MATHUSLA Signal and Background reconstruction simulations

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Special thanks to David and Miriam for helpful comments and suggestions :)

20 July 2022

Primary vs Secondary Physics Goals

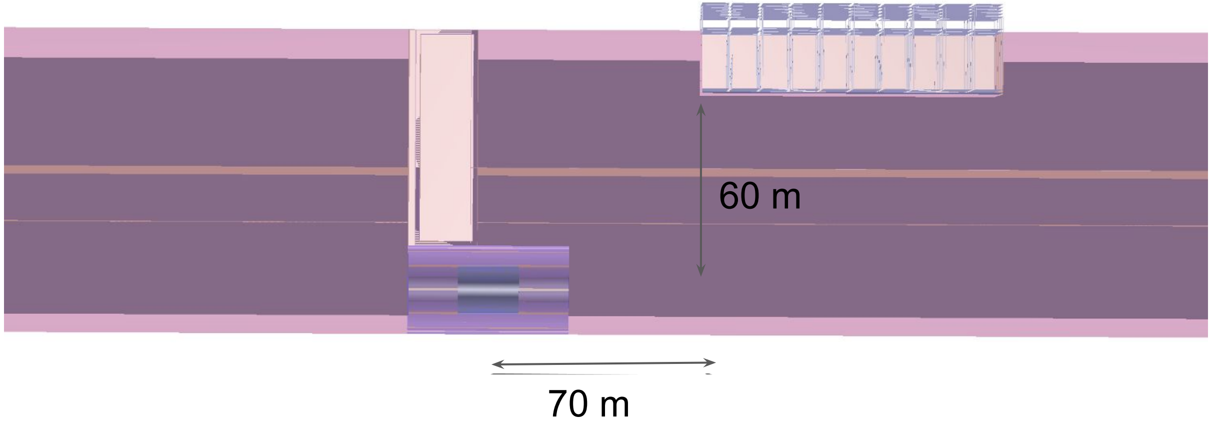
- MATHUSLA has a clear HIERARCHY of physics goals
- **PRIMARY** physics goal is LLPs > ~ 10 GeV decaying hadronically (high final state multiplicity), e.g. h->LLP etc. This is a blind spot for the main detectors and most motivated from higgs portal etc.
  - *For us, this means LLPs with O(10) tracks and boosts of O(1-10)*
- **SECONDARY** physics goal is LLPs < ~ 5 GeV produced in exotic meson decays. Regardless of portal, these tend to decay to low-multiplicity final states, ~ 2 tracks.
  - *For us, this means LLPs with 2 tracks and boosts of O(1-10)*
  - *This was NOT our original focus, but one that we started studying because we were asked to join the PBC group and study their benchmark models*
  - *This is a priori fixed target / FASER type territory, but it would be "nice to have" for MATHUSLA*
- **TERTIARY** physics goal is LLPs with much higher boost and decaying to 2 tracks
  - *High boost and leptonic decay tends to mean these events can be triggered at main detector, and reconstructed very well, so this is NOT our main target*
  - *This includes e.g. h->LLP with LLP << 10 GeV decaying leptonically*

Caveat

Up front: caveat that this is not full MVA optimization

Our studies are based on studying
1D distributions + physics judgement to get first idea

Global Geometry



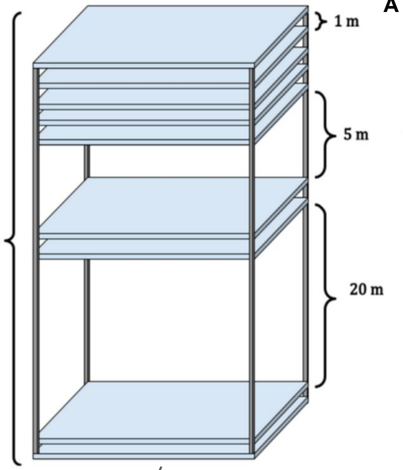
CMS cavern and access shaft modeled (concrete wall)

CMS detector treated as a 20m long hollow cylinder with ~10 interaction lengths of steel

Same rock modeling as test stand

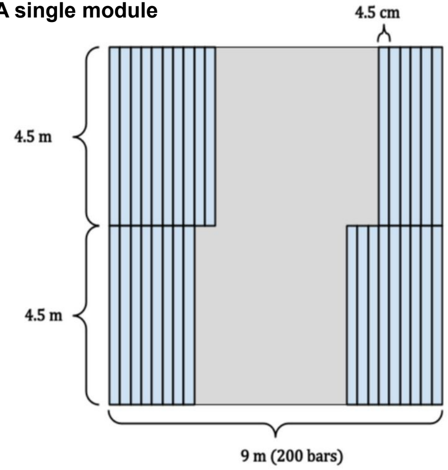
IP is 70 m from detector in z and 60 m in y

Detector Geometry



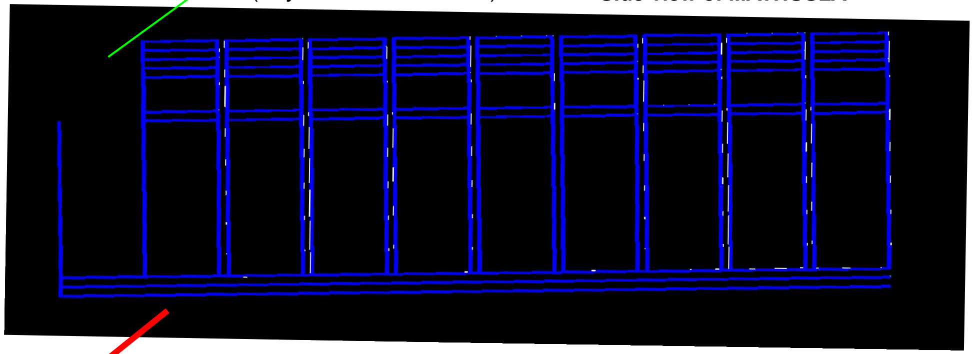
1 of 3 floor layers not pictured here

A single module



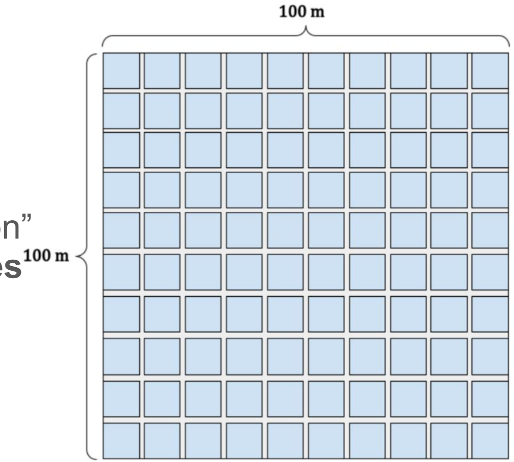
Modules closest to IP not shown for visibility of the **hermetic wall** (*only* for this visualization)

Side view of MATHUSLA



IP is ~ this way

- 100 (9m x 9m) modules with 1m gap
  - 2 + 5 tracker layers
  - 3 **gapless** floor layers in the simulation, but only one is “turned on”
    - Floor detectors are 50 cm x 50 cm **square scintillator tiles**
  - 1 **gapless** 20 m high wall layer facing the IP
- Tracker layers use 4.5m x 4.5 cm x 2.0 cm bars
- Aluminum casing for scintillator (1.0 cm thickness)
- 10 cm x 10 cm hollow Iron beam supports (2cm thick)



Top view of MATHUSLA

Simulation Methods of Signal and Background Samples

- Pythia 8 Generator for signal and background
  - Signal: h->aa
    - a's ctau=50 or 100m (doesn't matter, but will be made consistent in final version), mass from few GeV to 50 GeV
    - decays to either two muons or two up quarks
  - Background
    - W->u v
    - Generator-level cut to only propagate muons in geant with p>40 GeV pointing to the box
- Cosmic backgrounds
  - Using Parma and K_L particle guns, but none presented here

Follow Up From Previous Presentation

- As a result of the helpful questions and feedback on the Tracking Algorithm from our previous presentation, we have conducted studies, made some changes, and rerun our analysis with them implemented
    - Tracking **seed ordering has been adjusted**
        - Seeds are chosen in **order of least ds^2 / dr^2** (interval as a fraction of distance)
        - This prioritizes seeds that are most light-like in relative terms.
    - A typo on slide 9 point 3 (of this presentation) was found and corrected
    - **Merging has been removed** at the reconstruction stage
        - Merging criteria were previously being relaxed for tracks with 2 or more missing hits on layers between their first and last hits
            - this likely acted as a track quality cut before the analysis (which would be why Delta Ray tagging was not found to be helpful)
    - **Resolution Studies** for the Tracker have been conducted using a muon gun
    - **Resolution Studies** for the Vertexer have been conducted using the signal samples in the analysis
    - Number of reconstructed vertices **dependence on the following parameters** has been studied
        - Kalman Filter Momentum assumption (for multiple scattering variance)
        - Tracking Seed Interval Cut
- Summary: Performance is essentially unchanged, physics story is clearer

Outline of the Tracking Algorithm - Preparing

- **DIGITIZE** simulated (Geant4) hits (as in the Maximum Likelihood Linear Tracker)
  - Average close hits (within $20\mathrm{ns}$ and the same scintillator) weighted by their deposited energy
  - Smear time and position information according to position (~30 cm along bar) and timing uncertainty of 1ns
- **SEED** (as in the Maximum Likelihood Linear Tracker)
  - Look for pairs of hits close to lightlike separated, we require ds^2 / c^2 < 5 ns^2
  - Seeds are chosen in order of least ds^2 / dr^2 (interval as a fraction of distance)

After preparing the digitized hits and seeds, we loop over seeds and run the tracker on them until we run out of hits, or we run out of seeds (since hits used in tracking cannot be reused).

Outline of the Tracking Algorithm - Tracking

1. **FIND** hits by Kalman filtering in both directions from seed
   - Look for digitized hits that contribute the least to the chi^2 (provided chi^2_p < 200)
     - We allow lots of scattering (scattering momentum 0.5 GeV) for high acceptance
     - If no hits satisfy this, we propagate to the next layer
2. **FILTER** and **SMOOTH** over all hits found in previous step
3. **DROP** hits based on chi^2 increment and beta best estimate (from track fit)
   - Hits with chi increment with p-value that gets rounded to 1 by machine precision get dropped (small change from chi^2 > 150)
   - Hits must also satisfy **0.8 < beta (smoothed) < 1.2.**
     (Low-beta tracks would be detected in 2nd tracking pass to detect & reject low-beta tracks, which are associated with the neutrino BG.)
     - We assume momentum is high enough that signal tracks all have beta close to 1
4. **FIT:** Run filter and smoother without altering hits
   - **VETO** a track if
     - $\chi^2/\mathrm{ndof} \geq 15$
     - The track has fewer than 4 layers with hits

Outline of the Tracking Algorithm - Vertexing

1. **VERTEXER** used is a Maximum Likelihood fitter
    - Only use lowest (in vertical direction) hit and velocity vector to represent the track
    - **SEED** with pairs of tracks that have a closest approach distance < 100cm.
    - **For each seed:**
        - vertex position = midpoint of particle positions at the time of closest approach
        - Add other tracks to this vertex that have closest approach to vertex < 100cm
        - For all tracks belonging to vertex, get **chisq** for that track to go through the vertex.
        - **VETO** the vertex if $\chi^2/\mathrm{ndof} \geq 15$, where ndof = (# tracks) x 7 (posn, time, vel)

Track Resolution Studies

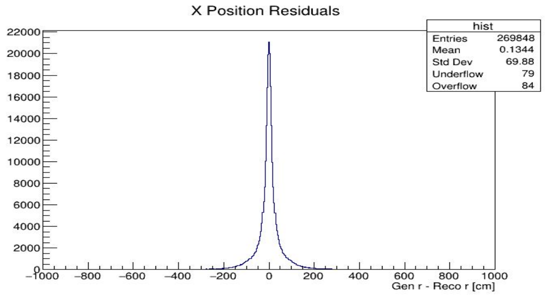
- 10 GeV muon gun
  - Fired from above and below the floor
  - Position in x and z is randomly sampled
  - Orientation is randomly sampled (polar angle from vertical is between 0 and 30 degrees)
- We compare generated and reconstructed track parameters
  - Project reconstructed and generated track parameters to the floor for position and time
  - Note that no effort was made to ignore **secondary tracks**
    - Ignoring these tracks would **increase** our resolution slightly
- Results:
  - Near 0 means for velocity residual distributions, with STDs: x, z ~ 0.03 - 0.04 c, y ~ 0.08 - 0.09 c
  - Mean Angular deviation of reconstructed from truth is ~ 0.025 rad
  - Position Resolution is not increased significantly by including floor hits
    - Floor tile geometry is comparable to resolution (square scintillators 50 cm x 50 cm)
  - Angular Track resolution gives **max boost LLP we can reconstruct will be *approximately*** b =~ 100 (LLP -> 2 leptons, m = 2 GeV is close to this limit)

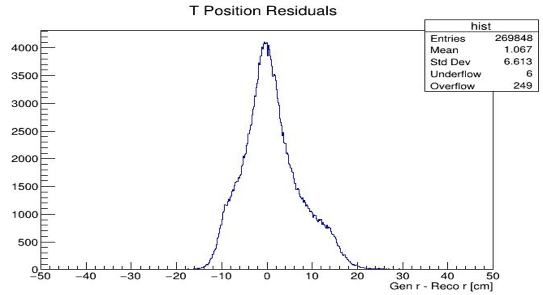
Track Resolution Above the Floor Results (269,848 Tracks Total)

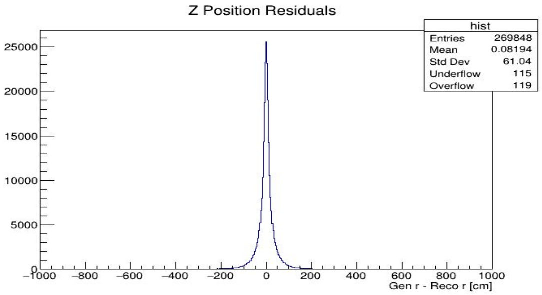
Y	Vertical
Z	Beamline
X	Other

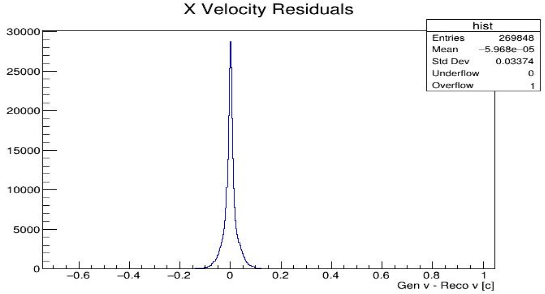
Residual / Statistic	x [cm]	t [ns]	z [cm]	vx [c]	vy [c]	vz [c]	Angle [rad]
Mean	0.1344	1.067	0.08194	-5.968e-5	0.007686	-6.802e-5	0.02409
STD	69.88	6.613	61.04	0.03374	0.09276	0.03135	0.03733
HWHM	11.56	4.721	11.56	0.006893	0.06773	0.006353	0.005780
65% Boundary (17.5%, 82.5%)	(-27.02, 27.33)	(-4.770, 7.047)	(-21.16, 21.15)	(-0.01754, 0.01740)	(-0.07394, 0.09335)	(-0.01407, 0.01407)	0.01994
95% Boundary (2.5%, 97.5%)	(-129.9, 130.9)	(-10.17, 15.35)	(-94.41, 94.54)	(-0.06312, 0.06274)	(-0.1697, 0.1810)	(-0.05076, 0.05038)	0.07385

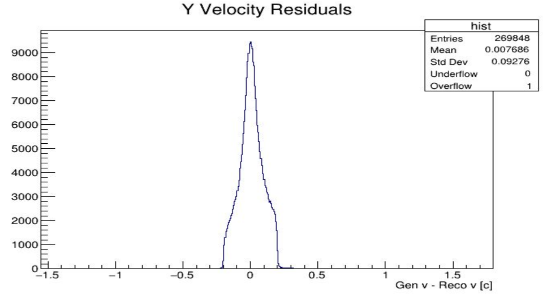
Track Residual Plots - Above the floor (linear scale)

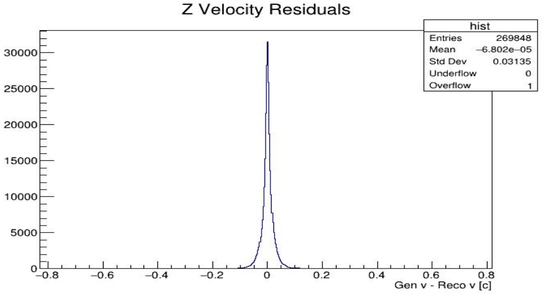


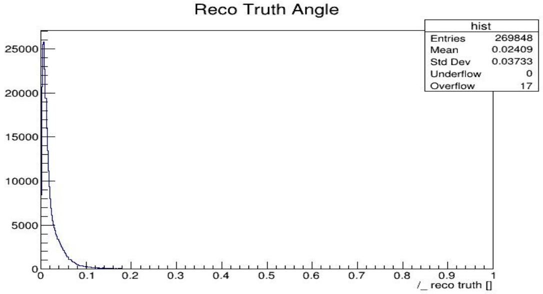






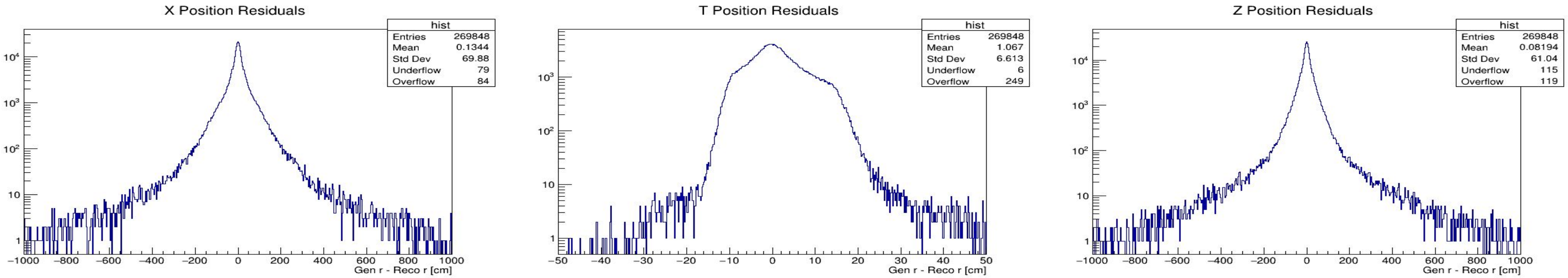


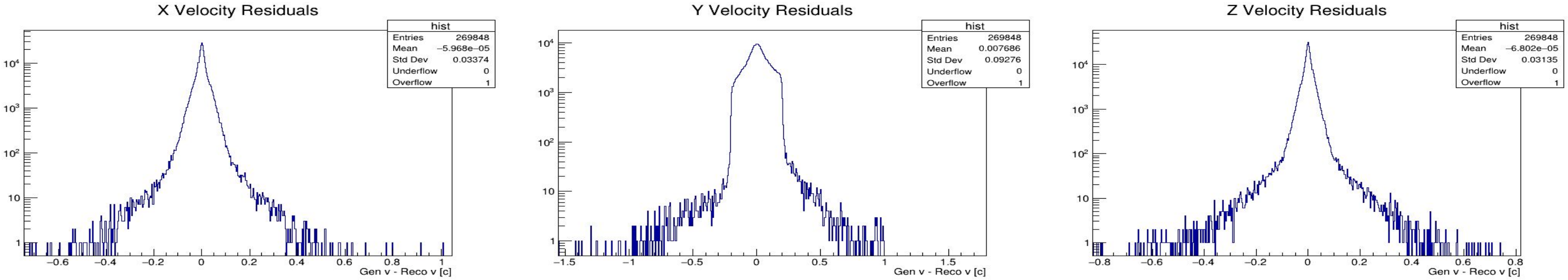


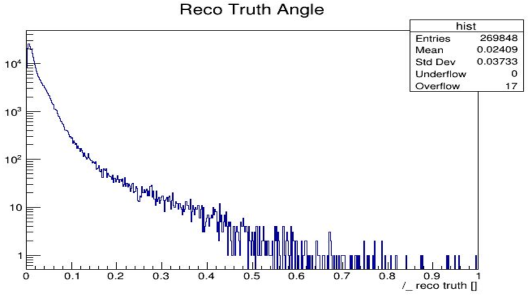


Y	Vertical
Z	Beamline
X	Other

Track Residual Plots - Above the floor (log scale)







