PDF 2.070 Implicit Differentiation

Until now, we have been determining derivatives when y is stated explicitly in terms of x.

However, sometimes an equation defines y implicitly as a differentiable function of x. When that happens, we need to differentiate each side of the equation in terms of x, and remember to use the chain rule when differentiating terms containing y.

We may need to use other rules of differentiation as well.

Intro to How Implicit Differentiation Works:

- The derivative of $5x^3$ with respect to x is $15x^2$. We can think of this slightly differently though and say that the derivative of $5x^3$ with respect to x is $15x^2 \frac{dx}{dx}$ and that since $\frac{dx}{dx} = 1$, therefore the derivative of $5x^3$ with respect to x is $15x^2$
- By the same way of thinking, the derivative of $5y^3$ with respect to x is $15y^2 \frac{dy}{dx}$

Example 1

If $x^2 + y^2 = 25$, determine $\frac{dy}{dx}$ and determine the slope of the tangent to the curve at the point $(3, -4)$.

Example 2

Determine $\frac{dy}{dx}$ for the relation $6y^3 - 3xy = 7$