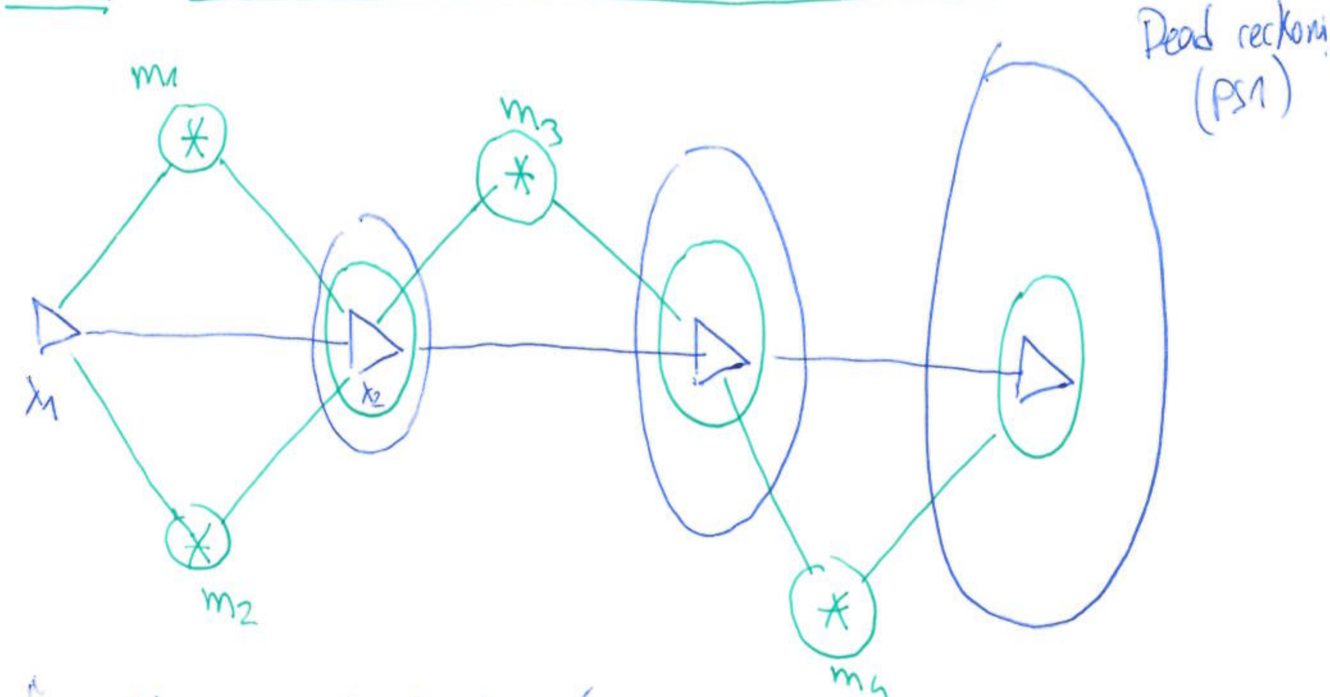
L10: SLAM with Known landmark correspondences



Simultaneous localization (x_t) and mapping (m)

Given: $u_{1:t} = \{ u_1, u_2, \dots, u_t \}$ actions

$z_{1:t} = \{ z_1, z_2, \dots, z_t \}$ observations

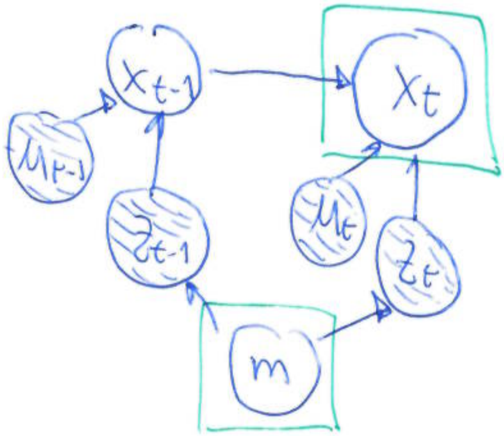
Calculate: $m = \left\{ \begin{bmatrix} m_{1,x} \\ m_{1,y} \end{bmatrix}, m_2, \dots, m_N \right\}$ Map of Landmarks

$x_{0:t} = \{ x_0, x_1, x_2, \dots, x_t \}$ Trajectory

- The association landmark-observation is not always Known.
- A single incorrect D.A. can ruin the map.

