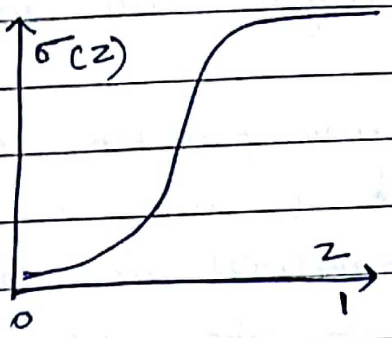
3. Sigmoid activation suffers from the vanishing gradients problem.



It can be seen from the figure that the slope of the function is very close to 0 which makes the change in weights very less at each epoch.

On the other hand, RELU activation function has derivative 0 or 1. Due to the large derivative, neural network learns much faster in this case but there is a risk (with deeper networks) of values getting too large, although we can say that the problem is now reduced.

Batch normalization can help in both cases. In this, we scale the input before feeding it to the next hidden layer. This way the scale of values becomes same ~~and~~ and hence reducing the epochs till the neural network reaches minima. It also reduces the effect of previous layers hence the data is not too large or small for the next layer.

We should initialize the weights randomly or else ~~as~~ all the neurons get the same signal and weights are updated in the same way. The initial weights must be

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small as large weights can cause neural network to overfit. Hence we can initialize the weights with random value b/w -1 to 1.

For a classification problem, cross entropy should be used as we model the output as probabilities and cross entropy loss is based on the probabilistic interpretation. Moreover cross entropy increases exponentially if the output becomes different from input (i.e misclassification is heavily penalized) something which can not be done with MSE. Also MSE can cause learning slowdown with sigmoid activation.

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4. The mean square error is given by -

$$c = (y-a)^2$$
where $a = \sigma(z)$

$$\Rightarrow \frac{dC}{dw_i} = 2(y-\sigma(z))\frac{da}{dw_i}$$

$$= 2(y-\sigma(z))\,\sigma'(z)\frac{dz}{dw_i}$$

$$= 2(y-\sigma(z))\,\sigma'(z)\,x_i$$

Let the target value be 1 and ~~sigma~~ $\sigma(z)$ be close to 0. We know that $\sigma'(z) = \sigma(z)(1-\sigma(z))$. Hence if $\sigma(z)$ is close to 0, then $\sigma'(z)$ is also very small due to which there is very less change in the weights when they are updated. Similarly, when $\sigma(z)$ is close to 1 and target value is 0, then again $\frac{\partial c}{\partial w_i}$ is very small.

On the other hand cross entropy for ~~binary~~ ~~case~~ is given by -

$$c = -\sum_{i=1}^{n} y_i \log a_i$$

where n = total no. of classes.

then

$$\frac{\partial c}{\partial w_j} = -\frac{y_i}{a_i}\frac{\partial a_i}{\partial w_j}$$

$$= -\frac{y_i}{a_i}\,\sigma'(z)\frac{\partial z}{\partial w_j}$$

$$= \frac{-y_i\,\sigma(z)(1-\sigma(z))}{\sigma(z)}\frac{\partial z}{\partial w_j}$$

$$= -y_i (1-\sigma(z)) \frac{\partial z}{\partial w_j}$$

$$= -y_i (1-\sigma(z)) x_j$$

Here the $\sigma(z)$ cancels out, and hence there is no learning slowdown due to absence of $\sigma'(z)$ term.

Similarly for the binary case,

$$c = -\left(y_i \log(a_i) + (1-y_i)\log(1-a_i)\right)$$

(for one sample)

$$\frac{\partial c}{\partial w_j} = -\left(\frac{y}{a_i}\frac{\partial(a_i)}{\partial w_j} - \frac{(1-y_i)}{(1-a_i)}\frac{\partial(a_i)}{\partial w}\right)$$

$$= -\left(\frac{y}{a_i} a_i(1-a_i)x_j - \frac{(1-y_i)}{(1-a_i)} x_j a_i (1-a_i)\right)$$

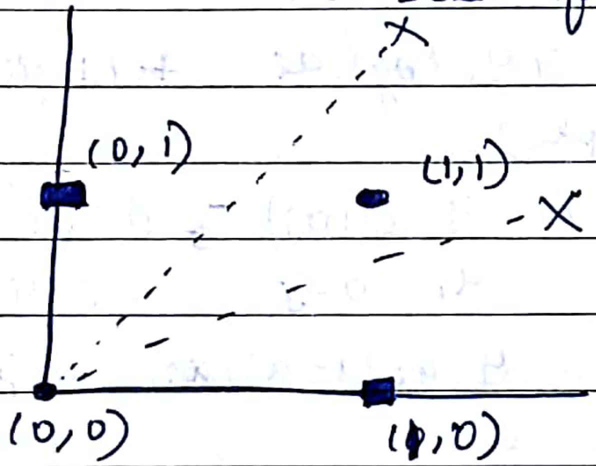
$$= -x_j \left(y_i(1-a_i) \cancel{x_j} - (1-y_i) \cancel{x_j} a_i\right)$$

$$= -x_j \left(y_i - \cancel{y_i a_i} - a_i + \cancel{y_i a_i}\right)$$

$$\boxed{\frac{\partial c}{\partial w_j} = (a_i - y_i) x_j}$$ where $a_i = \sigma(z)$

which again does not contain $\sigma'(z)$.

5. No, a neural network with just linear activations can not be used to create an arbitrary XOR truth table because a neural network with just linear activations is same as a neural network with linear activation and single layer. and a linear function can not be used to create a decision boundary that seperates the ~~points~~ 2 classes as can be seen from below diagram



Input	Target
(0, 0)	0
(0, 1)	1
(1, 0)	1
(1, 1)	0