[redacted]

1. Using first principles, determine the derivatives of each of the following. Using proper limit notation, but only when necessary. Then, evaluate that derivative at $x = 2$:

   a. $f(x) = \frac{1}{x}$

   b. $g(x) = 3x^2 + 2x - 1$

   c. $h(x) = \sqrt{2x-1}$

   d. $k(x) = \frac{3}{\sqrt{x-1}}$

   e $m(x) = \frac{x-1}{x}$

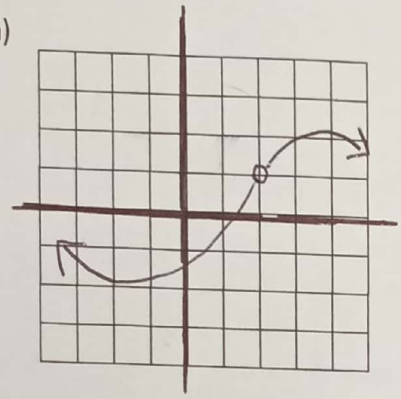
   e $p(x) = 4x + 5$

2. Prove the power rule. In other words, prove that if $f(x) = x^n$, then $f'(x) = nx^{n-1}$ (you can focus on cases when n is a positive integer, like we did in class)
3. Prove the product rule. In other words, prove that if $p(x) = f(x)g(x)$, then $p'(x) = f'(x)g(x) + f(x)g'(x)$
4. Prove the quotient rule. In other words, prove that if $h(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$, then $h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$

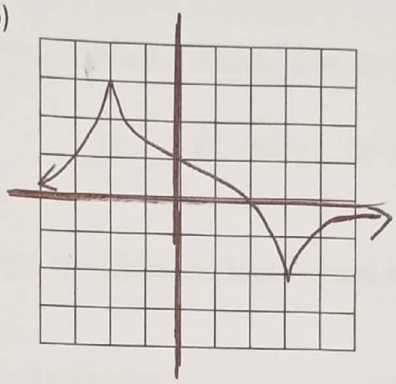
For the rest of the review, you can use any formula that you wish to answer the question.

5. For each of the following, state the domain on which f is differentiable.

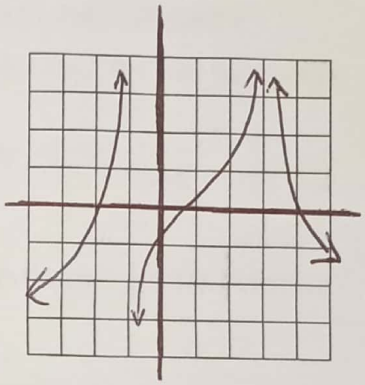
a)



b)



c)



6. Draw a function that is differentiable on the domain $\{x \,/\, x \neq -1, x \neq 3, x \in R\}$
7. Determine an expression for $f'(x)$, then evaluate $f'(a)$ for the given value of a for each of the following functions. If the value of $f'(a)$ does not exist, then state that. If the answer does exist, state the answer in reduced form without decimals.

   a. $f(x) = 5x^3 - 4x^2 + 2x - 1; \quad a = 3$

   b. $f(x) = \frac{3}{x} - \sqrt{x}; \quad a = 4$

   c.. $f(x) = -2(x+2)^4; \quad a = -4$

   d. $f(x) = 5x^{\frac{-5}{3}} - \sqrt{x+1} + \frac{1}{3\sqrt{3x+1}}; \quad a = 8$

   e.. $f(x) = 4(4x^2 + x - 4)^6(8x^3 - 15x^2 + x - 4)^2; \quad a = -1$

   f.. $f(x) = \frac{3x^2 - x + 1}{2x+5}; \quad a = 2$

   g. $f(x) = \left(\frac{x^3 - 4x}{x^2+3}\right)^4; \quad a = 1$

   h.. $f(x) = \frac{5x^3 - 12x^2 + 8x - 9}{x^2}; \quad a = -1$

8. Determine the equation of the line tangent to the curve $y = 4 - 3x + x^2$ where $x = 2$
9. Determine the equation of the line tangent to the curve $y = \frac{1-x^2}{x+2}$ where $x = 3$
10. Determine the equation of the normal line to the curve $y = 4x^2 - 9x + 1$ at $x = -3$
11. At what point(s) on the graph of $y = x^3 - 6x^2 - 13x + 1$ is the tangent line parallel to $2x - y + 14 = 0$?
12. Determine the points, if any, at which each of the following curves have horizontal tangent lines:

