Power analysis on skew normal fitting for right tail

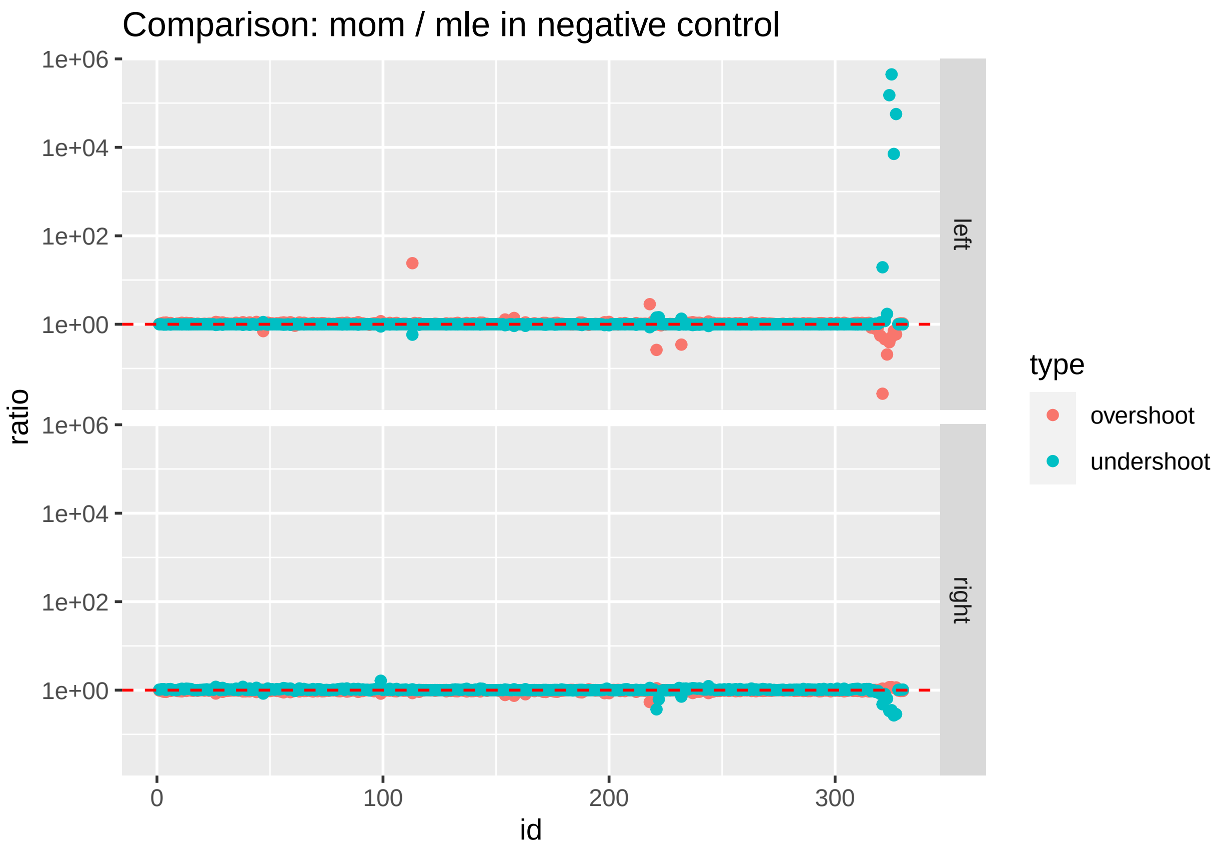
Some notation

In this report, define A = subsample fitted probability; B = subsample empirical tail probability; C = full sample fitted probability and D = full sample empirical tail probability. I consider the analysis for $A/D(D/A)$ and $A/B(B/A)$.

