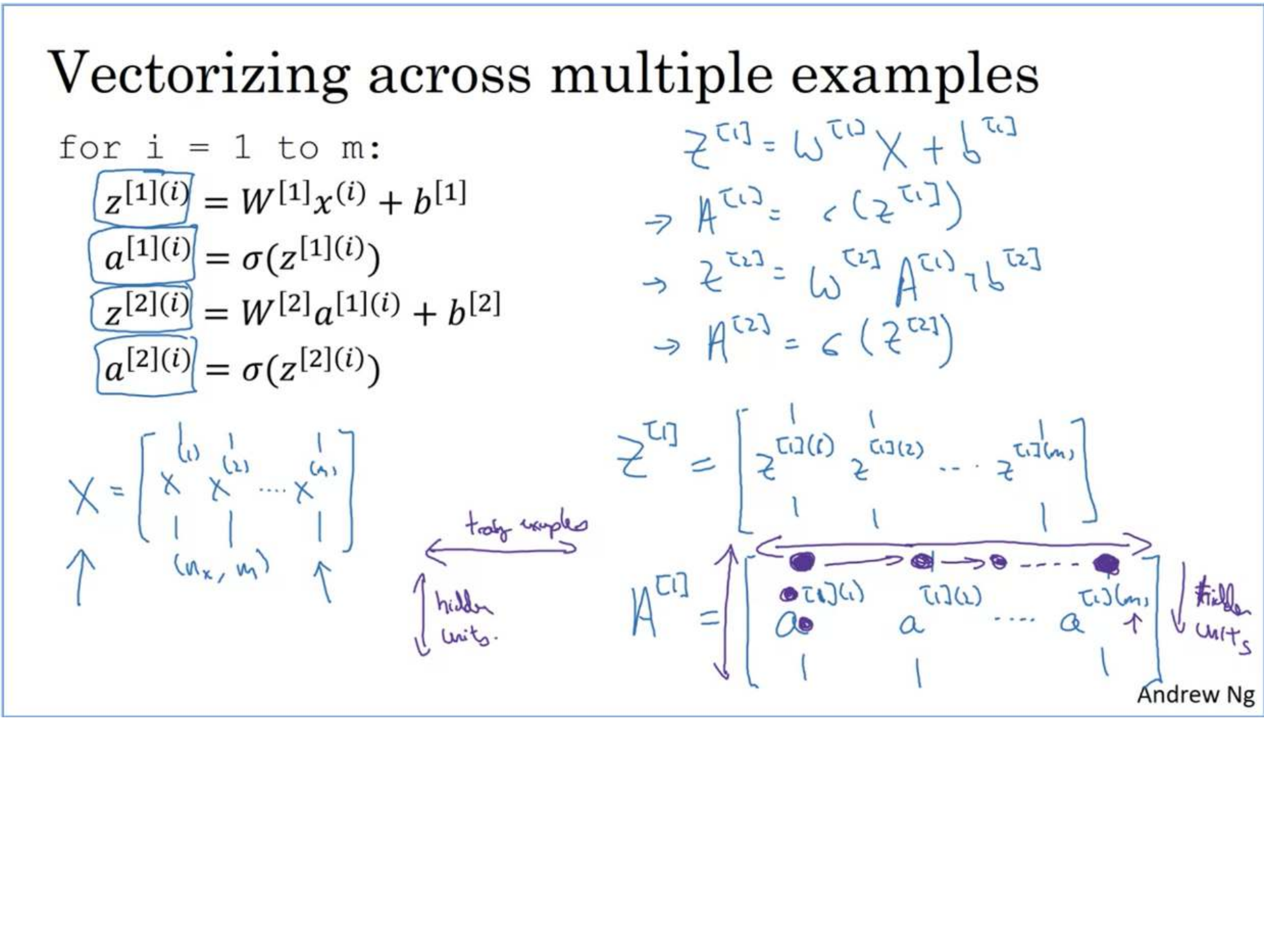
$$\rightarrow A^{[2]} = \sigma(Z^{[2]})$$

$$\underset{\uparrow}{X} = \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \cdots & x^{(m)} \\ | & | & & \underset{\uparrow}{|} \end{bmatrix}$$

(n_x, m)

$$Z^{[1]} = \begin{bmatrix} | & | & & | \\ z^{[1](1)} & z^{[1](2)} & \cdots & z^{[1](m)} \\ | & | & & | \end{bmatrix}$$

$$A^{[1]} = \begin{bmatrix} | & | & & | \\ a^{[1](1)} & a^{[1](2)} & \cdots & a^{[1](m)} \\ | & | & & | \end{bmatrix}$$



Vectorizing across multiple examples

```
for i = 1 to m:
    $\boxed{z^{[1](i)}} = W^{[1]}x^{(i)} + b^{[1]}$
    $\boxed{a^{[1](i)}} = \sigma(z^{[1](i)})$
    $\boxed{z^{[2](i)}} = W^{[2]}a^{[1](i)} + b^{[2]}$
    $\boxed{a^{[2](i)}} = \sigma(z^{[2](i)})$
```

$Z^{[1]} = W^{[1]} X + b^{[1]}$

$\rightarrow A^{[1]} = \sigma(Z^{[1]})$

$\rightarrow Z^{[2]} = W^{[2]} A^{[1]} + b^{[2]}$

$\rightarrow A^{[2]} = \sigma(Z^{[2]})$

$$X = \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \cdots & x^{(m)} \\ | & | & & | \end{bmatrix}$$

(n_x, m)

$$Z^{[1]} = \begin{bmatrix} | & | & & | \\ z^{[1](1)} & z^{[1](2)} & \cdots & z^{[1](m)} \\ | & | & & | \end{bmatrix}$$

$$A^{[1]} = \begin{bmatrix} | & | & & | \\ a^{[1](1)} & a^{[1](2)} & \cdots & a^{[1](m)} \\ | & | & & | \end{bmatrix}$$

training examples

hidden units.

#hidden units

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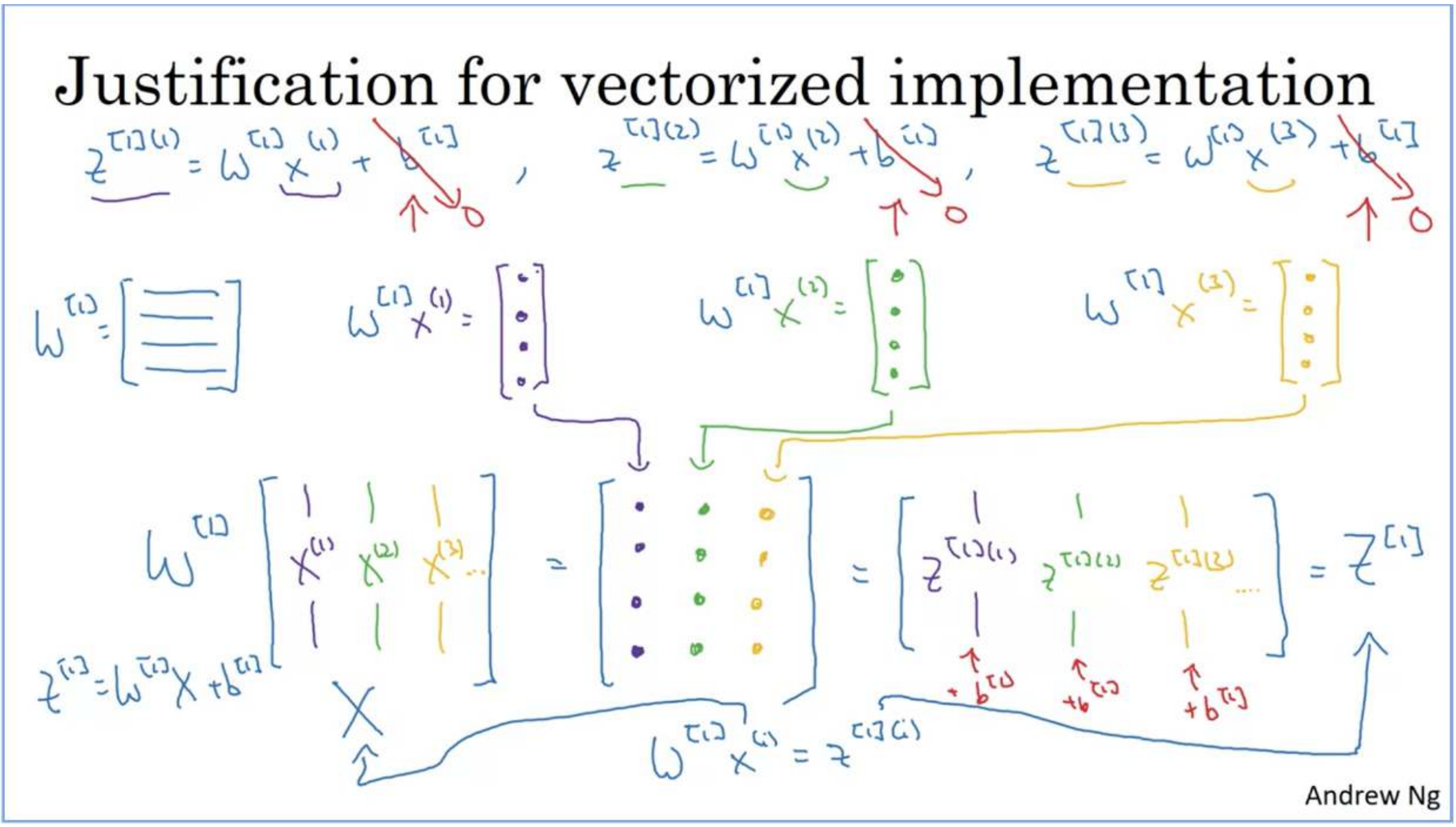

