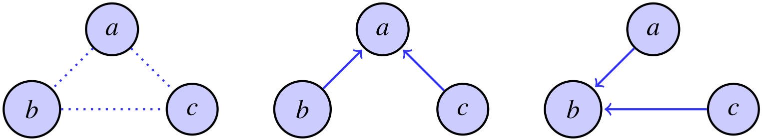
Directed and undirected networks
Causality and discovery



Network analysis

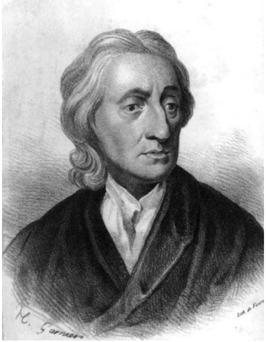
Lourens Waldorp

Universiteit van Amsterdam

Objectives

Get to know and have some intuition about

- Causality in philosophy (of science)
- Conditional independence
- Undirected and directed networks
- Causal discovery using `R`



David Hume (1711-1776)

Topics

Why causality is interesting

- Explanations
- Causality and probability

d-separation and conditional independence

- conditional independence
- d-separation
- When are models the same?

Discovering causal graphs

- Inductive Causation algorithm

Explanations

René Descartes

(1596 – 1650)

Explanations

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(1596 – 1650)

Naturalism:
Explanation of phenomena in terms of physical objects

Explanations

René Descartes

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- Example: the dials of a clock rotate by cogwheels and springs

Explanations

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- Example: the dials of a clock rotate by cogwheels and springs
- Divide mechanism into parts

Explanations

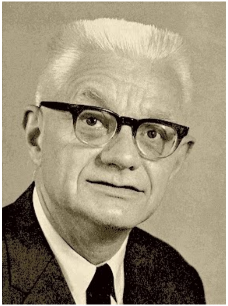
René Descartes

(1596 – 1650)

Naturalism:
Explanation of phenomena in terms of physical objects

- Example: the dials of a clock rotate by cogwheels and springs
- Divide mechanism into parts
- Not of mental processes like memory

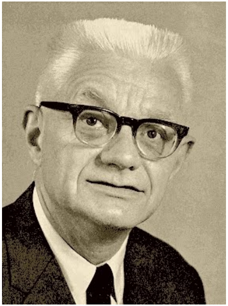
Explanations



Carl Hempel (1905 – 1977)

Logical positivists

Explanations



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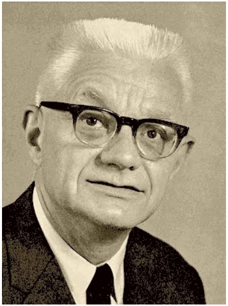
Logical positivists

Deductive-nomological model

Explanations



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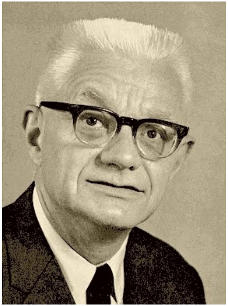
Logical positivists

Deductive-nomological model

- $C_1, \ldots, C_k$ (circumstances)
- $L_1, \ldots, L_n$ (laws)

$\vdash E$ (explanandum)

Explanations



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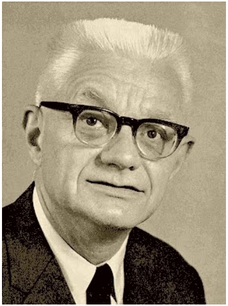

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Logical positivists

Deductive-nomological model

- $C_1, \ldots, C_k$ (circumstances)
- $L_1, \ldots, L_n$ (laws)

$\vdash E$ (explanandum)

- Necessarily true if C and L are true.
- *The* basis for all sciences are laws and logical deductions.

Explanations

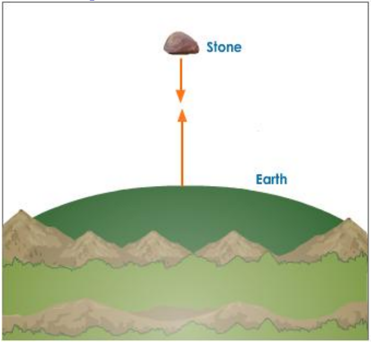
Isaac Newton $(1643 - 1727)$

Explanations

Isaac Newton (1643 – 1727)

You can fall back on the laws of Newton

Example



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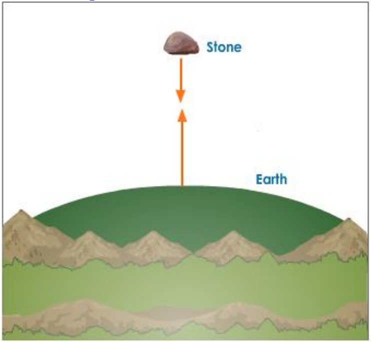
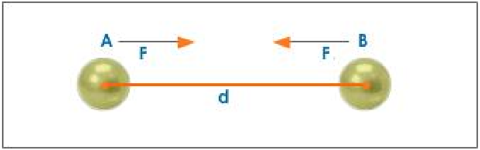
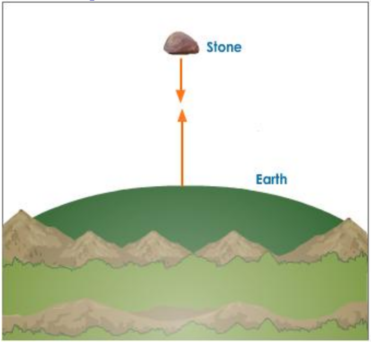
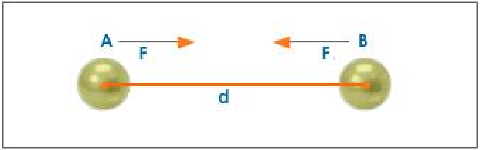




