Power analysis on skew normal fitting for left tail

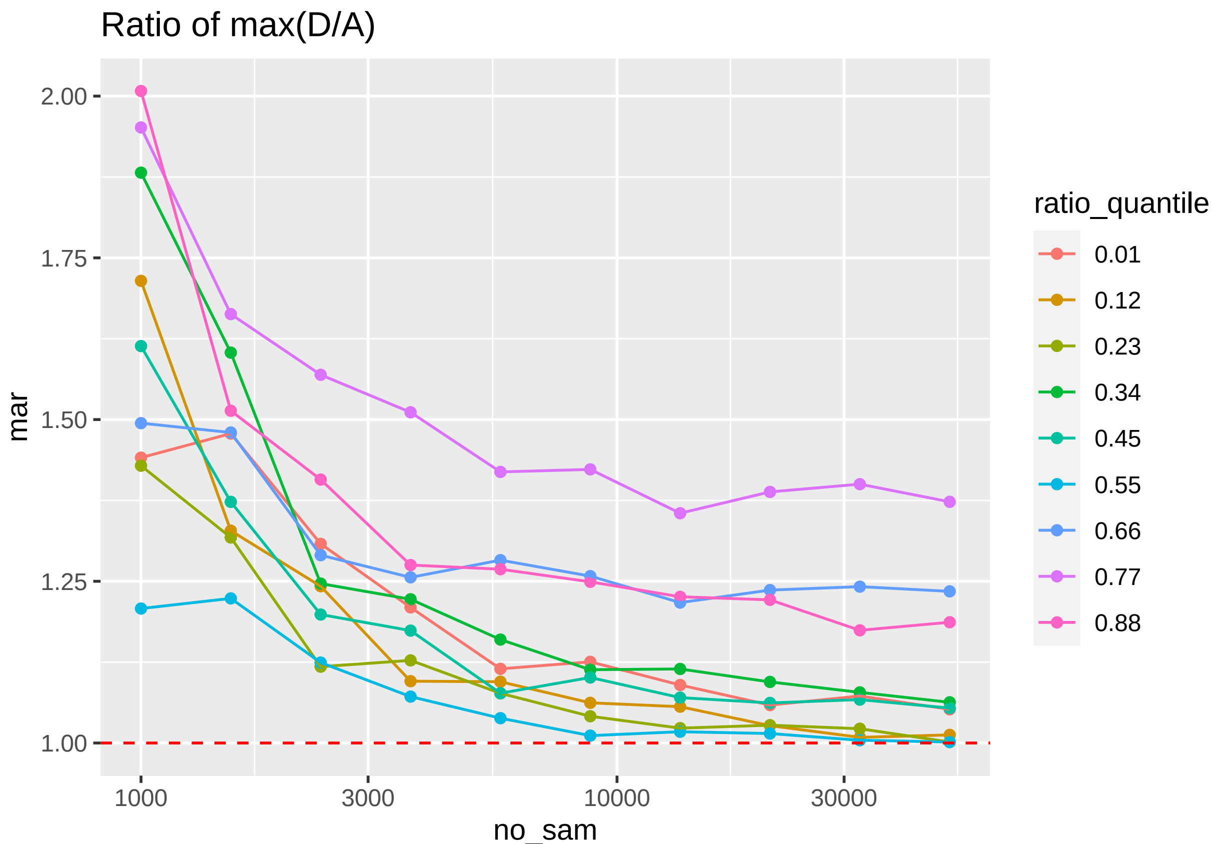
Some notation

In this report, define A = subsample fitted probability; B = subsample empirical tail probability; C = full sample fitted probability and D = full sample empirical tail probability. I consider the analysis for $A/D(D/A)$ and $A/B(B/A)$.

