

## Version 6: Cluster first

### Simulate data

```
I <- 100
K <- 10
S <- 3

set.seed(123)

pi <- rep(0.1, 10)
#z <- sample(1:K, size = I, replace = T, prob = pi)
z <- rep(1:10, each=10)
w <- matrix(c(0.98, 0.99, 0.97,
              0.98, 0.90, 0.82,
              0.55, 0.00, 0.80,
              0.20, 0.00, 0.50,
              0.30, 0.00, 0.30,
              0.43, 0.90, 0.00,
              0.30, 0.70, 0.00,
              0.20, 0.00, 0.00,
              0.00, 0.00, 0.30,
              0.00, 0.50, 0.00),
            byrow=T,
            nrow=K, ncol=S)

colnames(w) <- paste0("sample", 1:S)
w

##      sample1 sample2 sample3
## [1,]    0.98    0.99    0.97
## [2,]    0.98    0.90    0.82
## [3,]    0.55    0.00    0.80
## [4,]    0.20    0.00    0.50
## [5,]    0.30    0.00    0.30
## [6,]    0.43    0.90    0.00
## [7,]    0.30    0.70    0.00
## [8,]    0.20    0.00    0.00
## [9,]    0.00    0.00    0.30
## [10,]   0.00    0.50    0.00

tcn <- matrix(2, nrow=I, ncol=S)
m <- matrix(rep(sample(1:2, size = I, replace = T), S),
            nrow=I, ncol=S)
W <- w[z, ]
calcTheta <- function(m, tcn, w) {
  (m * w) / (tcn * w + 2*(1-w))
}

theta <- calcTheta(m, tcn, W)

n <- replicate(S, rpois(I, 100))
y <- matrix(NA, nrow=I, ncol=S)
```

```

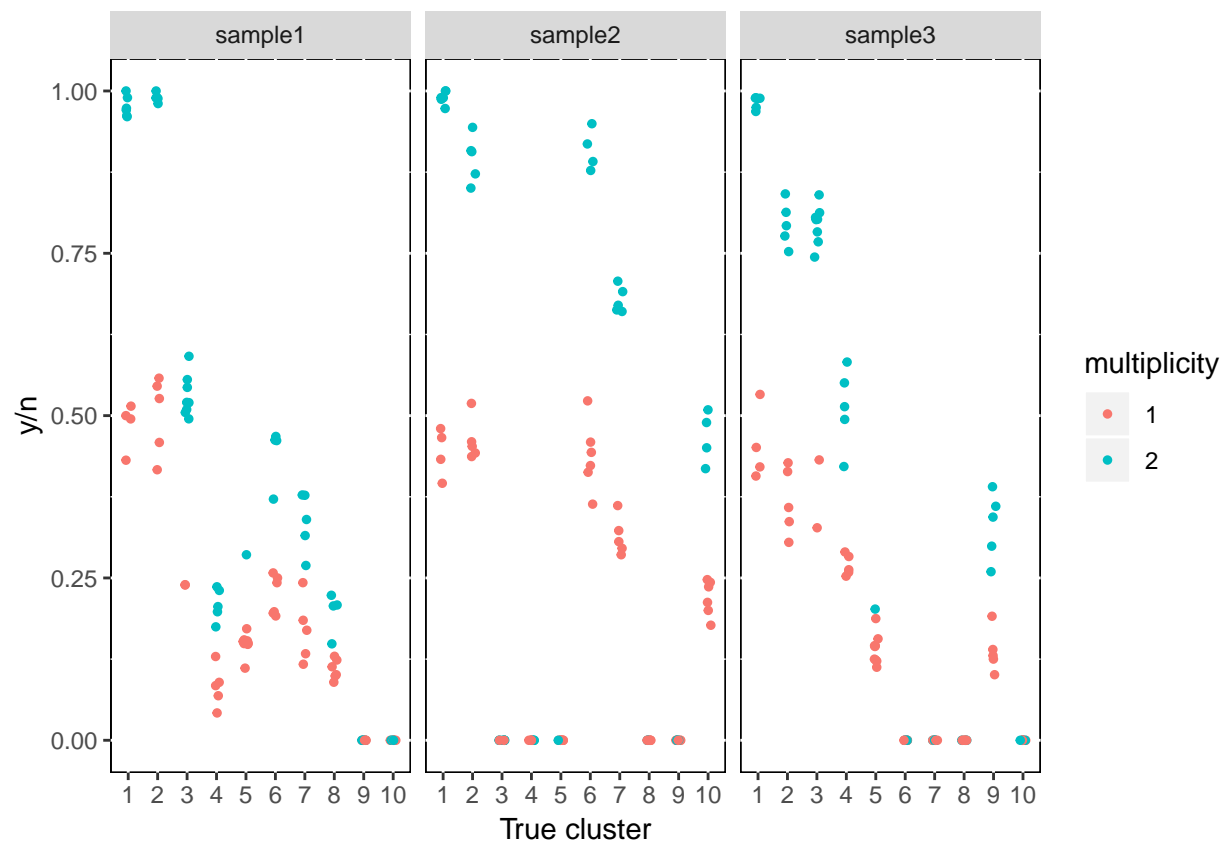
for (i in 1:I) {
  for (s in 1:S) {
    y[i, s] <- rbinom(1, n[i, s], theta[i,s])
  }
}

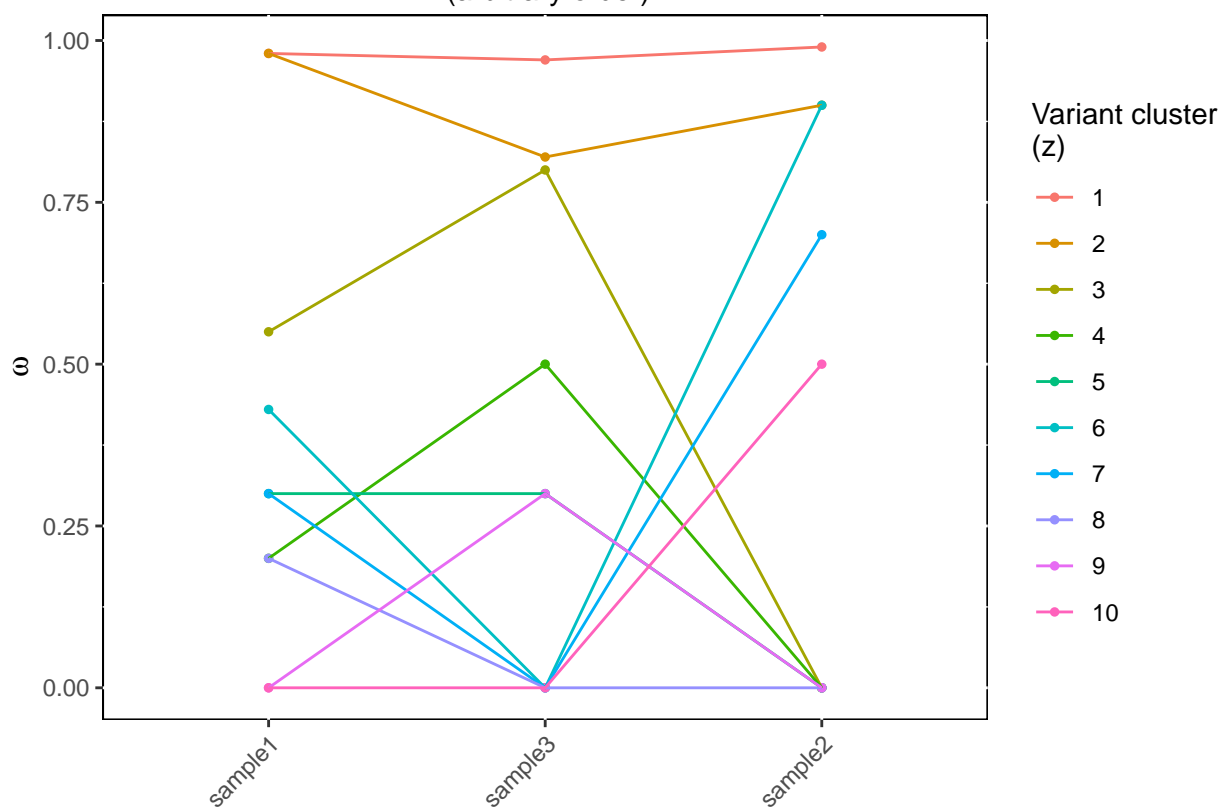
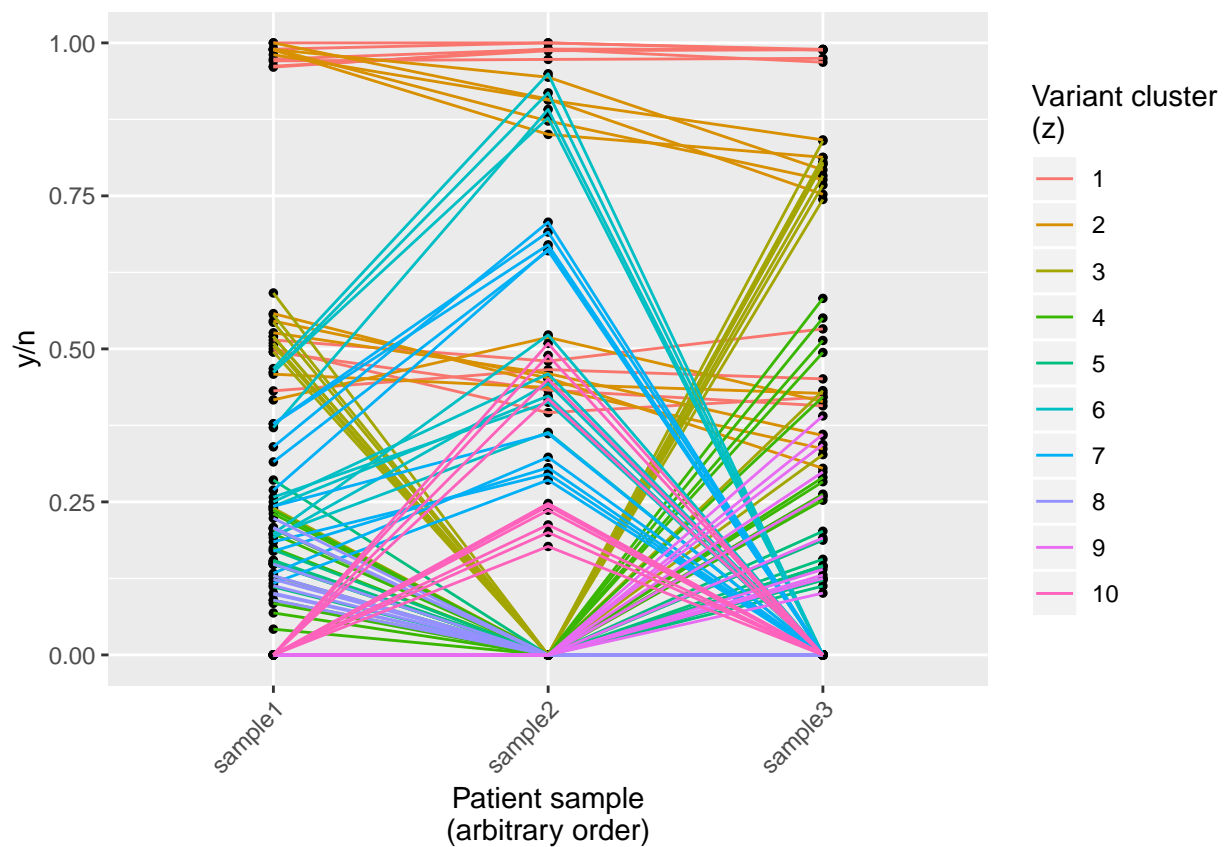
test.data <- list("I" = I, "S" = S, "K" = K,
                  "y" = y, "n" = n,
                  "m" = m, "tcn" = tcn)

