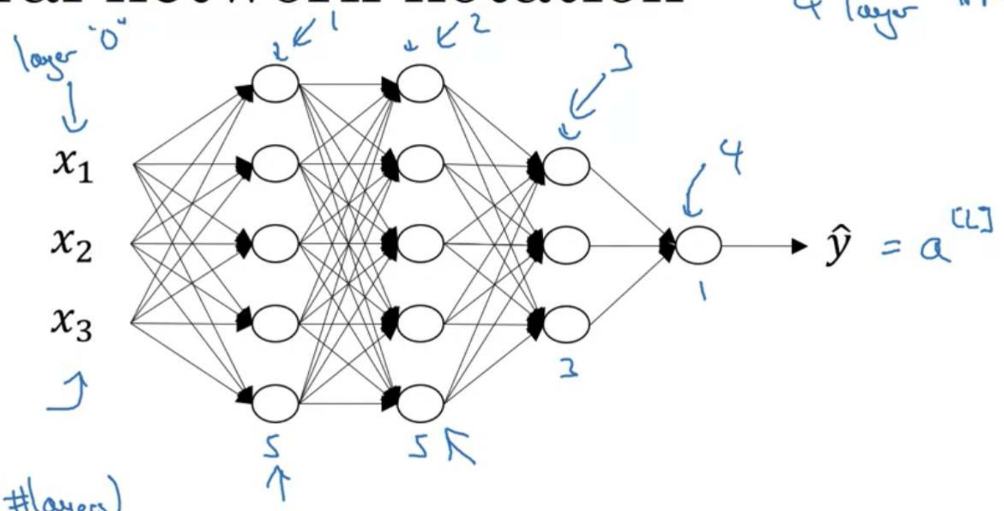
Deep neural network notation

4 layer NN



$X = a^{[0]}$

$L = 4$ (#layers)

$n^{[l]}$ = #units in layer l

$n^{[1]} = 5$, $n^{[2]} = 5$, $n^{[3]} = 3$, $n^{[4]} = n^{[L]} = 1$

$n^{[0]} = n_x = 3$

$a^{[l]}$ = activations in layer l

$a^{[l]} = g^{[l]}(z^{[l]})$, $W^{[l]}$ = weights for $z^{[l]}$

$b^{[l]}$

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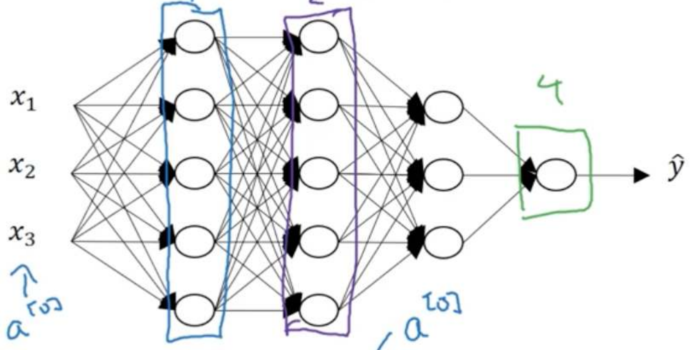
Forward propagation in a deep network



$a^{[0]}$

$X:\ z^{[1]} = W^{[1]} a^{[0]} + b^{[1]}$

$a^{[1]} = g^{[1]}(z^{[1]})$

$z^{[2]} = W^{[2]} a^{[1]} + b^{[2]}$

$a^{[2]} = g^{[2]}(z^{[2]})$

$\cdots$

$z^{[4]} = W^{[4]} a^{[3]} + b^{[4]}$, $a^{[4]} = g^{[4]}(z^{[4]}) = \hat{y}$

$$\begin{bmatrix} z^{[1](1)} & z^{[1](2)} & \cdots & z^{[1](m)} \end{bmatrix}$$

$$\boxed{\begin{aligned} Z^{[l]} &= W^{[l]} A^{[l-1]} + b^{[l]} \\ A^{[l]} &= g^{[l]}(Z^{[l]}) \end{aligned}}$$

