PDF 4.040 Concavity and Points of Inflection

Concave Up $f''(x) > 0$

The graph of $y = f(x)$ is concave up when the second derivative is positive, i.e., when $f''(x) > 0$

Notice that the tangent lines go from a slope of negative, to zero, to positive. In other words, the slopes are increasing, meaning that the rate of change of the rate of change is positive (ie, $f''(x) > 0$). When $f''(x) > 0$, the curve is above the tangent lines.

Concave Down $f''(x) < 0$

The graph of $y = f(x)$ is concave down when the second derivative is positive, i.e., when $f''(x) < 0$

Notice that the tangent lines go from a slope of positive, to zero, to negative. In other words, the slopes are decreasing, meaning that the rate of change of the rate of change is negative (ie, $f''(x) < 0$). When $f''(x) < 0$, the curve is below the tangent lines.

Second Derivative Test

If at a given point, $f'(x) = 0$ and $f''(x) > 0$, then there is a local minimum.

If at a given point, $f'(x) = 0$ and $f''(x) < 0$, then there is a local maximum.

Points of Inflection

Points at which $f''(x) = 0$ are called points of inflection.

Putting Together $f'(x)$ and $f''(x)$

$f'(x) > 0$ and $f''(x) > 0$	$f'(x) > 0$ and $f''(x) < 0$	$f'(x) < 0$ and $f''(x) > 0$	$f'(x) < 0$ and $f''(x) < 0$

Example 1

Sketch the graph of $y = x^3 - 3x^2 - 9x + 10$

Example 2

Sketch the graph of the function $f(x) = x^{\frac{1}{3}}$

Example 3

Determine any points of inflection on the graph of $f(x) = \frac{1}{x^2+3}$