

# Separable Least-Mean Squares Beamforming

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Fortaleza, 2021

# Outline

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# Introduction

- ▶ Comparisons with LMS, NLMS, ATLMS and TLMS.
- ▶ Convergence Analysis ?

# System Model

# Implemented Algorithms I

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## Algorithm 1 Tensor LMS algorithm

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**Require:** Step parameter  $\mu$ , sample size  $K$

- 1:  $k \leftarrow 1$
  - 2: Initialize  $\mathbf{w}_h[k]$  and  $\mathbf{w}_v[k]$  as  $[1, 0, \dots, 0]^T$
  - 3: **for**  $k = 1 : K$  **do**      $\triangleright$  Note we use MATLAB's notation
  - 4:      $\mathbf{u}_h[k] \leftarrow \mathbf{X}[k] \mathbf{w}_v^*[k]$
  - 5:      $\mathbf{u}_v[k] \leftarrow \mathbf{X}[k]^T \mathbf{w}_h^*[k]$
  - 6:      $e[k] \leftarrow s_d[k] - (\mathbf{w}_v[k] \otimes \mathbf{w}_h[k])^H \mathbf{x}[k]$
  - 7:      $\tilde{\mu}[k] \leftarrow \frac{\mu}{\|\mathbf{u}_h[k]\|_2^2 + \|\mathbf{u}_v[k]\|_2^2}$
  - 8:      $\mathbf{w}_h[k+1] \leftarrow \mathbf{w}_h[k] + \tilde{\mu}[k] \mathbf{u}_h[k] e^*[k]$
  - 9:      $\mathbf{w}_v[k+1] \leftarrow \mathbf{w}_v[k] + \tilde{\mu}[k] \mathbf{u}_v[k] e^*[k]$
  - 10:     Check convergence
  - 11: **end for**
  - 12: **return**  $\mathbf{w}_v[k+1] \otimes \mathbf{w}_h[k+1]$
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Figure TLMS algorithm from [1].

# Implemented Algorithms II

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**Algorithm 2** Alternating Tensor LMS algorithm

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**Require:** Step parameter  $\mu$ , sample parameters  $K$ ,  $K_h$ ,  $K_v$