Justification for vectorized implementation

$$z^{[1](1)} = W^{[1]}x^{(1)} + \cancel{b^{[1]}} \rightarrow 0, \quad z^{[1](2)} = W^{[1]}x^{(2)} + \cancel{b^{[1]}} \rightarrow 0, \quad z^{[1](3)} = W^{[1]}x^{(3)} + \cancel{b^{[1]}} \rightarrow 0$$

$$W^{[1]} = \begin{bmatrix} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{bmatrix} \qquad W^{[1]}x^{(1)} = \begin{bmatrix} \bullet \\ \bullet \\ \bullet \\ \bullet \end{bmatrix} \qquad W^{[1]}x^{(2)} = \begin{bmatrix} \bullet \\ \bullet \\ \bullet \\ \bullet \end{bmatrix} \qquad W^{[1]}x^{(3)} = \begin{bmatrix} \bullet \\ \bullet \\ \bullet \\ \bullet \end{bmatrix}$$

$$W^{[1]} \underbrace{\begin{bmatrix} | & | & | & \\ x^{(1)} & x^{(2)} & x^{(3)} & \cdots \\ | & | & | & \end{bmatrix}}_{X} = \begin{bmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{bmatrix} = \begin{bmatrix} | & | & | & \\ z^{[1](1)} & z^{[1](2)} & z^{[1](3)} & \cdots \\ | & | & | & \end{bmatrix} = Z^{[1]}$$

$+b^{[1]}$ $+b^{[1]}$ $+b^{[1]}$

$$Z^{[1]} = W^{[1]}X + b^{[1]}$$

$$W^{[1]}x^{(i)} = z^{[1](i)}$$

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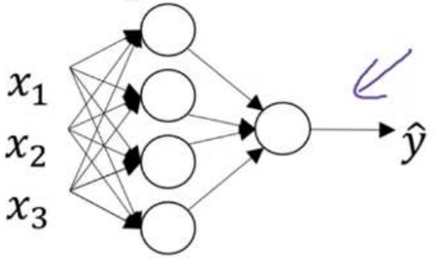
Recap of vectorizing across multiple examples



```
for i = 1 to m
    $\boxed{z^{[1](i)} = W^{[1]}x^{(i)} + b^{[1]}}$
    $a^{[1](i)} = \sigma(z^{[1](i)})$
    $z^{[2](i)} = W^{[2]}a^{[1](i)} + b^{[2]}$
    $a^{[2](i)} = \sigma(z^{[2](i)})$
```

$$X = \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \cdots & x^{(m)} \\ | & | & & | \end{bmatrix}$$

$$A^{[1]} = \begin{bmatrix} | & | & & | \\ a^{[1](1)} & a^{[1](2)} & \cdots & a^{[1](m)} \\ | & | & & | \end{bmatrix}$$

$X = a^{[0]}$ $x^{(i)} = a^{[0](i)}$

$$Z^{[1]} = W^{[1]}\overset{A^{[0]}}{X} + b^{[1]} \leftarrow W^{[1]}A^{[0]} + b^{[1]}$$

$$A^{[1]} = \sigma(Z^{[1]})$$

$$Z^{[2]} = W^{[2]}A^{[1]} + b^{[2]}$$

$$A^{[2]} = \sigma(Z^{[2]})$$

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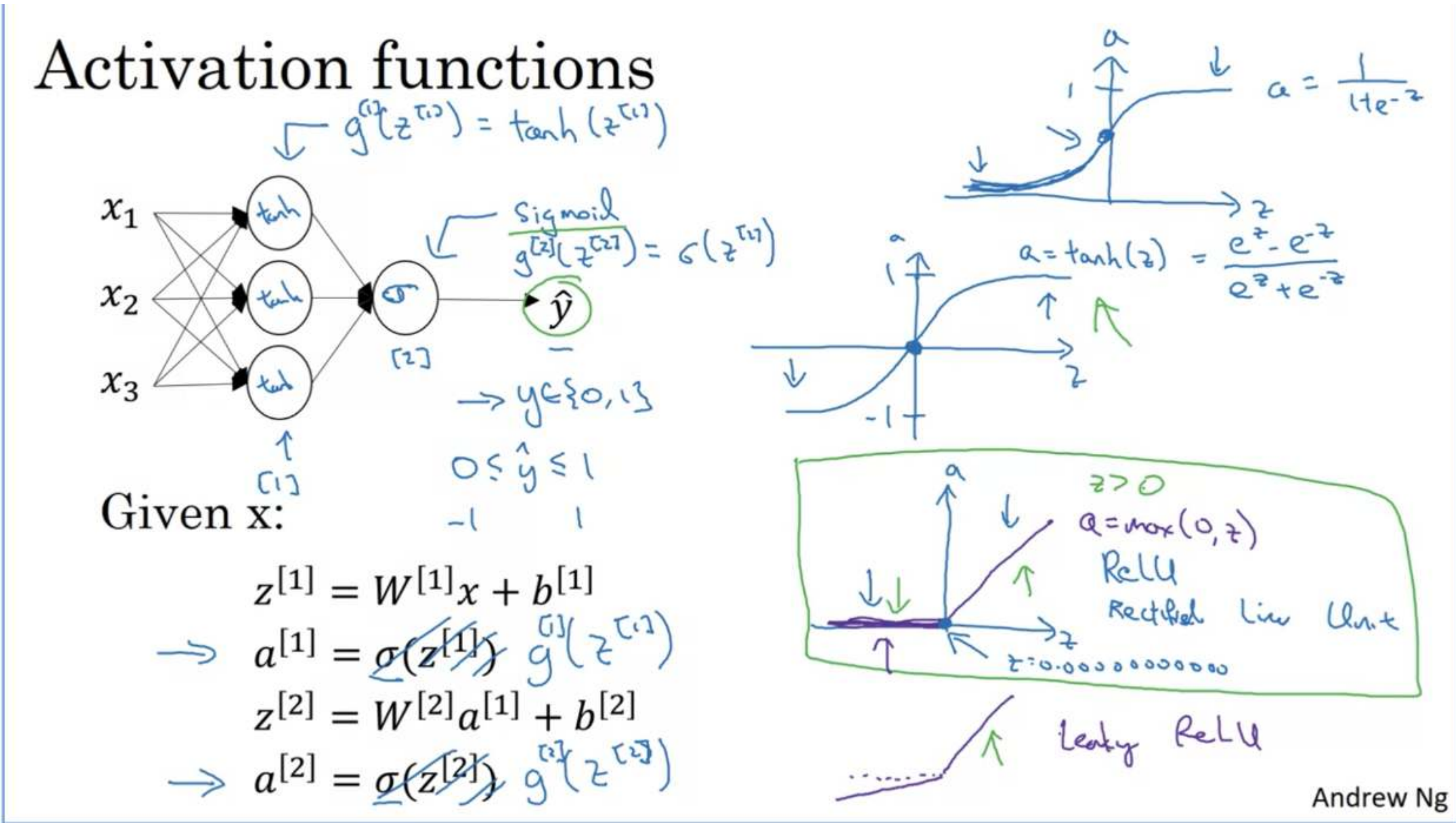
Activation functions