    a. $y = 4x^3 + 21x^2 - 24x - 17$

    b. $y = 7x^4 - 14x^2 + 11$

    c. $y = \frac{x^2 + 4x - 5}{x^2 + 6x + 5}$

13. Determine the coordinates of the point(s) on the curve $y = x^2 - 2x - 8$ that have tangent lines that extend through the point (3,-9)
14. Determine the coordinate(s) of the point(s) on the curve $y = x^2 + 2x + 3$ that have tangent lines that extend through the point (0,-1)
15. Show that the line $y = 8x - 2$ is tangent to the curve $y = x^2 + 6x - 1$
16. Solve for a if you know that the line $y = 12x + a$ is tangent to the curve $y = 2x^3 - 3x^2 + 1$
17. Solve for the values of a, b and c if you know that the function $y = ax^2 + bx + c$ has a horizontal tangent at the point (-7,-2) and passes through the point (-5, -14).
18. Given the following table, evaluate $h'(7)$ and $k'(4)$ given that $h(x) = f \circ g(x)$ and given that $k(x) = g \circ f(x)$

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
4	10	3	1	4
5	12	-1	15	-18
7	9	12	16	-3
10	23	0	-8	17
12	-6	-9	19	-22

19. The graph of $y = \frac{ax+b}{x^2-10x+21}$ has a horizontal tangent at the point (4,6). Determine the values of a and b.
20. Determine an expression for $\frac{dy}{dx}$ in each of the following relations and then state the equation of the tangent line at the given point.
    a. $3x^2y + 4xy^3 = 268$; (1,4) b. $x^3 - 4x^2y + 3y^4 = 147$; (3,-2)
    c.. $x = 2(x + y)^3$; (2, -1)
21. The circle $x^2 + y^2 = 169$ has two points on it with tangent lines that pass through the point (-7, 17). Determine the coordinates of those points.
22. Determine the following derivatives
    a. $\frac{d(5x^2-20x+25)}{dx}$ b. $\frac{d(5x^2-20x+25)}{d(5x^2-20x+25)}$ c. $\frac{d(5x^2-20x+25)}{d(x^2-4x+5)}$
    d.. $\frac{d(x-1)^2}{dx}$ e. $\frac{d(x^2-2x+1)}{dx}$ f. $\frac{d(x^2-2x+1)}{d(x-1)}$ g. $\frac{d(2x^2-4x+2)}{d(x-1)}$

Word Problems

In these word problems, you do not need to show your units of measurement in your solution, but you need to show them in the final answer (examples could include m/s or mL/hour, etc)

23. A 150 L gas tank has a leak. After t hours, the remaining volume, V in litres is given by $v(t) = 150\left(1 - \frac{t}{18}\right)^2$. How quickly is gas leaking from the gas tank when the tank is half-full? Include the proper units of measurement in your final answer.
24. A motorboat coasts towards a dock with its motor turned off. Its distance s, from the dock in metres t seconds after the engine is turned off is given by the function $s(t) = \frac{84-12t}{t+4}, 0 \le t \le 7$. Determine the speed of the boat when it bumps into the dock. Include proper units of measurement in your final answer.
25. The radius of a circular spot of blood on a paper towel is given by the function $r(t) = \frac{10+20t}{t+3}$ where r is measured in cm and t is measured in seconds. How quickly is the radius of the spot of blood increasing after 5 seconds. Include proper units of measurement in your final answer.
26. A population of bacteria grows according to the function $p(t) = 500\left(2 + \frac{9t}{40+2t^2}\right)$ where p represents the number of bacteria and t represents the number of hours that have passed. What is the rate of growth of bacteria after 3 hours. You must include proper units of measurement in your answer.