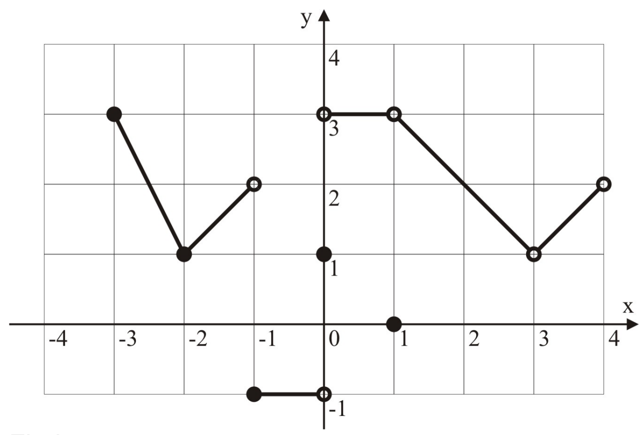
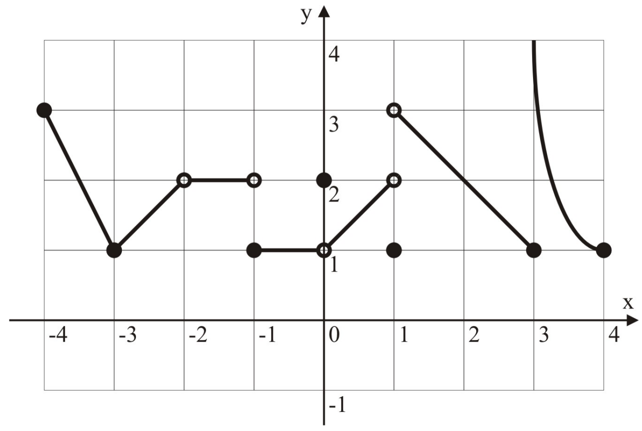
MCV4U Final Exam Review

Answer (or Solution)	Practice Questions
	1. Consider the function $f(x)$ defined by the following graph:  Find: a) $\lim\limits_{x\to -2} f(x)$ b) $\lim\limits_{x\to -1} f(x)$ c) $\lim\limits_{x\to 1} f(x)$ d) $\lim\limits_{x\to 3} f(x)$
	2. Evaluate the following limits. a) $\lim\limits_{x\to -1} (x^3 - 2x^2 + 3x - 1)$ b) $\lim\limits_{x\to 2} \dfrac{x + x^2}{x^3 - 2}$ c) $\lim\limits_{x\to \pi/2} \sqrt{\sin x}$ d) $\lim\limits_{x\to \pi} 2^{\cos x}$
	3. Consider the function: $f(x) = \begin{cases} x+1 & \text{if } x < -1 \\ x^2 & \text{if } -1 \le x \le 1 \\ \sqrt{x} & \text{if } x > 1 \end{cases}$ Find the following limits, if they exit. a) $\lim\limits_{x\to -1} f(t)$ b) $\lim\limits_{x\to 0} f(t)$ c) $\lim\limits_{x\to 1} f(t)$
	4. Evaluate. a) $\lim\limits_{x\to -2} \dfrac{x^2 - 4}{x + 2}$ b) $\lim\limits_{x\to 1} \dfrac{x^2 + x - 2}{x - 1}$ c) $\lim\limits_{x\to 2} \dfrac{x^3 - 8}{x^2 - 4}$ d) $\lim\limits_{x\to 3} \dfrac{x^2 - 2x - 3}{x^2 - 4x + 3}$ e) $\lim\limits_{x\to -3} \dfrac{x^3 + 27}{x + 3}$ f) $\lim\limits_{x\to 2} \dfrac{\frac{1}{x} - \frac{1}{2}}{x - 2}$
	5. Evaluate. a) $\lim\limits_{x\to 4} \dfrac{x - 4}{\sqrt{x} - 2}$ b) $\lim\limits_{x\to 1} \dfrac{\sqrt{x} - x}{x - 1}$ c) $\lim\limits_{x\to 2} \dfrac{\sqrt{6 - x} - 2}{\sqrt{3 - x} - 1}$ d) $\lim\limits_{t\to 0} \dfrac{\sqrt{3 + t} - \sqrt{3}}{t}$ e) $\lim\limits_{x\to 1} \dfrac{\frac{1}{\sqrt{x}} - 1}{x - 1}$ f) $\lim\limits_{t\to 0} \left(\dfrac{1}{t\sqrt{1 + t}} - \dfrac{1}{t} \right)$
	6. Evaluate. a) $\lim\limits_{x\to 8} \dfrac{x - 8}{\sqrt[3]{x} - 2}$ b) $\lim\limits_{x\to 4} \dfrac{x^{3/2} - 8}{x - 4}$ c) $\lim\limits_{x\to 1} \dfrac{\sqrt{x} - 1}{\sqrt[3]{x} - 1}$

