Exponentials and Trig Derivatives Test Wedn

Version 2

$\lim_{x \to 0} \frac{\sin x}{x}$	$\sin(a+b) = \sin a \cos b + \cos a \sin b$	$\cos(a+b) = \cos a \cos b - \sin a \sin b$	$\sin 2x = 2 \sin x \cos x$
	$\sin(a-b) = \sin a \cos b - \cos a \cos b$	$\cos(a-b) = \cos a \cos b + \sin a \sin b$	$\cos 2x = \cos^2 x - \sin^2 x$ $= 2\cos^2 x - 1$ $= 1 - 2\sin^2 x$

1. Given the function $f(x) = cos^3 x \sin(\tan 3x)$, determine an expression for the derivative $f'(x)$. You do not need to simplify your expression. /3

$f(x) = (\cos x)^3 \sin(\tan 3x)$

$f'(x) = 3(\cos x)^2 (-\sin x) \sin(\tan 3x) + (\cos^3 x) \cos(\tan 3x)$ [illegible]

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2. Evaluate each the following or state that the limit does not exist. Show sufficient work to justify your answer and use good form

a. $\lim_{x \to 0} \frac{\tan x - \sin x}{x \cos x}$ /3

[illegible]

$= 0$

3

$$\boxed{\lim_{x \to 0} \frac{\tan x - \sin x}{x \cos x} = 0}$$

b. $\lim_{x \to \frac{\pi}{2}} \frac{\cot x}{x - \frac{\pi}{2}}$ /3

[illegible]

3

$$\boxed{\lim_{x \to \frac{\pi}{2}} \frac{\cot x}{x - \frac{\pi}{2}} = -1}$$

3. You are given that $f(x) = \sin(\tan(\sin 2x))$.
Determine the value of the first derivative at $x = \pi$
You must use exact values in your solution and in the answer box. /4

[illegible]

4

$$\boxed{f'(\pi) = 2}$$

13

4. The displacement of a particle is given by $s(t) = 5\cos\left(2t + \frac{\pi}{6}\right)$

What is the maximum positive velocity of the particle over the domain $0 \le t \le \pi$? You do not need to plug in end values of the domain in your analysis. Use exact values in your solution and in your answer box. Just give a numerical answer. You do not need to worry about units of measurement in your answer. /5

$S'(t) = V(t) = 5[-\sin(2t+\frac{\pi}{6})](2)$

$= -10\sin(2t+\frac{\pi}{6})$.

$V'(t) = A(t) = -10\cos(2t+\frac{\pi}{6})(2)$

$= -20\cos(2t+\frac{\pi}{6})$.

$V(\frac{\pi}{6}) = -10\sin(\frac{\pi}{2}) = -10$

$V(\frac{2}{3}\pi) = -10\sin(\frac{3}{2}\pi) = 10$

$\because 0 \le t \le \pi$

$\therefore \frac{\pi}{6} \le 2t+\frac{\pi}{6} \le \frac{13}{6}\pi$

$\therefore V_{max} = 10$

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$A(t) = 0 \rightarrow -20\cos(2t+\frac{\pi}{6}) = 0$

$\cos(2t+\frac{\pi}{6}) = 0$

$2t+\frac{\pi}{6} = \cancel{\frac{\pi}{2}}, \frac{\pi}{2}, \frac{3}{2}\pi, \cancel{\frac{5}{2}\pi}$

$t = \frac{\pi}{6}, \frac{2}{3}\pi$

The maximum positive velocity of the particle over the domain given is (Give an exact value. Do not worry about units of measurement)	10

5. We know that $f(x) = 4^{6x-15} - 7(3)^{8x+1}$. Solve for x where $f'(x) = 0$

You can round your answer for x to two decimal places. /4

$f'(x) = 4^{6x-15}(\ln 4)\cdot(6) - 7(3)^{8x+1}(\ln 3)(8)$

$f'(x) = 0 \rightarrow 4^{6x-15}(\ln 4)(6) = 7(3)^{8x+1}(\ln 3)(8)$.

$\ln 4^{6x-15} + \ln(\ln 4) + \ln 6 = \ln 7 + \ln 3^{8x+1} + \ln(\ln 3) + \ln 8$

$(6x-15)\ln 4 - (8x+1)\ln 3 = \ln 7 + \ln(\ln 3) + \ln 8 - \ln(\ln 4) - \ln 6$

$6x\ln 4 - 8x\ln 3 = \ln 7 + \ln(\ln 3) + \ln 8 - \ln(\ln 4) - \ln 6 + 15\ln 4 + \ln 3$.