3rd law of Newton

Example

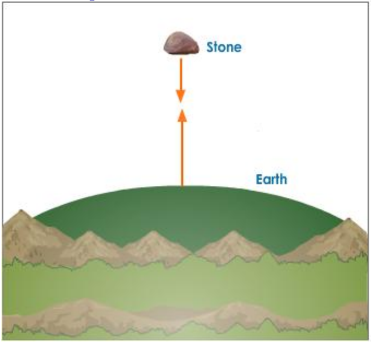


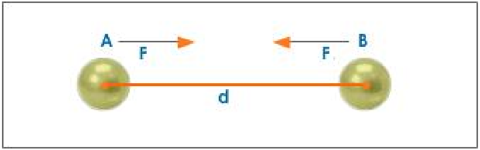


3rd law of Newton

- C : we are in the gravitational pull of the earth

Example





3rd law of Newton

- C : we are in the gravitational pull of the earth
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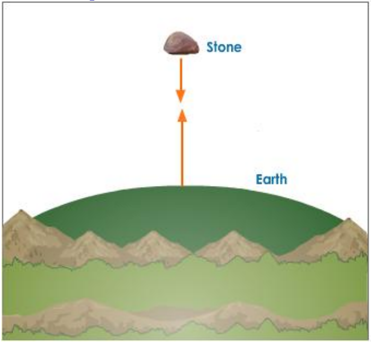

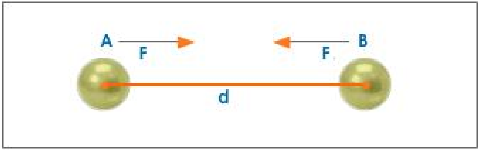

3rd law of Newton

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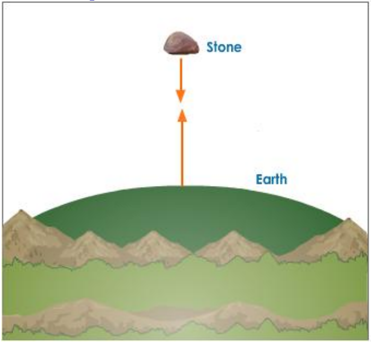
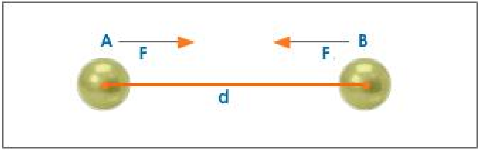
- L : 2nd and 3rd laws of Newton

$$F = ma \text{ and } F = \frac{g m_1 m_2}{d^2}$$

Example





3rd law of Newton

- C : we are in the gravitational pull of the earth
- L : 2nd and 3rd laws of Newton

$$F = ma \text{ and } F = \frac{gm_1m_2}{d^2}$$

$\vdash$ the rock falls to the earth

Explanations

So, what are these laws?

Explanations

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- Does the effect really always follow the cause?

Explanations

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- Does the effect really always follow the cause? No.

Explanations

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Explanations

So, what are these laws?

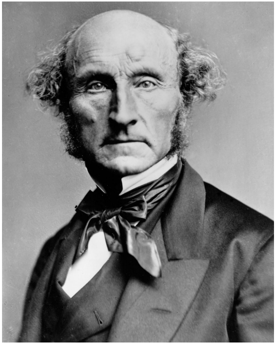
- Does the effect really always follow the cause? No.
- So, it is a regularity? Well, sort of...
- But it is a causal relation? That could be, but what is causality....
- This is what we need to figure out. It is the key to providing an explanation.

Causal relation and laws

John Stuart Mill's conditions

- A always co-occurs with B
- A occurs before B
- There is no alternative explanation for the co-occurrence of A and B

But this was unsatisfactory because A does not always occur with B; no universality.



John Stuart Mill (1806-1973)

Probabilities enter the stage

- Probability is essential to replace 'universal laws'
  Example

$$P(\text{catch fish} \mid \text{fishing rod}) \geq P(\text{catch fish})$$

- Conditional independence relations are the key to causal relations
  Example

$$P(\text{shark bite} \mid \text{ice cream}) \geq P(\text{shark bite})$$

$$P(\text{shark bite} \mid \text{ice cream, hot weather})$$
$$= P(\text{shark bite} \mid \text{hot weather})$$

Hans Reichenbach (1891-1953)

Probabilities enter the stage

- Conditional independence relations are the key to causal relations
  Example

$$P(\text{shark bite} \mid \text{ice cream}) \geq P(\text{shark bite})$$

$$P(\text{shark bite} \mid \text{ice cream, hot weather})$$

$$= P(\text{shark bite} \mid \text{hot weather})$$

- 'Screening off': search for nonredundent and sufficient 'variables' that define the relation (statistically relevant)

Wesley Salmon (1925-2001)

Probabilities enter the stage

- Spirtes and Glymour: Use screening off principle to identify unique relations from data
- Pearl: Semantics for difference between ‘do’ and ‘see’
  Example

$$P(\text{catch fish} \mid \text{rod}) \neq P(\text{catch fish} \mid \text{do(rod)})$$

- Causality definition:

$$E(\text{catch fish}) \neq E(\text{catch fish} \mid \text{do(rod)})$$

Judea Pearl (1936-)

Peter Spirtes (1956-)

Clark Glymour (1942-)

Topics

Why causality is interesting

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Discovering causal graphs

- Inductive Causation algorithm

What is a causal graph?

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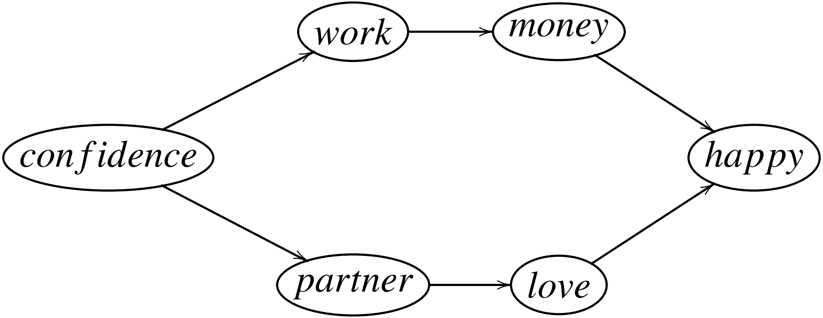

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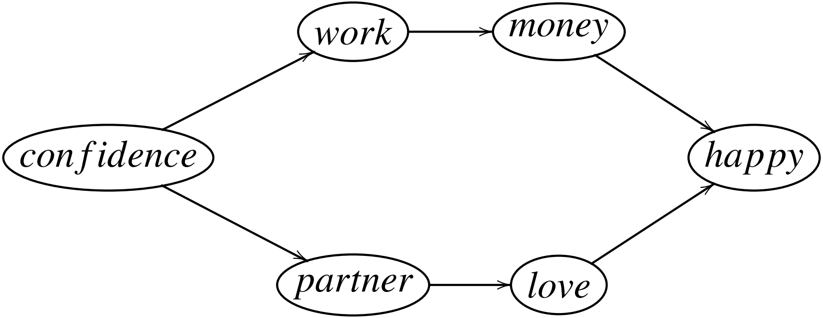

