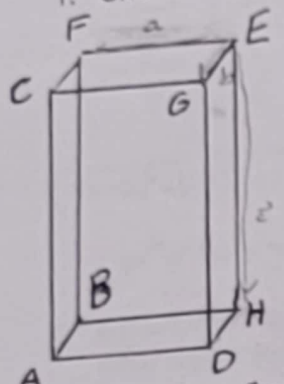
Vectors Test

1. Given the square based prism shown below, and given that $\overrightarrow{EF} = \vec{a}, \overrightarrow{EG} = \vec{b}$ and $\overrightarrow{EH} = \vec{c}$,



a. determine an expression in terms of $\vec{a}$, $\vec{b}$ and/or $\vec{c}$ equivalent to $\overrightarrow{HA}$ /1

$\overrightarrow{HA} = \vec{b} + \vec{a}$ ✓

b. determine an expression in terms of $\vec{a}$, $\vec{b}$ and/or $\vec{c}$ equivalent to $\overrightarrow{CE}$ /1

$\overrightarrow{CE} = -\vec{a} - \vec{b}$ ✓

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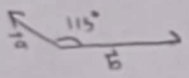
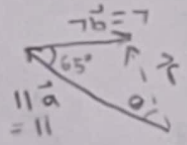
2. The vectors $\vec{a}$ and $\vec{b}$ are unit vectors (i.e., vectors with a magnitude of 1) that have an angle of 115^o between them. Determine the magnitude and direction of the vector $11\vec{a} + 7\vec{b}$ and state the direction of the resultant vector in terms of $\vec{a}$ and $\vec{b}$ (not in terms of $11\vec{a}$ and $7\vec{b}$.) Round both the magnitude and direction to one decimal place /4





$|7| = \sqrt{11^2 + 7^2 - 2(11)(7)\cos 65}$

≈ 10.2 units

$\frac{\sin\theta}{7} = \frac{\sin 65}{10.2}$

$\theta \approx 38.3°$

4

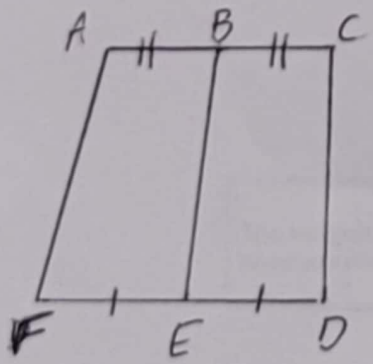
$|11\vec{a} + 7\vec{b}| =$ 10.2 units

Round your answer to one decimal place

The direction of $11\vec{a} + 7\vec{b}$ is 38.3° stated from $\vec{a}$ towards $\vec{b}$

Round your answer to one tenth of a degree (i.e., one decimal place).

3. Show that $\overrightarrow{FA} + \overrightarrow{DC} = 2\overrightarrow{EB}$. Show sufficient work to justify your answer. /3



$\overrightarrow{FA} = \overrightarrow{FE} + \overrightarrow{EB} + \overrightarrow{BA}$

$\overrightarrow{DC} = \overrightarrow{DE} + \overrightarrow{EB} + \overrightarrow{BC}$

$\overrightarrow{FA} + \overrightarrow{DC} = 0 + 2\overrightarrow{EB} + 0$

$\therefore \overrightarrow{FA} + \overrightarrow{DC} = 2\overrightarrow{EB}$ ✓

$\overrightarrow{FE}$ & $\overrightarrow{DE}$ equal & opposite = 0

$\overrightarrow{BA}$ & $\overrightarrow{BC}$ equal & opposite = 0

3

9

4. Three given points are $A(2,8,6)$ and $B(-1,4,11)$ and $P(w,0,0)$. Solve for w if we know that $|\overrightarrow{AP}| = |\overrightarrow{BP}|$.
/3

$\overrightarrow{AP} = \langle w-2, -8, -6 \rangle$

$\overrightarrow{BP} = \langle w+1, -4, -11 \rangle$

$|\overrightarrow{AP}| = |\overrightarrow{BP}|$

$\sqrt{(w-2)^2+(-8)^2+(-6)^2} = \sqrt{(w+1)^2+(-4)^2+(-11)^2}$ (square both sides + expand)

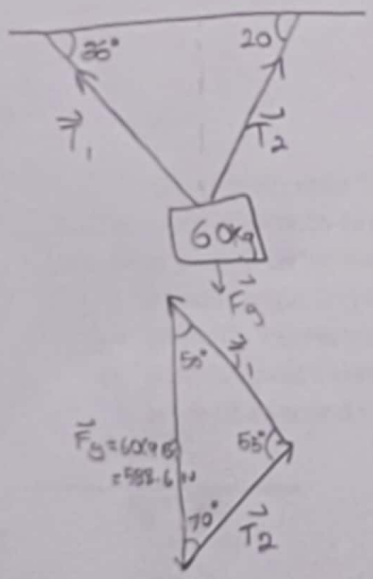
$w^2 - 4w + 4 + 64 + 36 = w^2 + 2w + 1 + 16 + 121$

$-34 = 6w$

$w = -17/3$

$w = -17/3$

5. A mass of 60 kg is suspended by two ropes. One of the ropes makes an angle of 20° with the ceiling and the other rope makes an angle of 35° with the ceiling. What is the magnitude of the tension in each of the ropes?
You can just state the magnitude of each tension; you do not have to state direction. Round your answer to the nearest tenth of a Newton (i.e., one decimal place). /5



$\frac{|\vec{T}_2|}{\sin 55} = \frac{|\vec{F}_g|}{\sin 55}$

$|\vec{T}_2| = \frac{588.6 \sin 55}{\sin 55}$

$= 588.6$ N

$\frac{|\vec{T}_1|}{\sin 70} = \frac{|\vec{F}_g|}{\sin 55}$

$|\vec{T}_1| = \frac{588.6 \sin 70}{\sin 55}$

$= 675.2$ N

The magnitudes of the tensions in the ropes are 588.6 N and 675.2 N.
Round your answers to one tenth of a Newton (i.e., one decimal place). You can state the magnitudes in either order.