$g^{[1]}(z^{[1]}) = \tanh(z^{[1]})$

Sigmoid

$g^{[2]}(z^{[2]}) = \sigma(z^{[2]})$

$\rightarrow y \in \{0,1\}$

$0 \le \hat{y} \le 1$

$-1 \qquad 1$

Given x:

$$z^{[1]} = W^{[1]}x + b^{[1]}$$

$$\rightarrow \quad a^{[1]} = \cancel{\sigma(z^{[1]})}\; g^{[1]}(z^{[1]})$$

$$z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$

$$\rightarrow \quad a^{[2]} = \cancel{\sigma(z^{[2]})}\; g^{[2]}(z^{[2]})$$

$a = \frac{1}{1+e^{-z}}$

$a = \tanh(z) = \frac{e^{z} - e^{-z}}{e^{z} + e^{-z}}$

$z > 0$

$a = \max(0, z)$

ReLU

Rectified Linear Unit

$z = 0.000000000000$

Leaky ReLU

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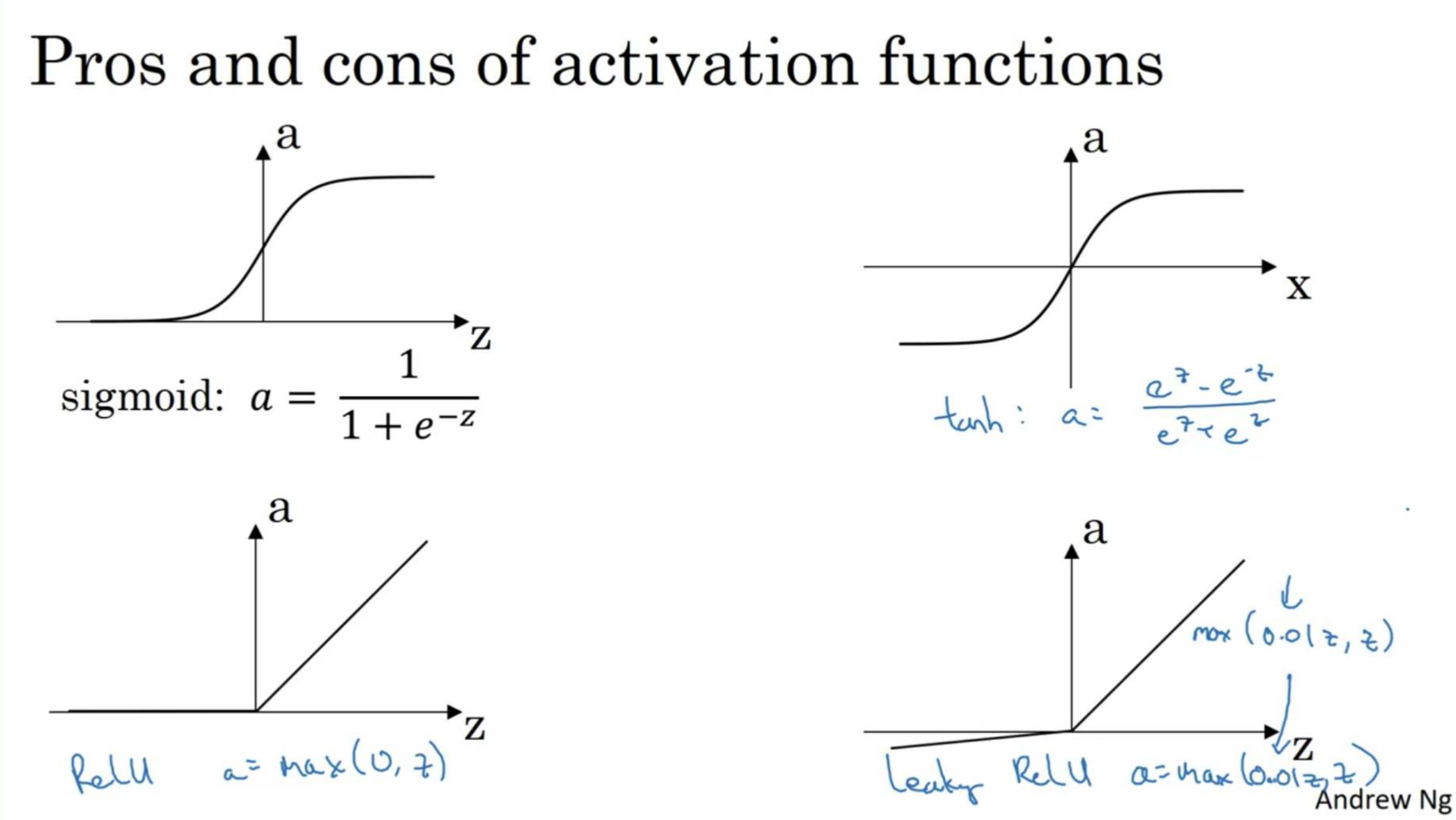

Pros and cons of activation functions

sigmoid: $a = \frac{1}{1 + e^{-z}}$

tanh: $a = \frac{e^{z} - e^{-z}}{e^{z} + e^{z}}$

ReLU $a = \max(0, z)$

$\max(0.01z, z)$

Leaky ReLU $a = \max(0.01z, z)$

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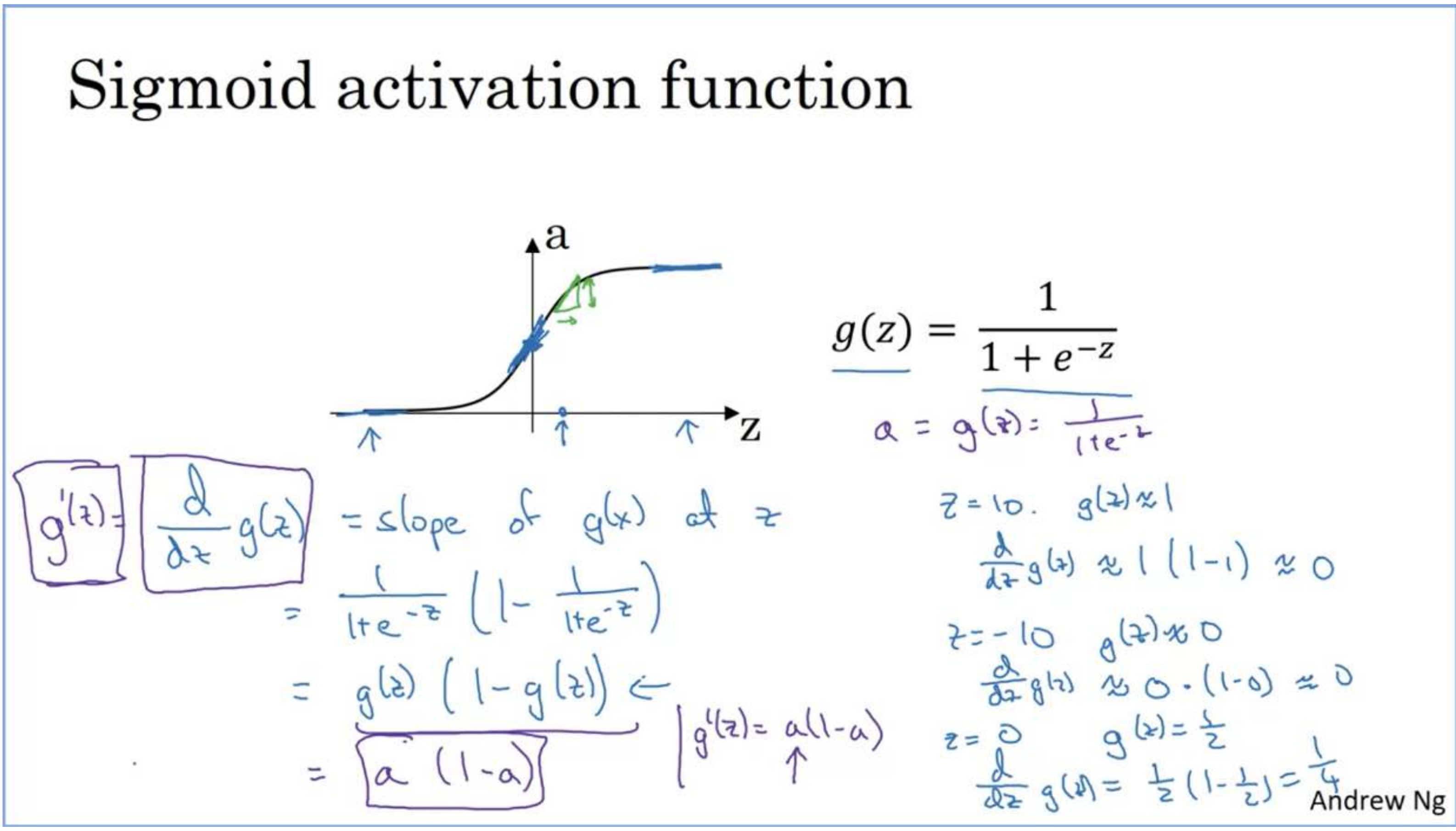