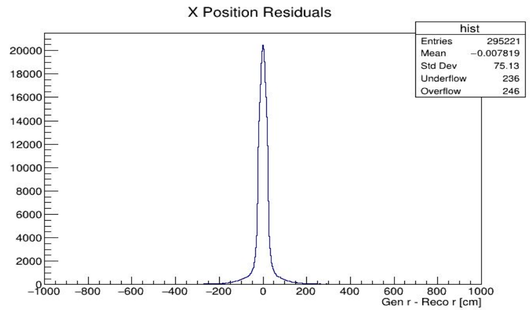
Y	Vertical
Z	Beamline
X	Other

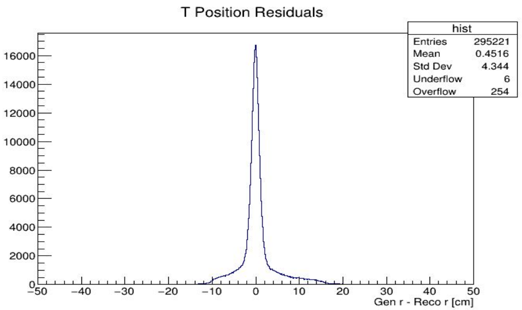
Track Resolution Below the Floor Results (295,221 Tracks Total)

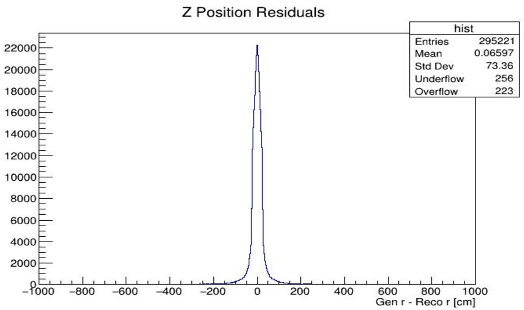
Y	Vertical
Z	Beamline
X	Other

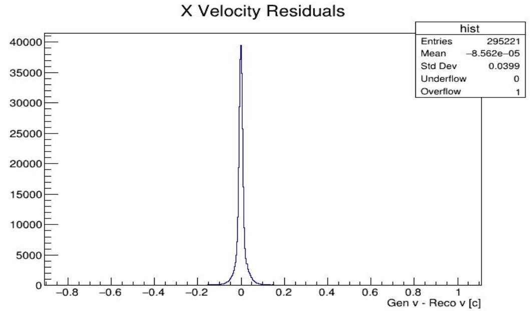
Residual / Statistic	x [cm]	t [ns]	z [cm]	vx [c]	vy [c]	vz [c]	Angle [rad]
Mean	-0.007819	0.4516	0.06597	-8,562e-5	0.00747	-1.505e-5	0.02542
STD	75.13	4.344	73.36	0.0399	0.0776	0.03952	0.05209
HWHM	22.10	1.013	22.10	0.007434	0.01996	0.006828	0.005524
65% Boundary (17.5%, 82.5%)	(-20.10, 20.18)	(-1.552, 1.855)	(-19.01, 18.94)	(-0.0117, 0.0116)	(-0.0462, 0.0658)	(-0.0108, 0.0108)	0.01575
95% Boundary (2.5%, 97.5%)	(-114.3, 114.0)	(-7.908, 12.51)	(-88.65, 88.91)	(-0.0585, 0.0584)	(-0.155, 0.174)	(-0.0523, 0.0520)	0.08929

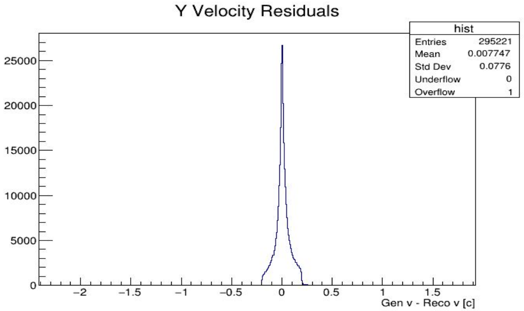
Tracker Resolution Studies - Below the floor (linear scale)

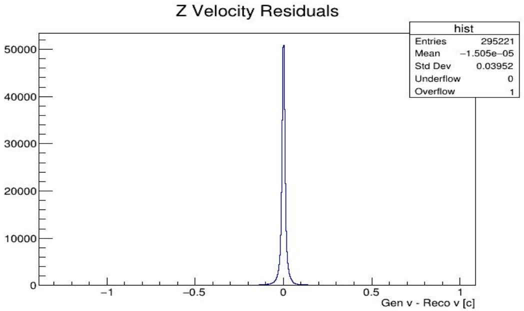


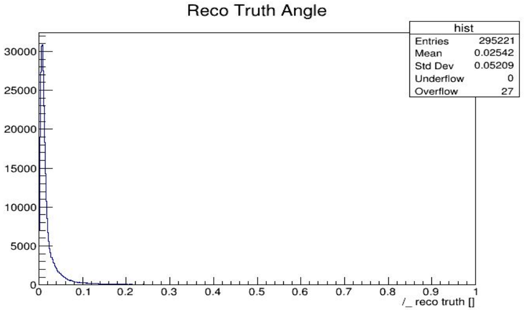






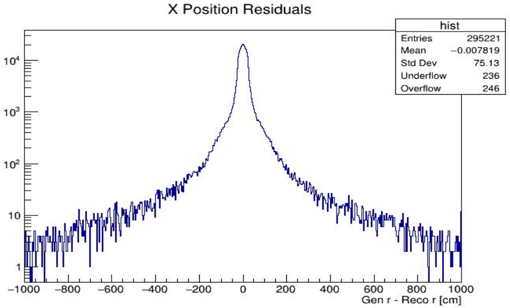


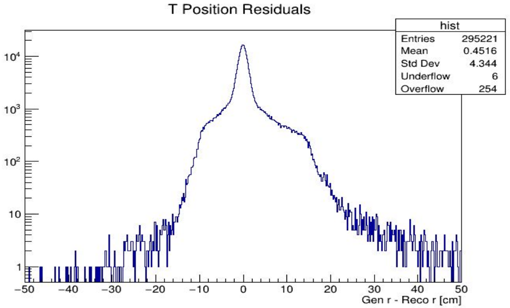


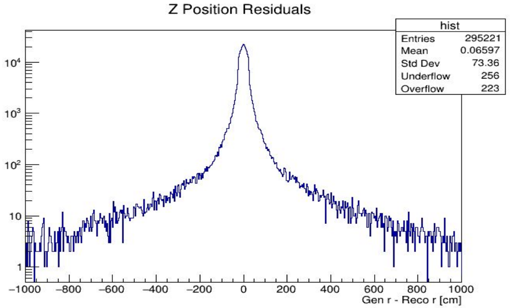


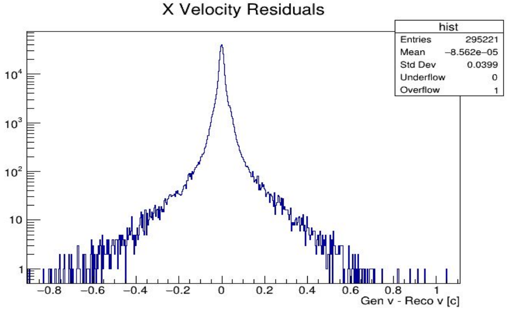
Y	Vertical
Z	Beamline
X	Other

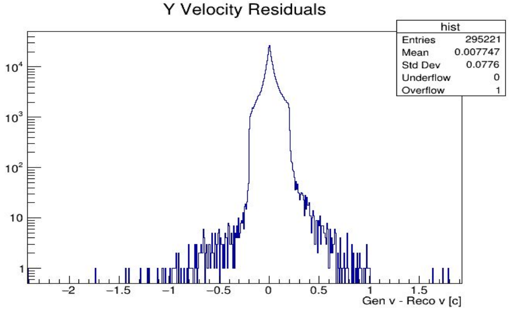
Tracker Resolution Studies - Below the floor (log scale)

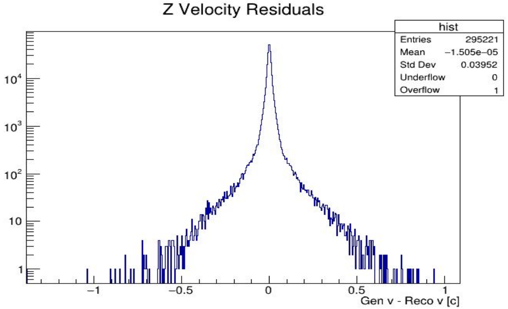


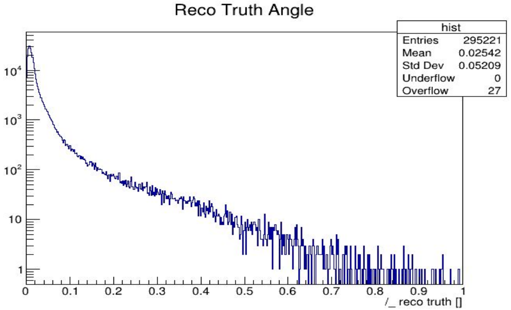








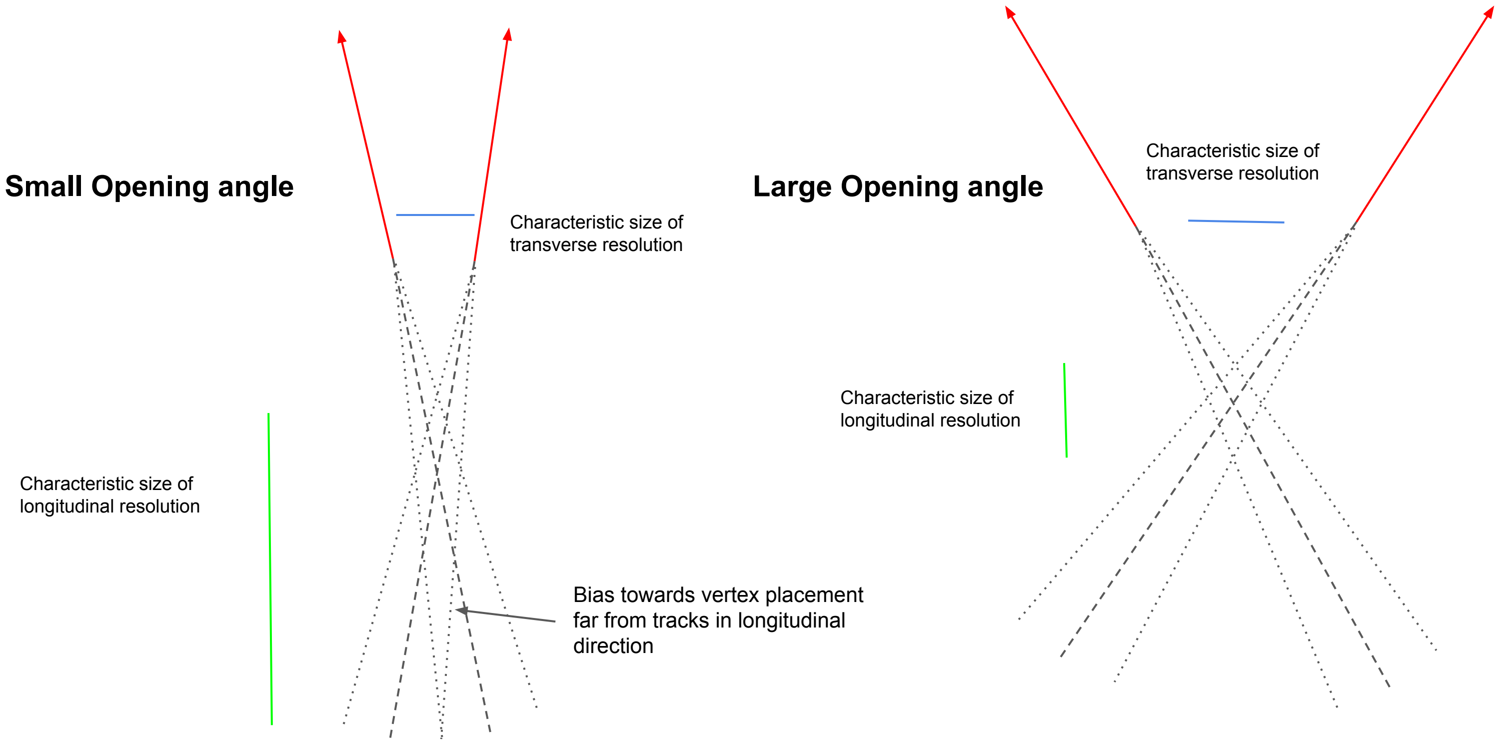




Y	Vertical
Z	Beamline
X	Other

Vertex Resolution Studies - Physics Story

- Assume near constant angular track resolution
- Vertex Resolution, especially along the track direction, is sensitive to the opening angle of tracks
- There will be a ***longitudinal bias*** for vertices with small opening angle
  - This can be ***compensated for*** in future upgrades of the vertexing algorithm



Vertex Resolution Study

- We compare the residual of the simulated (geant) vertex to the spatially closest *reconstructed vertex* ***before any analysis level cuts***
- We fit a gaussian to the core of the distribution to characterize the resolution of vertexing algorithm
- RESULTS:
    - Hadronic LLP vertex resolution (primary physics goal) is poor; this is due to highly sub-optimal vertexing for high-multiplicity vertices, can no doubt be optimized, but does not hugely impact our analysis.
    - Low-boost leptonic LLP vertex resolution (secondary physics goal) is O(1 m) at the core
    - The expected dependence of resolution on 2 track opening angle holds, i.e. worse for lighter (higher boost) LLPs

Vertex Resolution Results - Primary

Y	Vertical
Z	Beamline
X	Other

Sample	LLP->qqbar, m = 10 GeV, ctau = 50 m					
Residual / Statistic	x [cm]	y [cm]	z [cm]	t [ns]	Transverse [cm]	Longitudinal [cm]
Fit Mean	60.40	361.7	605.6	28.02	-28.76	398.9
Fit STD	445.0	556.9	1018	37.01	455.6	575.8
HWHM	235.3	635.2	181.7	34.92	251.2	1062.8
65% Boundary	(-506.9, 656.4)	(-2348, 1413)	(-3775, 1961)	(-165.8, 71.50)	(-1091, 481.1)	(-3833, 2281)
95% Boundary	(-10810, 7187)	(-1374, 4806)	(-29690, 8118)	(-1374, 233.1)	(-11750, 9082)	(-38730, 7408)

Vertex Resolution Results - Primary

Y	Vertical
Z	Beamline
X	Other

Sample	LLP->qqbar, m = 50 GeV, ctau = 100 m					
Residual / Statistic	x [cm]	y [cm]	z [cm]	t [ns]	Transverse [cm]	Longitudinal [cm]
Fit Mean	5.566	140.7	97.42	9.502	34.88	158.9
Fit STD	244.4	511.3	492.7	3.019	555.8	638.7
HWHM	143.9	324.8	285.7	16.25	328.6	257.3
65% Boundary	(-561.2, 711.5)	(-4651, 1135)	(-5151, 1224)	(-312.4, 59.41)	(-1316, 924.2)	(-7780, 1640)
95% Boundary	(-9028, 10750)	(-3236, 3915)	(-60600, 5914)	(-3236, 237.3)	(-69170, 5337)	(-18060, 19330)

Vertex Resolution Results - Secondary

Y	Vertical
Z	Beamline
X	Other

Sample	LLP -> 2 leptons, m = 35 GeV…. h35 - larger opening angle than Tertiary Goal Leptonic Samples (h2 and h10)					
Residual / Statistic	x [cm]	y [cm]	z [cm]	t [ns]	Transverse [cm]	Longitudinal [cm]
Fit Mean	0.5325	3.018	4.962	0.4021	3.73	3.657
Fit STD	28.30	45.57	55.09	2.522	83.10	107.6
HWHM	13.53	19.53	36.27	1.214	53.47	77.69
65% Boundary	(-117.7, 60.97)	(-108.7, 297.9)	(-186.2, 299.1)	(-8.119, 15.42)	(-454.8, 294.2)	(-208.0, 437.0)
95% Boundary	(-655.6, 128.7)	(-2219, 1870)	(-655.6, 2419)	(-15460, 3177)	(-17630, 3575)	(-10650, 12180)

Vertex Resolution Results - Tertiary

Y	Vertical
Z	Beamline
X	Other

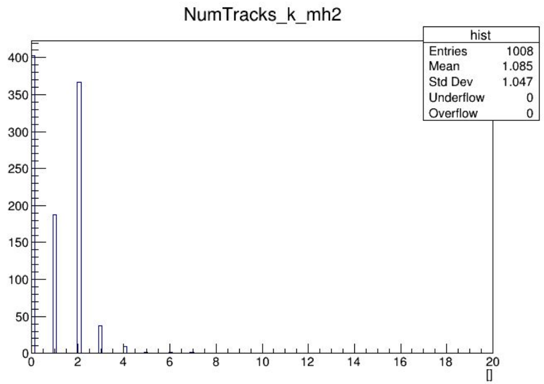
Sample	LLP -> 2 leptons, m = 10 GeV…. h10 - smaller opening angle than Secondary Goal Leptonic Sample (h35)					
Residual / Statistic	x [cm]	y [cm]	z [cm]	t [ns]	Transverse [cm]	Longitudinal [cm]
Fit Mean	-2.230	37.53	44.06	2.512	-1.110	60.27
Fit STD	33.39	97.89	114.7	5.637	31.74	152.0
HWHM	7.918	25.75	-	1.980	25.17	46.23
65% Boundary	(-65.88, 74.21)	(-118.8, 382.8)	(-144.2, 469.6)	(-6.886, 21.52)	(-62.89, 58.31)	(-191.4, 656.3)
95% Boundary	(-1410, 922.4)	(-616.5, 2043)	(-13230, 2539)	(-616.5, 116.7)	(-1713, 1325)	(-16500, 3306)

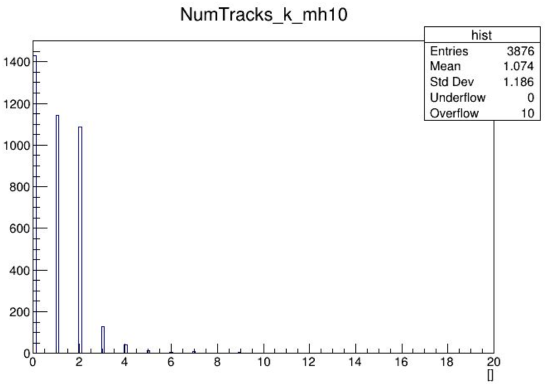
Vertex Resolution Results - Tertiary

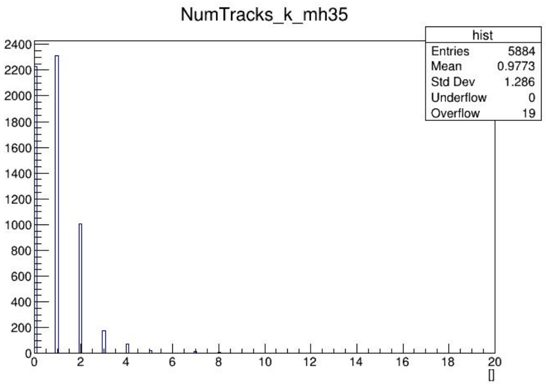
Y	Vertical
Z	Beamline
X	Other

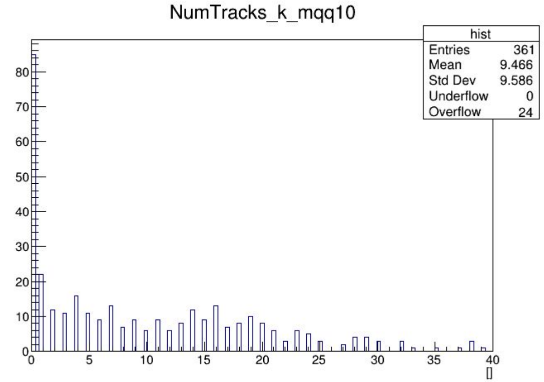
Sample	LLP -> 2 leptons, m = 2 GeV…. h2 - Smallest opening angle of Leptonic Signal Samples					
Residual / Statistic	x [cm]	y [cm]	z [cm]	t [ns]	Transverse [cm]	Longitudinal [cm]
Fit Mean	-4.577	225.4	301.4	11.38	-1.066	373.9
Fit STD	104.9	354.8	508.9	1.744	34.69	588.4
HWHM	19.63	296.4	368.2	16.44	21.00	476.6
65% Boundary	(-171.7, 162.1)	(-528.6, 791.1)	(-658.5, 1013)	(-29.41, 44.91)	(-50.55, 50.72)	(-845.3, 1322)
95% Boundary	(-1914, 1864)	(-1267, 2129)	(-24800, 3121)	(-1267, 126)	(-2896.6, 1353)	(-32570, 3756)

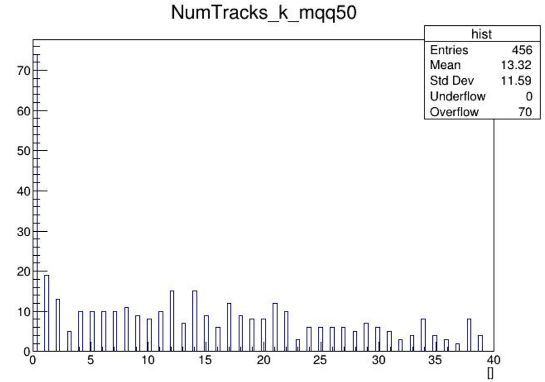
Efficiency Characterization - Tracks per Event