John Stuart Mill

- A (always) co-occurs with B
- A occurs before B
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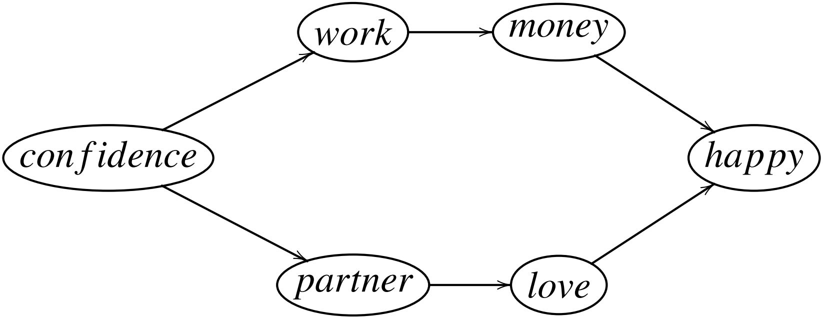

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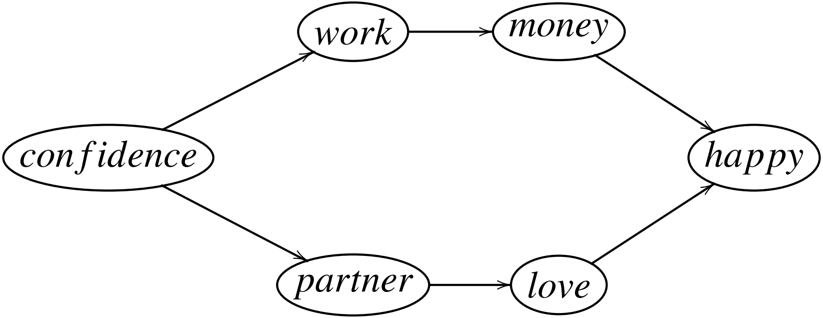

Causality by intervention

- If I wiggle A then B wiggles too (manipulation)
- There is no alternative explanation for the change in B as a result of the change in A

What is a causal graph?

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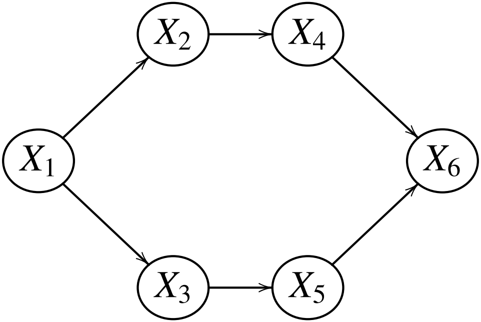

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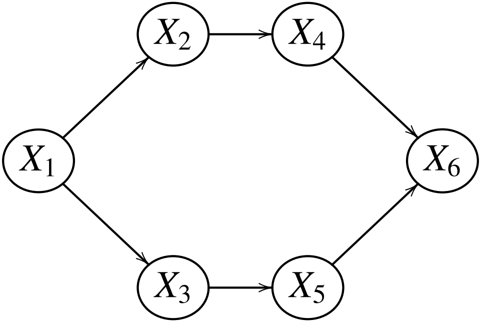

- A directed acyclic graph (DAG)
- A probability distribution over the nodes

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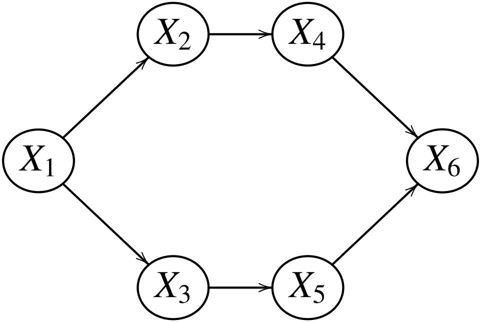

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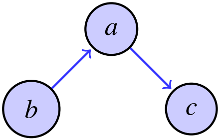
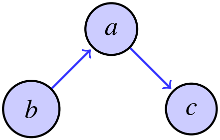
This forms the basis to infer causal relations between variables

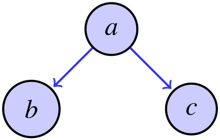
d-separation in directed graphs



chain



collider

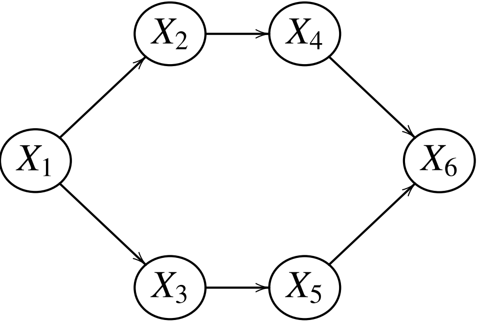


common cause

examples

- chain: healthy life style → exercise → recovery
- collider: healthy life style → recovery ← medication
- com. cause: exercise ← healthy life style → recovery

Graphs and probability



$$\Longleftrightarrow \quad P(X_1, X_2, X_3, X_4, X_5, X_6)$$

Graphs and probability

Theorem 1.2.5 (Pearl, 2000, p. 18)
For any three nodes (X, Y, Z) in a DAG G and for all probability functions P, we have

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(ii) if X and Y are independent conditional on Z in all distributions compatible with G, then X and Y are d-separated given Z.

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conditional independence

Affliction   vitamin K

	A	$\neg A$	total
K	14	56	70
$\neg K$	6	24	30
total	20	80	100

frequencies

	A	$\neg A$	total
K	0.14	0.56	0.67
$\neg K$	0.06	0.24	0.30
total	0.20	0.80	1.00

probabilities

$P(A) = 0.20 \qquad P(A, K) = 0.14$

conditional independence

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$$P(A \mid K) = \frac{P(A, K)}{P(K)} = \frac{0.14}{0.70} = 0.20$$

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$$P(A \mid K) = P(A \mid \neg K) = 0.20$$

So, A and K are independent or $A \perp\!\!\!\perp K$

conditional independence

Combined	A	$\neg A$		$P(A \mid (\neg)K)$	
Vit (K)	16	74	90	18%	$P(A \mid K) \neq P(A \mid \neg K)$
No vit ($\neg K$)	14	96	110	13%	
	30	170	200		

