Exponentials and Trig Derivatives Test

Version 1

$\lim_{x\to0}\frac{\sin x}{x}$	$\sin(a+b)=\sin a\cos b+\cos a\sin b$	$\cos(a+b)=\cos a\cos b-\sin a\sin b$	$\sin 2x = 2\sin x\cos x$
	$\sin(a-b)=\sin a\cos b-\cos a\cos b$	$\cos(a-b)=\cos a\cos b+\sin a\sin b$	$\cos 2x = \cos^2 x-\sin^2 x$ $=2\cos^2 x-1$ $=1-2\sin^2 x$

1. Given the function $f(x)=\cos^3 x\sin(\tan 3x)$, determine an expression for the derivative $f'(x)$. You do not need to simplify your expression. /3

$F(x) = (\cos x)^3 \sin(\tan 3x)$

$f'(x) = 3(\cos x)^2(-\sin x)\sin(\tan 3x) + (\cos x)^3\cos(\tan 3x)(\sec^2 3x)(3)$

3

2. Evaluate each of the following or state that the limit does not exist. Show sufficient steps to justify your answer and use good form.

a. $\lim_{x\to0}\frac{\csc x-\cot x}{\sin x}$ /3

$= \lim_{x\to0} \frac{\frac{1-\cos x}{\sin x}}{\sin x}$

$= \lim_{x\to0} \frac{(1-\cos x)(1+\cos x)}{\sin^2 x(1+\cos x)}$

$= \lim_{x\to0} \frac{\sin^2 x}{\sin^2 x(1+\cos x)}$

$= \frac{1}{2}$

3

$\boxed{\lim_{x\to0}\frac{\csc x-\cot x}{\sin x} = \frac{1}{2}}$

b. $\lim_{x\to0}\frac{\sin 2x}{5x^2+x}$ /3

$= \lim_{x\to0} \frac{2\sin x\cos x}{x(5x+1)}$

$= \lim_{x\to0} \frac{\sin x}{x}\left(\frac{2\cos x}{5x+1}\right)$

$= (1)\left(\frac{2}{1}\right)$

$= 2$

3

$\boxed{\lim_{x\to0}\frac{\sin 2x}{5x^2+x} = 2}$

3. You are given that $f(x)=\frac{1}{\cos^2(\pi x)+9\sin^2(\pi x)}$ $\to ((\cos(\pi x))^2 + (9\sin(\pi x))^2$

Determine the value of the first derivative at $x=\frac{1}{6}$

You must use exact values in your solution and in your answer box. If your final answer is a fraction, it must be reduced. /5

$f'(x) = \frac{0-(1)(2)(\cos(\pi x))(-\sin(\pi x))(\pi) - 18(\sin(\pi x))(\cos(\pi x))(\pi)}{[(\cos\pi x)^2 + 9\sin(\pi x)^2]^2}$

$f'\left(\frac{1}{6}\right) = \frac{2\cos\frac{\pi}{6}(\sin\frac{\pi}{6})\pi - 18\sin\frac{\pi}{6}(\cos\frac{\pi}{6})\pi}{[(\cos\frac{\pi}{6})^2 + 9\sin(\frac{\pi}{6})^2]^2}$

$= \frac{\frac{2\sqrt{3}}{2}\left(\frac{\pi}{2}\right) - 18\frac{\pi}{2}\left(\frac{\sqrt{3}}{2}\right)}{\left[\left(\frac{\sqrt{3}}{2}\right)^2 + 9\left(\frac{1}{2}\right)^2\right]^2}$

$= \frac{2\pi\sqrt{3} - 18\pi\sqrt{3}}{9(4)^2}$

$= \frac{\pi\sqrt{3} - 9\pi\sqrt{3}}{18}$

$4\frac{1}{2}$

$\boxed{f'\left(\frac{1}{6}\right) = \frac{\pi\sqrt{3} - 9\pi\sqrt{3}}{18} = -\frac{4\sqrt{3}\pi}{9}}$

(if your final answer is a fraction, it must be reduced)

$13\frac{1}{2}$

Determine the minimum value of the function $f(x) = \frac{6\sqrt{3}\cos x}{2+\sin x}$ over the domain $0 \le x \le 2\pi$. You do not need to plug in the end values of the domain as part of your analysis. Give exact values in your solution and in your answer box. /5

$$f'(x) = \frac{(2+\sin x)(6\sqrt{3})(-\sin x) - (6\sqrt{3}\cos x)(\cos x)}{(2+\sin x)^2} \qquad 0 \le x \le 2\pi$$

$\Rightarrow f'(x) = 0$

$$0 = -6\sqrt{3}\,(2\sin x + \sin^2 x + \cos^2 x)$$

$$0 = 2\sin x + \sin^2 x + (1 - \sin^2 x)$$

$$0 = 2\sin x + \sin^2 x + 1 - \sin^2 x$$

$\sin x = -\frac{1}{2}$

$x = \frac{-\pi}{6}$ out of domain

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

$f\left(\frac{7\pi}{6}\right) = -6$

$f\left(\frac{11\pi}{6}\right) = 6$

5 ✓

The ~~maximum~~ minimum value of the function in the domain given is -6
(give an exact value)