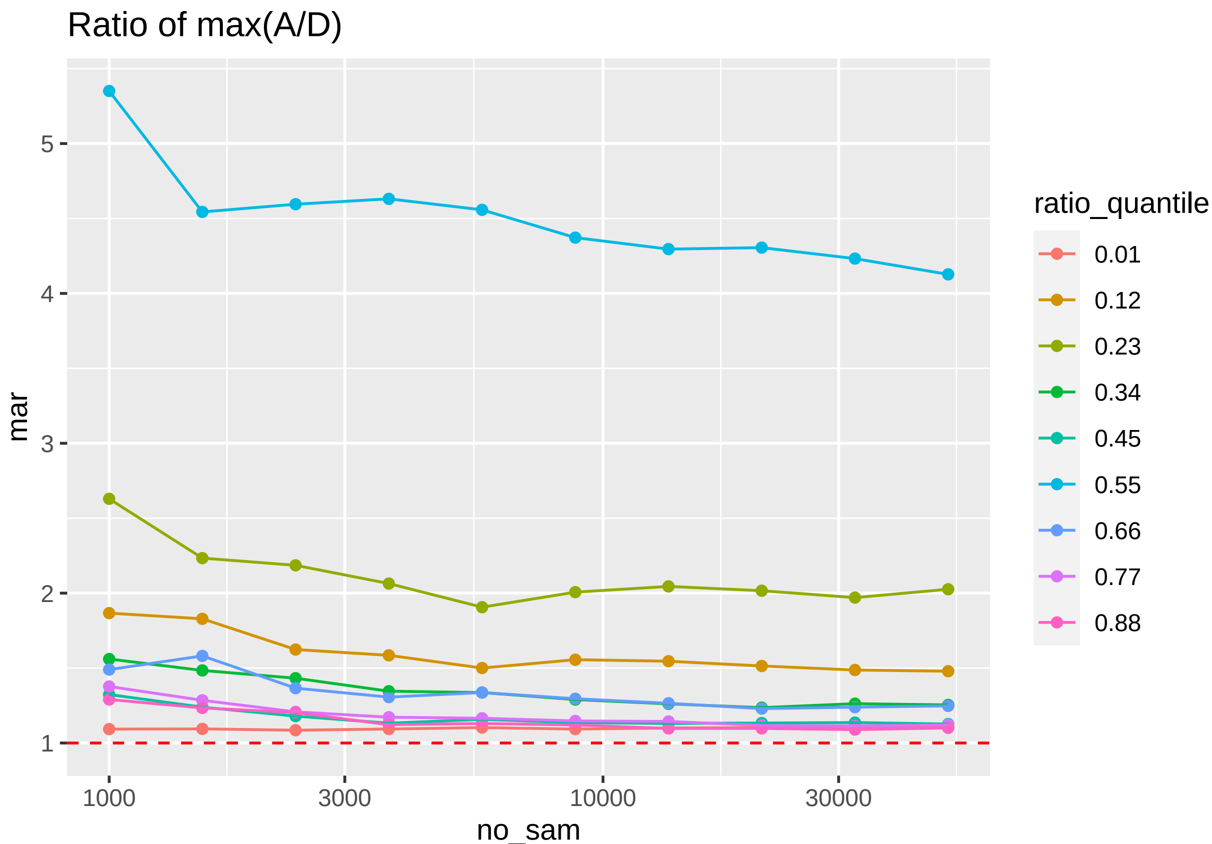
1.Load results for fitting comparison

```
library(dplyr)
library(ggplot2)
library(tidyverse)
undershoot <- read_csv("undershoot_left_fit.csv")[,-1]
overshoot <- read_csv("overshoot_left_fit.csv")[,-1]
quantile_list <- seq(0.01, 0.99, length.out = 10)
no_sam <- round(exp(seq(log(1e3), log(5e4), length.out = 10)))
# rearrange the data frame
B <- 100
undershoot_df <- data.frame(id = rep(1:B, 10*10),
                            ratio_value = 0,
                            no_sam = 0,
                            ratio_quantile = 0)
overshoot_df <- data.frame(id = rep(1:B, 10*10),
                           ratio_value = 0,
                           no_sam = 0,
                           ratio_quantile = 0)

# i: quantile; j: no of sample
for (i in 1:10) {
  for (j in 1:10) {
    start <- (j - 1 + (i-1)*10)*B +1
    end <- (j + (i-1)*10)*B
    undershoot_df[start:end, 2] <- as.vector(undershoot[(((j-1)*B+1) :(j*B)), (i-1)*3+2])[[1]]
    undershoot_df[start:end, 3] <- rep(no_sam[j], B)
    undershoot_df[start:end, 4] <- rep(quantile_list[i], B)
    overshoot_df[start:end, 2] <- as.vector(overshoot[(((j-1)*B+1) :(j*B)), (i-1)*3+2])[[1]]
    overshoot_df[start:end, 3] <- rep(no_sam[j], B)
    overshoot_df[start:end, 4] <- rep(quantile_list[i], B)
  }
}