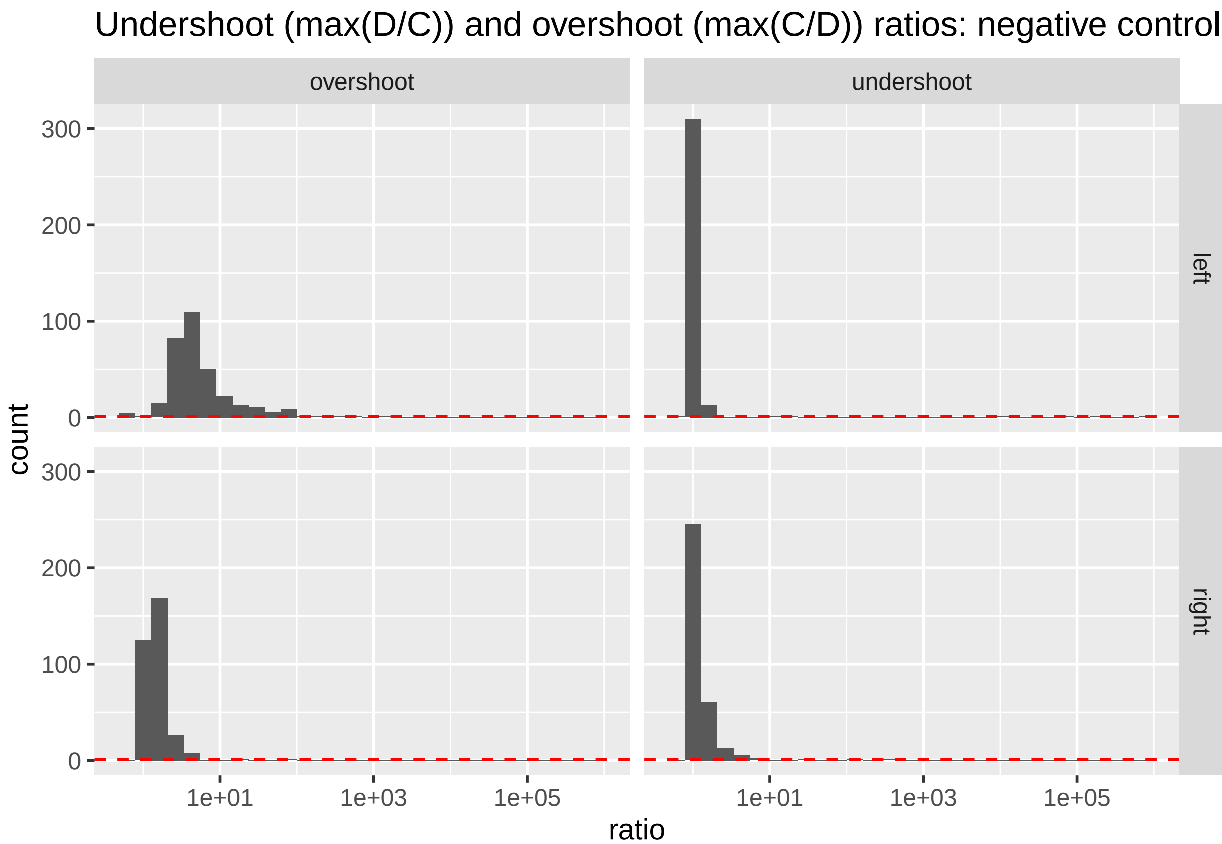
1.Load results for fitting comparison

```
library(dplyr)
library(ggplot2)
library(tidyverse)
undershoot <- read_csv("undershoot_refine_fit.csv")[,-1]
overshoot <- read_csv("overshoot_refine_fit.csv")[,-1]
quantile_list <- seq(0.01, 0.99, length.out = 10)
no_sam <- round(exp(seq(log(1e3), log(5e4), length.out = 10)))
# rearrange the data frame
B <- 100
undershoot_df <- data.frame(id = rep(1:B, 10*10),
                            ratio_value = 0,
                            no_sam = 0,
                            ratio_quantile = 0)
overshoot_df <- data.frame(id = rep(1:B, 10*10),
                           ratio_value = 0,
                           no_sam = 0,
                           ratio_quantile = 0)

# i: quantile; j: no of sample
for (i in 1:10) {
  for (j in 1:10) {
    start <- (j - 1 + (i-1)*10)*B +1
    end <- (j + (i-1)*10)*B
    undershoot_df[start:end, 2] <- as.vector(undershoot[(((j-1)*B+1) :(j*B)), (i-1)*3+2])[[1]]
    undershoot_df[start:end, 3] <- rep(no_sam[j], B)
    undershoot_df[start:end, 4] <- rep(quantile_list[i], B)
    overshoot_df[start:end, 2] <- as.vector(overshoot[(((j-1)*B+1) :(j*B)), (i-1)*3+2])[[1]]
    overshoot_df[start:end, 3] <- rep(no_sam[j], B)
