TIP8419 - Tensor Algebra
Homework 1

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Kronecker Product

Problem 1 For randomly generated $\mathbf{A} \in \mathbb{C}^{N \times N}$ and $\mathbf{B} \in \mathbb{C}^{N \times N}$, evaluate the computational performance (run time) of the following matrix inversion formulas:

(a) Method 1: $(\mathbf{A}_{N \times N} \otimes \mathbf{B}_{N \times N})^{-1}$
Method 2: $\mathbf{A}^{-1}_{N \times N} \otimes \mathbf{B}^{-1}_{N \times N}$
for $n \in \{2, 4, 8, 16, 32, 64\}$.

(b) Method 1: $\left(\mathbf{A}^{(1)}_{4\times 4} \otimes \mathbf{A}^{(2)}_{4\times 4} \otimes \ldots \otimes \mathbf{A}^{(K)}_{4\times 4}\right)^{-1} = \left(\bigotimes_{i=1}^{\mathrm{K}} \mathbf{A}^{(i)}_{4\times 4}\right)^{-1}$
Method 2: $\left(\mathbf{A}^{(1)}\right)^{-1}_{4\times 4} \otimes \left(\mathbf{A}^{(2)}\right)^{-1}_{4\times 4} \otimes \ldots \otimes \left(\mathbf{A}^{(K)}\right)^{-1}_{4\times 4} = \bigotimes_{i=1}^{\mathrm{K}} \left(\mathbf{A}^{(i)}\right)^{-1}_{4\times 4}$
for $K \in \{2, 4, 6, 8, 10\}$.

Problem 2 Let $eig(\mathbf{X})$ be the function that returns the matrix $\mathbf{\Sigma}_{K \times K}$ of eigenvalues of $\mathbf{X}$. Show algebraically that $eig(\mathbf{A} \otimes \mathbf{B}) = eig(\mathbf{A}) \otimes eig(\mathbf{B})$.

<u>Hint</u>: Use the property $(\mathbf{A} \otimes \mathbf{B})(\mathbf{C} \otimes \mathbf{D}) = \mathbf{AC} \otimes \mathbf{BD}$.

$\otimes$ Denotes the Kronecker Product.