Perpendicular Questions **Date:_____________________**

Finding the point that creates lines that are perpendicular

For questions 1-4, you only need to check to see if the answer that you got is already one of the points given in the question. If it is, it's not a valid third possible point of a triangle. If it isn't one of the points given in the question, then it is a valid third point of a triangle.

1. The line segment between A (-4,4) and B (3,1) is the hypotenuse of the right angled triangle ABC. The third point C is on the line $\vec{r} = (4,1) + t(3,2)$. Determine the coordinates of point C.

   Answer: $(1, -1)$ or $\left(\frac{28}{13}, \frac{-3}{13}\right)$

2. The line segment between J (3,1,-1) and K (1,9,9) is the hypotenuse of the right angled triangle JKL. The third point L is on the line $\vec{r} = (5,1,2) + t(1,-1,1)$. Determine the coordinates of point L.

   Answer: $(6,0,3)$ or $\left(\frac{2}{3}, \frac{16}{3}, \frac{-7}{3}\right)$

3. The line segment between D (-4,3) and E (3,-1) is the hypotenuse of the right angle triangle DEF. The third point F is on the line with parametric equations $x = 11 + 2t$ and $y = 3 + t$. Determine the coordinates of point F.

   Answer: $(-1, -3)$

   [note that you also obtain the point (3,-1), but it's already a point given so not a valid third point of the triangle ☺]

4. The line segment between T (2, 4, -1) and U (-3, 1, 5) is the hypotenuse of the right angled triangle TUV. The third point V is on the line with symmetric equation $x + 1 = -(y - 2) = -(z - 3)$. Determine the coordinates of point V.

   Hint: it may be helpful to consider what the direction vector is if you consider the symmetric equation of the line in more traditional symmetric equation form (i.e., "with denominators")

   Answer: (-3, 4, 5) or $\left(\frac{5}{3}, \frac{-2}{3}, \frac{1}{3}\right)$

5. Two skew lines in R^3 are given by

   $L_1: \vec{r} = (-3,2,1) + t(2,1,1)$ and $L_2: \vec{r} = (8,6,-1) + s(1,2,-1)$.

   Determine the coordinates of point A on L_1 and point B on L_2 such that the distance between points A and B is a minimum.

   Answer: A(3, 5, 4) and B(6, 2, 1)

6. Two skew lines in R^3 are given by

   $L_1: \frac{x+5}{2} = \frac{y+2}{4} = \frac{z}{-1}$ and $L_2: \vec{r} = (0,7,4) + t(3,-1,2)$.

   Determine the coordinates of point P on L_1 and point Q on L_2 such that the line segment joining P and Q is perpendicular to each of the two given lines.

   Answer: P(-1 , 6, -2) and Q(-3, 8, 2)