    overshoot_df[start:end, 4] <- rep(quantile_list[i], B)
  }
}

# load the oracle ratio
mle_param_nc <- read_csv("figures/power_exploration/sknorm_tail_prob_500000_resamples_0.96_percentile/pa
mom_param_nc <- read_csv("figures/power_exploration/sknorm_tail_prob_500000_resamples_0.96_percentile/mo
mle_param_twosides <- t(mle_param_nc[,-1])
mom_param_twosides <- t(mom_param_nc[,-1])
mle_overshoot_ratio <- as.numeric(mle_param_twosides[, 6])
```

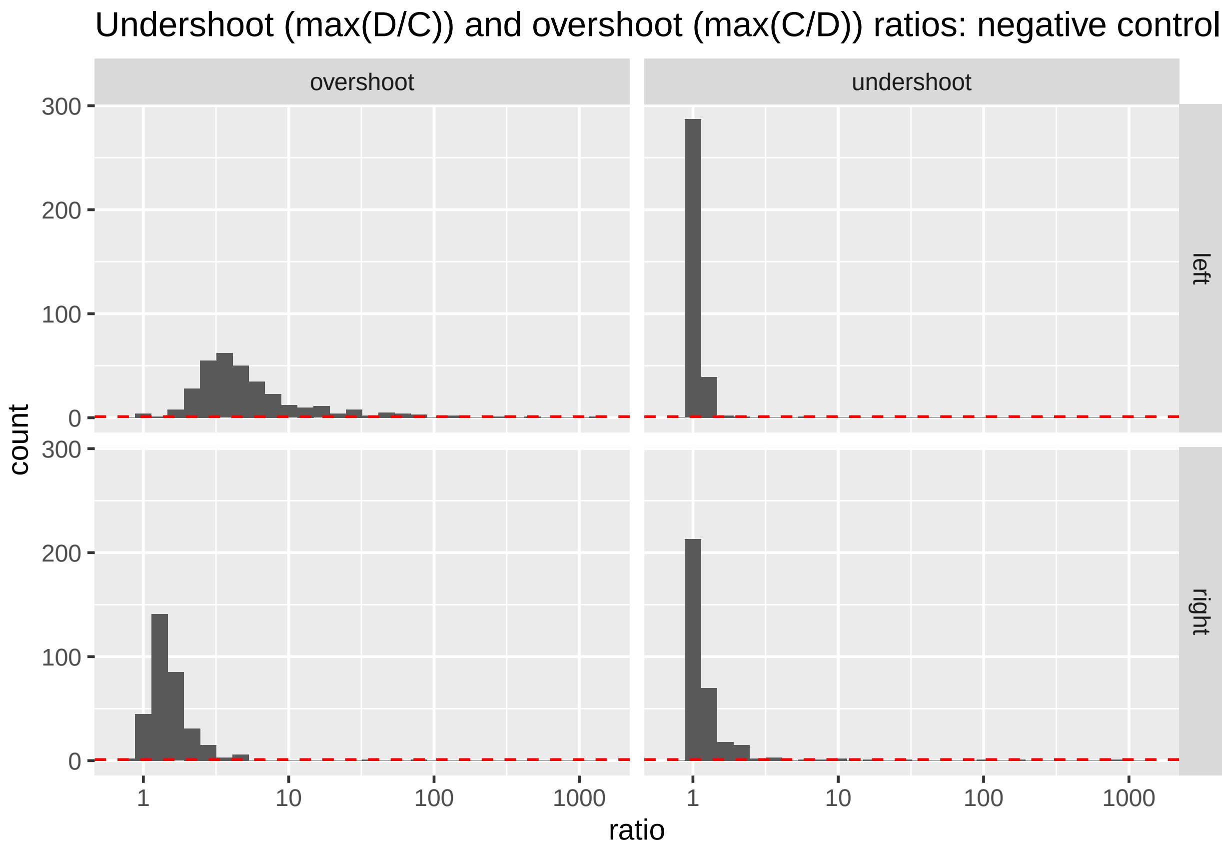
```r
mle_undershoot_ratio <- as.numeric(mle_param_twosides[, 7])
mom_overshoot_ratio <- as.numeric(mom_param_twosides[, 6])
mom_undershoot_ratio <- as.numeric(mom_param_twosides[, 7])
quantile_list <- seq(0.01, 0.99, length.out = 10)
overshoot_set <- data.frame(index = numeric(10), ratio = numeric(10))
undershoot_set <- data.frame(index = numeric(10), ratio = numeric(10))

# find distributions based on right tail
for (r in 1:10){
  dist <- abs(mom_overshoot_ratio[331:660] - quantile(mom_overshoot_ratio[331:660], quantile_list[r]))
  overshoot_set[r, 1] <- which(dist == min(dist))
  overshoot_set[r, 2] <- mom_overshoot_ratio[which(dist == min(dist)) + 330]
  dist <- abs(mom_undershoot_ratio[331:660] - quantile(mom_undershoot_ratio[331:660], quantile_list[r]))
  undershoot_set[r, 1] <- which(dist == min(dist))
