PDF 7.020 Addition and Subtraction of Vectors in R^2 and R^3

The unit vectors $\vec{\imath}$, $\vec{\jmath}$ and $\vec{k}$

Any vector in the two-dimensional plane can be expressed in terms of the unit vectors $\vec{\imath}$ and $\vec{\jmath}$ where $\vec{\imath} = \overrightarrow{(1,0)}$ and $\vec{\jmath} = \overrightarrow{(0,1)}$

For example, $\overrightarrow{(-3,4)} = -3\overrightarrow{(1,0)} + 4\overrightarrow{(0,1)} = -3\vec{\imath} + 4\vec{\jmath}$

Similarly, any vector in three dimensions can be expressed in terms of the unit vectors $\vec{\imath}$, $\vec{\jmath}$ and $\vec{k}$ where $\vec{\imath} = \overrightarrow{(1,0,0)}$, $\vec{\jmath} = \overrightarrow{(0,1,0)}$ and $\vec{k}$=$\overrightarrow{(0,0,1)}$

For example, $\overrightarrow{(5,-2,-3)} = 5\overrightarrow{(1,0,0)} - 2\overrightarrow{(0,1,0)} - 3\overrightarrow{(0,0,1)}$

$$= 5\vec{\imath} - 2\vec{\jmath} - 3\vec{k}$$

Distributive Property

As a general rule, if $\vec{v} = \overrightarrow{(a,b)}$, then $k\vec{v} = k\overrightarrow{(a,b)} = \overrightarrow{(ka,kb)}$

For example, $5\overrightarrow{(2,-3)} = \overrightarrow{(10,-15)}$

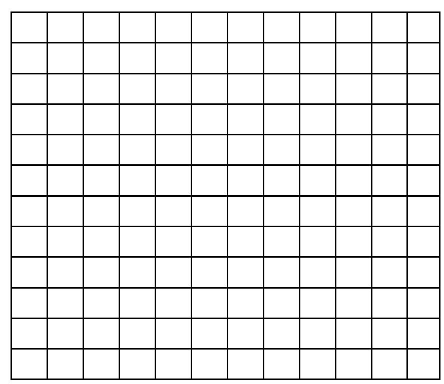
Similarly, if $\vec{v} = \overrightarrow{(a,b,c)}$, then $k\vec{v} = k\overrightarrow{(a,b,c)} = \overrightarrow{(ka,kb,kc)}$

For example, $-4\overrightarrow{(6,-1,2)} = \overrightarrow{(-24,4,-8)}$

Adding and Subtracting Vectors

Given $\vec{a} = \overrightarrow{OA} = \overrightarrow{(1,3)}$ and $\vec{b} = \overrightarrow{OB} = (4,-2)$,

a) graph $\overrightarrow{OA}$ and $\overrightarrow{OB}$
b) graph $\vec{a} + \vec{b}$ and state the components of that vector
c) graph $\vec{a} - \vec{b}$ and state the components of that vector



In short, if $\vec{a} = \overrightarrow{(x_1, y_1)}$ and if $\vec{b} = \overrightarrow{(x_2, y_2)}$, then

$$\vec{a} + \vec{b} = \overrightarrow{(x_1 + x_2, y_1 + y_2)}$$

This also works in 3 dimensions. In other words, if

$\vec{a} = \overrightarrow{(x_1, y_1, z_1)}$ and if $\vec{b} = \overrightarrow{(x_2, y_2, z_2)}$, then

$$\vec{a} + \vec{b} = \overrightarrow{(x_1 + x_2, y_1 + y_2, z_1 + z_2)}$$

Subtracting vectors: Notice that if we are to subtract vector $\vec{b}$ from vector $\vec{a}$, we can do so by the tip-to-tail method as well.

We add the opposite of $\vec{b}$ to $\vec{a}$. In other words, $\vec{a}-\vec{b}=\vec{a}+(-\vec{b})$

So, for example,

$$\overrightarrow{(1,3)}-\overrightarrow{(4,-2)}=\overrightarrow{(-3,5)}$$

In short, if $\vec{a}=\overrightarrow{(x_1,y_1)}$ and if $\vec{b}=\overrightarrow{(x_2,y_2)}$, then

$$\vec{a}-\vec{b}=\overrightarrow{(x_1-x_2,y_1-y_2)}$$

This also works in 3 dimensions. In other words, if $\vec{a}=\overrightarrow{(x_1,y_1,z_1)}$ and if $\vec{b}=\overrightarrow{(x_2,y_2,z_2)}$, then

$$\vec{a}-\vec{b}=\overrightarrow{(x_1-x_2,y_1-y_2,z_1-z_2)}$$

Magnitudes of Vectors

Given the points $A(x_1,y_1)$ and $B(x_2,y_2)$,

$\overrightarrow{AB}=\overrightarrow{(x_2-x_1,y_2-y_1)}$ and $|\overrightarrow{AB}|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Similarly, given the points $A(x_1,y_1,z_1)$ and $B(x_2,y_2,z_2)$,

$\overrightarrow{AB}=\overrightarrow{(x_2-x_1,y_2-y_1,z_2-z_1)}$ and

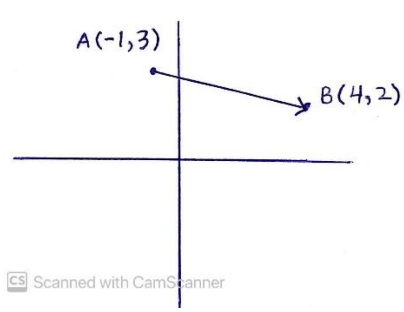
$$|\overrightarrow{AB}|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

Example 1

Determine $\overrightarrow{AB}$ and $|\overrightarrow{AB}|$ where the coordinates of A are (3,2,-5) and the coordinates of B are (-4,0,-3).

Example 2

For the vector shown at the right, determine the components of the position vector.



Example 3

Parallelogram OABC is determined by the vectors $\overrightarrow{OA} = \overrightarrow{(7,4)}$ and $\overrightarrow{OC} = \overrightarrow{(1,-6)}$.
Determine $\overrightarrow{OB}$, $\overrightarrow{AB}$, and $\overrightarrow{BC}$.

Example 4

Find the components of the unit vector in the direction opposite to $\overrightarrow{CD}$ where $\overrightarrow{OC} = \overrightarrow{(-3,-6)}$ and $\overrightarrow{OD} = \overrightarrow{(-11,9)}$.
Then, determine the measure of the angle between $\overrightarrow{OC}$ and $\overrightarrow{OD}$

Example 5

If $\vec{u} = -2\vec{\imath} + 9\vec{\jmath} + 6\vec{k}$, and if $\vec{v} = \vec{\imath} - \vec{\jmath} + \vec{k}$, determine $3\vec{u} - 2\vec{v}$ and $|3\vec{u} - 2\vec{v}|$

Example 6

A parallelepiped is formed by the vectors $\overrightarrow{OA} = \overrightarrow{(3,-1,4)}$, $\overrightarrow{OB} = \overrightarrow{(-1,8,-7)}$, and $\overrightarrow{OC} = \overrightarrow{(4,0,-9)}$.

Determine the coordinates of all the vertices for the parallelepiped.

Example 7

Determine the midpoint of the points A (-4, 3, 9) and B (2, -11, 7)