7. Consider the function $f(x)$ defined by the following graph:



For each value of $x = a$, classify the function $f(x)$ as continuous, ~~having a jump, a removable or an infinite discontinuity~~. or discontinuous

a) $a = -3$ b) $a = -1$ c) $a = 2$
d) $a = 0$ e) $a = 3$

8. For what value of the constant c is the function

$$f(x) = \begin{cases} x + c & \text{if } x < 2 \\ cx^2 + 1 & \text{if } x \geq 2 \end{cases}$$

continuous at every number?

9. For each function, calculate first the slope of the tangent line with the formula:

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

then find the equation of the tangent line at the given point.

a) $f(x) = x^2 + x$, at $P(-1,0)$
b) $f(x) = \dfrac{x-1}{x+1}$, at $P(1,0)$

10. For each case find the slope of the tangent line at the general point $P(a, f(a))$ using $m = \lim_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$.

a) $f(x) = x^2$ b) $f(x) = x^3$
c) $f(x) = x^2 - 2x + 1$ d) $f(x) = \dfrac{1}{x}$

11. For each case, find the ARC over the given interval.

a) $f(x) = x^4 - x^3 + x^2$, $[-1,1]$
b) $f(x) = \dfrac{2x-1}{2x+1}$, $[0,2]$

12. For each case, find the IRC at the given number.

a) $f(x) = x^4 - x^3$, at $x = 1$
b) $f(x) = \dfrac{x}{x^2+1}$, at $x = 0$

IRC means instantaneous rate of change

13. For each case, find the average velocity over the given interval.

a) $s(t) = t^2 + t$, $[0,2]$
b) $s(t) = t^3 - t^2$, $[1,2]$

14. For each case, find the instantaneous velocity at the given moment of time.

a) $s(t) = 2t^2 - t$, at $t = 1$
b) $s(t) = 2t^3 - 3t$, at $t = 0$

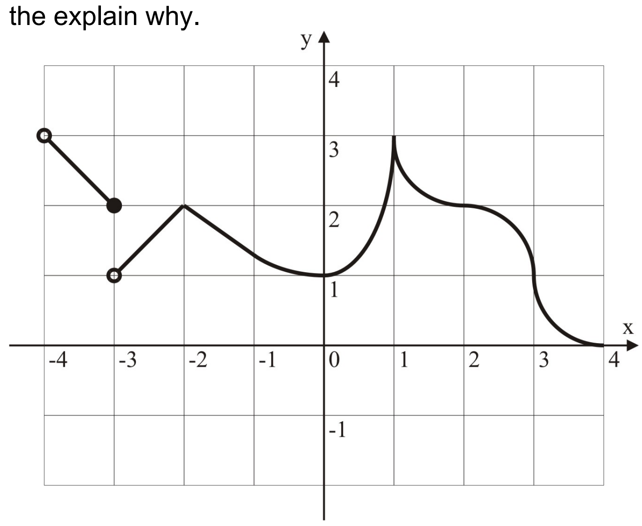
1. Use the first principles method to find the derivative of each function. State the domain of each function and its derivative.

a) $f(x)=3$ b) $f(x)=-2x+5$

c) $f(x)=3x^2-2x+1$ d) $f(x)=x^3+2x^2+3x+4$

e) $f(x)=\sqrt{x}$

2. For the function $y=f(x)$ defined graphically below, find the values where the function f is not differentiable and the explain why.



