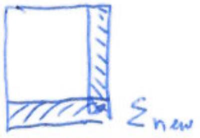
L09. Data Association

* Summary of L10 SLAM : Online SLAM using EKF

$p(\underbrace{x_t, m} | Z, \mathcal{U})$

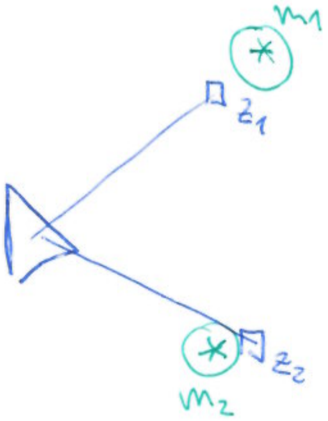
$g(y_t, u_t) = \begin{bmatrix} g^x \\ m_1 \\ \vdots \\ m_N \end{bmatrix}$, $z_t = h(y_t, c_t)$

$\bar{m}_{new} = h^{-1}(x_t, z_t)\Big|_{\bar{\mu}_t}$ $\mu_t = \begin{bmatrix} \mu_t \\ \mu_{new} \end{bmatrix}$ $\bar{\Sigma}_t =$



$H = [H^r, 0, \dots 0, H^j, 0 \dots, 0]$

* Euclidean nearest neighbour



$c_t^i = \arg\min \| m_j - z_t^i \|_2$

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for  $i = 1, \dots, K$   (z obs)
    for  $j = 1, \dots, N$
        $c_t^j = \min(\| m_j - z_t^i \|_2)$
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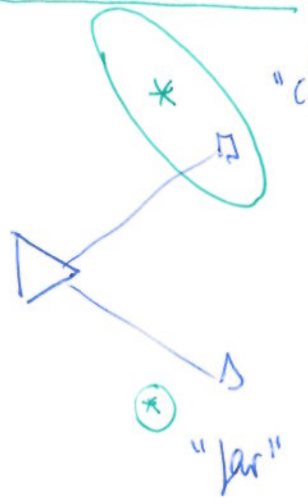
$O(K \cdot N)$

~ ✓ Matches each observation to closest landmark

✓ Easy and fast ∢ $O(KN)$

✗ Greedy data association.

* Mahalanobis NN



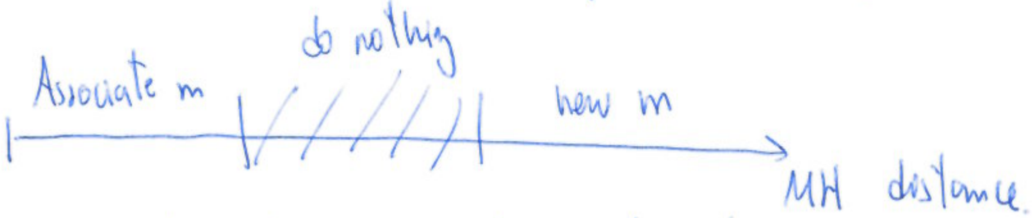
or $h(\mu_t)$

$h^{-1}(z^i, y_t)$

$$d_{ij}^2 = \| m_j - z^i \|_{\Sigma}^2$$

$$= (m_j - z^i)^T \Sigma^{-1} (m_j - z^i)$$

MH captures uncertainty.



New landmarks → need confirmation over several obs.

✓ More robust to noise.

✓ Easy and fast $O(K \cdot N)$

✗ Still greedy

* Maximum likelihood (ML)

from the Bayes filter, we should calculate the distribution of correspondences:

$$p(c_{1:t} \mid x_t, m, z)$$

very complicated!

sequence of all correspondences should be re-evaluated constantly for all observations.

* Aproximation 1: Solve DA incrementally

$p(c_t \mid z_t, y_t)$ The history of correspondences $c_{1:T}$ only depends on the last ~~[illegible]~~ correspondences.

