1. The displacement of an object moving on a horizontal line is given by $s(t) = t^{\frac{10}{3}}(26 - t)$. When does the object change direction? (Just state a positive time) (3 marks)

changes direction when $v(t) = 0$

$s(t) = 26t^{\frac{10}{3}} - t^{\frac{13}{3}}$

$v(t) = \frac{260}{3}t^{7/3} - \frac{13}{3}t^{10/3}$

$\Rightarrow v(t) = 0$

$\left[0 = \frac{260}{3}t^{7/3} - \frac{13}{3}t^{10/3}\right] \times 3$

$260t^{7/3} = 13t^{10/3}$

$(20t^{7/3})^3 = (t^{10/3})^3$

$8000t^7 - t^{10} = 0$

$t^7(8000 - t^3) = 0$

$t = 0 \quad t = 20$ sec

3

The object changes direction at a time of 20 seconds (just state a positive time)

2. Determine the maximum value of the function $f(x) = \frac{2x}{x^2+16}$ over the domain $-2 \le x \le 10$. You must show sufficient work to justify your answer and include the domain in your analysis. Your final answer should contain an exact value, simplified, with no decimal approximations. (5 marks)

$f'(x) = \frac{(x^2+16)(2) - (2x)(2x)}{(x^2+16)^2}$, $-2 \le x \le 10$

$\Rightarrow f'(x) = 0$

$0 = 2x^2 + 32 - 4x^2$

$2x^2 = 32$

$x^2 = 16$

$x = \pm 4$

outside domain

$f(-2) = -1/5$

$f(4) = 1/4$

$f(10) = 5/29$

5

The maximum value of the function on the interval is 1/4

8

3. Jolene is building a rectangular courtyard. The courtyard will have an area of $160\ m^2$. Three sides of t courtyard will have fencing that costs $\$50/m$, and the fourth side will use fencing that costs $\$200/m$. is the minimum possible cost for this project. You do not need to include a domain analysis, and you c round your answer to two decimal places. You do not need to do a domain analysis for this question.

(5 marks)

$lw = 160$

$l = \frac{160}{w}$

$Cost = 50(2w) + 50l + 200l$

$= 100w + 250l$

$C(w) = 100w + 250\left(\frac{160}{w}\right)$

$= 100w + 40000w^{-1}$

$C'(w) = 100 - 40\,000w^{-2}$

$\Rightarrow C'(w) = 0$

$0 = 100 - \frac{40\,000}{w^2}$

$40\,000 = 100w$

$w^2 = 400$

$w = 20$

min cost at $w = 20$ m

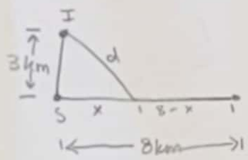
$C(20) = 100(20) + 40\,000$

$= \$4000$

5

The minimum possible cost for the project is $4000
(round your answer to two decimal places if necessary)

4. An island is located 3 km away from a straight shoreline and a school is located 8 km down the straight shoreline from a place directly across from the island. The Happy Mustang Internet Company wants to lay internet cable to get from the island to the school. The cost to lay cable underwater is $2000/km, and the cost to lay cable underground is $1000/km. What is the minimum possible cost to the company to lay the cable from the island to the school? You must include a domain analysis which includes the minimum and maximum possible values of your independent variable. You can round your answer to two decimal places if necessary. (7 marks)



$d = \sqrt{9+x^2}$

$cost = 2000(\sqrt{9+x^2}) + 1000(8-x),\ 0 \le x \le 8$

$C(x) = 2000(9+x^2)^{1/2} + 8000 - 1000x$

$C'(x) = 1000(9+x^2)^{-1/2}(2x) - 1000$

$\Rightarrow C'(x) = 0$

$0 = \frac{2000x}{\sqrt{9+x^2}} - 1000$

$1000\sqrt{9+x^2} = 2000x$

$\sqrt{9+x^2} = 2x$

$9 + x^2 = 4x^2$

$3x^2 = 9$

$x = \pm\sqrt{3},\ 1.73$ km

$C(0) = \$14000$

$C(1.73) = \$13196.15$

$C(8) = \$17088.0$

7

The minimum possible cost for the project is $13176.15
(round your answer to two decimal places if necessary)