3. Use the power rule to differentiate.

a) $f(x)=x$ b) $f(x)=x^2$ c) $f(x)=x^3$

d) $f(x)=x^0$ e) $f(x)=x^{-1}$ f) $f(x)=\dfrac{1}{x^2}$

g) $f(x)=\sqrt{x}$ h) $f(x)=\sqrt[3]{x}$ i) $f(x)=\sqrt{x^3}$

4. Differentiate.

a) $f(x)=-3\sin x$ b) $f(x)=5\cos x$

c) $f(x)=-4e^x$ d) $f(x)=-2\ln x$

5. Differentiate.

a) $f(x)=1-2x+3x^2$ b) $f(x)=x+\dfrac{2}{x}-\dfrac{3}{x^2}+x^3$

c) $f(x)=\sin x-\cos x$ d) $f(x)=-2e^x+3\ln x$

6. Find the equation of the tangent line to the curve $y=x^3-3x^2$ at the point $T(1,-2)$.

7. Find the equation of the tangent line(s) with the slope $m=-6$ to the curve $y=x^4-2x$.

8. At what points on the hyperbola $xy=12$ is the tangent line parallel to the line $3x+y=0$.

9. Differentiate.

a) $f(x)=(x-1)(x+2)$ b) $f(x)=\sqrt{x}(\sqrt{x}-1)$

c) $x\sin x$ d) xe^x e) $e^x \ln x$

10. Differentiate.

a) $f(x)=(2x-1)^2$ b) $f(x)=(x-\sqrt{x})^2$

c) $f(x)=(x^2+x+1)^{100}$

d) $f(x)=\sin^3 x$ e) $f(x)=(\ln x)^5$

11. Differentiate, then simplify.

a) $f(x)=\dfrac{x^2}{x-1}$ b) $f(x)=\dfrac{x^2-1}{x^2+1}$

c) $f(x)=\dfrac{\sin x}{\cos x}$ d) $f(x)=\dfrac{\sin x}{e^x}$

e) $f(x)=\dfrac{\sqrt{x}}{\ln x}$ f) $f(x)=\dfrac{\ln x}{\sin x}$

12. For each case, use $\dfrac{dy}{dx}=\dfrac{dy}{du}\dfrac{du}{dx}$ to find the derivative of $y=f(g(x))$.

a) $y=u^5$, $u=2x-1$ b) $y=\sqrt{u}$, $u=x^2+1$

13. Use chain rule to differentiate.

a) $y=[(x^2-1)^3+x^2]^2$ b) $y=\dfrac{(1-x^2)^2}{(1-x)^3}$

c) $y=\sqrt{x+\sqrt{x}}$

14. Use the generalized differentiation rules to find the derivative of each function.

a) $y=(1+x+x^2)^{10}$ b) $y=\sqrt{x^2-2x}$

c) $y=\sin\sqrt{x}$ d) $y=\sqrt{\cos x}$

e) $y=\ln\sqrt{x}$ f) $y=\sqrt{x+e^x}$

15. For each case, find the first and the second derivative.

a) $y=1-3x+4x^2$ b) $y=4-2x+x^2-2x^3$

c) $y=\dfrac{1}{x}$ d) $y=\dfrac{1}{x^2}$

e) $y=\sqrt{x}$ f) $y=\dfrac{x}{x+1}$

g) $y=\sin x$ h) $y=e^x$

i) $y=\ln x$ j) $y=\log x$

16. For each case, find the velocity and the acceleration functions.

a) $s(t)=2t^2-3t+1$ b) $s(t)=t^3+2t^2+t-3$

c) $s(t)=\dfrac{t}{t+1}$

17. For each case, find the moments of time at which the object is at rest.

a) $s(t)=t^3-t$ b) $s(t)=3t^4-6t^2$

	1. For each case, use the first derivative sign to find the intervals of increase or decrease. a) $f(x)=x^2-2x$ b) $f(x)=\frac{x}{x-1}$ c) $f(x)=\frac{x}{x^2+1}$ d) $f(x)=\sqrt{x}(x-1)$ **2.** Find the intervals of increase or decrease. a) $f(x)=x\ln x$ b) $f(x)=xe^x$ c) $f(x)=xe^{-x}$ d) $f(x)=x+\sin x$
critical point means $f'(x)=0$ or $f'(x)$ is undefined	**3.** For each case, find the critical points. a) $f(x)=x^2+2x$ b) $f(x)=2x^3+3x^2$ c) $f(x)=\|x\|$ d) $f(x)=\sqrt[3]{x}$ e) $f(x)=\frac{1}{x^2}$ f) $f(x)=\frac{x}{x^2+1}$ **4.** For each case, find the critical points. a) $f(x)=\sin x$ b) $f(x)=\tan x$ c) $f(x)=e^x$ d) $f(x)=\ln x$ e) $f(x)=x\ln x$ f) $f(x)=xe^x$