Assumes previous correspondences were correct.

$$p(c_t \mid z_t, y_t) \propto p(z_t \mid c_t, y_t)$$

posterior likelihood for a given c_t

$$c_t^* = \underset{c_t}{\arg\max} \{ p(z_t \mid c_t, y_t) \}$$

7 params. Maximum likelihood estimator.

$$\begin{aligned} p(z_t \mid c_t, y_t) = {} & p(z_t^1 \mid z_t^{2:K}, y_t, c_t) \\ & \cdot p(z_t^2 \mid z_t^{3:K}, y_t, c_t) \\ & \cdot p(z_t^3 \mid z^{4:K}, y_t, c_t) \\ & \vdots \\ & p(z_t^K \mid y_t, c_t) \end{aligned}$$

19.4

Approximation 2: Independence assumption

for each z^i choose 1 c^i single.

$$p(z_t \mid c_t, y_t) \simeq \prod_i^K p(z_t^i \mid c_t^{\text{\textcircled{i}}}, y_t)$$

$$c_t^* = \underset{c_t}{\operatorname{argmax}} \prod_i^K p(z_t^i \mid c_t^i, y_t)$$

where, $c_t = \{ c_t^1 = m_{j_1},\ c_t^2 = m_{j_2}, \dots c_t^i = m_{j_i}, \dots \}$

Since c_t are independent:

$$\max \left(\prod_i^K p(z_t^i \mid c_t^i, y_t) \right) = \prod_i^K \max_{c_t^i} p(z_t^i \mid c_t^i, y_t)$$

evaluate c_t^i individually!

$$z_t^i \sim N\left(z_t^i;\ h(\bar{\mu}_t, c^i),\ H_t^i \bar{\Sigma}_t H_t^{iT} + Q_t\right)$$

New landmark: (ProbRob 322)

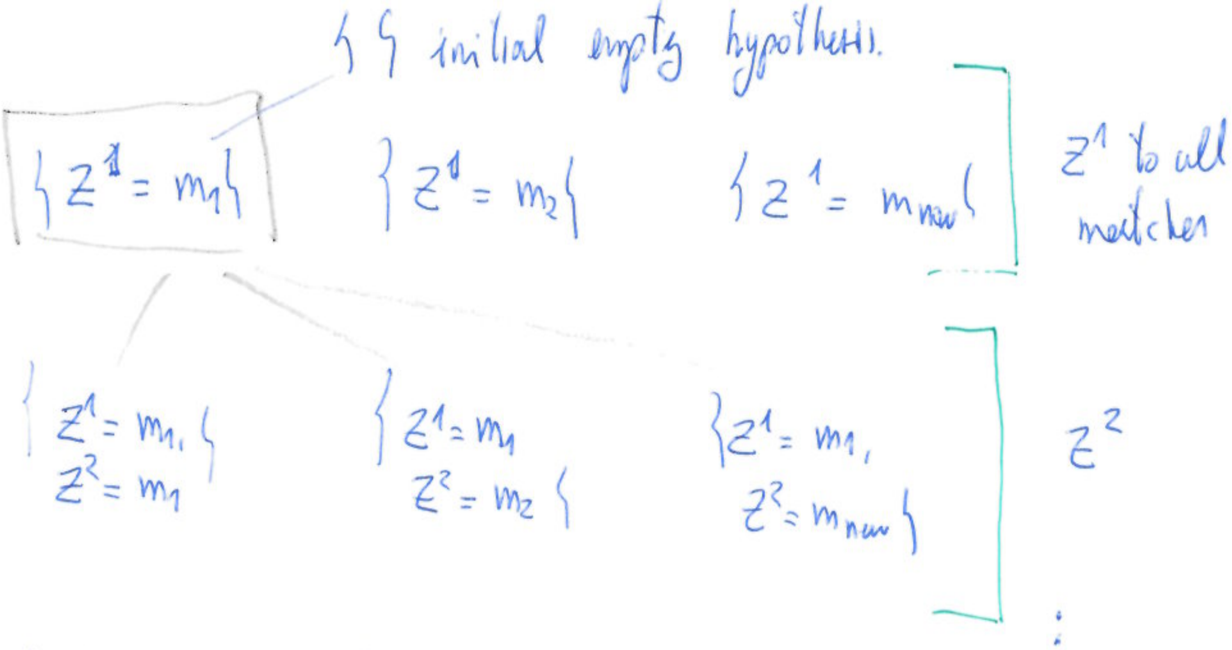
$$p(z_t^i \mid c_t^i = new, y_t) = \alpha$$

threshold value.
hard to tune.

If we don't set this L.H. to α, then the new landmark won't be created.

We create a landmark only if distance to all other landmarks is higher than α.

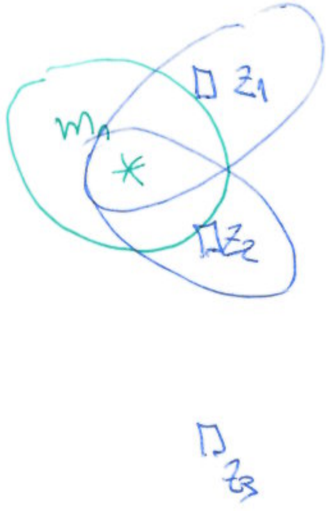
Efficient way to express DA: tree representation.