  undershoot_set[r, 2] <- mom_undershoot_ratio[which(dist == min(dist)) + 330]
}

# accuracy using 5e5 mom estimate
mle_groundtruth <- data.frame(ratio = c(mle_overshoot_ratio, mle_undershoot_ratio),
                              type = c(rep("overshoot", 660), rep("undershoot", 660)),
                              tail = c(rep(c(rep("left", 330),rep("right", 330)) , 2)),
                              id = c(rep(1:330, 2)))

mom_groundtruth <- data.frame(ratio = c(mom_overshoot_ratio, mom_undershoot_ratio),
                              type = c(rep("overshoot", 660), rep("undershoot", 660)),
                              tail = c(rep(c(rep("left", 330),rep("right", 330)) , 2)),
                              id = c(rep(1:330, 2)))

groundtruth_comp <- data.frame(ratio = c(mom_overshoot_ratio, mom_undershoot_ratio) / c(mle_overshoot_ra
                               type = c(rep("overshoot", 660), rep("undershoot", 660)),
                               tail = c(rep(c(rep("left", 330),rep("right", 330)) , 2)),
                               id = c(rep(1:330, 2)))

groundtruth_comp |>
  ggplot(aes_string(x = "id", y = "ratio", colour = "type")) +
  facet_grid(tail~.) +
  geom_point() +
  scale_y_log10() +
  geom_hline(yintercept = 1, linetype = "dashed", colour = "red") +