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1:  $k \leftarrow 1$ 
2:  $K_b \leftarrow \lfloor \frac{K}{K_h + K_v} \rfloor$ 
3: Initialize  $\mathbf{w}_h[k]$  and  $\mathbf{w}_v[k]$  as  $[1, 0, \dots, 0]^\top$ 
4: for  $k = 1 : K_h + K_v : K_b(K_h + K_v)$  do
5:   for  $k_h = k : k + K_h - 1$  do
6:      $\mathbf{u}_h[k_h] \leftarrow \mathbf{X}[k_h] \mathbf{w}_v^*[k_h]$ 
7:      $e[k_h] \leftarrow s_d[k_h] - (\mathbf{w}_v[k_h] \otimes \mathbf{w}_h[k_h])^\mathbf{H} \mathbf{x}[k_h]$ 
8:      $\tilde{\mu}_h[k_h] \leftarrow \frac{\mu}{\|\mathbf{u}_h[k_h]\|_2^2}$ 
9:      $\mathbf{w}_h[k_h + 1] \leftarrow \mathbf{w}_h[k_h] + \tilde{\mu}_h[k_h] \mathbf{u}_h[k_h] e^*[k_h]$ 
10:   end for
11:   for  $k_v = k + K_h : k + K_h + K_v - 1$  do
12:      $\mathbf{u}_v[k_v] \leftarrow \mathbf{X}[k_v]^\top \mathbf{w}_h[k_v]^*$ 
13:      $e[k_v] \leftarrow s_d[k_v] - (\mathbf{w}_v[k_v] \otimes \mathbf{w}_h[k_h + 1])^\mathbf{H} \mathbf{x}[k_v]$ 
14:      $\tilde{\mu}_v[k_v] \leftarrow \frac{\mu}{\|\mathbf{u}_v[k_v]\|_2^2}$ 
15:      $\mathbf{w}_v[k_v + 1] \leftarrow \mathbf{w}_v[k_v] + \tilde{\mu}_v[k_v] \mathbf{u}_v[k_v] e^*[k_v]$ 
16:   end for
17:   Check convergence
