Vectors Test 2 Version 2 Thursday, May 30, 2024

1. Are the vectors $\vec{a} = \overrightarrow{(-2,-1,4)}$, $\vec{b} = \overrightarrow{(3,4,2)}$ and $\vec{c} = \overrightarrow{(1,8,22)}$ coplanar?
   - Just give the answer yes or no, but you must show the work to justify your decision (just giving correct answer without the work is not worth any marks) (3 marks)

$(1,8,22) = n(-2,-1,4) + m(3,4,2)$

FROM X: $1 = -2n + 3m$

Y: $8 = -n + 4m \rightarrow n = 4m - 8$

$1 = -2(4m-8) + 3m$

$1 = -8m + 16 + 3m$

$-15 = -5m$

$m = 3$ $\quad n = 4(3) - 8$

$n = 4$

TEST Z

$22 = 4n + 2m$

$22 = 4(4) + 2(3)$

$22 = 22$ ✓

3

YES

Just write yes or no in the box.

2. Determine the angle made by the vector $\vec{v} = \overrightarrow{(-4,-1,9)}$ with the positive z-axis. Round your answer to one tenth of a degree (i.e., one decimal place) if necessary. (2 marks)

$\cos\gamma = \frac{c}{\sqrt{a^2+b^2+c^2}}$

$= \frac{9}{\sqrt{16+1+81}}$

$= \frac{9}{\sqrt{98}}$

$\gamma = 24.6°$

2

$\gamma = 24.6°$

Round your answer to the nearest tenth of a degree (i.e., one decimal place) if necessary.

3. Determine the vector projection of the vector $\vec{a} = \overrightarrow{(4,-5,2)}$ on the vector $\vec{b} = \overrightarrow{(-2,5,6)}$. (2 marks)

$\text{vect}_{\vec{b}}\vec{a} = \frac{\vec{a}\cdot\vec{b}}{\vec{b}\cdot\vec{b}}\vec{b}$

$= \frac{-8-25+12}{4+25+36}(\overrightarrow{-2,5,6})$

$= \frac{-21}{65}(\overrightarrow{-2,5,6})$

2

$\text{vect}_{\vec{b}}\vec{a} = \frac{-21}{65}(\overrightarrow{-2,5,6})$

4. A 75 Newton force is applied at the end of a wrench that is 50 cm long. The force is applied at an angle of 56° to the wrench. Calculate the magnitude of the torque created (not necessary to include direction). (2 marks)

$|\vec{T}| = |r||F|\sin\theta$

$= (0.5\text{m})(75\text{N})\sin 56$

$= 31.1$ J

2

The magnitude of the torque is 31.1 J

Round your answer to one tenth of a joule (i.e., one decimal place) if necessary.

/9

5. The vectors $\vec{u} = \overrightarrow{(k,-1,2)}$ and $\vec{v} = \overrightarrow{(1,4,-8)}$ are such that $\theta = cos^{-1}\left(\frac{-2}{3}\right)$, where $\theta$ is the angle between $\vec{u}$ and $\vec{v}$. Determine all possible value(s) for $k$. (6 marks)

$(k,-1,2) \cdot (1,4,-8) = \sqrt{k^2+1+4}\ \sqrt{1+16+64}\ \cos\theta$

$k - 4 - 16 = \sqrt{k^2+5}\ \sqrt{81}\ \cos\left(\cos^{-1}\left(\frac{-2}{3}\right)\right)$

$k - 20 = \sqrt{k^2+5}\ (9)\left(\frac{-2}{3}\right)$

$k - 20 = \sqrt{k^2+5}\ (-6)$

SQUARE BOTH SIDES

$(k-20)(k-20) = (k^2+5)(36)$

$k^2 - 40k + 400 = 36k^2 + 180$

$0 = 35k^2 + 40k - 220$

$0 = 5(7k^2 + 8k - 44)$

$0 = 5(k-2)(7k+22)$

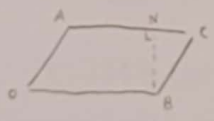
$k = 2 \quad \text{or} \quad 7k = -22$

$k = -22/7$

$k = 2$ or $\frac{-22}{7}$

State all possible solutions (could be one or more)

