Vectors Test 2 Version 1 Thursday, May 30, 2024

1. Are the vectors $\vec{a} = \overrightarrow{(-2,-1,4)}$, $\vec{b} = \overrightarrow{(3,4,2)}$ and $\vec{c} = \overrightarrow{(1,8,22)}$ coplanar?
   - Just give the answer yes or no, but you must show the work to justify your decision (just giving the correct answer without the work is not worth any marks) (3 marks)

$\overrightarrow{(1,8,22)} = x\,\overrightarrow{(-2,-1,4)} + y\,\overrightarrow{(3,4,2)}$

$1 = -2x + 3y$ $\quad 8 = -x + 4y$

$1 = -2(4y-8) + 3y$ $\quad x = 4y - 8$ $\quad 22 = 4x + 2y$

$1 = -8y + 16 + 3y$ $\quad x = 4(3) - 8$ $\quad 22 = 4(4) + 2(3)$

$-15 = -5y$ $\quad = 12 - 8$ $\quad 22 = 16 + 6$

$y = 3$ $\quad x = 4$ $\quad 22 = 22$ ✓

3 ✓

yes

Just write yes or no in the box.

2. Determine the angle made by the vector $\vec{v} = \overrightarrow{(-4,-1,9)}$ with the positive z-axis. Round your answer to one tenth of a degree (i.e., one decimal place) if necessary. (2 marks)

$\cos \gamma = \dfrac{c}{\sqrt{a^2+b^2+c^2}}$

$\cos \gamma = \dfrac{9}{\sqrt{(-4)^2+(-1)^2+(9)^2}}$

$\cos \gamma = \dfrac{9}{\sqrt{98}}$

$\gamma = 24.6°$

2 ✓

$\gamma = 24.6°$

Round your answer to the nearest tenth of a degree (i.e., one decimal place) if necessary.

3. Determine the vector projection of the vector $\vec{a} = \overrightarrow{(4,-5,2)}$ on the vector $\vec{b} = \overrightarrow{(-2,5,6)}$. (2 marks)

$vect\,\frac{\vec{a}}{\vec{b}} = \dfrac{\vec{a}\cdot\vec{b}}{\vec{b}\cdot\vec{b}}\,\vec{b}$

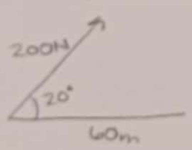
$= \dfrac{-8-25+12}{4+25+36}\,\overrightarrow{(-2,5,6)}$

$= \dfrac{-21}{65}\,\overrightarrow{(-2,5,6)}$

2 ✓

$vect_{\vec{b}}^{\vec{a}} = \dfrac{-21}{65}\,\overrightarrow{(-2,5,6)}$

4. Francesca pulls a toboggan with a rope that makes an angle of 20° with the horizontal ground. If Francesca applies a force of 200 N and pulls the toboggan forward 60 m, calculate the work done in Joules. Round your answer to one tenth of a joule (i.e., one decimal place) if necessary. (2 marks)



$W = \vec{f}\cdot\vec{s}$

$W = |\vec{f}||\vec{s}|\cos\theta$

$= (200)(60)\cos 20°$

$= 11276.3\ J$

2 ✓

The work done by Francesca is 11276.3 J

Round your answer to one tenth of a joule (i.e., one decimal place) if necessary.

9

5. The area of the parallelogram determined by the vectors $\vec{a} = \overrightarrow{(10,k,5)}$ and $\vec{b} = \overrightarrow{(-3,2,-2)}$ is 15 square units. Determine the value(s) of k. ~~(3 marks)~~

$|a \times b| = 15u^2$

$(-2k)-(10) = -2k-10$

$(-15)-(-20) = 5$

$(20)-(-3k) = 3k+20$

$\overrightarrow{a\times b} = (-2k-10, 5, 3k+20)$

$|a\times b| = 15 \Rightarrow 15 = \sqrt{(-2k-10)^2+(5)^2+(3k+20)^2}$

$15 = \sqrt{4k^2+40k+100+25+9k^2+120k+400}$

$15 = \sqrt{13k^2+160k+525}$ SBS

$225 = 13k^2+160k+525$

$0 = 13k^2+160k+300$

$k = \frac{-160 \pm \sqrt{25600-15600}}{26}$

$k = \frac{-160 \pm 100}{26}$

$k = -\frac{30}{13}$ or $k = -10$

4/4

$k = -\frac{30}{13}$ or -10

State all possible solutions (could be one or more)

6. The vectors $\vec{u} = \overrightarrow{(k,-1,2)}$ and $\vec{v} = \overrightarrow{(1,4,-8)}$ are such that $\theta = cos^{-1}\left(\frac{-2}{3}\right)$, where θ is the angle between $\vec{u}$ and $\vec{v}$. Determine all possible value(s) for k. (6 marks)

obtuse

$\vec{u}\cdot\vec{v} = |\vec{u}||\vec{v}|\cos\theta$

$\overrightarrow{(k,-1,2)}\cdot\overrightarrow{(1,4,-8)} = \sqrt{(k)^2+(-1)^2+(2)^2}\sqrt{(1)^2+(4)^2+(-8)^2}\left(-\frac{2}{3}\right)$