Gene C	A	$\neg A$		$P(A \mid C, (\neg)K)$	
Vit (K)	2	18	20	10%	
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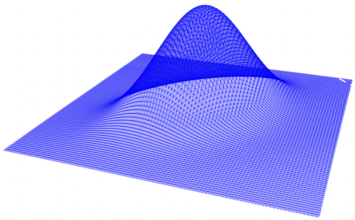
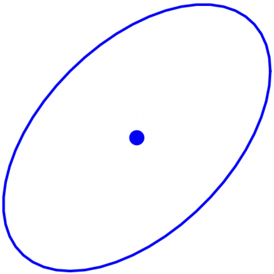
So: A is independent of K conditional on C, or $A \perp\!\!\!\perp K \mid C$

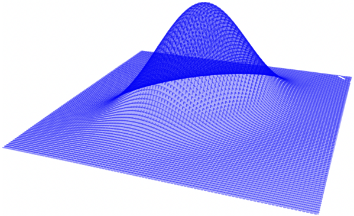
conditional independence

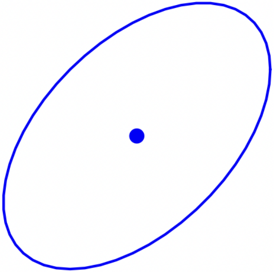




If the data are multivariate normal then we can use partial correlations for conditional independence

conditional independence





for multivariate normal data, the following are equivalent

(a) X and Y are independent conditional on Z

(b) the partial correlation between X and Y is 0 given Z ($\omega_{XY|Z} = 0$)

conditional independence

If the data are multivariate normal then we can use partial correlations for conditional independence

$$\Sigma = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 1.5 & 0.5 \\ -1 & 0.5 & 1.5 \end{pmatrix} \qquad \Sigma^{-1} = K = \begin{pmatrix} 1 & 0.5 & 0.5 \\ 0.5 & 1 & 0 \\ 0.5 & 0 & 1 \end{pmatrix}$$

covariances   partial covariances

partial correlation: $\omega_{XY|Z} = -\frac{\kappa_{XY}}{\sqrt{\kappa_{XX}\kappa_{YY}}}$ or $X \perp\!\!\!\perp Y \mid Z$

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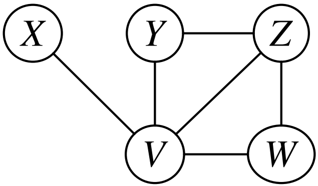
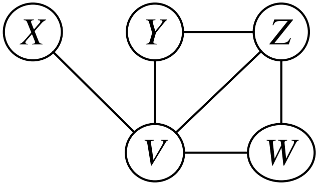
d-separation undirected graphs



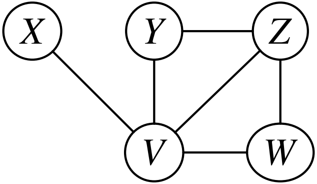
1. d-separation is a graphical (pictorial) tool to determine if variables are or can be separated.
2. For undirected graphs we simply check whether we can get from one node to another given a third (set).

d-separation undirected graphs



$(X \perp\!\!\!\perp Z \mid YW)$

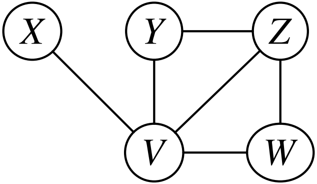
d-separation undirected graphs



$(X \perp\!\!\!\perp Z \mid YW)$

1. We block paths between X and Z with Y and W.

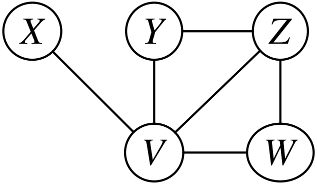
d-separation undirected graphs



$(X \perp\!\!\!\perp Z \mid YW)$

1. We block paths between X and Z with Y and W.
2. There is a path $X - V - Z$ that remains unblocked.

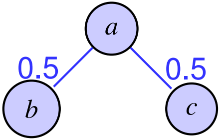
d-separation undirected graphs



$(X \perp\!\!\!\perp Z \mid YW)$

1. We block paths between X and Z with Y and W.
2. There is a path $X - V - Z$ that remains unblocked.
3. So: X and Z are not d-separated when blocking on Y and W $\cancel{(X \perp\!\!\!\perp Z \mid YW)}$.

graphs and conditional independence



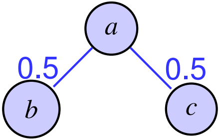
$G_{(a,b),(a,c)}$

$$\Sigma = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 1.5 & 0.5 \\ -1 & 0.5 & 1.5 \end{pmatrix} \qquad \Sigma^{-1} = K = \begin{pmatrix} 1 & 0.5 & 0.5 \\ 0.5 & 1 & 0 \\ 0.5 & 0 & 1 \end{pmatrix}$$

covariances — partial covariances

for multivariate normal data (Gaussian graphical model, ggm)

graphs and conditional independence



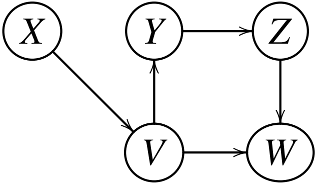
$G_{(a,b),(a,c)}$

$$\Omega = \begin{pmatrix} 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0 \\ 0.5 & 0 & 0 \end{pmatrix}$$

connectivity parameters

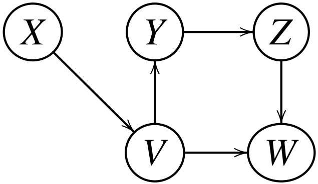
for multivariate binary data (Ising model, IsingFit)

d-separation in directed graphs



Lauritzen $(X \perp\!\!\!\perp Z \mid YW)$

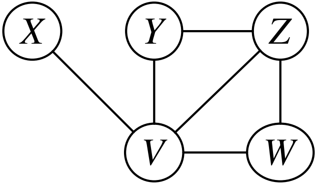
d-separation in directed graphs



Lauritzen $(X \perp\!\!\!\perp Z \mid YW)$

1. Make the *ancestral graph* (G_{an}): the variables of interest and all variables that have a directed path to those variables $\{W, X, Y, Z\}$.

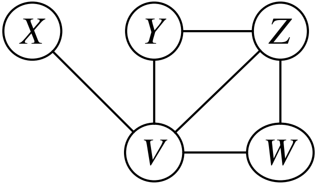
d-separation in directed graphs



Lauritzen $(X \perp\!\!\!\perp Z \mid YW)$

1. Make the *ancestral graph* (G_{an}): the variables of interest and all variables that have a directed path to those variables $\{W, X, Y, Z\}$.
2. Make the *ancestral graph moral* (G^m_{an}): marry all the parents that have a child in common *and* convert all arrows into undirected edges.

