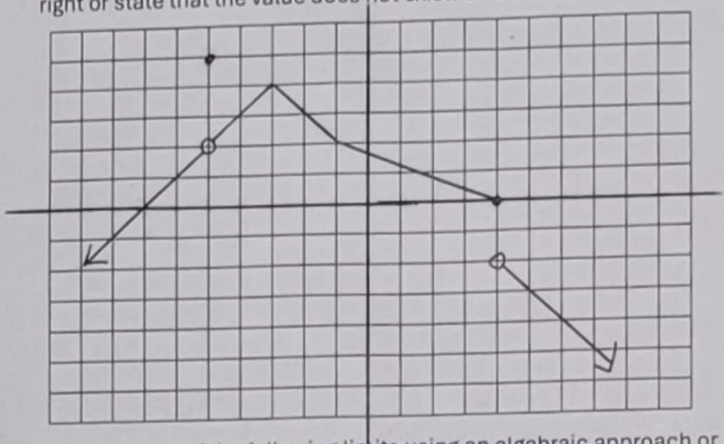
MCV4U Initial Qu

1. You are given the graph of the function $y = f(x)$ below. State the value of each of the values shown at the right or state that the value does not exist. One mark for each.



a) $\lim_{x \to -2^+} f(x) = 3$ ✓

b) $\lim_{x \to 4} f(x) =$ does not exist ✓

c) $\lim_{x \to -5} f(x) = 2$ ✓

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2. Evaluate each of the following limits using an algebraic approach or state that the limit does not exist. Use good technique. Use limit notation until it is no longer necessary in your solution and put equals signs at each step where they belong. No decimal approximations in final answer.

a. $\lim_{x \to -3} \frac{6x^2+33x+45}{x^2+5x+6}$ (3 marks)

$= \lim_{x \to -3} \frac{3(2x^2+11x+15)}{(x+3)(x+2)}$

$= \lim_{x \to -3} \frac{3\cancel{(x+3)}(2x+5)}{\cancel{(x+3)}(x+2)}$

$= \lim_{x \to -3} \frac{3(2x+5)}{x+2}$

$= \frac{3(2(-3)+5)}{-3+2}$

$= 3$

✓ 3

$\boxed{\lim_{x \to -3} \frac{6x^2+33x+45}{x^2+5x+6} = 3}$

b) $\lim_{x \to \frac{-3}{2}} \frac{4x+6}{2x^3-3x^2+5x+21}$ (4 marks)

$= \lim_{x \to -\frac{3}{2}} \frac{2(2x+3)}{2x^3-3x^2+5x+21}$

| | | | | |
|---|---|---|---|---|
| $-\frac{3}{2}$ | 2 | -3 | 5 | 21 |
| | ↓ | -3 | 9 | -21 |
| | 2 | -6 | 14 | 0 |

$= \lim_{x \to -3/2} \frac{4\cancel{(x+\frac{3}{2})}}{\cancel{(x+\frac{3}{2})}(2x^2-6x+14)}$

$= \lim_{x \to -3/2} \frac{4}{2x^2-6x+14}$

$= \frac{4}{2(-\frac{3}{2})^2-6(-\frac{3}{2})+14}$

$= \frac{8}{55}$

✓ 4

$= \frac{2(2x+3)}{(x+\frac{3}{2})(2x^2-6x+14)}$

$= \frac{4(x+\frac{3}{2})}{(x+\frac{3}{2})(2x^2-6x+14)}$

$=$

$\boxed{\lim_{x \to \frac{-3}{2}} \frac{4x+6}{2x^3-3x^2+5x+21} = \frac{8}{55}}$

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c) $\lim_{x \to 5} \dfrac{\sqrt{3x+1} - \sqrt{x+11}}{\sqrt{10x+14} - \sqrt{6x+34}}$ (6 marks)

