PDF 9.020 Systems of Equations

A linear system of equations can have zero, one, or an infinite number of solutions. Think of two lines. Those lines might be parallel and not intersect (no solutions), or they might not be parallel and intersect at one point (one solution), or they might be parallel and consistent (an infinite number of solutions).

Terms to Know

An ***inconsistent system*** is a system of equation that has zero solutions.

A ***consistent system*** is a system of equations that has one solution or an infinite number of solutions.

Two systems of equations are defined as ***equivalent systems of equations*** if every solution to one system is also a solution to the second system of equations, and vice versa

Elementary Row Operations

1. Multiply an equation by a nonzero constant.
2. Interchange any pair of equations.
3. Add a non-zero multiple of one equation to a second equation to replace the second equation.

Matrices

A matrix (plural matrices) is a very powerful tool in mathematics. For now, we will introduce matrices as a way to organize our solutions, keeping them neat and organized. Also, matrices will help us to see how we can use elementary row operations to solve a system of equations.

Matrices have many more uses than this, and there are entire university math courses dedicated to them.

Example 1

Solve the following system of equations using a matrix and elementary row operations.

$2x + y = -9$ ①

$x + 2y = -6$ ②

Example 2

Solve the following system of equations using a matrix and elementary row operations.

$3x + 4y = 8$ ①
$6x + 8y = 15$ ②

Example 3

Solve the following system of equations using a matrix and elementary row operations.

$5x - 2y = 11$ ①
$55x - 22y = 121$ ②

Examples with 3 Equations and 3 Unknowns

Example 4

Solve the following system of equations

$3x - 4y + 2z + 5 = 0$

$2x + y - 7z + 32 = 0$
$2x - 3y + 8z - 17 = 0$

Example 5

Solve the following system of equations. State whether there are 0 points of intersection, 1 point of intersection, or an infinite number of points of intersection.

$8x - 11y - 7z - 22 = 0$

$6x - 11y - 8z + 41 = 0$

$2x + 3y + 4z = 0$

Example 6

Solve the following system of equations. State whether there are 0 points of intersection, 1 point of intersection, or an infinite number of points of intersection.

$9x - 8y - 5z - 34 = 0$

$9x - 7y - 4z - 38 = 0$

$15x - 6y - z - 86 = 0$

Thinker

Example 7

Consider the following system of equations

$kx - 4y = 10$

$-2x + 8y = -20$

Solve for k such that this system has 0 solutions, 1 solution, or an infinite number of solutions