PDF 8.040 Vector and Parametric Equations of a Plane

We know that a point on a line and a direction vector define a line in R^3.

We also know that two non-zero, non-collinear vectors span a plane in R^3.

Putting these concepts together gives us this:

Two non-zero, non-collinear vectors and a point define a plane in R^3.

Example 1

State the vector and parametric equations of a plane in R^3 that contains the point $(8, -1, 4)$ and the direction vectors $\overrightarrow{m_1} = \overrightarrow{(3, -9, 4)}$ and $\overrightarrow{m_2} = \overrightarrow{(0, 1, -2)}$

Example 2

Given the vector equation $\vec{r} = \overrightarrow{(8, -1, 4)} + s\overrightarrow{(3, -9, 4)} + t\overrightarrow{(0, 1, -2)},\ s, t \in R$

and the parametric equations $x = 8 + 3s, y = -1 - 9s + t, z = 4 + 4s - 2t,\ s, t \in R$, state three other points on the plane.

Example 3

Given the vector equation $\vec{r} = \overrightarrow{(9, 2, -3)} + s\overrightarrow{(8, 5, 2)} + t\overrightarrow{(2, 1, -2)},\ s, t \in R$

and the parametric equations $x = 9 + 8s + 2t, y = 2 + 5s + t, z = -3 + 2s - 2t,\ s, t \in R$, determine whether the point (31, 16, 5) is on the plane

Example 4

Given the plane with vector equation

$\vec{r} = \overrightarrow{(1,5,-3)} + s\overrightarrow{(3,-4,6)} + t\overrightarrow{(4,5,-8)}$

and parametric equations $x = 1 + 3s + 4t$, $y = 5 - 4s + 5t$ and $z = -3 + 6s - 8t$, determine whether the point

(11, 2, 3) is on the plane.

In summary, to determine whether a given point is on a given plane when the plane is written in vector form or in parametric form, we need to get the plane into parametric form. Then, we can set the x-coordinate of the point in question equal to the parametric expression for x and we set the y-coordinate of the point in question equal to the parametric expression for y. Then, we usually use substitution or elimination to solve for the parameters (usually s and t). Then, we plug those parameter values into the parametric expression for z and see if we get the required value of the z-coordinate.

More Information About Planes

One and only one plane can be determined if we are given any of the following:

- a line and a point not on the line
- three non-collinear points (three points not on a line)
- two intersecting lines
- two parallel and non-coincident lines

Example 5

Determine a vector equation and parametric equations for the plane containing the points A (6, 2, 1), B (9, -4, 2) and

C (-1, -2, -3).

Example 6

Given the plane with parametric equations $x = 6 + 3s + 7t$, $y = 2 - 6s + 4t$, $z = 1 + s + 4t$, $s, t \in R$, determine the point where this plane intersects the x-axis

Example 7

Determine the vector equation of the plane that contains the point A(0,-3, -2) and the line $\vec{r} = \overrightarrow{(3,-2,9)} + t\overrightarrow{(11,2,-3)}$.

Example 8

Determine a vector equation of the plane that contains the intersecting lines

$L_1{:}\,\vec{r} = \overrightarrow{(-5,-13,-1)} + s\overrightarrow{(3,-9,4)}$ and

$L_2{:}\,\vec{r} = \overrightarrow{(-9,-13,-8)} + t\overrightarrow{(4,0,7)}$

Example 9

Determine the equation of the plane that contains the parallel and non-coincident lines

$L_1{:}\,\vec{r} = \overrightarrow{(-3,7,0)} + s\overrightarrow{(2,-7,1)}$ and $L_2{:}\,\vec{r} = \overrightarrow{(2,2,6)} + t\overrightarrow{(-4,14,-2)}$

Example 10

Determine two lines that lie on the plane

$$\vec{r} = \overrightarrow{(1,-9,3)} + s\overrightarrow{(3,-2,1)} + t\overrightarrow{(1,-2,8)}$$