Unit 3 Test – Higher Order Derivatives, Optimization Tuesday

Version 2

1. Determine the minimum value of the function $f(x) = 2x + \frac{\quad}{2x-60}$ over the interval $32 \le x \le 42$. Do not include a domain analysis in your solution.

~~(4 marks)~~

$f'(x) = 2 - \frac{144(2)}{(2x-60)^2}$

$f'(x) = 0$

$\Rightarrow 0 = 2 - \frac{288}{4x^2 - 240x + 3600}$

$8x^2 - 480x + 7200 = 288$

$0 = 8x^2 - 480x + 6912$

$0 = 8(x^2 - 60x + 864)$

$x = \frac{60 \pm \sqrt{3600 - 3456}}{2}$

$= \frac{60 \pm 12}{2}$

$x = 36$ or $x = 24$

↑ answer

$f(36) = 84$

$f(24) = 36$ minimum

2/2

The minimum value of the function over the given domain is 36 when $x = 24$

2. A rock slides on the ice and its displacement is given by $s(t) = 45t - 6t^{\frac{3}{2}}$ where $s(t)$ is the displacement measured in metres and t is measured in seconds. What is the displacement when the stone changes direction? $v(t) = 0$ /4
   - You can assume that time must be positive.
   - You don't have to give a direction with your displacement. Just the numerical value. Round your answer to one decimal place if necessary.

$v(t) = 45 - 9t^{1/2}$

$v(t) = 0$

$\Rightarrow 0 = 45 - 9t^{1/2}$

$\frac{45}{9} = t^{1/2}$

$(5)^2 = (t^{1/2})^2$

$25 = t$

$s(25) = 45(25) - 6(25)^{3/2}$

$= 375m$

4

When the stone changes direction, the displacement is 375 m.
(No direction necessary. Just a numerical value. Round your answer to one decimal place if necessary)

/6

3. A rectangular poster has an inner rectangular printing area and margins of 5 cm on the top and bottom, as well as margins of 3 cm on the left and right sides. If the poster has an overall area of 9375 cm² (including the inner rectangular printing area and the margins all around it), then what is the maximum possible area of the inner rectangular printing area?

- You do not need to include a domain analysis for this question. Round your answer to one decimal place if necessary.

$A = 9375$

$lw = 9375$

$l = \frac{9375}{w}$

$6 \le w \le 937.5$

$$A = (l-10)(w-6)$$
$$= lw - 6l - 10w + 60$$
$$A(w) = 9375 - 6\left(\frac{9375}{w}\right) - 10w + 60, \quad 6 \le w \le 937.5$$
$$= -56250w^{-1} - 10w + 9435$$

$A'(w) = 0$

$$\Rightarrow 0 = \frac{56250}{w^2} - 10$$
$$10w^2 = 56250$$
$$w^2 = 5625$$
$$w = 75\text{cm}$$

$$\boxed{A(75) = 7935\text{cm}^2}$$

5/5

The maximum possible area of the inner rectangular printing area is 7935 cm².
(Round your answer to one decimal place if necessary)

4. A cylindrical aluminum can with an open top has a volume of 4000 cm^3. (In other words, the cylinder has a bottom but no top). What radius of can will minimize the amount of aluminum needed to create the can? You must give an exact value.

- You must give an exact value. No decimal approximations. Reduce and simplify your fraction if necessary.
- No domain analysis is necessary.

~~10~~

$V = 4000$

$V = \pi r^2 h$

$\frac{4000}{\pi r^2} = h$

$$A = \pi r^2 + 2\pi r h$$

$$A(r) = \pi r^2 + 2\cancel{\pi} r\left[\frac{4000}{\cancel{\pi} r^{\cancel{2}}}\right]$$

$$= \pi r^2 + 8000 r^{-1}$$

$$A'(r) = 0$$

$$\Rightarrow 0 = 2\pi r - \frac{8000}{r^2}$$

$$2\pi r = \frac{8000}{r^2}$$

$$2\pi r^3 = 8000$$

$$r^3 = \frac{4000}{\pi}$$

$$r = \sqrt[3]{\frac{4000}{\pi}}$$

$$r = \frac{4000^{1/3}}{\pi^{1/3}}$$

5

5

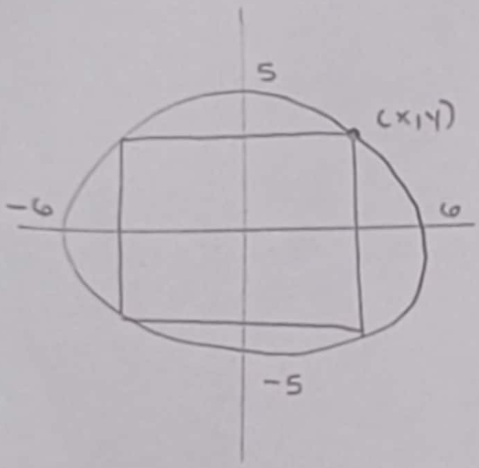
To minimize the surface area of the can, the radius of the cylinder should be $\sqrt[3]{\frac{4000}{\pi}}$ cm.

(Give an exact value. Reduce your fraction and simplify if necessary. No decimals).

5 A rectangle is inscribed inside the ellipse with the equation $25x^2 + 36y^2 = 900$. What is the maximum possible area of the rectangle? /7

- You must include a domain analysis in your answer. Round your answer to one decimal place if necessary.

$A = lw$
$= (2x)(2y)$
$= 4xy$



$A(x) = 4x\left[\frac{900-25x^2}{36}\right]^{1/2}, \quad 0 \le x \le 6$

$= 4x\left[25 - \frac{25}{36}x^2\right]^{1/2}$

$A'(x) = 0$

$\Rightarrow 0 = 4\left[25-\frac{25}{36}x^2\right]^{1/2} + (\cancel{4}^{2}x)\left[\frac{1}{\cancel{2}}\left(25-\frac{25}{36}x^2\right)^{-1/2}\left(\frac{-25}{18}x\right)\right]$

$0 = 4\sqrt{25-\frac{25}{36}x^2} - \frac{25x^2}{9\sqrt{25-\frac{25}{36}x^2}}$

$4\sqrt{25-\frac{25}{36}x^2} = \frac{25x^2}{9\sqrt{25-\frac{25}{36}x^2}}$

$36\left(25-\frac{25}{36}x^2\right) = 25x^2$

$900 - 25x^2 = 25x^2$

$900 = 50x^2$

$18 = x^2$

$x = 3\sqrt{2}$

$18 \mid 2, 9 \mid 3, 3 \mid 3, 1$

$25x^2 = 900$
$x = \pm 6$

$36y^2 = 900$
$y = \pm 5$

$25x^2 + 36y^2 = 900$

$36y^2 = 900 - 25x^2$

$y^2 = \frac{900-25x^2}{36}$

$y = \sqrt{\frac{900-25x^2}{36}}$

$0 \le x \le 6$

$A(0) = 0$

$\boxed{A(3\sqrt{2}) = 60}$ max

$A(6) = 0$

6/6

The maximum possible area of the rectangle is 60 square units.
(Round your answer to one decimal place if necessary)