Vectorized:

$X = A^{[0]}$

$Z^{[1]} = W^{[1]} A^{[0]} + b^{[1]}$

$A^{[1]} = g^{[1]}(Z^{[1]})$

$Z^{[2]} = W^{[2]} A^{[1]} + b^{[2]}$

$A^{[2]} = g^{[2]}(Z^{[2]})$

$\hat{Y} = g(Z^{[4]}) = A^{[4]}$

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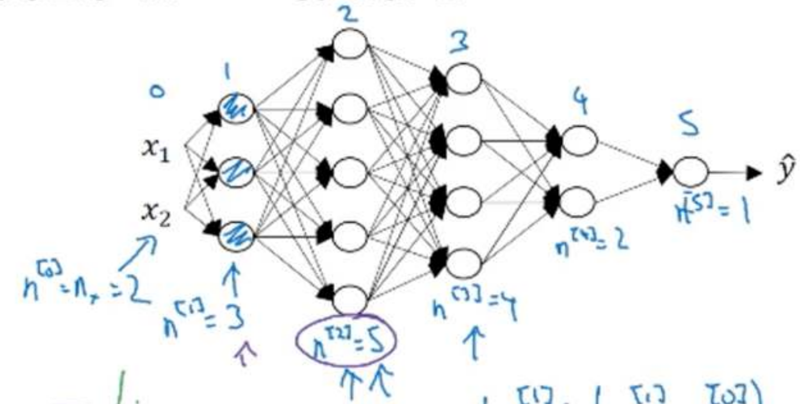
Parameters $W^{[l]}$ and $b^{[l]}$

$L = 5$

$z^{[l]} = g^{[l]}(a^{[l]})$

$a^{[l]}$



$W^{[l]} : (n^{[l]}, n^{[l-1]})$

$b^{[l]} : (n^{[l]}, 1)$

$dW^{[l]} : (n^{[l]}, n^{[l-1]})$

$db^{[l]} : (n^{[l]}, 1)$

$$z^{[1]} = \boxed{W^{[1]} \cdot x} + \boxed{b^{[1]}}$$

$(3,1) \leftarrow (3,2) \quad (2,1) \quad (3,1)$

$(n^{[1]},1) \quad (n^{[1]}, n^{[0]}) \quad (n^{[0]},1) \quad (n^{[1]},1)$

$$\begin{bmatrix} \cdot \\ \cdot \\ \cdot \end{bmatrix} = \begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \end{bmatrix} \begin{bmatrix} \cdot \\ \cdot \end{bmatrix}$$

$W^{[1]} : (n^{[1]}, n^{[0]})$

$W^{[2]} : (5,3) \quad (n^{[2]}, n^{[1]})$

$$z^{[2]} = \boxed{W^{[2]} \cdot a^{[1]}} + \boxed{b^{[2]}}$$

$(5,1) \quad (5,3) \quad (3,1) \quad (5,1)$

$(n^{[2]},1)$

$W^{[3]} : (4,5)$

$W^{[4]} : (2,4) \quad , \quad W^{[5]} : (1,2)$

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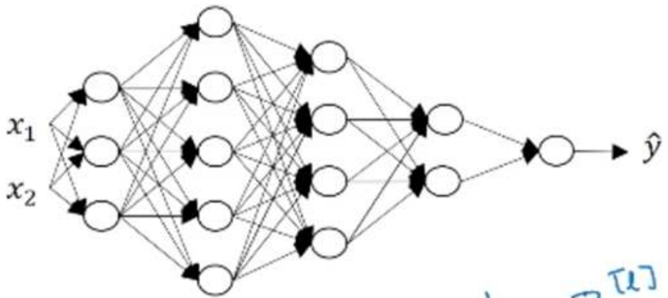
Vectorized implementation



