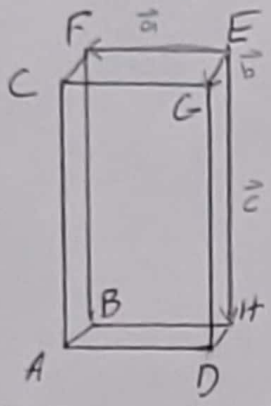
1. Given the square based prism shown below, and given that $\overrightarrow{EF} = \vec{a}, \overrightarrow{EG} = \vec{b}$ and $\overrightarrow{EH} = \vec{c}$,



a. determine an expression in terms of $\vec{a}, \vec{b}$ and/or $\vec{c}$ equivalent to $\overrightarrow{HA}$ /1

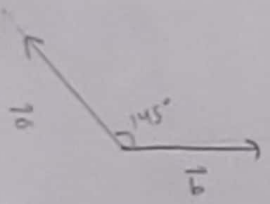
$\overrightarrow{HA} = \vec{b} + \vec{a}$ ✓

b. determine an expression in terms of $\vec{a}, \vec{b}$ and/or $\vec{c}$ equivalent to $\overrightarrow{CE}$ /1

$\overrightarrow{CE} = -\vec{a} - \vec{b}$ ✓

2

2. The vectors $\vec{a}$ and $\vec{b}$ are unit vectors (i.e., vectors with a magnitude of 1) that have an angle of 145^o between them. Determine the magnitude and direction of the vector $11\vec{a} + 7\vec{b}$ and state the direction of the resultant vector in terms of $\vec{a}$ and $\vec{b}$ (not in terms of $11\vec{a}$ and $7\vec{b}$.) Round both the magnitude and direction to one decimal place /4



$|\vec{r}|^2 = 11^2 + 7^2 - 2(11)(7)\cos 35°$

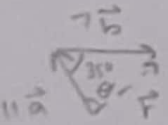
$|\vec{r}| = 6.6$

$7^2 = 11^2 + 6.6^2 - 2(11)(6.6)\cos\theta$

$\cos^{-1}\left(\frac{7^2 - 11^2 - 6.6^2}{-2(11)(6.6)}\right) = \theta$

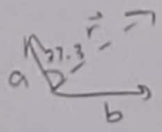
$\theta = 37.3°$

4



$|11\vec{a}| = 11$

$7|\vec{b}| = 7$



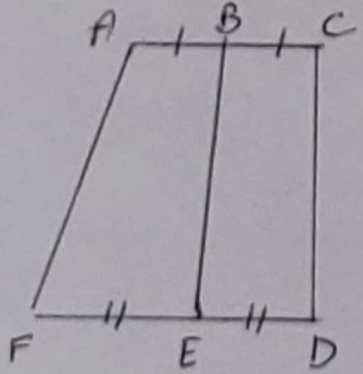
$|11\vec{a} + 7\vec{b}| = 6.6$

Round your answer to one decimal place

The direction of $11\vec{a} + 7\vec{b}$ is 37.3° rotated from $\vec{a}$ towards $\vec{b}$

Round your answer to one tenth of a degree (i.e., one decimal place).

3. Show that $\overrightarrow{FA} + \overrightarrow{DC} = 2\overrightarrow{EB}$. Show sufficient work to justify your answer. /3



$\overrightarrow{FA} = \overrightarrow{FE} + \overrightarrow{EB} + \overrightarrow{BA}$

$+ \quad \overrightarrow{DC} = \overrightarrow{DE} + \overrightarrow{EB} + \overrightarrow{BC}$

$\overrightarrow{FA} + \overrightarrow{DC} = \vec{0} + \overrightarrow{EB} + \overrightarrow{EB} + \vec{0}$

$\overrightarrow{FA} + \overrightarrow{DC} = 2\overrightarrow{EB}$

3

9

4. Three given points are $A(2,8,6)$ and $B(-1,4,11)$ and $P(0,w,0)$. Solve for w if we know that $|\overrightarrow{AP}| = |\overrightarrow{BP}|$.
/3

$\overrightarrow{AP} = (0-2, w-8, 0-6)$
$= (-2, w-8, -6)$

$\overrightarrow{BP} = (0+1, w-4, 0-11)$
$= (1, w-4, -11)$

$|\overrightarrow{AP}| = |\overrightarrow{BP}|$

$\sqrt{(-2)^2+(w-8)^2+(-6)^2} = \sqrt{(1)^2+(w-4)^2+(11)^2}$

$\sqrt{4+(w-8)^2+36} = \sqrt{1+(w-4)^2+121}$

$40+(w-8)(w-8) = 122+(w-4)(w-4)$

$40+w^2-16w+64 = 122+w^2-8w+16$

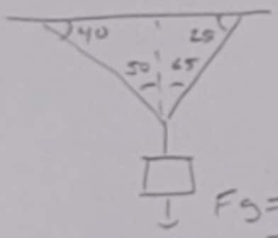
$104-16w = 138-8w$

$104-138 = 16w-8w$

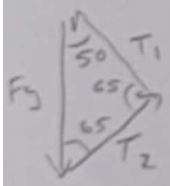
$-34 = 8w$

$w = -\frac{17}{4}$

5. A mass of 50 kg is suspended by two ropes. One of the ropes makes an angle of 25° with the ceiling and the other rope makes an angle of 40° with the ceiling. What is the magnitude of the tension in each of the ropes?
You can just state the magnitude of each tension; you do not have to state direction. Round your answer to the nearest tenth of a Newton (i.e., one decimal place).
/5



