

Version 3: 3 clusters

Simulate data

```
I <- 50
K <- 3
S <- 10

# choose diffuse priors for gamma
a_gamma <- 2
b_gamma <- 10

set.seed(123)

a <- matrix(NA, nrow=K, ncol=S)
b <- matrix(NA, nrow=K, ncol=S)
for (s in 1:S) {
  a[, s] <- rgamma(K, a_gamma, rate = 1/b_gamma)
  b[, s] <- rgamma(K, a_gamma, rate = 1/b_gamma)
}

# reorder a,b matrices to match ordering of means (U) in S1
U <- a/(a+b)
V <- a+b
U.ordered <- U[order(U[,1]), ]
a.ordered <- a[order(U[,1]), ]
b.ordered <- b[order(U[,1]), ]
V.ordered <- V[order(U[,1]), ]

pi <- as.vector(rdirichlet(1, rep(1, K)))
z <- sample(1:K, size = I, replace = T, prob = pi)

w <- matrix(NA, nrow=I, ncol=S)
for (s in 1:S) {
  w[, s] <- rbeta(I, a.ordered[,s][z], b.ordered[,s][z])
}

tcn <- matrix(2, nrow=I, ncol=S)
m <- matrix(rep(sample(1:2, size = I, replace = T), S), nrow=I, ncol=S)

calcTheta <- function(m, tcn, w) {
  (m * w) / (tcn * w + 2*(1-w))
}
theta <- calcTheta(m, tcn, w)

n <- replicate(S, rpois(I, 100))
y <- matrix(NA, nrow=I, ncol=S)
for (i in 1:I) {
  for (s in 1:S) {
    y[i, s] <- rbinom(1, n[i, s], theta[i,s])
  }
}
```

```
}
```

JAGS

```
jags.file <- file.path(models.dir, "v3_no_constraints.jags")

test.data <- list("I" = I, "S" = S, "K" = K,
                 "y" = y, "n" = n,
                 "m" = m, "tcn" = tcn)
jags.m <- jags.model(jags.file, test.data,
                    n.chains = 1,
                    inits = list(".RNG.name" = "base::Wichmann-Hill",
                                ".RNG.seed" = 123))
```

```
## Compiling model graph
##   Resolving undeclared variables
##   Allocating nodes
## Graph information:
##   Observed stochastic nodes: 500
##   Unobserved stochastic nodes: 611
##   Total graph size: 8511
##
## Initializing model
```

```
params <- c("z", "w", "U", "V")
samps <- coda.samples(jags.m, params, n.iter=15000, thin=7)
s <- summary(samps)
effectiveSize(samps)
```

```
##   U[1,1]  U[2,1]  U[3,1]  U[1,2]  U[2,2]  U[3,2]  U[1,3]  U[2,3]
## 2142.000 2142.000 2006.880 2142.000 2142.000 2142.000 1811.995 2142.000
##   U[3,3]  U[1,4]  U[2,4]  U[3,4]  U[1,5]  U[2,5]  U[3,5]  U[1,6]
## 1578.722 2426.304 2142.000 1570.756 2142.000 2142.000 1658.058 2142.000
##   U[2,6]  U[3,6]  U[1,7]  U[2,7]  U[3,7]  U[1,8]  U[2,8]  U[3,8]
## 2142.000 1987.063 1968.406 2197.227 2142.000 1996.482 2142.000 2175.500
##   U[1,9]  U[2,9]  U[3,9]  U[1,10] U[2,10] U[3,10]  V[1,1]  V[2,1]
## 2193.014 1483.625 2142.000 2142.000 1662.116 2142.000 1718.416 1972.182
##   V[3,1]  V[1,2]  V[2,2]  V[3,2]  V[1,3]  V[2,3]  V[3,3]  V[1,4]
## 1820.855 1452.636 1817.606 2142.000 1601.271 1640.141 2142.000 2163.888
##   V[2,4]  V[3,4]  V[1,5]  V[2,5]  V[3,5]  V[1,6]  V[2,6]  V[3,6]
## 1639.306 2004.973 1839.223 1823.634 2142.000 1806.398 1719.000 1802.482
##   V[1,7]  V[2,7]  V[3,7]  V[1,8]  V[2,8]  V[3,8]  V[1,9]  V[2,9]
## 1562.180 1896.964 1993.573 1793.223 1945.895 1803.795 1893.288 1879.145
##   V[3,9]  V[1,10] V[2,10] V[3,10]  w[1,1]  w[2,1]  w[3,1]  w[4,1]
## 2142.000 1743.050 1685.673 2142.000 1800.997 1959.039 1981.470 2142.000
##   w[5,1]  w[6,1]  w[7,1]  w[8,1]  w[9,1]  w[10,1] w[11,1] w[12,1]
## 2142.000 2519.824 2056.323 2142.000 2142.000 2649.001 2142.000 2142.000
##   w[13,1] w[14,1] w[15,1] w[16,1] w[17,1] w[18,1] w[19,1] w[20,1]
## 2142.000 2142.000 2142.000 2142.000 1956.397 2142.000 2142.000 2142.000
##   w[21,1] w[22,1] w[23,1] w[24,1] w[25,1] w[26,1] w[27,1] w[28,1]
## 2142.000 1983.856 2142.000 2428.275 2142.000 1891.294 2284.129 2142.000
##   w[29,1] w[30,1] w[31,1] w[32,1] w[33,1] w[34,1] w[35,1] w[36,1]
## 2142.000 2142.000 1781.407 2142.000 2142.000 2142.000 2142.000 2142.000
```

```

