PDF 8.030 Vector, Parametric and Symmetric Equations of a Line in R^3

Example 1

Determine the vector equation and parametric equations of a line that passes through the points A(2,-3,4) and B(8,1,-9)

Example 2

Show that the following two vector equations represent the same line

$$L_1: \vec{r} = \overrightarrow{(3,1,0)} + s\overrightarrow{(-9,6,3)}, s \in R$$

$$L_2: \vec{r} = \overrightarrow{(-6,7,3)} + t\overrightarrow{(3,-2,-1)}, t \in R$$

Example 3

Show that the following two vector equations represent different lines that are parallel

$$L_1: \vec{r} = \overrightarrow{(-2,2,3)} + s\overrightarrow{(1,5,-3)}, s \in R$$

$$L_2: \vec{r} = \overrightarrow{(0,12,-2)} + t\overrightarrow{(-2,-10,6)}, t \in R$$

Example 4

Does the line $\vec{r} = \overrightarrow{(8,4,-1)} + t\overrightarrow{(4,1,3)}, t \in R$ have a z-intercept?

Example 5

Determine the y-intercept of the line $\vec{r} = \overrightarrow{(6,5,-4)} + t\overrightarrow{(-3,-1,2)}, t \in R$

Symmetric Equation of a Line

Suppose $\vec{r} = \overrightarrow{(x_o, y_o, z_o)} + t\overrightarrow{(a,b,c)}, t \in R$ is the equation of a line in R^3. Further, suppose that none of a or b or c are zero (in other words, the components of the direction vector are all non-zero).

Then, the parametric equations of the line are

$x = x_o + ta$

$y = y_o + tb$

$z = z_o + tc,\ t \in R$

Let's solve for t in each of these three parametric equations

$x = x_o + ta$ $\quad$ $y = y_o + tb$ $\quad$ $z = z_o + tc$

$x - x_o = ta$ $\quad$ $y - y_o = tb$ $\quad$ $z - z_o = tc$

$\frac{x-x_o}{a} = t$ $\quad$ $\frac{y-y_o}{b} = t$ $\quad$ $\frac{z-z_o}{c} = t$

Now notice that in the last line, t is equal to all of those quantities. That must mean that each of those quantities is equal to each other. In other words

$$\frac{x-x_o}{a} = \frac{y-y_o}{b} = \frac{z-z_o}{c}$$

If the vector equation of a line is $\vec{r} = \overrightarrow{(x_o, y_o, z_o)} + t\overrightarrow{(a,b,c)}, t \in R$, then the symmetric equation of the line is

$$\frac{x-x_o}{a} = \frac{y-y_o}{b} = \frac{z-z_o}{c}, \quad a \neq 0, b \neq 0, c \neq 0$$

Example 6

Determine the symmetric equation of the line that goes through the points A(2,5,3) and B(9,6,8)

Example 7

Determine the symmetric equation of the line that goes through the points C(3,6,2) and D(-2,6,1)

Example 8

State the vector equation of the line

$$\frac{x-4}{2}=\frac{y+7}{-2}=\frac{z-11}{5}$$

Example 9

Determine the vector equation of the line

$$\frac{y+2}{3}=z, x=7$$

Example 10

There is a point A on $L_1: \vec{r} = \overrightarrow{(4,2,-1)} + s\overrightarrow{(1,-1,-2)}$ and a point B on $L_2: \vec{r} = \overrightarrow{(0,1,3)} + t\overrightarrow{(2,1,1)}$ such that the distance between the two lines is a minimum. Determine the coordinates of A and B.