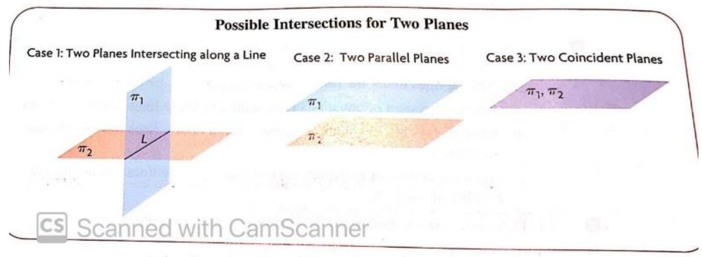
**PDF 9.030 Intersection of Two Planes: 3 Cases**

There are <u>3 cases</u> when it comes to the intersection of 2 planes
Case 1: Two planes intersect along a line (infinite points of intersection)
Case 2: Two parallel planes (no points of intersection)
Case 3: Two coincident planes (infinite points of intersection)




- It is not possible for two planes to intersect at only one point. There are either zero points of intersection (like in case 2) or there are an infinite number of points of intersection (a line like in case 1 or a plane like in case 3)

Method For Determining the Intersection of Two Planes

1. Make sure that the planes are in Cartesian form and compare the normal vectors.
2. If the normal vectors are collinear (i.e., if $\overrightarrow{n_1}$ is a scalar multiple of $\overrightarrow{n_2}$) then one of the following two situations is true.
    a. Perhaps the planes are parallel and coincident creating inifinite points of intersection. This is true if the scalar multiple between the normal vectors of the two planes is also the scalar multiple between the constant terms (i.e., the D values of the two planes)
    b. Perhaps the planes are parallel and non-coincident, creating zero points of intersection. This is true if the scalar multiple between the normal vectors of the two planes is not the scalar multiple between the constant terms (i.e., the D values of the two planes)
3. If the normal vectors are not collinear (i.e., if $\overrightarrow{n_1}$ is not a scalar multiple of $\overrightarrow{n_2}$) then the two planes intersect along a line and there are infinite points of intersection.
    a. Make a matrix with the Cartesian equations of the planes
    b. Use elementary row operations to get a zero in one of the three left-hand columns of one of the rows
    c. Using that row, sub in a value for one of the remaining columns, and determine the value for the other column
    d. Then, sub both of those values in to the other row to determine a value for the third column. You now have a position vector.
    e. The direction vector is the cross product of the normal vectors of the planes.
    f. You can now state the equation of the line in vector form.

2.

<u>Example 1</u>

Determine whether the following planes intersect. If they do intersect, state how they intersect, and include the equation of the line of intersection if necessary.

$\pi_1: 2x - 3y + 7z - 25 = 0$ and $\pi_2: 4x - 6y + 14z - 50 = 0$

Example 2

Determine whether the following planes intersect. If they do intersect, state how they intersect, and include the equation of the line of intersection if necessary.

$\pi_1: 9x - y + 4z - 16 = 0$ and $\pi_2: -18x + 2y - 8z + 8 = 0$

Example 3

Determine whether the following planes intersect. If they do intersect, state how they intersect, and include the equation of the line of intersection if necessary.

$\pi_1: 2x - y + 3z - 6 = 0$ and $\pi_2: 3x + 2y - z + 18 = 0$