Sigmoid activation function

$$g(z) = \frac{1}{1 + e^{-z}}$$

$$a = g(z) = \frac{1}{1+e^{-z}}$$

$$\boxed{g'(z)} = \boxed{\frac{d}{dz} g(z)} = \text{slope of } g(x) \text{ at } z$$

$$= \frac{1}{1+e^{-z}} \left(1 - \frac{1}{1+e^{-z}}\right)$$

$$= \underbrace{g(z)\left(1 - g(z)\right)} \leftarrow$$

$$= \boxed{a\,(1-a)}$$

$$g'(z) = \underset{\uparrow}{a}(1-a)$$

$z = 10.\quad g(z) \approx 1$

$$\frac{d}{dz} g(z) \approx 1\,(1-1) \approx 0$$

$z = -10 \quad g(z) \approx 0$

$$\frac{d}{dz} g(z) \approx 0 \cdot (1-0) \approx 0$$

$z = 0 \quad g(z) = \frac{1}{2}$

$$\frac{d}{dz} g(z) = \frac{1}{2}\left(1 - \frac{1}{2}\right) = \frac{1}{4}$$

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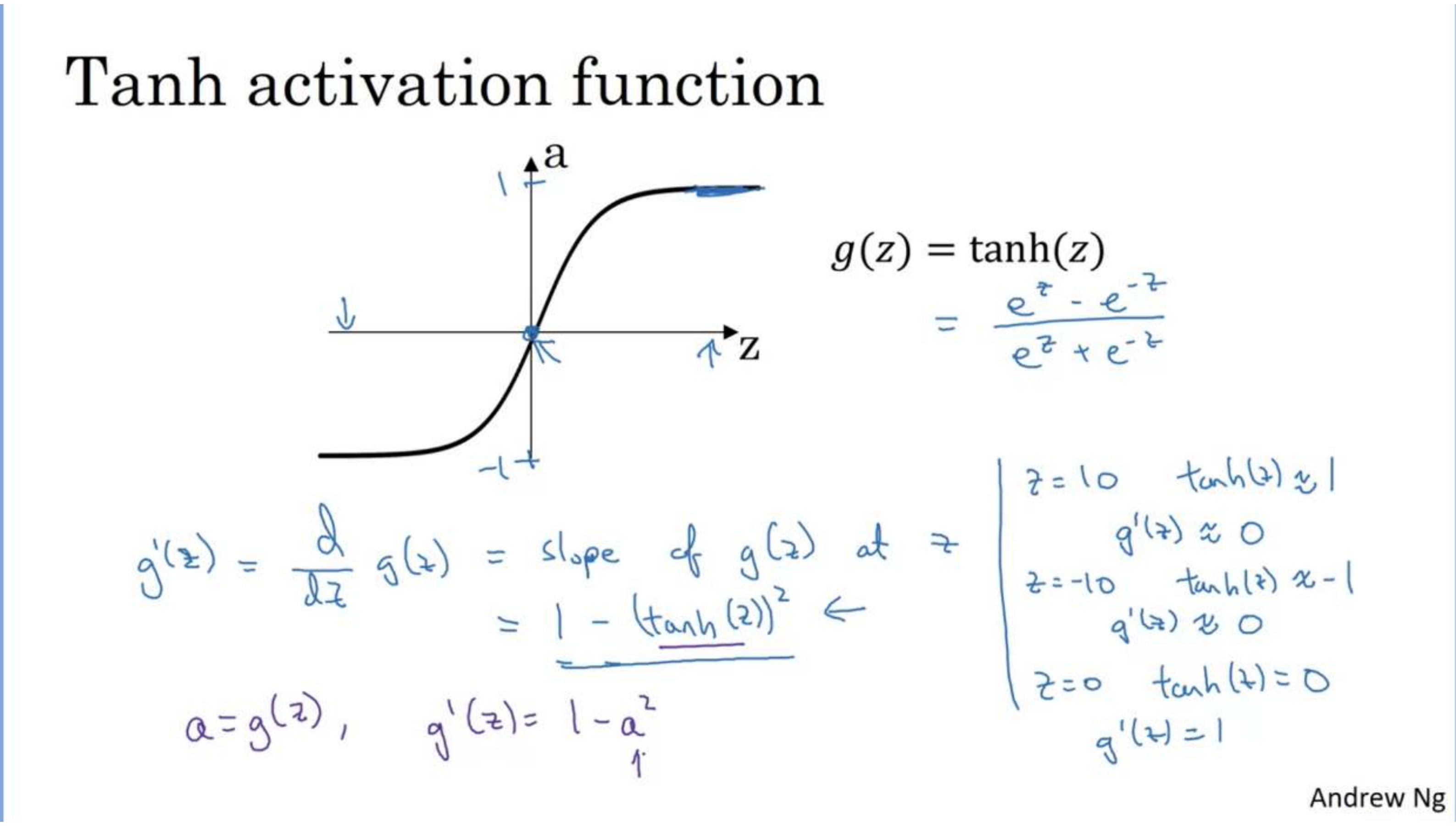

Tanh activation function

$g(z) = \tanh(z)$

$= \frac{e^{z} - e^{-z}}{e^{z} + e^{-z}}$

$g'(z) = \frac{d}{dz} g(z)$ = slope of $g(z)$ at z

$= \underline{1 - (\tanh(z))^2} \leftarrow$

$a = g(z), \quad g'(z) = 1 - a^2$

$z = 10 \quad \tanh(z) \approx 1$

$g'(z) \approx 0$

$z = -10 \quad \tanh(z) \approx -1$

$g'(z) \approx 0$

$z = 0 \quad \tanh(z) = 0$

$g'(z) = 1$

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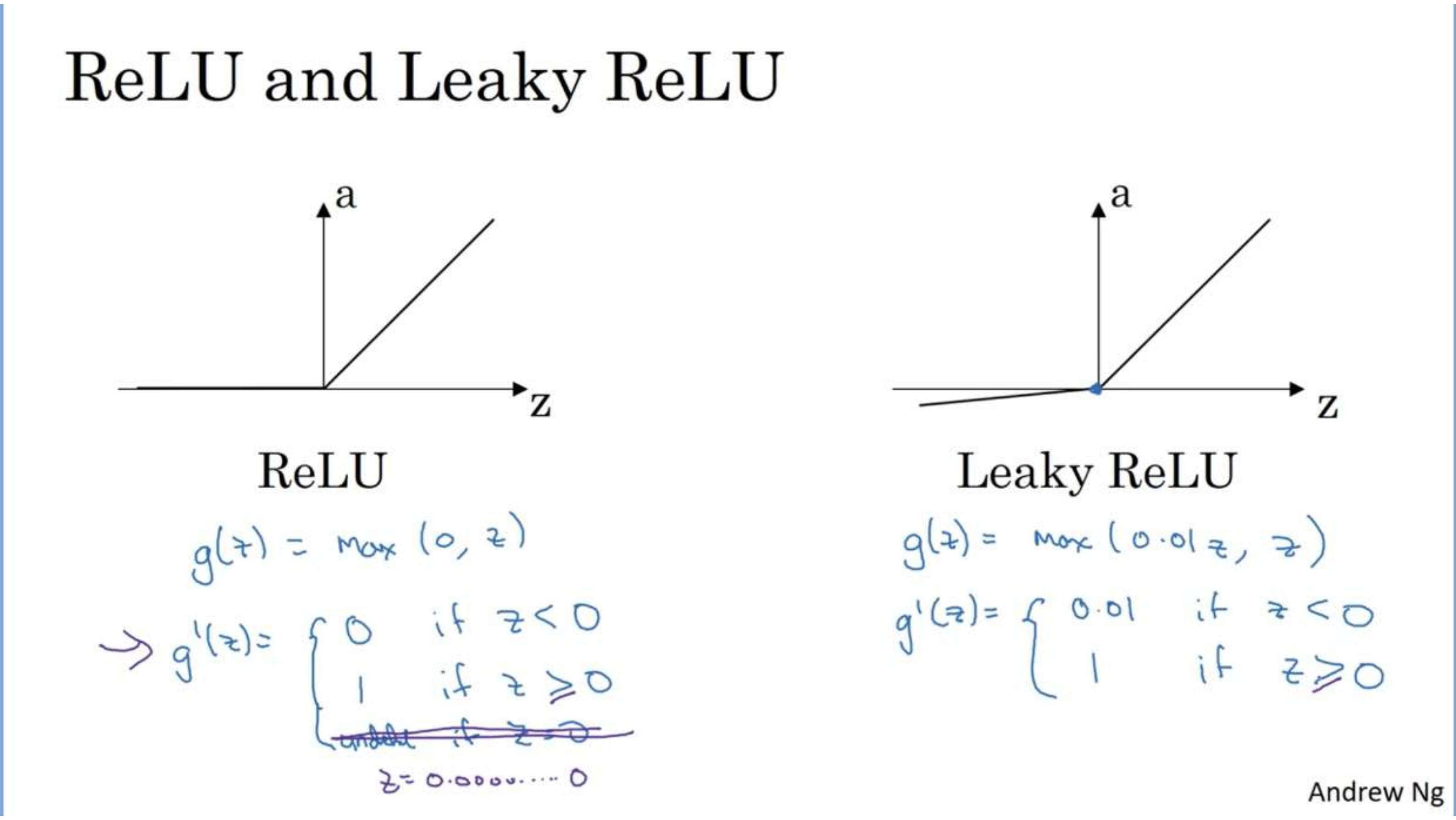

