TIP8419 - Tensor Algebra
Homework 9

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Multidimensional Least-Squares Khatri-Rao Factorization (MLS-KRF)

Problem 1 Let $\mathbf{X} = \mathbf{A}^{(1)} \diamond \mathbf{A}^{(2)} \cdots \diamond \mathbf{A}^{(N)} \in \mathbb{C}^{I_1 I_2 \dots I_N \times R}$ be a matrix generated from the Khatri-Rao product of N matrices $\mathbf{A}^{(n)} \in \mathbb{C}^{I_n \times R}$, with $n = 1, 2, \dots, N$. Considering $N = 3$ and choosing your own values for R and I_n, $n = 1, 2, 3$, implement the MLS-KRF algorithm to find the estimates of $\mathbf{A}^{(1)}, \mathbf{A}^{(2)}$ and $\mathbf{A}^{(3)}$ by solving the following problem:

$$(\hat{\mathbf{A}}^{(1)}, \hat{\mathbf{A}}^{(2)}, \hat{\mathbf{A}}^{(3)}) = \min_{\hat{\mathbf{A}}^{(1)}, \hat{\mathbf{A}}^{(2)}, \hat{\mathbf{A}}^{(3)}} \|\mathbf{X} - \mathbf{A}^{(1)} \diamond \mathbf{A}^{(2)} \diamond \mathbf{A}^{(3)}\|_F^2.$$

Compare the estimated matrices $\hat{\mathbf{A}}^{(1)}, \hat{\mathbf{A}}^{(2)}$ and $\hat{\mathbf{A}^{(3)}}$ with the original ones. What can you conclude? Explain the results.

Hint: Use the file "krf_matrix_3D.mat" to validate your result.

Problem 2 Assuming 1000 Monte Carlo experiments, generate $\mathbf{X}_0 = \mathbf{A} \diamond \mathbf{B} \diamond \mathbf{C} \in \mathbb{C}^{I_1 I_2 I_3 \times R}$, for randomly chosen $\mathbf{A} \in \mathbb{C}^{I_1 \times R}$, $\mathbf{B} \in \mathbb{C}^{I_2 \times R}$ and $\mathbf{C} \in \mathbb{C}^{I_3 \times R}$, with $R = 4$, whose elements are drawn from a normal distribution. Let $\mathbf{X} = \mathbf{X}_0 + \alpha \mathbf{V}$ be a noisy version of $\mathbf{X}_0$, where $\mathbf{V}$ is the additive noise term, whose elements are drawn from a normal distribution. The parameter α controls the power (variance) of the noise term, and is defined as a function of the signal to noise ratio (SNR), in dB, as follows

$$\text{SNR}_{\text{dB}} = 10\log_{10}\left(\frac{||\mathbf{X}_0||_F^2}{||\alpha \mathbf{V}||_F^2}\right). \tag{1}$$

Assuming the SNR range [0, 5, 10, 15, 20, 25, 30] dB, find the estimates $\hat{\mathbf{A}}$, $\hat{\mathbf{B}}$ and $\hat{\mathbf{C}}$ *via* the MLS-KRF algorithm, assuming $I_1 = 2$, $I_2 = 3$ and $I_3 = 4$.

Let us define the normalized mean square error (NMSE) measure as follows

$$\text{NMSE}(\mathbf{X}_0) = \frac{1}{1000} \sum_{i=1}^{1000} \frac{\|\hat{\mathbf{X}}_0(i) - \mathbf{X}_0(i)\|_F^2}{\|\mathbf{X}_0(i)\|_F^2}, \tag{2}$$

where $\mathbf{X}_0(i)$ e $\hat{\mathbf{X}}_0(i)$ represent the original data matrix and the reconstructed one at the ith experiment, respectively. For each SNR value and configuration, plot the NMSE vs. SNR curve. Discuss the obtained results.

Note: For a given SNR (dB), the parameter α to be used in your experiment is determined from equation (1).