## w[37,1] w[38,1] w[39,1] w[40,1] w[41,1] w[42,1] w[43,1] w[44,1]
## 2142.000 2142.000 1829.167 2142.000 2142.000 1853.383 1971.223 2370.350
## w[45,1] w[46,1] w[47,1] w[48,1] w[49,1] w[50,1] w[1,2] w[2,2]
## 2142.000 2142.000 2142.000 2142.000 2142.000 2328.024 1755.238 1940.938
## w[3,2] w[4,2] w[5,2] w[6,2] w[7,2] w[8,2] w[9,2] w[10,2]
## 2142.000 2142.000 1832.912 2303.186 2343.480 2142.000 1866.467 1807.309
## w[11,2] w[12,2] w[13,2] w[14,2] w[15,2] w[16,2] w[17,2] w[18,2]
## 2142.000 2142.000 2142.000 2142.000 1832.196 2142.000 2142.000 2142.000
## w[19,2] w[20,2] w[21,2] w[22,2] w[23,2] w[24,2] w[25,2] w[26,2]
## 1948.585 2142.000 2142.000 2142.000 2142.000 2271.819 1798.303 1541.334
## w[27,2] w[28,2] w[29,2] w[30,2] w[31,2] w[32,2] w[33,2] w[34,2]
## 2142.000 1861.504 2392.127 1996.999 1984.887 2015.571 2142.000 1986.299
## w[35,2] w[36,2] w[37,2] w[38,2] w[39,2] w[40,2] w[41,2] w[42,2]
## 2142.000 1972.838 2142.000 2142.000 1872.781 2142.000 2142.000 2207.614
## w[43,2] w[44,2] w[45,2] w[46,2] w[47,2] w[48,2] w[49,2] w[50,2]
## 1805.430 1849.735 2142.000 2142.000 2371.874 2142.000 2003.964 1572.811
## w[1,3] w[2,3] w[3,3] w[4,3] w[5,3] w[6,3] w[7,3] w[8,3]
## 2142.000 2142.000 2142.000 2171.721 2677.332 2142.000 2142.000 2142.000
## w[9,3] w[10,3] w[11,3] w[12,3] w[13,3] w[14,3] w[15,3] w[16,3]
## 2142.000 1729.736 2002.602 2142.000 2142.000 2142.000 2142.000 2142.000
## w[17,3] w[18,3] w[19,3] w[20,3] w[21,3] w[22,3] w[23,3] w[24,3]
## 2184.929 1873.625 2142.000 2142.000 2142.000 1913.038 2142.000 2142.000
## w[25,3] w[26,3] w[27,3] w[28,3] w[29,3] w[30,3] w[31,3] w[32,3]
## 2115.147 2142.000 1910.254 2142.000 2142.000 2142.000 1052.805 2142.000
## w[33,3] w[34,3] w[35,3] w[36,3] w[37,3] w[38,3] w[39,3] w[40,3]
## 2065.140 2142.000 2303.922 2142.000 2142.000 2005.041 2142.000 2142.000
## w[41,3] w[42,3] w[43,3] w[44,3] w[45,3] w[46,3] w[47,3] w[48,3]
## 1906.826 2142.000 2142.000 2142.000 2142.000 2142.000 2312.049 2142.000
## w[49,3] w[50,3] w[1,4] w[2,4] w[3,4] w[4,4] w[5,4] w[6,4]
## 2142.000 2142.000 2461.949 2142.000 2142.000 2142.000 2142.000 2142.000
## w[7,4] w[8,4] w[9,4] w[10,4] w[11,4] w[12,4] w[13,4] w[14,4]
## 2142.000 2142.000 2142.000 2142.000 2142.000 1567.555 2110.186 1786.340
## w[15,4] w[16,4] w[17,4] w[18,4] w[19,4] w[20,4] w[21,4] w[22,4]
## 1978.167 2142.000 2142.000 2142.000 2142.000 2142.000 2142.000 2142.000
## w[23,4] w[24,4] w[25,4] w[26,4] w[27,4] w[28,4] w[29,4] w[30,4]
## 2142.000 2180.977 2142.000 2142.000 2142.000 2142.000 2142.000 2142.000
## w[31,4] w[32,4] w[33,4] w[34,4] w[35,4] w[36,4] w[37,4] w[38,4]
## 2142.000 2183.922 2142.000 2319.362 2012.250 2142.000 2142.000 2142.000
## w[39,4] w[40,4] w[41,4] w[42,4] w[43,4] w[44,4] w[45,4] w[46,4]
## 2142.000 2142.000 2142.000 2142.000 2791.499 2142.000 2142.000 2142.000
## w[47,4] w[48,4] w[49,4] w[50,4] w[1,5] w[2,5] w[3,5] w[4,5]
## 2142.000 2142.000 2598.023 2142.000 1985.890 2142.000 2142.000 2142.000
## w[5,5] w[6,5] w[7,5] w[8,5] w[9,5] w[10,5] w[11,5] w[12,5]
## 2142.000 2142.000 2052.287 2142.000 2142.000 2142.000 2142.000 1863.392
## w[13,5] w[14,5] w[15,5] w[16,5] w[17,5] w[18,5] w[19,5] w[20,5]
## 2142.000 2142.000 2142.000 2011.041 1745.843 2142.000 2142.000 2142.000
## w[21,5] w[22,5] w[23,5] w[24,5] w[25,5] w[26,5] w[27,5] w[28,5]
## 2142.000 2142.000 2142.000 1492.224 2142.000 2142.000 2376.384 2142.000
## w[29,5] w[30,5] w[31,5] w[32,5] w[33,5] w[34,5] w[35,5] w[36,5]
## 2142.000 2354.851 1731.853 2142.000 2639.707 1787.353 2142.000 2142.000
## w[37,5] w[38,5] w[39,5] w[40,5] w[41,5] w[42,5] w[43,5] w[44,5]
## 2142.000 2142.000 2142.000 3391.601 2142.000 2498.970 2142.000 2142.000
## w[45,5] w[46,5] w[47,5] w[48,5] w[49,5] w[50,5] w[1,6] w[2,6]
## 2170.709 2124.086 2954.543 2142.000 2142.000 2142.000 2142.000 2142.000

