Derivatives Test Thursday March 7 2024 Name: [redacted]

Version 1

Volume of a Cone $= \frac{1}{3}\pi r^2 h$	Surface Area of a Sphere $= 4\pi r^2$	Volume of a Sphere $= \frac{4}{3}\pi r^3$

1. Determine the equation of the tangent line to the curve $y = 2x^3 + 4x - 9$ at the point where $x = 5$.

/3

$y' = 6x^2 + 4$

at $x = 5$

$y' = 6(5)^2 + 4$

$= 154$ ← slope

point at $x = 5$

$y = 2(5)^3 + 4(5) - 9$

$= 261$

$(5, 261)$

$y = mx + b$

$261 = 154(5) + b$

$b = -509$

$y = 154x - 509$

3

The equation of the tangent line is $y = 154x - 509$

2. Given $f(x) = \left[\frac{(7x-19)^5 - 30}{x^3 - 10x + 4}\right]^3 + 10x^2$, determine an expression for the derivative $f'(x)$. You do not need to simplify that expression. Then, evaluate $f'(3)$.

/5

$$f'(x) = 3\left[\frac{(7x-19)^5 - 30}{x^3 - 10x + 4}\right]^2 \left[\frac{(x^3 - 10x + 4)(5)(7x-19)^4(7) - [(7x-19)^5 - 30][3x^2 - 10]}{(x^3 - 10x + 4)^2}\right] + 20x$$

$$f(3) = 3\left[\frac{32 - 30}{1}\right]^2 \left[\frac{(1)(5)(16)(7) - (2)(17)}{1}\right] + 60$$

$= 3(4)(526) + 60$

$= 6312 + 60$

$= 6372$

5

$f'(3) =$ 6372

/8

3. Solve for a and b given that the function $f(x) = \frac{ax+b}{x^2-12x+20}$ has a horizontal tangent at $(-2, 2)$. /4

$f'(x) = \frac{(x^2-12x+20)(a) - (ax+b)(2x-12)}{(x^2-12x+20)^2}$

horiz tangent means $f'(x) = 0$

$0 = \frac{(4+24+20)a - (-2a+b)(-16)}{2304}$

$0 = 48a - (32a - 16b)$

$16a = -16b$
$a = -b$

at $(-2, 2)$

$2 = \frac{-2a+b}{48}$

$96 = -2a + b$

sub $a = -b$

$96 = -2(-b) + b$

$96 = 3b$

$b = 32$

$a = -32$

4

$a = -32 \qquad b = 32$

4. The function $y = 15x - (2x+3)^{\frac{3}{2}}$ has a horizontal tangent line at one point. Determine the coordinates of that point. /4

$y' = 15 - \frac{3}{\cancel{2}}(2x+3)^{\frac{1}{2}}(\cancel{2})$

$= 15 - 3(2x+3)^{1/2}$

$y' = 0$ at horiz tangent

$0 = 15 - 3(2x+3)^{1/2}$

$\frac{-15}{-3} = (2x+3)^{1/2}$

$(5)^2 = (\sqrt{2x+3})^2$

$25 = 2x+3$

$x = 11$

4

at $x = 11$

$y = 15(11) - (2(11)+3)^{3/2}$

$= 40$

$(11, 40)$

The point with a horizontal tangent is $(11, 40)$

8

5. Suppose that $y = 7u^6 - 4u^5 + 2$ and $u = 17x - 49$. Evaluate $\frac{dy}{dx}$ at $x = 3$ ~~17~~

$\frac{2}{2}$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= (42u^5 - 20u^4)(17)$$

when $x = 3$, $u = 2$

$$\frac{dy}{dx} = (42(2)^5 - 20(2)^4)(17)$$

$$= (1024)(17)$$

$$= 17408$$

At $x = 3$, $\frac{dy}{dx} =$ 17408

6. Given that $(x^2 + 2y)^2 + x = 172$, evaluate $\frac{dy}{dx}$ at the point (3,2). /4

4

$$2(x^2+2y)\left(2x + 2\frac{dy}{dx}\right) + 1 = 0$$

$$2x + 2\frac{dy}{dx} = \frac{-1}{2(x^2+2y)}$$

$$\frac{2dy}{dx} = \frac{-1}{2(x^2+2y)} - 2x$$

$$\frac{dy}{dx} = \frac{-1}{4(x^2+2y)} - x$$

sub (3,2) in

$$\frac{dy}{dx} = \frac{-1}{4[3^2+2(2)]} - 3$$

$$= \frac{-157}{52}$$

At the point (3,2), $\frac{dy}{dx} = \frac{-157}{52}$

/6

7. Determine an expression for $\dfrac{d\left[(3x^2+8x+1)^3+7(3x^2+8x+1)\ \ +8\right]}{d(3x^2+8x+1)}$

Your answer should not have any variable other than x in it, but it does not need to be simplified. /2

let $w = 3x^2+8x+1$

let $y = w^3+7w+8$

$\dfrac{dy}{dw} = 3w^2+7$

$= 3(3x^2+8x+1)^2+7$

2

$\dfrac{d\left[(3x^2+8x+1)^3+7(3x^2+8x+1)\ \ +8\right]}{d(3x^2+8x+1)} = 3(3x^2+8x+1)^2+7$

(your final answer should not have any variable other than x, but you don't need to expand it)

8. Water is being poured into an inverted right circular cone at a rate of $54\pi cm^3/min$. The height of the cone is equal to 3 times the radius of the cone. At what rate is the height of the water rising when the volume is $8\pi\ cm^3$. Include proper units of measurement in your answer. /5

given

$\dfrac{dV}{dt} = 54\pi cm^3/min$

$h = 3r$

$r = \frac{1}{3}h$

$\dfrac{dh}{dt} = ?$

$V = 8\pi\ cm^3$

5

$V = \frac{1}{3}\pi r^2 h$

$8\cancel{\pi} = \frac{1}{3}\cancel{\pi}(\frac{1}{3}h)^2 h$

$8 = \frac{1}{27}h^3$

$h = 6\ cm$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\ (\frac{1}{3}h)^2 h$

$V = \frac{1}{27}\pi h^3$

$\dfrac{dV}{dt} = \frac{1}{27}\pi(3)h^2\dfrac{dh}{dt}$

$\dfrac{dh}{dt} = \dfrac{54\cancel{\pi}(27)}{\cancel{\pi}\,3\,(6)^2}$

$= \dfrac{27}{2}\ cm/min$

At that time, the height of the water is increasing at a rate of $\frac{27}{2}$ cm/min.
(include proper units of measurement in your answer).

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