

TIP8419 - Tensor Algebra

Homework 5

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Kronecker Product Singular Value Decomposition (KPSVD)

Problem 1 Generate a block matrix according to the following structure

$$\mathbf{X} = \begin{pmatrix} \mathbf{X}_{1,1} & \cdots & \mathbf{X}_{1,N} \\ \vdots & \ddots & \vdots \\ \mathbf{X}_{M,1} & \cdots & \mathbf{X}_{M,N} \end{pmatrix}, \quad \mathbf{X}_{i,j} \in \mathbb{C}^{P \times Q}, \quad 1 \leq i \leq M, \quad 1 \leq j \leq N,$$

Implement the KPSVD for the matrix \mathbf{X} by computing σ_k , \mathbf{U}_k , and \mathbf{V}_k such that

$$\mathbf{X} = \sum_{k=1}^{r_{KP}} \sigma_k \mathbf{U}_k \otimes \mathbf{V}_k.$$

Problem 2 In the above problem, set $M = N = P = Q = 3$ and randomly generate $\mathbf{X}_{i,j} = \text{rand}(P, Q)$, $1 \leq i \leq M$, $1 \leq j \leq N$. Then compute the KPSVD and the Kronecker-rank r_{KP} of \mathbf{X} by using your KPSVD prototype function. Consider $r \leq r_{KP}$. Compute the nearest rank- r for the matrix \mathbf{X} .