MCV4U Derivatives Quiz Friday, March 1, 2024

Version 1

1. Prove the quotient rule. In other words, prove that if $h(x) = \frac{f(x)}{g(x)}$, then $h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$. Just like in class, you can assume that we know the product rule already. /4

$h(x) = \frac{f(x)}{g(x)}$

$h(x)\,g(x) = f(x)$ ∴ due to product rule $\Rightarrow$ $h'(x)g(x) + h(x)g'(x) = f'(x)$

rearrange: $h'(x) = \frac{f'(x) - h(x)g'(x)}{g(x)}$

$= \frac{f'(x) - \left[\frac{f(x)}{g(x)}\right] g'(x)}{g(x)}$

$= \frac{f'(x)\left[\frac{g(x)}{g(x)}\right] - \frac{f(x)g'(x)}{g(x)}}{g(x)}$

$= \frac{\frac{f'(x)g(x)}{g(x)} - \frac{f(x)g'(x)}{g(x)}}{g(x)}$

$h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$

4

2. Determine each of the following derivatives. You do not need to simplify. You can leave your answer the way it appears after your first line. /7 total

a. $f(x) = \underbrace{3(2x^3 - 19x^2 + 1)^6}\underbrace{(5x-4)^8}$

$f'(x) = 18(2x^3 - 19x^2 + 1)^5(6x^2 - 38x)(5x-4)^8 + (3)(2x^3 - 19x^2 + 1)^6(8)(5x-4)^7(5)$

4/4

Rule: $f'(x)g(x) + g'(x)$

b. $g(x) = \frac{3x+4}{(8x^2-7x+2)^5}$

$g'(x) = \frac{(8x^2-7x+2)^5(3) - (3x+4)(5)(8x^2-7x+2)^4(16x-7)}{(8x^2-7x+2)^{10}}$

3/3

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3. A 4000 litre tank can be drained in 50 minutes. The volume of water remaining in the tank after t minjutes is $V(t) = 4000\left(1 - \frac{t}{50}\right)^3$. At what rate is the water flowing out of the tank at a time of 25 minutes. Include proper units in your answer.

/3

$v(t) = 4000\left(-\frac{1}{50}t + 1\right)^3$

$v'(t) = 12000\left(-\frac{1}{50}t + 1\right)^2\left(-\frac{1}{50}\right)$

$v'(25) = 12000\left(-\frac{1}{50}(25) + 1\right)^2\left(-\frac{1}{50}\right)$

$= (12000)\left(\frac{1}{4}\right)\left(-\frac{1}{50}\right)$

$= \frac{-12000}{200}$

$= -60 \text{ L/min}$

due to losing volume in tank (out)

∴ water flows out at 60L/min

3

At a time of 25 minutes, the water is flowing out of the tank at a rate of 60 L/min
(include proper units in your answer)

4. The radius of a circular juice blot on a piece of paper towel t seconds after it was first seen is modeled by $r(t) = \frac{3+12t}{1+t}$ where r is measured in centimetres. At the time that the radius of the circular blot is 9 cm, what is the rate at which the radius is increasing. Include proper units in your answer.

r(t) find t

/4

$r(t) = \frac{3 + 12t}{1+t}$

$r'(t) = \frac{(1+t)(12) - (3+12t)(1)}{(1+t)^2}$

$r'(2) = \frac{(3)(12) - (27)(1)}{(3)^2}$

$= \frac{36 - 27}{9}$

$= 1 \text{ cm/s}$

$r(t) = 9$

$9 = \frac{3+12t}{1+t}$

$9 + 9t = 3 + 12t$

$0 = 3t - 6$

$0 = 3(t-2)$

$2 = t$

∴ radius growing @ 1cm/s when it is 9cm.

4

The radius is increasing at a rate of 1 cm/s.
(include proper units in your answer)

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