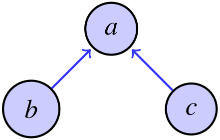
d-separation in directed graphs



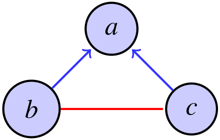
Lauritzen $(X \perp\!\!\!\perp Z \mid YW)$

1. Make the *ancestral graph* (G_{an}): the variables of interest and all variables that have a directed path to those variables $\{W, X, Y, Z\}$.
2. Make the *ancestral graph moral* (G^m_{an}): marry all the parents that have a child in common *and* convert all arrows into undirected edges.
3. Consider separating all paths in G^m_{an} between X and Z.

d-separation in directed graphs



true



spurious

example

- suppose that recovery (a) is because of healthy life style (b) or homeopathy (c)
- we know you are recovering, and you do not take homeopathy
- then, it must be a healthy life style that causes you to recover

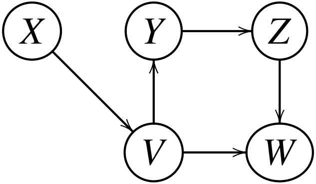
d-separation in directed graphs

$$(X \perp\!\!\!\perp Z \mid YW)$$

d-separation in directed graphs

$$(X \perp\!\!\!\perp Z \mid YW)$$

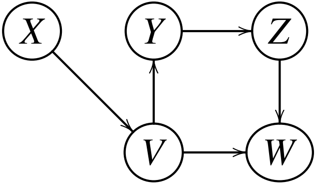
G



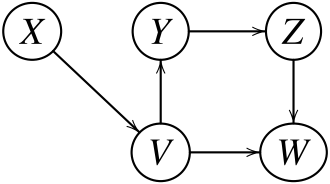
d-separation in directed graphs

$(X \perp\!\!\!\perp Z \mid YW)$

G



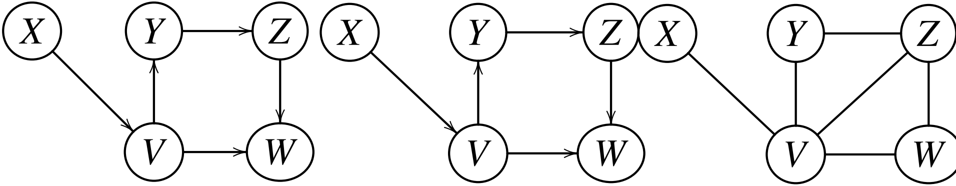
G_{an}



d-separation in directed graphs

$(X \perp\!\!\!\perp Z \mid YW)$

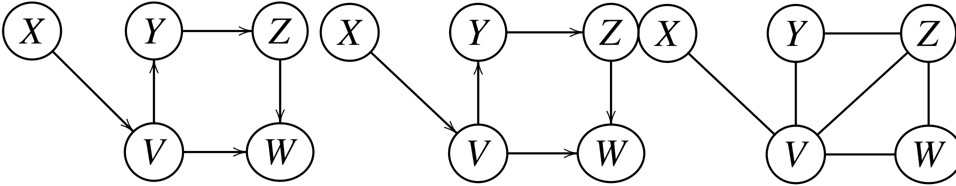
G $\qquad\qquad$ G_{an} $\qquad\qquad$ G_{an}^{m}



