Chap 1.040 Limits as x goes to Negative Infinity or Infinity

We previously discussed how to evaluate limits as x goes to infinity or as x goes to negative infinity using a graphing approach.

However, we can discuss how to evaluate these types of limits without using a graphing approach every time.

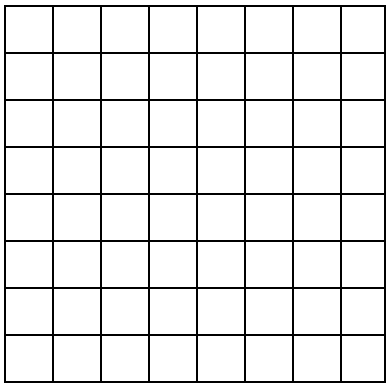
To discuss how to do this, we will look at some graphs.

It is NOT important to know the numbers of the scenarios that follow.

Scenario 1

Suppose $y = a^x,\ 0 < a < 1$

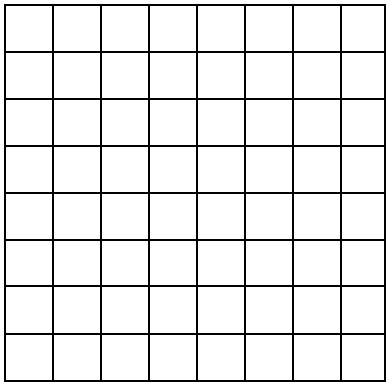
For example, suppose $y = \left(\frac{1}{2}\right)^x$



Scenario 2

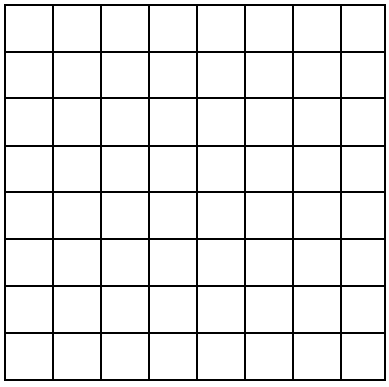
Suppose $y = a^x,\ a > 1$

For example, suppose $y = 2^x$



Scenario 3

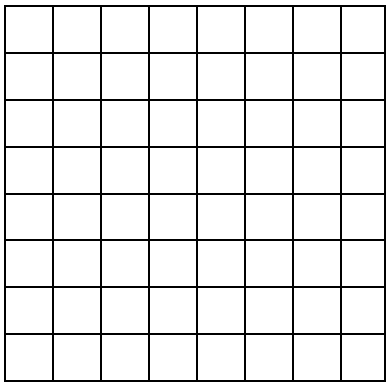
Suppose $y = a^x,\ a = 1$



Scenario 4

Suppose $y = x^a,\ a > 0$

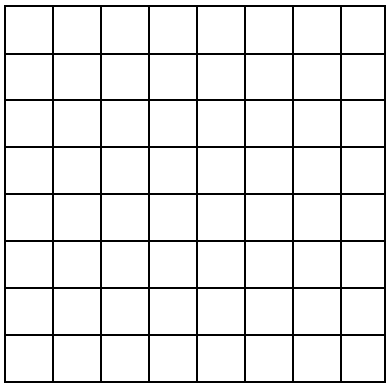
For example, suppose $y = x^{\frac{1}{2}}$ or $y = x^2$



Scenario 5

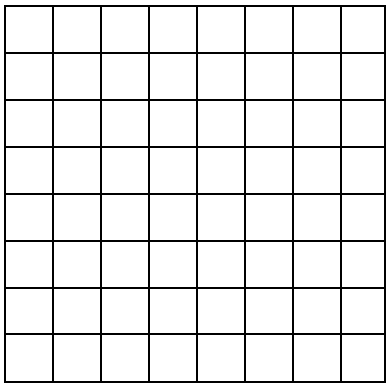
Suppose $y = x^a,\ a < 0$

For example, suppose $y = x^{\frac{-1}{2}}$ or $y = x^{-2}$



Scenario 6

Suppose $y = a^x,\ a = 0$



<u>Example 1</u>

Evaluate $\lim_{x \to -\infty} \frac{6^x}{7^x}$

Example 2

Evaluate $\lim_{x \to \infty} \frac{6^x}{7^x}$

Limits of Rational Functions as x goes to Infinity or x goes to Negative Infinity

When we are evaluating a limit as x goes to infinity or to negative infinity of a rational function (i.e., polynomial over polynomial), we can use the following approach.

1. Determine the degree of the numerator and the degree of the denominator. Factor x^n from both the numerator and denominator where n is either the greater of those two values or the lesser of those two values. If the degree of both numerator and denominator is the same, then n is equal to that degree. (Factor the same term from both numerator and denominator). Cancel out like factors.
2. Determine the limit of each term of the numerator and add those limits. Determine the limit of each term of the denominator and add those limits.
    a) a numerator with a limit that's finite over a denominator growing without bound produces a limit of 0
    b) a numerator with a limit that's finite over a denominator with a limit that's finite produces a limit equal to the quotient of those limits.
    c) a numerator with a limit that's finite over a denominator that is infinitesimally small produces a limit of infinity or negative infinity.
    d) a numerator growing without bound over a denominator that's finite or infinitesimally small produces a limit of infinity or negative infinity.
    e) a numerator that's infinitesimally small over a denominator that's finite or growing without bound produces a limit of 0

Example 3

Evaluate $\lim_{x \to \infty} \frac{5x^2+2x+1}{3x+4}$

Example 4

Evaluate $\lim_{x \to -\infty} \frac{-6x^3-3x+1}{2x^3+5x^2-10x+5}$

Example 5

Evaluate $\lim_{x \to \infty} \frac{2x^2+x-9}{6x^5-4x^4-8x}$

Example 6

Evaluate $\lim_{x \to \infty} \frac{3x-4}{\sqrt{x^2-3}}$

Example 7

Evaluate $\lim_{x \to -\infty} \frac{3x-4}{\sqrt{x^2-3}}$