  labs(title = "Comparison: mom / mle in negative control")
```



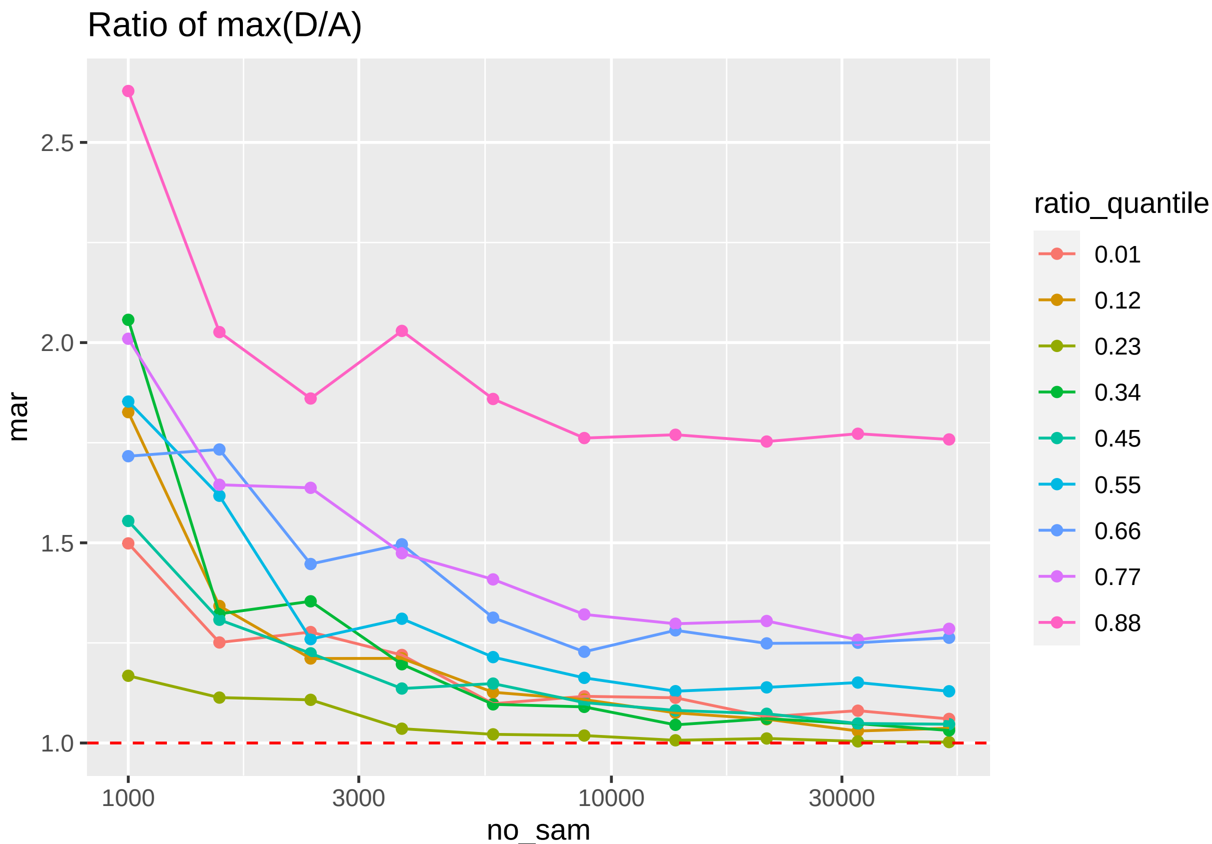

```r
mom_groundtruth |>
  ggplot(aes_string(x = "ratio")) +
  facet_grid(tail~type) +
  geom_histogram() +
  scale_x_log10() +
  geom_hline(yintercept = 1, linetype = "dashed", colour = "red") +
  labs(title = "Undershoot (max(D/C)) and overshoot (max(C/D)) ratios: negative control")
```



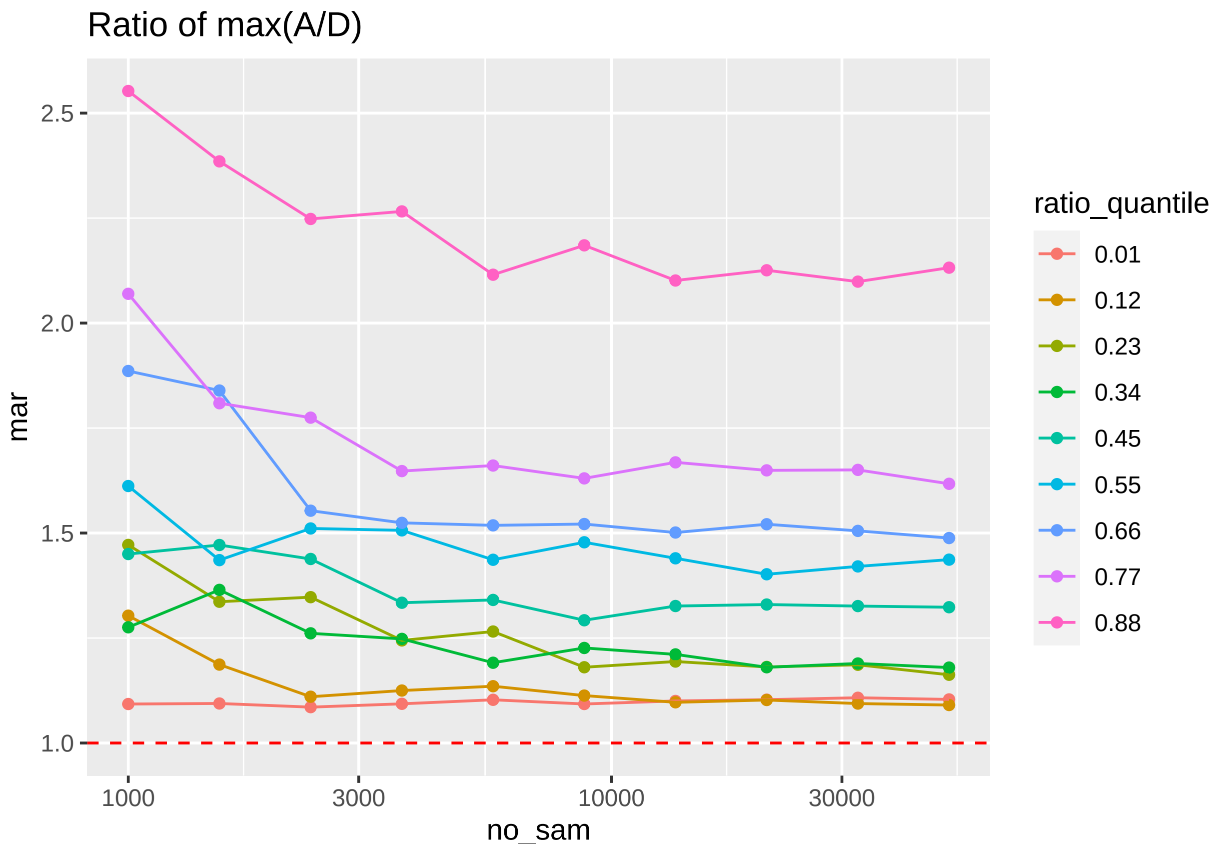

```r
mle_groundtruth |>
  ggplot(aes_string(x = "ratio")) +
  facet_grid(tail~type) +
  geom_histogram() +
  scale_x_log10() +
  geom_hline(yintercept = 1, linetype = "dashed", colour = "red") +
  labs(title = "Undershoot (max(D/C)) and overshoot (max(C/D)) ratios: negative control")
```




```
# accuracy matrix for undershoot matrix
undershoot_acc <- matrix(abs(undershoot_df$ratio_value), 100, 100)
undershoot_ame <- data.frame(mar = apply(undershoot_acc, 2, mean),
                             no_sam = rep(no_sam, 10),
                             ratio_quantile = as.character(round(rep(quantile_list, each = 10), 2)))
undershoot_ame |>
  filter(ratio_quantile != as.character(0.99)) |>
  ggplot(aes_string(x = "no_sam", y = "mar", colour = "ratio_quantile")) +
  scale_x_log10() +
  geom_point() +
