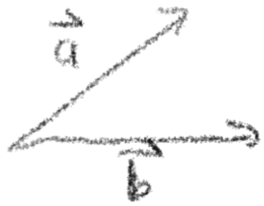
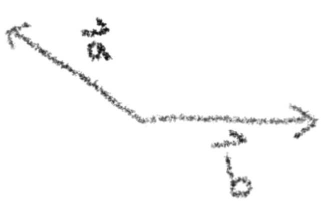
PDF 7.060 Scalar and Vector Projections

Refer to the diagram below:



The scalar projection of $\vec{a}$ on $\vec{b}$ is obtained by drawing a line from the head of $\vec{a}$ perpendicular to $\vec{b}$ or to an extension of $\vec{b}$.
Let's establish a formula for the scalar projection of $\vec{a}$ on $\vec{b}$ in this situation.

But what if the angle is obtuse?



In this case, we just extend $\vec{b}$ in its opposite direction.

We state that the scalar projection is negative. Luckily, the cosine value of an obtuse angle is negative.
Let's establish a formula for the scalar projection of $\vec{a}$ on $\vec{b}$ in this situation.

But, what if $\vec{b}$ is seemingly not long enough?

In that case we just extend $\vec{b}$

Let's establish a formula for the scalar projection of $\vec{a}$ on $\vec{b}$ in this situation.

In each of the cases shown, we see that the scalar projection has the same formula.

We will refer to the scalar projection of $\vec{a}$ on $\vec{b}$ as $scal_{\vec{b}}^{\vec{a}}$ and we will state our formula:

$$scal_{\vec{b}}^{\vec{a}} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$$

Vector Projections

When we evaluate the scalar projection of one vector onto another, we get a scalar quantity.

That projection is always on the vector being projected onto.

What if we want a vector projection instead?

We can multiply the scalar projection by a unit vector in the direction of the vector being projected onto to get a vector projection.

In other words,

$vect_{\vec{b}}^{\vec{a}} = scal_{\vec{b}}^{\vec{a}} \;\times$ (unit vector in the direction of $\vec{b}$)

We said in one of our earlier lessons that to get a unit vector in the direction of a vector, we simply divide that vector by its own magnitude.

In other words, (unit vector in the direction of $\vec{b}$) $= \frac{1}{|\vec{b}|}\vec{b}$

$$vect_{\vec{b}}^{\vec{a}} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} \times \frac{1}{|\vec{b}|}\vec{b}$$

$$vect_{\vec{b}}^{\vec{a}} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\vec{b}$$

But remember from our first lesson on the dot product, $|\vec{b}|^2 = \vec{b}\cdot\vec{b}$

Therefore,

$$\boxed{vect_{\vec{b}}^{\vec{a}} = \frac{\vec{a}\cdot\vec{b}}{\vec{b}\cdot\vec{b}}\vec{b}}$$

Example 1

Given the vectors $\vec{a} = \overrightarrow{(-3,4,5\sqrt{3})}$ and $\vec{b} = \overrightarrow{(-2,2,-1)}$, determine the scalar and vector projections of $\vec{a}$ on $\vec{b}$ and of $\vec{b}$ on $\vec{a}$

Determining Projections onto an Axis

To determine the scalar and vector projections made by a vector with one of the positive axes, you simply need to determine the scalar or vector projection of that vector with the associated unit vector, either $\vec{\imath}$, $\vec{\jmath}$ or $\vec{k}$.

Example 2

Determine the scalar projection of the vector $\vec{u} = (-3,7,4)$ on the vector $\vec{v} = \overrightarrow{(1,0,0)}$ and then on the vector $\vec{w} = \overrightarrow{(100,0,0)}$.

The scalar and vector projections of the vector $\vec{u}$ are the same on the vector $\overrightarrow{(1,0,0)}$ as they are on the vector $\overrightarrow{(100,0,0)}$.

This makes sense

We see that if we extend the floor on the first diagram, then the projections are the same.

Projecting onto an Axis

Let's extend the above discussion to projecting onto an axis. Therefore, if we are asked to determine the projection of a vector onto the positive x-axis, we can project that vector onto $\vec{\imath}$. If we are asked to determine the projection of a vector onto the negative x-axis, we can project that vector onto $-\vec{\imath}$.

Similarly, if we are asked to determine the projection of a vector onto the positive y-axis, we project that vector onto $\vec{\jmath}$. If we are asked to determine the projection of a vector onto the negative y-axis, we can project that vector onto $-j$.

Finally, if we are asked to determine the projection of a vector onto the positive z-axis, we project that vector onto $\vec{k}$. If we are asked to determine the projection of a vector onto the negative z-axis, we can project that vector onto $-\vec{k}$.

Direction Cosines

Suppose we want to know the angle made by the vector $\overrightarrow{(a,b,c)}$ with the positive x-axis

if $\vec{v} = \overrightarrow{(a,b,c)}$ is a vector in R^3, then

$\cos\alpha = \frac{a}{\sqrt{a^2+b^2+c^2}}$, $\cos\beta = \frac{b}{\sqrt{a^2+b^2+c^2}}$, and $\cos\gamma = \frac{c}{\sqrt{a^2+b^2+c^2}}$,

where α is the vector that $\vec{v}$ makes with the positive x-axis,

β is the vector that $\vec{v}$ makes with the positive y-axis, and

γ is the vector that $\vec{v}$ makes with the positive z-axis,

Example

Determine the angle made by the vector $\overrightarrow{(-2,-6,3)}$ with each of the positive axes.