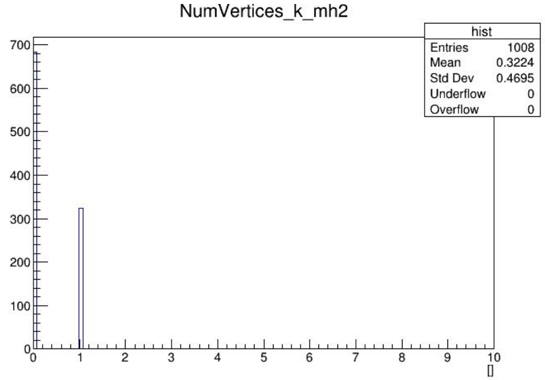


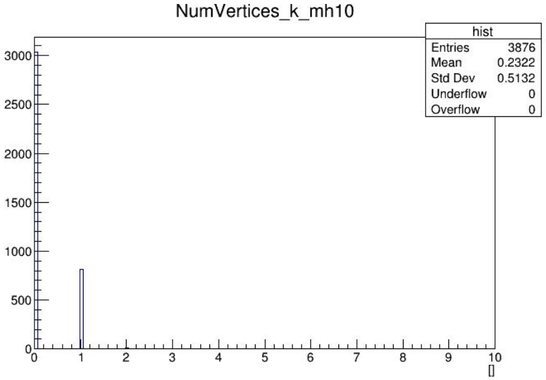


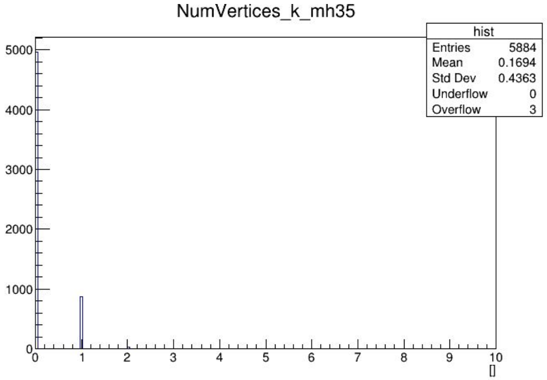


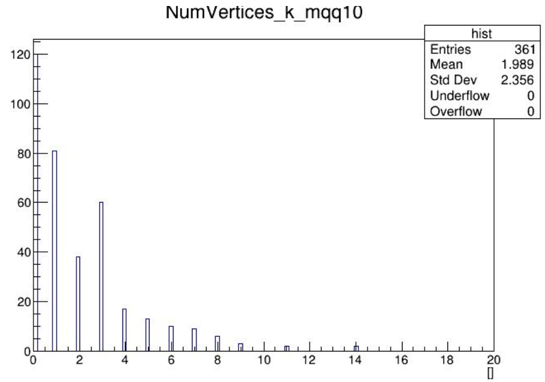
Refer to cutflow tables in each analysis for track reconstruction efficiency for tracks originating from LLP decays in MATHUSLA's volume

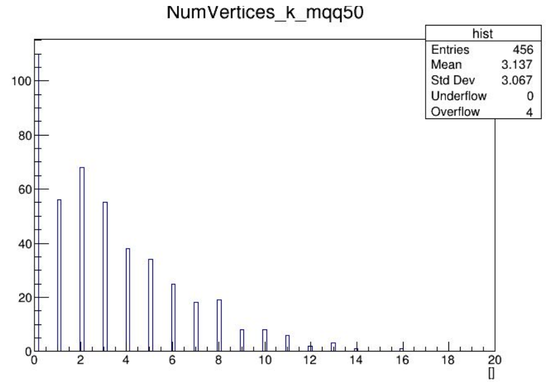
Efficiency Characterization - Vertices Per Event











Refer to cutflow tables in each analysis for vertexing efficiency

Studying dependence of vertex reconstruction on momentum assumption in tracking with Kalman filtering

No merge, relative seed ordering. Table entries give number of reconstructed vertices.

Momenta [MeV] / sample	h2	h10	W	Qq 10GeV, ctau = 50m	Qq 50GeV, ctau = 100m
100	326	901	127	732	1580
250	318	897	120	766	1625
500	325	900	110	714	1544
1000	294	896	110	700	1496
2500	283	842	115	668	1418

UPSHOT: very little dependence of reconstructed vertices on this parameter!

Studying dependence of vertex reconstruction on maximum space-time-hit separation for track seeds

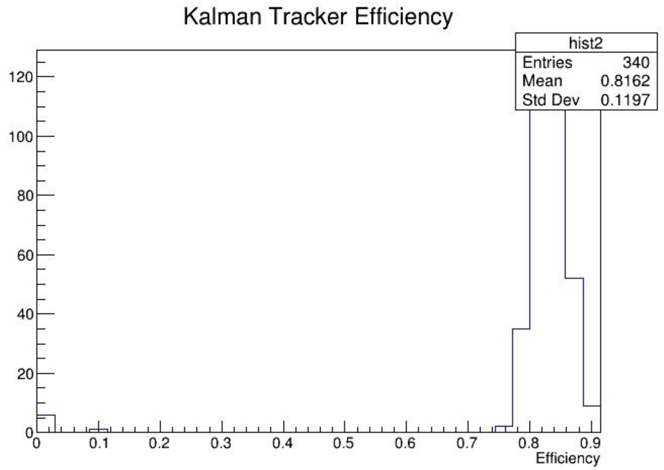
No merge, relative seed ordering. Table entries give number of reconstructed vertices.

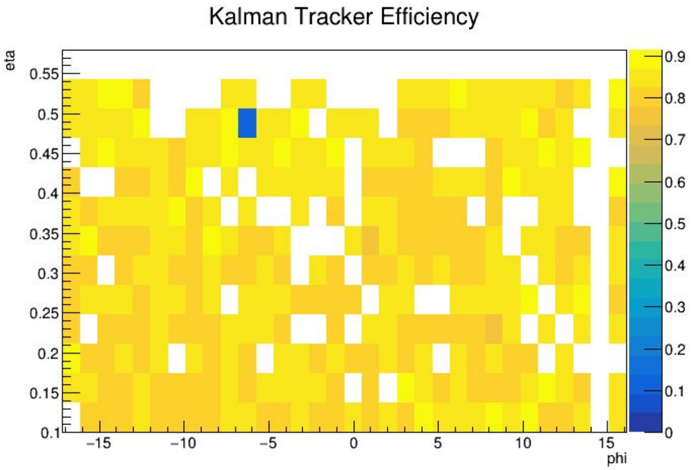
Interval [ns^2] / sample	h2	h10	W	Qq 10GeV, ctau = 50m	Qq 50GeV, ctau = 100m
4	283	802	98	690	1494
5	319	875	129	747	1555
5.5	314	914	126	744	1586
6.0	356	964	124	748	1599

UPSHOT: weak dependence, stick with 5 ns^2 (we run into memory crashes around 7.5 ns^2)

Tracker Reconstruction Efficiency Study

- ***Done With Previous Tracker Version***
  - This version used track merging, and absolute seed ordering (as in the previous talk on the analysis)
  - We do not expect either to play an important role when tracking single muons (especially for efficiency)
  - HENCE: we expect this to be ***a good representation of current tracker efficiency***





- Study details:
  - 10 GeV muon gun fired upwards at tracking layers from 5 m below for a range of eta and phi each with 1000 events
  - Efficiency is # of fired muons / # reconstructed tracks
- *UPSHOT: Reconstruction efficiency mean is >~ 80 % for tracker*

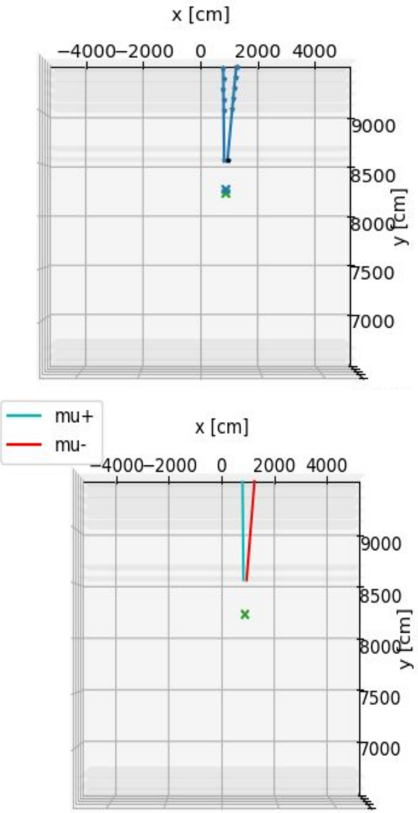
Vertex reconstruction and cuts for all analyses

Assume a 90% efficient muon veto from CMS

1. 2 or more tracks in the event (can require more tracks later)
2. 1 or more vertex
3. 1 or more vertex in decay volume, i.e. Fiducial vertex
4. Fiducial volume cuts: top corner that can't be cut by wall veto. **Note no fiducial cuts are necessary/helpful to exclude vertices caused by material interactions (see next slides)**.
5. Veto events with fewer than 30 tracks and at least 1 upward going track with hits in the floor
6. Veto on floor or wall hits that *might* be part of tracks even if they were not included in the track in the original reconstruction.
    i. If there is a hit in the floor or wall near (within 20 m) to where we expect a track, and it occurred in time before the vertex, veto the event
7. Veto events with only Delta Ray vertices (tagged by maximum deflection in tracks)

For Secondary and Tertiary Analysis (could be tagged as events with < 7 tracks **not done currently**), also do the following:

8. Veto multi track track vertices that couldn't have come from the IP
9. Veto 2 track vertices that couldn't have come from the IP (more aggressive than above)

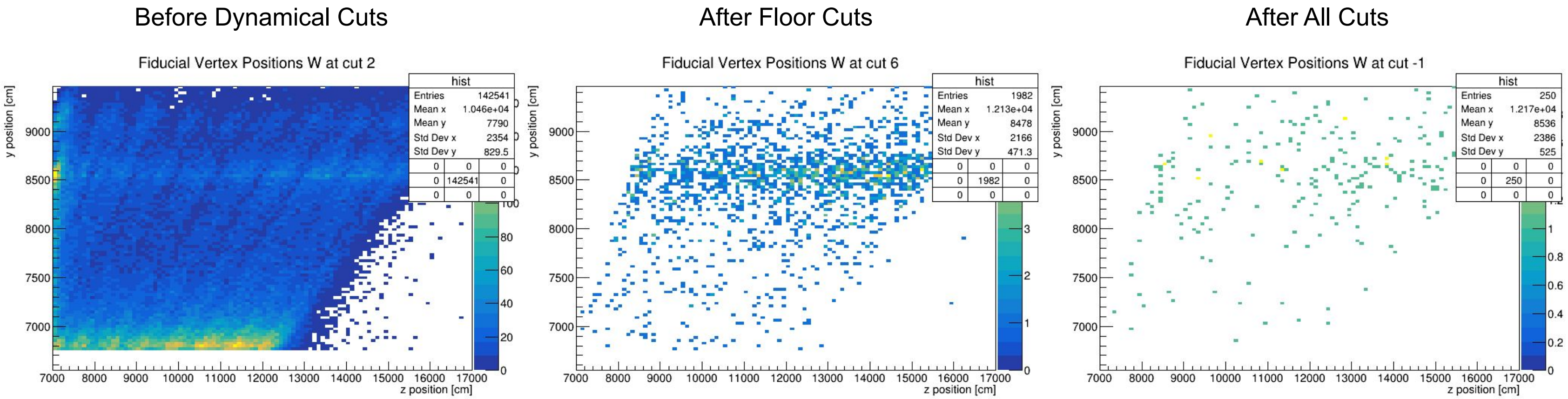


10 GeV Muon Signal

Material Interaction Fiducial Cuts?

- We can assume that the dominant source of background vertices are from particles created via material interactions in the detector
    - One would like to tag vertices reconstructed at the interaction prone materials and consider those regions non fiducial. Background vertices are dominantly produced by material interactions with aluminum support structure slabs under the scintillator layers, the hollow iron support columns are the next most dominant
        - volumes are 567 m^3 (floor + wall, support + tracker) vs 77 m^3 (columns), radiation lengths are 8.9 cm vs 13.84 cm respectively).
- This suggests that most background vertices should originate near floor/wall/layers, with smaller contribution from columns.
- As we show on next slides, the wall/floor is indeed dominant in BG vertex position map, but columns are indiscernible due to poor vertex spatial resolution.
- However, after wall/floor *hit* vetoes (even with 1E-2 inefficiency), those BG vertices that are near the floor/wall are efficiently removed. The remaining BG vertices have no discernable spatial preference inside the decay volume after angular and delta ray cuts.
- **Therefore, explicit fiducial volume cuts to remove material interaction are not useful, beyond the top front corner of the detector that is not covered by the wall.**

Preview: W BG vertex positions after cuts for secondary and tertiary physics analysis with 1e-2 Hit Inefficiency.



Upshot: the floor/wall veto cuts remove BG vertices caused by material interaction in wall/floor, so no explicit fiducial cut is necessary to exclude regions near floor/wall material.

1. PRIMARY PHYSICS GOAL ANALYSIS

LLPs decaying hadronically to many tracks, O(1–10 boost)

Produced in h->aa, a->uu, ma ~ 10, 50 GeV

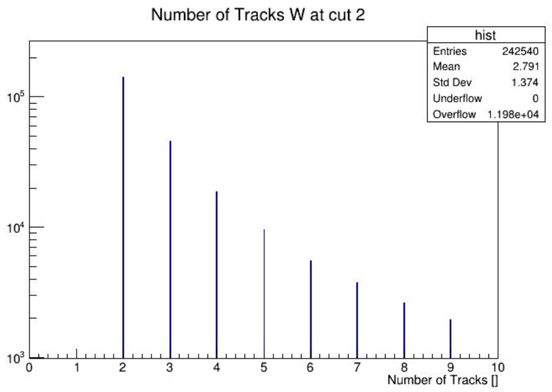
c \tau = 50 m, 100 m (this difference shouldn’t matter much)

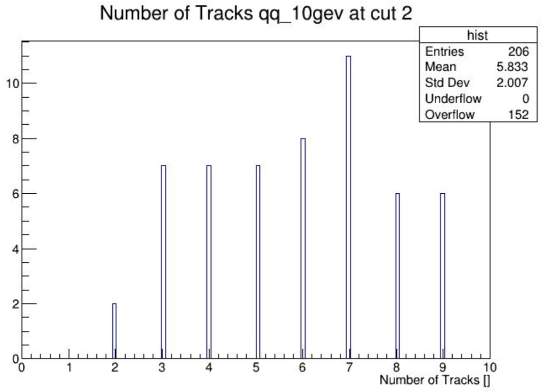
Primary Physics Goal - Relevant Background

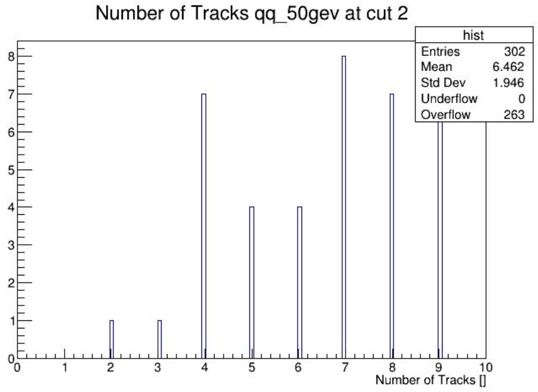
- We expect 6e10 W events at HL-LHC assuming 3/ab in total (no MATHUSLA-specific cuts).
- Ws contribute roughly ½ of the muons that reach MATHUSLA, with the others coming from other processes at the IP.
  So to get estimate of total muon rate at MATHUSLA, multiply rate from W by 2.
- Simulated 4.59E9 inclusive W events, so we simulated 8% of real-world intensity in MC. Given that this is half the total muon rate, and we use the 90% efficient CMS muon veto,
  multiplying our background rates by 2 will roughly give the real background rate in MATHUSLA.

- **The relevant background (from the IP) is W muons interacting and forming fake vertices**
  - The low track multiplicity of this background, compared to LLP->hadrons signal, gives a very strong handle for rejecting them

Events with >= 2 tracks







Primary Goal - W Background (1e-3 hit inefficiency)

43,143,321 triggered events (events with >=3 Geant (sim) hits somewhere in the Detector) out of 4.6e9 generated events

Cuts / Efficiency	2-3 Tracks in the Event	2-3 Fraction of Triggered	4-6 Tracks in the Event	4-6 Fraction of Triggered	>=7 Tracks in the Event	>=7 Fraction of Triggered
1. Tracks >= 2	960823	0.0223	83499	0.00194	37988	8.81e-4
2. Vertices >= 1	537931	0.0125	80161	0.00186	37954	8.80e-4
3. **Vertex in Decay Volume**	188399	0.00437	33840	7.84e-4	20301	4.71e-4
4. Non-Fiducial Corner Cut	158613	0.00368	29976	6.95e-4	18500	4.29e-4
5. Veto Events w/ Tracks that include Floor/Wall Hits	663	1.54e-5	27	6.26e-7	2120	4.91e-5
6. Veto Tracks w/ nearby unassociated Floor/Wall hits	318	7.37e-6	20	4.64e-7	13	3.01e-7
7. **Crinkle Cut**	95	2.20e-6	5	1.16e-7	5	1.16e-7
8. High Multiplicity Angular Consistency	32	7.42e-7	3	6.95e-8	-	-
9. 2 Track Angular Consistency	8	1.85e-7	3	6.95e-8	-	-

Primary Goal - uubar Signal (m_a =10 GeV, c \tau = 50 m)

* only for 2-pronged vertices

Cuts / Efficiency (269 Fiducial LLP Decays)	2-3 Tracks in the Event	2-3 Fraction of **Decays**	4-6 Tracks in the Event	4-6 Fraction of **Decays**	>=7 Tracks in the Event	>=7 Fraction of **Decays**
1. Tracks >= 2	23	0.0855	36	0.134	195	0.725
2. Vertices >= 1	12	0.0446	34	0.126	195	0.725
3. **Vertex in Decay Volume**	9	0.0335	22	0.0818	175	0.651
4. Non-Fiducial Corner Cut	8	0.0297	20	0.0743	165	0.613
5. Veto Events w/ Tracks that include Floor/Wall Hits	7	0.026	18	0.0669	149	0.554
6. Veto Tracks w/ nearby unassociated Floor/Wall hits	7	0.026	17	0.0632	124	0.461
7. **Crinkle Cut**	5	0.0186	16	0.0595	123	0.457
8. High Multiplicity Angular Consistency	1	0.00372	12	0.0446	-	-
9. 2 Track Angular Consistency	1	0.00372	12	0.0446	-	-

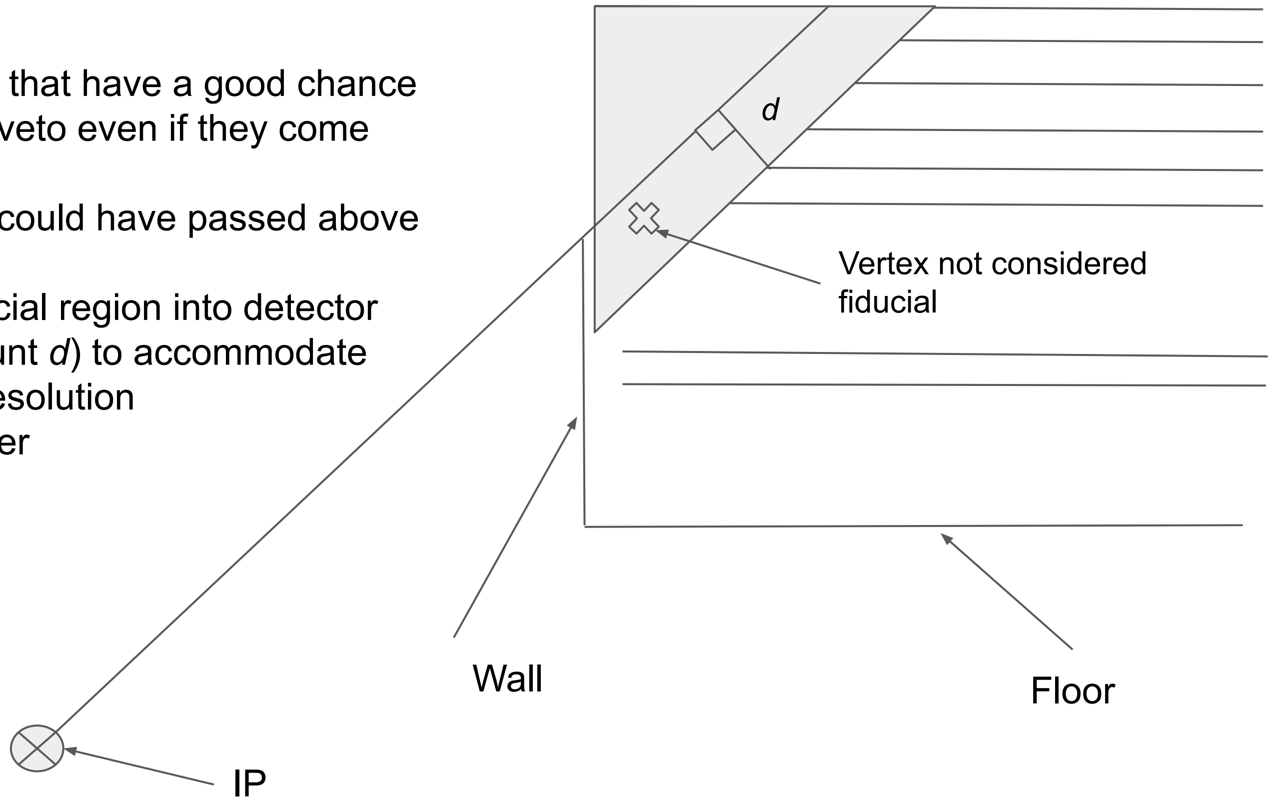
Primary Goal - uubar Signal (m_a =50 GeV, c \tau = 100 m)

