PDF 2.010 Instantaneous Rate of Change, aka Tangent Lines, Slope at a Point, Derivatives

Tangent Lines: A tangent line to a curve is the straight line that most resembles the graph near that point. By finding the slope of a tangent line, we can find the slope of the curve at the given point.

Here are some examples of tangent lines:

Up until now, we have talked about needing two points to determine the slope of a line. However, as we begin talking about derivatives, we need to talk about the slope of a curve at a certain point.

Suppose that we wish to determine the slope of the curve $y=f(x)$ at the point where $x=a$. This is the same as finding the slope of the line tangent to the curve $y=f(x)$ at $x=a$.

When $x=a$, $y=f(a)$. Therefore, we will be trying to determine the slope of the curve $y=f(x)$ at the point $(a,f(a))$.

The derivative of the function $y = f(x)$ is given by the formula

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Other symbols for the derivative include y' and $\frac{dy}{dx}$

The derivative of a function allows you to determine the slope of the line tangent to a curve at a given point. In other words, you can find the slope of the line when you only know one point on the line. The value of the derivative is often called the instantaneous rate of change.

Example 1

Determine the derivative of the function $y = x^2$ and evaluate the derivative at the point where x = 3.

Example 2

Determine the equation of the tangent line to the curve $y = x^2$ at the point where x = 3.

Example 3

Determine the derivative $f'(t)$ of the function $f(t) = \sqrt{t},\ t > 0$

Example 4

Given that $y = 3x^2 - 7x + 6$, determine the value of $\frac{dy}{dx}$ at $x = 5$.

Example 5

Given that $f(x) = \sqrt{3x + 4}$, determine $f'(7)$

Example 6

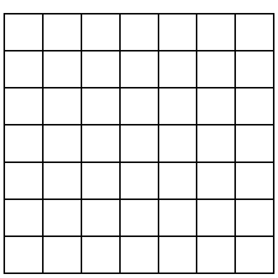
Determine the equation of the tangent line of the curve $f(x) = \frac{1}{x}$ at the point where $x = 2$

Example 7

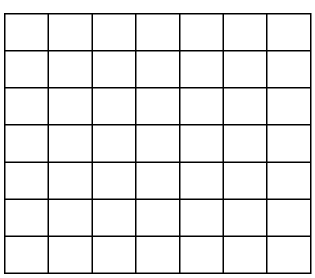
Determine an equation of the line that is perpendicular to the tangent to the graph of $f(x) = \frac{1}{x}$ at the point where $x = 2$ and that intersects it at the point of tangency. (this is called the normal)

The Existence of Derivatives

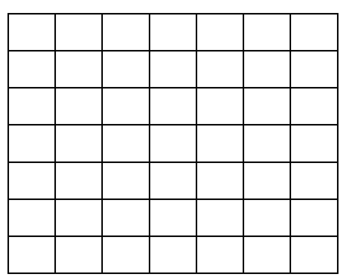
A function f is said to be differentiable at $x = a$ if $f'(a)$ exists. At points where f is not differentiable, we say that the derivative does not exist. Common ways for a derivative to not exist are shown.



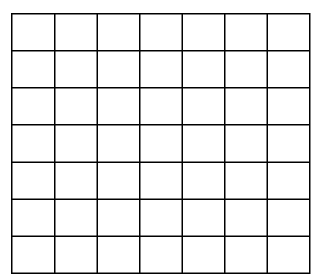
Cusp



Vertical Tangent



Discontinuity (hole or jump discontinuity as happens in some piecewise functions)



Corner