```

## Visualize densities of simulated data

Clustering is by  $\omega$





## functions

```
runMCMC <- function(data, K, jags.file, inits, params, n.iter, thin) {
  data$K <- K
  jags.m <- jags.model(jags.file, data,
    n.chains = 1,
    inits = inits,
    n.adapt = 1000)
  samps <- coda.samples(jags.m, params, n.iter=n.iter, thin=thin)
  samps
}

getParamChain <- function(samps, param) {
  chains <- do.call(rbind, samps)
  chain <- chains[, grep(param, colnames(chains))]
}

reshapeW <- function(w, S, K) {
  w.mat <- matrix(w, nrow = K)
  colnames(w.mat) <- paste0("sample", 1:S)
  w.mat
}

calcLogLik <- function(z.iter, w.iter, data) {
  W <- w.iter[z.iter, ]
  theta <- calcTheta(data$m, data$tcn, W)
  sum(dbinom(data$y, data$n, theta, log=T))
}

calcChainLogLik <- function(samps, data, K) {
  z.chain <- getParamChain(samps, "z")
  w.chain <- getParamChain(samps, "w")
  lik <- c()
  for(iter in 1:nrow(z.chain)) {
    z.iter <- z.chain[iter,]
    w.iter <- reshapeW(w.chain[iter,], data$S, K)
    lik <- c(lik, calcLogLik(z.iter, w.iter, data))
  }
  mean(lik)
}

calcBIC <- function(n, k, ll) log(n)*k - 2*ll
```

## Cluster – JAGS

```
jags.file <- file.path(models.dir, "model.jags")
inits <- list(".RNG.name" = "base::Wichmann-Hill",
  ".RNG.seed" = 123)
test.data <- list("I" = I, "S" = S,
  "y" = y, "n" = n,
  "tcn" = tcn)
params <- c("z", "w", "ystar", "m")
```

```

n.iter = 10000
thin = 7
K <- 10

samps <- runMCMC(test.data, K, jags.file, inits, params, n.iter, thin)

## Compiling model graph
##   Resolving undeclared variables
##   Allocating nodes
## Graph information:
##   Observed stochastic nodes: 300
##   Unobserved stochastic nodes: 731
##   Total graph size: 4638
##
## Initializing model

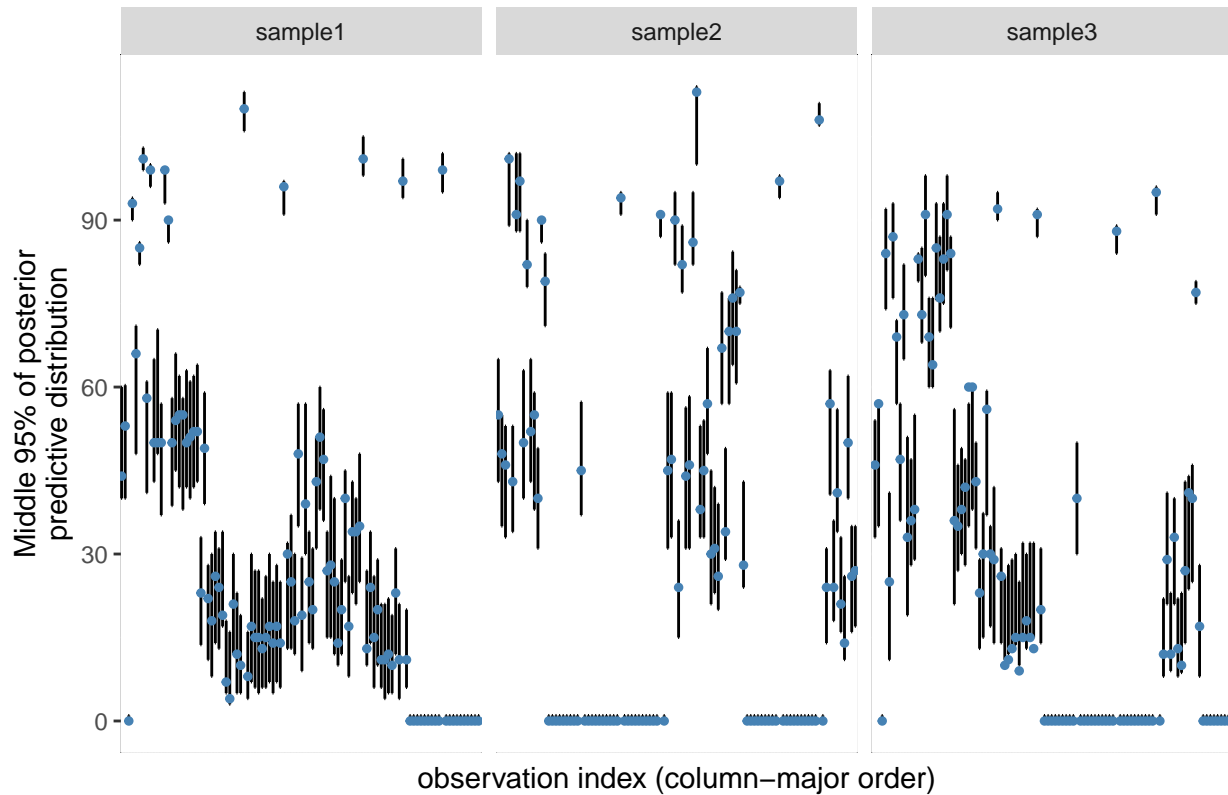
z.chain <- getParamChain(samps, "z")
w.chain <- getParamChain(samps, "w")
m.chain <- getParamChain(samps, "m")

mcmc_vals <- summary(samps)$statistics
mcmc_w <- mcmc_vals[substr(rownames(mcmc_vals), 1, 1) == "w", "Mean"]
mcmc_w <- matrix(mcmc_w, nrow=K)
colnames(mcmc_w) <- paste0("sample", 1:S)

