

## PDF 6.020 Forces

Generally speaking, force can be defined as that which changes, or tends to change, the state of rest, or uniform motion of a body.

The description of a force's magnitude, without also specifying its direction, has little practical value. Since force has a magnitude and direction, therefore **force is a vector**.

Often, there are a number of forces working on an object.

- The **resultant force** is the single force that would produce exactly the same effect as all of the forces acting together.
- **Equilibrium** is a state in which an object does not move (i.e., its net force is 0).
- The **equilibrant** of a number of forces is the single force that opposes the resultant of the forces; therefore when the equilibrant is applied to the object, the object is in a state of equilibrium. The equilibrant is equal to the resultant times negative 1.

The **Newton** is the unit of measurement for force. One Newton is equal to the amount of force necessary to cause a mass of one kg to accelerate at one metre per second squared. In other words  $1N = 1 \text{ kg} \times m/s^2$

### Example 1

Two children, James and Fred, are pushing on a rock. James pushes with a force of 80 N in an easterly direction and Fred pushes with a force of 60 N in the same direction. Determine the resultant and the equilibrant of these two forces.

- Of course, forces acting on an object are not always collinear. The manner that we determine the resultant force is to add the vectors representing each of the individual forces. We can use numerous different methods to do this, including
- the triangle law of addition of vectors, or
- the parallelogram law of addition, or
- adding components

The best way to approach the situation will often depend on the specific question, and it will be helpful for us to recall the sin law and the cos law.

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \quad \text{or} \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

\*\* Of course, we need to remember that the angle between two vectors refers to "tail-to-tail", but often in our calculations we use the angle between the tip and the tail, which is a different value.

### Example 2

Two forces of 20N and 40N act at an angle of 30° to each other. Determine the resultant of these two forces.

Equilibrium: A group of forces are said to be in equilibrium when there is no movement.

Two forces in equilibrium could look like this:	Three forces in equilibrium could look like this:	Four forces in equilibrium could look like this:
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As you may notice, when there are two forces in equilibrium, they are opposite vectors.

When there are more than two forces in equilibrium, they form a closed polygon.

Example 3

Is it possible for three forces of 2 N, 3 N and 4 N to be in equilibrium?

Example 4

Is it possible for three forces of 15 N, 18 N and 36 N to be in equilibrium?

Example 5

Given that three forces of 2 N, 3 N and 4 N are in equilibrium, determine the angle between the two smallest forces?

## Component Vectors

It is often convenient to break a vector down into one or more vectors that together comprise the original vector.

- In two dimensions, this is most commonly done by considering the horizontal and vertical components of a vector.
- In three dimensions, this is most commonly done by considering the components along each of the three axes.

Consider the vector  $\vec{f}$  shown at the right

We can break  $\vec{f}$  down into a horizontal and  
a vertical component vector



### Example 6

Kayla pulls on a rope attached to her sleigh with a force of  $200N$ . If the rope makes an angle of  $20^\circ$  with the horizontal, determine:

- a) the force that pulls the sleigh forward
- b) the force that tends to lift the sleigh

**Example 4 from page 359** A mass of  $20\text{ kg}$  is suspended from a ceiling by two lengths of rope that make angles of  $60^\circ$  and  $45^\circ$  with the ceiling. Determine the magnitude of the tension in each rope