PDF 4.030 Asymptotes etc.

Testing for Vertical Asymptotes and Holes

Given a rational function of the form $f(x) = \frac{p(x)}{q(x)}$, we should simplify by crossing out like factors from the numerator and the denominator.

1. There is a hole at x = c if the factor (x – c) was originally present in both the numerator and the denominator and if the degree of that factor in the numerator was greater than or equal to the degree of that factor in the denominator. In other words, if (x – c) is present in both the numerator and denominator and if, after simplifying, it is completely crossed out of the denominator, then there is a hole at x = c. To determine the y-coordinate of the hole, plug the x-value into the simplified rational expression.
2. There's a vertical asymptote at x = c if, AFTER SIMPLIFYING, the denominator equals 0 at x = c and the numerator does not equal 0 at x = c.

Test for Horizontal and Oblique Asymptotes

Given a rational function of the form $f(x) = \frac{p(x)}{q(x)}$, we should simplify by crossing out like factors from the numerator and the denominator. If the denominator is degree 0 (i.e, there are no more x's), then there are no horizontal or oblique asymptotes. If the denominator is not degree 0, then, AFTER SIMPLIFYING:

1. There's a horizontal asymptote at y = 0 if the degree of the denominator is greater than the degree of the numerator.
2. There's a horizontal asymptote at $y = \frac{lead\ coefficient\ of\ numerator}{lead\ coefficient\ of\ denominator}$ if the degree of the numerator equals the degree of the denominator. (Remember, the denominator must be at least degree 1 after simplifying)
3. There's an oblique asymptote if the degree of the numerator is greater than the degree of the denominator. To determine the equation of the oblique asymptote, we use synthetic division or long division.

Example 1

Determine the equations of any asymptotes and the location of any holes (including y-coordinates) of the function $y = \frac{2x-2}{x^2+3x-4}$

Example 2

Determine the equations of any asymptotes and the locations of any holes (including y-coordinate) of the function $y = \frac{5x+15}{2x+12}$

Example 3

Determine the equations of any asymptotes and the locations of any holes (including y-coordinate) of the function $y = \frac{3x^2+15x-18}{4x^2-20x-24}$

Example 4

Determine the equations of any asymptotes and the locations of any holes (including y-coordinate) of the function $y = \frac{x^2-2x+4}{x-3}$

Example 5

Find constants a and b such that the graph of the function $y = \frac{ax+8}{bx-1}$ has a horizontal asymptote at y = 8 and a vertical asymptote at x = -5