```

## PPD

K = 10



## Z

```

plot.z <- function(samps, z) {
  mcmc_vals <- summary(samps)$statistics
  mcmc_z <- as.vector(mcmc_vals[substr(rownames(mcmc_vals), 1, 1) == "z", "Mean"])
  plot(z, mcmc_z, type = "p")
  z_comp <- data.frame(z, mcmc_z)
}

z.chain.to.tb <- function(z.chain) {
  z.chain.tb <- z.chain %>%
    as_tibble() %>%
    mutate(iter=1:nrow(z.chain)) %>%
    gather(variant, mcmc_z, -c(iter))
  z.chain.tb <- z.chain.tb %>%
    mutate(variant = as.integer(gsub(".*\\[(.*)\\].*", "\\1", z.chain.tb$variant))) %>%
    mutate(true_z = rep(1:10, each=nrow(z.chain)*10)) %>%
    group_by(variant, mcmc_z) %>%
    mutate(count = n()) %>%
    ungroup() %>%
    mutate(iter = NULL)
  z.chain.tb_simp <- distinct(z.chain.tb)
  z.chain.tb_simp <- z.chain.tb_simp %>%
    group_by(variant) %>%
    mutate(prop = round(count/sum(count), 2))

  z.chain.tb_simp
}

z.chain.tb <- z.chain.to.tb(z.chain)
z.chain.tb

## # A tibble: 114 x 5
## # Groups:   variant [100]
##   variant mcmc_z true_z count prop
##   <int>   <dbl> <int> <int> <dbl>
## 1     1     6     1  1428     1
## 2     2     8     1  1428     1
## 3     3     6     1  1428     1
## 4     4     8     1  1428     1
## 5     5     8     1  1428     1
## 6     6     6     1  1428     1
## 7     7     8     1  1428     1
## 8     8     8     1  1428     1
## 9     9     8     1  1428     1
## 10    10     6     1  1428     1
## # ... with 104 more rows

z.seg.tb <- tibble(variant = numeric(),
                   mcmc_z_1 = numeric(),
                   mcmc_z_2 = numeric())
for (i in 1:ncol(z.chain)) {
  z.vals <- as.integer(names(table(z.chain[,i])))
  if (length(z.vals) > 1) {
    z.seg.tb[i, ] <- c(i, z.vals[1], z.vals[2])
  }
}

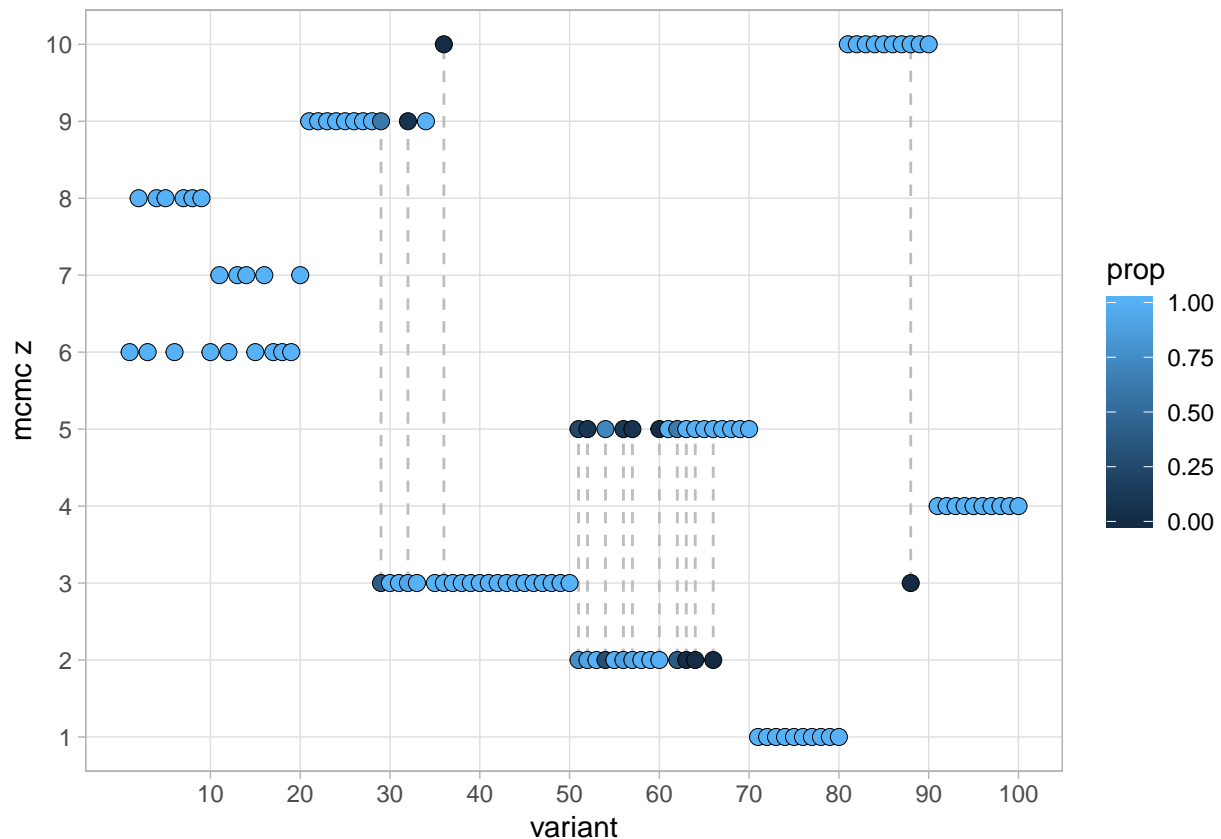
```

```

} else {
  z.seg.tb[i, ] <- c(i, z.vals, z.vals)
}
}
#z.seg.tb
z.plot <- ggplot(z.chain.tb, aes(variant, mcmc_z)) +
  ylab("mcmc z") +
  xlab("variant") +
  theme_light() +
  scale_y_continuous(breaks = 1:K, minor_breaks=NULL) +
  scale_x_continuous(breaks = seq(10,100,10), minor_breaks=NULL) +
  geom_segment(data = z.seg.tb,
    aes(x=variant, xend=variant,
        y=mcmc_z_1, yend=mcmc_z_2),
    color="gray", linetype=2) +
  geom_point(aes(y=mcmc_z, fill=prop),
    pch=21, size=3, stroke=0)