$k-20 = \sqrt{k^2+5}\,(\cancel{9}^{3})\left(-\frac{2}{\cancel{3}}\right)$

$(k-20) = (-6)\sqrt{k^2+5}$ SBS

$k^2-40k+400 = 36k^2+180$

$0 = 35k^2+40k-220$

$0 = 5(7k^2+8k-44)$

$k = \frac{-8 \pm \sqrt{64+1232}}{14}$

$k = \frac{-8 \pm 36}{14}$

$k = 2$

$k = -\frac{22}{7}$

$= -3.14$

$(2,-1,2)\cdot(1,4,-8)$

$= 2-4-16$

$= -18$

$(-3.14,-1,2)\cdot(1,4,-8)$

$= -3.14-4-16$

$= -\#$

$\therefore$ both soln work

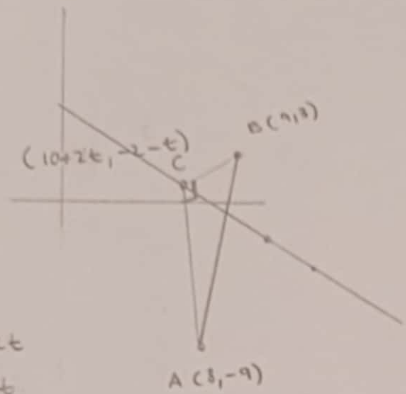
6

$k = 2$ or $-\frac{22}{7}$

State all possible solutions (could be one or more)

10

7. The segment joining the points $A(8,-9)$ and $B(9,3)$ is the hypotenuse of the right triangle ABC. The third vertex C, lies on the line with the vector equation $\vec{r} = \overrightarrow{(10,-2)} + t\,\overrightarrow{(2,-1)}, t \in R$. Determine the coordinates of point C. State all possible solutions. (7 marks)



$x = 10 + 2t$
$y = -2 - t$

$\vec{AC} = (10 + 2t - 8,\ -2 - t + 9)$
$= (2t + 2,\ -t + 7)$

$\vec{BC} = (10 + 2t - 9,\ -2 - t - 3)$
$= (2t + 1,\ -t - 5)$

$\vec{AC} \cdot \vec{BC} = 0$

$(2t+2,\ -t+7) \cdot (2t+1,\ -t-5) = 0$

$(2t+2)(2t+1) + (-t+7)(-t-5) = 0$

$4t^2 + 2t + 4t + 2 + t^2 + 5t - 7t - 35 = 0$

$5t^2 + 4t - 33 = 0$

$t = \dfrac{-4 \pm \sqrt{16 + 660}}{10}$

$= \dfrac{-4 \pm 26}{10}$

$t = -3$ $\qquad$ $t = \frac{11}{5}$

$x = 10 + 2(-3)$
$= 10 - 6$
$= 4$
$y = -2 - (-3)$
$= 1$

$C(4,1)$

$x = 10 + 2\left(\frac{11}{5}\right)$
$= \frac{50}{5} + \frac{22}{5}$
$= \frac{72}{5}$

$y = -2 - \left(\frac{11}{5}\right)$
$= -\frac{10}{5} - \frac{11}{5}$ $\quad C\left(\frac{72}{5}, -\frac{21}{5}\right)$
$= -21/5$

7

The coordinates of point C are $C(4,1)$ or $C\left(\frac{72}{5}, \frac{-21}{5}\right)$

Give all possible answers (could be one or more)

8. The vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = 2$ and $|\vec{b}| = 3$. The angle between $\vec{a}$ and $\vec{b}$ is 120°. Solve for k given that the vector $\vec{a} - 3\vec{b}$ is perpendicular to the vector $2\vec{a} + k\,\vec{b}$ (4 marks)

$(\vec{a} - 3\vec{b}) \cdot (2\vec{a} + k\vec{b}) = 0$

$2\vec{a}\cdot\vec{a} + k\vec{a}\cdot\vec{b} - 6\,\vec{a}\cdot\vec{b} - 3k\vec{b}\cdot\vec{b} = 0$

$2|\vec{a}|^2 + k\,\vec{a}\cdot\vec{b} - 6\vec{a}\cdot\vec{b} - 3k|\vec{b}|^2 = 0$

$2(2)^2 + k(-3) - 6(-3) - 3k(3)^2 = 0$

$8 - 3k + 18 - 27k = 0$

$-30k + 26 = 0$

$30k = 26$

$k = \frac{26}{30} = \frac{13}{15}$

$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos 120°$
$= (2)(3)\cos 120°$
$= -3$

3/3

$k = 13/15$

/10