Map $\rightleftarrows$ localize. Chicken and egg problem. Hard!

Initial approach: Use KF with known correspondances
↳ next lecture we will cover unknown D.A.



* Online SLAM

$p(x_t, m \mid z_{1:t}, u_{1:t})$

* full SLAM

$p(x_{1:t}, m \mid z_{1:t}, u_{1:t})$

EKF SLAM with known corresp. $\{c_t^i = j\} = c_t$

$p(x_t, m \mid z_{1:t}, u_{1:t}, c_t)$

Augmented state

$y_t = [x_t, \quad m_1, \quad m_2, \quad \dots, \quad m_N]^T$

$[\underbrace{x, y, \theta}_{\text{Robot pose}}, \underbrace{m_{1,x}, m_{1,y}}, \underbrace{m_{2,x}}, \dots \underbrace{m_{N,x}, m_{N,y}}]$

Robot pose Landmark positions

$$y_t \sim \mathcal{N}(\mu_t, \Sigma_t) = \mathcal{N}\left(\begin{bmatrix} \mu_t^x \\ \mu_t^m \end{bmatrix}, \begin{bmatrix} \Sigma_x & \Sigma_{xm} \\ \Sigma_{mx} & \Sigma_m \end{bmatrix} \right)$$

Since we use EKF, posterior is Gaussian.

$$\begin{array}{|c|c|l} \hline \Sigma_x & \Sigma_{xm} & 3 \\ \hline \Sigma_{mx} & \Sigma_m & \left.\vphantom{\begin{matrix} a \\ a \\ a \end{matrix}}\right\} 2N \\ \hline \end{array}$$

→ EKF (L07) SLAM (ProbRob 314)

$$\left.\begin{array}{ll} \text{I:} & \bar{\mu}_t = g(\mu_{t-1}, u_t) \\ \text{II:} & \bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + R_t \end{array}\right\} \text{Prediction}$$

$$\left.\begin{array}{ll} \text{III:} & K_t = \bar{\Sigma}_t H_t^T (H_t \bar{\Sigma}_t H_t^T + Q)^{-1} \\ \text{IV} & \mu_t = \bar{\mu}_t + K_t (z_t - h(\bar{\mu}_t)) \\ \text{V} & \Sigma_t = (I - K_t H_t) \bar{\Sigma}_t \end{array}\right\}$$

Multiple observations can be corrected (L08)

Sequential vs Batch

Prediction

$$y_t = [x, m_1, m_2, \dots, m_N]^T$$

$g(y_{t-1}, u_t) \rightarrow$ transition function
- Odometry model (PS3)
- Kinematic model (ProbRob)
- other models (car, etc.

→ Prediction

I: $g(\mu_{t-1}, u_t) = \begin{bmatrix} g_x(x_{t-1}, u_t) \\ m_1 \\ m_2 \\ \vdots \\ m_N \end{bmatrix} + \varepsilon_t = y_{t-1} + \begin{bmatrix} \delta_{trans} \cdot \cos(\theta_{t-1} + \delta_{rot1}) \\ \delta_{trans} \cdot \sin(\theta_{t-1} + \delta_{rot1}) \\ \delta_{rot1} + \delta_{rot2} \\ 0 \\ \vdots \\ 0 \end{bmatrix}$

odometry model (e.g.)

$\bar{\mu}_t = g(\mu_{t-1}, u_t)$

II: $\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + R_t$

$$G_t = \left.\frac{\partial g(y_{t-1}, u_t)}{\partial y_{t-1}}\right|_{\mu_{t-1}} = \begin{bmatrix} \frac{\partial g_x}{\partial x} & \frac{\partial g_x}{\partial m_1} & \cdots & \frac{\partial g_x}{\partial m_N} \\ \frac{\partial m_1}{\partial x} & \frac{\partial m_1}{\partial m_1} & & \vdots \\ \vdots & & \ddots & \vdots \\ \frac{\partial m_N}{\partial x} & \frac{\partial m_N}{\partial m_1} & \cdots & \frac{\partial m_N}{\partial m_N} \end{bmatrix} = \begin{bmatrix} G_t^x & 0 & 0 & \cdots & 0 \\ 0 & I & & & \vdots \\ 0 & & I & & \vdots \\ \vdots & & & \ddots & \\ 0 & \cdots & & & I \end{bmatrix}$$

$$G_t = \begin{bmatrix} G_t^x & 0 \\ 0 & I_{2N \times 2N} \end{bmatrix}$$

Robot noise in state space or action space.