$$z^{[1]} = W^{[1]} \cdot x + b^{[1]}$$

$(n^{[1]},1)$ $(n^{[1]},n^{[0]})$ $(n^{[0]},1)$ $(n^{[1]},1)$

$$\left[z^{[1](1)} \; z^{[1](2)} \; \cdots \; z^{[1](m)} \right]$$

$$Z^{[1]} = W^{[1]} \cdot X + b^{[1]}$$

$(n^{[1]},m)$ $(n^{[1]},n^{[0]})$ $(n^{[0]},m)$ $(n^{[1]},1)$

$(n^{[1]},m)$

$z^{[l]}, a^{[l]} : (n^{[l]},1)$

$Z^{[l]}, A^{[l]} : (n^{[l]},m)$

$l=0$ $A^{[0]} = X = (n^{[0]},m)$

$dZ^{[l]}, dA^{[l]} : (n^{[l]},m)$

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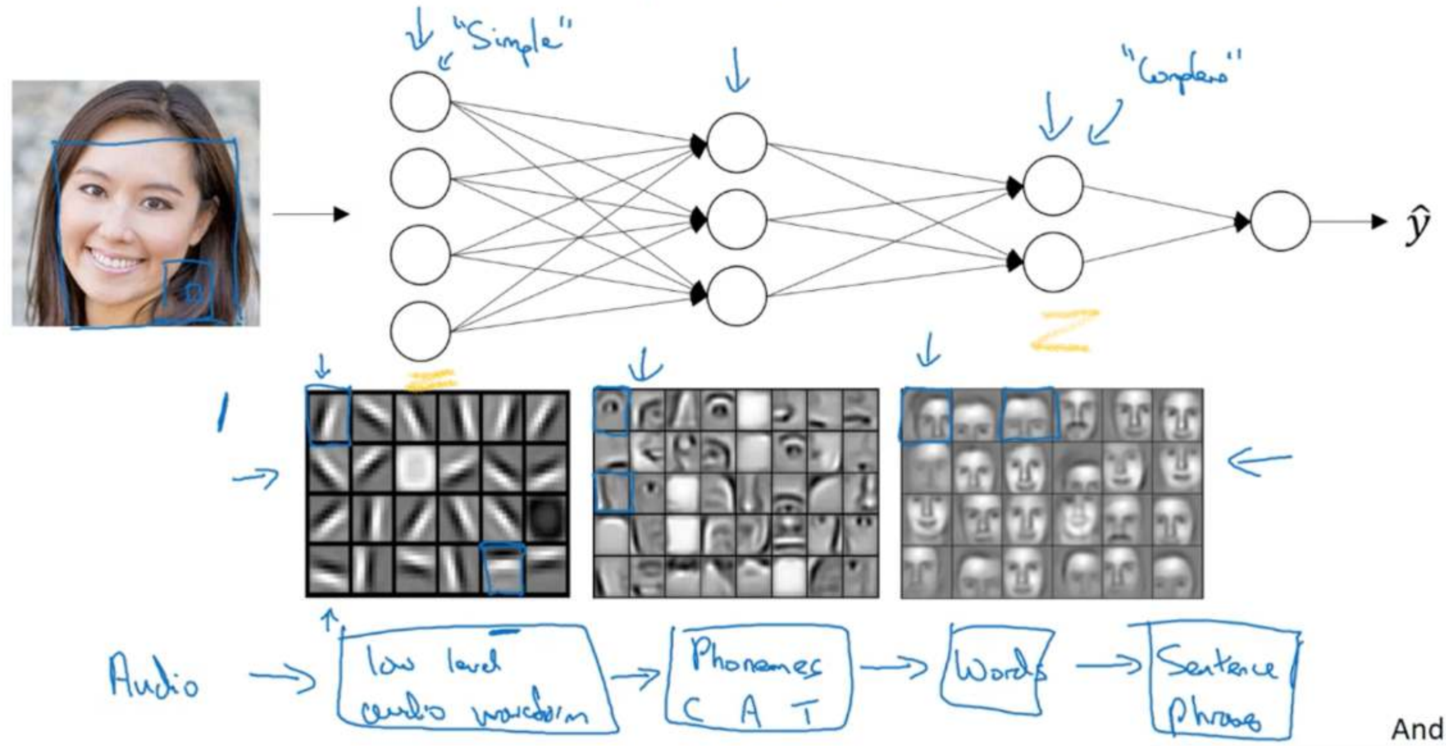
Intuition about deep representation



