$$= -\lim_{\frac{x}{2} \to 0} \frac{\sin \frac{x}{2}}{\frac{x}{2}} \lim_{x \to 0} \sin \frac{x}{2}$$
$$= -(1)(0)$$
$$= 0$$



The limit in Example 4 is an important result. It will be used to develop the derivative of $y = \sin x$ in Section 7.2.

Example 5 Evaluate
$$\lim_{x \to \pi} \frac{\sin x}{\pi - x}$$
.

Solution In order to use (1) we need a variable to approach 0.

Now
$$as x \to \pi, x - \pi \to 0$$
Therefore
$$\pi - x \to 0$$
Since
$$sin x = sin(\pi - x)$$
we get
$$\lim_{x \to \pi} \frac{\sin x}{\pi - x} = \lim_{\pi - x \to 0} \frac{\sin(\pi - x)}{\pi - x} = 1$$



EXERCISE 7.1

Use a calculator to estimate the value of each of the following B limits.

1.
$$\lim_{x \to 0} \frac{\sin 3x}{x}$$

$$2. \lim_{x \to 0} \frac{\sin 2x}{\sin 3x}$$

3.
$$\lim_{x \to 0} \frac{\sin^3 2x}{\sin^3 3x}$$

1.
$$\lim_{x \to 0} \frac{\sin 3x}{x}$$
 2. $\lim_{x \to 0} \frac{\sin 2x}{\sin 3x}$ 3. $\lim_{x \to 0} \frac{\sin^3 2x}{\sin^3 3x}$ 4. $\lim_{x \to 0} \frac{1 - \cos^2 x}{x^2}$ 5. $\lim_{x \to 0} \frac{1 - \cos x}{\tan x}$ 6. $\lim_{x \to 0} \frac{\sin(\cos x)}{\sec x}$

5.
$$\lim_{x \to 0} \frac{1 - \cos x}{\tan x}$$

6.
$$\lim_{x \to 0} \frac{\sin(\cos x)}{\sec x}$$

Evaluate each of the following limits.

7.
$$\lim_{x \to 0} \frac{\sin 3x}{x}$$

8.
$$\lim_{x \to 0} \frac{\sin ax}{\sin bx}$$

8.
$$\lim_{x \to 0} \frac{\sin ax}{\sin bx}$$
 9.
$$\lim_{x \to 0} \frac{\sin^3 2x}{\sin^3 3x}$$

10.
$$\lim_{x \to 0} \frac{1 - \cos x}{x}$$

11.
$$\lim_{x\to 0} (x^2 + \cos x)$$

10.
$$\lim_{x\to 0} \frac{1-\cos x}{x}$$
 11. $\lim_{x\to 0} (x^2+\cos x)$ 12. $\lim_{x\to \frac{\pi}{3}} (\sin x-\cos x)$

$$13. \lim_{x \to \frac{\pi}{4}} \frac{\sin x}{3x}$$

13.
$$\lim_{x \to \frac{\pi}{4}} \frac{\sin x}{3x}$$
 14. $\lim_{x \to -3\pi} x^3 \sin^4 x$ 15. $\lim_{x \to 0} \frac{\sin 5x}{5}$

15.
$$\lim_{x \to 0} \frac{\sin 5x}{5}$$

$$16. \lim_{x \to \frac{\pi}{4}} \frac{\tan x}{4x}$$

16.
$$\lim_{x \to \frac{\pi}{4}} \frac{\tan x}{4x}$$
 17. $\lim_{x \to 0} \frac{\tan 3x}{3 \tan 2x}$ 18. $\lim_{x \to 0} \frac{\sin^2 3x}{x^2}$

18.
$$\lim_{x \to 0} \frac{\sin^2 3x}{x^2}$$

$$21. \lim_{x\to 0} \frac{\cos x - 1}{\sin x}$$

24.
$$\lim_{x \to 0} \frac{1 - \cos x}{2x^2}$$

22.
$$\lim_{x \to 0} \frac{\tan x}{4x}$$
 23. $\lim_{x \to 0} \frac{x^3}{\tan^3 2x}$ 24. $\lim_{x \to 0} \frac{1 - \cos x}{2x^2}$ 25. $\lim_{x \to 0} \frac{x}{\sin \frac{x}{2}}$ 26. $\lim_{x \to 0} \frac{2 \tan x}{x \sec x}$ 27. $\lim_{x \to 0} \frac{1 - \cos^2 x}{x^2}$

27.
$$\lim_{x \to 0} \frac{1 - \cos^2 x}{x^2}$$

28.
$$\lim_{x \to 0} \frac{1 - \cos 2x}{x^2}$$

28.
$$\lim_{x \to 0} \frac{1 - \cos 2x}{x^2}$$
 29. $\lim_{x \to \frac{\pi}{2}} \frac{\cot x}{\frac{\pi}{2} - x}$ 30. $\lim_{x \to \pi} \frac{\sin x}{x - \pi}$

$$30. \quad \lim_{x \to \pi} \frac{\sin x}{x - \pi}$$

31.
$$\lim_{x \to 0} \frac{\sin^2 x \cos x}{1 - \cos x}$$

32.
$$\lim_{x \to 0} \frac{\sin x}{\tan x}$$

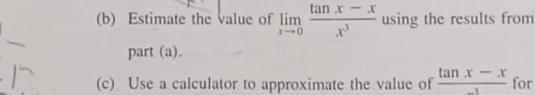
34.
$$\lim_{x \to 0} \frac{\tan x - \sin x}{x \cos x}$$

31.
$$\lim_{x \to 0} \frac{\sin^2 x \cos x}{1 - \cos x}$$
32. $\lim_{x \to 0} \frac{\sin x}{\tan x}$
33. $\lim_{x \to 0} \frac{2 \sin x - \sin 2x}{x \cos x}$
34. $\lim_{x \to 0} \frac{\tan x - \sin x}{x \cos x}$
35. $\lim_{x \to 0} \frac{1 - \cos x}{\tan x}$
36. $\lim_{x \to 0} \frac{\csc x - \cot x}{\sin x}$
37. $\lim_{x \to 0} \frac{\sin 2x}{2x^2 + x}$
38. $\lim_{x \to 0} \frac{\sin(\cos x)}{\sec x}$

$$36. \lim_{x \to 0} \frac{\csc x - \cot x}{\sin x}$$

38.
$$\lim_{x \to 0} \frac{\sin(\cos x)}{\sec x}$$

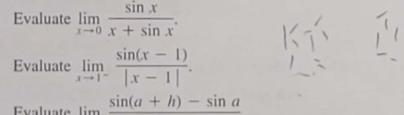
= $\left| \frac{1}{x} \right| = \frac{39}{x}$. (a) Use a calculator to approximate the value of $\frac{\tan x - x}{x^3}$ for x = 0.1, 0.001, and 0.0001.



(c) Use a calculator to approximate the value of $\frac{\tan x - x}{x^3}$ for x = 0.00001, 0.000001, and 0.0000001 and examine your answer to part (b). Can you explain what went wrong?

C 40. Does the $\lim_{x\to 0} \frac{\sin x}{|x|}$ exist? If so, what is it? If not, why not?

41. Evaluate
$$\lim_{x \to 0} \frac{\sin x}{x + \sin x}$$
.





- 43. Evaluate $\lim_{h\to 0} \frac{\sin(a+h) \sin a}{h}$.
- **44.** Evaluate $\lim_{h\to 0} \frac{\cos(a+h) \cos a}{h}$.