	5. For each case, find any local extrema using the first derivative test. a) $f(x)=x^4-2x^2+1$ b) $f(x)=x^2(3-2x)$ c) $f(x)=\frac{1+x}{1-x}$ d) $f(x)=x-\sqrt{x}$ e) $f(x)=x^2-\sqrt{x}$ f) $f(x)=\|x^2-4\|$
	6. For each case, find the absolute extrema (maximum or minimum) points. a) $f(x)=2x^3+3x^2-12x+1,\quad for\ x\in[-3,2]$ b) $f(x)=\frac{5x}{x+1},\quad for\ x\in[0,4]$ c) $f(x)=x+\frac{4}{x},\quad for\ x\in[1,4]$ d) $f(x)=\cos x,\quad for\ x\in[-\pi/2,2\pi]$ e) $f(x)=x\log x,\quad for\ x\in[1,10]$ f) $f(x)=xe^{-x},\quad for\ x\in[-1,2]$ g) $f(x)=x+\sin x,\quad for\ x\in[0,2\pi]$
	7. For each case, find the intervals of concavity. a) $f(x)=x^4-6x^2$ b) $f(x)=(x^2-1)^3$ c) $f(x)=\frac{x}{x^2-1}$ d) $f(x)=(x-1)(x+1)^3$ e) $f(x)=x^2e^x$ f) $f(x)=x\ln x$ g) $f(x)=x^2\ln x$ h) $f(x)=x+\cos x$

8. For each case, find the points of inflection.

a) $f(x) = x^3 - x$ b) $f(x) = x + \frac{1}{x^2}$

c) $f(x) = (x+1)^{5/3}$ d) $f(x) = (1-x)^2(1+x)^2$

e) $f(x) = x^2 \ln x$ f) $f(x) = x - \sin x$

9. Find c given that the graph of $f(x) = cx^2 + 1/x^2$ has a point of inflection at $(1, f(1))$.

10. Use the second derivative test to find the local maximum and minimum values of each function.

a) $f(x) = x^3 - 6x^2$ b) $f(x) = x^4 - 6x^2 - 5$

c) $f(x) = \frac{x}{x^2+1}$ d) $f(x) = \frac{x}{(x-1)^2}$

11. Find the local minimum and maximum values for:

a) $y = x^3$ b) $y = x^4$

6. Second Derivative

⇨ compute $f''(x)$

⇨ find points where $f''(x) = 0$ or $f''(x)$ DNE

⇨ find points of inflection

⇨ find intervals of concavity upward/downward

⇨ check the local extrema using the second derivative test

7. Sketching

⇨ use broken lines to draw the asymptotes

⇨ plot x- and y- intercepts, extrema, and inflection points

⇨ draw the curve near the asymptotes

⇨ sketch the curve

12. Sketch the graph of the following polynomial functions.

a) $f(x) = 2x^3 - 3x^2 - 36x$ b) $f(x) = 3x^5 - 5x^3$

c) $f(x) = (x-1)^3$ d) $f(x) = x^2(x+3)$

e) $f(x) = (x^2-3)(x^2-5)$

13. Sketch the graph of the following rational functions.

a) $f(x) = \frac{x-1}{x+1}$ b) $f(x) = \frac{x}{x^2-1}$

c) $f(x) = \frac{x^2+1}{x^2-1}$ d) $f(x) = \frac{x^3}{x^2+1}$

14. A rectangle has a perimeter of $100m$. What length and width should it have so that its area is a maximum. What is the maximum value of its area?

15. If $2700cm^2$ of material is available to make a box with a square base and open top, find the dimensions of the box that give the largest volume of the box. What is the maximum value of the volume?