d-separation in directed graphs

$(X \perp\!\!\!\perp Z \mid YW)$

G $\qquad$ G_{an} $\qquad$ G_{an}^{m}



$\cancel{(X \perp\!\!\!\perp Z \mid YW)}$

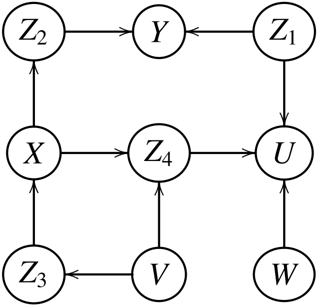
d-separation in directed graphs

$(X \perp\!\!\!\perp Y \mid Z_2, Z_3)$

d-separation in directed graphs

$$(X \perp\!\!\!\perp Y \mid Z_2, Z_3)$$

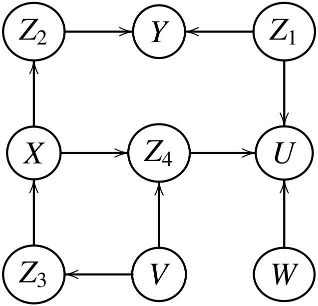
G



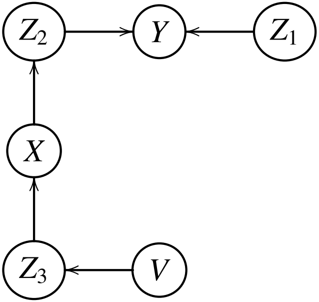
d-separation in directed graphs

$(X \perp\!\!\!\perp Y \mid Z_2, Z_3)$

G

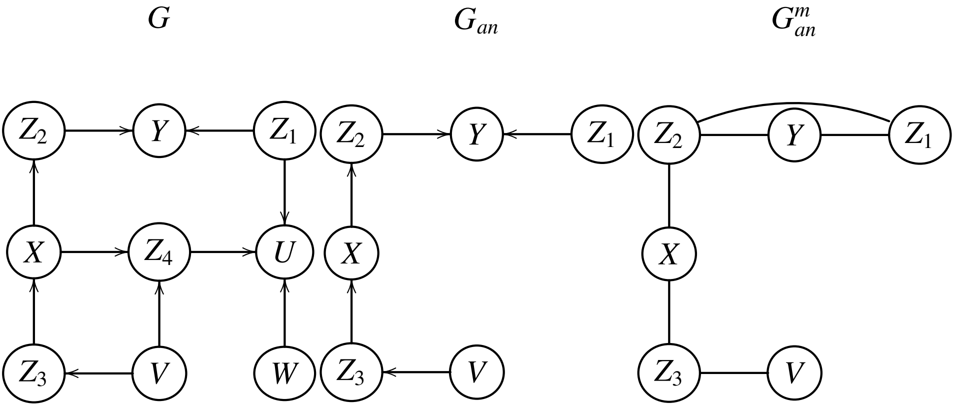


G_{an}



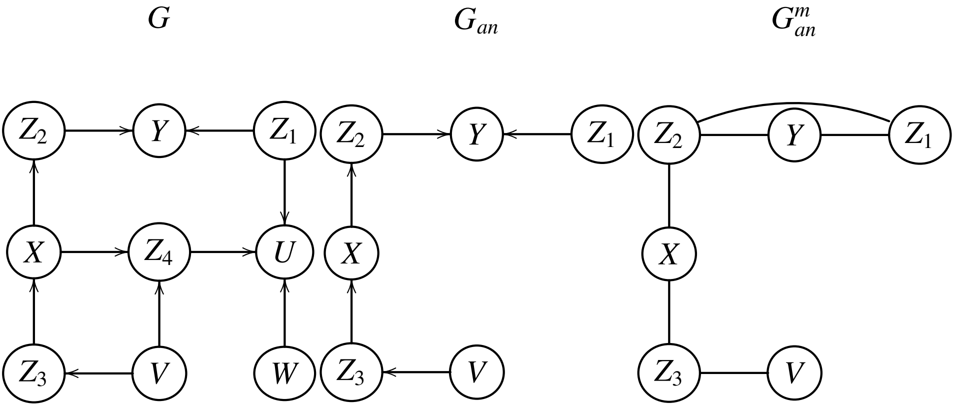
d-separation in directed graphs

$(X \perp\!\!\!\perp Y \mid Z_2, Z_3)$



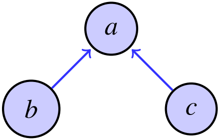
d-separation in directed graphs

$(X \perp\!\!\!\perp Y \mid Z_2, Z_3)$

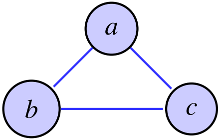


$(X \perp\!\!\!\perp Y \mid Z_2, Z_3)$

d-separation in directed graphs



directed



undirected

edge in directed graph

- causal relation $b \rightarrow a$

edge in undirected graph

- causal relation $a \rightarrow b$
- causal relation $b \rightarrow a$
- reciprocal $a - b$

Today

Why causality is interesting

Explanations

Causality and probability

d-separation and conditional independence

conditional independence

d-separation

When are models the same?

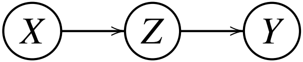
Discovering causal graphs

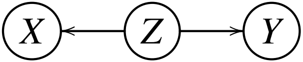
Inductive Causation algorithm

When are models the same?

$$X \rightarrow Z \rightarrow Y \qquad\qquad X \leftarrow Z \rightarrow Y$$

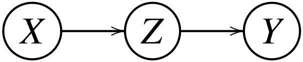
When are models the same?

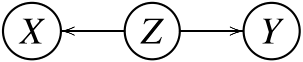




d-separation

When are models the same?





d-separation

$$(X \perp\!\!\!\perp Y \mid Z)$$

When are models the same?

$$X \rightarrow Z \rightarrow Y \qquad X \leftarrow Z \rightarrow Y \qquad X \rightarrow Z \leftarrow Y$$

When are models the same?

$X \rightarrow Z \rightarrow Y \qquad X \leftarrow Z \rightarrow Y \qquad X \rightarrow Z \leftarrow Y$

$(X \perp\!\!\!\perp Y \mid Z)$

When are models the same?

$$X \rightarrow Z \rightarrow Y \qquad X \leftarrow Z \rightarrow Y \qquad X \rightarrow Z \leftarrow Y$$

$$(X \perp\!\!\!\perp Y \mid Z) \qquad\qquad (X \perp\!\!\!\perp Y \mid Z)$$

When are models the same?

$X \rightarrow Z \rightarrow Y$ $\quad$ $X \leftarrow Z \rightarrow Y$ $\quad$ $X \rightarrow Z \leftarrow Y$

$(X \perp\!\!\!\perp Y \mid Z)$ $\qquad$ $(X \perp\!\!\!\perp Y \mid Z)$ $\qquad$ $(X \not\!\perp\!\!\!\perp Y \mid Z)$

When are models the same?

$X \rightarrow Z \rightarrow Y$ $\quad$ $X \leftarrow Z \rightarrow Y$ $\quad$ $X \rightarrow Z \leftarrow Y$

$(X \perp\!\!\!\perp Y \mid Z)$ $\quad$ $(X \perp\!\!\!\perp Y \mid Z)$ $\quad$ $(X \not\perp\!\!\!\perp Y \mid Z)$

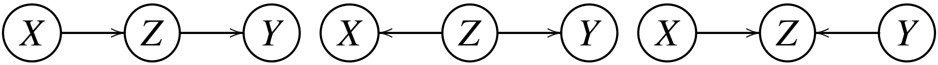
$(X \perp\!\!\!\perp Y)$

When are models the same?

$$X \rightarrow Z \rightarrow Y \qquad X \leftarrow Z \rightarrow Y \qquad X \rightarrow Z \leftarrow Y$$

$(X \perp\!\!\!\perp Y \mid Z)$	$(X \perp\!\!\!\perp Y \mid Z)$	$(X \not\perp\!\!\!\perp Y \mid Z)$
$(X \not\perp\!\!\!\perp Y)$	$(X \not\perp\!\!\!\perp Y)$	$(X \perp\!\!\!\perp Y)$

When are models the same?



$(X \perp\!\!\!\perp Y \mid Z)$	$(X \perp\!\!\!\perp Y \mid Z)$	$(X \not\perp\!\!\!\perp Y \mid Z)$
$(X \not\perp\!\!\!\perp Y)$	$(X \not\perp\!\!\!\perp Y)$	$(X \perp\!\!\!\perp Y)$

Models are the same when the d-separations are the same!

Today

Why causality is interesting
- Explanations
- Causality and probability

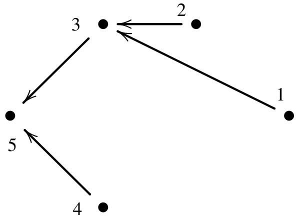
d-separation and conditional independence
- conditional independence
- d-separation
- When are models the same?