```

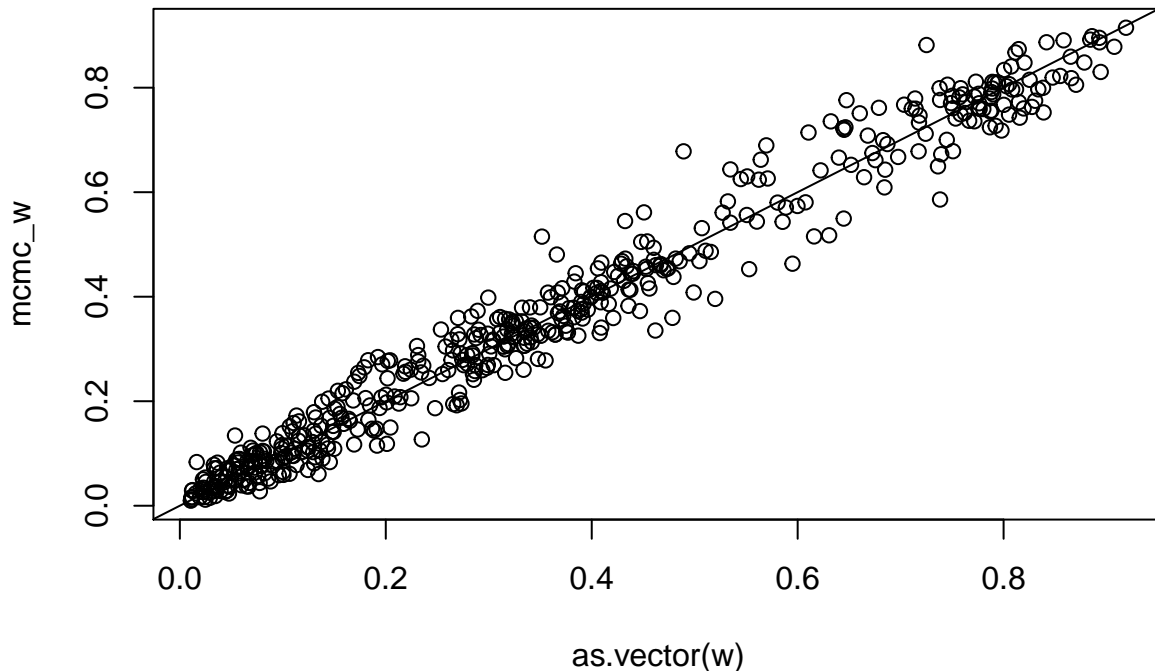
##	w[3,6]	w[4,6]	w[5,6]	w[6,6]	w[7,6]	w[8,6]	w[9,6]	w[10,6]
##	1982.180	1962.134	1971.074	2142.000	2142.000	2142.000	2142.000	2142.000
##	w[11,6]	w[12,6]	w[13,6]	w[14,6]	w[15,6]	w[16,6]	w[17,6]	w[18,6]
##	2142.000	2197.534	2142.000	2142.000	2142.000	2184.238	2142.000	2142.000
##	w[19,6]	w[20,6]	w[21,6]	w[22,6]	w[23,6]	w[24,6]	w[25,6]	w[26,6]
##	2142.000	1779.004	2142.000	2015.093	2210.947	1970.288	2142.000	1770.205
##	w[27,6]	w[28,6]	w[29,6]	w[30,6]	w[31,6]	w[32,6]	w[33,6]	w[34,6]
##	1984.322	1924.590	2142.000	2288.599	2142.000	2142.000	1975.540	2297.189
##	w[35,6]	w[36,6]	w[37,6]	w[38,6]	w[39,6]	w[40,6]	w[41,6]	w[42,6]
##	1907.643	2142.000	2142.000	2142.000	1580.768	1735.494	2142.000	1991.563
##	w[43,6]	w[44,6]	w[45,6]	w[46,6]	w[47,6]	w[48,6]	w[49,6]	w[50,6]
##	2142.000	1997.550	2446.214	2142.000	2308.908	2142.000	2221.283	2310.555
##	w[1,7]	w[2,7]	w[3,7]	w[4,7]	w[5,7]	w[6,7]	w[7,7]	w[8,7]
##	2142.000	2142.000	2142.000	1919.233	1975.089	2142.000	2000.950	2142.000
##	w[9,7]	w[10,7]	w[11,7]	w[12,7]	w[13,7]	w[14,7]	w[15,7]	w[16,7]
##	1981.249	2142.000	2579.469	2142.000	2296.497	2142.000	2142.000	2153.435
##	w[17,7]	w[18,7]	w[19,7]	w[20,7]	w[21,7]	w[22,7]	w[23,7]	w[24,7]
##	2002.483	2142.000	1836.664	2142.000	2142.000	2142.000	2142.000	2076.000
##	w[25,7]	w[26,7]	w[27,7]	w[28,7]	w[29,7]	w[30,7]	w[31,7]	w[32,7]
##	1978.407	2142.000	2142.000	2006.511	2142.000	2142.000	2142.000	2142.000
##	w[33,7]	w[34,7]	w[35,7]	w[36,7]	w[37,7]	w[38,7]	w[39,7]	w[40,7]
##	2142.000	2142.000	2142.000	2142.000	2142.000	2142.000	2142.000	2142.000
##	w[41,7]	w[42,7]	w[43,7]	w[44,7]	w[45,7]	w[46,7]	w[47,7]	w[48,7]
##	2142.000	2142.000	1887.613	2142.000	2433.912	2142.000	2329.361	2486.092
##	w[49,7]	w[50,7]	w[1,8]	w[2,8]	w[3,8]	w[4,8]	w[5,8]	w[6,8]
##	2142.000	2142.000	2142.000	2142.000	1974.748	2142.000	1973.238	2142.000
##	w[7,8]	w[8,8]	w[9,8]	w[10,8]	w[11,8]	w[12,8]	w[13,8]	w[14,8]
##	2413.512	2142.000	1963.196	2142.000	2142.000	2142.000	3400.756	2142.000
##	w[15,8]	w[16,8]	w[17,8]	w[18,8]	w[19,8]	w[20,8]	w[21,8]	w[22,8]
##	1915.823	2142.000	2142.000	2496.505	2142.000	2142.000	2142.000	2142.000
##	w[23,8]	w[24,8]	w[25,8]	w[26,8]	w[27,8]	w[28,8]	w[29,8]	w[30,8]
##	2142.000	2142.000	2347.632	2142.000	2351.477	1922.427	2142.000	2142.000
##	w[31,8]	w[32,8]	w[33,8]	w[34,8]	w[35,8]	w[36,8]	w[37,8]	w[38,8]
##	2142.000	2142.000	2142.000	2142.000	2142.000	2142.000	2693.401	2142.000
##	w[39,8]	w[40,8]	w[41,8]	w[42,8]	w[43,8]	w[44,8]	w[45,8]	w[46,8]
##	2296.933	2142.000	2142.000	2142.000	2142.000	2142.000	2151.750	2142.000
##	w[47,8]	w[48,8]	w[49,8]	w[50,8]	w[1,9]	w[2,9]	w[3,9]	w[4,9]
##	1972.162	2142.000	2485.688	2142.000	2142.000	2142.000	2142.000	2021.215
##	w[5,9]	w[6,9]	w[7,9]	w[8,9]	w[9,9]	w[10,9]	w[11,9]	w[12,9]
##	2142.000	2142.000	2142.000	1920.995	2142.000	2142.000	2142.000	2142.000
##	w[13,9]	w[14,9]	w[15,9]	w[16,9]	w[17,9]	w[18,9]	w[19,9]	w[20,9]
##	2142.000	2142.000	1901.021	2297.457	1926.658	2142.000	2142.000	2142.000
##	w[21,9]	w[22,9]	w[23,9]	w[24,9]	w[25,9]	w[26,9]	w[27,9]	w[28,9]
##	2142.000	2142.000	2142.000	2142.000	2142.000	2012.179	1952.010	2294.422
##	w[29,9]	w[30,9]	w[31,9]	w[32,9]	w[33,9]	w[34,9]	w[35,9]	w[36,9]
##	2142.000	2142.000	2142.000	2142.000	2142.000	2142.000	2142.000	2142.000
##	w[37,9]	w[38,9]	w[39,9]	w[40,9]	w[41,9]	w[42,9]	w[43,9]	w[44,9]
##	2277.041	2142.000	1899.353	2142.000	2352.514	2142.000	2142.000	2142.000
##	w[45,9]	w[46,9]	w[47,9]	w[48,9]	w[49,9]	w[50,9]	w[1,10]	w[2,10]
##	2142.000	2214.531	2142.000	2142.000	2142.000	2142.000	2048.270	2142.000
##	w[3,10]	w[4,10]	w[5,10]	w[6,10]	w[7,10]	w[8,10]	w[9,10]	w[10,10]
##	2142.000	1959.550	2142.000	2142.000	2142.000	2329.004	2142.000	2142.000
##	w[11,10]	w[12,10]	w[13,10]	w[14,10]	w[15,10]	w[16,10]	w[17,10]	w[18,10]
##	1907.791	2142.000	2142.000	3383.611	2135.752	2142.000	2820.534	1939.852

```
## w[19,10] w[20,10] w[21,10] w[22,10] w[23,10] w[24,10] w[25,10] w[26,10]
## 1881.902 2142.000 2142.000 2142.000 2575.129 2007.821 2142.000 2142.000
## w[27,10] w[28,10] w[29,10] w[30,10] w[31,10] w[32,10] w[33,10] w[34,10]
## 2216.867 1978.739 2142.000 2142.000 1933.267 2201.143 2142.000 2142.000
## w[35,10] w[36,10] w[37,10] w[38,10] w[39,10] w[40,10] w[41,10] w[42,10]
## 2142.000 2142.000 2555.469 2142.000 2142.000 1627.877 1982.515 2142.000
## w[43,10] w[44,10] w[45,10] w[46,10] w[47,10] w[48,10] w[49,10] w[50,10]
## 1566.237 2136.387 2142.000 2142.000 2142.000 2142.000 1972.924 2194.389
##      z[1]      z[2]      z[3]      z[4]      z[5]      z[6]      z[7]      z[8]
##      0.000      0.000      0.000      0.000      0.000      0.000      0.000      0.000
##      z[9]      z[10]     z[11]     z[12]     z[13]     z[14]     z[15]     z[16]
##      0.000      0.000      0.000      0.000      0.000      0.000      0.000      0.000
##      z[17]     z[18]     z[19]     z[20]     z[21]     z[22]     z[23]     z[24]
##      0.000      0.000      0.000      0.000      0.000      0.000      0.000      0.000
##      z[25]     z[26]     z[27]     z[28]     z[29]     z[30]     z[31]     z[32]
##      0.000      0.000      0.000      0.000      0.000      0.000      0.000      0.000
##      z[33]     z[34]     z[35]     z[36]     z[37]     z[38]     z[39]     z[40]
##      0.000      0.000      0.000      0.000      0.000      0.000      0.000      0.000
##      z[41]     z[42]     z[43]     z[44]     z[45]     z[46]     z[47]     z[48]
##      0.000      0.000      0.000      0.000      0.000      0.000      0.000      0.000
##      z[49]     z[50]
##      0.000      0.000
```

```
pdf(file.path(trace.dir, paste0(runName, "_trace.pdf")))
plot(samps)
dev.off()
```

```
## pdf
## 2
```