```

z.plot



```
ggsave(file.path(figs.dir, "z_plot.pdf"), z.plot, width=14, height=6)
```

```

z.map.tb <- z.chain.tb %>%
  group_by(variant) %>%
  filter(prop == max(prop))
z.map.tb

```

```
## # A tibble: 100 x 5
## # Groups:   variant [100]
##   variant mcmc_z true_z count prop
##   <int>   <dbl> <int> <int> <dbl>
## 1       1     6     1  1428     1
## 2       2     8     1  1428     1
## 3       3     6     1  1428     1
## 4       4     8     1  1428     1
## 5       5     8     1  1428     1
## 6       6     6     1  1428     1
## 7       7     8     1  1428     1
## 8       8     8     1  1428     1
## 9       9     8     1  1428     1
## 10      10     6     1  1428     1
## # ... with 90 more rows
```

```
z.map <- z.map.tb$mcmc_z
z.map
```

```
## [1] 6 8 6 8 8 6 8 8 8 6 7 6 7 7 6 7 6 6 6 7 9 9 9
## [24] 9 9 9 9 9 9 3 3 3 3 9 3 3 3 3 3 3 3 3 3 3 3
## [47] 3 3 3 3 2 2 2 5 2 2 2 2 2 2 5 5 5 5 5 5 5 5
## [70] 5 1 1 1 1 1 1 1 1 1 1 10 10 10 10 10 10 10 10 10 4 4
## [93] 4 4 4 4 4 4 4 4
```

```
z.map.ind <- which(apply(z.chain, 1, function(x) all(x == z.map)))
```

