PDF 3.010 Higher Order Derivatives, Velocity, Acceleration, etc

The second derivative of $y = f(x)$ is the derivative of $y = f'(x)$

In Newton notation, the second derivative is $f''(x)$

In Leibniz notation, the second derivative is $\frac{d^2y}{dx^2}$

Displacement: The position of an object with respect to time, usually referred to as $s(t)$.

Velocity: The rate of change of displacement over time. Therefore, the first derivative of the displacement function with respect to time is velocity.

$v(t) = s'(t)$ or $v(t) = \frac{ds}{dt}$

Acceleration: The rate of change of velocity over time. Therefore, the first derivative of the velocity function with respect to time, or the second derivative of displacement with respect to time, is acceleration.

$a(t) = v'(t) = s''(t)$ or $a(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2}$

Some Points to Ponder About Velocity and Acceleration

- Negative velocity means that an object is moving in a negative direction (left or down) while positive velocity means the opposite
- Zero velocity means that an object is stationary and that a possible change in direction may occur.
- Negative acceleration means that the velocity is decreasing, while positive acceleration means that the velocity is increasing.
- Zero acceleration means that the velocity is constant
- The speed of an object is the magnitude of its velocity.
    - Speed = $|v(t)| = |s'(t)|$
- An object is speeding up (increasing speed) when its velocity and acceleration have the same signs. However, an object is slowing down (decreasing speed) when its velocity and acceleration have opposite signs.
- If the displacement is measured in metres and time is measured in seconds, then velocity is measured in m/s and the acceleration is measured in m/s^2.

Example 1

Determine the second derivative, simplified, of $f(x) = \frac{x}{1+x}$. Then, determine the value of the second derivative when x = 1.

Example 2

A baseball is hit vertically upward. The position function, $s(t)$, in metres, of the ball above the ground is $s(t) = -5t^2 + 30t + 1$, where t is in seconds.

a) Determine the maximum height reached by the ball.
b) Determine the velocity of the ball when it is caught 1 m above the ground.

Example 3

The position of an object moving on a line is given by $s(t) = 6t^2 - t^3$, $t \geq 0$ where $s(t)$ is in metres and t is in seconds.

a) Determine the velocity and acceleration of the object at $t = 2$
b) At what time(s) is the object at rest?
c) In which direction is the object moving at $t = 5$?
d) * is it moving towards or away from the starting point at that time?
e) When is the object moving in a positive direction?
f) When does the object return to its initial position?