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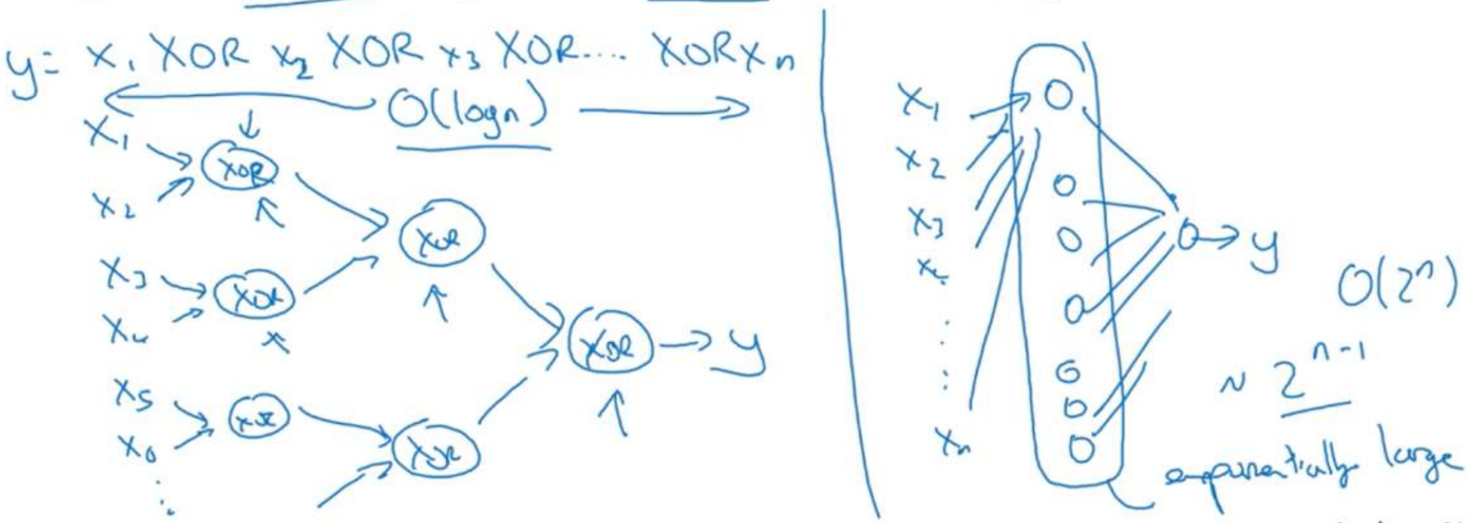
Circuit theory and deep learning

Informally: There are functions you can compute with a "small" L-layer deep neural network that shallower networks require exponentially more hidden units to compute.



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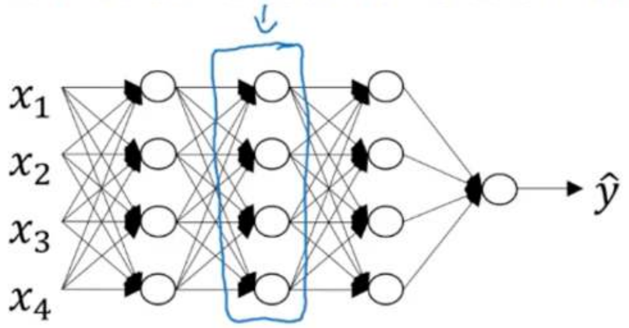
Forward and backward functions