**m**

```
m.tb <- m %>%
  as_tibble() %>%
  mutate(variant = 1:nrow(m)) %>%
  gather(sample, m, -c(variant)) %>%
  mutate(sample = as.integer(gsub("sample", "", .$sample))) %>%
  group_by(variant, sample)

m.chain.tb <- m.chain %>%
  as_tibble() %>%
  mutate(iter = 1:nrow(m.chain)) %>%
  gather(label, mcmc_m, -c(iter))

m.chain.tb <- m.chain.tb %>%
  mutate(variant = as.integer(gsub(".*\\[(.*)\\].*", "\\1", m.chain.tb$label))) %>%
  mutate(sample = as.integer(gsub(".*\\((.*)\\].*", "\\1", m.chain.tb$label))) %>%
  mutate(true_m = rep(m.tb$m, each = nrow(m.chain)))

m.chain.tb.simp <- m.chain.tb %>%
  group_by(variant, sample) %>%
  mutate(prop.m.correct = round(sum(mcmc_m == true_m) / nrow(m.chain), 2)) %>%
  ungroup() %>%
  mutate(iter = NULL) %>%
  mutate(mcmc_m = NULL)

m.chain.tb.simp <- distinct(m.chain.tb.simp) %>%
  mutate(z.map = rep(z.map, times = S))

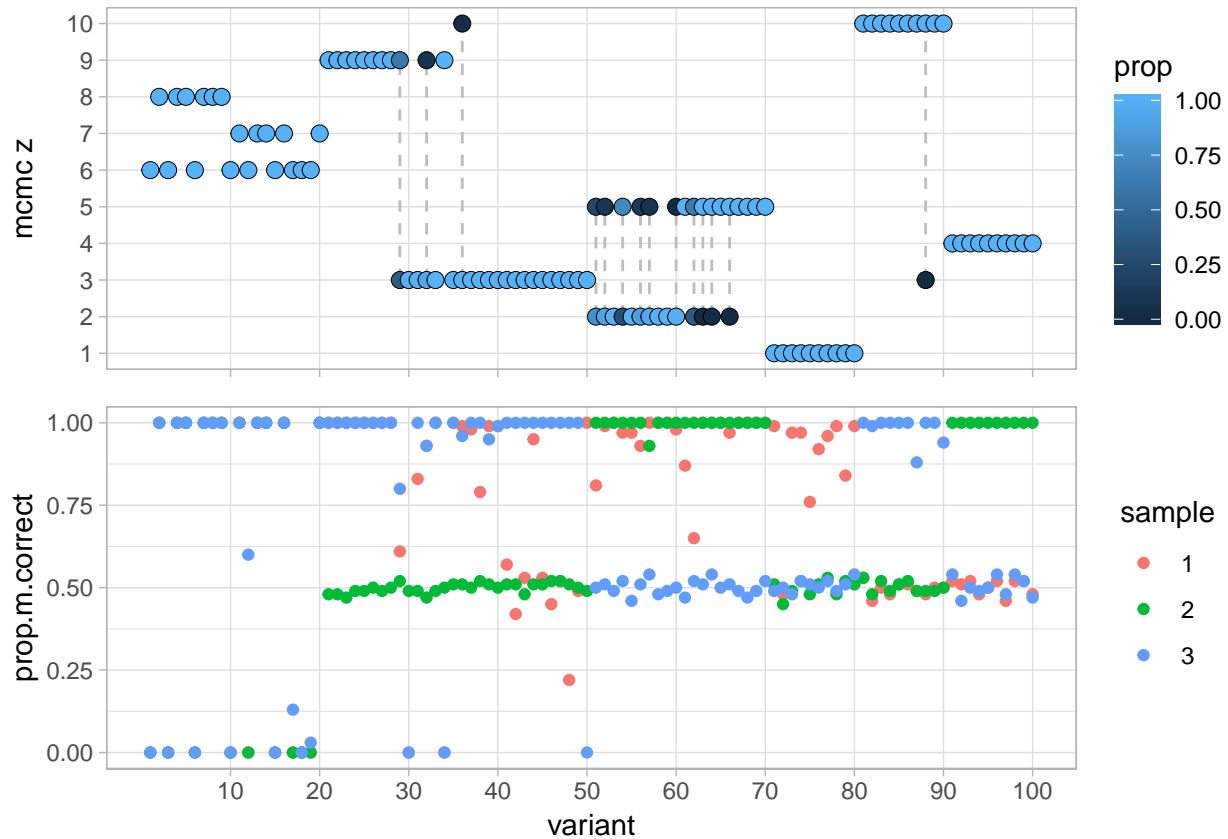
m.chain.tb.simp <- m.chain.tb.simp %>% mutate(sample = as.character(m.chain.tb.simp$sample))
```



```

m.plot <- ggplot(m.chain.tb.simp, aes(x=variant, prop.m.correct, colour = sample)) +
  geom_point() +
  scale_x_continuous(breaks = seq(10,100,10), minor_breaks=NULL) +
  theme_light()
z.plot.2 <- z.plot + theme(axis.title.x = element_blank(), axis.text.x = element_blank())
grid.newpage()
grid.draw(rbind(ggplotGrob(z.plot.2), ggplotGrob(m.plot), size = "last"))

```



$\omega$

```

mcmc_vals <- summary(samps)$statistics
mcmc_w <- mcmc_vals[substr(rownames(mcmc_vals), 1, 1) == "w", "Mean"]
mcmc_w <- matrix(mcmc_w, nrow=K)
colnames(mcmc_w) <- paste0("sample", 1:S)
round(mcmc_w, 2)

```

```

##      sample1 sample2 sample3
## [1,]    0.21    0.00    0.00
## [2,]    0.45    0.91    0.00
## [3,]    0.21    0.00    0.45
## [4,]    0.00    0.46    0.00
## [5,]    0.33    0.67    0.00
## [6,]    0.49    0.45    0.42
## [7,]    0.99    0.89    0.79
## [8,]    0.97    0.99    0.98
## [9,]    0.52    0.00    0.79

