1. Determine the maximum value of function $f(x) = \frac{324}{x} + 12 + 4x$ over the interval $-12 \le x \le -6$. ~~(4 marks)~~

   Do not include a domain analysis in your solution.

$f(x) = 324x^{-1} + 12 + 4x$

$f'(x) = -324x^{-2} + 4$

$f'(x) = 0 \to 4 = 324x^{-2}$

$x^{-2} = \frac{1}{81}$

$x = 9 / -9$

(X)

$\therefore x = -9$

$f(-12) = -63$

$f(-9) = -60$

$f(-6) = -66$

$\therefore f(x)_{max} = f(-9) = -60$

3/3

The maximum value of the function over the given domain is -60

2. A rock slides on the ice and its displacement is given by $s(t) = 20t - t^{\frac{5}{2}}$ where $s(t)$ is the displacement measured in metres and t is measured in seconds. What is the displacement when the stone changes direction?
   - You can assume that time must be positive.
   - You do not need to give a direction with the displacement. Just the numerical value. Round your answer to one decimal place if necessary. /4

~~S(t)~~ $S'(t) = V(t) = -\frac{5}{2}t^{\frac{3}{2}} + 20.$

Direction change → $V = 0$

$\therefore -\frac{5}{2}t^{\frac{3}{2}} + 20 = 0$

$-\frac{5}{2}t^{\frac{3}{2}} = -20$

$t^{\frac{3}{2}} = 8$

$t^3 = 64$

$t = 4$

$S(4) = 80 - 4^{\frac{5}{2}}$

$= 48$

4

When the stone changes direction, the displacement is 48 m. (Not necessary to give a direction. Just a numerical value. Round your answer to one decimal place if necessary)

/1

3. A rectangular poster has an inner rectangular printing area and margins of 5 cm on the top and bottom, as well as margins of 3 cm on the left and right sides. If the inner rectangular printing area of the poster has an area of 4335 cm^2, then what is the minimum possible area of the entire poster (including the inner rectangular printing area and the margins all around it)?

- You do not need to include a domain analysis for this question. Round your answer to one decimal place if necessary.

$\cancel{20}$

$$l \cdot w = 4335 \ cm^2. \quad \rightarrow \quad L = \frac{4335}{w}$$

$$A = (l+10)(w+6)$$
$$= lw + 10w + 6l + 60$$
$$= 4335 + 10w + \frac{26010}{w} + 60$$
$$= 26010w^{-1} + 10w + 4395$$

$$A' = -26010w^{-2} + 10$$

$$A' = 0 \rightarrow \quad 10 = 26010w^{-2}$$
$$w^{-2} = \frac{1}{2601}$$
$$w = 51 \ / -51 \ (\times)$$
$$\therefore w = 51 \ cm$$

$$A = 510 + 510 + 4395$$
$$= 5415 \ cm^2$$

5
5

The minimum possible area of the entire poster is 5415 cm^2.
(Round your answer to one decimal place if necessary)

4. A cylindrical aluminum can with an open top has a volume of 2000 cm^3. (In other words, the cylinder has a bottom but no top). What radius of can will minimize the amount of aluminum needed to create the can? You must give an exact value.

- You must give an exact value. No decimal approximations. Reduce and simplify your fraction if necessary.
- No domain analysis is necessary.

~~15~~

$V = \pi r^2 \cdot h = 2000 \text{ cm}^3 \rightarrow h = \frac{2000}{\pi r^2}$

$SA = \pi r^2 + 2\pi r h$

$= \pi r^2 + 2\pi r \left(\frac{2000}{\pi r^2}\right)$

$= \pi r^2 + 4000 r^{-1}$

5/5

$SA' = -4000 r^{-2} + 2\pi r$

$SA' = 0 \rightarrow 2\pi r = 4000 r^{-2}$

$\pi r = \frac{2000}{r^2}$

$\pi r^3 = 2000$

$r^3 = \frac{2000}{\pi}$

$r = \sqrt[3]{\frac{2000}{\pi}}$ ✓

To minimize the surface area of the can, the radius of the cylinder should be $\sqrt[3]{\frac{2000}{\pi}}$ cm.
(Give an exact value. Reduce your fraction and simplify if necessary. No decimals).

A wire with a length of 400 cm is divided into two parts. One part of the wire forms a rectangle with a length three times greater than the width. The other part of the wire forms a circle. What is the minimum possible combined area of the two shapes?

- You do not need to include a domain analysis in your answer. You can use decimal approximations rounded to two decimal places throughout your solution. Then you can round your final answer to one decimal place.

x, $3x$, r

$P = 2\pi r + 8x = 400$ cm $\rightarrow 8x = 400 - 2\pi r$

$x = 50 - \frac{1}{4}\pi r$.

6/6

$A = \pi r^2 + 3x^2$

$= \pi r^2 + 3(50 - \frac{1}{4}\pi r)^2$

$= \pi r^2 + 3(2500 + \frac{1}{16}\pi^2 r^2 - 25\pi r)$

$= 7500 + \frac{3}{16}\pi^2 r^2 - 75\pi r + \pi r^2$

$A' = \frac{3}{8}\pi^2 r - 75\pi + 2\pi r$

$= (\frac{3}{8}\pi + 2)\pi r - 75\pi$

$A' = 0 \rightarrow (\frac{3}{8}\pi + 2)\pi r = 75\pi$

$r = \frac{75}{(\frac{3}{8}\pi + 2)}$

$r \approx 23.60$ cm

$\therefore A = 7500 + \frac{3}{16}\pi^2 (23.60)^2 - 75\pi(23.60) + \pi(23.60)^2$

≈ 4719.8 cm^2

The minimum possible combined area of the two shapes is 4719.8 cm².
(Round your final answer to one decimal place if necessary)