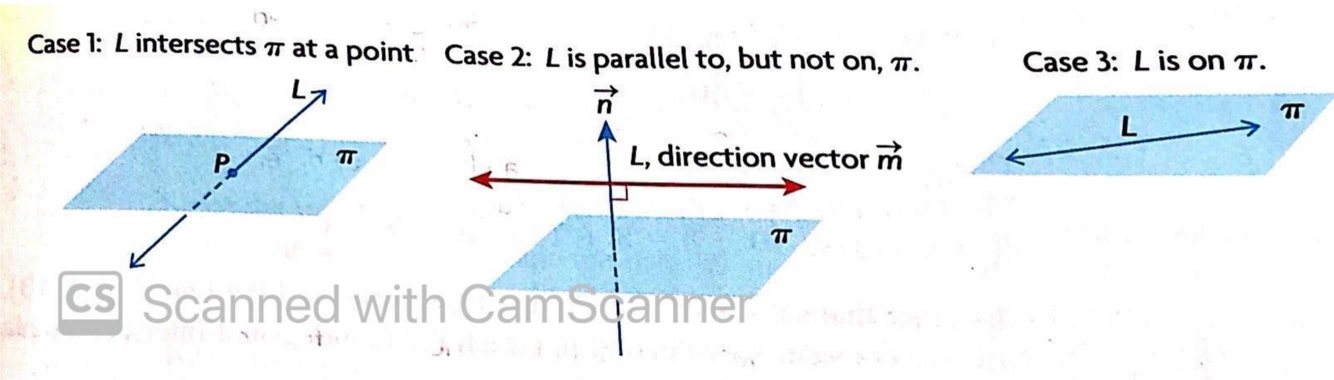
PDF 9.010 Intersection of a Line with a Plane or Two Lines Intersecting

Part 1: Intersection of a Line with a Plane: 3 Cases

1. The line L intersects the plane π at exactly one point, P.
2. The line L does not intersect the plane so it is parallel to the plane. There are no points of intersection.
3. The line L lies on the plane π. Every point on L intersects the plane. There are an infinite number of points of intersection.



Procedure For Determining Intersection(s) of a Line with a Plane (if any exist)

1. Set the parametric expressions for x, y and z into the Cartesian equation Ax + By + Cz + D = 0.
2. Let's assume the parameter is t. Solve the equation we get in part 1 above. If we get to 0t = non-zero, then there are no solutions (because it is impossible for 0 times a real number to equal a non-zero quantity). Therefore there are no points of intersection, and we have a situation like case 2 on the previous slide (the line L is parallel to, but not on, the plane π)
3. If we get 0t = 0, then there are an infinite number of solutions, meaning that we have a situation like case 3 on the previous slide (the line L is on the plane π)
4. If we get a non-zero quantity times t equaling a real number, then we have one solution, and we have a situation like case 1 on the previous slide (the line L intersects the plane π at a single point)

Example 1

Determine the point(s) of intersection, if any, between the line

$L: \vec{r} = \overrightarrow{(3,1,2)} + s\overrightarrow{(1,-4,-8)}$ and the plane $\pi: 4x + 2y - z - 8 = 0$

<u>Example 2</u>

Determine the points of intersection, if any exist, between the line $L: x = 2 + t, y = 2 + 2t, z = 9 + 8t$ and the plane

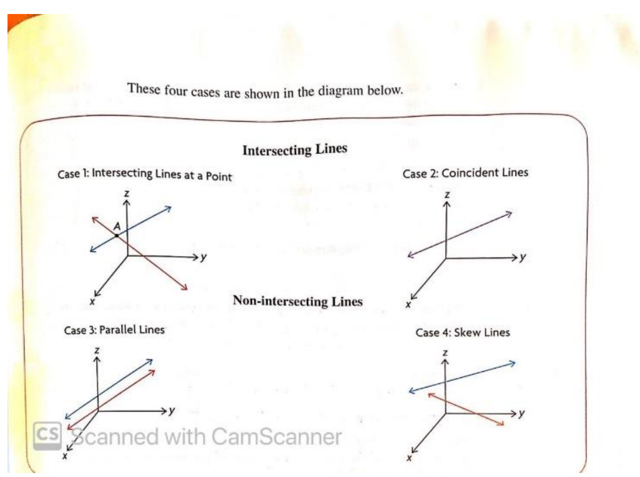
$\pi: \vec{r} = \overrightarrow{(3,1,-5)} + m\overrightarrow{(1,1,1)} + n\overrightarrow{(1,2,8)}$

<u>Example 3</u>

Determine the point(s) of intersection, if any exist, between the line L: $\vec{r} = \overrightarrow{(3,-2,1)} + s\overrightarrow{(14,-5,-3)}$ and the plane

$\pi: x + y + 3z - 4 = 0$

<u>Part 2: The Intersection of Two Lines (4 Cases)</u>

1. Two lines intersect at a single point if - The direction vectors are not collinear, and - We algebraically determine a point of intersection 2. Two lines are coincident if - The direction vectors are collinear, and - We algebraically determine a point of intersection 3. Two lines are parallel & non-coincident (no points of intersection) if - The direction vectors are collinear, and - We determine that one point on one of the lines is not on the other line 4. Two lines are skew (no points of intersection) if - The direction vectors are not collinear, and - We algebraically determine that there are no points of intersection.	

Example 4

Determine the point(s) of intersection, if any, of the lines $L_1: \vec{r} = \overrightarrow{(9,1,0)} + s\overrightarrow{(4,-1,-2)}, s \in R$ and

$L_2: \vec{r} = \overrightarrow{(-4,-1,10)} + t\overrightarrow{(5,4,-6)},\ t \in R$

Example 5

Determine the point(s) of intersection, if any, of the lines $L_1: \vec{r} = \overrightarrow{(3,0,-1)} + s\overrightarrow{(1,-2,-3)}, s \in R$ and

$L_2: \vec{r} = \overrightarrow{(-2,-3,-8)} + t\overrightarrow{(8,1,0)}, t \in R$

Example 6

Determine the point(s) of intersection, if any, of the lines $L_1: \vec{r} = \overrightarrow{(-4,-1,-6)} + s\overrightarrow{(3,2,-5)}, s \in R$ and

$L_2: \vec{r} = \overrightarrow{(8,7,-26)} + t\overrightarrow{(-6,-4,10)}, t \in R$

Example 7

Determine the point(s) of intersection, if any, of the lines $L_1: \vec{r} = \overrightarrow{(-4,-1,-6)} + s\overrightarrow{(3,2,-5)}, s \in R$ and

$L_2: \vec{r} = \overrightarrow{(8,7,-16)} + t\overrightarrow{(-6,-4,10)}, t \in R$