Vectors Quiz 2 Version 2 Thursday, May 23, 2024 Name: /13

1. Solve for k if we know that $\vec{a} = \overrightarrow{(3,-8,2)}$, $\vec{b} = \overrightarrow{(-4,7,1)}$ and $\vec{c} = \overrightarrow{(14,-41,k)}$ are coplanar /4

$\overrightarrow{(14,-41,k)} = m\overrightarrow{(3,-8,2)} + n\overrightarrow{(-4,7,1)}$

x $14 = 3m - 4n \Rightarrow 14 + 4n = 3m$

y $-41 = -8m + 7n \Rightarrow -41 + 8m = 7n$

z $k = 2m + n \Rightarrow k - 2m = n$

$-41 + 8m = 7(k - 2m)$

$-41 + 8m = 7k - 14m$

$22m = 7k + 41$

$m = \frac{7k + 41}{22}$

$14 + 4(k - 2m) = 3m$

$14 + 4k - 8m = 3m$

$14 + 4k = 11m$

$\frac{14 + 4k}{11} = m$

$\frac{7k + 41}{22} = \frac{14 + 4k}{11}$

$77k + 451 = 308 + 88k$

$\frac{143}{11} = \frac{11k}{11}$

$13 = k$

$k = 13$

4

2. Give possible values for m and n such that the vectors $\vec{u} = \overrightarrow{(3,m,5)}$ and $\vec{v} = \overrightarrow{(-2,8,n)}$ are perpendicular. There are many different possible answers here. (No decimal approximations).
/3

$\overrightarrow{(3,m,5)} \cdot \overrightarrow{(-2,8,n)} = 0$

$-6 + 8m + 5n = 0$

let $m = 1$

$-6 + 8(1) + 5n = 0$

$2 = -5n \Rightarrow -\frac{2}{5} = n$

3

A possible pair of values making the vectors perpendicular are $m =$ 1 and $n =$ $\frac{-2}{5}$
(There are many different possible answers; do not use decimal approximations.)

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3. Determine the measure of the angle between the vectors $\vec{c} = \overrightarrow{(-4,-1,-2)}$ and $\vec{d} = \overrightarrow{(1,5,-2)}$. Round your answer to the nearest tenth of a degree (i.e., one decimal place) if necessary. /3

$-4 - 5 + 4 = \sqrt{(4)^2 + (-1)^2 + (-2)^2}\sqrt{1^2 + 5^2 + (-2)^2} \cos\theta$

$-5 = \sqrt{21}\sqrt{30} \cos\theta$

$\frac{-5}{\sqrt{21}\sqrt{30}} = \cos\theta$

$\cos^{-1}(-0.19) = \theta$

$101.49° = \theta$

3

$\theta = 101.5°$

Round your answer to one tenth of a degree if necessary (i.e., one decimal place).

4. The vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are mutually perpendicular (i.e., each of the vectors is perpendicular with each of the other vectors). If we know that $|\vec{a}| = 4$, $|\vec{b}| = 10$ and $|\vec{c}| = 5$, evaluate $(\vec{a} - \vec{c}) \cdot (\vec{c} + \vec{b} - \vec{a})$ /3

$\vec{a} \cdot \vec{c} + \vec{a} \cdot \vec{b} - |\vec{a}|^2 - |\vec{c}|^2 - \vec{b} \cdot \vec{c} + \vec{a} \cdot \vec{c} = 0$

Perpendicular = 0

$0 + 0 - (4)^2 - (5)^2 = 0$

$-16 - 25 = 0$

$-41 = 0$

$\vec{a} \cdot \vec{c} + \vec{a} \cdot \vec{b} - \vec{a} \cdot \vec{a} - \vec{c} \cdot \vec{c} - \vec{b} \cdot \vec{c} + \vec{a} \cdot \vec{c}$

3

$(\vec{a} - \vec{c}) \cdot (\vec{c} + \vec{b} - \vec{a}) = -41$