Chap 1.010 Evaluating Limits Graphically

Imagine someone is standing 10 metres from a wall. We tell that person to move halfway to the wall. We then tell them to again move halfway to the wall. We then tell them to again move halfway to the wall, and we continue to do this. Theoretically, the person will never reach the wall. However, we can say that as the number of trials moves to infinity, the limit of where the person is standing will be the wall.

Of course, the person will never reach the wall (theoretically), but we could still refer to the wall as the limit.

Another way to think about limits is to think about sequences.

Consider the sequence $f(n)$ where n is a positive integer referring to the ordinal position of the term in the sequence and $f(n)$ is the value of that term.

$$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$$

The values are getting closer and closer to 0. We know that the values will never reach 0, but they will get infinitely close to 0. We can say that the limit of the sequence as the number of terms goes to infinity is 0

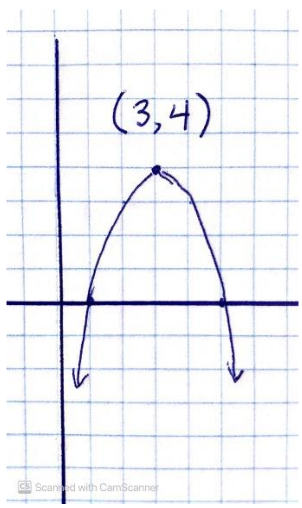
$$\lim_{n \to \infty} f(n) = 0$$

In mathematics, a limit is the value that a function (or sequence) "approaches" as the input (or index) "approaches" some value. Limits are essential to calculus (and mathematical analysis in general) and are used to define continuity, derivatives, and integrals.

When dealing with functions, we can let the x-value inputted to the function grow closer and closer to some set value. We then watch the pattern of y-values to see if there is a set y-value that we grow infinitely close to. If there is such a value, we say that that is the value of the limit.

Example 1

Suppose that we wish to determine $\lim_{x \to 3}(-x^2 + 6x - 5)$. We can draw the graph of $y = -x^2 + 6x - 5$

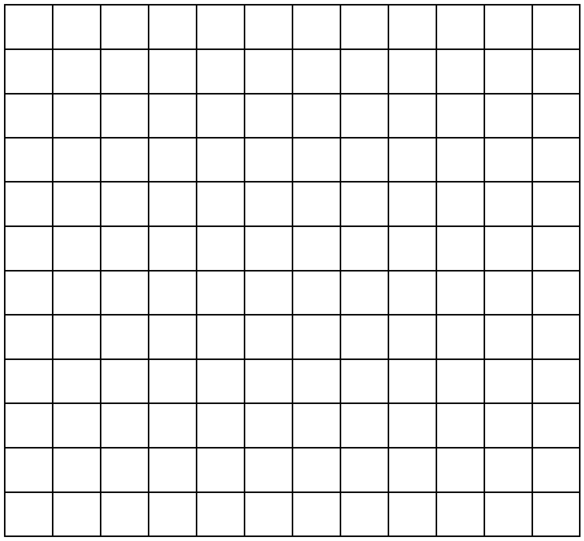


When determining whether the limit as x approaches a particular value of a function exists, we compare the two one-sided limits and see if they are the same. If they are, then the limit exists, and is equal to that common value.

If $\lim_{x \to a^-} f(x) = b$ and if $\lim_{x \to a^+} f(x) = b$ then $\lim_{x \to a} f(x) = b$
However, if $\lim_{x \to a^-} f(x)$ exists and if $\lim_{x \to a^+} f(x)$ exists but $\lim_{x \to a^-} f(x) \neq \lim_{x \to a^+} f(x)$, then $\lim_{x \to a} f(x)$ does not exist.

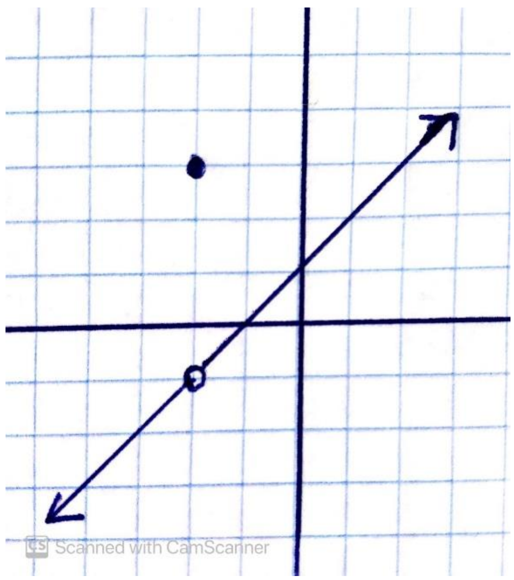
Example 2

Given that $f(x) = \frac{x^2+3x+2}{x+2}$, use a graphing approach to determine $\lim_{x \to -2} f(x)$ or justify that it does not exist.



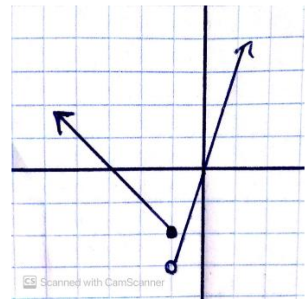
Example 3

Given that $f(x) = \begin{cases} \frac{x^2+3x+2}{x+2}, & x \neq -2 \\ 3, & x = -2 \end{cases}$, use a graphing approach to determine $\lim_{x \to -2} f(x)$ or justify that it does not exist.