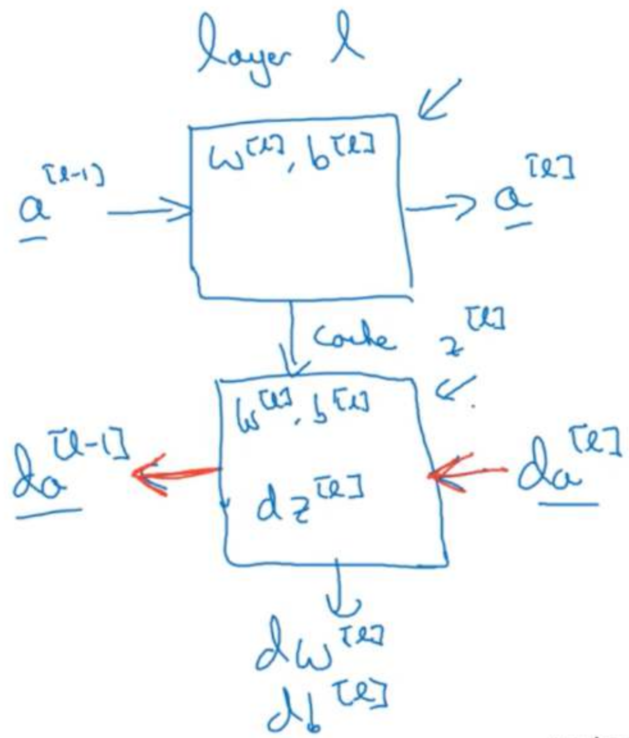
Layer l: $W^{[l]}, b^{[l]}$

→ Forward: Input $a^{[l-1]}$, output $a^{[l]}$

$z^{[l]} = W^{[l]} a^{[l-1]} + b^{[l]}$ cache $z^{[l]}$

$a^{[l]} = g^{[l]}(z^{[l]})$

→ Backward: Input $da^{[l]}$, cache($z^{[l]}$) output $da^{[l-1]}$, $dW^{[l]}$, $db^{[l]}$



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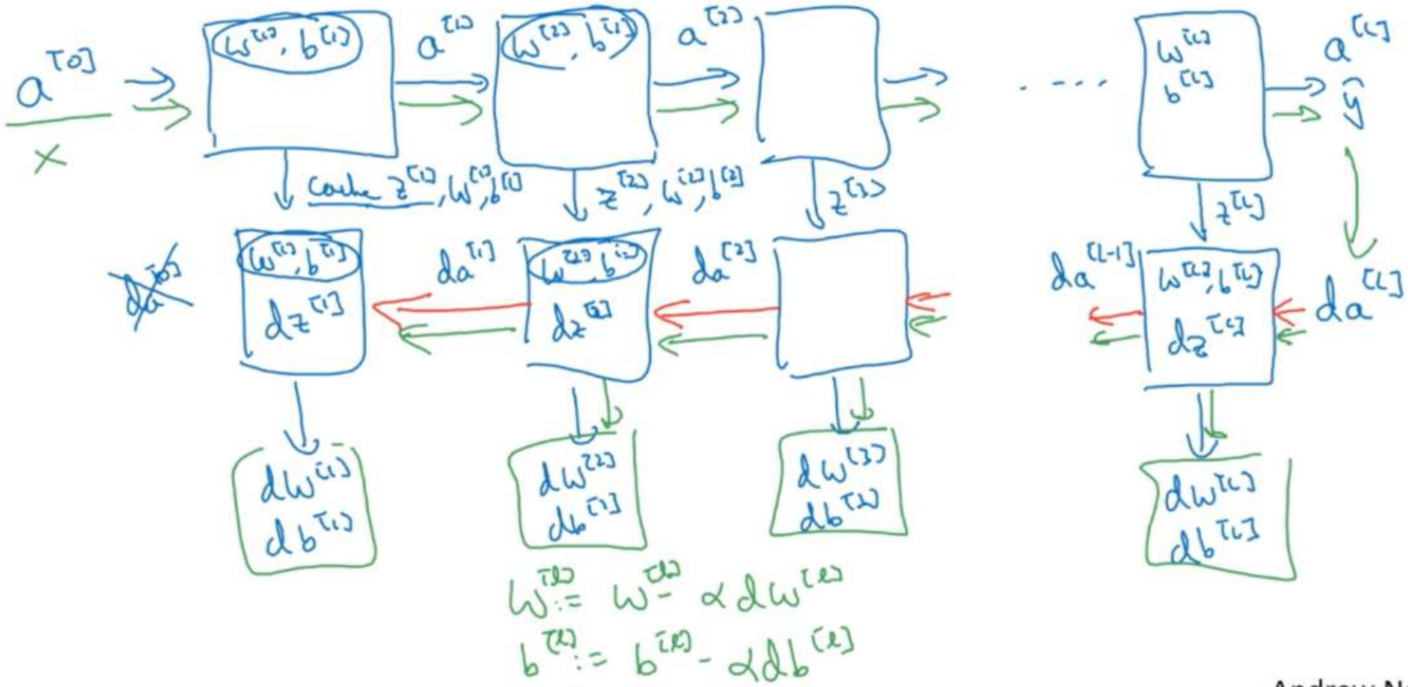
Forward and backward functions



