$$= -\lim_{\frac{x}{2}\to 0} \frac{\sin \frac{x}{2}}{\frac{x}{2}} \lim_{x\to 0} \sin \frac{x}{2}$$
$$= -(1)(0)$$
$$= 0$$

The limit in Example 4 is an important result. It will be used to develop the derivative of $y = \sin x$ in Section 7.2.

**Example 5** Evaluate $\lim_{x\to\pi} \frac{\sin x}{\pi - x}$.

**Solution** In order to use ① we need a variable to approach 0.

Now as $x \to \pi$, $x - \pi \to 0$

Therefore $\pi - x \to 0$

Since $\sin x = \sin(\pi - x)$

we get $\lim_{x\to\pi} \frac{\sin x}{\pi - x} = \lim_{\pi - x\to 0} \frac{\sin(\pi - x)}{\pi - x} = 1$

## EXERCISE 7.1

B Use a calculator to estimate the value of each of the following limits.

**1.** $\lim_{x\to 0} \frac{\sin 3x}{x}$ **2.** $\lim_{x\to 0} \frac{\sin 2x}{\sin 3x}$ **3.** $\lim_{x\to 0} \frac{\sin^3 2x}{\sin^3 3x}$

**4.** $\lim_{x\to 0} \frac{1 - \cos^2 x}{x^2}$ **5.** $\lim_{x\to 0} \frac{1 - \cos x}{\tan x}$ **6.** $\lim_{x\to 0} \frac{\sin(\cos x)}{\sec x}$

Evaluate each of the following limits.

**7.** $\lim_{x\to 0} \frac{\sin 3x}{x}$ **8.** $\lim_{x\to 0} \frac{\sin ax}{\sin bx}$ **9.** $\lim_{x\to 0} \frac{\sin^3 2x}{\sin^3 3x}$

**10.** $\lim_{x\to 0} \frac{1 - \cos x}{x}$ **11.** $\lim_{x\to 0} (x^2 + \cos x)$ **12.** $\lim_{x\to \frac{\pi}{3}} (\sin x - \cos x)$

**13.** $\lim_{x\to \frac{\pi}{4}} \frac{\sin x}{3x}$ **14.** $\lim_{x\to -3\pi} x^3 \sin^4 x$ **15.** $\lim_{x\to 0} \frac{\sin 5x}{5}$

**16.** $\lim_{x\to \frac{\pi}{4}} \frac{\tan x}{4x}$ **17.** $\lim_{x\to 0} \frac{\tan 3x}{3 \tan 2x}$ **18.** $\lim_{x\to 0} \frac{\sin^2 3x}{x^2}$

19. $\lim_{x \to \frac{\pi}{6}} \sqrt{\sin x}$

20. $\lim_{x \to 0} \dfrac{\sin 6x}{\cos 4x}$

21. $\lim_{x \to 0} \dfrac{\cos x - 1}{\sin x}$

22. $\lim_{x \to 0} \dfrac{\tan x}{4x}$

23. $\lim_{x \to 0} \dfrac{x^3}{\tan^3 2x}$

24. $\lim_{x \to 0} \dfrac{1 - \cos x}{2x^2}$

25. $\lim_{x \to 0} \dfrac{x}{\sin \frac{x}{2}}$

26. $\lim_{x \to 0} \dfrac{2 \tan x}{x \sec x}$

27. $\lim_{x \to 0} \dfrac{1 - \cos^2 x}{x^2}$

28. $\lim_{x \to 0} \dfrac{1 - \cos 2x}{x^2}$

29. $\lim_{x \to \frac{\pi}{2}} \dfrac{\cot x}{\frac{\pi}{2} - x}$

30. $\lim_{x \to \pi} \dfrac{\sin x}{x - \pi}$

31. $\lim_{x \to 0} \dfrac{\sin^2 x \cos x}{1 - \cos x}$

32. $\lim_{x \to 0} \dfrac{\sin x}{\tan x}$

33. $\lim_{x \to 0} \dfrac{2 \sin x - \sin 2x}{x \cos x}$

34. $\lim_{x \to 0} \dfrac{\tan x - \sin x}{x \cos x}$

35. $\lim_{x \to 0} \dfrac{1 - \cos x}{\tan x}$

36. $\lim_{x \to 0} \dfrac{\csc x - \cot x}{\sin x}$

37. $\lim_{x \to 0} \dfrac{\sin 2x}{2x^2 + x}$

38. $\lim_{x \to 0} \dfrac{\sin(\cos x)}{\sec x}$

39. (a) Use a calculator to approximate the value of $\dfrac{\tan x - x}{x^3}$ for $x = 0.1, 0.01, 0.001$, and $0.0001$.

(b) Estimate the value of $\lim_{x \to 0} \dfrac{\tan x - x}{x^3}$ using the results from part (a).

(c) Use a calculator to approximate the value of $\dfrac{\tan x - x}{x^3}$ for $x = 0.00001, 0.000001$, and $0.0000001$ and examine your answer to part (b). Can you explain what went wrong?

C 40. Does the $\lim_{x \to 0} \dfrac{\sin x}{|x|}$ exist? If so, what is it? If not, why not?

41. Evaluate $\lim_{x \to 0} \dfrac{\sin x}{x + \sin x}$.

42. Evaluate $\lim_{x \to 1^-} \dfrac{\sin(x - 1)}{|x - 1|}$.

43. Evaluate $\lim_{h \to 0} \dfrac{\sin(a + h) - \sin a}{h}$.

44. Evaluate $\lim_{h \to 0} \dfrac{\cos(a + h) - \cos a}{h}$.