  geom_line() +
  geom_hline(yintercept = 1, linetype = "dashed", colour = "red") +
  labs(title = "Ratio of max(D/A)")
```




```r
# accuracy matrix for overshoot matrix
overshoot_acc <- matrix(abs(overshoot_df$ratio_value), 100, 100)
overshoot_ame <- data.frame(mar = apply(overshoot_acc, 2, mean),
                            no_sam = rep(no_sam, 10),
                            ratio_quantile = as.character(round(rep(quantile_list, each = 10), 2)))
overshoot_ame |>
  filter(ratio_quantile != as.character(0.99)) |>
  ggplot(aes_string(x = "no_sam", y = "mar", colour = "ratio_quantile")) +
  scale_x_log10() +
  geom_point() +
  geom_line() +
  geom_hline(yintercept = 1, linetype = "dashed", colour = "red") +
  labs(title = "Ratio of max(A/D)")
```



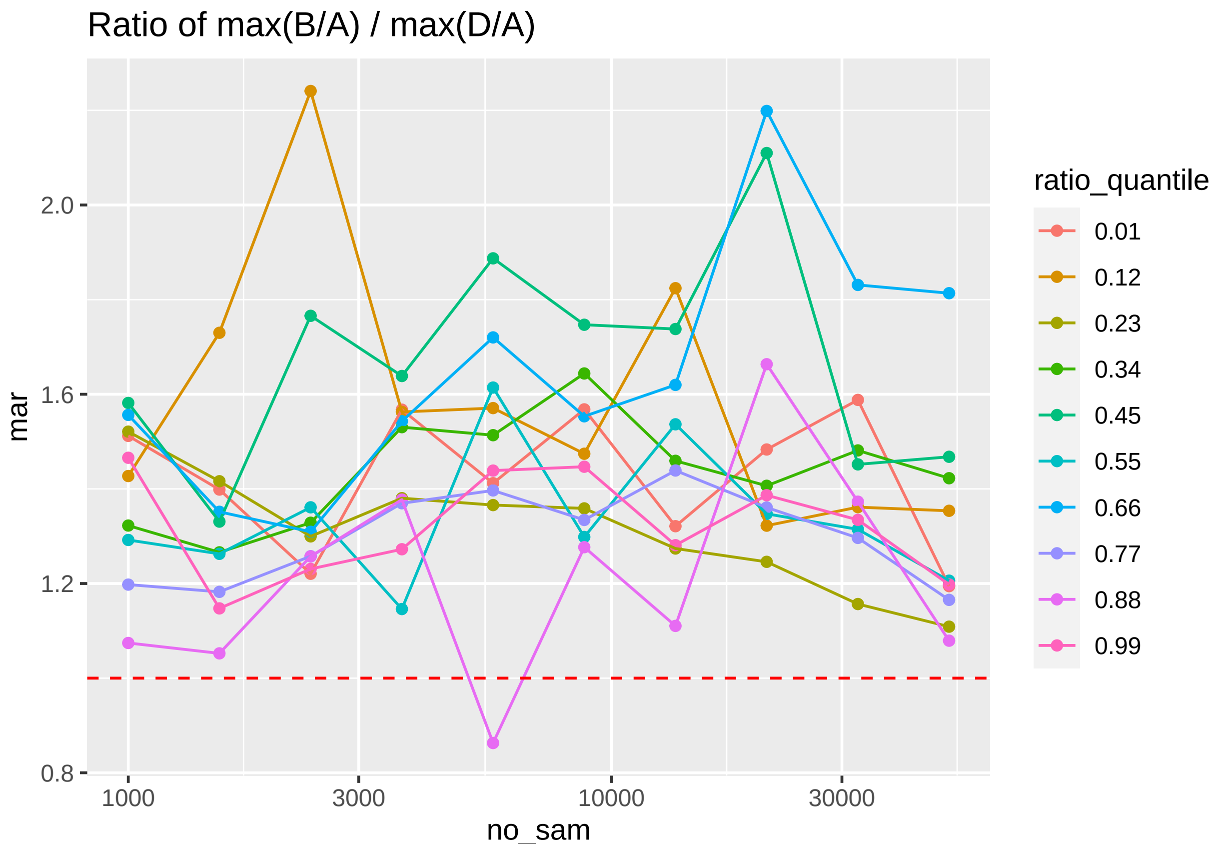

```r
# store D/A (A/D)
gt_overshoot_curve <- overshoot_df$ratio_value
gt_undershoot_curve <- undershoot_df$ratio_value
```

Here we clearly see the ratio $\max[A/D](max[D/A])$ approaches $\max[C/D](\max[D/C])$ very fast. This indicates the error decrease rather fast in estimating the tail probability at least in average sense. In the next section, we mainly consider the changes for the ratio $\max[A/B](max[B/A])$ and how does this approach $\max[C/D](\max[D/C])$.