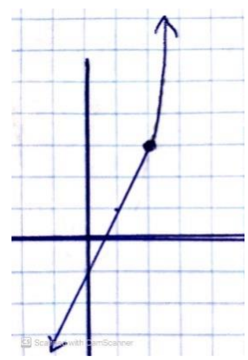
Example 4

Given that $f(x) = \begin{cases} -x-3, & x \le -1 \\ 3x, & x > -1 \end{cases}$, use a graphing approach to determine $\lim_{x\to -1} f(x)$ or justify that it does not exist.



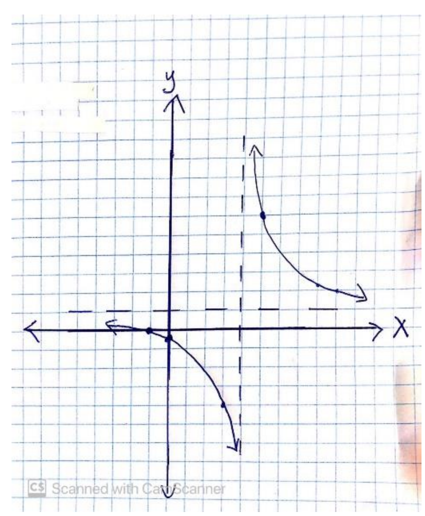
Example 5

Given $f(x) = \begin{cases} 2x-1, & x < 2 \\ x^2-1, & x \ge 2 \end{cases}$, use a graphing approach to determine $\lim_{x\to 2} f(x)$ or justify that it does not exist.



Example 6

Given that $f(x) = \frac{x+1}{x-4}$, use a graphing approach to determine $\lim_{x\to 4} f(x)$ or justify that it does not exist.



Note that in the last example, we expressed that limits were equal to ∞ and to −∞. Technically, this is not true. Nothing can equal ∞ or −∞, so one could immediately say that $\lim_{x \to 4} \left(\frac{x+1}{x-4}\right)$ does not exist as soon as one of the one-sided limits is equal to ∞ or to −∞. However, in this course we will say that a one sided limit can equal ∞ or −∞ (and this will include limits as x is going to ∞ or to −∞, which can only be addressed from one direction). However, if we are seeking the limit of a function as x approaches a real number (a two-sided limit) and if one of the one-sided limits is equal to ∞ or to −∞, then we will state that the limit of the function as x approaches the real number does not exist.

Limits that can only be approached from one side

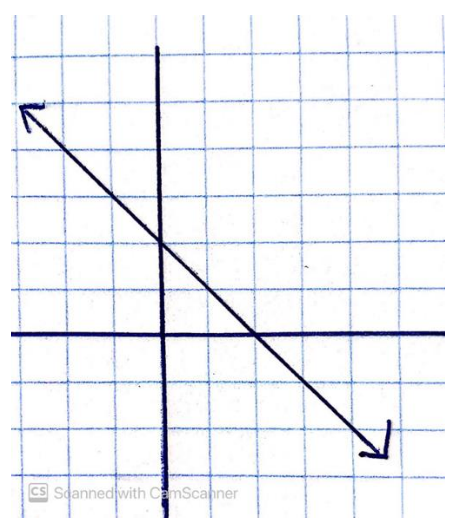
Sometimes, we will come across a limit that can only be approached from the left or from the right. The most common type of limit of this form is when we seek to find the limit of a function as x approaches ∞ or as x approaches −∞.

If we want to evaluate the limit of a function as x approaches −∞, we can only approach it from the right, because we can't be to the left of −∞.

Similarly, if we want to evaluate the limit of a function as x approaches ∞, we can only approach it from the left, because we can't be to the right of ∞.

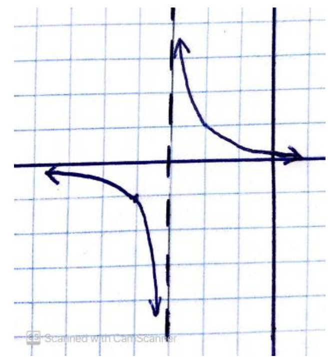
Example 7

Using a graphing approach, evaluate $\lim_{x \to \infty} (-x + 2)$ or state that it does not exist.



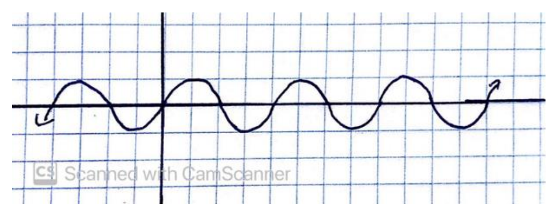
Example 8

Using a graphing approach, evaluate $\lim_{x \to -\infty} \left(\frac{1}{x+3}\right)$ or state that it does not exist.



Example 9

Use a graphing approach to evaluate $\lim_{x \to \infty} \sin x$ or state that it does not exist.



Example 10

Evaluate the following using a graphing approach

$\lim_{x \to 2} (\sqrt{x-2} + 3)$