5. We know that $f(x) = 4^{6x-11} - 7(3)^{2x+1}$. Solve for x where $f'(x) = 0$
You can round your answer for x to two decimal places if necessary. /4

$$f(x) = 4^{6x-11} - 7(3)^{2x+1}$$

$$f'(x) = 4^{6x-11}(\ln 4)(6) - [7(3)^{2x+1}(\ln 3)(2)]$$

$$f'(x) = 0 \Rightarrow 4^{6x-11}(6\ln 4) - (14\ln 3)(3)^{2x+1} = 0$$

$$(6x-11)(\ln 4) + \ln(6\ln 4) = \ln(14\ln 3) + (2x+1)(\ln 3)$$ ✓

$$6x\ln 4 - 11\ln 4 + \ln(6\ln 4) = \ln(14\ln 3) + 2x\ln 3 + \ln 3$$

$$6x\ln 4 - 2x\ln 3 = \ln(14\ln 3) + \ln 3 + 11\ln 4 - \ln(6\ln 4)$$

$$x = \frac{\ln(14\ln 3) + \ln 3 + 11\ln 4 - \ln(6\ln 4)}{(6\ln 4 - 2\ln 3)}$$

$x = 2.77$

✓

4

$x = 2.77$
(Round your answer to two decimal places if necessary)

$8\frac{1}{2}$

6. We are given that $f(x) = (2x^2 - 7x + 5)^{x^3 - 24}$
Evaluate the first derivative at $x = 3$
Round your answer to one decimal place if necessary.

/4

$f(x) = (2x^2 - 7x + 5)^{x^3-24}$

let $y = f(x)$

$\ln y = (x^3 - 24)\ln(2x^2 - 7x + 5)$

$\left(\frac{1}{y}\right)\frac{dy}{dx} = (3x^2)(\ln(2x^2-7x+5)) + \frac{(x^3-24)(4x-7)}{(2x^2-7x+5)}$

$\frac{dy}{dx} = \left[(2x^2-7x+5)^{x^3-24}\right]\left[3x^2\ln(2x^2-7x+5) + \frac{(x^3-24)(4x-7)}{(2x^2-7x+5)}\right]$

$f'(3) = (8)(26.215)$

$= 209.72$

4 ✓

$f'(3) =$ 209.72

(Round your answer to one decimal place if necessary)

7. You are given that $y = \log_4(6x^5 - 8x^2 + 11)$
Determine an expression for $\frac{dy}{dx}$

/2

$\frac{dy}{dx} = \frac{(30x^4 - 16x)}{\ln 4\,(6x^5 - 8x^2 + 11)}$

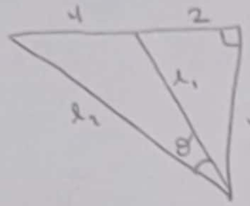
2

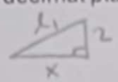
✓

$\frac{dy}{dx} = \frac{(30x^4 - 16x)}{\ln 4\,(6x^5 - 8x^2 + 11)}$

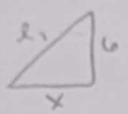
/6

8. Determine the maximum value of the angle θ in the following diagram. (You might want to start by finding the value of x that maximizes θ). You do not need to include a domain value. Round your answer to the nearest tenth of a degree (i.e., one decimal place). /7





$l_1 = \sqrt{x^2+4}$



$l_2 = \sqrt{x^2+36}$

$\sqrt{x^2+4}\,\sqrt{x^2+36}$

$\sqrt{x^4+40x^2+144}$

$c^2 = a^2 + b^2 - 2ab\cos C$

$4^2 = \sqrt{x^2+4}^{\,2} + \sqrt{x^2+36}^{\,2} - 2\sqrt{x^2+4}\,\sqrt{x^2+36}\,\cos\theta$

$16 = x^2+4+x^2+36 - 2\sqrt{x^2+4}\,\sqrt{x^2+36}\,\cos\theta$

$\dfrac{\cancel{2}(x^2+12)}{\cancel{2}\sqrt{x^2+4}\,\sqrt{x^2+36}} = \cos\theta$ let $y = \cos\theta$

$y' = \left[\sqrt{x^4+40x^2+144}\,(2x)\right] - \left[(x^2+12)\left(\tfrac{1}{2}\right)(x^4+40x^2+144)^{-1/2}\right]$
$\downarrow$
$(4x^3+80x)$

$y' = 0$
$\Downarrow$

$2x\sqrt{x^4+40x^2+144} = \dfrac{(x^2+12)(4x^3+80x)}{2\sqrt{x^4+40x^2+144}}$

$4x(x^4+40x^2+144) = (x^2+12)(4x^3+80x)$

$\cancel{4x^5} + 160x^3 + 576x = \cancel{4x^5} + 80x^3 + 48x^3 + 960x$

$32x^3 - 384x = 0$

$32x(x^2-12) = 0$

$\downarrow$ $\cancel{x=0}$ — cannot be "–" (distance)

$\downarrow$ $x = \pm\sqrt{12}$

$x = 3.5$ m

$\downarrow$ plug into Eqn $= 0.866$

$\cos\theta = 0.866$

$\theta = 30.0^\circ$

The maximum value of the angle θ is 30.0°
Round your answer to the nearest tenth of a degree (i.e., one decimal point)

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