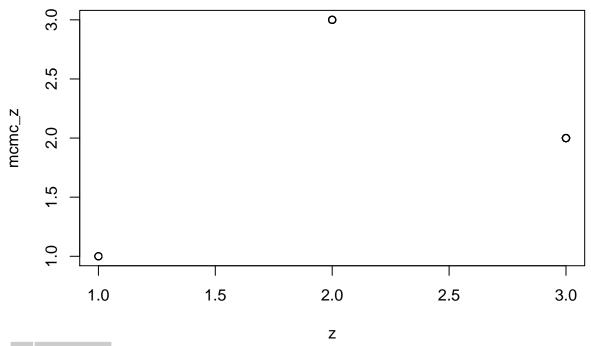
Version 3: 3 clusters

Simulate data

```
I <- 50
K <- 3
S <- 10
# choose diffuse priors for gamma
a_gamma <- 2
b_gamma <- 10
set.seed(123)
a <- matrix(NA, nrow=K, ncol=S)
b <- matrix(NA, nrow=K, ncol=S)</pre>
for (s in 1:S) {
  a[, s] <- rgamma(K, a_gamma, rate = 1/b_gamma)
  b[, s] <- rgamma(K, a_gamma, rate = 1/b_gamma)
colnames(a) <- colnames(b) <- paste0("sample", seq_len(ncol(a)))</pre>
# reorder a,b matrices to match ordering of means (U) in S1
U \leftarrow a/(a+b)
V <- a+b
ix <- order(U[, 1])</pre>
U.ordered <- U[ix, ]
a.ordered <- a[ix, ]
b.ordered <- b[ix, ]</pre>
V.ordered <- V[ix, ]</pre>
#pi <- as.vector(rdirichlet(1, rep(1, K)))</pre>
pi \leftarrow c(0.2, 0.3, 0.5)
z <- sample(1:K, size = I, replace = T, prob = pi)
w <- matrix(NA, nrow=I, ncol=S)
for (s in 1:S) {
  w[, s] <- rbeta(I, a.ordered[,s][z], b.ordered[,s][z])
tcn <- matrix(2, nrow=I, ncol=S)</pre>
m <- matrix(rep(sample(1:2, size = I, replace = T), S),</pre>
             nrow=I, ncol=S)
calcTheta <- function(m, tcn, w) {</pre>
  (m * w) / (tcn * w + 2*(1-w))
theta <- calcTheta(m, tcn, w)
n <- replicate(S, rpois(I, 100))</pre>
y <- matrix(NA, nrow=I, ncol=S)
for (i in 1:I) {
```

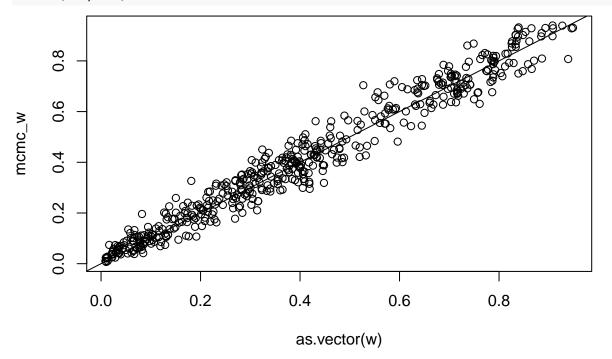
JAGS

```
jags.file <- file.path(models.dir, "v3_no_constraints.jags")</pre>
inits <- list(".RNG.name" = "base::Wichmann-Hill",</pre>
               ".RNG.seed" = 123)
jags.m <- jags.model(jags.file, test.data,</pre>
                      n.chains = 1,
                      inits = inits,
                      n.adapt = 1000)
## Compiling model graph
##
      Resolving undeclared variables
##
      Allocating nodes
## Graph information:
      Observed stochastic nodes: 500
##
##
      Unobserved stochastic nodes: 611
##
      Total graph size: 8531
##
## Initializing model
params <- c("z", "w", "U", "V")
samps <- coda.samples(jags.m, params, n.iter=10000, thin=7)</pre>
jags_df <- ggs(samps)</pre>
s <- summary(samps)</pre>
#effectiveSize(samps)
\#pdf(file.path(trace.dir, pasteO(runName, "\_trace.pdf")))
#plot(samps)
#dev.off()
```



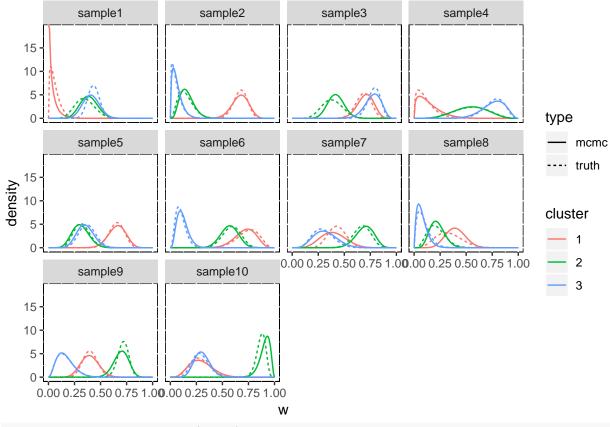
Z	mcmc_z
1	1
2	3
3	2

```
mcmc_vals <- s$statistics
mcmc_w <- mcmc_vals[substr(rownames(mcmc_vals), 1, 1) == "w", "Mean"]
plot(as.vector(w), mcmc_w, type = "p")
abline(a=0, b=1)</pre>
```



```
\# mcmc_U \leftarrow mcmc_vals[substr(rownames(mcmc_vals), 1, 1) == "U", "Mean"]
