

1. For each of the following, determine all critical points and whether the point represents a local maximum, a local minimum, or neither

a.  $y = 8 - \sqrt{x^2 + 12x + 45}$

$$y = -(x^2 + 12x + 45)^{\frac{1}{2}} + 8$$

$$y' = \left(-\frac{1}{2}\right)(x^2 + 12x + 45)^{-\frac{1}{2}}(2x + 12)$$

$$0 = \frac{(2x + 12)}{(-2)(x^2 + 12x + 45)^{\frac{1}{2}}}$$

$$0 = -(x + 6)$$

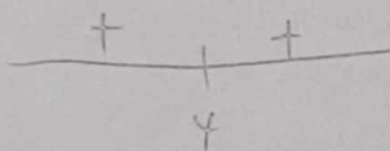
$$y' = 0 \text{ when } x = -6$$

b.  $y = (2x - 8)^{\frac{2}{3}}$

$$y = \frac{2}{3}(2x - 8)^{-\frac{1}{3}}(2)$$

$$0 = \frac{2}{3(2x - 8)^{\frac{2}{3}}}$$

$y'$  is undefined when  $x = 4$



$$y = (8 - 8)^{\frac{1}{2}} = 0$$

$(-6, 5)$

$$x^2 + 12x + 45 = 0$$

$$x = \frac{-12 \pm \sqrt{144 - 4(45)}}{2}$$

critical point  $(-6, 5)$

is a local maximum

$$y = 8 - \sqrt{36 + 12(-6) + 45}$$

$$= 8 - 3$$

$$= 5$$

$\therefore$  the point  $(-6, 5)$  is neither a local maximum nor a local minimum.

2. For the following function, determine the equations of all asymptotes and the coordinates of all points that are either a local maximum or a local minimum

$$y' = \frac{(1)(x^2 - 9) - (x - 5)(2x)}{(x^2 - 9)^2}$$

$$0 = \frac{x^2 - 9 - 2x^2 + 10x}{(x^2 - 9)^2}$$

$$0 = \frac{-x^2 + 10x - 9}{(x^2 - 9)^2}$$

$$0 = -\frac{(x - 1)(x - 9)}{[(x + 3)(x - 3)]^2}$$

$$(x^2 - 9)^2 = 0$$

$$x^2 - 9 = 0$$

$$x^2 = 9$$

$$x = \pm 3$$

$$y = \frac{x - 5}{x^2 - 9}$$

$$y = \frac{x - 5}{(x + 3)(x - 3)}$$

$$x = 1:$$

$$y = \frac{1 - 5}{1 - 9} = \frac{-4}{-8} = \frac{1}{2}$$

Equations of asymptotes:

V.A. at  $x = -3$  and  $x = 3$

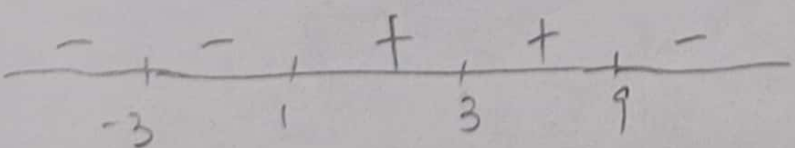
H.A. at  $y = 0$

All local maximum points (if none, leave blank)

$(9, \frac{1}{8})$

All local minimum points (if none, leave blank)

$(1, \frac{1}{2})$



$$x = 9$$

$$y = \frac{9 - 5}{81 - 9} = \frac{4}{72} = \frac{1}{18}$$

3. Determine the equations of any asymptotes and the locations of any holes, including y-coordinate, for the curve

$$y = \frac{x^3 - 3x^2 + 6x}{2x^2 - 2x}$$

$$y = \frac{x(x^2 - 3x + 6)}{(2x)(x-1)}$$

$$y = \frac{x^2 - 3x + 6}{2(x-1)}$$

hole at  $x=0$

$$y = \frac{0 - 0 + 6}{2(-1)}$$

$$= \frac{6}{-2}$$

$$= -3$$

$$(0, -3)$$

/5

$$y = x - 2$$

$$\begin{array}{r} \frac{1}{2}x - 1 \\ 2x - 2 \overline{) x^2 - 3x + 6} \\ \underline{-(x^2 - x)} \phantom{+ 6} \\ -2x + 6 \\ \underline{-(-2x + 2)} \\ 4 \end{array}$$

Equation of vertical asymptote(s)  
(if none, leave blank)

$$x = 1$$

Location of hole including y-coordinate (if none, leave blank)

$$(0, -3)$$

Equation of horizontal asymptote(s)  
(if none, leave blank)

Equation of oblique asymptote  
(if none, leave blank)

$$y = \frac{1}{2}x - 1$$

4. The function  $f(x) = ax^3 + 12x^2 + 36x + b$  has a critical point at  $(3, 125)$ . Determine the values of  $a$  and  $b$ .

$$f(x) = 3ax^2 + 24x + 36$$

$$f'(3) = 27a + 72 + 36$$

$$108 = 27a + 108$$

$$-108 = 27a$$

$$a = -4$$

$$f(x) = -4x^3 + 12x^2 + 36x + b$$

$$125 = -4(27) + 12(9) + 36(3) + b$$

$$125 = -108 + 108 + 108 + b$$

$$125 = 108 + b$$

$$b = 17$$

$$a = -4$$

$$b = 17$$