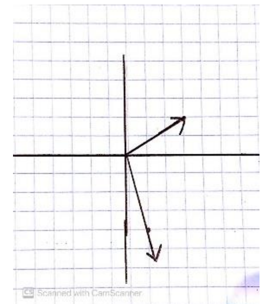
PDF 7.070 Cross Product (aka the Vector Product)

If you are given two non-zero vectors in R^2, it is not necessarily possible to find a vector perpendicular to both of the given vectors.

For example, consider the two vectors at the right.



However, it is a different story in R^3…

If you are given any two non-zero vectors in R^3, it is possible to find a third vector perpendicular to both of the given vectors. One way of doing this is called the cross-product.

The reason that it is always possible in R^3 is because you have a third dimension. (Think of the perpendicular vector as coming out of the page, or going into the page). Perhaps it's helpful to think of the x and y and z axes. That third dimension provides us with an opportunity to find a vector perpendicular to each of the given vectors.

Cross Product of Two Vectors in R^3

To determine a vector perpendicular to each of two different vectors in R^3, we use the cross product.

Suppose $\vec{a} = \overrightarrow{(a_1, a_2, a_3)}$ and $\vec{b} = \overrightarrow{(b_1, b_2, b_3)}$.

If we let $\vec{v} = \overrightarrow{(a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1)}$, then we can say that $\vec{v}$ is the cross product of $\vec{a}$ and $\vec{b}$, and we can write this as

$$\vec{v} = \vec{a} \times \vec{b}$$

$\vec{a} \times \vec{b} = \overrightarrow{(a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1)}$

* We develop the cross product formula in the powerpoint

Example 1

Show that $\vec{v}$ (from the formula above) is perpendicular to both $\vec{a}$ and $\vec{b}$.

Example 2

Given that $\vec{p} = \overrightarrow{(-1,3,2)}$ and that $\vec{q} = \overrightarrow{(2,-5,6)}$, determine $\vec{p} \times \vec{q}$ and $\vec{q} \times \vec{p}$. Then determine that each of them is perpendicular to each of $\vec{p}$ and $\vec{q}$.