# mcmc_U <- matrix(mcmc_U, nrow=K)</pre>
# mcmc_V <- mcmc_vals[substr(rownames(mcmc_vals), 1, 1) == "V", "Mean"]</pre>
# mcmc_V <- matrix(mcmc_V, nrow=K)</pre>
\# p \leftarrow seq(0, 1, length = 100)
# colors <- c("#000000", "#DCA200", "#8FA7ED", "#9D847A", "#A47901")
# for (s in 1:S) {
   for (k in 1:K) {
#
      if (k == 1) {
#
        # plot mcmc mean U, V
#
        plot(p, dbeta(p, mcmc\_U[k,s] * mcmc\_V[k,s], (1-mcmc\_U[k,s])*mcmc\_V[k,s]),
#
             main = pasteO("S", s),
             ylab = "density", xlab = "w", type = "l", col = colors[k],
#
#
             ylim = c(0, 20))
#
        # plot truth
#
        lines(p, dbeta(p, a.ordered[k,s], b.ordered[k,s]), type = "l", col = colors[k], lty=2)
#
        # add legend
#
        allU <- round(as.vector(rbind(mcmc_U[,s], U.ordered[,s])), digits = 2)
        legend(x = "topleft",
#
#
                legend = pasteO(c("mean k", "true k"), rep(1:K, each=2), ", U=", allU),
#
                col = colors[rep(1:K, each=2)],
#
                lty = rep(1:2, K),
#
                cex=0.8)
#
      } else {
#
        # plot mcmc mean U, V
#
        lines(p, dbeta(p, mcmc_U[k,s] * mcmc_V[k,s], (1-mcmc_U[k,s])*mcmc_V[k,s]),
#
               type = "l", col = colors[k])
#
        # plot truth
#
        lines(p, dbeta(p, a.ordered[k,s], b.ordered[k,s]), type = "l", col = colors[k], lty=2)
#
    }
#
# }
mcmc U <- mcmc vals[substr(rownames(mcmc vals), 1, 1) == "U", "Mean"]</pre>
mcmc U <- matrix(mcmc U, nrow=K)</pre>
colnames(mcmc U) <- paste0("sample", 1:S)</pre>
mcmc_V <- mcmc_vals[substr(rownames(mcmc_vals), 1, 1) == "V", "Mean"]</pre>
mcmc_V <- matrix(mcmc_V, nrow=K)</pre>
colnames(mcmc_V) <- paste0("sample", 1:S)</pre>
mcmc_U2 <- mcmc_U %>%
    as_tibble() %>%
    mutate(cluster=z_mapping$mcmc_z,
           type="mcmc") %>%
    gather("sample", "U", -c(cluster, type)) %>%
    mutate(sample=factor(sample, levels=colnames(mcmc U)))
mcmc_betas <- mcmc_V %>%
    as_tibble() %>%
    mutate(cluster=z_mapping$mcmc_z,
           type="mcmc") %>%
    gather("sample", "V", -c(cluster, type)) %>%
    mutate(sample=factor(sample, levels=colnames(mcmc_U))) %>%
```

```
left_join(mcmc_U2, by=c("cluster", "type", "sample"))
mcmc_betas$a <- mcmc_betas$U * mcmc_betas$V</pre>
mcmc_betas$b <- (1 - mcmc_betas$U) * mcmc_betas$V</pre>
a2 <- a.ordered %>%
    as tibble() %>%
    mutate(cluster=1:K,
