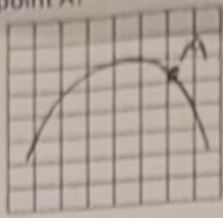
(1 mark)

d 1. Which of the following is true regarding the point A?

a. $f'(x) > 0$ and $f''(x) > 0$
b. $f'(x) > 0$ and $f''(x) < 0$
c. $f'(x) < 0$ and $f''(x) > 0$
d. $f'(x) < 0$ and $f''(x) < 0$
e. None of the above

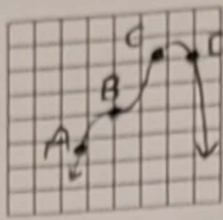


2

(1 mark)

b 2. Which point on the graph below is a point of inflection?

a. A
b. B
c. C
d. D



3. A critical point is a point at which the first derivative is either equal to zero or is undefined. Determine all critical points on the curve $y = x + 3(1-x)^{\frac{1}{3}}$ (5 marks)

$y' = 1 + (1-x)^{-\frac{2}{3}}(-1)$

$= 1 + \frac{(-1)}{\sqrt[3]{(1-x)^2}}$

$= \frac{\sqrt[3]{(1-x)^2} - 1}{\sqrt[3]{(1-x)^2}}$

$y' = 0 \rightarrow \sqrt[3]{(1-x)^2} = 1$

$1 - x = \pm 1$

$x = 2 / 0$

y' is undefined at $x = 1 \rightarrow (1, 1)$.

~~∴ (2, 2+3∛(3-2))~~.

$(0, 3)$

$(2, -1)$.

5

The critical points are

$(0,3)$, $(2,-1)$, $(1,1)$.

4. Determine the values of a, b and c given that $y = 3x^4 + ax^3 + bx^2 - 70x + c$ has a y-intercept of 8 and a point of inflection at $(-2,4)$. /5

$y' = 12x^3 + 3ax^2 + 2bx - 70$

$y'' = 36x^2 + 6ax + 2b$

$y'' = 0 \rightarrow 36(-2)^2 + 6a(-2) + 2b = 0$

$144 - 12a + 2b = 0$

$-12a + 2b = -144$

$-6a + b = -72$. ①

② - ① → $4a = 24$

$a = 6$

$-36 + b = -72$

$b = -36$

$y(-2) = 4 \rightarrow 48 - 8a + 4b + 140 + c = 4$.

$-8a + 4b + c = -184$.

$y(0) = 8 \rightarrow c = 8$

∴ $-8a + 4b = -192$

$-2a + b = -48$ ②

5

$a = 6$ $b = -36$ $c = 8$

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Sketch a function $y = f(x)$ that meets the following conditions

