Graphing Quiz Version 1 Friday, April 14, 2023 Name: [redacted]

1. For each of the following, determine all critical points and whether the point represents a local maximum, a local minimum, or neither

a. $y = 8 - \sqrt{x^2 + 12x + 45}$ /5

$y = -(x^2+12x+45)^{\frac{1}{2}} + 8$

$y' = (-\frac{1}{2})(x^2+12x+45)^{-\frac{1}{2}}(2x+12)$

$0 = \frac{(2x+12)}{(-2)(x^2+12x+45)^{\frac{1}{2}}}$

$0 = \frac{-(x+6)}{(x^2+12x+45)^{\frac{1}{2}}}$

$y' = 0$ when $x = -6$

$x^2+12x+45 = 0$

$x = \frac{-12 \pm \sqrt{144-4(45)}}{2}$

X

critical point $(-6, 5)$

$y = 8 - \sqrt{36+12(-6)+45}$

$= 8 - 3$

$= 5$

+ | − (at $x = -6$)

is a local maximum

b. $y = (2x-8)^{\frac{1}{3}}$ /5

$y = \frac{1}{3}(2x-8)^{-\frac{2}{3}}(2)$

$0 = \frac{2}{3(2x-8)^{\frac{2}{3}}}$

y' is undefined when $x = 4$

+ | + (at $x = 4$)

$y = (8-8)^{\frac{1}{3}} = 0$

$(4, 0)$

∴ the point $(4,0)$ is neither a local maximum nor a local minimum.

2. For the following function, determine the equations of all asymptotes and the coordinates of all points that are either a local maximum or a local minimum /5

$y = \frac{x-5}{x^2-9}$

$y = \frac{x-5}{(x+3)(x-3)}$

$y' = \frac{(1)(x^2-9) - (x-5)(2x)}{(x^2-9)^2}$

$0 = \frac{x^2 - 9 - 2x^2 + 10x}{(x^2-9)^2}$

$0 = \frac{-x^2+10x-9}{(x^2-9)^2}$

$0 = \frac{-(x-1)(x-9)}{[(x+3)(x-3)]^2}$

$(x^2-9)^2 = 0$

$x^2 - 9 = 0$

$x^2 = 9$

$x = \pm 3$

− | − | + | + | − (at -3, 1, 3, 9)

$x = 1$:

$y = \frac{1-5}{1-9} = \frac{-4}{-8} = \frac{1}{2}$

$x = 9$:

$y = \frac{9-5}{81-9} = \frac{4}{72} = \frac{1}{18}$

Equations of asymptotes:
V.a. at $x = -3$ and at $x = 3$
h.a. at $y = 0$

All local maximum points (if none, leave blank)
$(9, \frac{1}{18})$

All local minimum points (if none, leave blank)
$(1, \frac{1}{2})$

/15

3. Determine the equations of any asymptotes and the locations of any holes, including y-coordinate, for the curve $y = \frac{x^3-3x^2+6x}{2x^2-2x}$ /5

$y = \frac{x(x^2-3x+6)}{2x(x-1)}$

$y = \frac{x^2-3x+6}{2(x-1)}$

hole at $x=0$

$y = \frac{0-0+6}{2(-1)}$

$= \frac{6}{-2}$

$= -3$

$(0,-3)$

	1	-3	6
1		1	-2
	1	-2	4

$y = x-2$

5

$\frac{\frac{1}{2}x - 1}{2x-2 \overline{)\, x^2-3x+6}}$

$-(x^2 - x)$

$-2x+6$

$-(-2x+2)$

4

Equation of vertical asymptote(s) (if none, leave blank)	Location of hole including y-coordinate (if none, leave blank)
$x=1$ ✓	$(0,-3)$ ✓
Equation of horizontal asymptote(s) (if none, leave blank)	**Equation of oblique asymptote (if none, leave blank)**
	$y = \frac{1}{2}x - 1$ ✓

4. The function $f(x) = ax^3 + 12x^2 + 36x + b$ has a critical point at (3, 125). Determine the values of a and b. /4

$f'(x) = 3ax^2 + 24x + 36$

$f'(3) = 27a + 72 + 36$

$0 = 27a + 108$

$-108 = 27a$

$a = -4$

$f(x) = -4x^3 + 12x^2 + 36x + b$

$125 = -4(27) + 12(9) + 36(3) + b$

$125 = -108 + 108 + 108 + b$

$125 = 108 + b$

$b = 17$

4 ✓

$a =$	-4	$b =$	17

9