

1. Determine the maximum value of function $f(x) = \frac{324}{x} + 12 + 4x$ over the interval $-12 \leq x \leq -6$. (4 marks)

Do not include a domain analysis in your solution.

$$f(x) = 324x^{-1} + 12 + 4x$$

$$f'(x) = -324x^{-2} + 4$$

$$f'(x) = 0 \rightarrow 4 = 324x^{-2}$$

$$x^{-2} = \frac{1}{81}$$

$$x = 9 / -9$$

(x)

$$\therefore x = -9$$

$$f(-12) = -63$$

$$f(-9) = -60$$

$$f(-6) = -66$$

$$\therefore f(x)_{\max} = f(-9) = -60$$

The maximum value of the function over the given domain is -60

2. A rock slides on the ice and its displacement is given by $s(t) = 20t - t^{\frac{5}{2}}$ where $s(t)$ is the displacement measured in metres and t is measured in seconds. What is the displacement when the stone changes direction?

- You can assume that time must be positive.
- You do not need to give a direction with the displacement. Just the numerical value. Round your answer to one decimal place if necessary.

/4

$$S'(t) = V(t) = -\frac{5}{2}t^{\frac{3}{2}} + 20$$

Direction change $\rightarrow V = 0$

$$\therefore -\frac{5}{2}t^{\frac{3}{2}} + 20 = 0$$

$$-\frac{5}{2}t^{\frac{3}{2}} = -20$$

$$t^{\frac{3}{2}} = 8$$

$$t^3 = 64$$

$$t = 4$$

$$S(4) = 80 - 4^{\frac{5}{2}} = 48$$

When the stone changes direction, the displacement is 48 m.
(Not necessary to give a direction. Just a numerical value. Round your answer to one decimal place if necessary)

3. A rectangular poster has an inner rectangular printing area and margins of 5 cm on the top and bottom, as well as margins of 3 cm on the left and right sides. If the inner rectangular printing area of the poster has an area of ~~4335~~ cm^2 , then what is the minimum possible area of the entire poster (including the inner rectangular printing area and the margins all around it)?

- You do not need to include a domain analysis for this question. Round your answer to one decimal place if necessary.

$$l \cdot w = 4335 \text{ cm}^2 \quad \rightarrow \quad l = \frac{4335}{w}$$

$$\begin{aligned} A &= (l+10)(w+6) \\ &= lw + 10w + 6l + 60 \\ &= 4335 + 10w + \frac{26010}{w} + 60 \\ &= 26010w^{-1} + 10w + 4395 \end{aligned}$$

$$A' = -26010w^{-2} + 10$$

$$A' = 0 \rightarrow 10 = 26010w^{-2}$$

$$w^{-2} = \frac{1}{2601}$$

$$w = 51 \text{ or } -51 \text{ (X)}$$

$$\therefore w = 51 \text{ cm}$$

$$\begin{aligned} A &= 510 + 510 + 4395 \\ &= 5415 \text{ cm}^2 \end{aligned}$$

The minimum possible area of the entire poster is 5415 cm^2 .
(Round your answer to one decimal place if necessary)

4. A cylindrical aluminum can with an open top has a volume of 2000 cm^3 . (In other words, the cylinder has a bottom but no top). What radius of can will minimize the amount of aluminum needed to create the can? You must give an exact value.

- You must give an exact value. No decimal approximations. Reduce and simplify your fraction if necessary.
- No domain analysis is necessary.

$$V = \pi r^2 \cdot h = 2000 \text{ cm}^3 \rightarrow h = \frac{2000}{\pi r^2}$$

$$SA = \pi r^2 + 2\pi r h$$

$$= \pi r^2 + 2\pi r \left(\frac{2000}{\pi r^2} \right)$$

$$= \pi r^2 + 4000 r^{-1}$$

$$SA' = -4000 r^{-2} + 2\pi r$$

$$SA' = 0 \rightarrow 2\pi r = 4000 r^{-2}$$

$$\pi r = \frac{2000}{r^2}$$

$$\pi r^3 = 2000$$

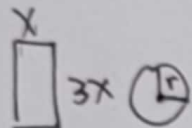
$$r^3 = \frac{2000}{\pi}$$

$$r = \sqrt[3]{\frac{2000}{\pi}}$$

To minimize the surface area of the can, the radius of the cylinder should be $\sqrt[3]{\frac{2000}{\pi}}$ cm.
(Give an exact value. Reduce your fraction and simplify if necessary. No decimals).

A wire with a length of 400 cm is divided into two parts. One part of the wire forms a rectangle with a length three times greater than the width. The other part of the wire forms a circle. What is the minimum possible combined area of the two shapes?

- You do not need to include a domain analysis in your answer. You can use decimal approximations rounded to two decimal places throughout your solution. Then you can round your final answer to one decimal place.



$$P = 2\pi r + 8x = 400 \text{ cm} \rightarrow 8x = 400 - 2\pi r$$

$$x = 50 - \frac{1}{4}\pi r$$

$$\begin{aligned}
 A &= \pi r^2 + 3x^2 \\
 &= \pi r^2 + 3\left(50 - \frac{1}{4}\pi r\right)^2 \\
 &= \pi r^2 + 3\left(2500 + \frac{1}{16}\pi^2 r^2 - 25\pi r\right) \\
 &= 7500 + \frac{3}{16}\pi^2 r^2 - 75\pi r + 3\pi r^2
 \end{aligned}$$

$$A' = \frac{3}{8}\pi r - 75\pi + 2\pi r$$

$$= \left(\frac{3}{8}\pi + 2\right)\pi r - 75\pi$$

$$A' = 0 \rightarrow \left(\frac{3}{8}\pi + 2\right)\pi r = 75\pi$$

$$r = \frac{75}{\left(\frac{3}{8}\pi + 2\right)}$$

$$r \approx 23.60 \text{ cm}$$

$$\therefore A = 7500 + \frac{3}{16}\pi^2 (23.60)^2 - 75\pi (23.60) + \pi (23.60)^2$$

$$\approx 4719.8 \text{ cm}^2$$

The minimum possible combined area of the two shapes is 4719.8 cm².
(Round your final answer to one decimal place if necessary)