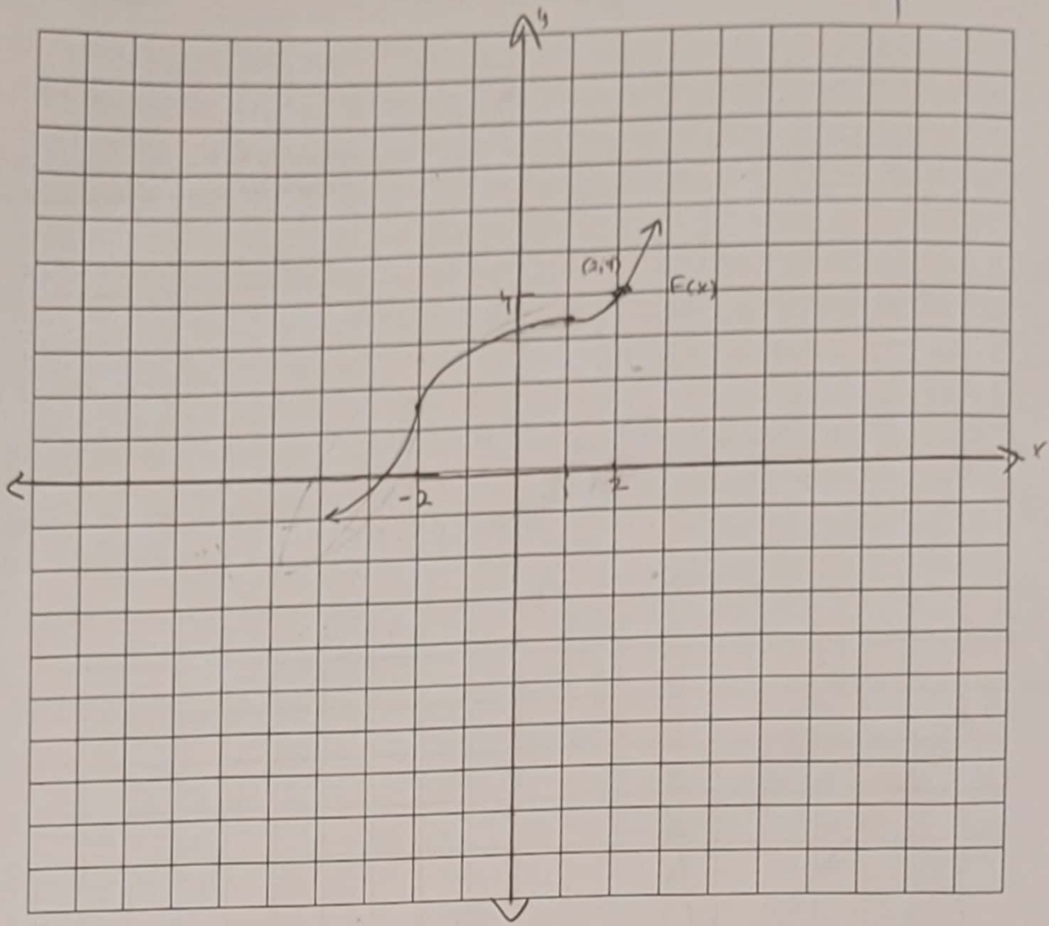
- $f'(x) > 0$ where $x < -2, -2 < x < 1, x > 1$
- $f'(x) = 0$ at $x = 1$
- $f'(x)$ is undefined at $x = -2$
- Concave up where $x < -2, x > 1$
- Concave down where $-2 < x < 1$
- $f(2) = 4$

(5 ~~marks~~)

$f'(x)$: $+ \;\; -2 \;\; + \;\; 1 \;\; +$

$f''(x)$: $+ \;\; -2 \;\; - \;\; 1 \;\; +$

4/4



6. Determine intervals of concavity for the function $y = (2-2x)^{\frac{3}{5}}$

In other words, state the interval(s) over which $f(x)$ is concave up and the interval(s) over which $f(x)$ is concave down. State your answer in interval notation. (4 marks)

$$y' = \frac{3}{5}(2-2x)^{-2/5}(-2)$$

$$= -\frac{6}{5}(2-2x)^{-2/5}$$

$$y'' = \frac{12}{25}(2-2x)^{-7/5}(-2)$$

$$= \frac{-24}{25(2-2x)^{7/5}}$$

y'' never $= 0$

y'' undef @ $x = 1$

y'': $- \;\; 1 \;\; +$

The function is concave up over the following interval(s)

$x \in (1, \infty)$

(State your answer in interval notation)

The function is concave down over the following interval(s)

$x \in (-\infty, 1)$

(State your answer in interval notation)

4

8

7. Fully analyze the following and graph the function $y = \frac{x^2-5x+7}{x-3}$

Do a first derivative analysis but not a second derivative analysis. (7 marks)

$$y = \frac{x^2-5x+7}{x-3}$$

$$\begin{array}{c|ccc} 3 & 1 & -5 & 7 \\ & 0 & 3 & -6 \\ \hline & 1 & -2 & 1 \end{array}$$

$y = x - 2 + \frac{1}{x-3}$

VA : $x = 3$

OA : $y = x - 2$

no x int $\sqrt{(-5)^2 - 4(7)(1)} = \sqrt{-}$

y int $(0, -7/3)$

$y' = 0$ @ $x = 2$ $x = 4$

$(2, -1)$ $(4, 3)$

y' undefined @ $x = 3$ (va)