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Forward propagation for layer l

- Input $a^{[l-1]}$
- Output $a^{[l]}$, cache $(z^{[l]})$

$W^{[l]}, b^{[l]}$

$$z^{[l]} = W^{[l]} \cdot a^{[l-1]} + b^{[l]}$$

$$a^{[l]} = g^{[l]}(z^{[l]})$$

Vectorized:

$$Z^{[l]} = W^{[l]} \cdot A^{[l-1]} + b^{[l]}$$

$$A^{[l]} = g^{[l]}(Z^{[l]})$$

$a^{[0]}$

$A^{[0]}$

$$X = A^{[0]} \rightarrow \square \rightarrow \square \rightarrow \square \rightarrow$$

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Backward propagation for layer l

→ Input $da^{[l]}$

→ Output $\boxed{da^{[l-1]}}, dW^{[l]}, db^{[l]}$

$$dz^{[l]} = da^{[l]} * g^{[l]\prime}(z^{[l]})$$

$$dW^{[l]} = dz^{[l]} \cdot a^{[l-1]}$$

$$db^{[l]} = dz^{[l]}$$

$$\boxed{da^{[l-1]}} = W^{[l]T} \cdot dz^{[l]}$$

$$dz^{[l]} = W^{[l+1]T} dz^{[l+1]} * g^{[l]\prime}(z^{[l]})$$