$$= \lim_{x \to 5} \frac{(\sqrt{3x+1} - \sqrt{x+11})(\sqrt{3x+1} + \sqrt{x+11})(\sqrt{10x+14} + \sqrt{6x+34})}{(\sqrt{10x+14} - \sqrt{6x+34})(\sqrt{10x+14} + \sqrt{6x+34})(\sqrt{3x+1} + \sqrt{x+11})}$$

$$= \lim_{x \to 5} \frac{(3x+1-x-11)(\sqrt{10x+14} + \sqrt{6x+34})}{(10x+14-6x-34)(\sqrt{3x+1} + \sqrt{x+11})}$$

$$= \lim_{x \to 5} \frac{(2x-10)(\sqrt{10x+14} + \sqrt{6x+34})}{(4x-20)(\sqrt{3x+1} + \sqrt{x+11})}$$

$$= \lim_{x \to 5} \frac{2\cancel{(x-5)}(\sqrt{10x+14} + \sqrt{6x+34})}{4\cancel{(x-5)}(\sqrt{3x+1} + \sqrt{x+11})}$$

$$= \frac{2(\sqrt{10(5)+14} + \sqrt{6(5)+34})}{4(\sqrt{3(5)+1} + \sqrt{5+11})}$$

$$= 1$$

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$$\boxed{\lim_{x \to 5} \frac{\sqrt{3x+1} - \sqrt{x+11}}{\sqrt{10x+14} - \sqrt{6x+34}} = 1}$$

d) $\lim_{x \to -3} \dfrac{\sqrt[5]{9x-5} + 2}{x^2+8x+15}$ let $a = (9x-5)^{1/5}$, $b = 2$ (5 marks)

$$= \lim_{x \to -3} \frac{a+b}{(x+3)(x+5)}$$

$$= \lim_{x \to -3} \frac{(a+b)(a^4 - a^3b + a^2b^2 - ab^3 + b^4)}{(x+3)(x+5)(a^4 - a^3b + a^2b^2 - ab^3 + b^4)}$$

$$= \lim_{x \to -3} \frac{a^5 + b^5}{(x+3)(x+5)(a^4 - a^3b + a^2b^2 - ab^3 + b^4)}$$

$$= \lim_{x \to -3} \frac{9x-5+32}{(x+3)(x+5)[(9x-5)^{4/5} - 2(9x-5)^{3/5} + 4(9x-5)^{2/5} - 8(9x-5)^{1/5} + 16]}$$

$$= \lim_{x \to -3} \frac{9x+27}{(x+3)(x+5)\,(\text{long factor})}$$

$$= \lim_{x \to -3} \frac{9\cancel{(x+3)}}{\cancel{(x+3)}(x+5)(\text{long factor})}$$

$$= \frac{9}{(-3+5)(16+16+16+16+16)}$$

$$= \frac{9}{160}$$

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$$\boxed{\lim_{x \to -3} \frac{\sqrt[5]{9x-5} + 2}{x^2+8x+15} = \frac{9}{160}}$$

e) $\lim_{x \to 3} \left[\left(\dfrac{1}{x-3}\right)\left(\dfrac{1}{x+1} - \dfrac{2}{x+5}\right)\right]$ (5 marks)

$$= \lim_{x \to 3} \left[\left(\frac{1}{x-3}\right)\left(\frac{x+5-2(x+1)}{(x+1)(x+5)}\right)\right]$$

$$= \lim_{x \to 3} \left(\frac{1}{x-3}\right)\left(\frac{x+5-2x-2}{(x+1)(x+5)}\right)$$

$$= \lim_{x \to 3} \left(\frac{1}{\cancel{x-3}}\right)\left(\frac{-\cancel{(x-3)}}{(x+1)(x+5)}\right)$$

$$= (1)\left(\frac{-1}{(3+1)(3+5)}\right)$$

$$= -\frac{1}{32}$$

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$$\boxed{\lim_{x \to 3} \left[\left(\frac{1}{x-3}\right)\left(\frac{1}{x+1} - \frac{2}{x+5}\right)\right] = -\frac{1}{32}}$$

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