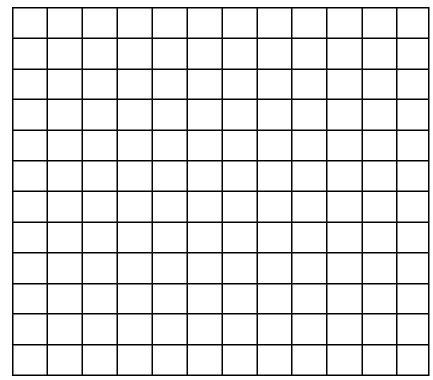
PDF 8.010 Vector and Parametric Equations of a Line in $\boldsymbol{R^2}$

To find the vector and/or parametric equations of a line in R^2, we must be given two points on the line, or a point on the line and a direction vector.

A direction vector is defined to be a vector $\vec{m} = \overrightarrow{(a,b)}$ that is parallel to (i.e., collinear with) the line in question.

Example 1

Sketch the line that has the point (3,-2) on it, with the direction vector $\vec{m} = \overrightarrow{(4,-1)}$.



Therefore, to travel from the origin to any point on the line, we need to travel along the following journey:

$$\vec{r} = \vec{r_o} + t\vec{m}, t\epsilon R$$

This is the vector equation

In this example, the vector equation is $\vec{r} = \overrightarrow{(3,-2)} + t\overrightarrow{(4,-1)}, t \in R$, where $\overrightarrow{(3,-2)}$ is called the position vector, $\overrightarrow{(4,-1)}$ is called the direction vector, and t is called a parameter

The vector equation of a line in R^2 is
$$\vec{r} = \vec{r_o} + t\vec{m},\ t \in R$$
where
$\vec{r_o}$ is called the position vector
$\vec{m}$ is called the direction vector
t is called a parameter

How to Develop the Parametric Equations

If we are given the vector equation

$$\vec{r} = \overrightarrow{(3,-2)} + t\overrightarrow{(4,-1)},\ t \in R$$

we can think of that as

$$\overrightarrow{(x,y)} = \overrightarrow{(3,-2)} + t\overrightarrow{(4,-1)},\ t \in R$$

In turn, we can think of that as

$x = 3 + 4t, y = -2 - t, t \in R$

These are the parametric equations of the line

Example 1

Determine the vector and parametric equations of a line passing through the points $(5, -6)$ and $(12, -8)$.

Example 2a

Given the line that you just found in the previous example, i.e., with vector equation $\vec{r} = \overrightarrow{(5, -6)} + t(7, -2), t \in R$ and parametric equations $x = 5 + 7t, y = -6 - 2t, t \in R$, determine two more points on the line.

Example 2b

Given the line that you just found in the previous example, i.e., with vector equation $\vec{r} = \overrightarrow{(5, -6)} + t(7, -2), t \in R$ and parametric equations $x = 5 + 7t, y = -6 - 2t, t \in R$, determine if the points (26, -12) and (-23, 3) are on the line.

Example 2c

Given the line that you just found in the previous example, i.e., with vector equation $\vec{r} = \overrightarrow{(5, -6)} + t(7, -2), t \in R$ and parametric equations $x = 5 + 7t, y = -6 - 2t, t \in R$, determine the x and y intercepts of the line

Example 2d

Given the line that you just found in the previous example, i.e., with vector equation $\vec{r} = \overrightarrow{(5,-6)} + t\overrightarrow{(7,-2)}, t \in R$ and parametric equations $x = 5 + 7t, y = -6 - 2t, t \in R,$ if the vector equation $\vec{r} = \overrightarrow{(19,-10)} + t\overrightarrow{\left(\frac{-7}{2},1\right)}$ could also represent the line.

Example 3

Determine the equation of a line that is perpendicular to the line $\vec{r} = \overrightarrow{(5,-9)} + t\overrightarrow{(3,-5)}$ and that passes through the point (-5, -1).