$$dZ^{[l]} = dA^{[l]} * g^{[l]\prime}(Z^{[l]})$$

$$dW^{[l]} = \frac{1}{m} dZ^{[l]} \cdot A^{[l-1]T}$$

$$db^{[l]} = \frac{1}{m} np.sum(dZ^{[l]}, axis=1, keepdims=True)$$

$$\boxed{dA^{[l-1]}} = W^{[l]T} \cdot dZ^{[l]}$$

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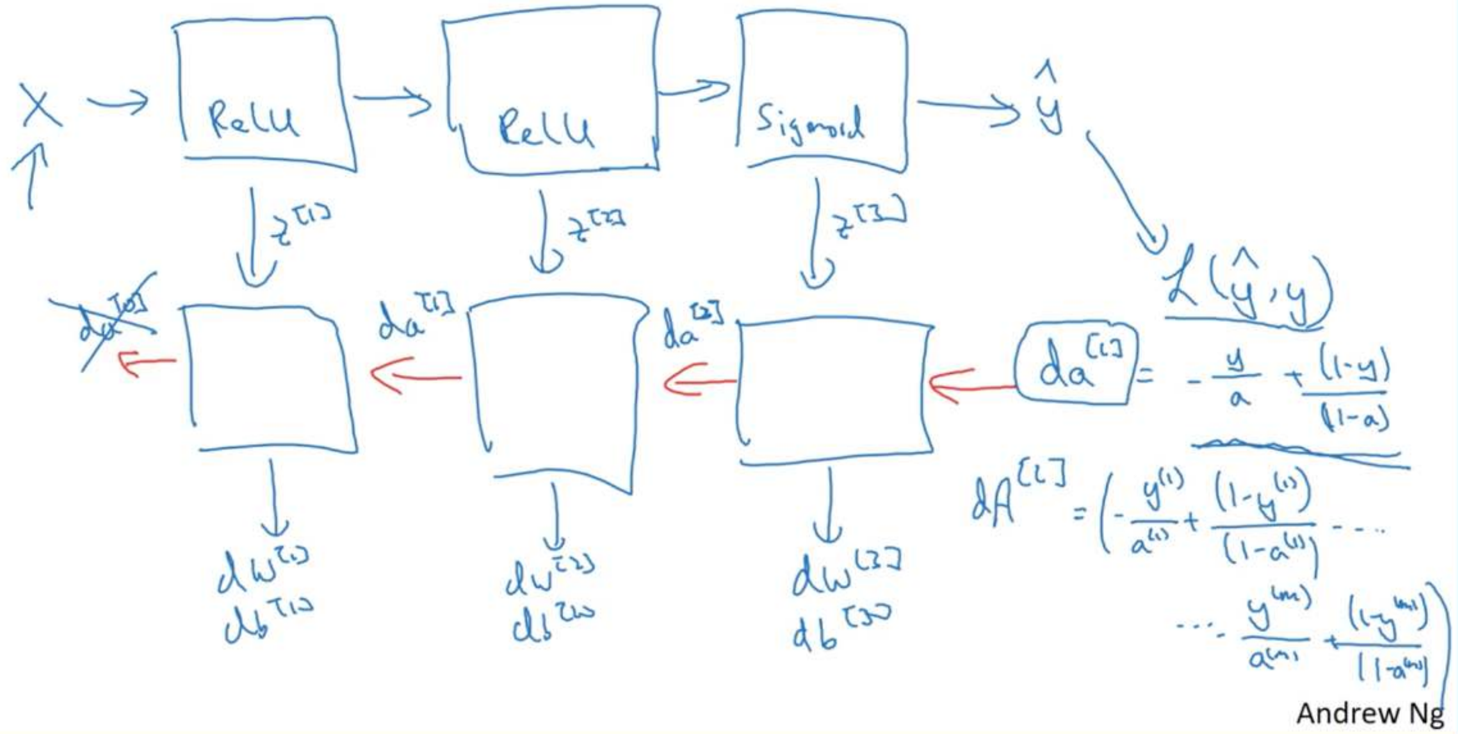
Summary



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What are hyperparameters?

Parameters: $W^{[1]}, b^{[1]}, W^{[2]}, b^{[2]}, W^{[3]}, b^{[3]}$ ...

Hyperparameters: learning rate α
#iterations
#hidden layers L
#hidden units $n^{[1]}, n^{[2]}, \ldots$
choice of activation function

Later: Momentum, mini-batch size, regularizations, ...

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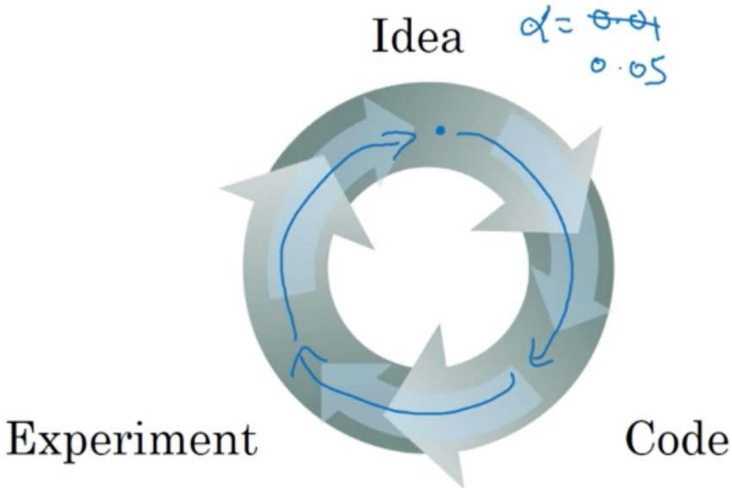
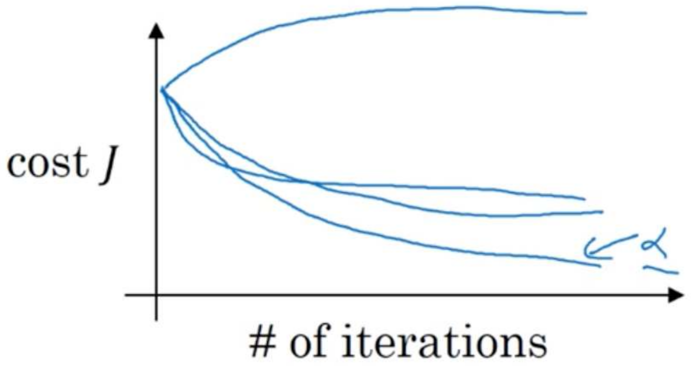
Applied deep learning is a very empirical process