```

```
## [10,]    0.00    0.00    0.32

mcmc_w_sd <- mcmc_vals[substr(rownames(mcmc_vals), 1, 1) == "w", "SD"]
mcmc_w_sd <- matrix(mcmc_w_sd, nrow=K)
colnames(mcmc_w_sd) <- paste0("sample", 1:S)

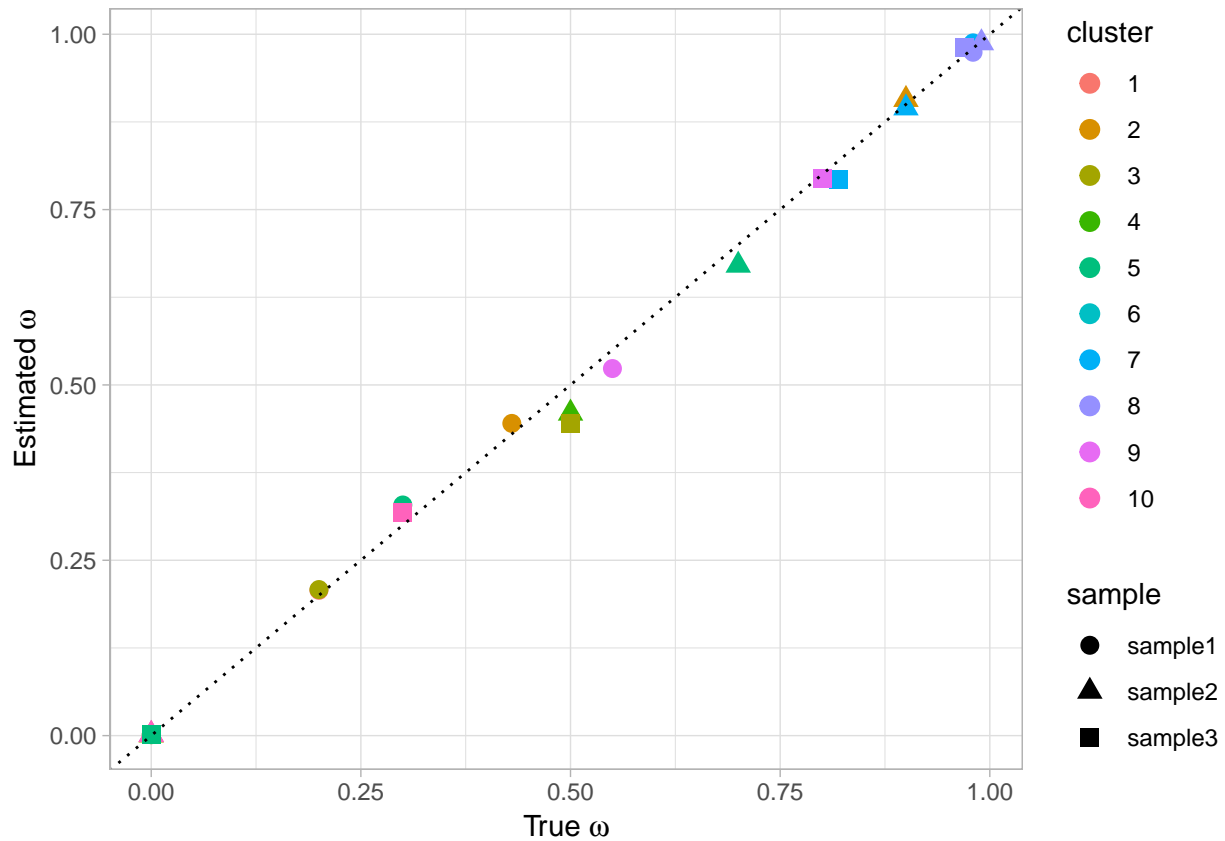
# order true w based on mcmc cluster numbering
mcmc_cluster_numbering <- matrix(z.map, nrow = 10)
get_mode <- function(v) {
  uniqv <- unique(v)
  uniqv[which.max(tabulate(match(v, uniqv)))]
}
mcmc_cluster_numbering <- apply(mcmc_cluster_numbering, 2, get_mode)
mcmc_cluster_numbering

## [1] 8 7 9 3 3 2 5 1 10 4

true_to_mcmc_w_ordering <- match(1:K, mcmc_cluster_numbering)
w_ordered <- w[true_to_mcmc_w_ordering, ]

# scatter
mcmc_w_tb <- mcmc_w %>%
  as_tibble() %>%
  mutate(cluster=1:K) %>%
  gather("sample", "mcmc_w", -c(cluster))
w_master <- w_ordered %>%
  as_tibble() %>%
  mutate(cluster=1:K) %>%
  gather("sample", "true_w", -c(cluster)) %>%
  left_join(mcmc_w_tb, by=c("cluster", "sample")) %>%
  mutate(cluster=factor(cluster),
         sample=factor(sample))
ggplot(w_master, aes(true_w, mcmc_w)) +
  geom_point(size=3, aes(color = cluster, shape = sample)) +
  geom_abline(slope=1, intercept=0, linetype="dotted") +
  xlab(expression("True " * omega)) +
  ylab(expression("Estimated " * omega)) +
  theme_light()

## Warning: Removed 3 rows containing missing values (geom_point).
```



```
#ggsave(file.path(figs.dir, "w_plot.pdf"), height = 7, width = 8)
```