$$R_t = \begin{bmatrix} R_t^x & 0 \\ 0 & \underbrace{0_{2N \times 2N}}_{\text{landmark do not propagate} \Rightarrow \text{no noise!}} \end{bmatrix} = \begin{bmatrix} V_t^x M_t^x (V_t^x)^T & 0 \\ 0 & 0 \end{bmatrix}$$

$$\bar{\Sigma}_t = \begin{bmatrix} \Sigma_x & \Sigma_{xm} \\ \Sigma_{mx} & \Sigma_m \end{bmatrix}$$

← updated

Q: show this

remains the same. Σ_m^{t-1}

(odometry) recall covariance for noise on transition in the action space:

$$M_t^x = \begin{bmatrix} \alpha_1 \delta_{rot1}^2 + \alpha_2 \delta_{trans}^2 & 0 & 0 \\ 0 & \alpha_3 \delta_{trans}^2 + \alpha_4(\delta_{rot1}^2 + \delta_{rot2}^2) & 0 \\ 0 & 0 & \alpha_1 \delta_{rot1}^2 + \alpha_2 \delta_{trans}^2 \end{bmatrix}$$

→ **Correction I:** $\{ c_t^i = j \Rightarrow z_i \to m_j \}$

New landmark is observed. We must initialize it.

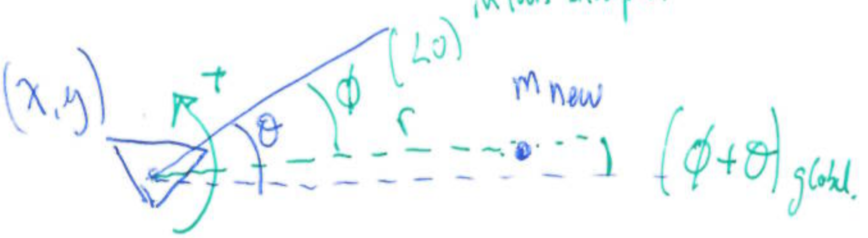
$$z_t = h(y_t, j) + \eta_t = \begin{bmatrix} \sqrt{(m_{j,x} - x)^2 + (m_{j,y} - y)^2} \\ \operatorname{atan2}(m_{j,y} - y,\ m_{j,x} - x) - \theta \end{bmatrix} + \eta_t$$

(L06) 2D model

Inverse observation model (1 landmark)

$$m_{j\,new} = h^{-1}(z_t, \bar{y}_t)\Big|_{\bar{\mu}_t} = \underbrace{\begin{bmatrix} \bar{\mu}_{t,x} \\ \bar{\mu}_{t,y} \end{bmatrix}_{2\times1}}_{\text{(coordinates)}} + r_t \begin{bmatrix} \cos(\phi_t + \bar{\theta}_t) \\ \sin(\phi_t + \bar{\mu}_{t,\theta}) \end{bmatrix}$$

$(\bar{\mu}_{t,\theta})$



New landmark is expected w.r.t. robot
$m_{new} = h^{-1}(z, y)$

$$\bar{y}_t = \begin{bmatrix} \bar{y}_{t(old)} \\ m_{new} \end{bmatrix}$$ } We have augmented the state vector.

Q: and $\bar{\Sigma}_t$? what is the new augmented covariance?

$$\left[\begin{array}{cc|c} \Sigma_x & \Sigma_{xm} & \\ \Sigma_{mx} & \Sigma_m & \Sigma_{y,new} \\ \hline \multicolumn{2}{c|}{\Sigma_{new,y}} & \Sigma_{m,new} \end{array}\right]$$

h^{-1} non-linear

$\Downarrow$

Covariance projection

$$h^{-1}(z_t, y_t) \simeq h^{-1}(z_t, \mu_t) + L(y_t - \mu_t) + W(z_t - \hat{z}_t)$$

$$L = \frac{\partial h^{-1}}{\partial y_t} = \left.\frac{\partial h^{-1}}{\partial x_t}\right|_{\bar{\mu}_t^x} = \begin{array}{c} \begin{bmatrix} 1 & 0 & -r_t \sin(\phi_t + \bar{\mu}_{t,\theta}) \\ 0 & 1 & r_t \cdot \cos(\phi_t + \bar{\mu}_{t,\theta}) \end{bmatrix} \\ \begin{array}{ccc} \partial/\partial x & \partial/\partial y & \partial/\partial\theta \end{array} \end{array}$$