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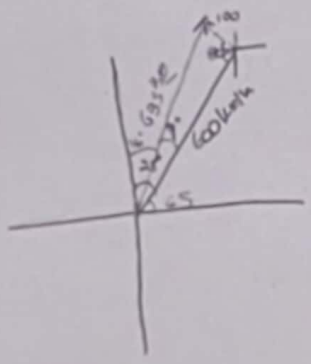
6. A plane is traveling with a component velocity of 600 km/h, N 25° E (component vector) when it encounters a wind (component vector) with a speed of 100 km/h. The resultant velocity of the airplane is 635 km/h, N 16° E. What is the direction of the wind (i.e. what is the direction of the wind vector)? Round your answer to the nearest tenth of a degree (i.e., one decimal place) /5



$635^2 = 600^2 + 100^2 - 2(600)(100)\cos\theta$

$\theta = \cos^{-1}\left(\frac{635^2 - 600^2 - 100^2}{-2(600)(100)}\right)$

$= 106.1°$

$\text{angle} = 106.1° - 65°$

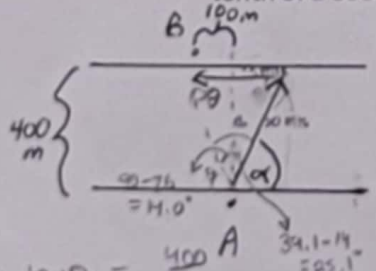
$= 41.1°$

The direction of the wind vector is

W41.1° N

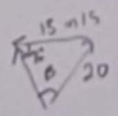
Round your answer to the nearest tenth of a degree (i.e., one decimal place)

7. A river that is 400 m wide flows with a current of 13 m/s. Franklin can row 20 m/s in still water. Franklin wishes to travel directly to a point 100 m downstream from his starting point (i.e., he wishes to travel from point A to point B on the map shown)
   a. At what angle to the bank should Franklin steer his boat (i.e., what is the value of α in the diagram shown?). Round your answer to the nearest tenth of a degree (i.e., one decimal place) if necessary.
   b. How long will it take Franklin to travel from point A to point B.? Round your answer to the nearest tenth of a second (i.e., one decimal place) if necessary. (total 7)



a) $\tan\theta = \frac{400}{100}$

$\theta \approx 76.0°$



$\frac{\sin\beta}{13} = \frac{\sin 76}{20}$

$\beta = 39.1°$

$\alpha = 180 - \theta - \beta$

$= 180 - 76 - 39.1°$

$= 64.9°$

$\alpha =$ 64.9°

Round your answer to the nearest one tenth of a degree (i.e., one decimal place)

b) $t = \frac{\text{distance}}{v_{\text{vertical}}}$

$\cos 25° = \frac{\text{vertical speed}}{20}$

$v_{\text{vert}} = 20\cos 25.1$

$= 20\sin 64.9$

$= \frac{400}{20\sin 64.9}$

$= 22.1$ seconds

It will take Frankling 22.1 seconds to get from A to B

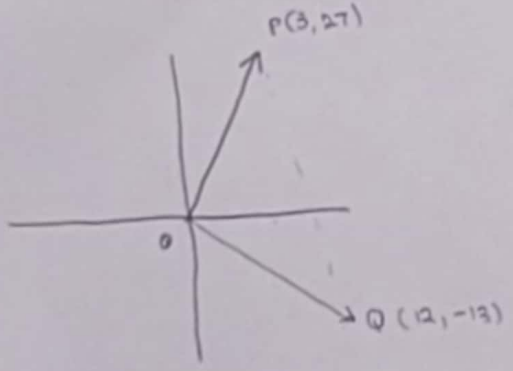
Round your answer to the nearest tenth of a second (i.e., one decimal place)

/12

8. We know O represents the point (0,0). We also know that the vector $\overrightarrow{OP} = \overrightarrow{(3,27)}$ and the vector $\overrightarrow{OQ} = \overrightarrow{(12,-13)}$. Determine:

   a. the components of the vector $\overrightarrow{PQ}$

   b. the components of a unit vector in the same direction as $\overrightarrow{PQ}$. /3



a) $\overrightarrow{PQ} = (12-3, -13-27)$

$= \overrightarrow{(9, -40)}$

b) $|\overrightarrow{PQ}| = \sqrt{9^2 + 40^2}$

$= 41$

unit vector $\frac{1}{41}\overrightarrow{(9, -40)}$

$= \left(\frac{9}{41}, \frac{-40}{41}\right)$

$\overrightarrow{PQ} = \overrightarrow{(9, -40)}$
A unit vector in the same direction as $\overrightarrow{PQ}$ is $\left(\frac{9}{41}, \frac{-40}{41}\right)$

3