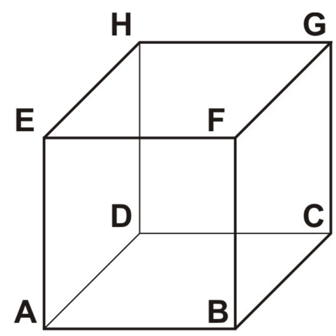
16. A rectangular piece of paper with perimeter $100cm$ is to be rolled to form a cylindrical tube. Find the dimensions of the paper that will produce a tube with maximum volume.

17. A farmer wants to fence an area of $240{,}000m^2$ in a rectangular field and divide it in half with a fence parallel to one of the sides of the rectangle. How can be done so as to minimize the cost of the fence?

18. A metal cylinder container with an open top is to hold $1ft^3$. If there is no waste in construction, find the dimensions that require the least amount of material.

1. Consider the cube $ABCDEFGH$ with the side length equal to $10cm$. Find the magnitude of the following vectors:

a) $\overrightarrow{AB}$ b) $\overrightarrow{BD}$ c) $\overrightarrow{BH}$



2. Prove or disprove each statement.

a) If $\vec{a} = \vec{b}$ then $|\vec{a}| = |\vec{b}|$.

b) If $|\vec{a}| = |\vec{b}|$ then $\vec{a} = \vec{b}$.

3. Two vectors are defined by $\vec{a} = 4N[E]$ and $\vec{b} = 5N[090°]$. Find the sum vector $\vec{s} = \vec{a} + \vec{b}$ the difference vector $\vec{d} = \vec{a} - \vec{b}$.

4. Two vectors are defined by $\vec{a} = 2km[W]$ and $\vec{b} = 4km[S]$. Find the sum vector $\vec{s} = \vec{a} + \vec{b}$ the difference vector $\vec{d} = \vec{a} - \vec{b}$.

5. Two vectors are defined by $\vec{a} = 20m[E]$ and $\vec{b} = 30m[150°]$. Find the sum vector $\vec{s} = \vec{a} + \vec{b}$ the difference vector $\vec{d} = \vec{a} - \vec{b}$.

6. Given $\vec{a} = 2\vec{i} - 3\vec{j} + \vec{k}$, $\vec{b} = -\vec{i} + \vec{j} + 2\vec{k}$, simplify the following expressions:

a) $\vec{a} + \vec{b}$ b) $\vec{a} - 2\vec{b}$ c) $2\vec{a} - 3\vec{b}$

7. Find a unit vector parallel to the sum between $\vec{a} = 2m[E]$ and $\vec{b} = 3m[N]$.

8. Given $\vec{u} = 8m[W]$ and $\vec{v} = 10m[S30°W]$, determine the magnitude and the direction of the vector $2\vec{u} - 3\vec{v}$.

9. Adam can swim at the rate of $2km/h$ in still water. At what angle to the bank of a river must he head if he wants to swim directly across the river and the current in the river moves at the rate of $1km/h$?

10. A plane is heading due north with an air speed of $400km/h$ when it is blown off course by a wind of $100km/h$ from the northeast. Determine the resultant ground velocity of the airplane (magnitude and direction).

11. A car is travelling at $\overrightarrow{v_{car}} = 100km/h[E]$, a motorcycle is travelling at $\overrightarrow{v_{moto}} = 80km/h[W]$, a truck is travelling at $\overrightarrow{v_{truck}} = 120km/h[N]$ and an SUV is travelling at $\overrightarrow{v_{SUV}} = 100km/h[SW]$. Find the relative velocity of the car relative to:

a) motorcycle
b) truck
c) SUV

1. Find the algebraic vector $\overrightarrow{AB}$ in ordered triplet notation and unit vector notation where $A(2,-3,4)$ and $B(0,-2,3)$.

2. Find the magnitude of the vector $\vec{v} = -2\vec{i} + \vec{j} - 3\vec{k}$.

3. Given $\vec{a} = (-1,2,-3)$, $\vec{b} = 2\vec{i} - \vec{j} + \vec{k}$, and $\vec{c} = \vec{i} + \vec{j}$ do the required operations:
a) $2\vec{a} - \vec{b} + 3\vec{c}$
b) $3(\vec{a} + 2\vec{b}) - 2(\vec{a} - \vec{c})$

4. Given $A(1,-2,3)$, $B(-2,3,-4)$, and $C(0,1,-1)$, find the coordinates of a point $D(x,y,z)$ such that $ABCD$ is a parallelogram.

