PDF 7.030 Linear Combinations and Spanning Sets

A set of two vectors forms a spanning set for R^2 if every vector in R^2 can be written as a linear combination of those two vectors.

If $\vec{u}$, $\vec{v}$ and $\vec{w}$ are vectors and if $\vec{w} = a\vec{u} + b\vec{v}$ where a and b are scalars, then $\vec{w}$ can be written as a linear combination of $\vec{u}$ and $\vec{v}$

For example, the vectors $\vec{\imath}$ and $\vec{\jmath}$ span R^2 because every vector in R^2 can be written as a linear combination of $\vec{\imath}$ and $\vec{\jmath}$. To illustrate.

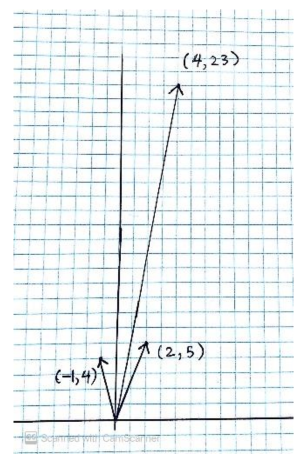
$$\overrightarrow{(4,-3)} = 4\vec{\imath} - 3\vec{\jmath}$$

<u>Example 1</u>

Show that $\vec{v} = \overrightarrow{(4,23)}$ can be written as a linear combination of the set of vectors $\left\{\overrightarrow{(-1,4)}\,,\ \overrightarrow{(2,5)}\right\}$.

Solution:

In other words, we want to show that we can get from the origin to the point (4, 23) using only the two vectors shown.

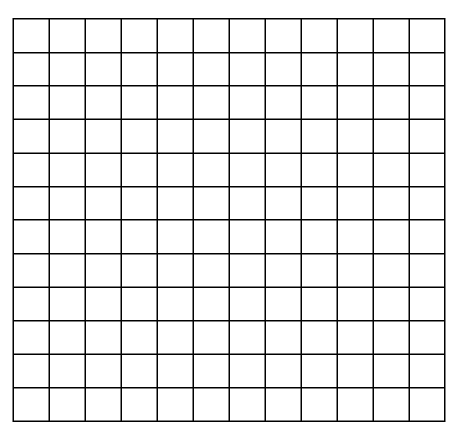


<u>Term to Know: Spanning Set</u>

A ***spanning set*** of R^2 exists if every vector in R^2 can be written as a linear combination of those vectors.

<u>Example 2</u>

Show that the set of vectors $\left\{\overrightarrow{(2,3)},\ \overrightarrow{(-4,\ -6)}\right\}$ is not a spanning set of R^2



Example 3

Show that the set of vectors $\{(2,\ 1),\ (-3,\ -1)\}$ is a spanning set for R^2

Example 4

Using the expressions for *a* and *b* that we determined in the previous question, show that the vector $\overrightarrow{(-7,22)}$ is a linear combination of the vectors $\overrightarrow{(2,1)}$ and $\overrightarrow{(-3,-1)}$

Term to Know: Collinear Vectors

Two vectors are collinear vectors if they are parallel and lie on the same straight line. We can tell algebraically if two vectors are collinear by determining whether they are scalar multiples of each other.

Examples:

1. $\overrightarrow{(-3,2)}$ and $\overrightarrow{(12,-8)}$ are collinear.
2. $\overrightarrow{(5,15,-30)}$ and $\overrightarrow{(1,3,-6)}$ are collinear.
3. $\overrightarrow{(9,4)}$ and $\overrightarrow{(3,2)}$ are not collinear.
4. $\overrightarrow{(-8,-12,4)}$ and $\overrightarrow{(2,-3,-1)}$ are not collinear.

General Rule Regarding Spanning Sets

Any two non-zero, non-collinear vectors in R^2 span R^2.

Any two collinear vectors in R^2 do not span R^2.

Another Similar and Related Rule Regarding Spanning Sets

Any two non-zero, non-collinear vectors in R^3 span a plane (two-dimensional) in R^3. This means that there will be many other vectors in R^3 on that plane and many other vectors in R^3 not on that plane.

As an example, think of the vectors $\vec{\imath} = \overrightarrow{(1,0,0)}$ and $\vec{\jmath} = \overrightarrow{(0,1,0)}$. These vectors are not collinear, yet they span the entire plane consisting of all vectors with a z-component of 0. (we will talk in great detail later in the course about the equations of planes)

Any two collinear vectors in R^3 do not span a plane in R^3.

Example 5

Given the two vectors $\vec{a} = \overrightarrow{(-1,-2,1)}$ and $\vec{b} = \overrightarrow{(3,-1,1)}$, does the vector $\vec{c} = (-9,-4,1)$ lie on the plane spanned by $\vec{a}$ and $\vec{b}$?

Example 6

Given the two vectors $\vec{a} = \overrightarrow{(-1,-2,1)}$ and $\vec{b} = \overrightarrow{(3,-1,1)}$, does the vector $\vec{c} = (-9,-4,2)$ lie on the plane spanned by $\vec{a}$ and $\vec{b}$?