PDF 5.040 Derivatives of Exponential Functions

Three key rules for this section are

- The derivative of ln x with respect to x is 1/x
- The derivative of e^x with respect to x is e^x
- The derivative of b^x with respect to x is (b^x)(ln b)

These three rules are proven in your powerpoint.

Putting the last rule together with the chain rule, we get that

the derivative of $b^{f(x)}$ is $b^{f(x)}(\ln b)\, f'(x)$

In other words, to differentiate an exponential function, restate the function, multiply by the natural logarithm (ln) of the base, and then multiply by the derivative of the exponent. This is assuming that there is no x in the base as well. We will see later that there is a different process when there is an x in both the base and in the exponent.

Example 1

Determine the derivative of $g(x) = e^{x^2-x}$

Example 2

Determine the derivative of the curve $f(x) = x^2e^x$

Example 3

Determine the equation of the tangent line to $y = \frac{e^x}{x^2}$ where x=1

Example 4

Differentiate the function $f(x) = 5^x$

Example 5

Differentiate the function $g(x) = 5^{3x-2}$

Example 6

Differentiate the function h(x) =$(8)7^{9x^2-5x+1}$

Word Problem 1

On January 1, 1850, the population of Goldrushtown was 50000. Since then the population of Goldrushtown can be expressed as P(t)=50000 $(0.98)^t$.

a) What was the population of Goldrushtown on January 1, 1900?
b) At what rate was the population changing on January 1, 1900?

Word Problem 2

A radioactive substance decays exponentially. The percent, P, of the material left after after t years is

P(t) = 100 $(1.015)^{-t}$.

a) What is the half-life?

b) How fast is the substance decaying at that time?