* only for 2-pronged vertices

Cuts / Efficiency (315 Fiducial LLP Decays)	2-3 Tracks in the Event	2-3 Fraction of Triggered	4-6 Tracks in the Event	4-6 Fraction of Triggered	>=7 Tracks in the Event	>=7 Fraction of Triggered
1. Tracks >= 2	18	0.0571	30	0.0952	315	1
2. Vertices >= 1	4	0.0127	27	0.0857	315	1
3. **Vertex in Decay Volume**	2	0.00635	15	0.0476	285	0.905
4. Non-Fiducial Corner Cut	2	0.00635	13	0.0413	264	0.838
5. Veto Events w/ Tracks that include Floor/Wall Hits	1	0.00317	9	0.0286	208	0.660
6. Veto Tracks w/ nearby unassociated Floor/Wall hits	1	0.00317	8	0.0254	139	0.441
7. **Crinkle Cut**	1	0.00317	6	0.019	139	0.441
8. High Multiplicity Angular Consistency	0	0	1	0.00317	-	-
9. 2 Track Angular Consistency	0	0	1	0.00317	-	-

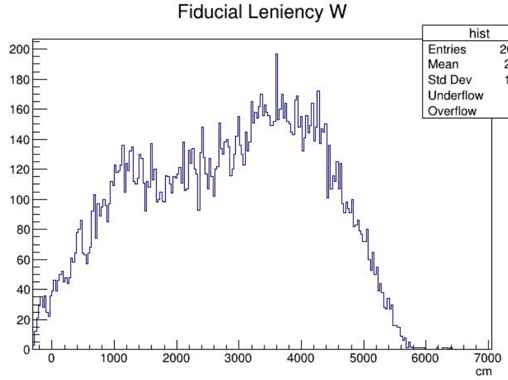
4. Non-Fiducial Corner Cut Details

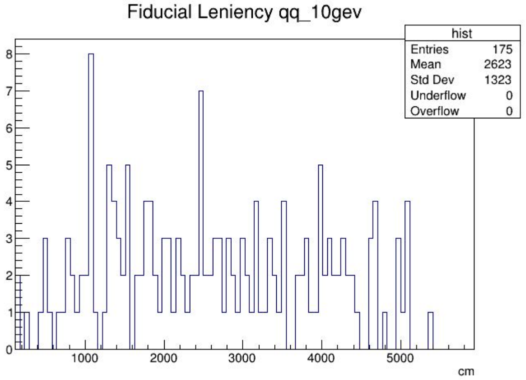
- This vetoes vertices that have a good chance of avoiding the wall veto even if they come from a muon
- Ignore vertices that could have passed above the wall
- Extend the non-fiducial region into detector volume (by an amount *d*) to accommodate imperfect tracking resolution
- *d* is the cut parameter
    - *d* = 7.5 m

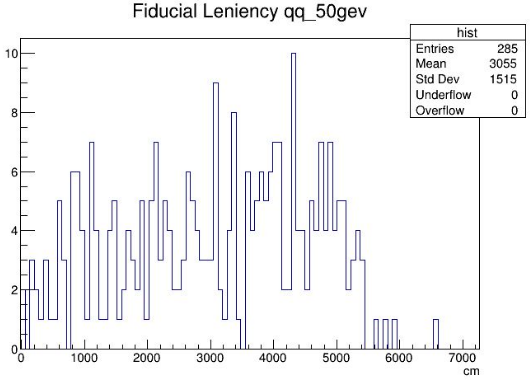


Primary Goal - 4. Non-Fiducial Corner Cut Distributions (>= 7 Tracks, 1e-3 hit inefficiency)

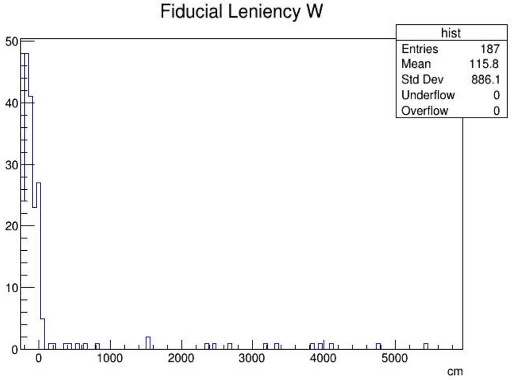
Before floor cuts

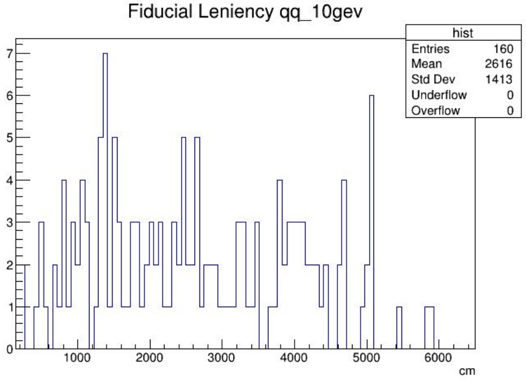


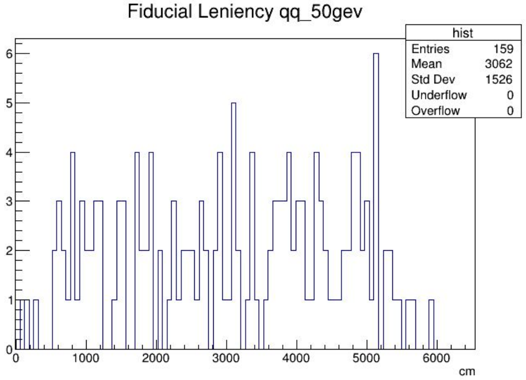




After floor cuts



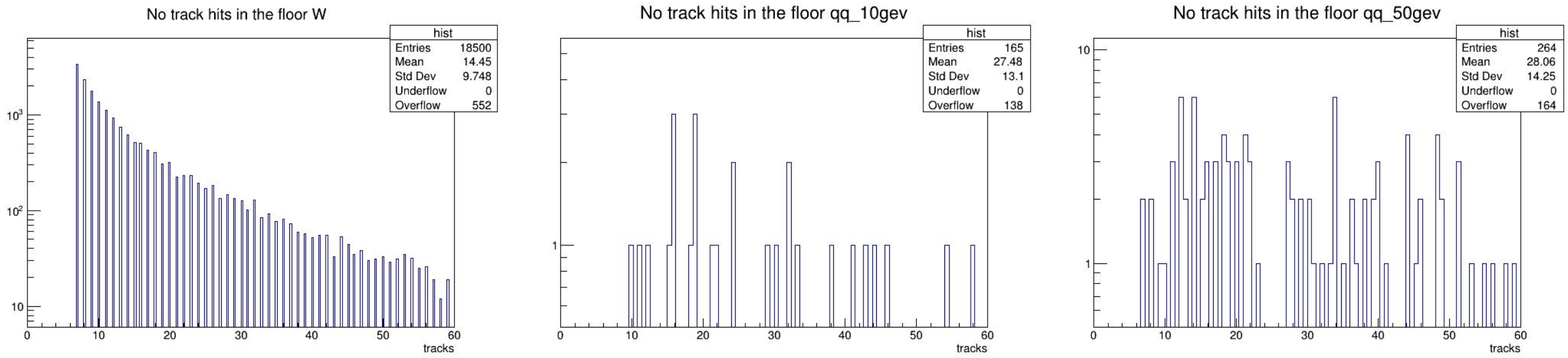




5. Veto Events w/ Upward Tracks that include Floor/Wall Hits

- **The event gets vetoed if Num_tracks < 30 AND the event contains at least one upward going track with a floor hit.**
  - *The upward going and Ntracks < 30 requirement prevents the cut from being too aggressive on the signal, which due to large hadron multiplicity includes many backwards traveling tracks.*
- **This cut reduces BG by 1-2 orders of magnitude depending on track number cut, with ~95% signal efficiency.**

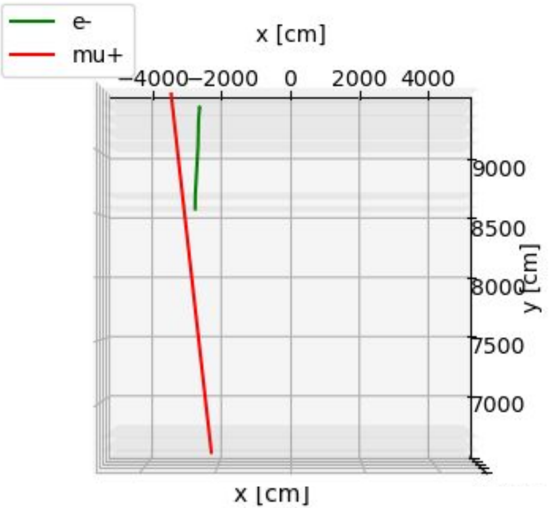
Distributions for number of tracks in events that have at least one upward going track with a floor hit (cut 5, >= 7 tracks)

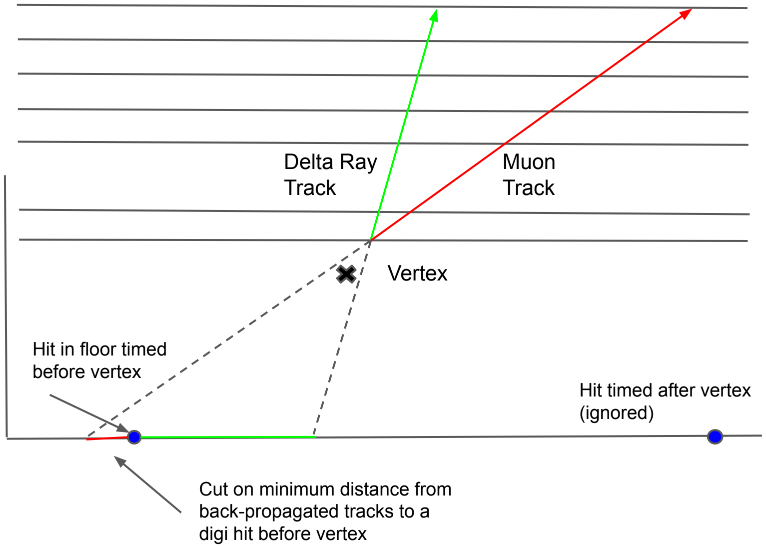


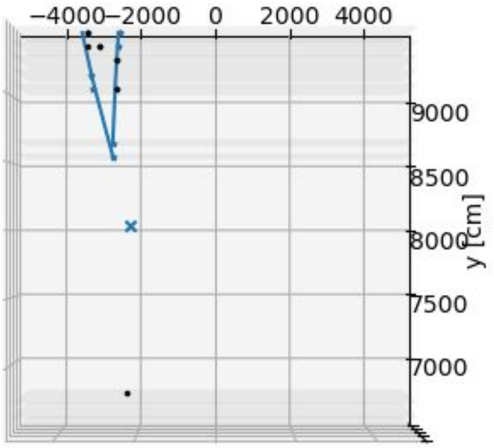
This shows that the W events which survive the cut can have lots of tracks

6. Veto Tracks w/ nearby unassociated Floor/Wall hits

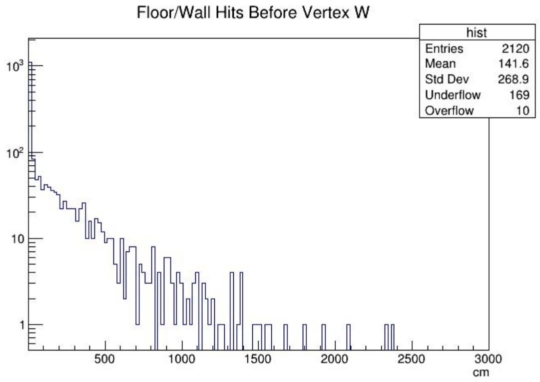
- Propagate vertex tracks backwards into the plane of the floor and wall
  - Compute the distance (in the plane) to all hits before the vertex **in time** from each back propagated position
- Cut on the minimum of these distances
  - Events where this parameter is less than 20 m are cut
    - This value is larger than one might expect primarily from hit selection inaccuracies
- **This cut removes a lot of BG but almost no signal, surprisingly good.**
- *Simulation has no detector noise right now, so this may have to be retuned with noise*



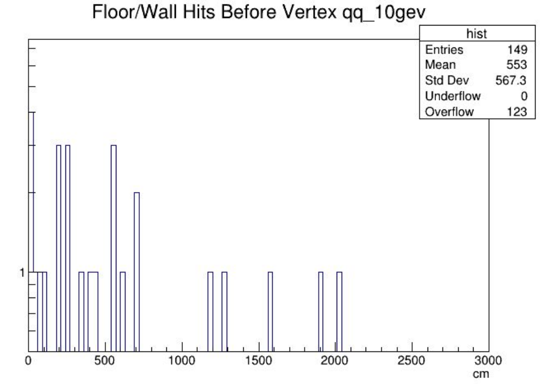


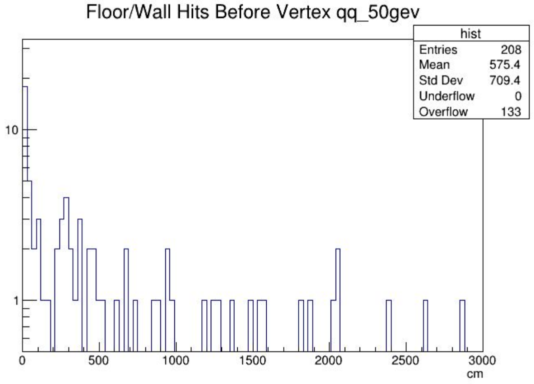


Primary Physics Goal Analysis: 6. Veto Tracks w/ nearby unassociated Floor/Wall hits Distributions (>= 7 Tracks)



- Nearly all the W’s in this plot get cut
- For signal, note the overflow numbers which contain most of the signal events. These survive the cut.

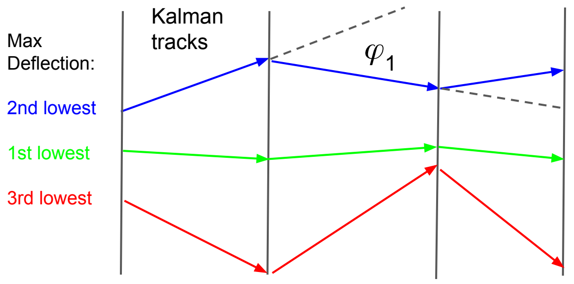




7. Crinkle Cut Details: VETO BAD DELTA RAY TRACKS

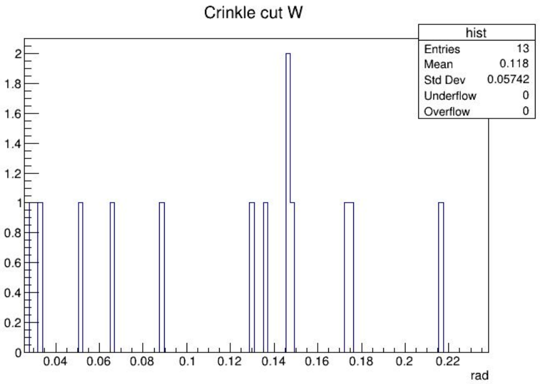
- Assume that most of the ***second*** tracks in W BG are Delta Rays, or other low energy particle (first track is will be a muon for both Leptonic Signal and BG).
  - ***Low energy => more deflection traversing scintillator layers***
  - One would like to tag and reject these events
- Psuedo-code:
  - Collect ***maximum*** deflection in the track for each track in a vertex
  - Collect the value for the ***second lowest*** track deflection
  - Cut on the ***lowest*** of these values across vertices in the event
    - We cut **0.1 rad**
    - **In other words, a vertex must have at least two tracks with max deflection angle < 0.1 rad**

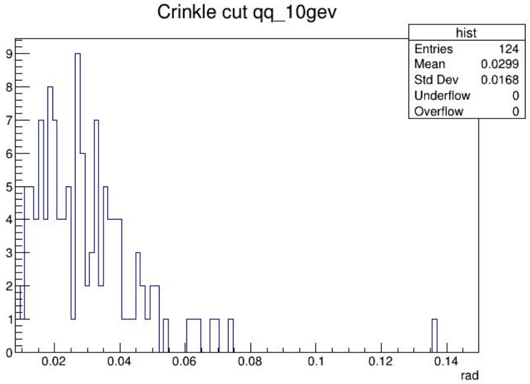
Crinkle Cut

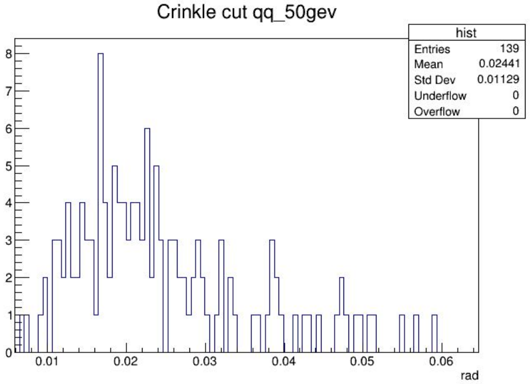


Cut on φ_1

Primary Goal - 7. Crinkle Cut Distributions (>= 7 tracks)







8.+9. Angular Consistency Cuts [IRRELEVANT HERE]

- We ignore these cuts for this analysis to retain more signal efficiency
    - They are not as helpful for background rejection at high multiplicity
- Explained in Secondary/Tertiary physics analysis slides.

Primary Physics Goal - Results with 10E-3 hit inefficiency

- High track multiplicity requirement likely optimal for this signal.
- Below numbers are estimates for *total* number of BG events @ MATHUSLA.
- Currently generating events for 10E-2 inefficiency.
- UPSHOT ~O(10) BG and >~ 50% signal efficiency possible for primary physics case
- *Signal efficiency can likely be improved with several obvious optimizations for high-multiplicity vertices.*

Sample / Tracks	2-3 Tracks Survivors	2-3 Tracks Fraction	4-6 Tracks Survivors	4-6 Tracks Fraction	>=7 Tracks Survivors	>=7 Tracks Fraction
W Background (includes additional x2 to get numbers at HL-LHC)	16	1.85e-7	6	6.95e-8	**10**	1.16e-7
10 GeV, c \tau = 50 m (denominator = 269 fiducial LLP Decays)	1	0.00372	12	0.0446	123	**0.457**
50 GeV, c \tau = 100 m (denominator = 315 fiducial LLP Decays)	0	0	1	0.00317	139	**0.441**

2. SECONDARY PHYSICS GOAL ANALYSIS

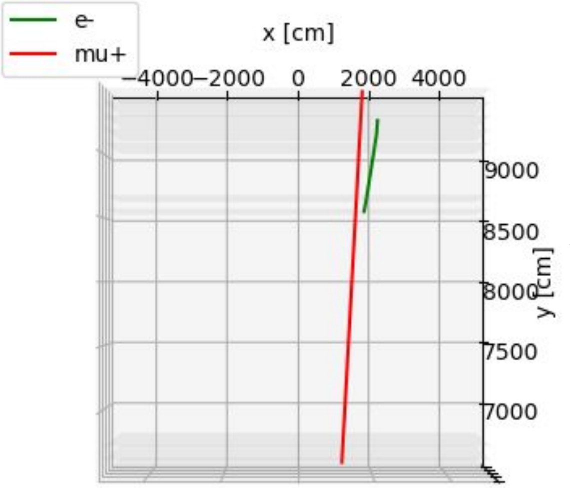
LLPs decaying to two tracks, O(1-10) boost

Produced in h->aa, a->ll, ma = 35 GeV
ctau= 100 m

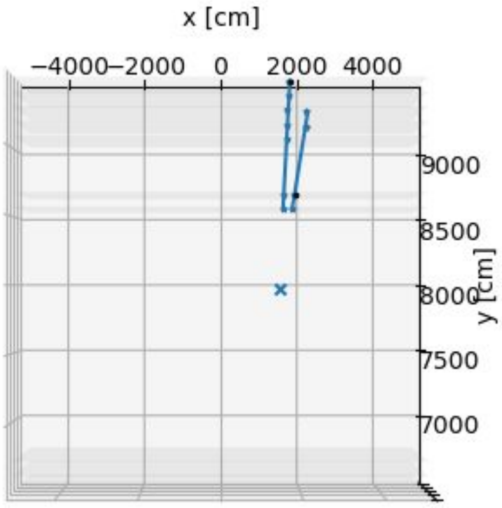
This is a stand-in for the PBC models with GeV-scale LLPs from B decays, similar boost distribution and purely geometric reconstruction efficiency (from Jai’s studies, to be released later)

Secondary Physics Goal - Relevant Backgrounds

- W's are main background. Same normalization as in Primary analysis.
- Distinguishing from secondary signal is difficult due low multiplicity of LLP decay tracks, which the BG can mimic via Delta Ray production.

