Homework 7

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1. Take for example $\phi(12)$. The current pseudocode will find that 2 divides 12, so it will then compute $\phi(2)$ and $\phi(6)$. In the call to $\phi(6)$, it will find that 2 again divides 6, and call $\phi(2)$ and $\phi(3)$. In this sense, when the argument has duplicate primes in its prime factorization, it will call ϕ on each of those duplicates. This means that memoization would be a good candidate for this algorithm.

2. Since the function only takes one positive integral argument, a growing array would be the correct data structure to use for memoization.

3. Since all positive integers are co-prime with all least 1 (1 is co-prime with itself since it has *no* primes in its prime factorization), then $\phi(n)$ is positive for all positive n. A sentinel value should be chosen that is not in this co-domain of ϕ. A good choice is 0.

4. Below is a memoized version of the provided `EulerPhi` according to (2) and (3).

Input: n: a positive integer
Output: $\phi(n)$
Function: eulerPhi
$\quad$ *memos* $= [\,]$
$\quad$ **return** memoPhi(n, *memos*)

Input: n: a positive integer
Input: *memos*: array of memoized results
Output: $\phi(n)$
Function: memoPhi

`// Extend the array and fill the new elements with 0's:`
for $i = \text{length}(\mathit{memos})$ **to** $n - 1$ **do**
$\quad$ $\mathit{memos}[i] = 0$

`// Check if function has been called before`
if $\mathit{memos}[n-1] \neq 0$ **then**
$\quad$ **return** $\mathit{memos}[n-1]$

for $a = 2$ to $\lfloor \sqrt{n} \rfloor$ **do**
$\quad$ **if** a divides n **then**
$\quad\quad$ $b = n/a$
$\quad\quad$ $g = \gcd(a, b)$
$\quad\quad$ **return** $\text{memoPhi}(a) \cdot \text{memoPhi}(b) \cdot g/\text{memoPhi}(g)$

return $n - 1$