## Admat functions

```
rand.admat <- function(admat) {
  for(col in 1:ncol(admat)) {
    ind.0 <- which(admat[,col] == 0) # possible positions (0's)
    rand.ind <- sample(ind.0, size=1)
    admat[rand.ind,col] <- 1
  }

  while (sum(admat[1, ]) == 0) {
    admat <- mutate.admat(admat)
  }

  admat
}

base.admat <- function(w, zero.thresh=0.01) {
  cluster.sample.presence <- apply(w, 1, function(x) which(x>zero.thresh))
  K <- nrow(w)
  S <- ncol(w)
  all.samples <- 1:S
  admat <- matrix(data=0, nrow=(1+K), ncol=K) # rows=from is root + 1:K, cols=to is 1:K

  # fill in restraints
```

```

# can go from root to anyone, skip and start at nrow=2 (cluster 1)
for (from in 2:(K+1)) {
  for (to in 1:K) {

    # can't go to self
    if ((from-1) == to) {
      admat[from, to] <- NA
      #print(c(from, to, "self"))
      next
    }
    # hierarchy restraints
    from.samples <- cluster.sample.presence[[from-1]]
    to.samples <- cluster.sample.presence[[to]]

    ## no restraints if same sample presence
    if (setequal(from.samples, to.samples)) {
      #print(c(from, to, "same"))
      next
    }

    ## restraint if # from.samples < # to.samples
    if(length(from.samples) < length(to.samples)) {
      #print(c(from, to, "from set is smaller than to set"))
      admat[from, to] <- NA
      next
    }

    ## no restraints if to.samples is subset of from.samples
    if (all(to.samples %in% from.samples)) {
      #print(c(from, to, "subset"))
      next
    } else {
      #print(c(from, to, "not subset"))
      admat[from, to] <- NA
    }
  }
}
admat
}

init.admat <- function(w, zero.thresh) {
  base <- base.admat(w, zero.thresh)
  rand.admat(base)
}

mutate.admat <- function(admat, ncol.to.mutate) {

  # choose a column(s) to mutate
  K <- ncol(admat)
  rand.ks <- sample(1:K, size=ncol.to.mutate)

  # mutate columns

```

```

new.admat <- admat
for (rand.k in rand.ks) {
  ## possible positions (0's)
  possiblePos <- which(!is.na(admat[, rand.k]) & admat[, rand.k] != 1)
  ## current position with 1
  ind.1 <- which(admat[, rand.k] == 1)
  ## select new position
  if (length(possiblePos) == 1) {
    new.1 <- possiblePos
  } else {
    new.1 <- sample(possiblePos, size=1)
  }

  new.admat[ind.1, rand.k] <- 0
  new.admat[new.1, rand.k] <- 1
}

while (sum(new.admat[1, ]) == 0) {
  new.admat <- mutate.admat(admat)
}
new.admat
}

```

## SCHISM tree scoring

```

decide.ht <- function(pval, alpha=0.05) {
  # 1 signals rejection event for null of i -> j
  if (pval <= alpha) return(1)
  else return(0)
}

create.cpov <- function(mcmc_w, mcmc_w_sd, alpha=0.05) {
  cpov <- base.admat(mcmc_w, zero.thresh = 0.01)
  S <- ncol(mcmc_w) # number of samples

  # root can go to anyone -- all 0's (default base admat value)

  for (r in 2:nrow(cpov)) {
    for (c in 1:ncol(cpov)) {

      if (is.na(cpov[r,c])) next # skip restricted position

      from <- r-1 # 'from' cluster node
      to <- c # 'to' cluster node

      statistic <- 0
      pval <- 0

      for(s in 1:S) {
        d <- mcmc_w[from,s] - mcmc_w[to,s]
        d_sd <- sqrt((mcmc_w_sd[from,s])^2 + (mcmc_w_sd[to,s])^2)
        I <- sum(d < 0)
      }
    }
  }
}

```

```

    statistic <- statistic + (d / d_sd)^2 * I

    for (k in 0:S) {
      pval <- pval + ((1 - pchisq(statistic, k)) * choose(S, k) / (2^S))
    }
  }
  cpov[r,c] <- decide.ht(pval, alpha)
}
}
cpov
}

calc.topology.cost <- function(admat, cpov) {

  TC <- 0
  edges <- which(admat == 1, arr.ind=T)
  for (i in 1:nrow(edges)) {
    TC <- TC + cpov[edges[i,1], edges[i,2]]
  }

  TC
}

calc.mass.cost <- function(admat, mcmc_w) {

  numChildren <- rowSums(admat, na.rm = T)
  nodes <- which(numChildren > 0, arr.ind = T) # not leaves
  mc.node <- rep(0, length(nodes))

  for (i in 1:length(nodes)) {
    node <- nodes[i]

    # root node: MCF = 1
    parent.w <- rep(1, ncol(mcmc_w))
    # not root node: look up MCF in mcmc_w
    if (node != 1) {
      parent.w <- mcmc_w[node-1,]
    }

    kids <- which(admat[node,] == 1, arr.ind = T)
    if (numChildren[node] > 1) {
      children.w <- colSums(mcmc_w[kids,])
    } else {
      children.w <- mcmc_w[kids,]
    }

    mc.s <- ifelse(parent.w >= children.w, 0, children.w - parent.w)
    mc.node[i] <- sqrt(sum(mc.s^2))
  }

  sum(mc.node)
}

```

```
calc.tree.fitness <- function(admat, cpov, mcmc_w, scaling.coeff=5) {  
  TC <- calc.topology.cost(admat, cpov)  
  MC <- calc.mass.cost(admat, mcmc_w)  
  Z <- TC + MC  
  fitness <- exp(-scaling.coeff * Z)  
  fitness  
}
```

Tree MCMC

Mutate 2 columns

Mutate 3 columns

Mutate 4 columns