TIP8419 - Tensor Algebra
Homework 10

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Multidimensional Least-Squares Kronecker Factorization (MLS-KronF)

On practice 4 we have implemented the LS-KronF(Least Square Kronecker Factorization) algorithm; now we will implement its generalization to the N-dimensional case, called MLS-KronF (Multidimensional Least Square Kronecker Factorization) algorithm. Let $\mathbf{X} \in \mathbb{C}^{I_1 I_2 \dots I_N \times J_1 J_2 \dots J_N}$ be a matrix that we wish to approximate as $\mathbf{X} \approx \mathbf{A}^{(1)} \otimes \mathbf{A}^{(2)} \cdots \otimes \mathbf{A}^{(N)}$, that is, as Kronecker product of N matrices $\mathbf{A}^{(n)} \in \mathbb{C}^{I_n \times J_n}$, with $n = 1, 2, \dots, N$. For $N = 3$ and arbitrary I_n and J_n, implement the MLS-KronF algorithm that estimates $\mathbf{A}^{(1)}, \mathbf{A}^{(2)}$ and $\mathbf{A}^{(3)}$ by solving the following problem

$$(\hat{\mathbf{A}}^{(1)}, \hat{\mathbf{A}}^{(2)}, \hat{\mathbf{A}}^{(3)}) = \underset{\mathbf{A}^{(1)}, \mathbf{A}^{(2)}, \mathbf{A}^{(3)}}{\arg\min} \|\mathbf{X} - \mathbf{A}^{(1)} \otimes \mathbf{A}^{(2)} \otimes \mathbf{A}^{(3)}\|_F^2$$

using either the truncated HOSVD or the HOOI initialized with the HOSVD (you should implement both versions).

Test the algorithms on a matrix that exactly follows the model. Compare the estimated matrices $\hat{\mathbf{A}}^{(1)}, \hat{\mathbf{A}}^{(2)}$ and $\hat{\mathbf{A}}^{(3)}$ with the original ones. What can you conclude? Explain the results.

Hint: Use the file “Practice_10_kronf_matrix_3D.mat” to validate your code.

Problem 1 Perform a Monte Carlo experiment with 1000 realizations as follows. For each realization, generate $\mathbf{X}_0 = \mathbf{A} \otimes \mathbf{B} \otimes \mathbf{C} \in \mathbb{C}^{I_1 I_2 I_3 \times J_1 J_2 J_3}$, for randomly chosen $\mathbf{A} \in \mathbb{C}^{I_1 \times J_1}$, $\mathbf{B} \in \mathbb{C}^{I_2 \times J_2}$ and $\mathbf{C} \in \mathbb{C}^{I_3 \times J_3}$, whose elements are drawn from a standard normal distribution. Let $\mathbf{X} = \mathbf{X}_0 + \alpha \mathbf{V}$ be a noisy version of $\mathbf{X}_0$, where $\mathbf{V}$ is the additive noise term, whose elements are also drawn from a standard normal. The parameter α controls the power (variance) of the noise term, and is defined as a function of the signal-to-noise ratio (SNR), in dB, as follows

$$\text{SNR}_{\text{dB}} = 10\log_{10}\left(\frac{||\mathbf{X}_0||_F^2}{||\alpha\mathbf{V}||_F^2}\right). \quad (1)$$

Assuming the SNR range $\{0,\ 5,\ 10,\ 15,\ 20,\ 25,\ 30\}$ dB, find the estimates $\hat{\mathbf{A}}$, $\hat{\mathbf{B}}$ and $\hat{\mathbf{C}}$ obtained with the MLS-KRF algorithm for the following configurations:

1. $(I_1, I_2, I_3) = (J_1, J_2, J_3) = (2, 2, 2)$;
2. $(I_1, I_2, I_3) = (J_1, J_2, J_3) = (5, 5, 5)$;
3. $(I_1, I_2, I_3) = (J_1, J_2, J_3) = (2, 3, 4)$;
4. $(I_1, I_2, I_3) = (2, 3, 4)$ and $(J_1, J_2, J_3) = (5, 6, 7)$.

Let us define the normalized mean square error (NMSE) measure as follows

$$\text{NMSE}(\mathbf{X}_0) = \frac{1}{1000} \sum_{i=1}^{1000} \frac{\|\hat{\mathbf{X}}_0(i) - \mathbf{X}_0(i)\|_F^2}{\|\mathbf{X}_0(i)\|_F^2}, \tag{2}$$

where $\mathbf{X}_0(i)$ e $\hat{\mathbf{X}}_0(i)$ represent the original data matrix and the reconstructed one at the ith experiment, respectively. For each configuration, plot the NMSE vs. SNR curve obtained with each version of the algorithm (plot the two curves together with different colors). Discuss the obtained results.

Note: For a given SNR (dB), the parameter α to be used in your experiment is determined from equation (1).