ReLU and Leaky ReLU

ReLU

$$g(z) = \max(0, z)$$

$$\rightarrow g'(z) = \begin{cases} 0 & \text{if } z < 0 \\ 1 & \text{if } z \geq 0 \\ \cancel{\text{undef} \quad \text{if } z = 0} \end{cases}$$

$$z = 0.0000 \cdots\cdots 0$$

Leaky ReLU

$$g(z) = \max(0.01z, z)$$

$$g'(z) = \begin{cases} 0.01 & \text{if } z < 0 \\ 1 & \text{if } z \geq 0 \end{cases}$$

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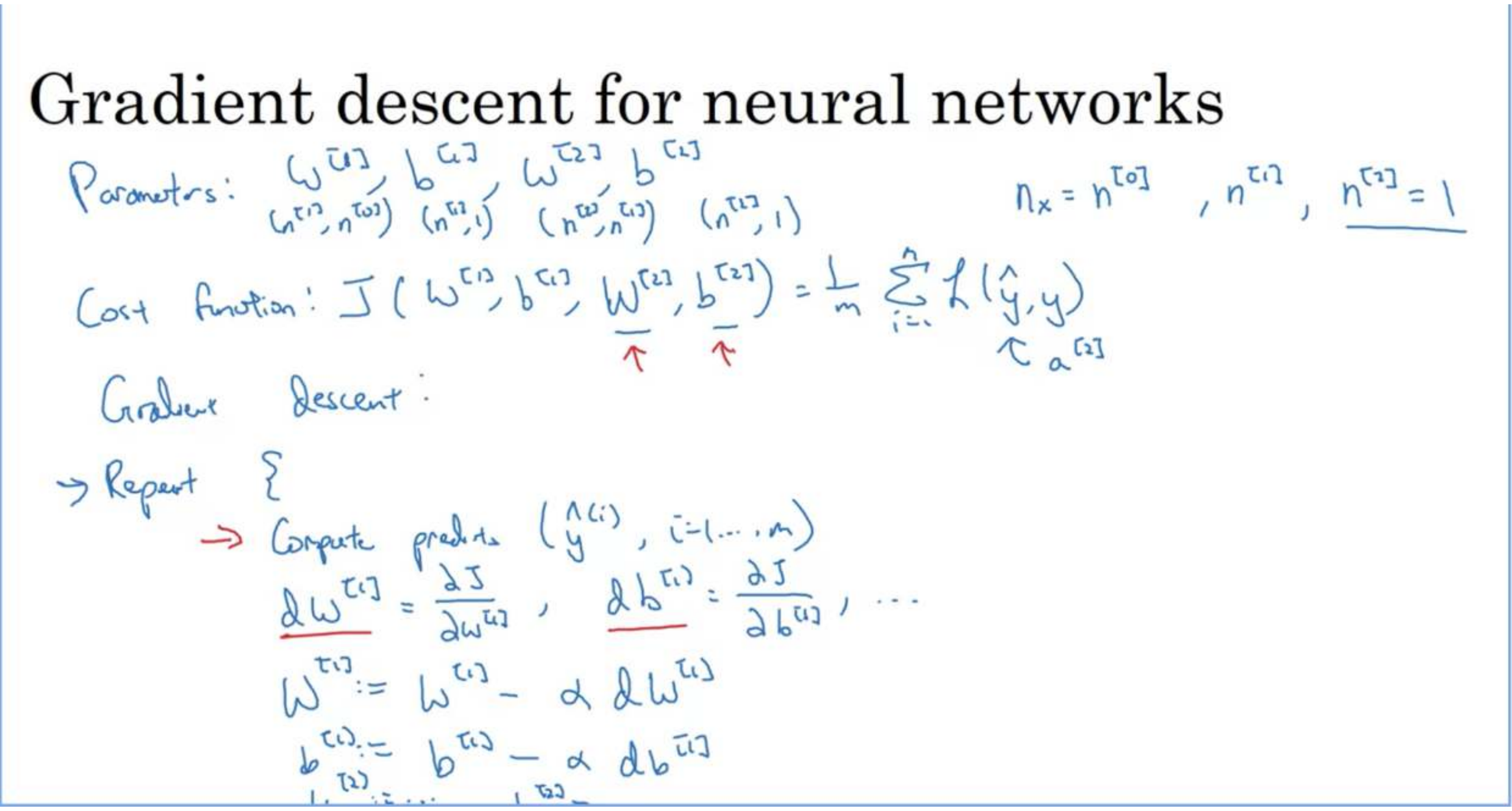


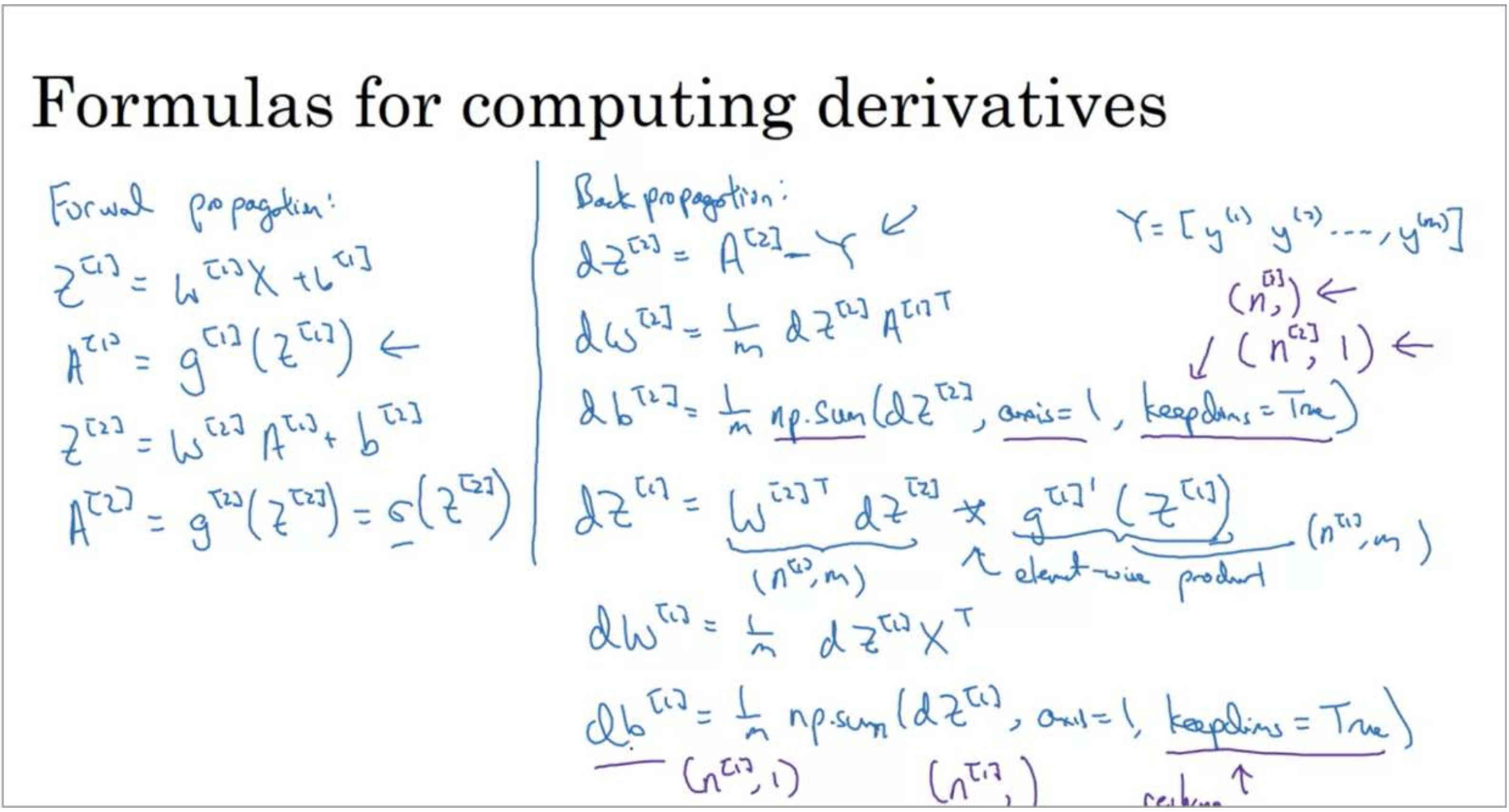
Gradient descent for neural networks

Parameters: $\underset{(n^{[1]}, n^{[0]})}{W^{[1]}}, \underset{(n^{[1]}, 1)}{b^{[1]}}, \underset{(n^{[2]}, n^{[1]})}{W^{[2]}}, \underset{(n^{[2]}, 1)}{b^{[2]}}$ $\quad n_x = n^{[0]}, n^{[1]}, \underline{n^{[2]} = 1}$

Cost function: $J(W^{[1]}, b^{[1]}, \underset{\uparrow}{\underline{W^{[2]}}}, \underset{\uparrow}{\underline{b^{[2]}}}) = \frac{1}{m} \sum_{i=1}^{n} \mathcal{L}(\underset{\uparrow a^{[2]}}{\hat{y}}, y)$