# load the oracle ratio
param_nc <- read_csv("figures/power_exploration/sknorm_tail_prob_500000_resamples_0.96_percentile/param
param_twosides <- t(param_nc[,-1])
overshoot_ratio <- as.numeric(param_twosides[, 6])
undershoot_ratio <- as.numeric(param_twosides[, 7])
quantile_list <- seq(0.01, 0.99, length.out = 10)
```

```r
overshoot_set <- data.frame(index = numeric(10), ratio = numeric(10))
undershoot_set <- data.frame(index = numeric(10), ratio = numeric(10))

# find distributions based on right tail
for (r in 1:10){
  dist <- abs(overshoot_ratio[1:330] - quantile(overshoot_ratio[1:330], quantile_list[r]))
  overshoot_set[r, 1] <- which(dist == min(dist))
  overshoot_set[r, 2] <- overshoot_ratio[which(dist == min(dist))]
  dist <- abs(undershoot_ratio[1:330] - quantile(undershoot_ratio[1:330], quantile_list[r]))
  undershoot_set[r, 1] <- which(dist == min(dist))
  undershoot_set[r, 2] <- undershoot_ratio[which(dist == min(dist))]
}


# accuracy matrix for undershoot matrix
undershoot_acc <- matrix(abs(undershoot_df$ratio_value), 100, 100)
undershoot_ame <- data.frame(mar = apply(undershoot_acc, 2, mean),
                             no_sam = rep(no_sam, 10),
                             ratio_quantile = as.character(round(rep(quantile_list, each = 10), 2)))
undershoot_ame |>
  filter(ratio_quantile != as.character(0.99)) |>