$x = \frac{\ln 7 + \ln(\ln 3) + \ln 8 - \ln(\ln 4) - \ln 6 + 15\ln 4 + \ln 3}{6\ln 4 - 8\ln 3}$

$x \approx -50.72$

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$x =$ -50.72
(Round your answer to two decimal places)

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6. You are given that $f(x) = x(\ln x)^2$. Determine the exact value of x at the point where this function has a horizontal tangent. You must use exact values in your solution and in your answer box. /3

$f'(x) = (\ln x)^2 + x(2)(\ln x)\left(\frac{1}{x}\right)$

$f'(x) = 0 \rightarrow 0 = (\ln x)^2 + 2\ln x$

$= \ln x(\ln x + 2)$

$\ln x = 0 \qquad \ln x = -2$

$x = 1 \qquad x = e^{-2}$

3

The function has a horizontal tangent at the point where $x =$ $1, e^{-2}$ (Give an exact value)

7. We are given that $f(x) = (2x^2 - 7x + 5)^{x^3-23}$ $\quad f(3) = 2^4 = 16$
Evaluate the first derivative at $x = 3$
Round your answer to one decimal place /4

$y = (2x^2 - 7x + 5)^{x^3-23}$

$\ln y = \ln\left[(2x^2 - 7x + 5)^{x^3-23}\right]$

$\ln y = (x^3 - 23)(\ln(2x^2 - 7x + 5))$

$\frac{1}{y} y' = (3x^2)(\ln(2x^2 - 7x + 5)) + (x^3 - 23)\left(\frac{4x-7}{2x^2-7x+5}\right)$

$y' = f(x)\left[3x^2(\ln(2x^2 - 7x + 5)) + \frac{(x^3-23)(4x-7)}{2x^2-7x+5}\right]$

$f'(3) = f(3)\left[3(9)(\ln 2) + \frac{4(5)}{2}\right]$

$= 16(27\ln 2 + 10)$

$= 459.439582$

4

$f'(3) =$ 459.4   (Round your answer to one decimal place)

8. You are given that $y = \log_4(6x^5 - 8x^2 + 11)$
Determine an expression for $\frac{dy}{dx}$ /2

$4^y = 6x^5 - 8x^2 + 11$

$y\ln 4 = \ln(6x^5 - 8x^2 + 11)$

$y = \frac{1}{\ln 4}\left(\ln(6x^5 - 8x^2 + 11)\right)$

$y' = \frac{(30x^4 - 16x)}{(\ln 4)(6x^5 - 8x^2 + 11)}$

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$\frac{dy}{dx} = \frac{30x^4 - 16x}{(\ln 4)(6x^5 - 8x^2 + 11)}$

/9

$\frac{}{20}$ $\frac{}{20}$

9. Carmela has to study for a chemistry test and for a history test. Because of the nature of the courses, the measure of study effectiveness on a numerical scale for her chemistry course is $E_1 = 10\left(3 + te^{\frac{-t}{30}}\right)$ while the measure of effectiveness for her history course is $E_2 = 20\left(4 + te^{\frac{-t}{20}}\right)$. Carmela is prepared to spend up to 50 hours, in total, studying for the exams. The total effectiveness is given by $E_1 + E_2$. If Carmela maximizes her overall studying effectiveness, how much time will she have spent studying for chemistry? Your solution needs to justify that your answer leads to the most overall studying effectiveness (considering both courses) /6

$$Eff = E_1 + E_2$$
$$= E_1(t) + E_2(50-t)$$
$$= 10(3 + te^{-\frac{t}{30}}) + 20(4 + (50-t)e^{-\frac{(50-t)}{20}})$$
$$= 30 + 10te^{-\frac{t}{30}} + 20(4 + 50e^{\frac{t-50}{20}} - te^{\frac{t-50}{20}})$$
$$E(t) = 30 + 10te^{-\frac{t}{30}} + 80 + 1000e^{\frac{t-50}{20}} - 20te^{\frac{t-50}{20}}$$
$$E'(t) = 10e^{-\frac{t}{30}} + 10te^{-\frac{t}{30}}(-\tfrac{1}{30}) + 1000e^{\frac{t-50}{20}}(\tfrac{1}{20}) - [20e^{\frac{t-50}{20}} + 20te^{\frac{t-50}{20}}(\tfrac{1}{20})]$$
$$0 = 10e^{-\frac{t}{30}} - \tfrac{1}{3}te^{-\frac{t}{30}} + 50e^{\frac{t-50}{20}} - 20e^{\frac{t-50}{20}} - te^{\frac{t-50}{20}}$$
$$= 10e^{\frac{t}{30}} - \tfrac{1}{3}te^{-\frac{t}{30}} + 30e^{\frac{t-50}{20}} - te^{\frac{t-50}{20}}$$
$$= e^{\frac{t}{30}}(10 - \tfrac{1}{3}t) + 3e^{\frac{t-50}{20}}(10 - \tfrac{1}{3}t)$$
$$= (10 - \tfrac{1}{3}t)(e^{\frac{t}{30}} + 3e^{\frac{t-50}{20}})$$

$$10 - \tfrac{1}{3}t = 0$$
$$10 = \tfrac{1}{3}t$$
$$t = 30$$

To maximize her overall studying effectiveness (i.e., both courses combined), Carmela should spend 30 hours studying for chemistry.

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