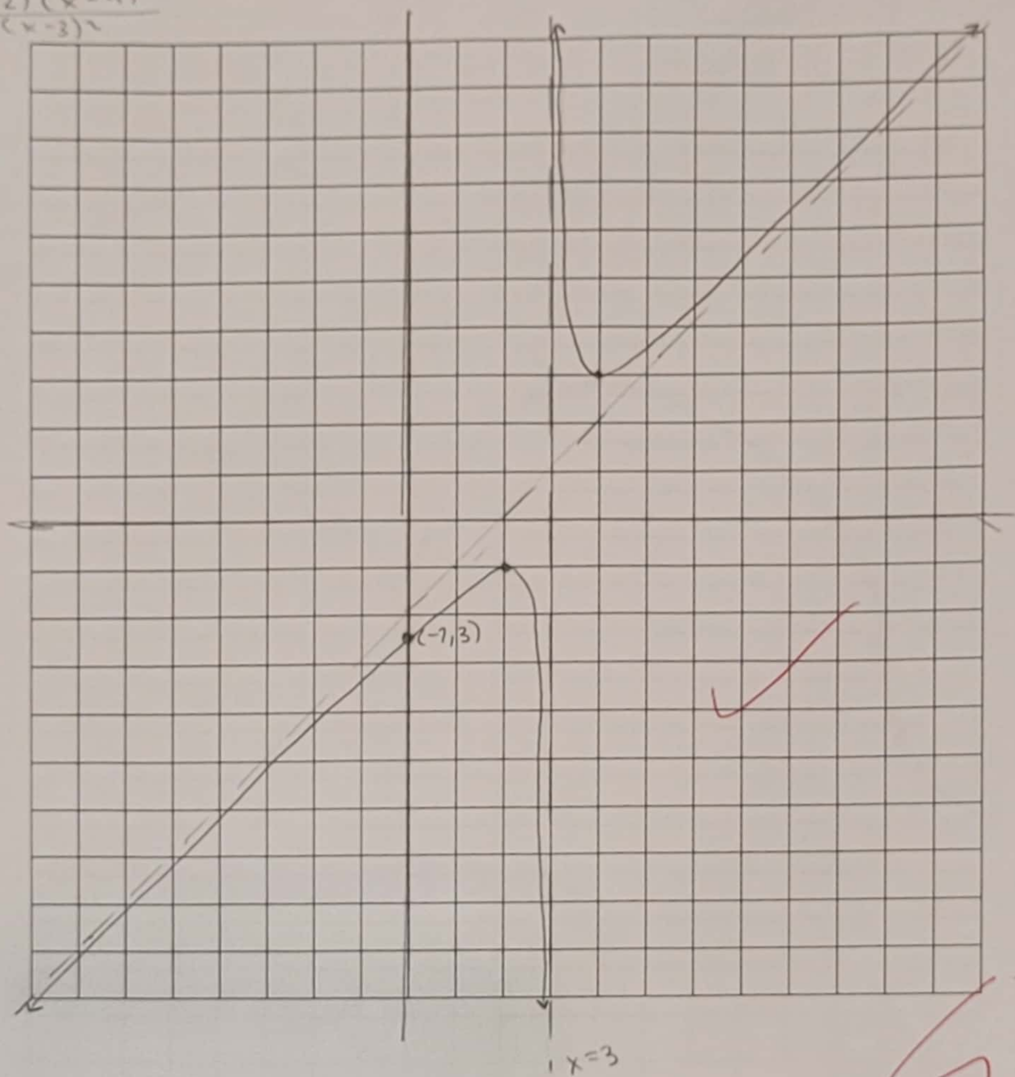
$$y' = \frac{(x-3)(2x-5) - (x^2-5x+7)}{(x-3)^2}$$

$$= \frac{2x^2 - 5x - 6x + 15 - x^2 + 5x - 7}{(x-3)^2}$$

$$= \frac{x^2 - 6x + 8}{(x-3)^2}$$

$$= \frac{(x-2)(x-4)}{(x-3)^2}$$

$+ \; 2 \; - \; 3 \; - \; 4 \; +$ $\quad y'$



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8. Fully analyze the following and graph the function $y = -(x+2)^{\frac{2}{5}}(x-8)^{\frac{3}{5}}$

The following information about derivatives will be helpful

$y' = \frac{-x+2}{(x+2)^{\frac{3}{5}}(x-8)^{\frac{2}{5}}}$ and $y'' = \frac{24}{(x+2)^{\frac{8}{5}}(x-8)^{\frac{7}{5}}}$ (8 marks)

you can use decimal approximations if necessary for intercepts and/or maximum or minimum points.

