PDF 2.080 Related Rates

Imagine that a stone is thrown into a still pond, creating a ripple. Imagine that the radius of the ripple increases at the rate of 1 metre per second.

- After 1 second, the circular ripple would have an area of $\pi(1)^2 = \pi m^2$
- After 2 seconds, the circular ripple would have an area of $\pi(2)^2 = 4\pi m^2$ (an increase of $3\pi m^2$)
- After 3 seconds, the circular ripple would have an area of $\pi(3)^2 = 9\pi m^2$ (an increase of $5\pi m^2$)
- After 4 seconds, the circular ripple would have an area of $\pi(4)^2 = 16\pi m^2$ (an increase of $7\pi m^2$)

We see that although the rate of increase of the radius remains constant, the rate of increase of the area is changing. The rate of increase of the area is related to the rate of increase of the radius. We call it a "related rate". The rate of increase of the area is a function of time and it is not a constant.

Examples of Related Rates in Calculus

There are many types of related rates in Calculus. For example,

- Imagine a stone in a pond causing a circular ripple. The rate of increase of the area of the circular ripple will be related to the rate of increase of the radius of the ripple. If the rate of increase of the radius of the circular ripple is constant, the rate of increase of the area of the circular ripple will not be.
- Imagine blowing air into a spherical balloon. The rate of increase of the radius of the sphere will be related to the rate of increase of the volume of the sphere. If the rate of increase of the volume of air in the sphere is constant, the rate of increase of the radius of the sphere will not be.
- Imagine filling a cone shaped container with liquid. The rate of increase of the height of the liquid will be related to the rate of increase of the volume of the liquid in the cone. If the rate of increase of the volume of the liquid is constant, the rate of increase of the height of the liquid will not be.

There are many more examples.

How To Approach a Related Rates Problem

1. Some people like to draw a diagram.
2. Assign a variable to each quantity in the problem that is a function of the independent variable. (We'll assume for the remainder of this discussion that the independent variable is time)
3. Develop an equation that associates the variables with one another.
4. Differentiate (possibly using implicit differentiation).
5. Substitute in given information and solve for the required rate of change.

<u>Example 1</u>

When a raindrop falls into a small puddle, it creates a circular ripple that spreads out from the point where the raindrop hit. The radius of the circle grows at a rate of 3 cm/s.

a) determine the rate of increase of the circumference of the circle with respect to time
b) determine the rate of increase of the area of the circle when its area is $81\pi cm^2$.

Example 2

Water is pouring into an inverted right circular cone at a rate of π m^3/min. The height and the diameter of the base of the cone are both 10 m. How fast is the water level rising when the depth of the water is 8m?