           type="truth") %>%
    gather("sample", "a", -c(cluster, type)) %>%
    mutate(sample=factor(sample, levels=colnames(a.ordered)))
true_betas <- b.ordered %>%
    as_tibble() %>%
    mutate(cluster=1:K,
           type="truth") %>%
    gather("sample", "b", -c(cluster, type)) %>%
    mutate(sample=factor(sample, levels=colnames(b.ordered))) %>%
    left_join(a2, by=c("cluster", "type", "sample"))
betas <- bind_rows(true_betas, mcmc_betas)</pre>
dens_list <- list()</pre>
for(row.ix in 1:nrow(betas)) {
 x \leftarrow seq(0, 1, length = 100)
 val <- dbeta(x, shape1=betas[row.ix, ]$a, shape2=betas[row.ix, ]$b)</pre>
 dens_list[[row.ix]] <- data.frame(x=x, val=val,</pre>
                                   type=betas[row.ix, ]$type,
                                   cluster=betas[row.ix, ]$cluster,
                                   sample=betas[row.ix, ]$sample,
                                   stringsAsFactors = F)
}
dens_df <- bind_rows(dens_list)</pre>
dens_df$grp <- paste0("k", dens_df$cluster, ",", dens_df$sample, ",", dens_df$type)</pre>
dens_df$cluster <- as.factor(dens_df$cluster)</pre>
## beta distributions by sample
beta.plot <- ggplot(dens_df, aes(x=x, y=val, group=grp)) +
  geom_line(aes(color=cluster, lty=type)) +
  facet_wrap(~sample) +
 ylab("density") +
 xlab("w") +
  theme(panel.background=element_rect(fill="white", color="black"))
beta.plot
```



```
samples_matrix <- as.matrix(samps)</pre>
U_chains <- samples_matrix[, which(substr(colnames(samples_matrix), 1, 1) == "U")]</pre>
pdf(file.path(working.dir, paste0(runName, "_Uplots.pdf")))
for (s in 1:S) {
  s.char <- paste(s)</pre>
  if (nchar(s.char) == 1) {
    U_sub <- U_chains[, which(substr(colnames(U_chains), 5, 6) == paste0(s.char, "]"))]</pre>
  } else {
    U_sub <- U_chains[, which(substr(colnames(U_chains), 5, 7) == paste0(s.char, "]"))]</pre>
 plot(density(as.vector(U_sub)),
       main = paste0("S", s, " U density; K=", K))
  legend(x = "topright",
         legend = paste0("k", 1:K, ", true U = ", round(U[,s], 2)))
}
dev.off()
pdf(file.path(working.dir, paste0(runName, "_Uplots_withTruth.pdf")))
for (s in 1:S) {
  s.char <- paste(s)</pre>
  if (nchar(s.char) == 1) {
    U_sub <- U_chains[, which(substr(colnames(U_chains), 5, 6) == paste0(s.char, "]"))]</pre>
 } else {
    U_sub <- U_chains[, which(substr(colnames(U_chains), 5, 7) == paste0(s.char, "]"))]</pre>
```