

# TIP8419 - Tensor Algebra

## Homework 8

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### High Order Order Orthogonal Iteration (HOOI)

**Problem 1** For a third-order tensor  $\mathcal{X} \in \mathbb{C}^{I \times J \times K}$  implement the higher order singular orthogonal iteration (HOOI) method, using the following prototype function:

$$[\mathcal{S}, \mathbf{U}^{(1)}, \mathbf{U}^{(2)}, \mathbf{U}^{(3)}] = \text{hooi}(\mathcal{X}) \quad (1)$$

Compare the results with the HOSVD algorithm.

Hint: Use the file “hosvd\_test.mat” to validate your result.

**Problem 2** Consider the two third-order tensors  $\mathcal{X} \in \mathbb{C}^{8 \times 4 \times 10}$  and  $\mathcal{Y} \in \mathbb{C}^{5 \times 5 \times 5}$  provided in the data file “hosvd\_denoising.mat”. By using your HOOI prototype function, find a low multilinear rank approximation for these tensors, defined as  $\tilde{\mathcal{X}} \in \mathbb{C}^{R_1 \times R_2 \times R_3}$  and  $\tilde{\mathcal{Y}} \in \mathbb{C}^{P_1 \times P_2 \times P_3}$ . Then, calculate the normalized mean square error (NMSE) between the original tensor and its approximation, i.e.,:

$$\text{NMSE}(\tilde{\mathcal{X}}) = \frac{\|\tilde{\mathcal{X}} - \mathcal{X}\|_F^2}{\|\mathcal{X}\|_F^2}, \quad \text{NMSE}(\tilde{\mathcal{Y}}) = \frac{\|\tilde{\mathcal{Y}} - \mathcal{Y}\|_F^2}{\|\mathcal{Y}\|_F^2}$$

Hint: The multilinear ranks of  $\mathcal{X}$  and  $\mathcal{Y}$  can be found by analysing the profile of the 1-mode, 2-mode and 3-mode singular values of these tensors.