Discovering causal graphs
- Inductive Causation algorithm

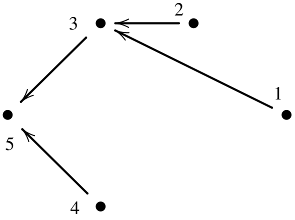
Inductive Causation algorithm

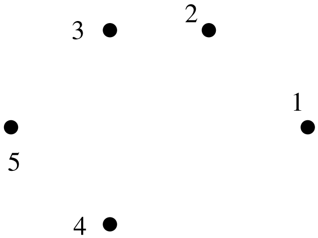
Inductive Causation algorithm

Inductive Causation algorithm

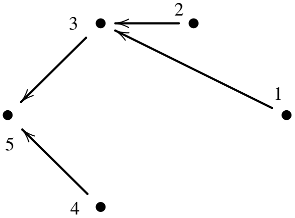


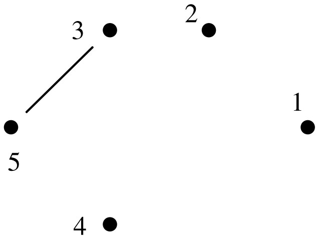
Inductive Causation algorithm





Inductive Causation algorithm





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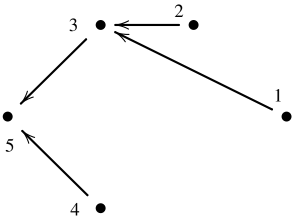
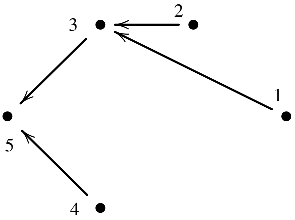
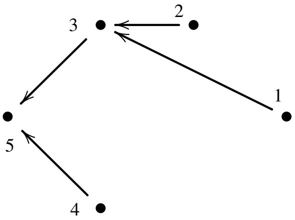


Inductive Causation algorithm



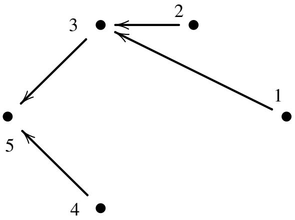
	$r(3,5)$	$r(3,5 \vert 1)$	$r(3,5 \vert 2)$	$r(3,5 \vert 4)$	$r(3,5 \vert 1,2)$	$\cdots$
$3-5$						

Inductive Causation algorithm



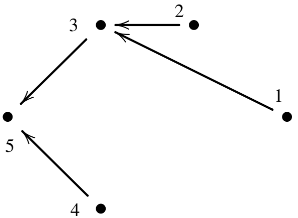
	$r(3,5)$	$r(3,5\|1)$	$r(3,5\|2)$	$r(3,5\|4)$	$r(3,5\|1,2)$	$\cdots$
$3 - 5$	TRUE					

Inductive Causation algorithm



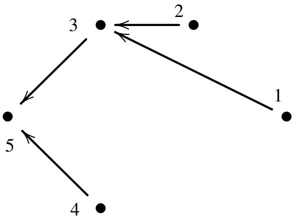
	$r(3,5)$	$r(3,5\|1)$	$r(3,5\|2)$	$r(3,5\|4)$	$r(3,5\|1,2)$	$\cdots$
$3-5$	TRUE	TRUE				

Inductive Causation algorithm



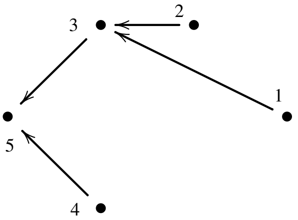
	$r(3,5)$	$r(3,5\|1)$	$r(3,5\|2)$	$r(3,5\|4)$	$r(3,5\|1,2)$	$\cdots$
$3-5$	TRUE	TRUE	TRUE			

Inductive Causation algorithm



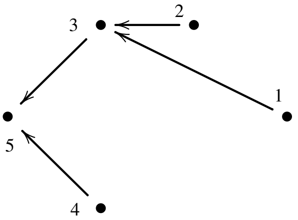
	$r(3,5)$	$r(3,5\|1)$	$r(3,5\|2)$	$r(3,5\|4)$	$r(3,5\|1,2)$	$\cdots$
$3-5$	TRUE	TRUE	TRUE	TRUE		

Inductive Causation algorithm



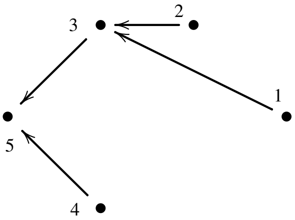
	$r(3,5)$	$r(3,5\|1)$	$r(3,5\|2)$	$r(3,5\|4)$	$r(3,5\|1,2)$	$\cdots$
$3-5$	TRUE	TRUE	TRUE	TRUE	TRUE	

Inductive Causation algorithm



	$r(3,5)$	$r(3,5\|1)$	$r(3,5\|2)$	$r(3,5\|4)$	$r(3,5\|1,2)$	$\cdots$
$3-5$	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE

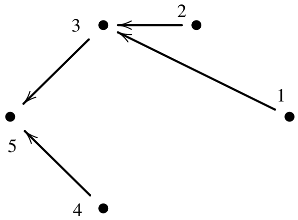
Inductive Causation algorithm



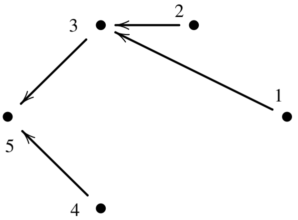
	$r(3,5)$	$r(3,5 \vert 1)$	$r(3,5 \vert 2)$	$r(3,5 \vert 4)$	$r(3,5 \vert 1,2)$	$\cdots$
$3 - 5$	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE

There is no (set of) node(s) x such that $(3 \perp\!\!\!\perp 5 \mid x)$ holds; and so connection $3 - 5$ is TRUE

Inductive Causation algorithm

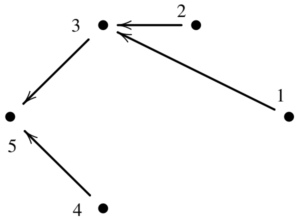


Inductive Causation algorithm



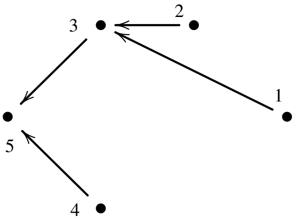
	$r(3,4)$	$r(3,4\|1)$	$r(3,4\|2)$	$r(3,4\|5)$	$r(3,4\|1,2)$	$\cdots$
$3-4$						

Inductive Causation algorithm



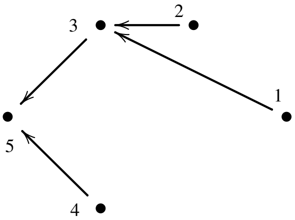
	$r(3,4)$	$r(3,4\|1)$	$r(3,4\|2)$	$r(3,4\|5)$	$r(3,4\|1,2)$	$\cdots$
3 – 4	FALSE					