  ggplot(aes_string(x = "no_sam", y = "mar", colour = "ratio_quantile")) +
  scale_x_log10() +
  geom_point() +
  geom_line() +
  geom_hline(yintercept = 1, linetype = "dashed", colour = "red") +
  labs(title = "Ratio of max(D/A)")
```




```r
# accuracy matrix for overshoot matrix
overshoot_acc <- matrix(abs(overshoot_df$ratio_value), 100, 100)
overshoot_ame <- data.frame(mar = apply(overshoot_acc, 2, mean),
                            no_sam = rep(no_sam, 10),
                            ratio_quantile = as.character(round(rep(quantile_list, each = 10), 2)))
overshoot_ame |>
  filter(ratio_quantile != as.character(0.99)) |>
  ggplot(aes_string(x = "no_sam", y = "mar", colour = "ratio_quantile")) +
  scale_x_log10() +
  geom_point() +
  geom_line() +
  geom_hline(yintercept = 1, linetype = "dashed", colour = "red") +
  labs(title = "Ratio of max(A/D)")
```



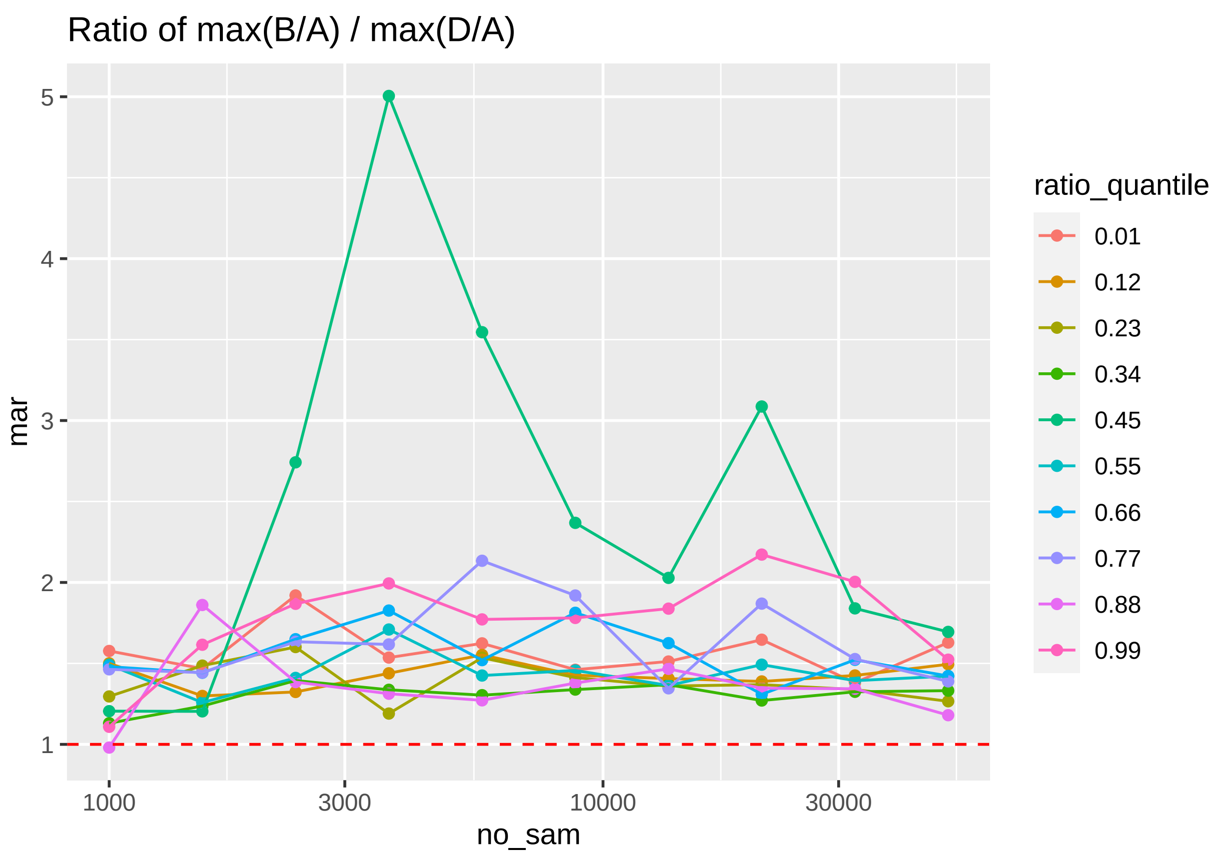

```
# store D/A (A/D)
gt_overshoot_curve <- overshoot_df$ratio_value
gt_undershoot_curve <- undershoot_df$ratio_value
```

Here we clearly see the ratio $\max[A/D](max[D/A])$ approaches $\max[C/D](\max[D/C])$ very fast. This indicates the error decrease rather fast in estimating the tail probability at least in average sense. In the next section, we mainly consider the changes for the ratio $\max[A/B](max[B/A])$ and how does this approach $\max[C/D](\max[D/C])$.