6. Parallelogram OBCA has its sides determined by $\overrightarrow{OA} = \overrightarrow{(3,5,6)}$ and $\overrightarrow{OB} = \overrightarrow{(-2,1,2)}$. The fourth vertex of the parallelogram is at point C. A line is drawn from point B perpendicular to side AC of the parallelogram to intersect AC at the point N. Determine the length of the line segment BN. (4 marks)




① Area = (base)(height)

= (3) (BN)

= 3BN

② Area = $|\vec{a} \times \vec{b}|$

$|\overrightarrow{OB}| = \sqrt{4+1+4}$

$= \sqrt{9}$

$= 3$

$3(BN) = \sqrt{509}$

$BN = \frac{\sqrt{509}}{3}$

$(4, -18, 13)$

$A = \sqrt{16+324+169}$

$= \sqrt{509}$

The length of the line segment BN is $\frac{\sqrt{509}}{3}$

7. The segment joining the points $A(8,-9)$ and $B(9,3)$ is the hypotenuse of the right triangle ABC. The third vertex C, lies on the line with the vector equation $\vec{r} = \overrightarrow{(10,-2)} + t\,\overrightarrow{(2,-1)}, t \in R$. Determine the coordinates of point C. State all possible solutions. (7 marks)

$x = 10 + 2t$

$y = -2 - t$

$C(10+2t, -2-t)$

$\vec{AC} = (10+2t-8, -2-t-(-9))$

$= (2+2t, 7-t)$

$\vec{BC} = (10+2t-9, -2-t-3)$

$= (1+2t, -5-t)$

$\vec{AC} \cdot \vec{BC} = 0$

$(2+2t)(1+2t) + (7-t)(-5-t) = 0$

$2 + 4t + 2t + 4t^2 - 35 - 7t + 5t + t^2 = 0$

$5t^2 + 4t - 33 = 0$

$(t+3)(5t-11) = 0$

| 5 | -33 | 4 |
|---|---|---|
| | 3 | 15 |
| 5 | -11 | -11 |

$t = -3$ $\qquad 5t = 11$

$t = 11/5$

$10 + 2(-3) = 4$ $\qquad -2-(-3) = 1$

$C(4,1)$

$10 + 2(11/5) = \frac{72}{5}$ $\qquad -2-(11/5) = -21/5$

$C\left(\frac{72}{5}, \frac{-21}{5}\right)$

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The coordinates of point C are $(4,1)$ and $\left(\frac{72}{5}, \frac{-21}{5}\right)$

Give all possible answers (could be one or more)

8. The vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = 12$ and $|\vec{b}| = 7$. If we know that the angle between $\vec{a}$ and $\vec{b}$ is 36°, then evaluate $(5\vec{a} - 2\vec{b}) \cdot (7\vec{a} - 3\vec{b})$. Round your answer to the nearest tenth (i.e., to one decimal place). (3 marks)

$(5\vec{a} - 2\vec{b}) \cdot (7\vec{a} - 3\vec{b})$

$= 35\vec{a}\cdot\vec{a} - 15\vec{a}\cdot\vec{b} - 14\vec{a}\cdot\vec{b} + 6\vec{b}\cdot\vec{b}$

$= 35|\vec{a}|^2 - 29\vec{a}\cdot\vec{b} + 6|\vec{b}|^2$

$= 35(12)^2 - 29(|\vec{a}||\vec{b}|\cos 36^\circ) + 6(7)^2$

$= 5040 - 1970.765 + 294$

$= 3363.235$

5334

3

$(5\vec{a} - 2\vec{b}) \cdot (7\vec{a} - 3\vec{b}) =$ 3363.2

Round your answer to one decimal place if necessary.

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