$F_g = -g$
$= 50(9.8)$
$= 490.5\,N$



T_1

$\frac{F_g}{\sin 65} = \frac{T_1}{\sin 65}$

$T_1 = \frac{490.5 \sin 65}{\sin 65}$

$= 490.5\,N$

$\frac{F_g}{\sin 65} = \frac{T_2}{\sin 50}$

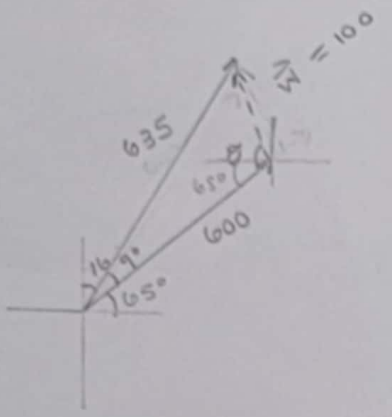
$T_2 = \frac{490.5 \sin 50}{\sin 65}$

$= 414.6$

The magnitudes of the tensions in the ropes are 490.5 N and 414.6 N.
Round your answers to one tenth of a Newton (i.e., one decimal place). You can state the magnitudes in either order.

8

6. A plane is traveling with a component velocity of 600 km/h, N 25° E (component vector) when it encounters a wind (component vector) with a speed of 100 km/h. The resultant velocity of the airplane is 635 km/h, N 16° E. What is the direction of the wind (i.e. what is the direction of the wind vector)? Round your answer to the nearest tenth of a degree (i.e., one decimal place) /5



$\cos^{-1}\left(\frac{635^2 - 100^2 - 600^2}{-2(100)(600)}\right) = \theta$

$\theta = 106.1°$

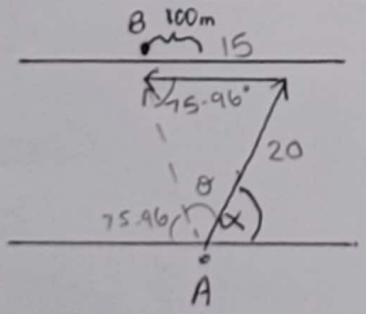
$106.1° - 65° = 41.1°$

∴ direction of wind is W 41.1° N, meaning that it is coming from E 41.1° S.

The direction of the wind vector is W 41.1° N

Round your answer to the nearest tenth of a degree (i.e., one decimal place)

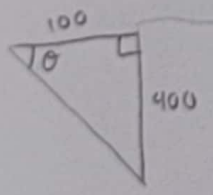
7. A river that is 400 m wide flows with a current of 15 m/s. Franklin can row 20 m/s in still water. Franklin wishes to travel directly to a point 100 m downstream from his starting point (i.e., he wishes to travel from point A to point B on the map shown)
   a. At what angle to the bank should Franklin steer his boat (i.e., what is the value of α in the diagram shown?). Round your answer to the nearest tenth of a degree (i.e., one decimal place) if necessary.
   b. How long will it take Franklin to travel from point A to point B.? Round your answer to the nearest tenth of a second (i.e., one decimal place) if necessary. (total 7)



$\frac{\sin 75.96}{20} = \frac{\sin\theta}{15}$

$\theta = 46.685°$

$180° - 46.685° - 75.96° = 57.35°$



$\tan\theta = \frac{400}{100}$

$\theta = 75.96°$

$\alpha = 57.4°$ from the bank

Round your answer to the nearest one tenth of a degree (i.e., one decimal place)

$\text{time} = \frac{\text{vertical distance}}{\text{vertical speed}}$

$= \frac{400}{20\sin(57.35)}$

$= 23.75$ s

It will take Frankling 23.8 seconds to get from A to B

Round your answer to the nearest tenth of a second (i.e., one decimal place)

7/12

8. We know O represents the point (0,0). We also know that the vector $\overrightarrow{OP} = \overrightarrow{(3,27)}$ and the vector $\overrightarrow{OQ} = \overrightarrow{(8,15)}$. Determine:

   a. the components of the vector $\overrightarrow{PQ}$

   b. the components of a unit vector in the same direction as $\overrightarrow{PQ}$. /3

a.) $\vec{PQ} = \vec{OQ} - \vec{OP}$

$\vec{PQ} = (8-3, 15-27)$

$= (\overrightarrow{5, -12})$

b.) $\frac{1}{|(\overrightarrow{5,-12})|} (\overrightarrow{5,-12})$

$= \frac{1}{\sqrt{(5)^2 + (-12)^2}} (\overrightarrow{5,-12})$

$= \frac{1}{13} (\overrightarrow{5,-12})$

$= (\frac{\overrightarrow{5}}{13}, \frac{-12}{13})$

$\overrightarrow{PQ} = (\overrightarrow{5,-12})$

A unit vector in the same direction as $\overrightarrow{PQ}$ is $(\frac{\overrightarrow{5}}{13}, \frac{-12}{13})$

/3