Vision, Speech, NLP, Ad, Search, Recommendation.

Clarification about What does this have to do with the brain video

Note that the formulas shown in the next video have a few typos. Here is the correct set of formulas.

$dZ^{[L]} = A^{[L]} - Y$

$dW^{[L]} = \frac{1}{m} dZ^{[L]} A^{[L-1]^T}$

$db^{[L]} = \frac{1}{m} np.sum(dZ^{[L]}, axis = 1, keepdims = True)$

$dZ^{[L-1]} = W^{[L]^T} dZ^{[L]} * g'^{[L-1]}(Z^{[L-1]})$

Note that * denotes element-wise multiplication)

$\vdots$

$dZ^{[1]} = W^{[2]} dZ^{[2]} * g'^{[1]}(Z^{[1]})$

$dW^{[1]} = \frac{1}{m} dZ^{[1]} A^{[0]^T}$

Note that $A^{[0]^T}$ is another way to denote the input features, which is also written as X^T

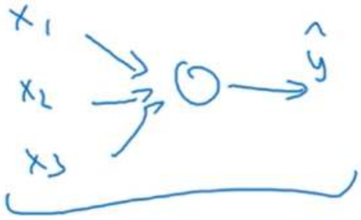
$db^{[1]} = \frac{1}{m} np.sum(dZ^{[1]}, axis = 1, keepdims = True)$

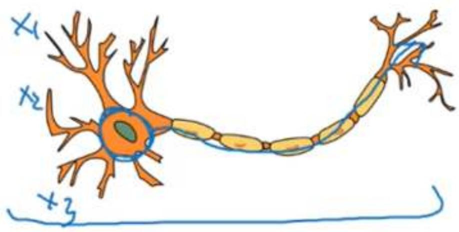
Forward and backward propagation

$$
\begin{aligned}
Z^{[1]} &= W^{[1]}X + b^{[1]} \\
A^{[1]} &= g^{[1]}(Z^{[1]}) \\
Z^{[2]} &= W^{[2]}A^{[1]} + b^{[2]} \\
A^{[2]} &= g^{[2]}(Z^{[2]}) \\
&\vdots \\
A^{[L]} &= g^{[L]}\left(Z^{[L]}\right) = \hat{Y}
\end{aligned}
$$

$$
\begin{aligned}
dZ^{[L]} &= A^{[L]} - Y \\
dW^{[L]} &= \frac{1}{m} dZ^{[L]} A^{[L]^T} \\
db^{[L]} &= \frac{1}{m} np.\mathrm{sum}(\mathrm{d}Z^{[L]}, axis = 1, keepdims = True) \\
dZ^{[L-1]} &= dW^{[L]^T} dZ^{[L]} g'^{[L]}(Z^{[L-1]}) \\
&\vdots \\
dZ^{[1]} &= dW^{[L]^T} dZ^{[2]} g'^{[1]}(Z^{[1]}) \\
dW^{[1]} &= \frac{1}{m} dZ^{[1]} A^{[1]^T} \\
db^{[1]} &= \frac{1}{m} np.\mathrm{sum}(\mathrm{d}Z^{[1]}, axis = 1, keepdims = True)
\end{aligned}
$$

"It's like the brain."





$X \longrightarrow y$

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