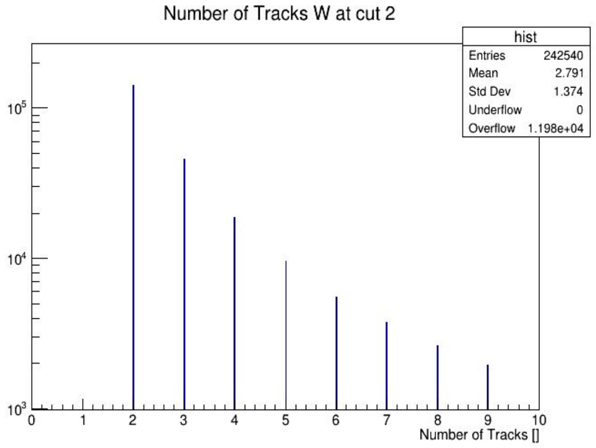


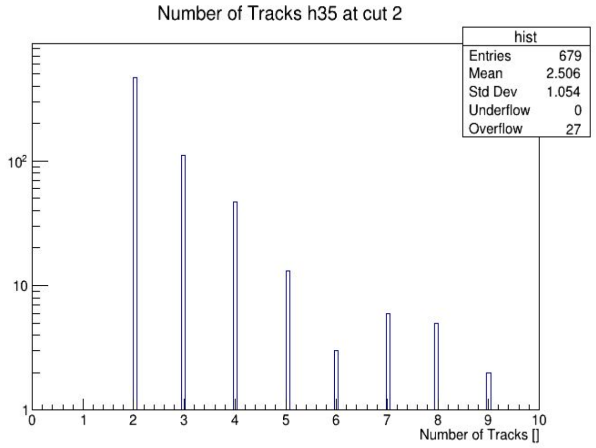
Missed hits can make W background look like signal



Additional Difficulties with Secondary Goal Background Rejection

- Some handles for the primary physics goal are unavailable here or begin to cut into signal (eg. track multiplicity)





Analysis Performance - W Background

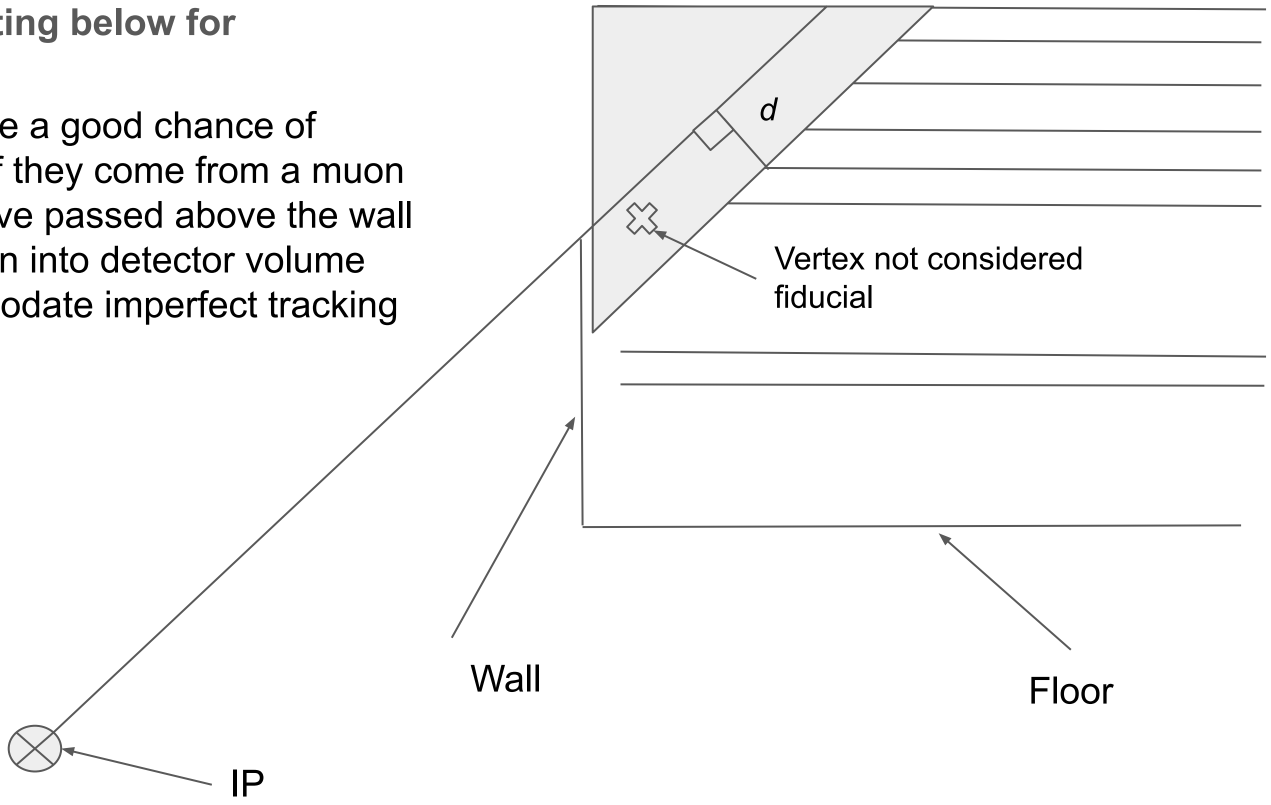
Cuts / Efficiency (Triggered Events) (Simulated Events)	Full Eff. (41,104,047) (4.3e9)	Full Eff. Fraction	1e-3 (43,143,321) (4.6e9)	1e-3 Fraction	1e-2 (29,941,414) (3.2e9)	1e-2 Fraction
1. Tracks >= 2	909277	0.0221	1082310	0.0251	577489	0.0193
2. Vertices >= 1	571990	0.0139	656046	0.0152	363532	0.0121
3. **Vertex in Decay Volume**	186335	0.00453	242540	0.00562	120018	0.00401
4. Non-Fiducial Corner Cut	159169	0.00387	207089	0.00480	102628	0.00343
5. Veto Events w/ Tracks that include Floor/Wall Hits	2331	5.67e-5	2810	6.51e-5	3166	1.06e-4
6. Veto Tracks w/ nearby unassociated Floor/Wall hits	36	8.76e-7	351	8.14e-6	1743	5.82e-5
7. **Crinkle Cut**	30	7.3e-7	105	2.43e-6	1474	4.92e-5
8. High Multiplicity Angular Consistency	5	1.22e-7	39	9.04e-7	648	2.16e-5
9. 2 Track Angular Consistency	5	1.22e-7	15	3.48e-7	230	7.68e-6

Analysis Performance - 35 GeV, c \tau = 100 m, Muon Signal

Cuts / Efficiency (3742 Fiducial LLP Decays)	Full Eff.	Full Eff. Fraction	1e-3	1e-3 Fraction	1e-2	1e-2 Fraction
1. Tracks >= 2	1370	0.366	1342	0.359	1347	0.36
2. Vertices >= 1	938	0.251	919	0.246	903	0.241
3. **Vertex in Decay Volume**	678	0.181	679	0.181	657	0.176
4. Non-Fiducial Corner Cut	583	0.156	578	0.154	567	0.152
5. Veto Events w/ Tracks that include Floor/Wall Hits	554	0.148	545	0.146	526	0.141
6. Veto Tracks w/ nearby unassociated Floor/Wall hits	545	0.146	538	0.144	519	0.139
7. **Crinkle Cut**	313	0.0836	506	0.135	492	0.131
8. High Multiplicity Consistency	404	0.108	394	0.105	375	0.1
9. 2 Track Angular Consistency	392	0.105	374	0.0999	359	0.0959

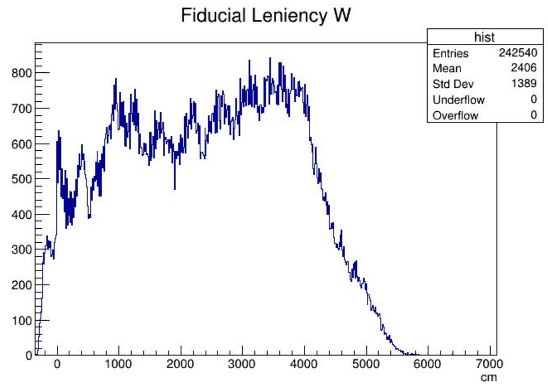
4. Non-Fiducial Corner Cut Details

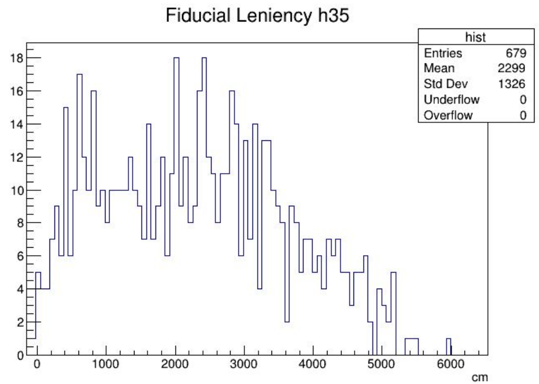
- **SAME AS PRIMARY, repeating below for convenience**
- This vetoes vertices that have a good chance of avoiding the wall veto even if they come from a muon
- Ignore vertices that could have passed above the wall
- Extend the non-fiducial region into detector volume (by an amount *d*) to accommodate imperfect tracking resolution
- *d* is the cut parameter
  - *d* = 7.5 m



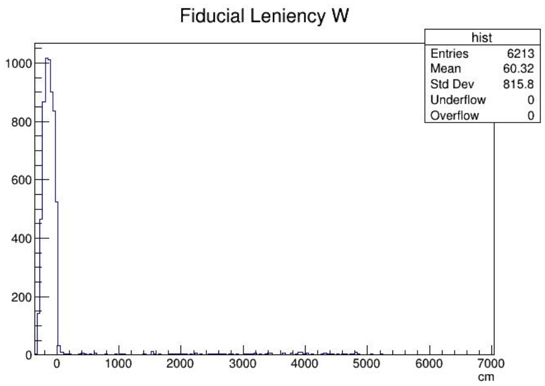
Secondary Goal - 4. Non-Fiducial Corner Cut Distributions (1e-3 Inefficiency)

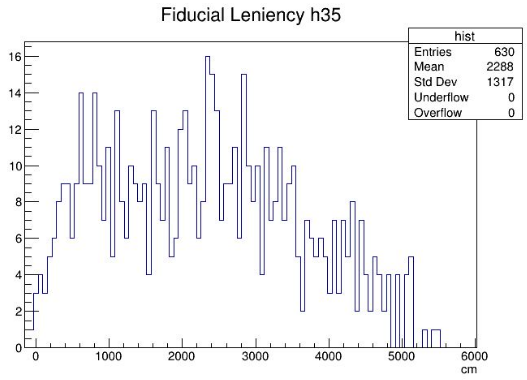
Before floor cuts





After floor cuts



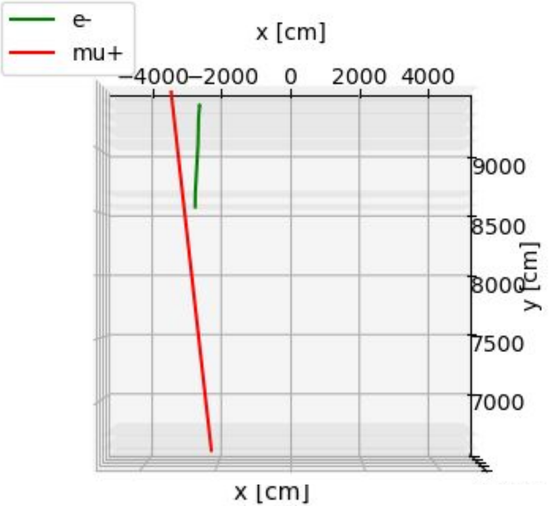


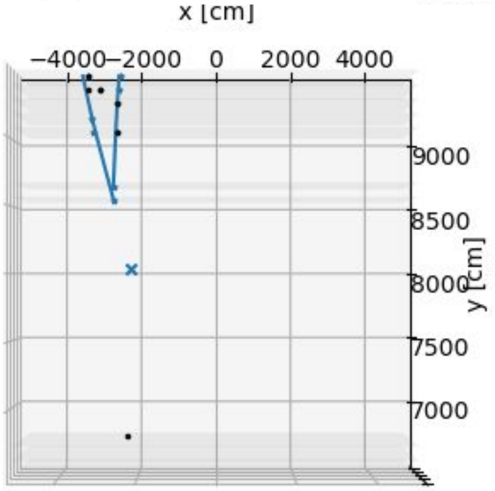
5. Veto Events w/ Upward Tracks that include Floor/Wall Hits

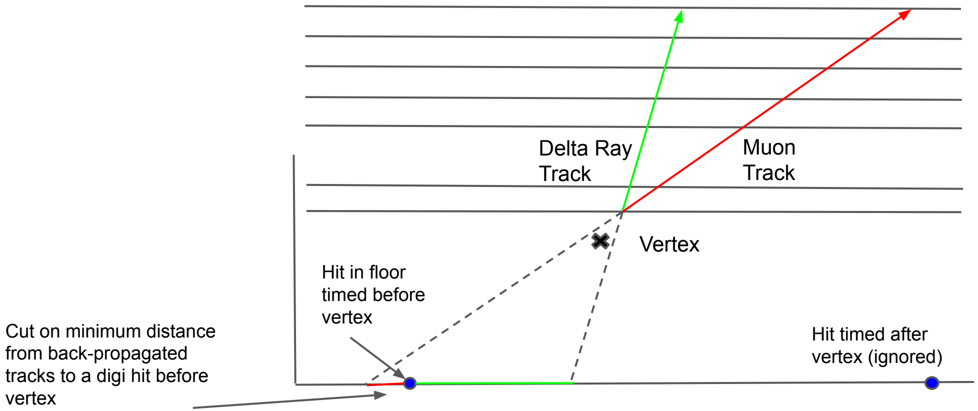
- *Unlike in the primary analysis, the Num_tracks and upwards requirements are irrelevant for this veto.*
- **This cut reduces BG by several orders of magnitude depending on hit inefficiency, with close-to-1 signal efficiency**

6. Veto Tracks w/ nearby unassociated Floor/Wall hits

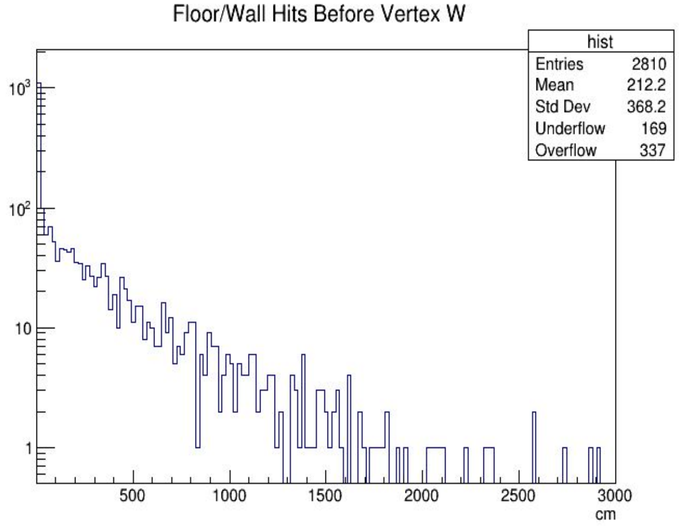
- **SAME AS PRIMARY, repeating below for convenience**
- Propagate vertex tracks backwards into the plane of the floor and wall
  - Compute the distance (in the plane) to all hits before the vertex **in time** from each back propagate position
- Cut on the minimum of these distances
  - Events where this parameter is less than 20 m are cut
    - This value is larger than one might expect primarily from hit selection inaccuracies
- **This cut removes a lot of BG but almost no signal, surprisingly good.**
- *Simulation has no detector noise right now, so this may have to be retuned with noise*

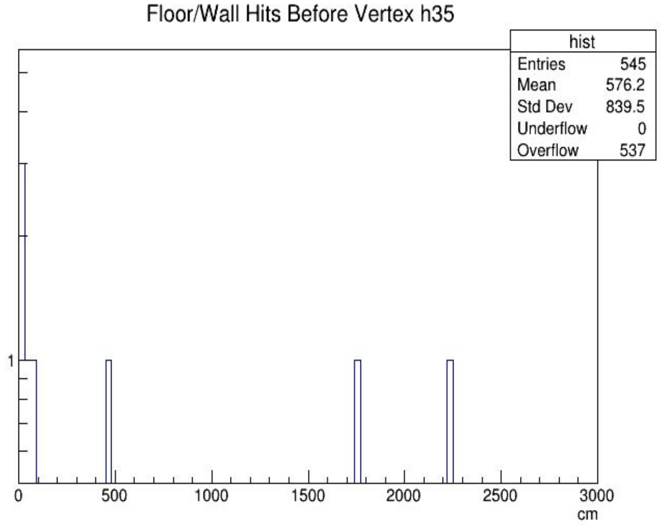






Secondary Goal - 6. Veto Tracks w/ nearby unassociated Floor/Wall hits Distributions (1e-3 Inefficiency)

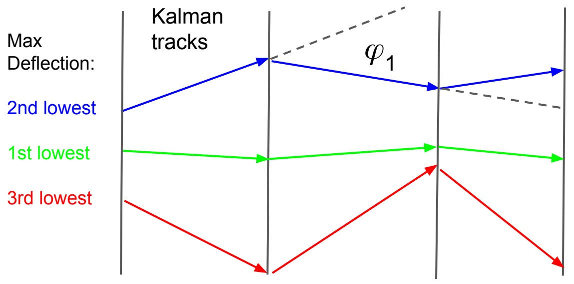




7. Crinkle Cut Details: VETO BAD DELTA RAY TRACKS

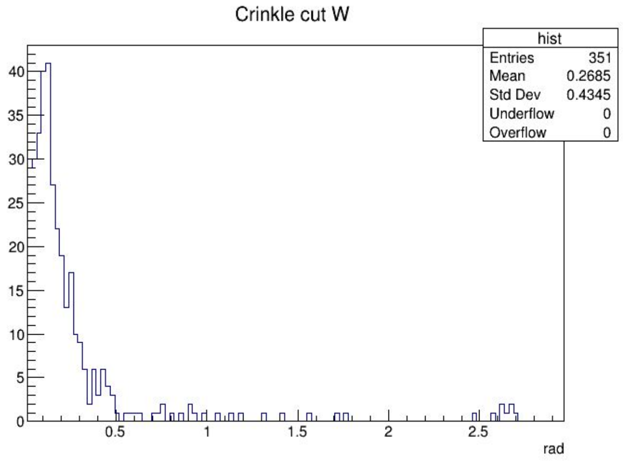
- **SAME AS PRIMARY, repeating below for convenience**
- Assume that most of the ***second*** tracks in W BG are Delta Rays, or other low energy particle (first track is will be a muon for both Leptonic Signal and BG).
  - ***Low energy => more deflection traversing scintillator layers***
  - One would like to tag and reject these events
- Psuedo-code:
  - Collect ***maximum*** deflection in the track for each track in a vertex
  - Collect the value for the ***second lowest*** track deflection
  - Cut on the ***lowest*** of these values across vertices in the event
    - We cut **0.1 rad**
    - **In other words, a vertex must have at least two tracks with max deflection angle < 0.1 rad**

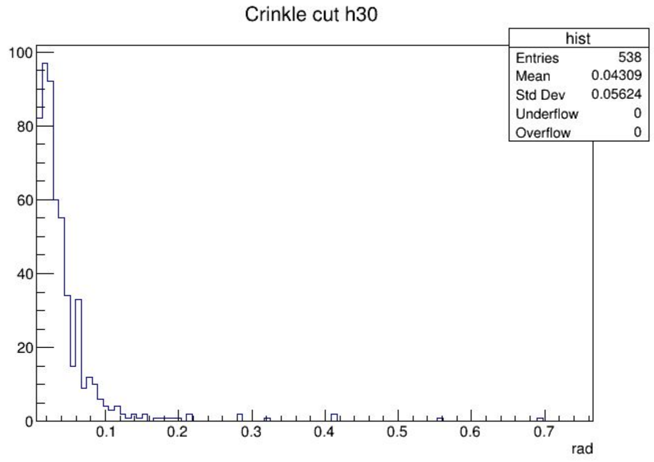
Crinkle Cut



Cut on φ_1

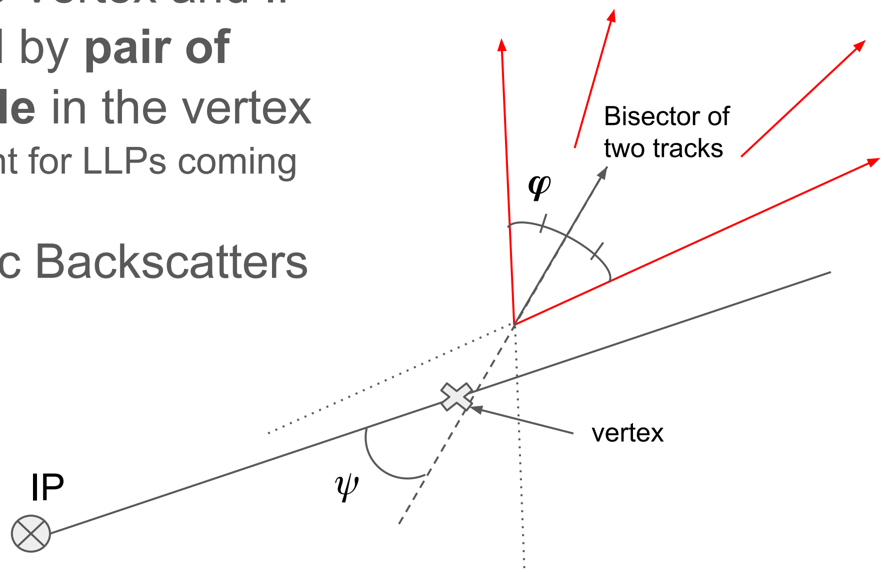
Secondary Goal - 7. Crinkle Cut (1e-3 Hit Inefficiency)