Gradient descent:

```
→ Repeat {
    → Compute predicts ($\hat{y}^{(i)}$, $i=1, \dots, m$)
    $\underline{dW^{[1]}} = \frac{\partial J}{\partial W^{[1]}}$, $\underline{db^{[1]}} = \frac{\partial J}{\partial b^{[1]}}$, ...
    $W^{[1]} := W^{[1]} - \alpha\, dW^{[1]}$
    $b^{[1]} := b^{[1]} - \alpha\, db^{[1]}$
    $b^{[2]} := \dots \; b^{[2]} -$
```



Formulas for computing derivatives

Forward propagation:

$Z^{[1]} = W^{[1]}X + b^{[1]}$

$A^{[1]} = g^{[1]}(Z^{[1]})$ ←

$Z^{[2]} = W^{[2]}A^{[1]} + b^{[2]}$

$A^{[2]} = g^{[2]}(Z^{[2]}) = \underline{\sigma}(Z^{[2]})$

Back propagation:

$dZ^{[2]} = A^{[2]} - Y$ ↙ $\quad Y = [y^{(1)}\ y^{(2)} \dots, y^{(m)}]$

$dW^{[2]} = \frac{1}{m} dZ^{[2]} A^{[1]T}$ $\quad (n^{[2]},)$ ←

$db^{[2]} = \frac{1}{m}\, \underline{np.sum}(dZ^{[2]}, \underline{axis=1}, \underline{keepdims = True})$ ↙ $(n^{[2]}, 1)$ ←

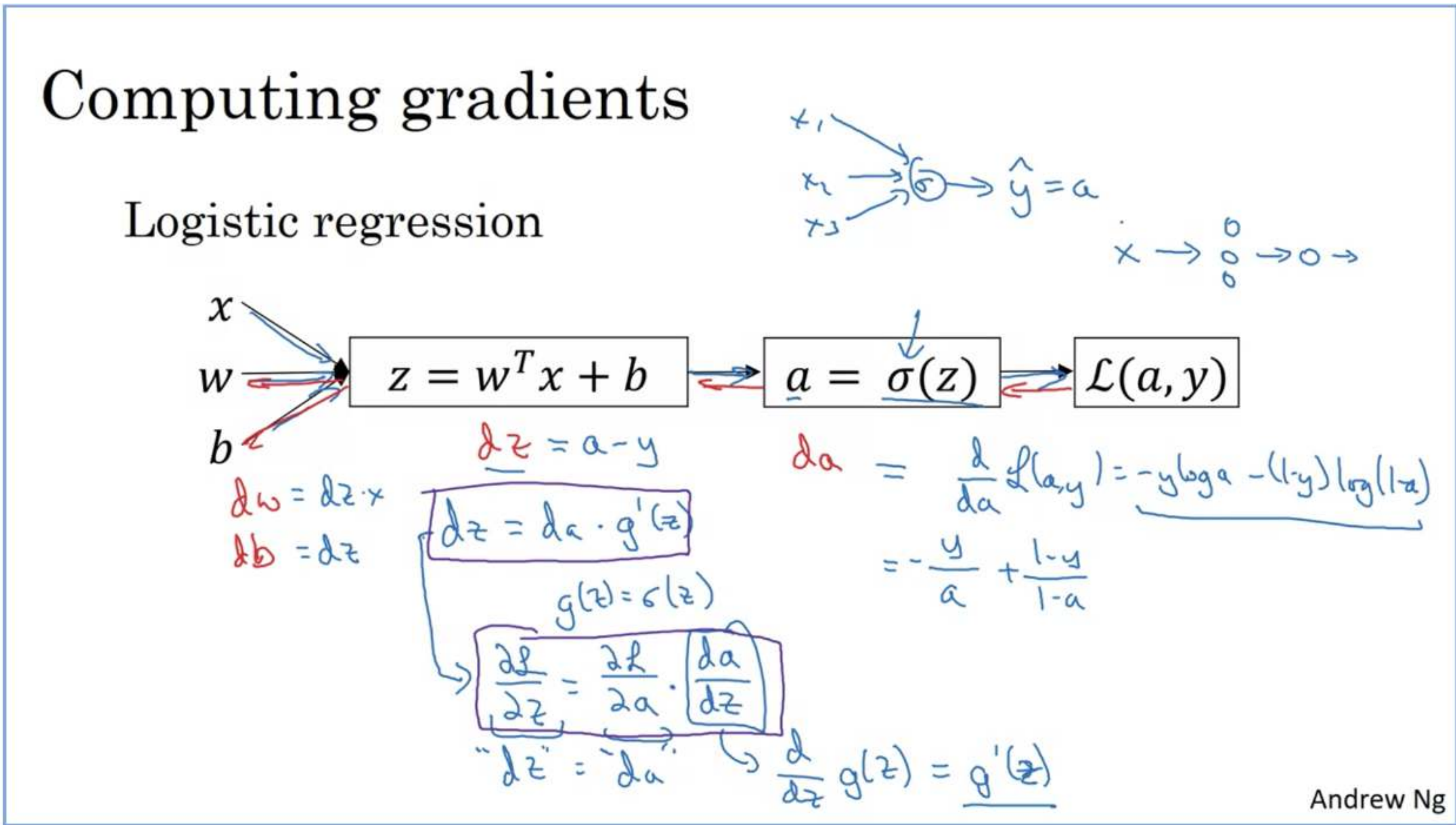
$dZ^{[1]} = \underbrace{W^{[2]T} dZ^{[2]}}_{(n^{[1]}, m)} * \underbrace{g^{[1]'}(Z^{[1]})}_{(n^{[1]}, m)}$

↑ element-wise product

$dW^{[1]} = \frac{1}{m} dZ^{[1]} X^T$

$\underline{db^{[1]}} = \frac{1}{m}\, np.sum(dZ^{[1]}, axis=1, \underline{keepdims = True})$

$(n^{[1]}, 1) \qquad (n^{[1]},)$ $\qquad$ reshape ↑



Computing gradients

Logistic regression

$$z = w^T x + b \rightarrow a = \sigma(z) \rightarrow \mathcal{L}(a, y)$$

$dz = a - y$

$dw = dz \cdot x$

$db = dz$

$\boxed{dz = da \cdot g'(z)}$

$g(z) = \sigma(z)$

$$\boxed{\underbrace{\frac{\partial \mathcal{L}}{\partial z}}_{\text{"}dz\text{"}} = \underbrace{\frac{\partial \mathcal{L}}{\partial a}}_{\text{"}da\text{"}} \cdot \boxed{\frac{da}{dz}}}$$

$\frac{d}{dz} g(z) = \underline{g'(z)}$

$$da = \frac{d}{da} \mathcal{L}(a,y) = \underbrace{-y \log a - (1-y) \log(1-a)}$$

$$= -\frac{y}{a} + \frac{1-y}{1-a}$$

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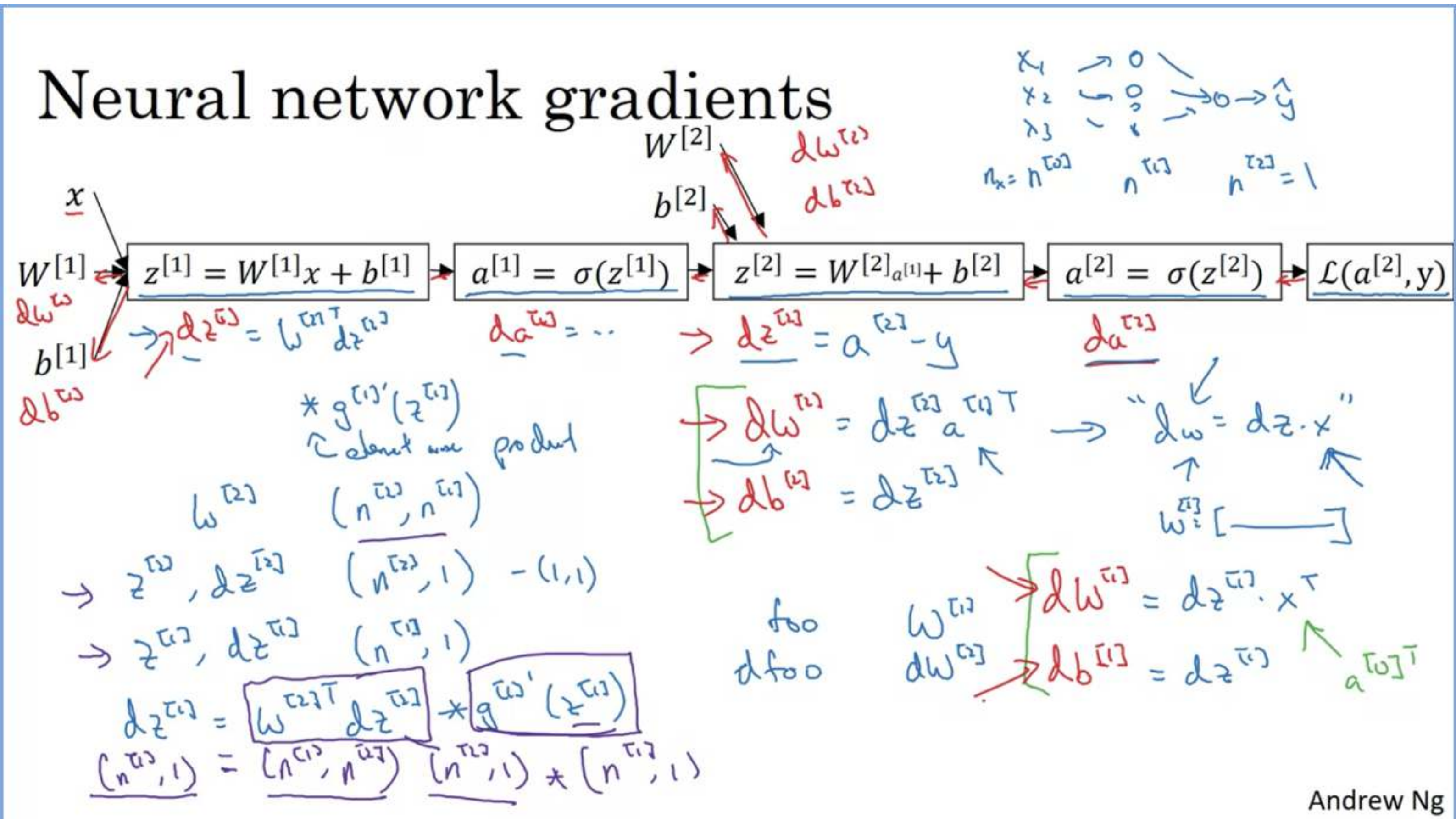