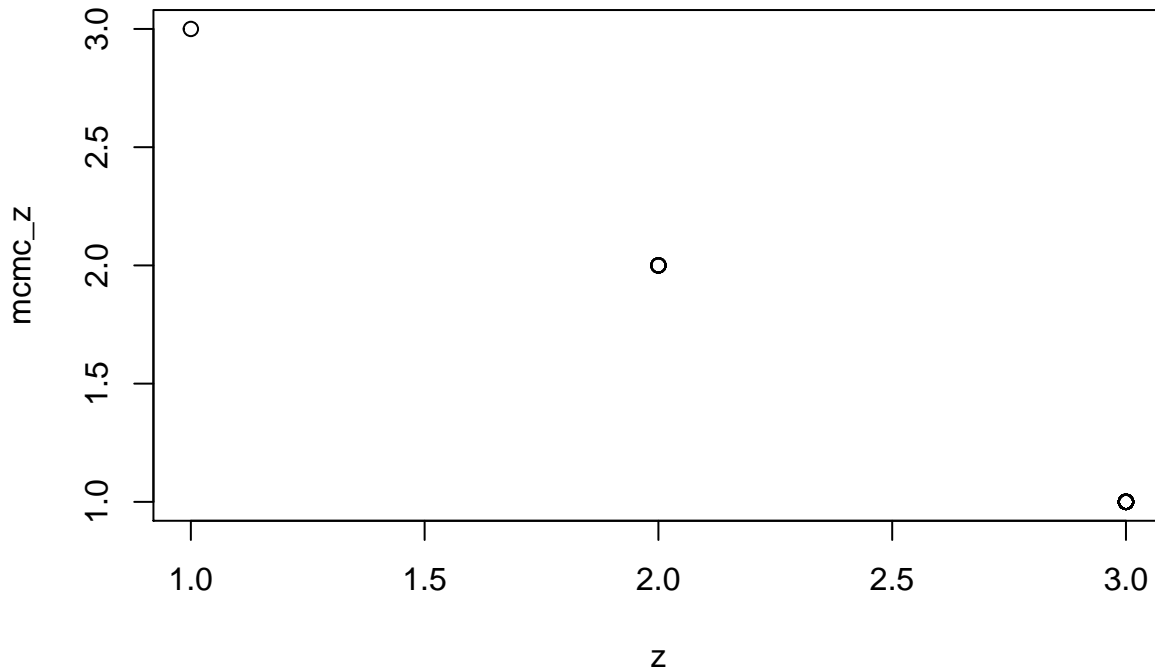
```
mcmc_vals <- s$statistics
mcmc_w <- mcmc_vals[substr(rownames(mcmc_vals), 1, 1) == "w", "Mean"]
plot(as.vector(w), mcmc_w, type = "p")
abline(a=0, b=1)
```



```

mcmc_z <- as.vector(mcmc_vals[substr(rownames(mcmc_vals), 1, 1) == "z", "Mean"])
#mcmc_z <- round(mcmc_z, 0)
plot(z, mcmc_z, type = "p")

```



```

mcmc_U <- mcmc_vals[substr(rownames(mcmc_vals), 1, 1) == "U", "Mean"]
mcmc_U <- matrix(mcmc_U, nrow=K)
mcmc_V <- mcmc_vals[substr(rownames(mcmc_vals), 1, 1) == "V", "Mean"]
mcmc_V <- matrix(mcmc_V, nrow=K)

p <- seq(0, 1, length = 100)
colors <- c("#000000", "#DCA200", "#8FA7ED", "#9D847A", "#A47901")
for (s in 1:S) {
  for (k in 1:K) {
    if (k == 1) {
      # plot mcmc mean U,V
      plot(p, dbeta(p, mcmc_U[k,s] * mcmc_V[k,s], (1-mcmc_U[k,s])*mcmc_V[k,s]),
            main = paste0("S", s),
            ylab = "density", xlab = "w", type = "l", col = colors[k],
            ylim = c(0, 20))
      # plot truth
      lines(p, dbeta(p, a.ordered[k,s], b.ordered[k,s]), type = "l", col = colors[k], lty=2)
      # add legend
      allU <- round(as.vector(rbind(mcmc_U[s], U.ordered[,s])), digits = 2)
      legend(x = "topleft",
             legend = paste0(c("mcmc k", "true k"), rep(1:K, each=2), ", U=", allU),
             col = colors[rep(1:K, each=2)],
             lty = rep(1:2, K),
             cex=0.8)
    } else {
      # plot mcmc mean U,V
      lines(p, dbeta(p, mcmc_U[k,s] * mcmc_V[k,s], (1-mcmc_U[k,s])*mcmc_V[k,s]),
            type = "l", col = colors[k])
    }
  }
}

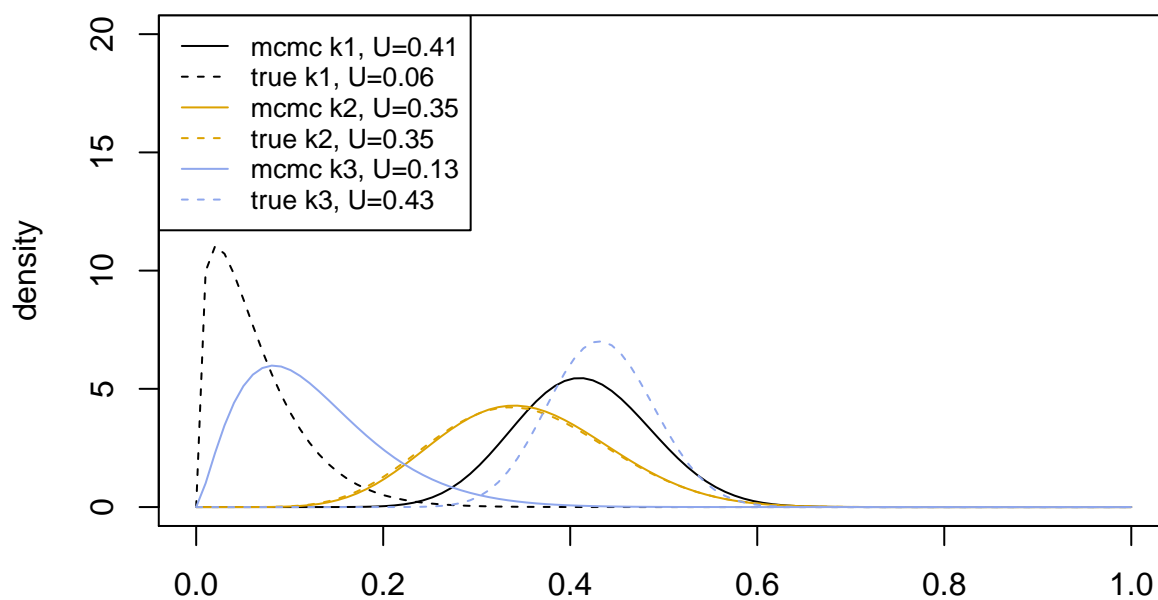
```

```

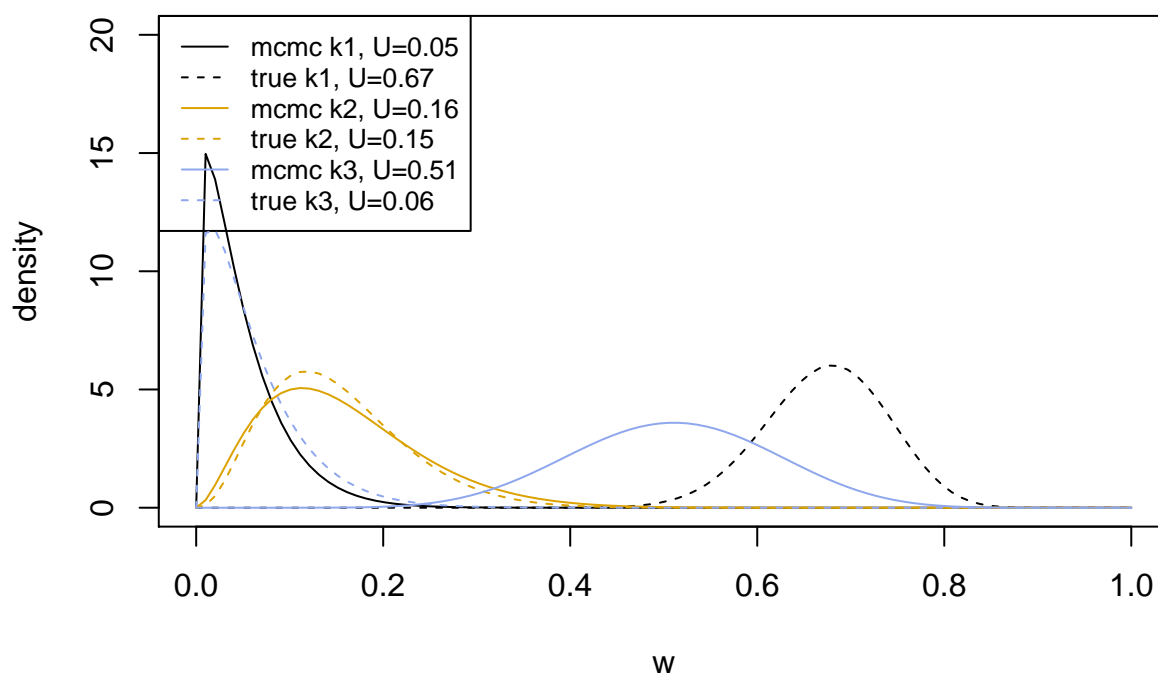
# plot truth
lines(p, dbeta(p, a.ordered[k,s], b.ordered[k,s]), type = "l", col = colors[k], lty=2)
}
}
}

