TIP8419 - Tensor Algebra
Practice 11

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Tensor Kronecker Product Singular Value Decomposition (TKPSVD)

Problem 1 On practice 5 we implement the KPSVD(Kronecker Product Singular Value Decomposition) algorithm, now we will go to implement the generalization of that to tensors, namely, TKPSVD(Tensor Kronecker Product Singular Value Decomposition) algorithm. Consider the N-order tensor $\mathcal{X} \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_N}$. Then, $\mathcal{X} = \sum_{j=1}^{R} \sigma_j \mathcal{A}_j^{(d)} \otimes \mathcal{A}_j^{(d-1)} \otimes \cdots \otimes \mathcal{A}_j^{(1)}$ is the TKPSVD of tensor $\mathcal{X}$, where the tensors $\mathcal{A}_j^{(i)} \in \mathbb{R}^{I_1^{(i)} \times I_2^{(i)} \times \cdots \times I_N^{(i)}}$ satisfy

$$||\mathcal{A}_j^{(i)}||_F = 1, \ \prod_{i=1}^{d} I_k^{(i)} = I_k, \ 1 \leq k \leq N.$$

For $d = 5, \ N = 3$ and I_n arbitrary implement the TKPSVD for that estimate $\mathcal{A}_j^{(1)}, \mathcal{A}_j^{(2)}, \mathcal{A}_j^{(3)}, \mathcal{A}_j^{(4)}$ and $\mathcal{A}_j^{(5)}, \ 1 \leq j \leq R$. What can you conclude? Explain the results.

Problem 2 Consider the $4000 \times 6000 \times 3$ tensor $\mathcal{X}$. Compute the degree-5 TKPSVD of $\mathcal{X}$ of the following form

$$(\hat{\mathcal{A}}_j^{(5)}, \hat{\mathcal{A}}_j^{(4)}, \cdots, \hat{\mathcal{A}}_j^{(1)}) = \min_{\mathcal{A}_j^{(5)}, \mathcal{A}_j^{(4)}, \cdots, \mathcal{A}_j^{(1)}} ||\mathcal{X} - \mathcal{A}_j^{(5)} \otimes \mathcal{A}_j^{(4)} \otimes \cdots \otimes \mathcal{A}_j^{(1)}||_F^2$$

with dimensions

$$(250 \times 375 \times 3) \otimes (2 \times 2 \times 1) \otimes (2 \times 2 \times 1) \otimes (2 \times 2 \times 1) \otimes (2 \times 2 \times 1)$$

using your TKPSVD prototype function.