5. The magnitudes of two vectors $\vec{a}$ and $\vec{b}$ are $|\vec{a}| = 2$ and $|\vec{b}| = 3$ respectively, and the angle between them is $\alpha = 60°$. Find the value of the dot product of these vectors.

6. Find the dot product of the vectors $\vec{a}$ and $\vec{b}$ where $\vec{a} = (1,-2,0)$ and $\vec{b} = \vec{i} - 2\vec{j} - \vec{k}$.

7. For what values of *k* are the vectors $\vec{a} = (6,3,-4)$ and $\vec{b} = (3,k,-2)$

a) perpendicular (orthogonal)?
b) parallel (collinear)?

8. Find the angle between the vectors $\vec{a}$ and $\vec{b}$ where $\vec{a} = (1,-2,-1)$ and $\vec{b} = -2\vec{j} + \vec{k}$.

9. A triangle is defined by three points $A(0,1,2)$, $B(1,0,2)$, and $C(-1,2,0)$. Find the angles $\angle A$, $\angle B$, and $\angle C$ of this triangle.

10. Given the vector $\vec{a} = (2,-3,4)$, find the scalar projection:
a) of $\vec{a}$ onto the unit vector $\vec{i}$
b) of $\vec{a}$ onto the vector $\vec{i} - \vec{j}$
c) of $\vec{a}$ onto the vector $\vec{b} = -\vec{i} + 2\vec{j} + \vec{k}$
d) of the unit vector $\vec{i}$ onto the vector $\vec{a}$

11. Given two vectors $\vec{a} = (0,1,-2)$ and $\vec{b} = (-1,0,3)$, find:
a) the vector projection of the vector $\vec{a}$ onto the vector $\vec{b}$
b) the vector projection of the vector $\vec{b}$ onto the vector $\vec{a}$
c) the vector projection of the vector $\vec{a}$ onto the unit vector $\vec{k}$
d) the vector projection of the vector $\vec{i}$ onto the vector $\vec{a}$

12. The magnitudes of two vectors $\vec{a}$ and $\vec{b}$ are $|\vec{a}| = 2$ and $|\vec{b}| = 3$ respectively, and the angle between them is $\alpha = 60°$. Find the magnitude of the cross product of these vectors.

easy with components but easy to make a mistake	**13.** For each case, find the cross product of the vectors $\vec{a}$ and $\vec{b}$. a) $\vec{a}=(1,-2,0)$, $\vec{b}=(0,-1,2)$ b) $\vec{a}=-\vec{i}+2\vec{j}$, $\vec{b}=\vec{i}-2\vec{j}-\vec{k}$ **14.** Use the cross product properties to prove the following relations: a) $(\vec{a}-\vec{b})\times(\vec{a}+\vec{b})=2(\vec{a}\times\vec{b})$ b) $(\vec{a}\times\vec{b})\cdot(\vec{a}\times\vec{b})+(\vec{a}\cdot\vec{b})(\vec{a}\cdot\vec{b})=(\vec{a}\cdot\vec{a})(\vec{b}\cdot\vec{b})$ **15.** Find an unit vector perpendicular to both $\vec{a}=(0,1,1)$ and $\vec{b}=(1,1,0)$.
	16. Find the area of the parallelogram defined by the vectors $\vec{a}=(1,-1,0)$ and $\vec{b}=(0,1,2)$.
	17. Find the area of the triangle defined by the vectors $\vec{a}=(1,2,3)$ and $\vec{b}=(3,2,1)$.
	18. Find the volume of the parallelepiped defined by the vectors $\vec{a}=(0,1,1)$, $\vec{b}=(0,1,0)$ and $\vec{c}=(1,0,1)$.
	19. Consider the following vectors: $\vec{a}=\vec{i}+\vec{j}-\vec{k}$, $\vec{b}=3\vec{i}-2\vec{j}$, and $\vec{c}=3\vec{i}-2\vec{k}$. Compute the required operations in terms of the unit vectors $\vec{i}$, $\vec{j}$, and $\vec{k}$. a) $\vec{a}+\vec{b}$ b) $\vec{a}-2\vec{b}$ c) $\vec{a}\cdot\vec{b}$ d) $\vec{b}\times\vec{c}$ e) $(\vec{a}\times\vec{b})\cdot\vec{c}$ f) $(\vec{a}\times\vec{b})\times\vec{c}$ g) $Proj(\vec{a}\ onto\ \vec{b})$