```

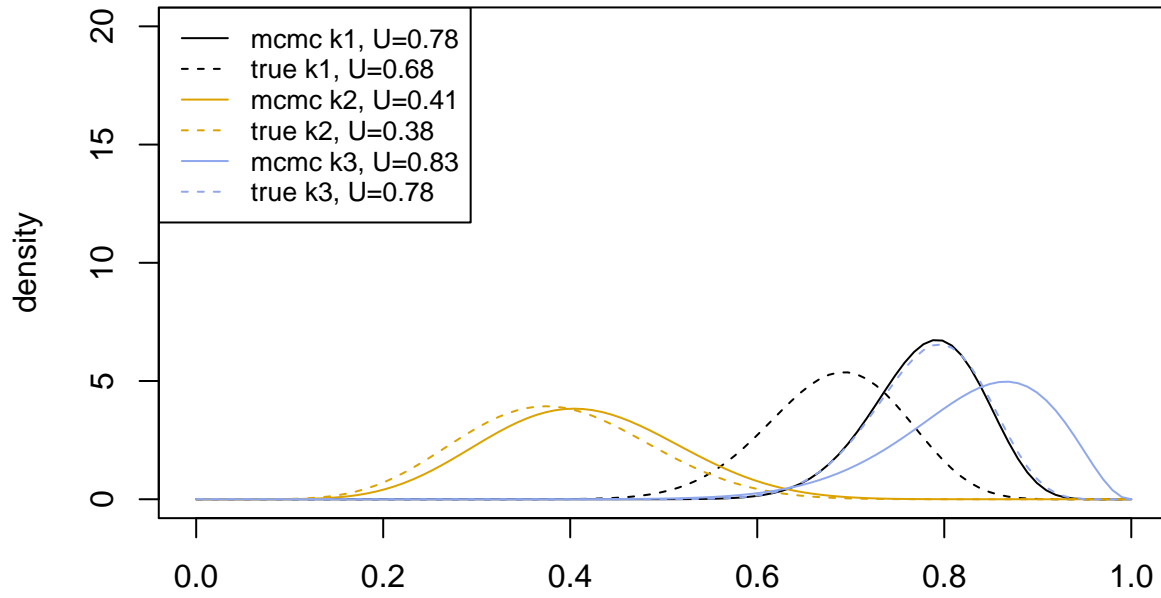
S1



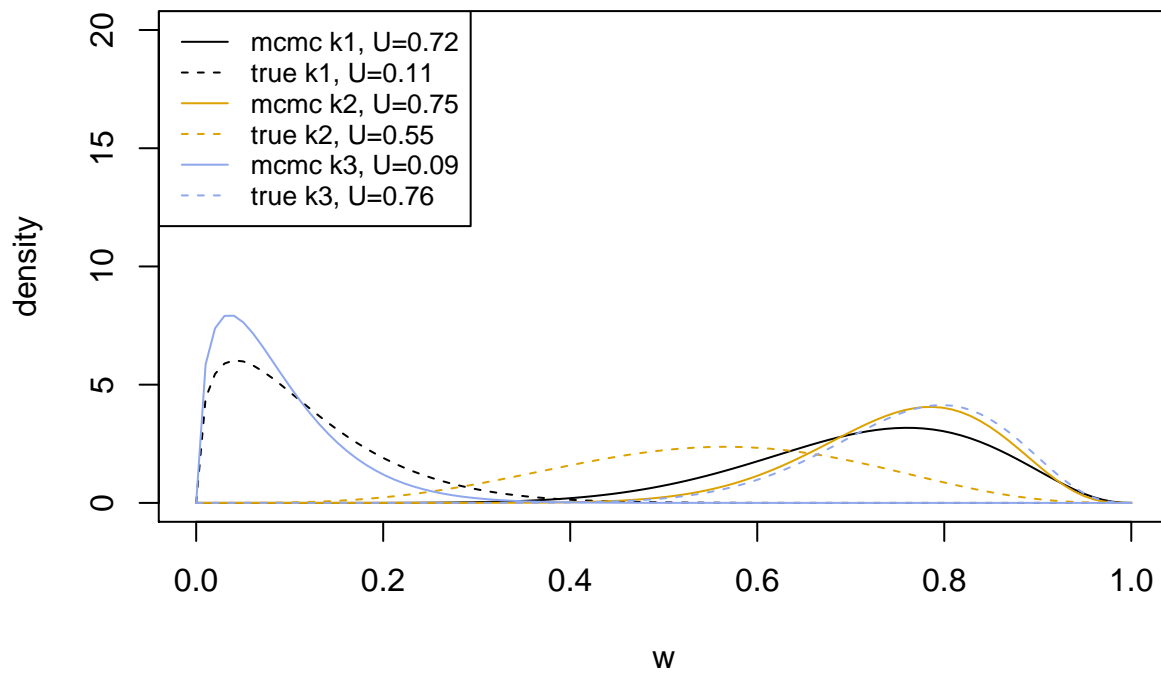
S2



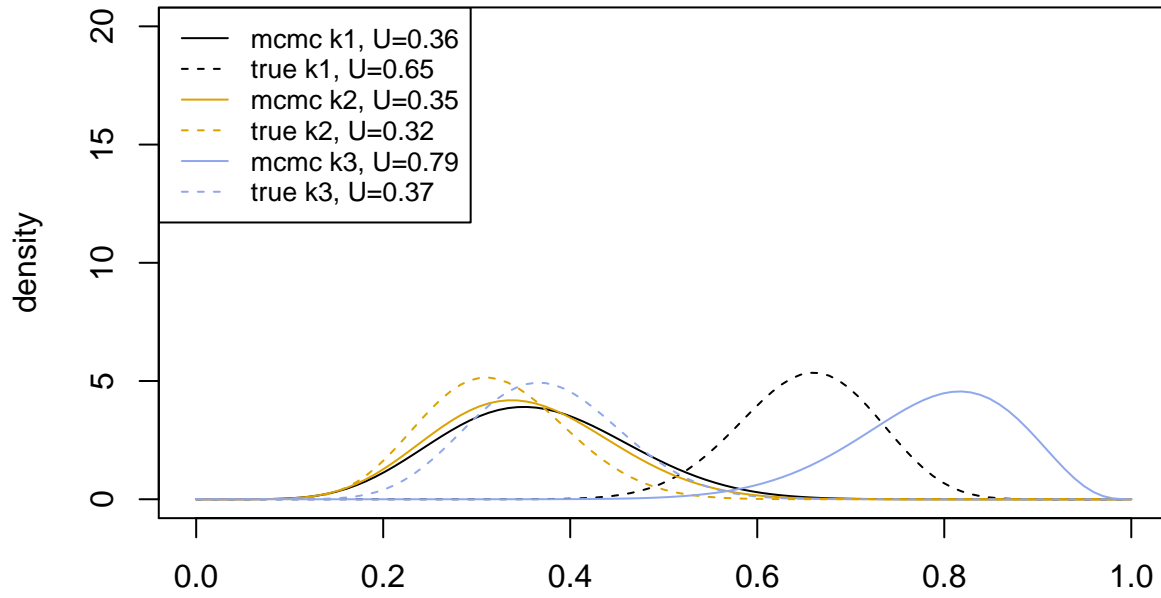
S3



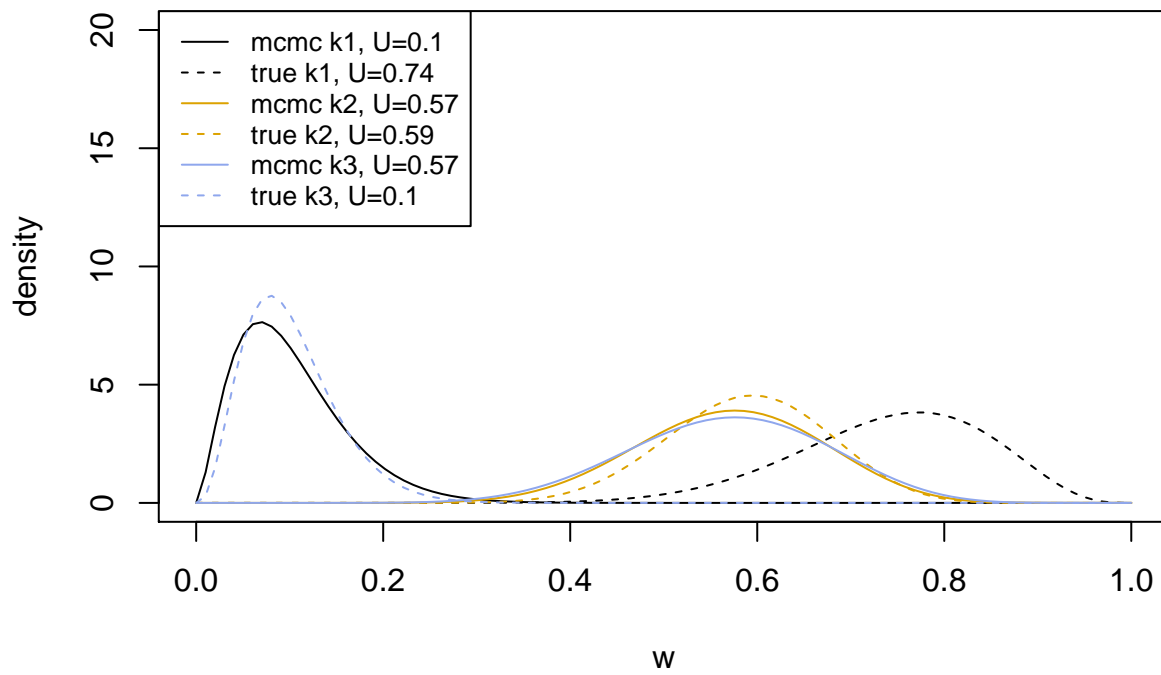
S4