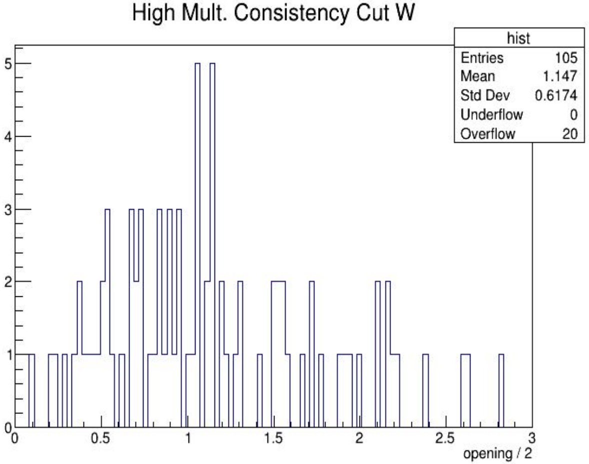


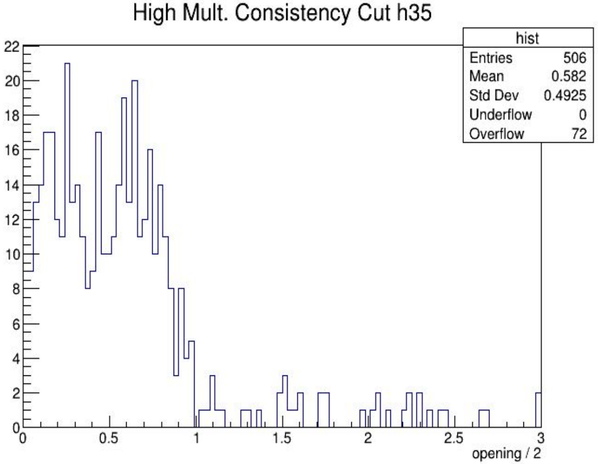
8. High Multiplicity Angular Consistency

- Cut events where the line between the Vertex and IP is not contained in the cone generated by **pair of tracks with the largest opening angle** in the vertex
    - This is a momentum conservation constraint for LLPs coming from the IP
- This is a very strong handle for Cosmic Backscatters into MATHUSLA
- Cut the event if $\psi / \boldsymbol{\varphi} > 1$



Secondary - 8. High Multiplicity Consistency Cut



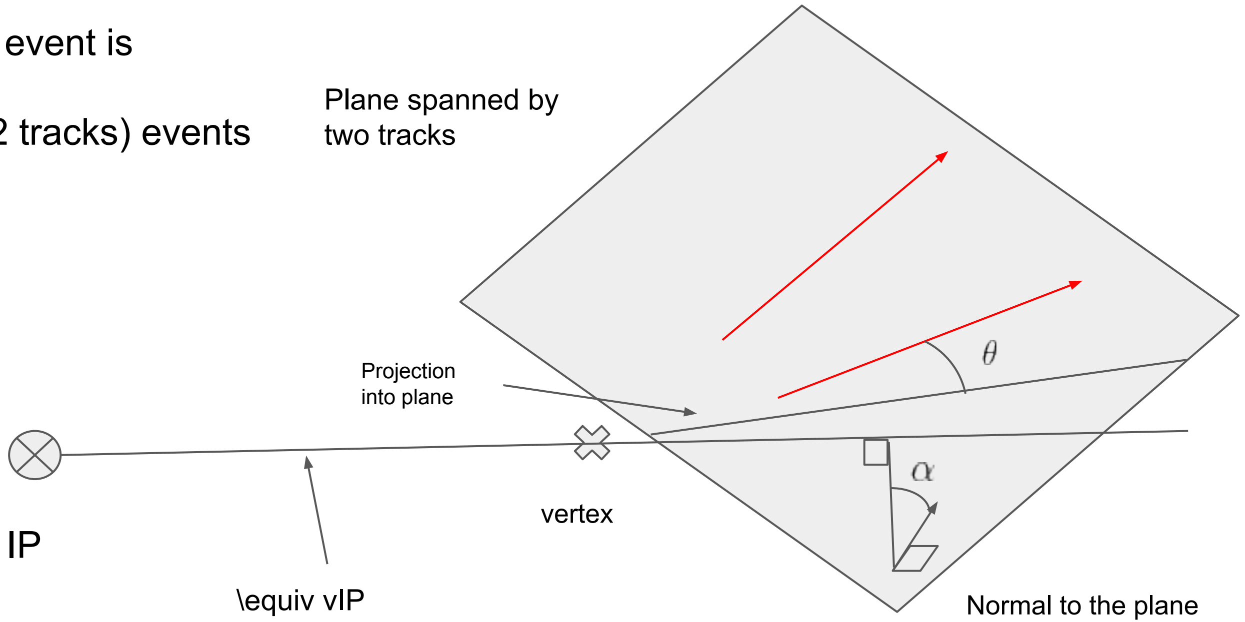


9. 2 Track Angular Consistency Cuts

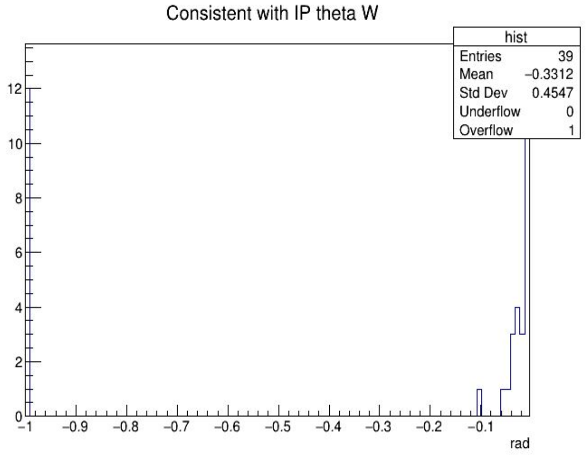
- Assume that all the energy in the event is accounted for by the two tracks
- Restrict to 1 vertex (with exactly 2 tracks) events
- Cut the event if
  - Theta > -0.05

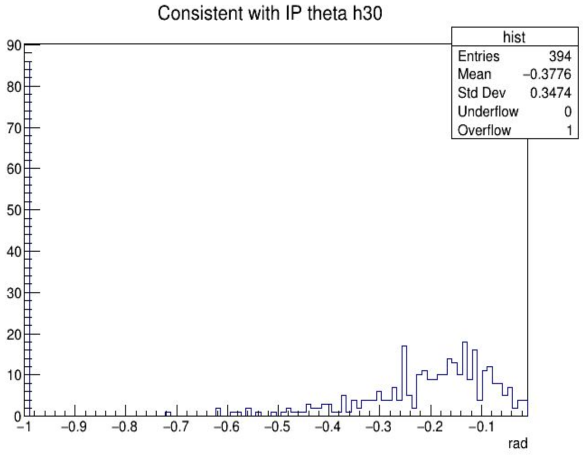
Theta pseudo-code:

- Compute cross product of track velocities
- Project out component of vIP parallel to cross product
- Theta is minimum angle between a track and this projection (negative if between them)



Secondary Goal - 9. 2 Track Angular Consistency Cut Distributions (1e-3 Inefficiency)





Secondary Physics Goal - Results

- To have good signal acceptance for low mass mumu signal
  - Hermetic coverage of the decay volume is essential for eliminating W background during Secondary Goal analysis
  - **We may have to realistically expect O(10-100) background events for this type of analysis.**
  - **Vertex reconstruction is pretty good after cuts: ‘geometric only’ efficiency is ~15-20%, and we get 10% signal efficiency.**

Sample / Tracks	Full Eff. Survivors	Full Eff. Fraction	1e-3 Survivors	1e-3 Fraction	1e-2 Survivors	1e-2 Fraction
W Background (Survivors include additional x2 to get numbers at HL-LHC)	10	1.22e-7	**30**	3.48e-7	460	7.68e-6
35 GeV, c \tau = 100 m (denominator = 3742 fiducial LLP decays)	392	0.105	374	**0.0999**	359	0.0959

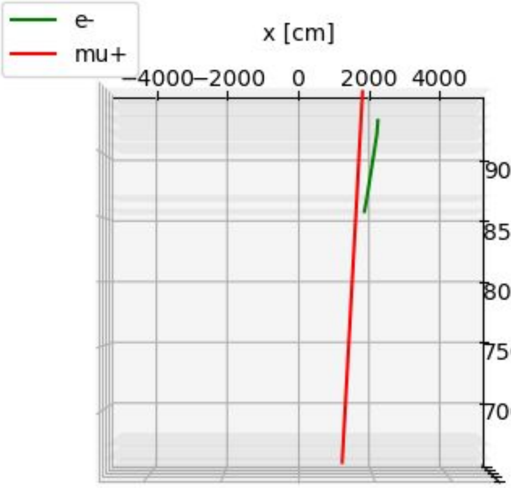
3. TERTIARY PHYSICS GOAL ANALYSIS

LLPs decaying to two tracks, boost > 10

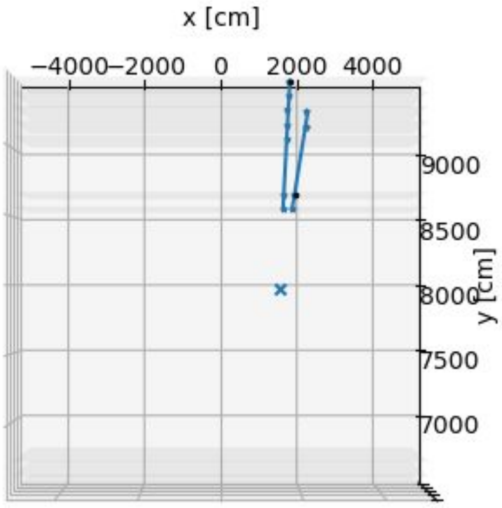
Produced in h->aa, a->ll, ma = 2, 10 GeV (“h2”, “h10”)
c \tau = 50 m

Tertiary Physics Goal - Relevant Backgrounds

- W's are main background. Same normalization as in Primary and Secondary analysis.
- Distinguishing from tertiary signal is difficult due to small opening angle and low multiplicity of LLP decay tracks, which the BG can mimic via Delta Ray production.

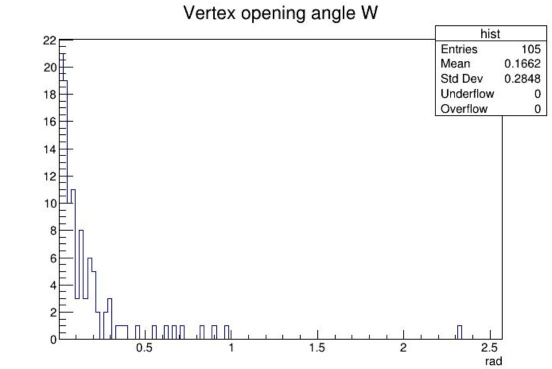


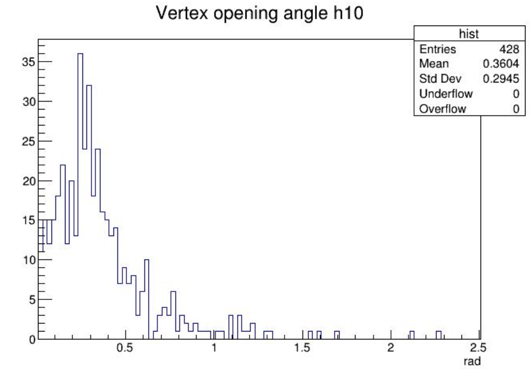
Missed hits can make W background look like signal

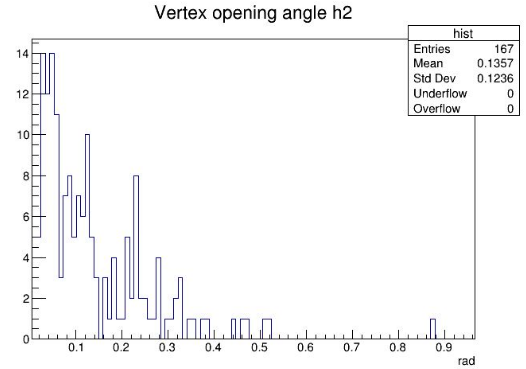


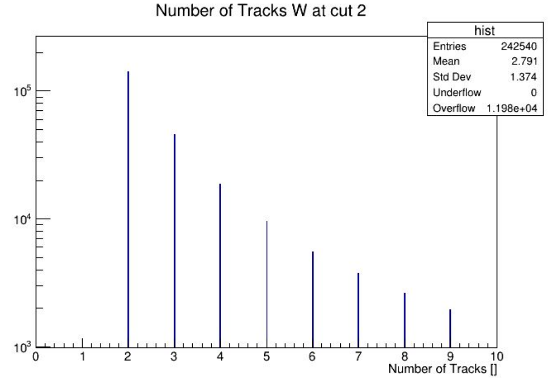
Additional Difficulties with Tertiary Goal Background Rejection

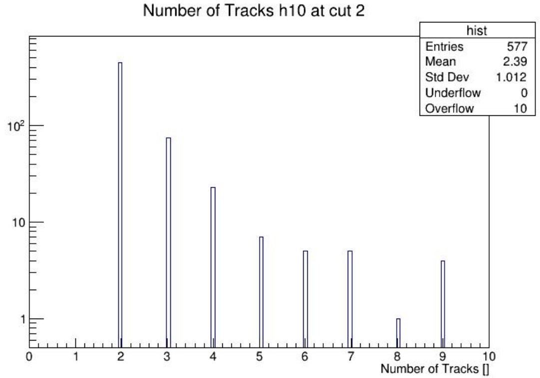
- Many handles for the primary physics goal are unavailable here or begin to cut into signal (track multiplicity, BG and signal have similar opening angle distributions, etc.)

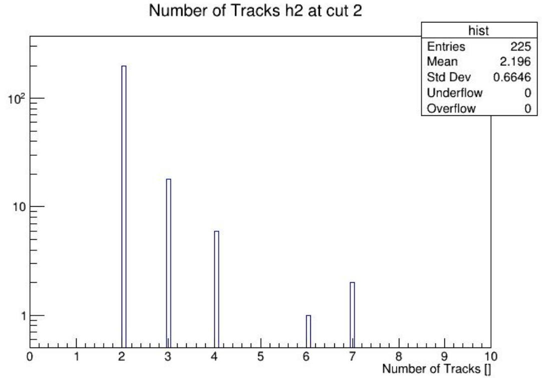












Analysis Performance - W Background

SAME AS SECONDARY, repeating for convenience

Cuts / Efficiency (Triggered Events) (Simulated Events)	Full Eff. (41,104,047) (4.3e9)	Full Eff. Fraction	1e-3 (43,143,321) (4.6e9)	1e-3 Fraction	1e-2 (29,941,414) (3.2e9)	1e-2 Fraction
1. Tracks >= 2	909277	0.0221	1082310	0.0251	577489	0.0193
2. Vertices >= 1	571990	0.0139	656046	0.0152	363532	0.0121
3. **Vertex in Decay Volume**	186335	0.00453	242540	0.00562	120018	0.00401
4. Non-Fiducial Corner Cut	159169	0.00387	207089	0.00480	102628	0.00343
5. Veto Events w/ Tracks that include Floor/Wall Hits	2331	5.67e-5	2810	6.51e-5	3166	1.06e-4
6. Veto Tracks w/ nearby unassociated Floor/Wall hits	36	8.76e-7	351	8.14e-6	1743	5.82e-5
7. **Crinkle Cut**	30	7.3e-7	105	2.43e-6	1474	4.92e-5
8. High Multiplicity Angular Consistency	5	1.22e-7	39	9.04e-7	648	2.16e-5
9. 2 Track Angular Consistency	5	1.22e-7	15	3.48e-7	230	7.68e-6

Analysis Performance - 10 GeV, c \tau = 50 m, Muon Signal

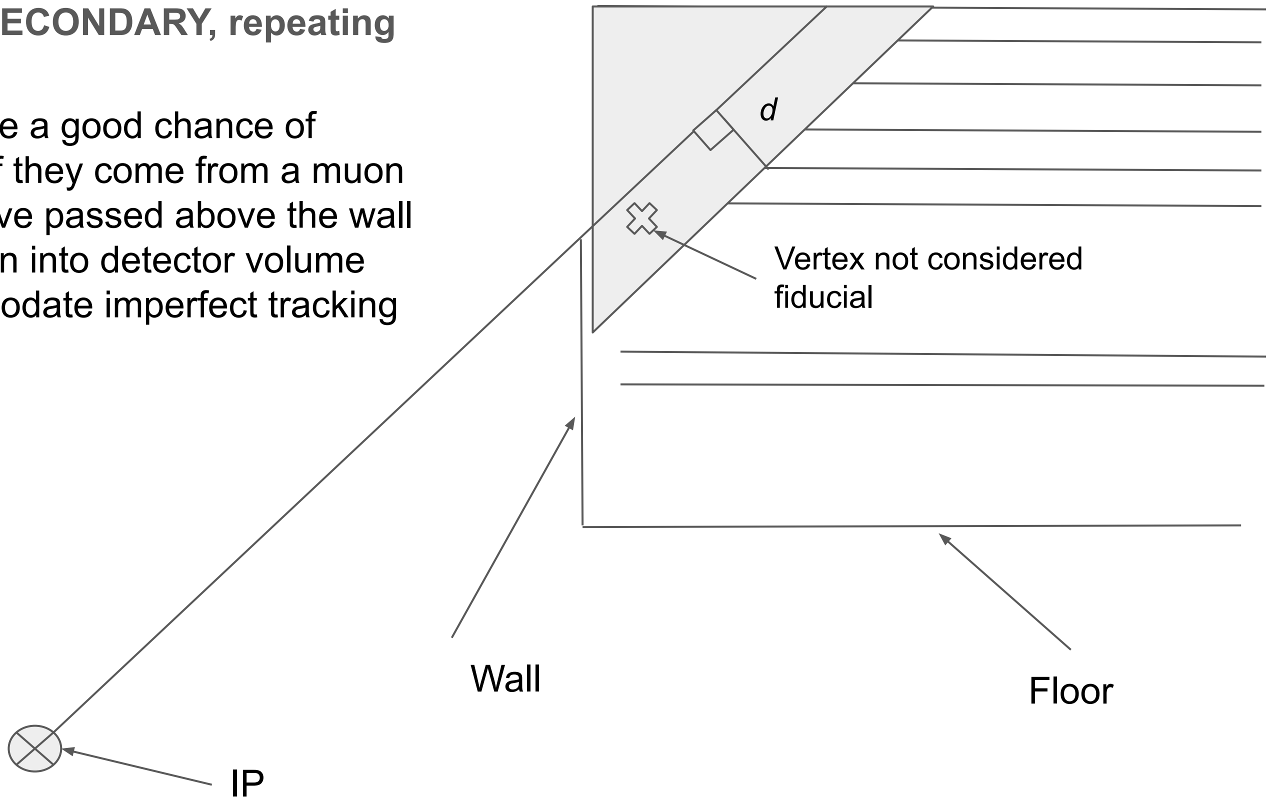
Cuts / Efficiency (2582 Fiducial LLP Decays)	Full Eff.	Full Eff. Fraction	1e-3	1e-3 Fraction	1e-2	1e-2 Fraction
1. Tracks >= 2	1285	0.498	1302	0.504	1294	0.501
2. Vertices >= 1	839	0.325	837	0.324	860	0.333
3. **Vertex in Decay Volume**	587	0.227	577	0.223	609	0.236
4. Non-Fiducial Corner Cut	508	0.197	498	0.193	531	0.206
5. Veto Events w/ Tracks that include Floor/Wall Hits	479	0.186	468	0.181	506	0.196
6. Veto Tracks w/ nearby unassociated Floor/Wall hits	474	0.184	464	0.180	498	0.193
7. **Crinkle Cut**	437	0.169	428	0.166	461	0.179
8. High Multiplicity Consistency	364	0.141	348	0.135	383	0.148
9. 2 Track Angular Consistency	276	0.107	260	0.101	281	0.109