$y = -(x+2)^{2/5}(x-8)^{3/5}$

xint $(-2,0)$ $(8,0)$

yint $(0, 4.6)$

no holes / asymptotes

$y' = 0$ @ $x = 2$ $(2, 5.1)$

y' undefined @ $x = -2$ $(-2,0)$

$x = 8$ $(8,0)$

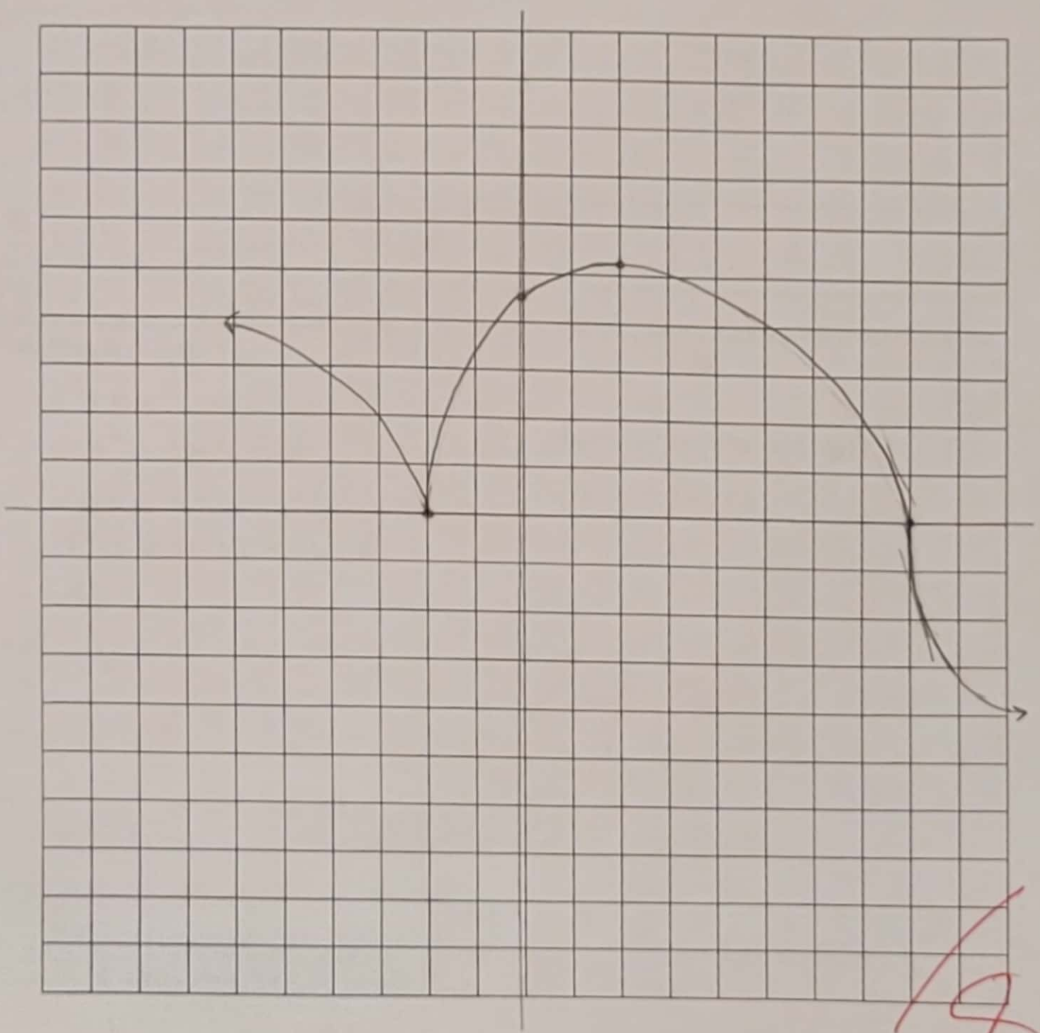
$y'' \neq 0$ $24 \neq 0$

y'' undefined @ $x = -2$ $(-2,0)$

$x = 8$ $(8,0)$

y': $-$ -2 $+$ 2 $-$ 8 $-$

y'': $-$ -2 $-$ 8 $+$



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