Best first exploration (from graph search). Greedy but efficient

True dimensionality of the tree: exponential $O((N+1)^k)$!!

* Individual compatibility



xi squared test. (α confidence level 90, 95, 99...)

$d_{11}^2 = \| m_1 - z^1 \|_\Sigma < \chi_\alpha^2$ ✓

$d_{12}^2 < \chi_\alpha^2$ ✓

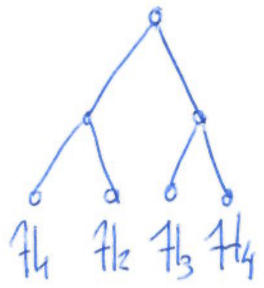
$d_{13}^2 = \| m_1 - z^3 \|_\Sigma \not< \chi_\alpha^2$ ✗

fast filter to reject clearly incorrect hypothesis.

* Joint Compatibility (JC) (Neira and Tardós 2001)

Context provides more accurate solutions

Ex: Stars vs constellations



Set of hypothesis → size exponential (later)

$$\mathcal{H}_i = \{ \underset{(c^1)}{z_1 = m_1}, \underset{(c^2)}{z_2 = m_3}, \underset{(c^3)}{z_3 = m_{new}} \}$$

We will evaluate joint candidates based on their joint compatibility (JC)

$$\text{if} \quad \boxed{d^2_{\mathcal{H}_e} < \chi^2_\alpha}$$

→ for Individual compatibility: (IC)

$$f_{ij}(\underset{\text{state}}{y}, \underset{\text{obs}}{z}) = z^i - h(y, j)$$

L05

Innovation vector (ideally ≈ 0)

→ JC

$$f_{\mathcal{H}_e}(y,z) = \begin{bmatrix} f_{\mathcal{H}_e^1}(y,z) \\ f_{\mathcal{H}_e^2}(y,z) \\ \vdots \\ f_{\mathcal{H}_e}(y,z) \end{bmatrix}$$

hypothesis

$$\mathcal{H}_e = \{ \underbrace{\underbrace{\underbrace{\underbrace{c^1}_{\mathcal{H}_e^1}, c^2}_{\mathcal{H}_e^2}, c^3}_{\mathcal{H}_e^3}, \dots, c^k}_{\mathcal{H}_e} \}$$

If we rewrite $f_{\mathcal{H}_e}(y,z)$ incrementally:

$$f_{\mathcal{H}_e^i}(y,z) = \begin{bmatrix} f_{\mathcal{H}_e^{i-1}}(y,z) \\ f_{\mathcal{H}_e^i}(y,z) \end{bmatrix}$$

$$\simeq f_{\mathcal{H}_e^i}(\hat{y},\hat{z}) + G_{\mathcal{H}_e^i}(y-\hat{y}) + H_{\mathcal{H}_e^i}(z-\bar{z})$$

($H \neq \mathcal{H} \Rightarrow$ from paper

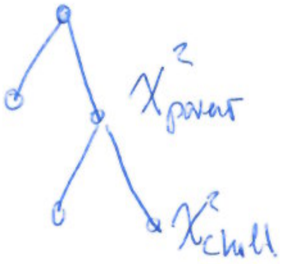
Covariance of the joint innovation

$$C_{\mathcal{H}_e^i} = H_{\mathcal{H}_e^i} S H_{\mathcal{H}_e^i}^T + G_{\mathcal{H}_e^i} \Sigma G_{\mathcal{H}_e^i}$$

$$\Rightarrow d^2_{\mathcal{H}_e^i} = f(\bar{y},z)^T \boxed{C^{-1}_{\mathcal{H}_e^i} f(\bar{y},z) < \chi^2_{d,\alpha}} \quad , \; d = \dim(\mathcal{H}_e^i)$$

this test is carried out incrementally (see paper)

* JC branch and Bound.



$\chi^2_{parent} < \chi^2_{child}$

This allows us to discard branches without evaluating!

$C_{\mathcal{H}_e^i}$ captures crosscorrelations of the observations, while ML doesn't (assumed independence)

More accurate and probabilistically more complete

Slow (not exponential, but it is on the worst-case)

↳ pre filter!