PDF 2.030 Product Rule

The Product Rule States that

If $p(x) = f(x)g(x)$, then

$p'(x) = f'(x)g(x) + f(x)g'(x)$

Product Rule in Newton Notation

$\frac{d}{dx}(uv) = \left(\frac{du}{dx}\right)v + u\left(\frac{dv}{dx}\right)$

Product Rule in Leibniz Notation

Proof of the Product Rule

Example 1

Differentiate $h(x) = (x^3 - 2x)(3x^4 + 2x + 8)$

Example 2

Determine the value of $f'(-1)$ given that $f(x) = (3x^4 - 12x^2 + 4x - 9)(6x^7 - 4x^4 + 18)$

Example 3

Determine the value of $f'(1)$ given that $f(x) = (4x^2 - 3x + 1)^7$

Because of the power rule, we are able to say that the derivative of $kf(x)$ where k is a scalar is $kf'(x)$. In other words, we are able to just leave the scalar alone in front and determine the derivative of the function.

An example of this would be that if $y = 7(3x^2 + 2x + 6)$, then

$\frac{dy}{dx} = 7(6x + 2)$

In other words, the derivative would equal 7 times the derivative of the polynomial.

How do we know this? By the power rule. Here's the explanation.