PDF 7.080 Applications of the Dot Product and the Cross Product

Application 1: Work

Suppose that $\vec{f}$ is a constant force operating on an object at O so that this force moves the object from O to A.

Let $\vec{s}$ be the vector from O to A, and let $s = \left|\overrightarrow{OA}\right|$ be the scalar representing the distance that the object is displaced.

We know from physics that work is a scalar quantity which is equal to the magnitude of the force applied to the object in the direction of the movement, times the distance moved. The distance moved is the magnitude of the displacement, $|\vec{s}|$

The magnitude of the force applied is $\left|\vec{f}\right|$, and the magnitude of the force applied in the direction of the displacement is $\left|\vec{f}\right| \cos\theta$

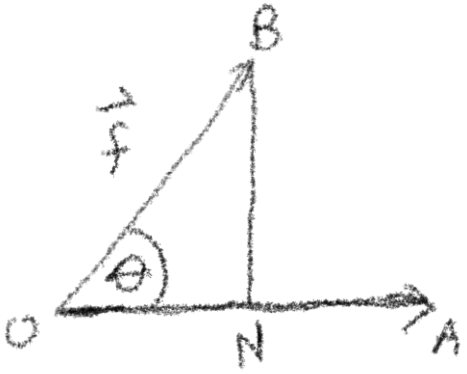
Work = (force in the direction of movement) x (distance traveled)

$W = \left|\vec{f}\right| \cos\theta \times |\vec{s}|$

$W = \left|\vec{f}\right||\vec{s}| \cos\theta$

$W = \vec{f} \cdot \vec{s}$

When force is measured in Newtons and displacement is measured in metres, then work is a scalar quantity measured in Joules (J). $1\ N \cdot m = 1$ J



$$W = \vec{f} \cdot \vec{s}$$

Example 1

Marianna is pulling her daughter in a toboggan and is exerting a force of 40 N acting at 24° to the ground. If Marianna pulls the child a distance of 100 m, how much work was done?

Application 2: Area

The area of a parallelogram determined by the vectors $\vec{a}$ and $\vec{b}$ is equal to $\left|\vec{a} \times \vec{b}\right|$

Since a triangle is one half of a parallelogram, therefore,

the area of a triangle determined by the vectors $\vec{a}$ and $\vec{b}$ is equal to $\frac{1}{2}\left|\vec{a} \times \vec{b}\right|$

* The area of a parallelogram formula is developed in the powerpoint

Example 2

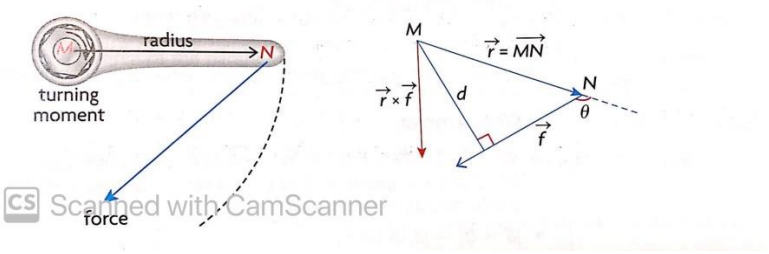
Determine the area of the parallelogram determined by the vectors $\vec{a} = \overrightarrow{(4,-7,5)}$ and $\vec{b} = \overrightarrow{(-1,0,2)}$

Example 3

Determine the area of the triangle determined by the vectors $\vec{u} = \overrightarrow{(-6,-1,-4)}$ and $\vec{v} = \overrightarrow{(15,2,1)}$

Application 3: Torque

Torque is the turning effect (twisting effect) of a force about a point



In the diagram shown, the torque of the force $\vec{f}$ about the point *M* is defined to be the vector $\vec{r} \times \vec{f}$. The magnitude of the torque is the product of the magnitude of the force (i.e., $|\vec{f}|$) and the distance d.
The force $\vec{f}$ is measured in Newtons, and the distance d is measured in metres, so the unit of magnitude for torque is Newton metres, or Joules (J), which is the same unit that work is measured in.

Example 4

A 20 N force is applied at the end of a wrench that is 40 cm in length. The force is applied at an angle of 60º to the wrench. Calculate the magnitude of the torque about the point of rotation *M*.