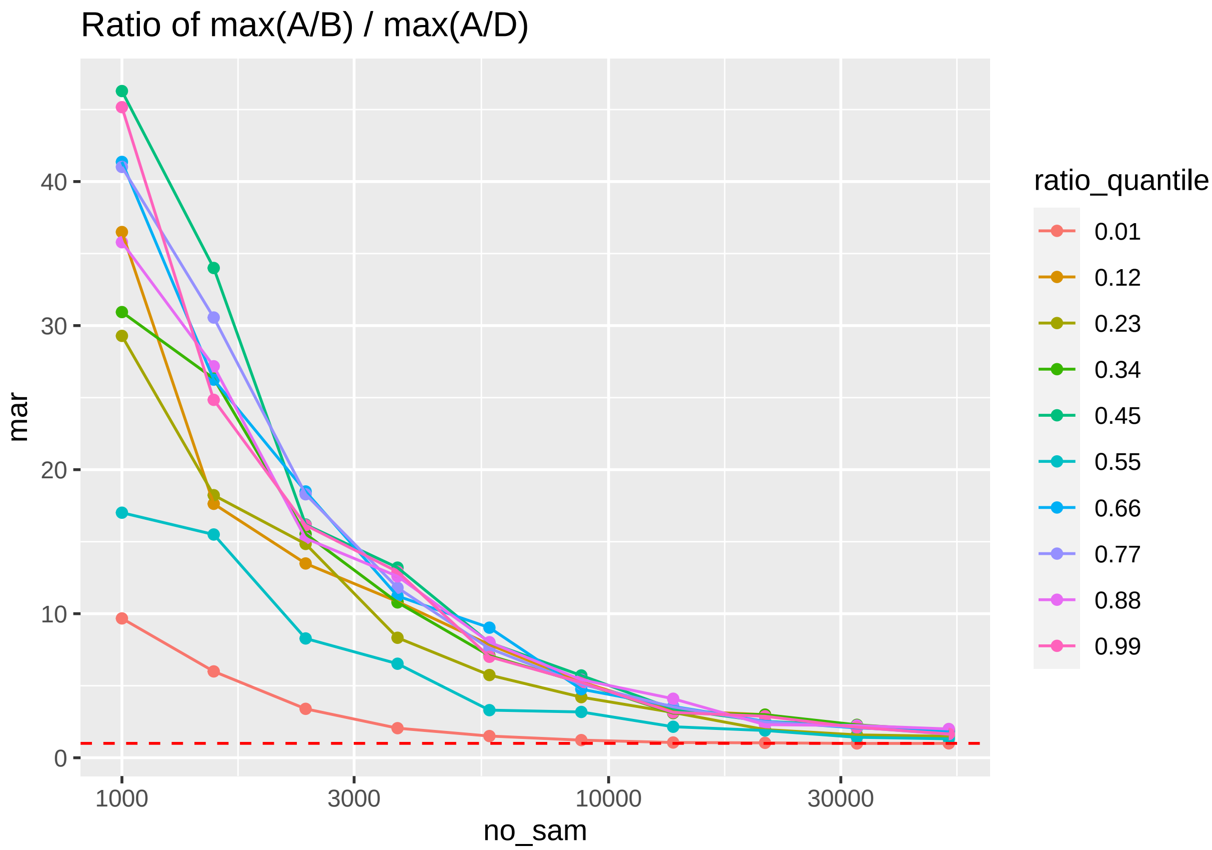
1.Load results for power comparison

```
undershoot <- read_csv("undershoot_left_power.csv")[,-1]
overshoot <- read_csv("overshoot_left_power.csv")[,-1]
quantile_list <- seq(0.01, 0.99, length.out = 10)
no_sam <- round(exp(seq(log(1e3), log(5e4), length.out = 10)))
# rearrange the data frame
B <- 100
undershoot_df <- data.frame(id = rep(1:B, 10*10),
                            ratio_value = 0,
                            no_sam = 0,
                            ratio_quantile = 0)
overshoot_df <- data.frame(id = rep(1:B, 10*10),
                           ratio_value = 0,
                           no_sam = 0,
                           ratio_quantile = 0)

# i: quantile; j: no of sample
```

```r
for (i in 1:10) {
  for (j in 1:10) {
    start <- (j - 1 + (i-1)*10)*B +1
    end <- (j + (i-1)*10)*B
    undershoot_df[start:end, 2] <- as.vector(undershoot[(((j-1)*B+1) :(j*B)), (i-1)*3+2])[[1]]
    undershoot_df[start:end, 3] <- rep(no_sam[j], B)
    undershoot_df[start:end, 4] <- rep(quantile_list[i], B)
    overshoot_df[start:end, 2] <- as.vector(overshoot[(((j-1)*B+1) :(j*B)), (i-1)*3+2])[[1]]
    overshoot_df[start:end, 3] <- rep(no_sam[j], B)
    overshoot_df[start:end, 4] <- rep(quantile_list[i], B)
  }
}

param_nc <- read_csv("figures/power_exploration/sknorm_tail_prob_500000_resamples_0.96_percentile/param_
param_twosides <- t(param_nc[,-1])
overshoot_ratio <- as.numeric(param_twosides[, 6])
undershoot_ratio <- as.numeric(param_twosides[, 7])
quantile_list <- seq(0.01, 0.99, length.out = 10)
overshoot_set <- data.frame(index = numeric(10), ratio = numeric(10))
undershoot_set <- data.frame(index = numeric(10), ratio = numeric(10))

# find distributions based on right tail
for (r in 1:10){
