Chap 1.020 Evaluating Limits Algebraically

Assume that we are trying to evaluate $\lim_{x \to c} f(x), c \in R$

We often don't use a graphing approach to evaluate a limit. Rather, we can use an algebraic approach if we remember the following steps.

1. If the curve is continuous at x = c, then we can simply evaluate f(c). In other words, $\lim_{x \to c} f(x) = f(c)$
    - This happened in the first example in the powerpoint about evaluating limits graphically. When we were seeking to evaluate $\lim_{x \to 3}(-x^2 + 6x - 5)$, we could have recognized that this was a continuous function at x = 3 and simply plugged in an x-value of 3.
2. However, sometimes direct substitution leads to a result of 0/0. Obviously, this result is undefined. However, the limit may still exist because there may be a hole at x = c in an otherwise continuous curve.
    - This is what happened in some of our examples in the powerpoint on graphically evaluating limits. For example, in the example where we sought $\lim_{x \to -2}\left(\frac{x^2+3x+2}{x+2}\right)$, there was a hole and the value of the limit equaled the y-coordinate of the location of the hole.

   When we get the indeterminate form of 0/0, we can try the following:

i. Factor numerator and denominator, and the offending factor may cancel out of both
ii. Rationalize the numerator and/or denominator and see if this leads you to be able to directly substitute
iii. Simplify the function prior to substituting to see if that allows you to directly substitute
iv. Introduce a new factor that allows the numerator and/or denominator to become a difference or sum of nth powers, which then creates the offending factor which can then be canceled from both numerator and denominator (you may wish to introduce a new variable to do this)

Examples where direct substitution leads to the correct answer

Example 1

Evaluate $\lim_{x \to 5} x^2 + 2x - 3$

Example 2

Evaluate $\lim_{x \to 5} 11$

Example 3

Evaluate $\lim_{x \to 0} \frac{x-1}{x+1}$

Example 4

Evaluate $\lim_{x \to 3} \frac{\sqrt{x-1} - \sqrt{x+1}}{\sqrt{x-3} - \sqrt{x+3}}$

2a) Examples where Direct Substitution Leads to 0/0 But you can then factor, cross out, and then sub in

Example 5

Evaluate $\lim_{x \to 3} \frac{x-3}{x^2+x-12}$

Example 6

Evaluate $\lim_{h \to 0} \frac{(2+h)^3 - 8}{h}$

Example 7

Evaluate $\lim_{x \to -2} \frac{x^4 - 16}{x + 2}$

Example 8

Evaluate $\lim_{x \to 25} \frac{x - 25}{\sqrt{x} - 5}$

Example 9

Evaluate $\lim_{x \to 2} \frac{x^2 - 4}{x^3 - 8}$

Difference of nth Powers Factoring

Example

Evaluate $\lim\limits_{x \to 3} \frac{2x^4 - 162}{-x^5 + 243}$

Example

Evaluate $\lim\limits_{x \to -2} \frac{x^7 + 128}{10x + 20}$

2b) Examples where direct substitution leads to 0/0 but we can rationalize the numerator and/or denominator, then perhaps factor, then cross out then substitute

Example

Evaluate $\lim\limits_{h \to 0} \frac{\sqrt{16+h} - 4}{h}$

Example

Evaluate $\lim\limits_{x \to 2} \frac{x - 2}{\sqrt{27 - x} - 5}$

Example

Evaluate $\lim_{x \to 8} \frac{3 - \sqrt{x+1}}{\sqrt{24-x} - 4}$

2c) Examples where direct substitution leads to 0/0 but we can simplify the expression and then directly substitute

Example

Evaluate $\lim_{x \to 2} \frac{\frac{1}{x} - \frac{1}{2}}{x - 2}$

<u>2d) Examples where direct substitution leads to 0/0 but we can create a difference of nth powers or sum of nth powers (if n is odd) and then factor and cross out and substitute</u>

**** Some people find it beneficial to introduce a new variable in these questions but it is not necessary**

<u>Example</u>

Evaluate $\lim_{x \to 64} \frac{\sqrt[3]{x} - 4}{x - 64}$

Example

Evaluate $\lim_{x \to -121} \frac{3x+363}{\sqrt[5]{2x-1} + 3}$

Example

Evaluate $\lim_{x \to 56} \frac{\sqrt[3]{x+8}-4}{\sqrt{x-40}-4}$