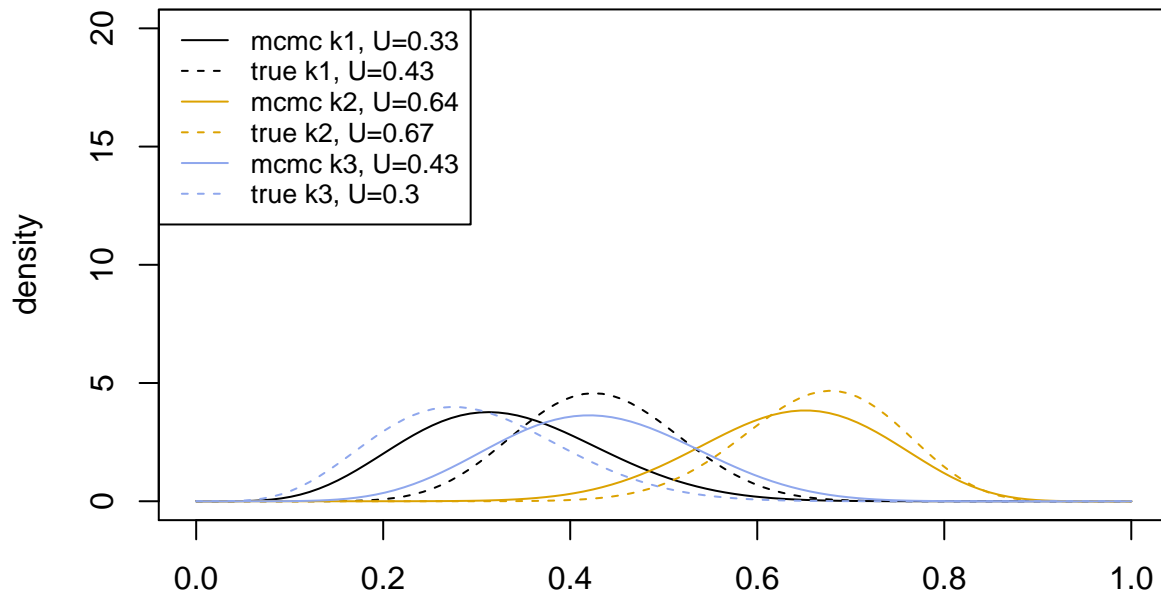
S5



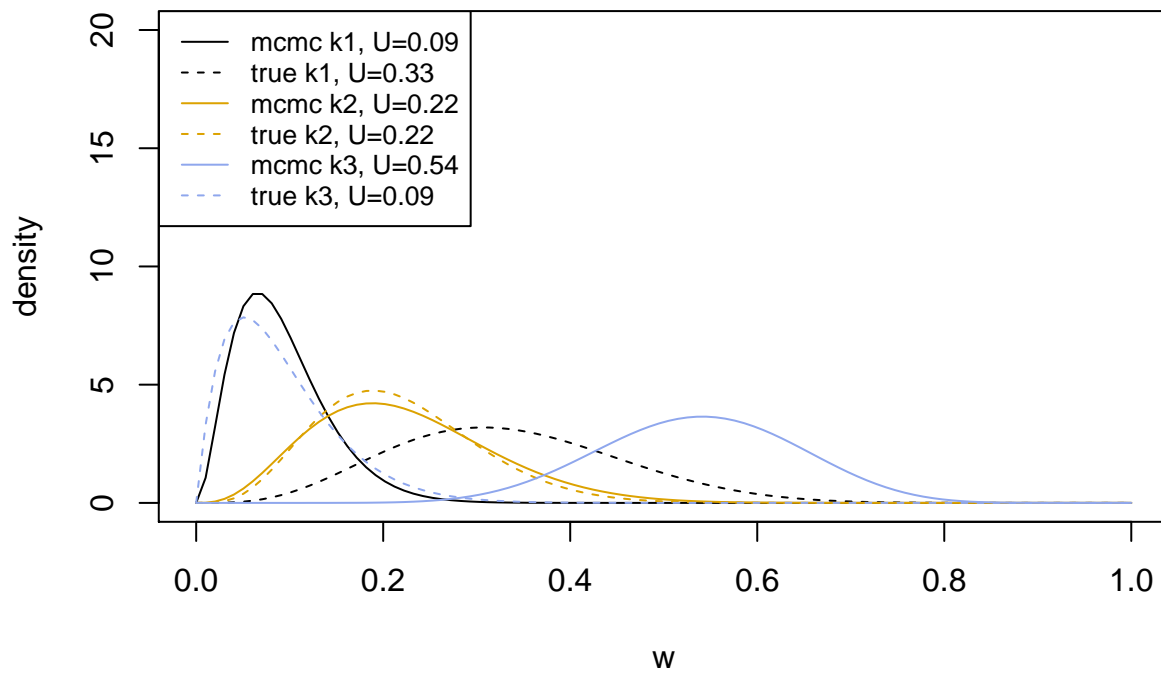
S6



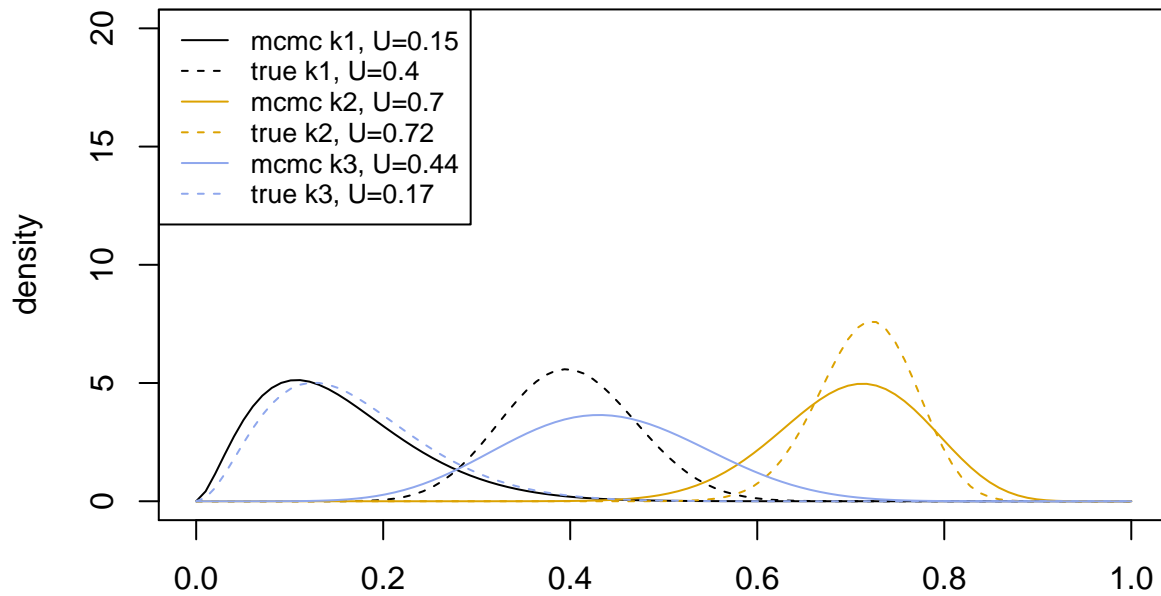
S7



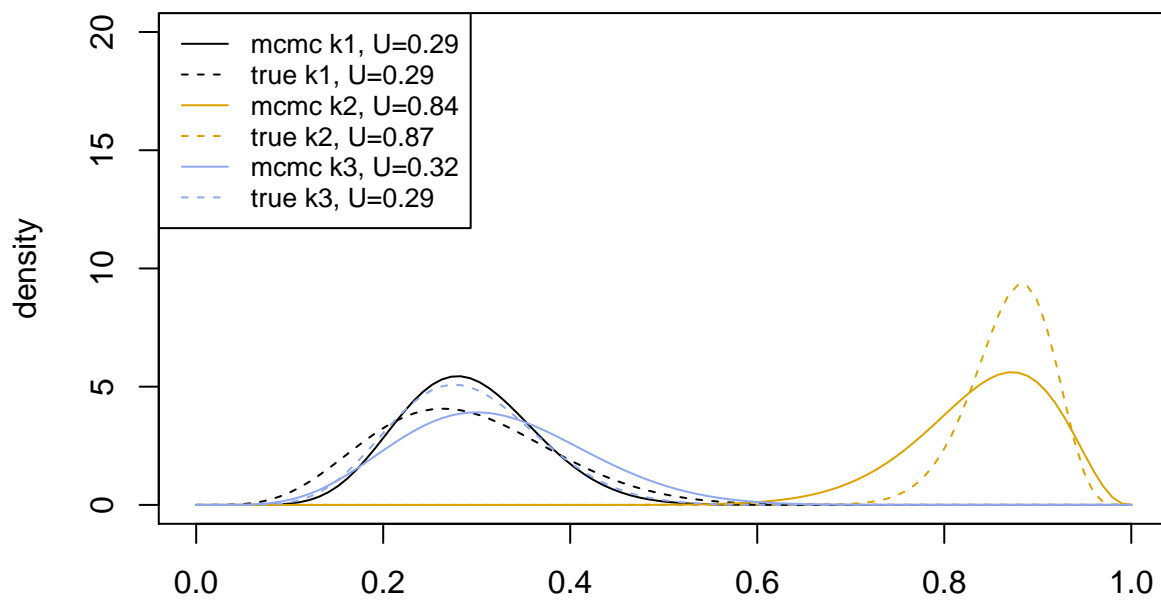
S8



S9



S10



