TIP8419 - Tensor Algebra
Homework 6

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2019.2

High Order Singular Value Decomposition (HOSVD)

Problem 1 For a third-order tensor $\mathcal{X} \in \mathbb{C}^{I \times J \times K}$ implement the truncated high-order singular value decomposition (HOSVD), using the following prototype function:

$$[\mathcal{S}, \mathbf{U}^{(1)}, \mathbf{U}^{(2)}, \mathbf{U}^{(3)}] = hosvd(\mathcal{X}) \tag{1}$$

Hint: Use the file "hosvd_test.mat" to validate your results.

Problem 2 Consider the two third-order tensors $\mathcal{X} \in \mathbb{C}^{8 \times 4 \times 10}$ and $\mathcal{Y} \in \mathbb{C}^{5 \times 5 \times 5}$ provided in the data file "hosvd_denoising.mat". By using your HOSVD prototype function, find a low multilinear rank approximation for these tensors, defined as $\tilde{\mathcal{X}} \in \mathbb{C}^{R_1 \times R_2 \times R_3}$ and $\tilde{\mathcal{Y}} \in \mathbb{C}^{P_1 \times P_2 \times P_3}$. Then, calculate the normalized mean square error (NMSE) between the original tensor and its approximation, i.e.,:

$$\text{NMSE}(\tilde{\mathcal{X}}) = \frac{\|\tilde{\mathcal{X}} - \mathcal{X}\|_F^2}{\|\mathcal{X}\|_F^2}, \quad \text{NMSE}(\tilde{\mathcal{Y}}) = \frac{\|\tilde{\mathcal{Y}} - \mathcal{Y}\|_F^2}{\|\mathcal{Y}\|_F^2}$$

Hint: The multilinear ranks of $\mathcal{X}$ and $\mathcal{Y}$ can be found by analysing the profile of the 1-mode, 2-mode and 3-mode singular values of these tensors.