Only depends on x_t

$(\partial h^{-1}/\partial m_j = 0)$

$$W = \frac{\partial h^{-1}}{\partial z} = \begin{array}{c} \begin{bmatrix} \cos(\phi_t + \bar{\mu}_{t,\theta}) & -r_t \sin(\phi_t + \bar{\mu}_{t,\theta}) \\ \sin(\phi_t + \bar{\mu}_{t,\theta}) & r_t \cos(\phi_t + \bar{\mu}_{t,\theta}) \end{bmatrix} \\ \begin{array}{cc} \partial/\partial r & \partial/\partial\phi \end{array} \end{array}$$

After linearizing the new landmark inverse model, we calculate all the covariances by cov. projection.

$$\Sigma_{m_{new}} = E\{ (m_{new} - \mu_{new})(m_{new} - \mu_{new})^T \}$$

$$= E\{ (L \cdot \Delta x + W \eta_t)(L \cdot \Delta x + W \eta_t)^T \}$$

$$= \boxed{L \Sigma_x L^T + W Q W^T}$$

$$\Sigma_{y,new} = E\{ (y_t - \mu_t)(m_{new} - \mu_{new})^T \}$$

$$= E\{ \underbrace{\Delta y}_{[x, m_1, \dots, m_N]^T} (L \cdot \Delta x + \underbrace{W \eta_t}_{\text{uncorrelated with } y})^T \} = \begin{bmatrix} \Sigma_x L^T \\ \Sigma_{m,x} L^T \end{bmatrix}$$

$$\Sigma_{new,y} = L \cdot \begin{bmatrix} \Sigma_x \\ \Sigma_{m,x} \end{bmatrix}^T = L [\Sigma_x , \Sigma_{x,m}]$$

$$\Sigma_t = \left[\begin{array}{c|c|c} \Sigma_x & \Sigma_{x,m} & \Sigma_x L^T \\ \hline \Sigma_{m,x} & \Sigma_m & \Sigma_{m,x} \cdot L^T \\ \hline L\Sigma_x & L\Sigma_{x,m} & L\Sigma_x L^T + WQW^T \end{array}\right]$$

$\ast$ **Correction** $(c_t^i = j)$ correspondences.

$$z_t^i = h(y_t, c_t^i) + \eta_t = \begin{bmatrix} \sqrt{(m_{j,x}-x)^2 + (m_{j,y}-y)^2} \\ \operatorname{atan2}(m_{j,y}-y, m_{j,x}-x) - \theta \end{bmatrix}$$

$$h(y_t, c_t) \simeq h(\bar{\mu}_t, c_t) + H_t \cdot \Delta y_t$$

$$H_t = \frac{\partial h(y_t, \overset{(j)}{c_t})}{\partial y_t} = \left[\frac{\partial h}{\partial x}, \underset{0}{\cancel{\frac{\partial h}{\partial m_1}}}, \dots, \underset{\text{not zero}}{\frac{\partial h}{\partial m_j}}, \dots, \underset{0 \dots 0}{\cancel{\frac{\partial h}{\partial m_N}}} \right]$$

$$= \begin{array}{c} \\ \left[\begin{array}{ccc:ccccc} \frac{-(m_{j,x}-x)}{\sqrt{q}} & \frac{-(m_{j,y}-y)}{\sqrt{q}} & 0 & 0 \dots 0 & \frac{m_{j,x}-x}{\sqrt{q}} & \frac{m_{j,y}-y}{\sqrt{q}} & 0 \dots 0 \\ \frac{m_{j,y}-y}{q} & \frac{-(m_{j,x}-x)}{q} & -1 & 0 \dots 0 & \frac{-(m_{j,y}-y)}{q} & \frac{m_{j,x}-x}{q} & 0 \dots 0 \end{array} \right] \\ \begin{array}{ccccccc} \partial h/\partial x & \partial h/\partial y & \partial h/\partial \theta & & \partial h/\partial m_{j,x} & \partial h/\partial m_{j,y} & \end{array} \end{array}$$

$$\Rightarrow H_t = [H^r, 0, 0, \dots, 0, H^j, 0, \dots, 0].$$

Correction for 1 observation.