1. A car races down a straight road. The finish line is 1200 m from the starting line. The distance traveled by the car from the starting line, in metres, is given by $s(t) = 8t^2 + 4t$, where $s(t)$ is measured in t in metres. How fast is the car traveling when it crosses the finish line. Include proper units of measurement in second your answer. $t \geq 0$
(4 marks)

$8t^2+4t=1200$

$2t^2+t-300=0$

$t=\frac{-1\pm\sqrt{1+2400}}{4}$

$=\frac{-1\pm 49}{4}$

$t=12\,s$ / -2.5 ⊗

$s'(t)=16t+4$

$s'(12)=16\cdot 12+4$

$=196\ m/s$

✓ 4

The car is traveling at a speed of 196 m/s when it crosses the finish line
(include proper units of measurement in your answer)

2. Determine the maximum value of the function $f(x) = \frac{4}{x^2-6x+12}$ over the domain $-1 \leq x \leq 6$. You must show sufficient work to justify your answer and include the domain in your analysis. Your final answer should contain an exact value, simplified, with no decimal approximations. (5 marks)

$f(x) = 4(x^2-6x+12)^{-1}$, $-1\leq x\leq 6$

$f'(x) = -4(x^2-6x+12)^{-2}(2x-6)$

$=\frac{-4(2x-6)}{(x^2-6x+12)^2}$

$f'(x)=0$

$-4(2x-6)=0$

$2x-6=0$

$x=3$

$f(-1)\approx 0.21 = \frac{4}{19}$

$f(6)=0.\bar{3}=\frac{1}{3}$

$f(3)=1.\bar{3}=\frac{4}{3}$

✓ 5

The maximum value of the function on the given interval is $\frac{4}{3}$
(exact value, no decimals in final answer)

3. You are given a rectangular piece of sheet metal with a length of 90 cm and a width of 60 cm. You cut a square shaped piece out of each corner so that you can fold up the sides and create a rectangular box with an open top. What is the maximum possible volume of the box? You must include a domain analysis which includes the maximum and minimum possible values of your independent variable. You can round your answer to one decimal place. Include proper units of measurement in your answer. (6 marks)

$L=90\,cm \quad W=60\,cm$

$V=(90-2x)(60-2x)x \qquad V'=12x^2-600x+5400$

$=(5400-120x-180x+4x^2)x \qquad 60-2x>0 \rightarrow \cancel{x>30}$ $x<30$.

$=4x^3-300x^2+5400x \qquad (0<x<30)$.

$V'=0$

$\cancel{4x^3-300x^2+5400x=0}$

$12x^2-600x+5400=0$

$2x^2-100x+900=0$

$x^2-50x+450=0$

$x=\frac{50\pm\sqrt{2500-1800}}{2}$

$=\frac{50\pm\sqrt{700}}{2}$

$\simeq 38.2$ / 11.8

& $V(0)=0$

$V(11.8)\simeq 28520.1\ cm^3$

$V(30)=0$

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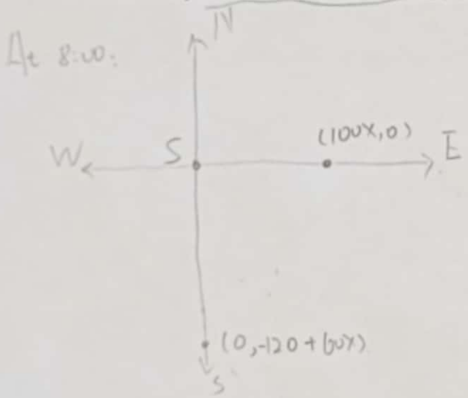
The maximum possible volume of the box is $28520.1\ cm^3$ (include correct units of measurement)

4. A train leaves a station at 8:00 a.m. and heads directly east at a speed of 100 km/h. Another train, which has been heading north at a speed of 60 km/h, reaches the station at 10:00 a.m. What is the minimum distance between the two trains? Round your answer to the nearest tenth of a km (i.e., one decimal place) if necessary. You do not need to include a domain analysis. (7 marks)

At 8:00:



$d=\sqrt{(100x)^2+(60x-120)^2}$

$=[10000x^2+3600x^2+14400-14400x]^{\frac{1}{2}}$

$=(13600x^2-14400x+14400)^{\frac{1}{2}}$

$d'=\frac{1}{2}(13600x^2-14400x+14400)^{-\frac{1}{2}}(27200x-14400)$

$=\frac{13600x-7200}{(13600x^2-14400x+14400)^{\frac{1}{2}}}$

$d'=0 \rightarrow 13600x-7200=0$

$136x=72$

$x\simeq 0.5\ h$

$\simeq 102.9563$

$d(0.5)\simeq 103.0\ km$

7

The minimum possible distance between the trains is 103.0 km (round your answer to one decimal place if necessary)