PDF 4.020 Critical Points, Local Maxima and Local Minima

First Derivative Test

If f ’(x) goes from positive to 0 to negative, then there is a local maximum at the point where f ’(x) = 0

Similarly, if f ’(x) goes from positive to undefined to negative, and if f(x) is defined at the point where f ’(x) is undefined, then there is a local maximum at the point where f ’(x) = 0

If f ’(x) goes from negative to 0 to positive, then there is a local minimum at the point where f ’(x) = 0

Similarly, if f ’(x) goes from negative to undefined to positive, and if f(x) is defined at the point where f ’(x) is undefined, then there is a local minimum at the point where f ’(x) = 0

If f’(x) goes from positive to 0 to positive, then there is neither a maximum nor a minimum.

If f ’(x) goes from negative to 0 to negative, then there is neither a maximum nor a minimum.

Critical Points and Critical Numbers

The points at which f(x) is defined and f ’(x) = 0 or f ’(x) is undefined are called critical points.

The values of x at which f ’(x) = 0 or f ’(x) is undefined are called critical numbers.

Example 1

For the function $y = x^4 - 8x^3 + 18x^2$, determine all the critical points. Determine whether each of these values of x gives a local maximum, a local minimum or neither.

Example 2

Determine whether the function $f(x) = x^3$ has a maximum or a minimum or neither at (c, f(c)) where f ’(c) = 0.

Example 3

Determine the critical numbers for the function $y = \sqrt{x^2 - 4x + 8}$ and determine whether each one represents a local maximum, a local minimum, or neither.

Example 4

Determine the critical numbers for $y = (x + 3)^{\frac{1}{3}}$ and then determine whether each critical number represents a maximum or a minimum or neither.

Example 5

Determine the critical numbers for $y = (x + 3)^{\frac{2}{3}}$ and then determine whether each critical number represents a maximum or a minimum or neither.

Example 6

Determine the values of a, b and c such that $f(x) = ax^2 + bx + c$ has a local maximum at (5, 6) and passes through (8, -12).

Example 7

Given the curve of y = f(x) shown below, graph the curve y = f ’(x)