18: end for
19: return  $\mathbf{w}_v[k_v + 1] \otimes \mathbf{w}_h[k_h + 1]$ 
```

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Figure ATLMS algorithm from [1].

# Numerical Results I

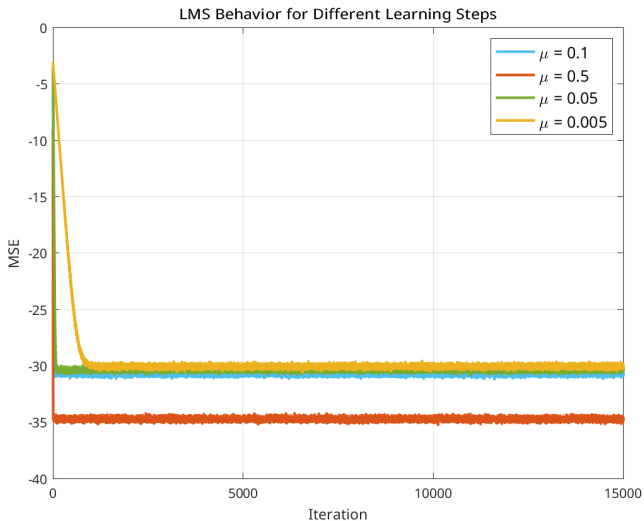


Figure Monte Carlo Experiment with 2500 runs for LMS algorithm.

## Numerical Results II

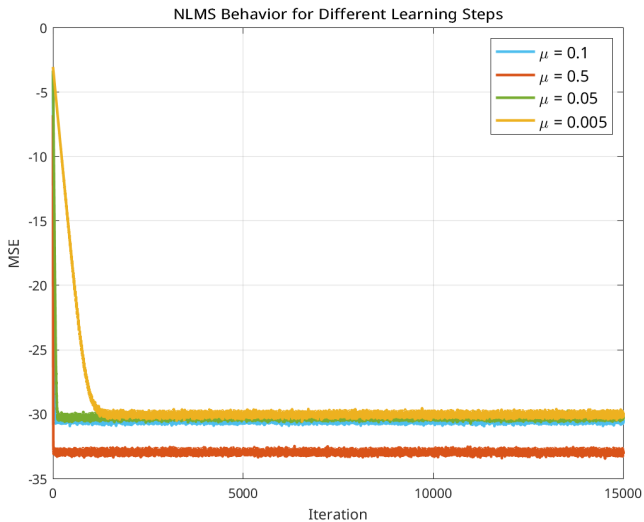


Figure Monte Carlo Experiment with 2500 runs for LMS algorithm.



## Numerical Results III

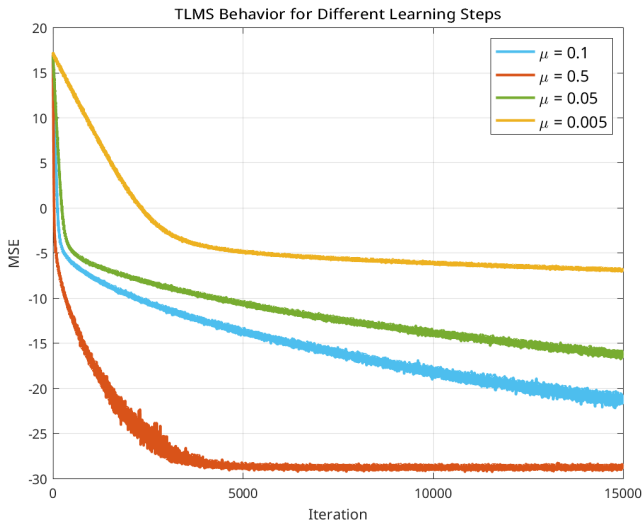


Figure Monte Carlo Experiment with 2500 runs for LMS algorithm.

## Numerical Results IV

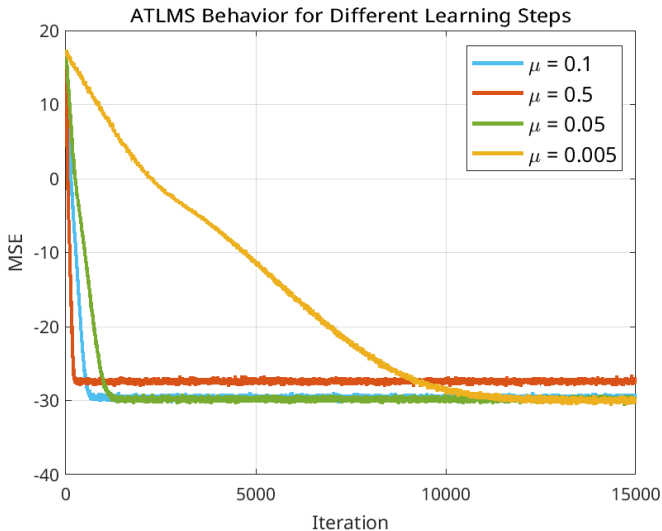


Figure Monte Carlo Experiment with 2500 runs for LMS algorithm.

## Numerical Results V

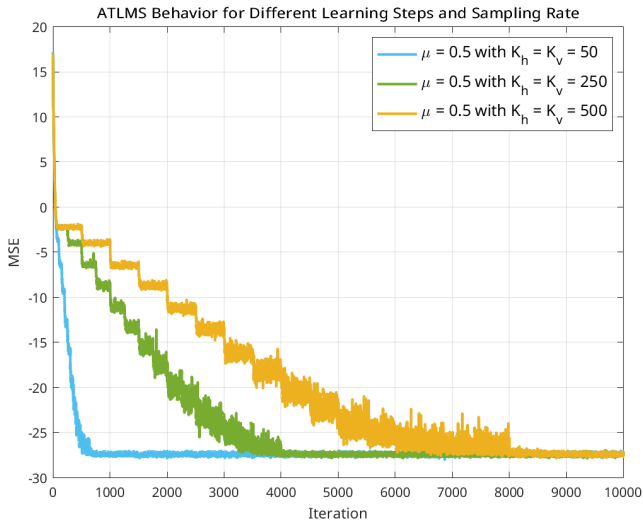


Figure Monte Carlo Experiment with 2500 runs for the ATLMS with different sampling intervals.

## Numerical Results VI

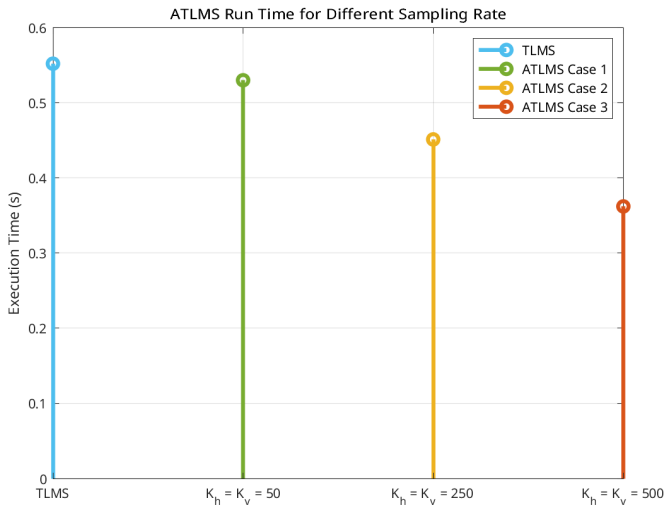


Figure Run time process for ATLMS with different sampling intervals.

# References

- [1] L. N. Ribeiro, B. Sokal, A. L. de Almeida, and J. C. M. Mota, “Separable least-mean squares beamforming,”

Thank you for your  
presence!