Analysis Performance - 2 GeV, c \tau = 50 m, Muon Signal

Cuts / Efficiency (704 Fiducial LLP Decays)	Full Eff.	Full Eff. Fraction	1e-3	1e-3 Fraction	1e-2	1e-2 Fraction
1. Tracks >= 2	409	0.581	417	0.592	418	0.594
2. Vertices >= 1	303	0.43	324	0.460	315	0.447
3. **Vertex in Decay Volume**	226	0.321	225	0.320	246	0.349
4. Non-Fiducial Corner Cut	203	0.288	202	0.287	223	0.317
5. Veto Events w/ Tracks that include Floor/Wall Hits	185	0.263	190	0.270	201	0.286
6. Veto Tracks w/ nearby unassociated Floor/Wall hits	185	0.263	190	0.270	201	0.286
7. **Crinkle Cut**	170	0.241	167	0.237	185	0.263
8. High Multiplicity Consistency	116	0.165	122	0.173	141	0.2
9. 2 Track Angular Consistency	29	0.0412	35	0.0497	52	0.0739

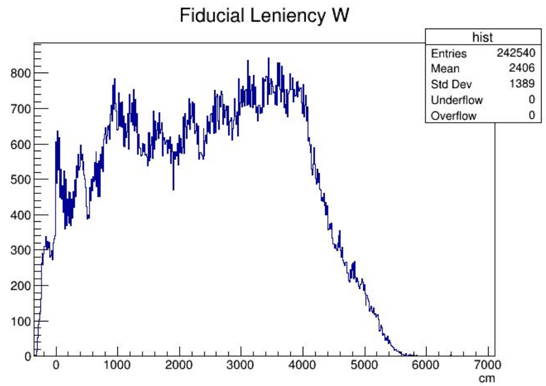
4. Non-Fiducial Corner Cut Details

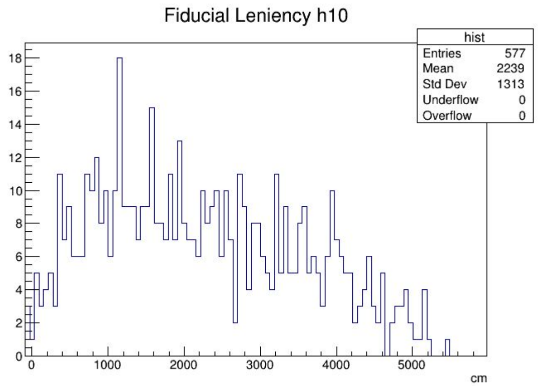
- **SAME AS PRIMARY AND SECONDARY, repeating below for convenience**
- This vetoes vertices that have a good chance of avoiding the wall veto even if they come from a muon
- Ignore vertices that could have passed above the wall
- Extend the non-fiducial region into detector volume (by an amount *d*) to accommodate imperfect tracking resolution
- *d* is the cut parameter
  - *d* = 7.5 m

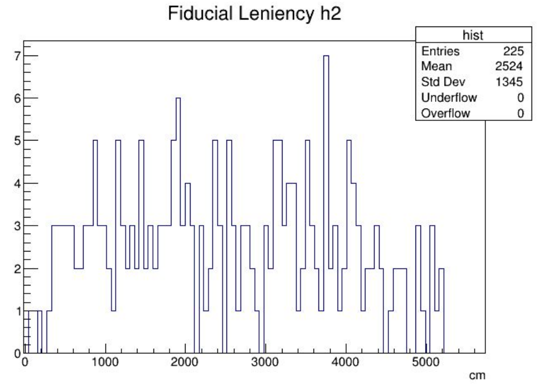


Tertiary Goal - 4. Non-Fiducial Corner Cut Distributions (1e-3 Inefficiency)

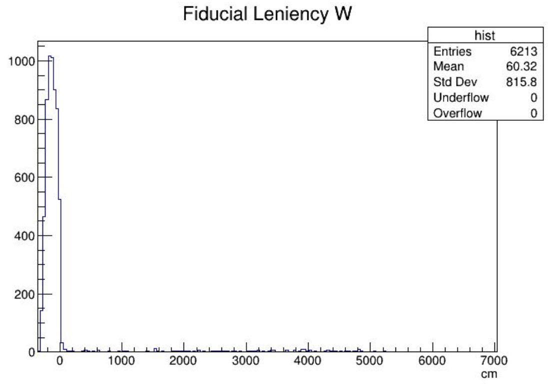
Before floor cuts

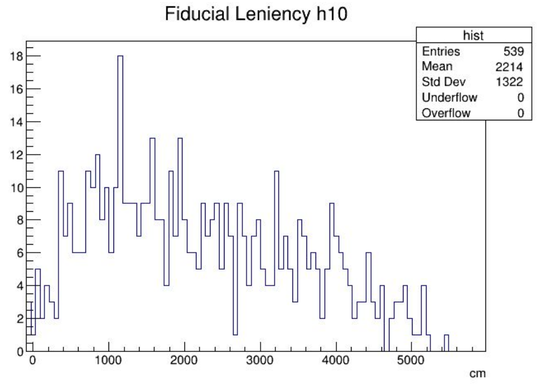


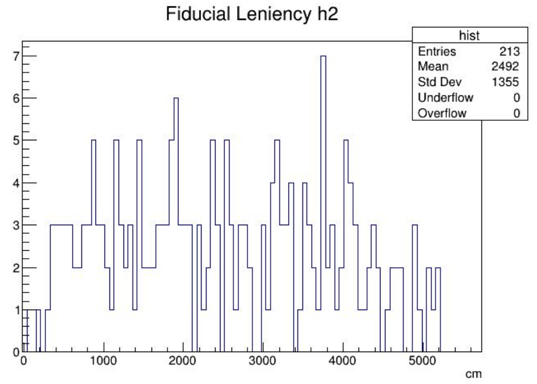




After floor cuts







5. Veto Events w/ Upward Tracks that include Floor/Wall Hits

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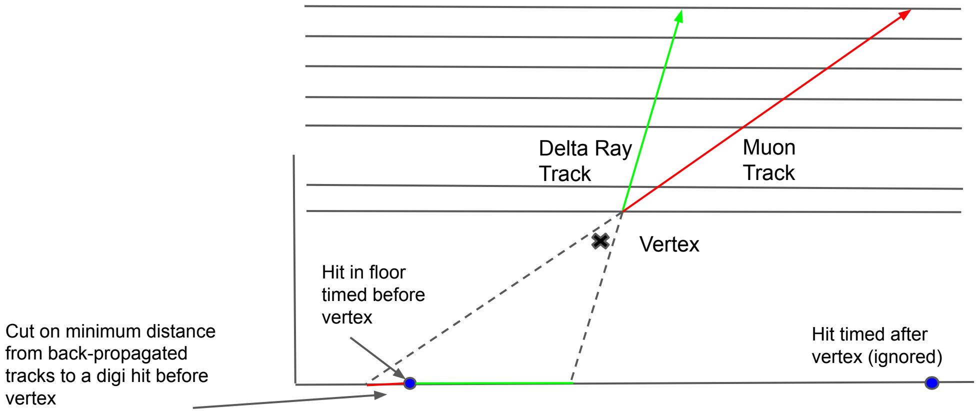
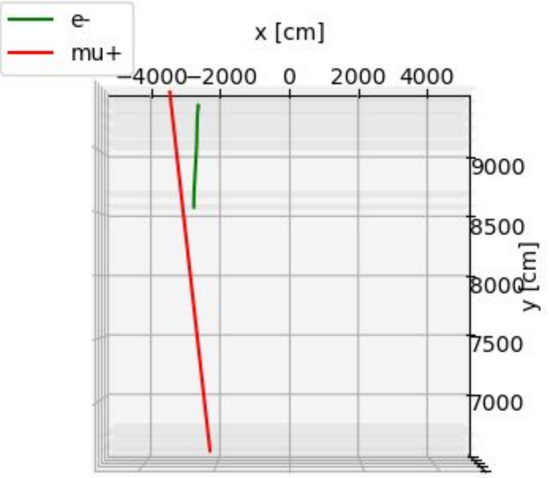
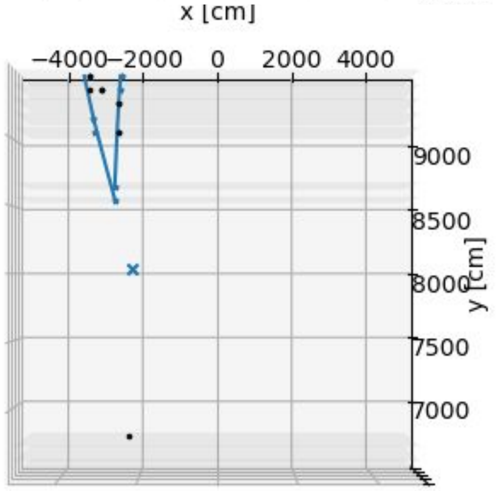
- *Unlike in the primary analysis, the Num_tracks and upwards requirements are irrelevant for this veto.*
- **This cut reduces BG by several orders of magnitude depending on hit inefficiency, with close-to-1 signal efficiency**

6. Veto Tracks w/ nearby unassociated Floor/Wall hits

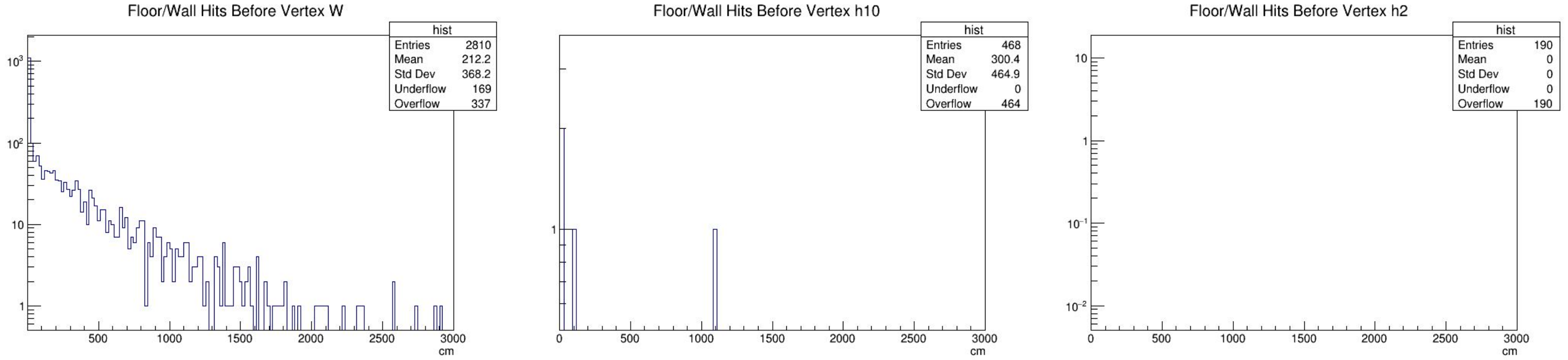
- **SAME AS PRIMARY AND SECONDARY, repeating below for convenience**
- Propagate vertex tracks backwards into the plane of the floor and wall
  - Compute the distance (in the plane) to all hits before the vertex **in time** from each back propagated position
- Cut on the minimum of these distances
  - Events where this parameter is less than 20 m are cut
    - This value is larger than one might expect primarily from hit selection inaccuracies
- **This cut removes a lot of BG but almost no signal, surprisingly good.**
- *Simulation has no detector noise right now, so this may have to be retuned with noise*







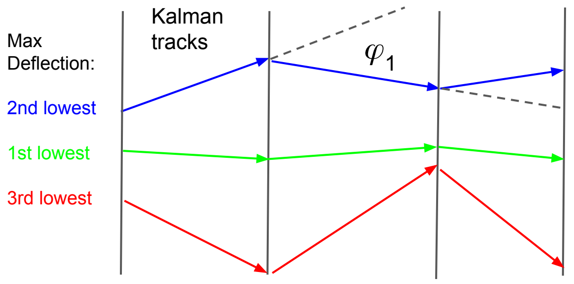
Tertiary Goal - 6. Veto Tracks w/ nearby unassociated Floor/Wall hits Distributions (1e-3 Inefficiency)



7. Crinkle Cut Details: VETO BAD DELTA RAY TRACKS

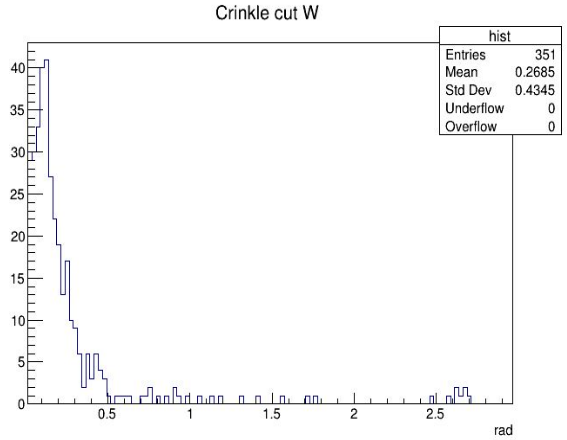
- **SAME AS PRIMARY AND SECONDARY, repeating below for convenience**
- Assume that most of the ***second*** tracks in W BG are Delta Rays, or other low energy particle (first track is will be a muon for both Leptonic Signal and BG).
    - ***Low energy => more deflection traversing scintillator layers***
    - One would like to tag and reject these events
- Psuedo-code:
    - Collect ***maximum*** deflection in the track for each track in a vertex
    - Collect the value for the ***second lowest*** track deflection
    - Cut on the ***lowest*** of these values across vertices in the event
        - We cut **0.1 rad**
        - **In other words, a vertex must have at least two tracks with max deflection angle < 0.1 rad**

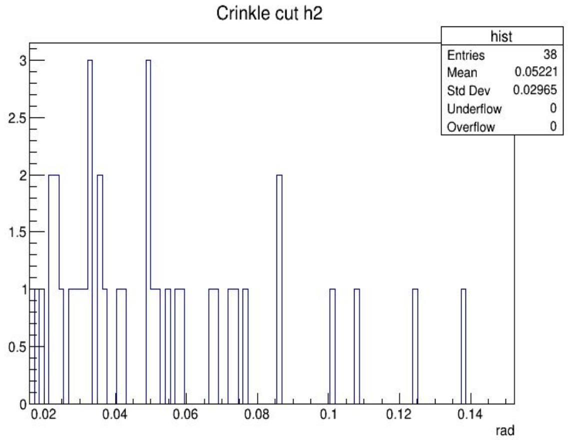
Crinkle Cut

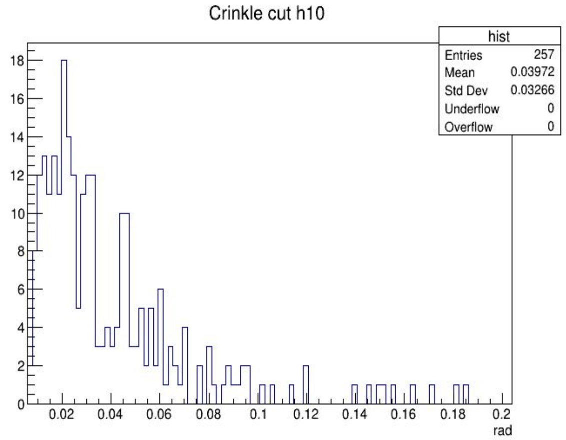


Cut on φ_1

Tertiary Goal - 7. Crinkle Cut (1e-3 Hit Inefficiency)

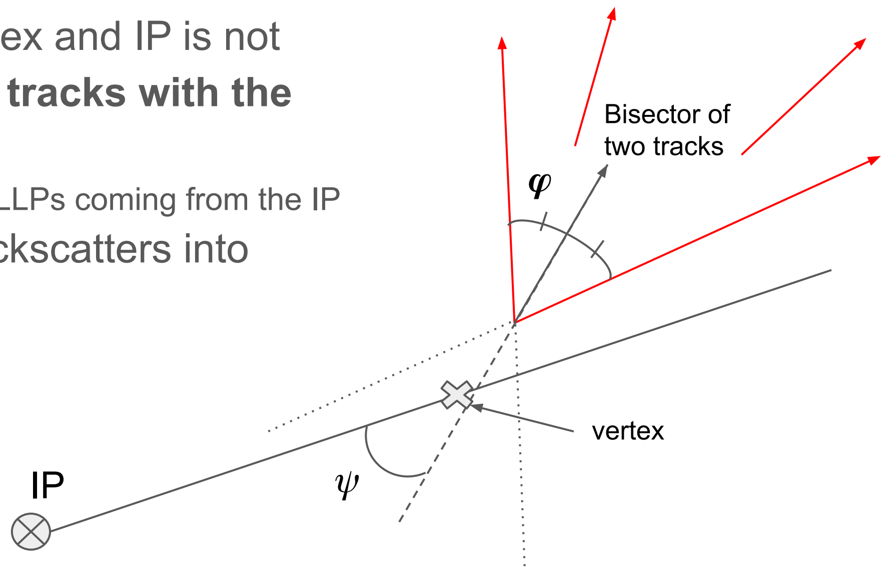




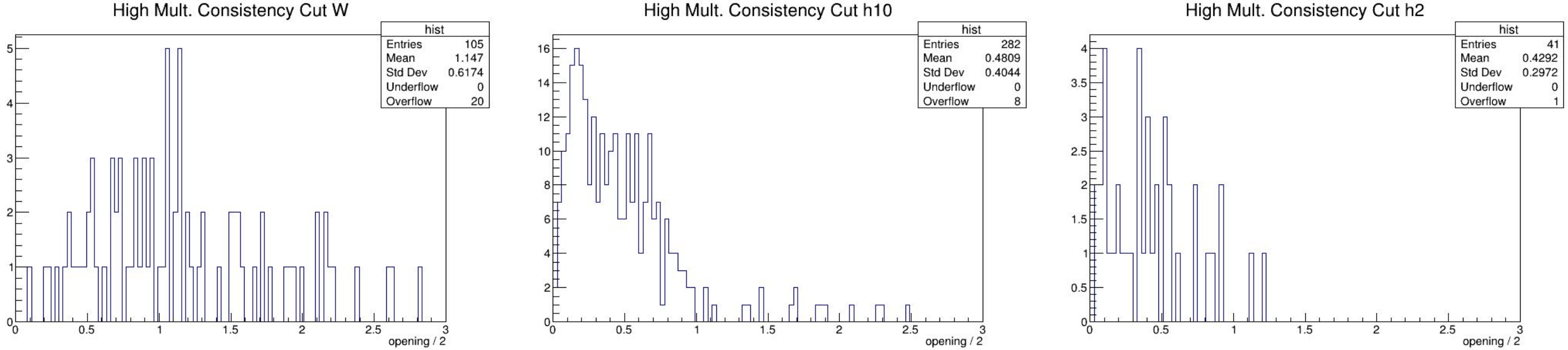


8. High Multiplicity Angular Consistency

- **SAME AS SECONDARY, repeating below for convenience**
- Cut events where the line between the Vertex and IP is not contained in the cone generated by **pair of tracks with the largest opening angle** in the vertex
  - This is a momentum conservation constraint for LLPs coming from the IP
- This is a very strong handle for Cosmic Backscatters into MATHUSLA
- Cut the event if ψ / φ > 1



Tertiary - 8. High Multiplicity Consistency Cut

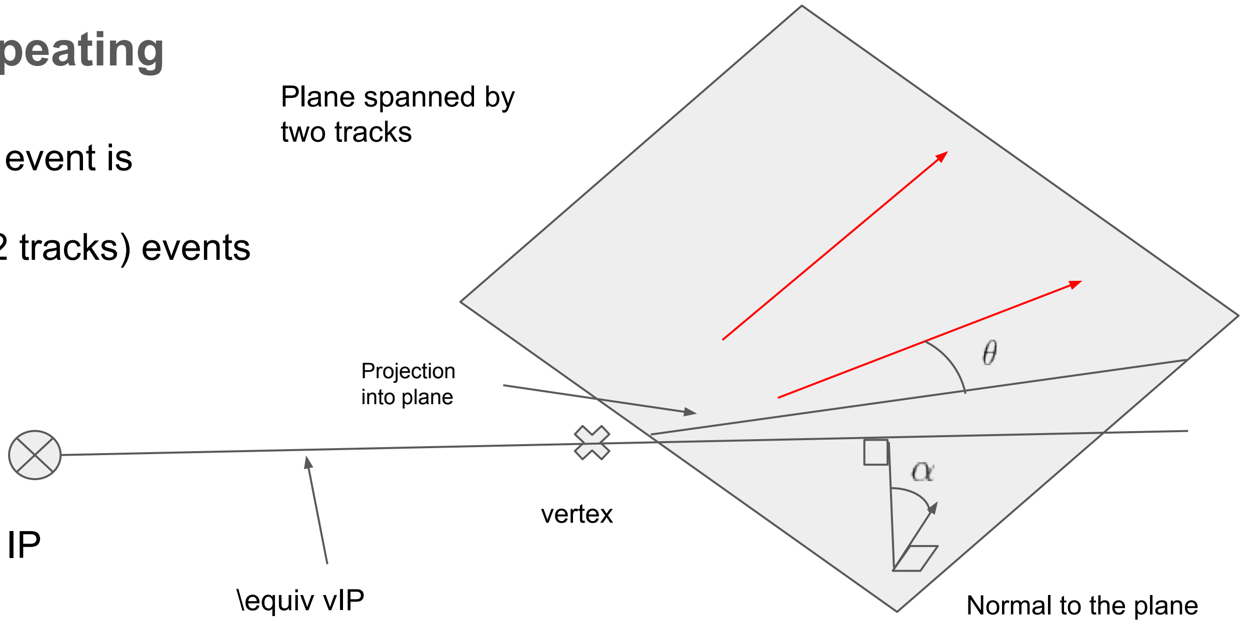


9. Angular Consistency Cuts

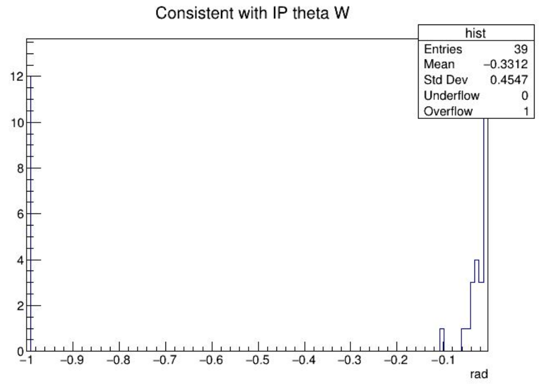
- **SAME AS Secondary, repeating below for convenience**
- Assume that all the energy in the event is accounted for by the two tracks
- Restrict to 1 vertex (with exactly 2 tracks) events
- Cut the event if
  - Theta > -0.05