Neural network gradients

x
$W^{[1]}$
$b^{[1]}$
$dW^{[1]}$
$db^{[1]}$
$z^{[1]} = W^{[1]}x + b^{[1]}$
$a^{[1]} = \sigma(z^{[1]})$
$W^{[2]}$
$b^{[2]}$
$dW^{[2]}$
$db^{[2]}$
$z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$
$a^{[2]} = \sigma(z^{[2]})$
$\mathcal{L}(a^{[2]}, y)$
x_1
x_2
x_3
$\hat{y}$
$n_x = n^{[0]}$
$n^{[1]}$
$n^{[2]} = 1$

$dz^{[1]} = W^{[2]T} dz^{[2]} * g^{[1]\prime}(z^{[1]})$

element wise product

$da^{[1]} = \cdots$

$dz^{[2]} = a^{[2]} - y$

$da^{[2]}$

$dW^{[2]} = dz^{[2]} a^{[1]T}$

$db^{[2]} = dz^{[2]}$

"$dw = dz \cdot x$"

$w^{[2]}: [\text{———}]$

$W^{[2]}$ $(n^{[2]}, n^{[1]})$

$\to z^{[2]}, dz^{[2]}$ $(n^{[2]}, 1)$ $- (1,1)$

$\to z^{[1]}, dz^{[1]}$ $(n^{[1]}, 1)$

$$dz^{[1]} = \boxed{W^{[2]T} dz^{[2]}} * \boxed{g^{[1]\prime}(z^{[1]})}$$

$$(n^{[1]}, 1) = (n^{[1]}, n^{[2]}) \; (n^{[2]}, 1) * (n^{[1]}, 1)$$

foo $W^{[1]}$

dfoo $dW^{[1]}$

$dW^{[1]} = dz^{[1]} \cdot x^T$

$db^{[1]} = dz^{[1]}$

$a^{[0]T}$

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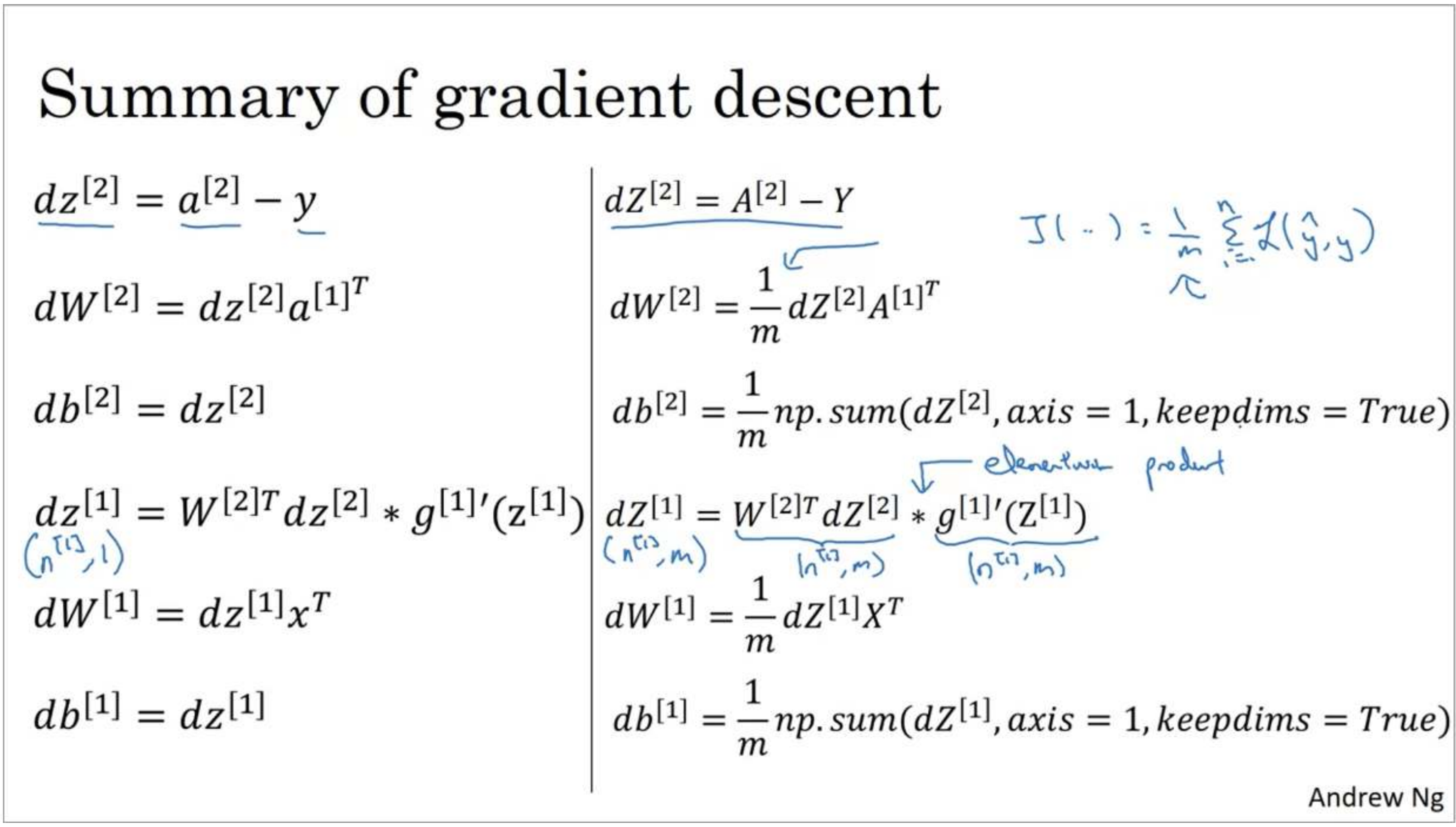

Summary of gradient descent

$$\underline{dz^{[2]}} = \underline{a^{[2]}} - \underline{y}$$

$$dW^{[2]} = dz^{[2]} a^{[1]^T}$$

$$db^{[2]} = dz^{[2]}$$

$$\underset{(n^{[1]},1)}{dz^{[1]}} = W^{[2]T} dz^{[2]} * g^{[1]\prime}(\mathrm{z}^{[1]})$$

$$dW^{[1]} = dz^{[1]} x^T$$

$$db^{[1]} = dz^{[1]}$$

$$\underline{dZ^{[2]} = A^{[2]} - Y}$$

$$J(\ldots) = \frac{1}{m} \sum_{i=1}^{n} \mathcal{L}(\hat{y}, y)$$

$$dW^{[2]} = \frac{1}{m} dZ^{[2]} A^{[1]^T}$$

$$db^{[2]} = \frac{1}{m} np.\,sum(dZ^{[2]}, axis = 1, keepdims = True)$$

elementwise product

$$\underset{(n^{[1]},m)}{dZ^{[1]}} = \underbrace{W^{[2]T} dZ^{[2]}}_{(n^{[1]},m)} * \underbrace{g^{[1]\prime}(\mathrm{Z}^{[1]})}_{(n^{[1]},m)}$$

