

# TIP8419 - Tensor Algebra

## Homework 2

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### Khatri-Rao Product

**Problem 1** Generate  $\mathbf{X} = \mathbf{A} \diamond \mathbf{B} \in \mathbb{R}^{I \times R}$ , for randomly chosen  $\mathbf{A} \in \mathbb{C}^{I \times R}$  and  $\mathbf{B} \in \mathbb{R}^{I \times R}$ . Compute the left pseudo-inverse of  $\mathbf{X}$  and obtain a graph that shows the run time vs. number of rows ( $I$ ) for the following methods

- (a) Method 1: Matlab/Octave function  $\text{pinv}(\mathbf{X}) = \text{pinv}(\mathbf{A} \diamond \mathbf{B})$
- (b) Method 2:  $\mathbf{X}^\dagger = (\mathbf{X}^\top \mathbf{X})^{-1} \mathbf{X}^\top = [(\mathbf{A} \diamond \mathbf{B})^\top (\mathbf{A} \diamond \mathbf{B})]^{-1} (\mathbf{A} \diamond \mathbf{B})^\top$
- (c) Method 3:  $\mathbf{X}^\dagger = [(\mathbf{A} \diamond \mathbf{B})^\top (\mathbf{A} \diamond \mathbf{B})]^{-1} (\mathbf{A} \diamond \mathbf{B})^\top = [(\mathbf{A}^\top \mathbf{A}) \odot (\mathbf{B}^\top \mathbf{B})]^{-1} (\mathbf{A} \diamond \mathbf{B})^\top$

Note: Consider the range of values  $I \in \{2, 4, 8, 16, 32, 64, 128, 256\}$  and plot the curves for  $R = 2$  and  $R = 4$ .

**Problem 2** Generate  $\mathbf{X} = \bigodot_{n=1}^N \mathbf{A}_{(n)} = \mathbf{A}_{(1)} \diamond \cdots \diamond \mathbf{A}_{(N)}$ , where every  $\mathbf{A}_{(n)}$  has dimensions  $4 \times 2$ ,  $n = 1, \dots, N$ . Evaluate the run time associated with the computation of the Khatri-Rao product as a function of the number  $N$  of matrices for the above methods.

Note: Consider the range of values  $N \in \{2, 4, 6, 8, 10\}$ .

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$\odot$  Denotes the Hadamard Product.  
 $\diamond$  Denotes the Khatri-Rao Product.