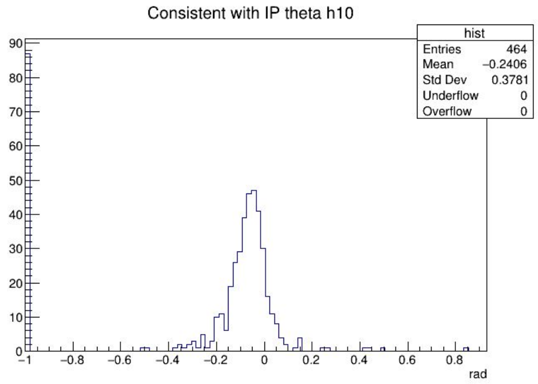
Theta pseudo-code:

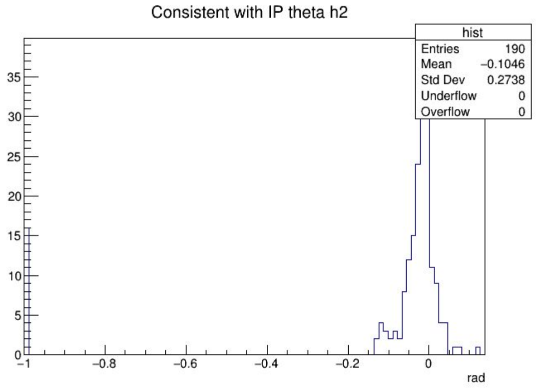
- Compute cross product of track velocities
- Project out component of vIP parallel to cross product
- Theta is minimum angle between a track and this projection (negative if between them)



Tertiary Goal - 8. Angular Consistency Cut Distributions (1e-3 Inefficiency)





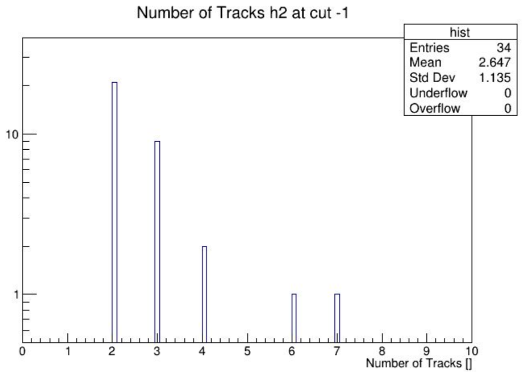


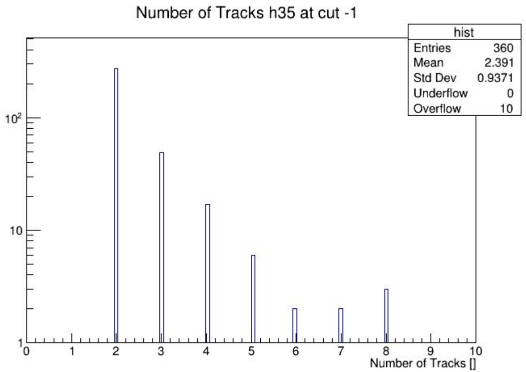
Tertiary Physics Goal - Results

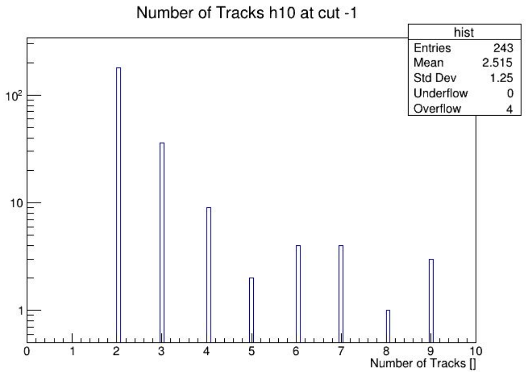
- To have good signal acceptance for low mass mumu signal
  - Hermetic coverage of the decay volume is essential for eliminating W background during Tertiary Goal analysis
  - We may have to realistically expect O(10-100) background events for this type of analysis.
  - Surprising: vertex efficiency is ~10%, similar to secondary physics goal despite higher boost.
  - 2 GeV LLP from higgs decays is probably at lower mass limit for our track resolution

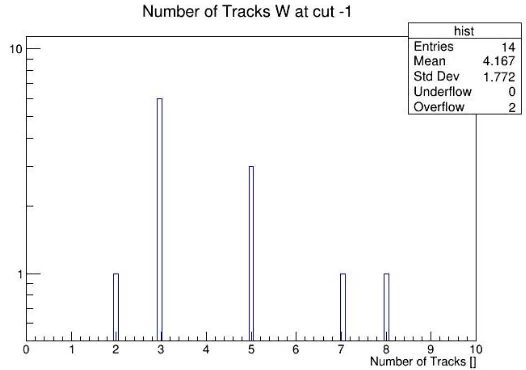
Sample / Tracks	Full Eff. Survivors	Full Eff. Fraction	1e-3 Survivors	1e-3 Fraction	1e-2 Survivors	1e-2 Fraction
W Background (Survivors include additional x2 to get numbers at HL-LHC)	10	1.22e-7	**30**	3.48e-7	460	7.68e-6
10 GeV, c \tau = 50 m (denominator = 2582 fiducial LLP decays)	276	0.107	260	**0.101**	281	0.109
2 GeV, c \tau = 50 m (denominator = 704 fiducial LLP decays)	29	0.0412	35	**0.0497**	52	0.0739

Track Multiplicity at End of Flows (1e-3 Hit Inefficiency)





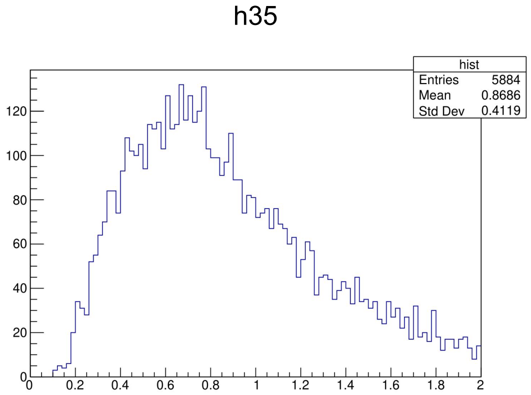


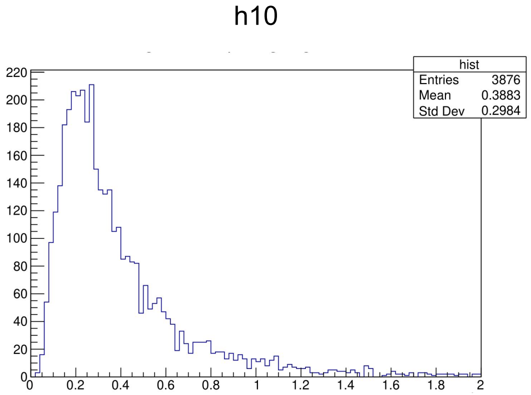


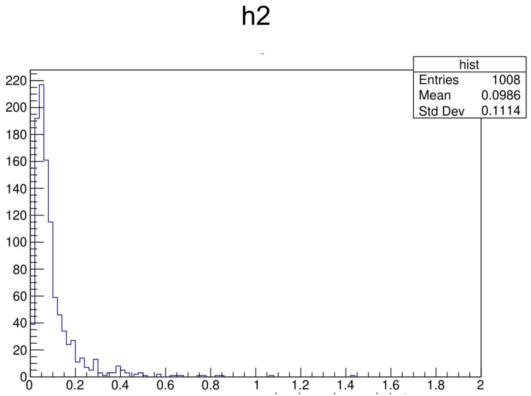
Cutting at < 7 tracks can be a way to factor the analysis into
Primary goal vs Secondary and Tertiary goal

BACKUP SLIDES

Backup - Truth Level Vertex Opening Angle for Leptonic LLP Signal

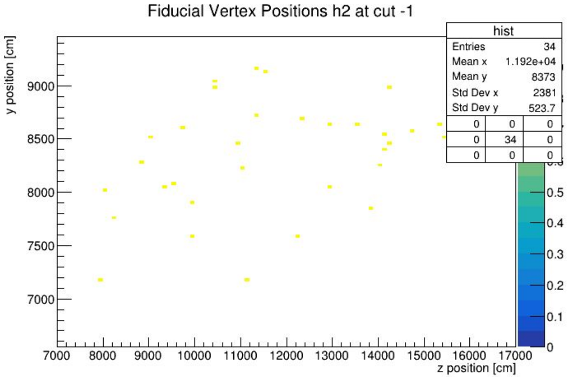


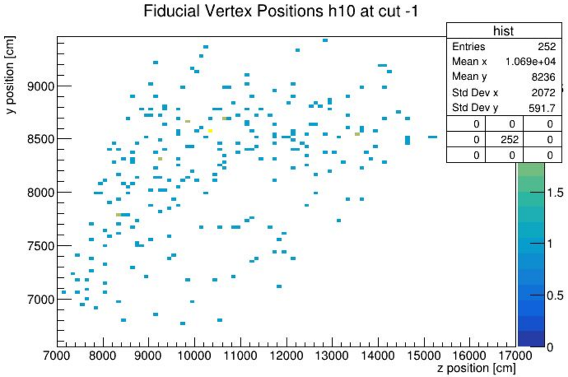


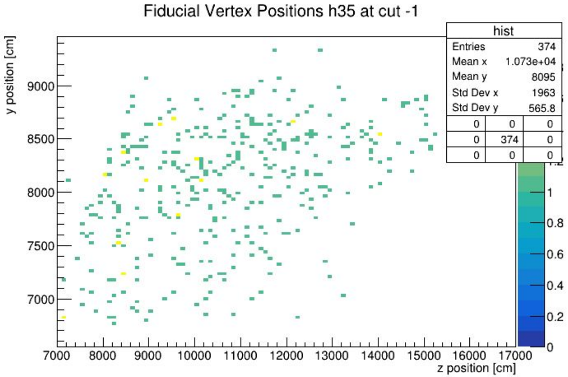


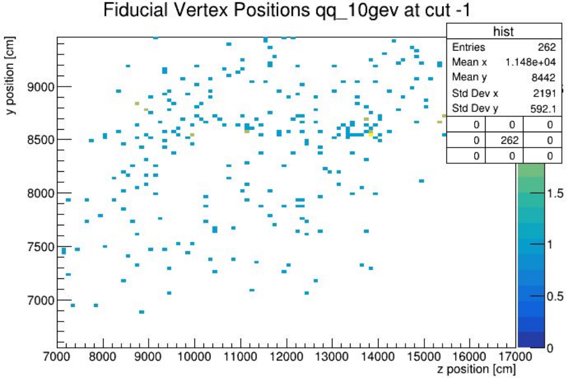
Opening angle (all in rad) between muon decay products of LLPs

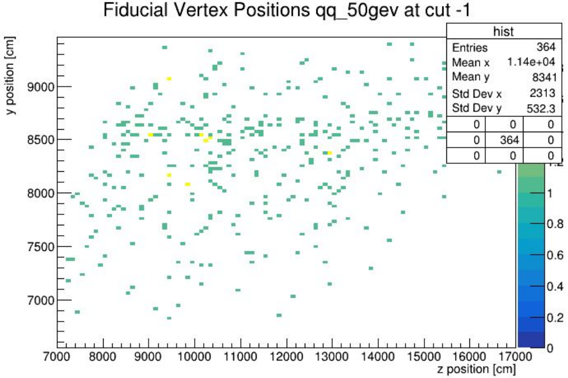
Backup - Surviving (after all cuts) Signal Vertex Positions (1e-3 hit inefficiency)











Backup - What is a Kalman Filter?

A Kalman Filter is a linear, recursive, flexible fitting algorithm that provides the optimal fit given gaussian uncertainties. Following [1] we can make a filter with these steps.

- Describe the Measured Data
    - Choose a Measurement Matrix H_k (for us this projects out the velocity in the filtered state vector)

Measurement of a hit $\longleftarrow m_k = H_k \; x_k \longrightarrow$ Filtered State

$[x_m, t_m, z_m]$ $\quad x_{\text{state}} = [x, t, z, v_x, v_y, v_z]$

k indexes the detector layers with chosen hits

- Predict
    - Choose a Prediction Matrix F_k (for us this propagates the state to the next layer)
    - Predict the next state vector and covariance based on the current state vector and covariance
        - Use this prediction to choose which data to add to the fit (for us these are the digitized hits m_k)

Predicted State $\longleftarrow x_k^{k-1} = F_k \; x_{k-1} \longrightarrow$ Filtered State

$$C_k^{k-1} = F_{k-1} C_{k-1} F_{k-1}^T + Q_{k-1}$$

Predicted Covariance

Filtered Covariance

Process Noise Covariance

This is where we include scattering!

Backup - What is a Kalman Filter? - Filtering and Smoothing

- Filter
  - Update the predicted state to include the newly chosen data in the fit

Filtered Covariance

$$C_k = [I - C_k^{k-1} H_k^T (V_k + H_k C_k^{k-1} H_k^T)^{-1} H_k] C_k^{k-1}$$

Filtered State

$$x_k = x_k^{k-1} + C_k H_k V_k^{-1} (m_k - H_k x_k^{k-1})$$

Covariance of m_k

- Smooth
  - After ALL data has been chosen and filtered, propagate later states information to earlier ones

Smoothed State

$$x_k^n = x_k + A_k (x_{k+1}^n - x_{k+1}^k)$$

Smoothed Covariance

$$C_k^n = C_k + A_k (C_{k+1}^n - C_{k+1}^k) A_k^T$$

Smoother Gain Matrix

$$A_k = C_k F_k^T (C_{k+1}^k)^{-1}$$

Where n is the number of measurements

Backup - Computing χ^2s

To compute the contribution to the χ^2 for a chosen hit $(\chi^2_p,\ \chi^2_f,\ \chi^2_s)$, generically called χ^2_+, we use the following formulas. These assume there are no correlations between states on successive layers.

χ^2_+ calculated at prediction

$$\chi^2_p = (r_k^{k-1})^T (V_k + H_k C_k^{k-1} H_k^T)^{-1} r_k^{k-1}$$

$$r_k^{k-1} = (m_k - H_k x_k^{k-1})$$

residual

$$\chi^2_f = \chi^2_p$$

χ^2_+ calculated at filter

χ^2_s, the χ^2 increment calculated at each smoothing step, carries over with $k-1 \rightarrow n$.

The total χ^2 used to calculate the χ^2/ndof and pass or veto the track is the sum over all χ^2_s

Backup - Multiple Scattering - Covariance Matrix Computation

To calculate Q (following [3]) we parameterize the scattering by two orthogonal uncorrelated angles θ_1 and θ_2, and fix $\beta = 1$

$$\sigma(\theta_{\text{proj}}) = \frac{13.6}{p}\sqrt{\frac{L_{rad}}{\sin\phi}}\left[1 + 0.038\ln\left(\frac{L_{rad}}{\sin\phi}\right)\right] \qquad L_{rad} \equiv \sum_i \frac{X_i}{X_{0,i}}$$

We choose $p = 500\text{MeV}$ for high acceptance

ϕ is the inclination angle of the track

$$= \sqrt{Var(\theta_1)} = \sqrt{Var(\theta_2)}$$

For two covariance matrices, whose variables are related by the functions f_i, we can approximate as

$$Q_{ij}(\hat{y}) \approx \left[\frac{\partial f_i}{\partial y_n}\frac{\partial f_j}{\partial y_m}V_{nm}\right]_{\hat{y}}$$

Letting V be the covariance matrix for the scattering angles and $P_i \equiv (x_k^{k-1})_i$, we find

$$Q_{ij} = \langle P_i, P_j \rangle = \sigma^2(\theta_{\text{proj}})\left(\frac{\partial P_i}{\partial\theta_1}\frac{\partial P_j}{\partial\theta_1} + \frac{\partial P_i}{\partial\theta_2}\frac{\partial P_j}{\partial\theta_2}\right)$$

Backup - Multiple Scattering - Explicit Components

Letting Δy be the difference in y between the current layer and the one we are predicting to

$$x_{\text{state}} \doteq P_i = \left[\frac{\Delta y\,\alpha_3}{\beta_3} + x_0, \frac{\Delta y}{c\,\beta_3} + t_0, \frac{\Delta y\,\gamma_3}{\beta_3} + z_0, c\,\alpha_3, c\,\beta_3, c\,\gamma_3\right]_i$$

$$\alpha_3 \equiv \frac{v_x}{c} \qquad \beta_3 \equiv \frac{v_y}{c} \qquad \gamma_3 \equiv \frac{v_z}{c}$$

Recall that we parametrise by y instead of t since it is the most precise parameter we have.

$$Q = \sigma^2(\theta_{\text{proj}}) \begin{pmatrix} \frac{\Delta y^2\left({\beta_3}^2+{\alpha_3}^2\right)}{{\beta_3}^4} & \frac{\Delta y^2\,\alpha_3}{c\,{\beta_3}^4} & \frac{\Delta y^2\,\alpha_3\gamma_3}{{\beta_3}^4} & \frac{c\,\Delta y}{\beta_3} & -\frac{c\,\Delta y\,\alpha_3}{{\beta_3}^2} & 0 \\ \frac{\Delta y^2\,\alpha_3}{c\,{\beta_3}^4} & \frac{\Delta y^2\left(1-{\beta_3}^2\right)}{c^2\,{\beta_3}^4} & \frac{\Delta y^2\,\gamma_3}{c\,{\beta_3}^4} & \frac{\Delta y\,\alpha_3}{\beta_3} & -\frac{\Delta y\left(1-{\beta_3}^2\right)}{{\beta_3}^2} & \frac{\Delta y\,\gamma_3}{\beta_3} \\ \frac{\Delta y^2\,\alpha_3\gamma_3}{{\beta_3}^4} & \frac{\Delta y^2\,\gamma_3}{c\,{\beta_3}^4} & \frac{\Delta y^2\left({\gamma_3}^2+{\beta_3}^2\right)}{{\beta_3}^4} & 0 & -\frac{c\,\Delta y\,\gamma_3}{{\beta_3}^2} & \frac{c\,\Delta y}{\beta_3} \\ \frac{c\,\Delta y}{\beta_3} & \frac{\Delta y\,\alpha_3}{\beta_3} & 0 & c^2\,\left(1-{\alpha_3}^2\right) & -c^2\,\alpha_3\beta_3 & -c^2\,\alpha_3\gamma_3 \\ -\frac{c\,\Delta y\,\alpha_3}{{\beta_3}^2} & -\frac{\Delta y\left(1-{\beta_3}^2\right)}{{\beta_3}^2} & -\frac{c\,\Delta y\,\gamma_3}{{\beta_3}^2} & -c^2\,\alpha_3\beta_3 & c^2\,\left(1-{\beta_3}^2\right) & -c^2\,\beta_3\gamma_3 \\ 0 & \frac{\Delta y\,\gamma_3}{\beta_3} & \frac{c\,\Delta y}{\beta_3} & -c^2\,\alpha_3\gamma_3 & -c^2\,\beta_3\gamma_3 & c^2\,\left(1-{\gamma_3}^2\right) \end{pmatrix}$$

While it was convenient to fix $\beta = 1$ for computing the scattering angle variance, in the rest of the algorithm we let it float and cut on it instead.

References for Backup Slides

1. Frühwirth, R. (1987). Application of Kalman filtering to track and vertex fitting. Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment, 262(2-3), 444-450. doi:10.1016/0168-9002(87)90887-4
2. Lynch, G. R., Dahl, O. I. (1991). Approximations to multiple Coulomb scattering. Nuclear Instruments and Methods in Physics Research Section B: Beam Interactions with Materials and Atoms, 58(1), 6-10.doi:10.1016/0168-583x(91)95671-y
3. E. J. Wolin and L. L. Ho, Nucl. Instrum. Meth. A329, 493-500 (1993) doi:10.1016/0168-9002(93)91285-U