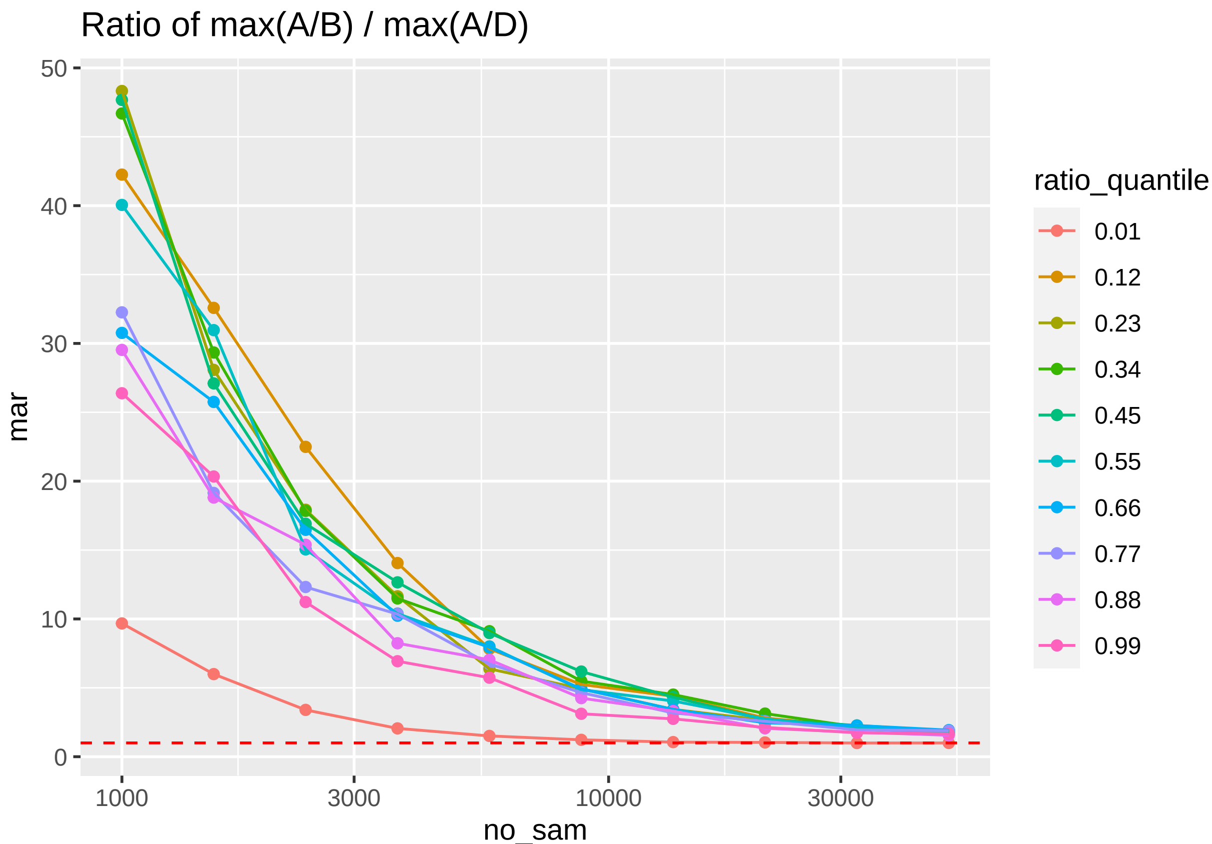
1.Load results for power comparison

```r
undershoot <- read_csv("undershoot_refine_power.csv")[,-1]
overshoot <- read_csv("overshoot_refine_power.csv")[,-1]
quantile_list <- seq(0.01, 0.99, length.out = 10)
no_sam <- round(exp(seq(log(1e3), log(5e4), length.out = 10)))
# rearrange the data frame
B <- 100
undershoot_df <- data.frame(id = rep(1:B, 10*10),
                            ratio_value = 0,
                            no_sam = 0,
                            ratio_quantile = 0)
overshoot_df <- data.frame(id = rep(1:B, 10*10),
                           ratio_value = 0,
                           no_sam = 0,
                           ratio_quantile = 0)

# i: quantile; j: no of sample
```

```r
for (i in 1:10) {
  for (j in 1:10) {
    start <- (j - 1 + (i-1)*10)*B +1
    end <- (j + (i-1)*10)*B
    undershoot_df[start:end, 2] <- as.vector(undershoot[(((j-1)*B+1) :(j*B)), (i-1)*3+2])[[1]]
    undershoot_df[start:end, 3] <- rep(no_sam[j], B)
    undershoot_df[start:end, 4] <- rep(quantile_list[i], B)
    overshoot_df[start:end, 2] <- as.vector(overshoot[(((j-1)*B+1) :(j*B)), (i-1)*3+2])[[1]]
    overshoot_df[start:end, 3] <- rep(no_sam[j], B)
    overshoot_df[start:end, 4] <- rep(quantile_list[i], B)
  }
}

param_nc <- read_csv("figures/power_exploration/sknorm_tail_prob_500000_resamples_0.96_percentile/param_
param_twosides <- t(param_nc[,-1])
overshoot_ratio <- as.numeric(param_twosides[, 6])
undershoot_ratio <- as.numeric(param_twosides[, 7])
quantile_list <- seq(0.01, 0.99, length.out = 10)
overshoot_set <- data.frame(index = numeric(10), ratio = numeric(10))
undershoot_set <- data.frame(index = numeric(10), ratio = numeric(10))

# find distributions based on right tail
for (r in 1:10){
  dist <- abs(overshoot_ratio[331:660] - quantile(overshoot_ratio[331:660], quantile_list[r]))
  overshoot_set[r, 1] <- which(dist == min(dist))
  overshoot_set[r, 2] <- overshoot_ratio[which(dist == min(dist)) + 330]
  dist <- abs(undershoot_ratio[331:660] - quantile(undershoot_ratio[331:660], quantile_list[r]))
  undershoot_set[r, 1] <- which(dist == min(dist))
  undershoot_set[r, 2] <- undershoot_ratio[which(dist == min(dist)) + 330]
}

# load the oracle ratio
undershoot_acc <- matrix(abs(undershoot_df$ratio_value / gt_undershoot_curve), 100, 100)
undershoot_ame <- data.frame(mar = apply(undershoot_acc, 2, mean),
                             no_sam = rep(no_sam, 10),