$$dW^{[1]} = \frac{1}{m} dZ^{[1]} X^T$$

$$db^{[1]} = \frac{1}{m} np.\,sum(dZ^{[1]}, axis = 1, keepdims = True)$$

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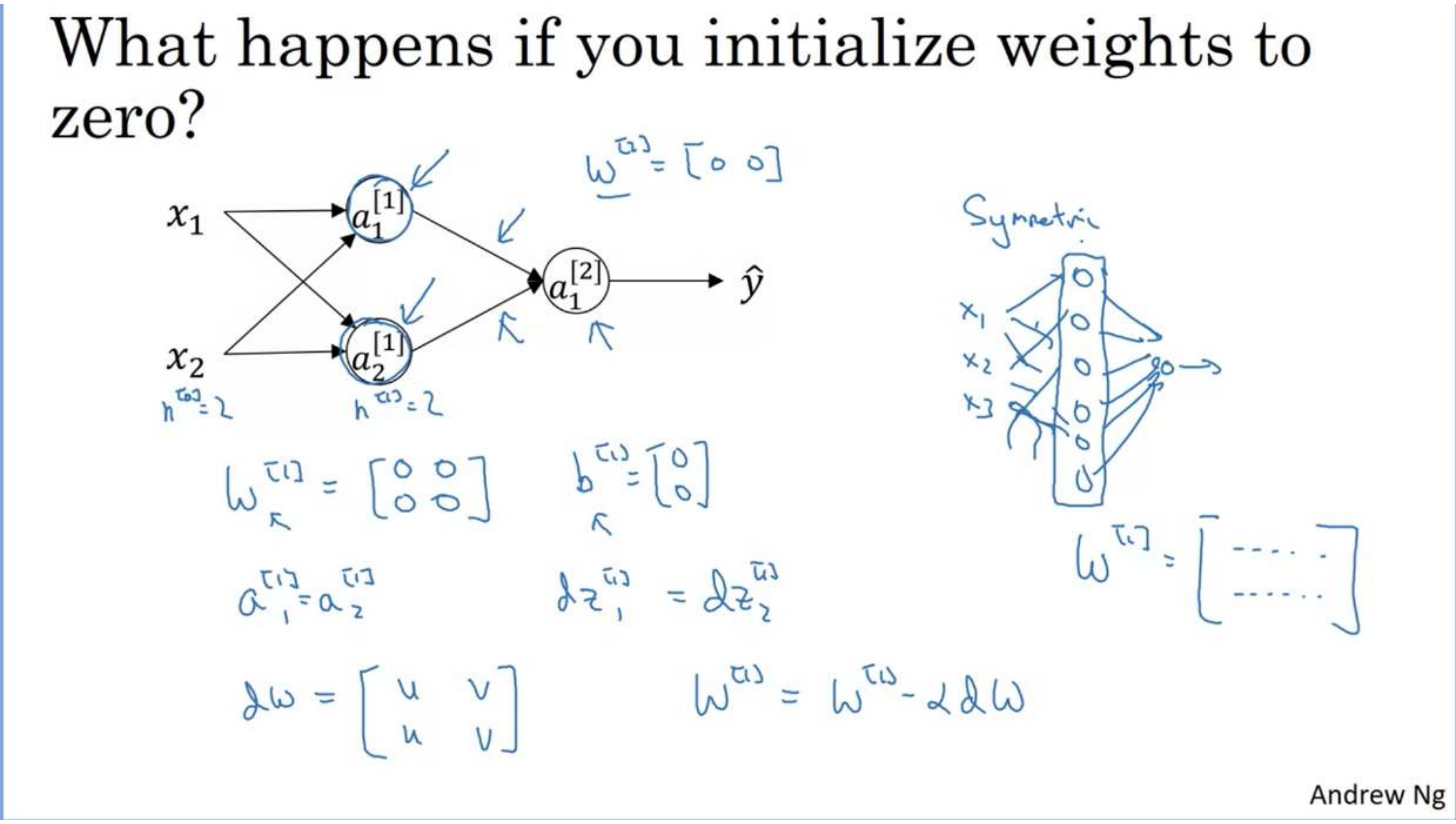

What happens if you initialize weights to zero?

$W^{[2]} = [0 \ 0]$

Symmetric

$n^{[0]} = 2$ $n^{[1]} = 2$

$W^{[1]} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $b^{[1]} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$W^{[1]} = \begin{bmatrix} \cdots\cdots \\ \cdots\cdots \end{bmatrix}$

$a_1^{[1]} = a_2^{[1]}$ $dz_1^{[1]} = dz_2^{[1]}$

$dw = \begin{bmatrix} u & v \\ u & v \end{bmatrix}$ $W^{[1]} = W^{[1]} - \alpha dW$

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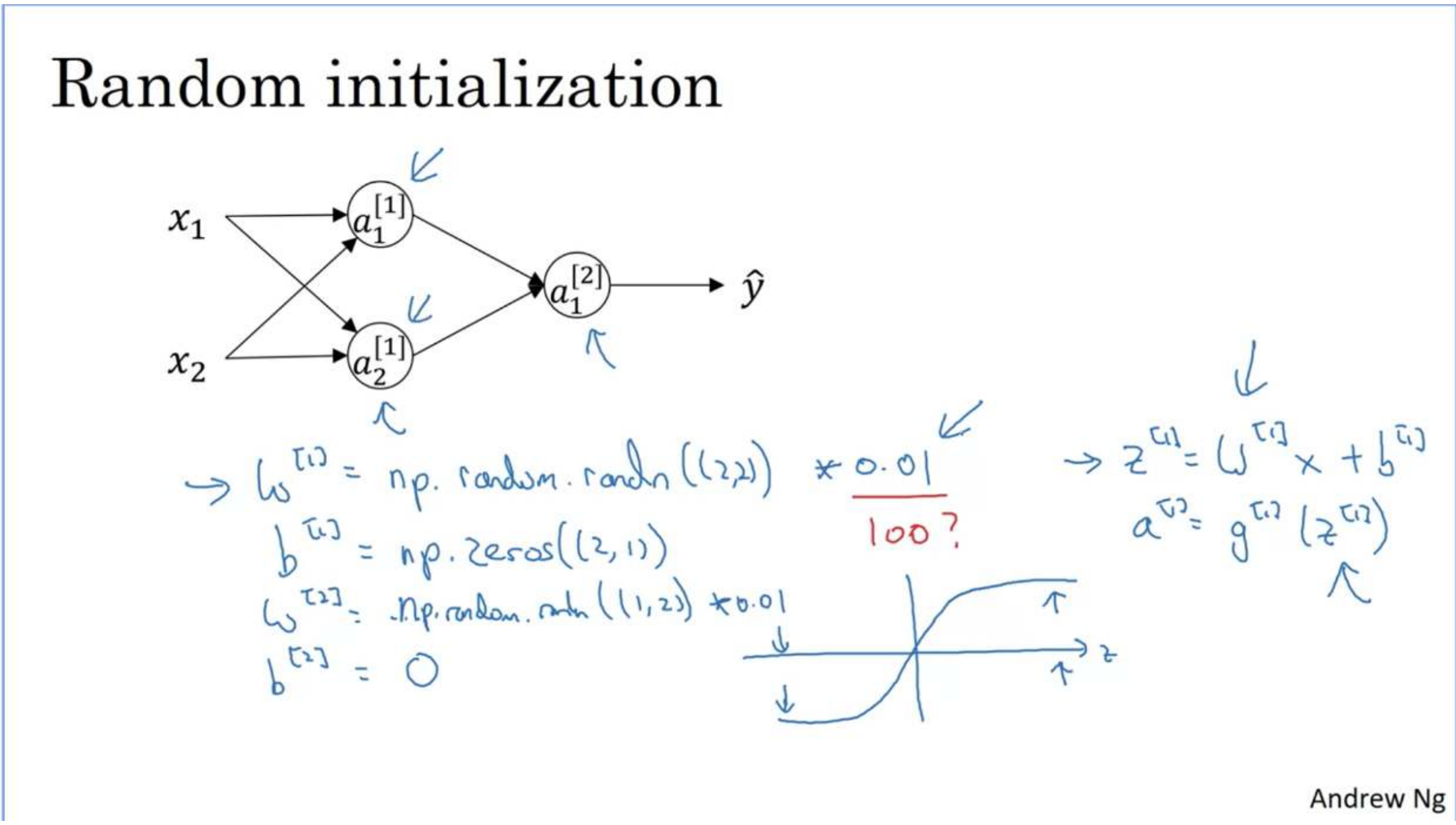

Random initialization

→ $W^{[1]}$ = np.random.randn((2,2)) $*$ 0.01

100?

$b^{[1]}$ = np.zeros((2,1))

$W^{[2]}$ = np.random.randn((1,2)) $*$0.01

$b^{[2]}$ = 0

→ $z^{[1]} = W^{[1]}x + b^{[1]}$

$a^{[1]} = g^{[1]}(z^{[1]})$

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