	1. Find the equation of a 2D line which a) passes through the points $A(0,-2)$ and $B(-3,1)$ b) passes through the point $A(1,-3)$ and is parallel to the vector $\vec{v}=(-2,-3)$ c) passes through the point $A(-2,3)$ and is perpendicular to the vector $\vec{v}=(3,-4)$ d) passes through the point $A(1,1)$ and is parallel to the line $y=-2+3x$ e) passes through the point $A(-2,-1)$ and is perpendicular to the line $2x-3y+4=0$
	2. Find the point(s) of intersection between the two given lines. a) $\vec{r}=(1,2)+t(3,1)$ and $\begin{cases}x=2-3s\\y=1-2s\end{cases}$ b) $\frac{x-1}{-2}=\frac{y+2}{4}$ and $y=-2x$ c) $y=-3x+1$ and $6x+2y-3=0$

	3. Find the equation of the perpendicular line to the given line through the given point. a) $\vec{r} = (0,1) + t(2,1)$, $B(2,-4)$ b) $\dfrac{x+1}{3} = \dfrac{y-2}{-2}$, $B(0,2)$ C) $2x - 3y + 4 = 0$, $B(3,1)$
	4. Find the distance from the given point to the given line. a) $\vec{r} = (-1,-2) + t(-1,2)$, $B(0,2)$ b) $\dfrac{x-3}{-2} = \dfrac{y+2}{3}$, $B(1,3)$ c) $x + 2y - 3 = 0$, $B(0,0)$
	5. Find the vector equation of a line that: a) passes through the points $A(0,1,2)$ and $B(-2,3,1)$ b) passes through the point $A(2,-1,4)$ and is perpendicular on the xy plane c) passes trough the point $A(3,-2,1)$ and is parallel to the y-axis d) passes through the point $A(2,-2,3)$ and is parallel to the vector $\vec{u} = (3,-2,1)$ e) passes through the origin O and is parallel to the vector $\vec{i} - 2\vec{j}$
	6. Convert the equation(s) of the line from the vector form to the parametric form or conversely: a) $\vec{r} = (0,1,2) + t(3,4,5)$ b) $\begin{cases} x = -1 - 2t \\ y = 1 + 3t \\ z = 2 \end{cases}$
	7. Convert each form of the equation(s) of the line to the other two equivalent forms. a) $\vec{r} = (0,1,2) + t(1,0,3)$ b) $\begin{cases} x = -1 - t \\ y = 2 - 3t \\ z = -4 \end{cases}$ c) $\dfrac{x-1}{2} = \dfrac{y+2}{-1} = \dfrac{z+3}{-2}$
	8. Find the x-int, y-int, and z-int for the line $\vec{r} = (1,-2,3) + t(1,-2,4)$ if they exist. **9.** Find the xy-int, yz-int, and zx-int for the line $\vec{r} = (-2,3,4) + t(-1,1,-2)$ if they exist.
	10. Find if the lines are parallel or not. a) $\vec{r} = (1,2,3) + t(1,-2,3)$, $\vec{r} = (3,2,1) + s(-2,4,-6)$ b) $\vec{r} = (1,2,3) + t(2,1,3)$, $\vec{r} = (3,2,1) + s(4,2,-6)$ c) $\vec{r} = (5,0,5) + t(-3,3,-6)$, $\vec{r} = (3,2,1) + s(1,-1,2)$
find a point on one, ← then find distance from that point to the line	**11.** In the case the lines are parallel and distinct, find the distance between the lines. a) $\vec{r} = (1,2,3) + t(1,-2,3)$, $\vec{r} = (3,2,1) + s(-2,4,-6)$ b) $\vec{r} = (1,2,3) + t(2,1,3)$, $\vec{r} = (3,2,1) + s(4,2,-6)$ c) $\vec{r} = (5,0,5) + t(-3,3,-6)$, $\vec{r} = (3,2,1) + s(1,-1,2)$