  dist <- abs(overshoot_ratio[1:330] - quantile(overshoot_ratio[1:330], quantile_list[r]))
  overshoot_set[r, 1] <- which(dist == min(dist))
  overshoot_set[r, 2] <- overshoot_ratio[which(dist == min(dist))]
  dist <- abs(undershoot_ratio[1:330] - quantile(undershoot_ratio[1:330], quantile_list[r]))
  undershoot_set[r, 1] <- which(dist == min(dist))
  undershoot_set[r, 2] <- undershoot_ratio[which(dist == min(dist))]
}

# load the oracle ratio
undershoot_acc <- matrix(abs(undershoot_df$ratio_value / gt_undershoot_curve), 100, 100)
undershoot_ame <- data.frame(mar = apply(undershoot_acc, 2, mean),
                             no_sam = rep(no_sam, 10),
                             ratio_quantile = as.character(round(rep(quantile_list, each = 10), 2)))
undershoot_ame |>
  ggplot(aes_string(x = "no_sam", y = "mar", colour = "ratio_quantile")) +
  scale_x_log10() +
  geom_point() +
  geom_line() +
  geom_hline(yintercept = 1, linetype = "dashed", colour = "red") +
  labs(title = "Ratio of max(B/A) / max(D/A)")
```




```r
# accuracy matrix for overshoot matrix
overshoot_acc <- matrix(abs(overshoot_df$ratio_value / gt_overshoot_curve), 100, 100)
overshoot_ame <- data.frame(mar = apply(overshoot_acc, 2, mean),
                            no_sam = rep(no_sam, 10),
                            ratio_quantile = as.character(round(rep(quantile_list, each = 10), 2)))
overshoot_ame |>
  ggplot(aes_string(x = "no_sam", y = "mar", colour = "ratio_quantile")) +
  scale_x_log10() +
  geom_point() +
  geom_line() +
  geom_hline(yintercept = 1, linetype = "dashed", colour = "red") +
  labs(title = "Ratio of max(A/B) / max(A/D)")
```


Ratio of max(A/B) / max(A/D)
mar
no_sam
ratio_quantile
0.01
0.12
0.23
0.34
0.45
0.55
0.66
0.77
0.88
0.99
0
10
20
30
40
1000
3000
10000
30000