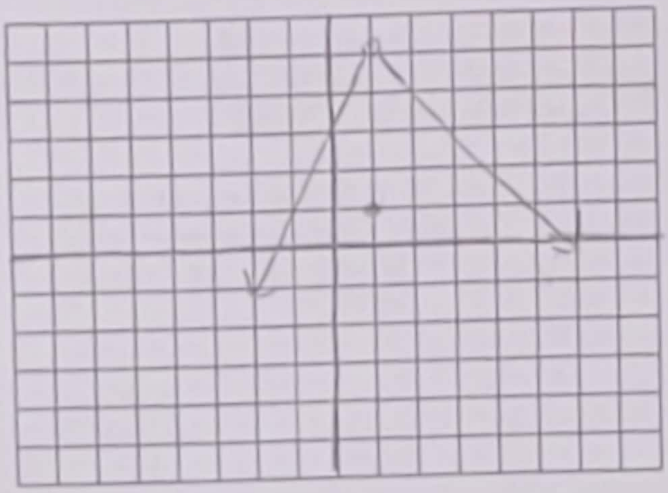
MCV4U Limits and Derivatives Test

Version 1

[redacted]

1. Graph the curve $f(x) = \begin{cases} 2x+3, & x<1 \\ 1, & x=1 \\ -x+6, & x>1 \end{cases}$ and then determine the values of the following limits or state that the limit does not exist (5 marks total)



a) $\lim_{x\to 1^-} f(x) =$ 5

b) $\lim_{x\to 1} f(x) =$ 5

5

2. Evaluate each of the following limits using algebraic technique and using good form as discussed in class, or state that the limit does not exist. Show sufficient work. Good algebraic technique includes putting brackets where they belong within your solution, including limit notation for as long as necessary, and putting equals signs where they belong within your solution.

a. $\lim_{x\to \frac{-5}{2}} \dfrac{6x^3+17x^2+3x-5}{12x+30}$ /4

$$= \lim_{x\to -\frac{5}{2}} \frac{\left(x+\frac{5}{2}\right)(6x^2+2x-2)}{6(2x+5)}$$

$$= \lim_{x\to -\frac{5}{2}} \frac{(x+5/2)\,2(3x^2+x-1)}{6(2x+5)}$$

$$= \lim_{x\to -\frac{5}{2}} \frac{\cancel{(2x+5)}(3x^2+x-1)}{6\cancel{(2x+5)}}$$

$$= \frac{3\left(-\frac{5}{2}\right)^2+\left(-\frac{5}{2}\right)-1}{6}$$

$$= \frac{61}{24}$$

$-\frac{5}{2}$	6	17	3	-5
		-15	-5	5
	6	2	-2	0

4

$$\boxed{\lim_{x\to \frac{-5}{2}} \frac{6x^3+17x^2+3x-5}{12x+30} = \frac{61}{24}}$$

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b. $\lim_{x \to -2} \frac{\sqrt{2-7x} - \sqrt{x+18}}{\sqrt[3]{2x-4} + 2}$ /6

$$= \lim_{x \to -2} \frac{(\sqrt{2-7x} - \sqrt{x+18})(\sqrt{2-7x} + \sqrt{x+18})(\text{long diff cub... Fac...})}{[(2x-4)^{1/3}+2][(2x-4)^{2/3} + 2(2x-4)^{1/3} + 4)(\sqrt{2-7x} + \sqrt{x+1...}}$$

$$= \lim_{x \to -2} \frac{-8(\cancel{x+2})\ (\text{long Factor})}{2(\cancel{x+2})(\sqrt{2-7x} + \sqrt{x+18})}$$

$$= \frac{-8(4+4+4)}{2(\sqrt{16} + \sqrt{16})}$$

$$= \frac{-96}{16}$$

$$= -6$$

6

$$\lim_{x \to -2} \frac{\sqrt{2-7x} - \sqrt{x+18}}{\sqrt[3]{2x-4} + 2} = -6$$

c. $\lim_{x \to -\infty} \frac{\sqrt{9x^2+4x-3}}{x+8}$ /3

$$= \lim_{x \to -\infty} \frac{\sqrt{x^2}\sqrt{9 + \frac{4}{x} - \frac{3}{x^2}}}{x(1 + \frac{8}{x})}$$

$$= \lim_{x \to -\infty} \frac{|x|\sqrt{9 + \frac{4}{x} - \frac{3}{x^2}}}{x(1 + \frac{8}{x})}$$

$$= \lim_{x \to -\infty} \frac{(-1)(\cancel{x})\sqrt{9 + \frac{4}{x} - \frac{3}{x^2}}}{(\cancel{x})(1 + \frac{8}{x})}$$

$$= \frac{(-1)(\sqrt{9+0-0})}{1+0}$$

$$= \frac{-3}{1}$$

$$= -3$$

3

$$\lim_{x \to -\infty} \frac{\sqrt{9x^2+4x-3}}{x+8} = -3$$

9

3. Determine the values of a and b such that the following function is continuous over all real numbers

$$f(x) = \begin{cases} 3x^2+1, & x \le -2 \\ ax-b, & -2 < x < 1 \\ 4x+25, & x \ge 1 \end{cases}$$

