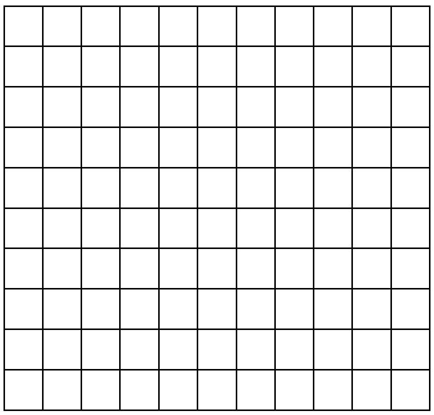
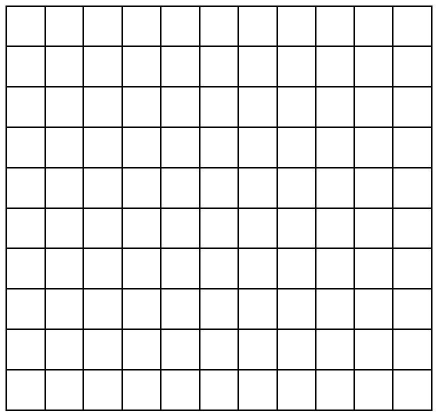
PDF 1.050 Continuity

The idea of continuity can be thought of informally as the idea of being able to draw a graph without lifting one's pencil.

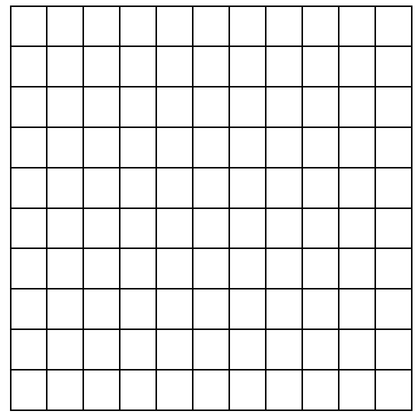
Three types of discontinuity are illustrated below.



Hole



Jump Discontinuity (common in piecewise functions)



Vertical Asymptote

Now for a more formal definition of continuity:

The function f is continuous at $x = a$ if $f(a)$ is defined and if $f(a) = \lim_{x \to a} f(x)$

<u>Example 1</u>

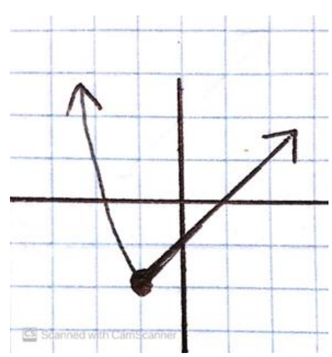
a) Graph the function $f(x) = \begin{Bmatrix} x^2 - 3, x \leq -1 \\ x - 1, \;\; x > -1 \end{Bmatrix}$

b) Determine $f(-1)$

c) Determine $\lim_{x \to -1} f(x)$

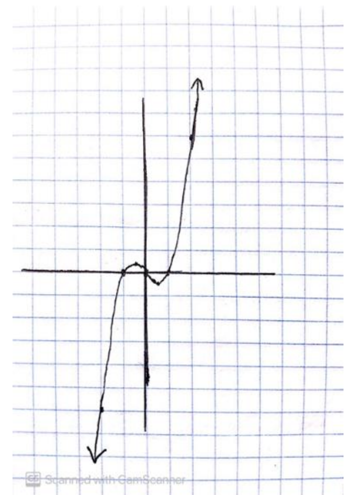
d) Is f continuous at $x = -1$

Solution:



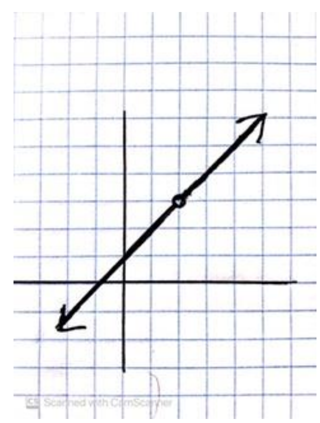
Example 2

Is the function $f(x) = x^3 - x$ continuous at $x = 2$?



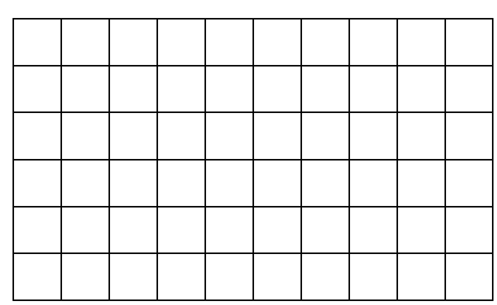
Example 3

Is the function $f(x) = \frac{x^2-x-2}{x-2}$ continuous at $x = 2$?



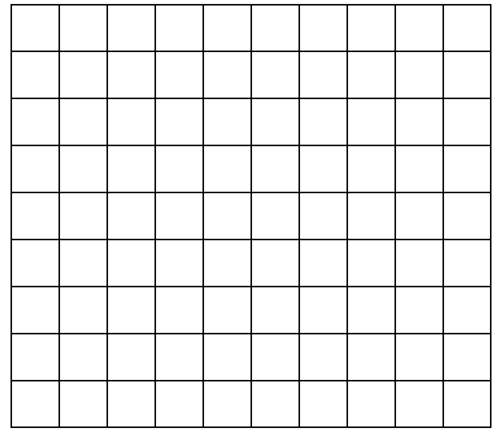
Example 4

Is the function $f(x) = \begin{Bmatrix} \frac{x^2-x-2}{x-2}, & x \neq 2 \\ 3, & x = 2 \end{Bmatrix}$ continuous at $x = 2$?



Example 5

Is the function $f(x) = \begin{cases} 5 - x^2, x < 1 \\ 3, \quad\quad\quad x \geq 1 \end{cases}$ continuous at $x = 1$?



General Observations Regarding Continuity

1. A function that is not continuous has some type of break in its graph. This break is the result of a hole, jump, or vertical asymptote.
2. All polynomial functions are continuous for all real numbers.
3. A rational function $h(x) = \frac{f(x)}{g(x)}$ is continuous at $x = a$ if $g(a) \neq 0$
4. A rational function in simplified for has a discontinuity at the zeros of the denominator.
5. When the one-sided limits are not equal to each other, then the limit at this point does not exist and the function is not continuous at this point.

Example 6

Determine the value of k such that the function $f(x) = \begin{cases} 3 - kx, x < 4 \\ x^2, \quad\quad\quad x \geq 4 \end{cases}$ is continuous over $\{x \in R\}$