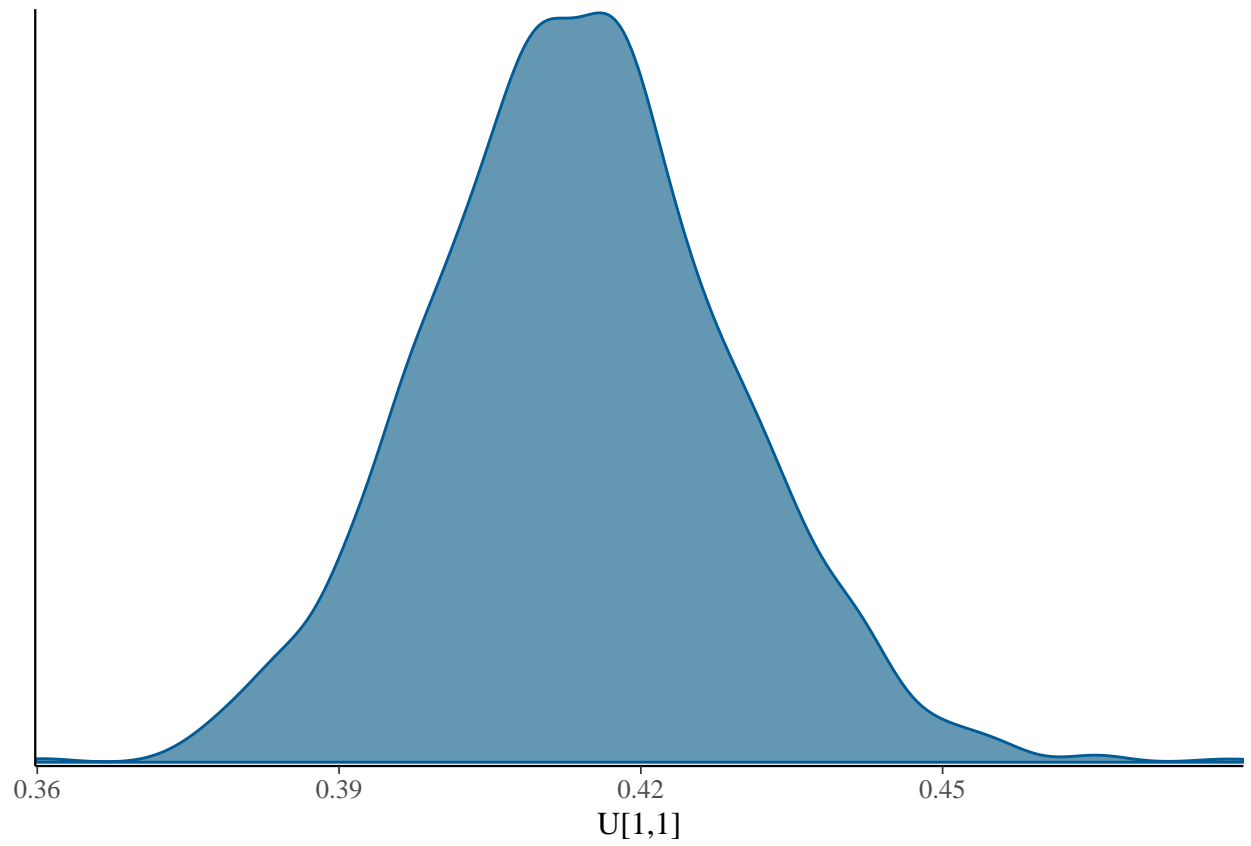
w

```
# https://cran.r-project.org/web/packages/bayesplot/vignettes/plotting-mcmc-draws.html
posterior <- as.array(samps)
```

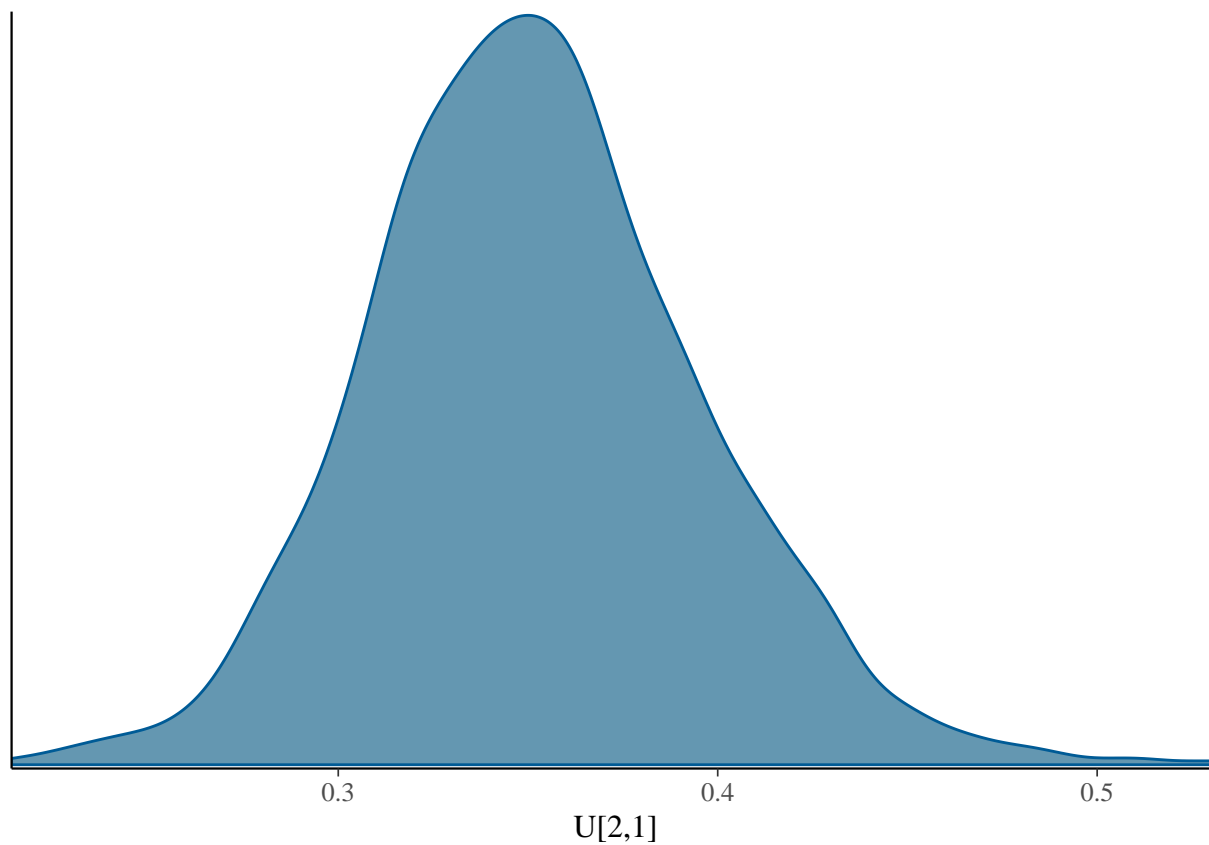
```
U.ordered[,1]
```

```
## [1] 0.0638265 0.3492084 0.4330499
```

```
mcmc_dens(posterior, pars = "U[1,1]")
```



```
mcmc_dens(posterior, pars = "U[2,1]")
```



```
mcmc_dens(posterior, pars = "U[3,1]")
```