$x=-2 \quad ax-b = 3x^2+1$

$-2a-b = 13$

$b = -13-2a$

$x=1 \quad ax-b = 4x+25$

$a-(-13-2a) = 4+25$

$a+13+2a = 29$

$3a = 16 \quad a = \frac{16}{3}$

$b = -13-2\left(\frac{16}{3}\right) = -\frac{71}{3}$

$f(-2) = 13$
$f(1) = 29$

a — plugging in a & b. to verify

$\lim_{x \to -2^-} f(x) = 13$

$\lim_{x \to -2^+} f(x) = 13$

$\lim_{x \to 1^-} f(x) = 29$

$\lim_{x \to 1^+} f(x) = 29$

3/3

$\boxed{a = \frac{16}{3} \qquad b = -\frac{71}{3}}$

4. Using first principles, determine the derivative of the function $f(x) = \sqrt{2x+6}$. You must use good form and show sufficient work to justify that you didn't just use the power rule. Good algebraic technique includes putting brackets where they belong within your solution, including limit notation for as long as necessary, and putting equals signs where they belong within your solution. /4

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$= \lim_{h \to 0} \frac{\sqrt{2(x+h)+6} - \sqrt{2x+6}}{h}$

$= \lim_{h \to 0} \frac{(\sqrt{2x+2h+6} - \sqrt{2x+6})(\sqrt{2x+2h+6} + \sqrt{2x+6})}{h(\sqrt{2x+2h+6} + \sqrt{2x+6})}$

$= \lim_{h \to 0} \frac{(\cancel{2x}+2h+\cancel{6}) - (\cancel{2x}+\cancel{6})}{h(\text{long factor})}$

$= \lim_{h \to 0} \frac{2(\cancel{h})}{(\cancel{h})(\sqrt{2x+2h+6} + \sqrt{2x+6})}$

plug in 0

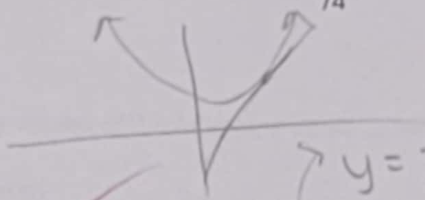
$= \frac{2}{2\sqrt{2x+6}}$

$= \frac{1}{\sqrt{2x+6}}$

4

$\boxed{f'(x) = \frac{1}{\sqrt{2x+6}}}$

5. Determine the equation of the line tangent to the curve $y = 2x^{-1} - 4$ at the point on the curve where $x = 2$ /4



$y = 2x^{-1} - 4$

$y' = -2x^{-2}$

$= \dfrac{-2}{x^2} \rightarrow m$

$y = 2(2)^{-1} - 4$

$= -3 \quad (2, -3)$

$y' = \dfrac{-2}{2^2}$ slope

$= \dfrac{-1}{2}$

$y = \dfrac{-1}{2}x + b$

$-3 = \dfrac{-1}{2}(2) + b$

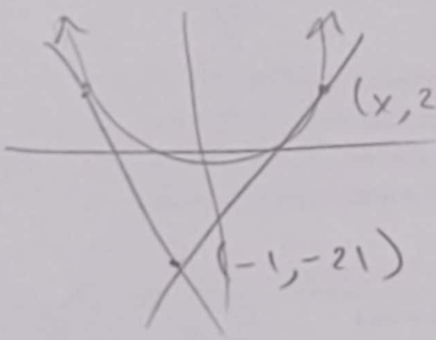
$b = -2$

$y = \dfrac{-1}{2}x - 2$

4

The equation of the tangent line is $y = \dfrac{-1}{2}x - 2$

6. There are two points on the parabola $f(x) = 2x^2 + 12x + 7$ that each have a tangent line that ultimately extends through the point $(-1, -21)$. What are the coordinates of those two points? /6



$f(x) = 2x^2 + 12x + 7$

$f'(x) = 4x + 12$ Deriv.

$m = \dfrac{(2x^2 + 12x + 7) - (-21)}{x + 1}$

$= \dfrac{2x^2 + 12x + 28}{x + 1}$

6

$m =$ deriv.

$\dfrac{2x^2 + 12x + 28}{x + 1} = 4x + 12$

$2x^2 + 12x + 28 = (4x + 12)(x + 1)$

$2x^2 + 12x + 28 = 4x^2 + 4x + 12x + 12$

$0 = 2x^2 + 4x - 16$

$0 = 2(x^2 + 2x - 8)$

$0 = 2(x - 2)(x + 4)$

$x = 2$ or -4

when $x = 2$, $f(x) = 2(2)^2 + 12(2) + 7$

$= 39 \quad \boxed{(2, 39)}$

when $x = -4$, $f(x) = 2(-4)^2 + 12(-4) + 7$

$= -9 \quad \boxed{(-4, -9)}$

The points on the parabola are $(2, 39)$ and $(-4, -9)$

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