                             ratio_quantile = as.character(round(rep(quantile_list, each = 10), 2)))
undershoot_ame |>
  ggplot(aes_string(x = "no_sam", y = "mar", colour = "ratio_quantile")) +
  scale_x_log10() +
  geom_point() +
  geom_line() +
  geom_hline(yintercept = 1, linetype = "dashed", colour = "red") +
  labs(title = "Ratio of max(B/A) / max(D/A)")
```




```r
# accuracy matrix for overshoot matrix
overshoot_acc <- matrix(abs(overshoot_df$ratio_value / gt_overshoot_curve), 100, 100)
overshoot_ame <- data.frame(mar = apply(overshoot_acc, 2, mean),
                            no_sam = rep(no_sam, 10),
                            ratio_quantile = as.character(round(rep(quantile_list, each = 10), 2)))
overshoot_ame |>
  ggplot(aes_string(x = "no_sam", y = "mar", colour = "ratio_quantile")) +
  scale_x_log10() +
  geom_point() +
  geom_line() +
  geom_hline(yintercept = 1, linetype = "dashed", colour = "red") +
  labs(title = "Ratio of max(A/B) / max(A/D)")
```



From the above figure, we notice it is much harder to approximate $\max[A/D](\max[D/A])$. Due to the large fluctuation of B in the estimate $\max[A/B](\max[B/A])$, the convergence as the increase of sample is not obvious in mean absolute ratio (mar).

Here are my several comments:

(a) It looks like skew normal fit can do a very decent job on estimating the tail probability even with thousands of data when the maximum tail ratio is not huge. (The resampling distribution tail is behaving okay.) This can be reflected in figures 1 and 2.

(b) When it comes to account for the variation of downsample variation, it becomes significantly hard to get a direct decent estimate of the p-value ratio. This can be reflected in figure 3 and 4. Also there is a discrepancy between the overshoot and undershoot behaviors. For the undershoot, we see most of the curves are bumping while some of them do decrease as the resample increases. For the overshoot, it can be seen that when using small size of resamples, it always far overestimate the ratio. But the difference between the estimate and ground truth decreases much faster.

(c) The last point is the validity of the maximal measure. We should notice that until now the results we have are all under the measure which is maximum of the ratios among the tail_list. It is reasonable to conjecture that such measure converge slower than the other mean-based measure due to uniform nature.