	12. For each case, find the distance between the given line and the given point. a) $\vec{r}=(1,2,-3)+t(2,-1,-2), \quad M(3,-2,1)$ b) $\begin{cases} x=-2+2t \\ y=3+t \\ z=1-2t \end{cases}, \qquad E(0,2,-3)$ c) $\frac{x-2}{3}=\frac{y-1}{2}=\frac{z}{-1}, \qquad B(-2,1,-3)$
	13. Find the point of intersection if it exists. a) $\vec{r}=(1,2,3)+t(1,-2,3)$, $\vec{r}=(1,1,1)+s(2,-1,0)$ b) $\vec{r}=(1,3,5)+t(0,1,2)$, $\vec{r}=(0,2,4)+s(-1,0,1)$

	1. Find the vector equation of a plane a) passing through the point $A(-1,2,-3)$ and parallel to the vectors $\vec{u}=(-2,1,0)$ and $\vec{v}=(2,-3,-1)$ b) passing through the points $A(2,3,2)$ and $B(2,1,5)$ and $C(3,-1,0)$ c) passing through the origin and containing the line $\vec{r}=(1,-3,2)+t(1,1,1)$ d) passing through the point $A(2,-1,3)$ and is parallel to the yz-plane.
	2. Convert the vector equation for a plane to the parametric equations or conversely. a) $\vec{r}=(0,-1,2)+t(1,-2,3)+s(2,-3,4)$ b) $\begin{cases} x=1-2t+3s \\ y=t-2s \\ z=-2+4t \end{cases}$
	3. Find the scalar equation of a plane that: a) passes through the point $(1,2,3)$ and is perpendicular to the y-axis b) passes through the point $(1,0,-1)$ and is parallel to the yz-plane c) passes through the origin and is perpendicular to the vector $(1,-2,4)$
	4. Find the intersection with the coordinate axes for the plane $\pi:-2x+3y-6z+12=0$.
same philosophy as # 11 above	**5.** For each case, find the distance between the given plane and the given point. a) $\vec{r}=(1,0,2)+t(0,1,2)+s(2,0,1)$, $\qquad B(2,3,0)$ b) $2x-3y+z-6=0$, $\qquad R(-2,0,3)$

	6. Find the intersection between the given line and the given plane. a) $\pi : 9x + 13y - 2z = 29$, $L : \begin{cases} x = 5 + 2t \\ y = -5 - 5t \\ z = 2 + 3t \end{cases}$ b) $\pi : 4x - y + 11z + 1 = 0$, $L : \vec{r} = (-2,4,1) + t(3,1,-1)$
	7. Find the equation of the line of intersection for each pair of planes (if it exists). a) $\pi_1 : 2x - 3y + z - 1 = 0$, $\pi_2 : 4x - 6y + 2z - 2 = 0$ b) $\pi_1 : 3x + 6y - 9z - 3 = 0$, $\pi_2 : 2x + 4y - 6z - 4 = 0$ c) $\pi_1 : x + 2y + 3z + 1 = 0$, $\pi_2 : x + 2y + z + 2 = 0$
	8. Find the angle between each pair of planes. a) $\pi_1 : x + 2y + 3z + 1 = 0$, $\pi_2 : 3x + 2y + z + 2 = 0$ b) $\pi_1 : x + y + z + 1 = 0$, $\pi_2 : x - y - 1 = 0$
	9. Solve the following system of equations. Give a geometric interpretation of the result. 1) $\begin{cases} x - 3y - 2z = -9 \\ 2x - 5y + z = 3 \\ -3x + 6y + 2z = 8 \end{cases}$ 2) $\begin{cases} x + y + 2z = -2 \\ 3x - y + 14z = 6 \\ x + 2y = -5 \end{cases}$ 3) $\begin{cases} x - y + z + 1 = 0 \\ -2x + 2y - 2z - 2 = 0 \\ 3x - 3y + 3z + 3 = 0 \end{cases}$ 4) $\begin{cases} x + y + z - 2 = 0 \\ x - y + z - 1 = 0 \\ 2x + 2y + 2z - 3 = 0 \end{cases}$