Limits Questions from ireallylovemath.com

(Video Solutions online)

Part 1: Determining Limits Graphically

1. Using a graphing approach, determine the following
   a. $\lim_{x\to 4^-} 2x-5$ b. $\lim_{x\to 4^+} 2x-5$ c. $\lim_{x\to 4} 2x-5$

2. Using a graphing approach, determine the following
   a. $\lim_{x\to 2^-} \frac{x^2-3x+2}{x-2}$ b. $\lim_{x\to 2^+} \frac{x^2-3x+2}{x-2}$ c. $\lim_{x\to 2} \frac{x^2-3x+2}{x-2}$

3. Using a graphing approach, determine the following where $g(x)=\begin{cases} x+2, x<2 \\ 3, x=2 \\ x^2, x>2 \end{cases}$
   a. $\lim_{x\to 2^-} g(x)$ b. $\lim_{x\to 2^+} g(x)$ c. $\lim_{x\to 2} g(x)$

4. Using a graphing approach, determine the following
   a. $\lim_{x\to -3^-} \frac{x+1}{x+3}$ b. $\lim_{x\to -3^+} \frac{x+1}{x+3}$ c. $\lim_{x\to -3} \frac{x+1}{x+3}$

Part 2: Evaluating Limits Algebraically

Algebraically evaluate each of the limits in part 2.

Evaluating Limits Using Factoring

1. $\lim_{x\to 2} \frac{x^2-5x+6}{x-2}$
2. $\lim_{x\to 3} \frac{x-3}{x^2-9}$
3. $\lim_{x\to -2} \frac{2x+4}{x^2+8x+12}$
4. $\lim_{x\to 0} \frac{x^3-8x^2}{15x^2}$
5. $\lim_{x\to \frac{1}{5}} \frac{25x-5}{5x^2-26x+5}$
6. $\lim_{x\to 4} \frac{x-4}{x^2-8x+16}$
7. $\lim_{x\to 5} \frac{2x^3+7x^2-100x+75}{x^3-9x^2+13x+35}$
8. $\lim_{x\to \frac{2}{3}} \frac{6x^3+29x^2-34x+8}{3x-2}$

Evaluating Limits Through Elementary Fraction Work

9. $\lim_{x\to 4} \frac{\frac{1}{x}-\frac{2}{x+4}}{x-4}$
10. $\lim_{x\to 3} \frac{\frac{x}{3}-\frac{4}{x+1}}{x-3}$

Evaluating Limits by Conjugating Expressions with Square Roots

11. $\lim_{x\to 4} \frac{\sqrt{x}-2}{x^2-5x+4}$
12. $\lim_{x\to 3} \frac{\sqrt{x+6}-3}{x^2-3x}$
13. $\lim_{x\to -5} \frac{\sqrt{x+6}-1}{2-\sqrt{x+9}}$

Evaluating Limits by Completing the Difference of Cubes

14. $\lim_{x\to 5} \frac{x-5}{\sqrt[3]{x+3}-2}$
15. $\lim_{x\to 17} \frac{\sqrt[3]{x+10}-3}{x-17}$

Evaluating Limits by Completing the Difference of Squares and Completing the Difference of Cubes

16. $\lim_{x\to 1} \frac{\sqrt[3]{x}-1}{\sqrt{x}-1}$
17. $\lim_{x\to 11} \frac{\sqrt{x+5}-4}{\sqrt[3]{x-3}-2}$
18. $\lim_{x\to 26} \frac{\sqrt[3]{x+1}-3}{\sqrt{x-22}-2}$

Evaluating Limits by Completing the Difference of nth Powers

19. $\lim_{x \to 64} \frac{x-64}{\sqrt[6]{x} - 2}$

20. $\lim_{x \to 24} \frac{\sqrt[5]{x+8} - 2}{2x-48}$

Evaluating Limits as x Goes to Infinity or x Goes to Negative Infinity

21. $\lim_{x \to \infty} \frac{5x^2+9x-4}{x^2+11}$

22. $\lim_{x \to -\infty} \frac{3x^3+13x-4}{2x^3-12x^2}$

23. $\lim_{x \to \infty} \frac{4x^2+8x-1}{3x+5}$

24. $\lim_{x \to -\infty} \frac{9x^2-8x+2}{6x^3+8x+4}$

25. $\lim_{x \to \infty} \frac{\sqrt{x^4+1}}{x^2-5}$

26. $\lim_{x \to -\infty} \frac{\sqrt{x^2-4x+8}}{x-7}$

27. $\lim_{x \to -\infty} \frac{\sqrt{2x^4+11x-1}}{x^2+4x-6}$

Part 3: Continuity

1. Find the value(s) of x at which the function $f(x) = \frac{x^2-4}{x+2}$ is discontinuous
2. Find the value(s) of x at which the function $f(x) = \frac{2x-1}{x-3}$ is discontinuous
3. Find the value(s) of x at which the function $f(x) = \begin{cases} 2x-2, x \leq 2 \\ 4-x, x > 2 \end{cases}$ is discontinuous
4. Find the value(s) of x at which the function $f(x) = \begin{cases} x+4, x \leq 1 \\ x^2+3, x > 1 \end{cases}$ is discontinuous
5. Find the value(s) of x at which the function $f(x) = \begin{cases} \frac{1}{2}x+3, x < 2 \\ 4, x = 2 \\ 6-x, x > 2 \end{cases}$ is discontinuous
6. Find the value(s) of x at which the function $f(x) = \begin{cases} 3-x, x < -1 \\ 2, x = -1 \\ x+5, x > -1 \end{cases}$ is discontinuous
7. Determine values for a and b such that the function $f(x) = \begin{cases} ax^2+b, x \leq 0 \\ 2x+3, 0 < x < 5 \\ bx-3a, x \geq 5 \end{cases}$ is continuous for all $x \in R$.

Answers:

Part 1:

1a) 3 b) 3 c) 3; 2a) 1 b) 1 c) 1; 3a) 4 b) 4 c) 4; 4a) ∞ (or does not exist) b) $-\infty$ (or dne) c) does not exist

Part 2:

$1.-1 \quad 2.\frac{1}{6} \quad 3.\frac{1}{2} \quad 4.\frac{-8}{15} \quad 5.\frac{-25}{24} \quad 6.\ does\ not\ exist \quad 7.-60 \quad 8.\frac{38}{9} \quad 9.\frac{-1}{32} \quad 10.\frac{7}{12} \quad 11.\frac{1}{12} \quad 12.\frac{1}{18} \quad 13.-2$

$14.12 \quad 15.\frac{1}{27} \quad 16.\frac{2}{3} \quad 17.\frac{3}{2} \quad 18.\frac{4}{27} \quad 19.192 \quad 20.\frac{1}{160} \quad 21.5 \quad 22.\frac{3}{2} \quad 23.\infty \quad 24.0 \quad 25.1 \quad 26.-1 \quad 27.\sqrt{2}$

Part 3:

$1.x = -2 \quad 2.x = 3 \quad 3.none \quad 4.x = 1 \quad 5.none \quad 6.x = -1 \quad 7.a = \frac{2}{3}, b = 3$