Inductive Causation algorithm



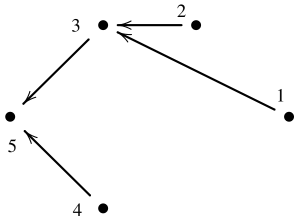
	$r(3,4)$	$r(3,4\|1)$	$r(3,4\|2)$	$r(3,4\|5)$	$r(3,4\|1,2)$	$\cdots$
$3 - 4$	FALSE	FALSE				

Inductive Causation algorithm



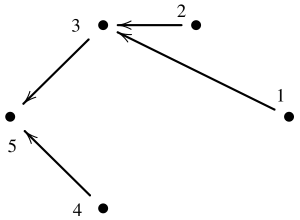
	$r(3,4)$	$r(3,4\|1)$	$r(3,4\|2)$	$r(3,4\|5)$	$r(3,4\|1,2)$	$\cdots$
3 – 4	FALSE	FALSE	FALSE			

Inductive Causation algorithm



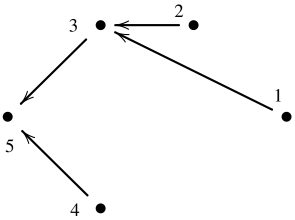
	$r(3,4)$	$r(3,4 \mid 1)$	$r(3,4 \mid 2)$	$r(3,4 \mid 5)$	$r(3,4 \mid 1,2)$	$\cdots$
$3 - 4$	FALSE	FALSE	FALSE	TRUE		

Inductive Causation algorithm



	$r(3,4)$	$r(3,4\|1)$	$r(3,4\|2)$	$r(3,4\|5)$	$r(3,4\|1,2)$	$\cdots$
$3 - 4$	FALSE	FALSE	FALSE	TRUE	FALSE	

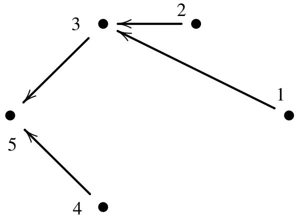
Inductive Causation algorithm



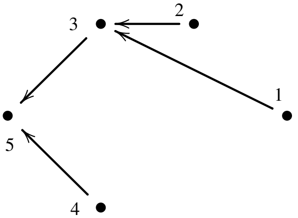
	$r(3,4)$	$r(3,4\|1)$	$r(3,4\|2)$	$r(3,4\|5)$	$r(3,4\|1,2)$	$\cdots$
$3 - 4$	FALSE	FALSE	FALSE	TRUE	FALSE	

No correlation between 3 and 4 and so no (direct) connection $3 - 4$, but conditioning on 5 gives correlation $r(3,4)$ and so a collider $3 \rightarrow 5 \leftarrow 4$.

Inductive Causation algorithm

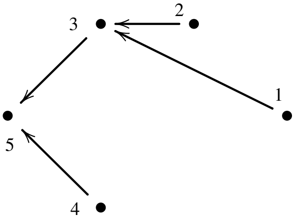


Inductive Causation algorithm



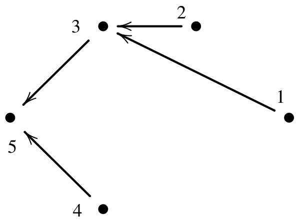
	$r(1,4)$	$r(1,4\|2)$	$r(1,4\|3)$	$r(1,4\|5)$	$r(1,4\|3,5)$
$1-4$					

Inductive Causation algorithm



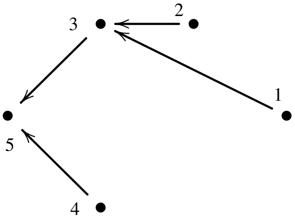
	$r(1,4)$	$r(1,4\|2)$	$r(1,4\|3)$	$r(1,4\|5)$	$r(1,4\|3,5)$
$1-4$	FALSE				

Inductive Causation algorithm



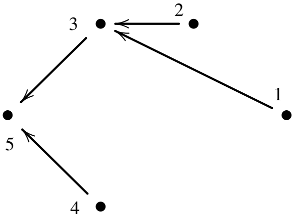
	$r(1,4)$	$r(1,4\|2)$	$r(1,4\|3)$	$r(1,4\|5)$	$r(1,4\|3,5)$
$1-4$	FALSE	FALSE			

Inductive Causation algorithm



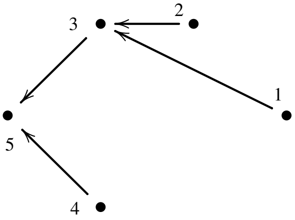
	$r(1,4)$	$r(1,4\|2)$	$r(1,4\|3)$	$r(1,4\|5)$	$r(1,4\|3,5)$
$1 - 4$	FALSE	FALSE	FALSE		

Inductive Causation algorithm



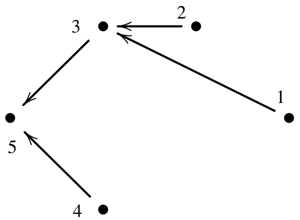
	$r(1,4)$	$r(1,4\|2)$	$r(1,4\|3)$	$r(1,4\|5)$	$r(1,4\|3,5)$
$1-4$	FALSE	FALSE	FALSE	TRUE	

Inductive Causation algorithm



	$r(1,4)$	$r(1,4 \mid 2)$	$r(1,4 \mid 3)$	$r(1,4 \mid 5)$	$r(1,4 \mid 3,5)$
$1 - 4$	FALSE	FALSE	FALSE	TRUE	

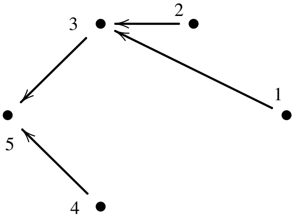
Inductive Causation algorithm



	$r(1,4)$	$r(1,4\|2)$	$r(1,4\|3)$	$r(1,4\|5)$	$r(1,4\|3,5)$
$1-4$	FALSE	FALSE	FALSE	TRUE	

There is no correlation between 1 and 4, and so no (direct) connection $1-4$.

Inductive Causation algorithm



	$r(1,4)$	$r(1,4 \mid 2)$	$r(1,4 \mid 3)$	$r(1,4 \mid 5)$	$r(1,4 \mid 3,5)$
$1-4$	FALSE	FALSE	FALSE	TRUE	FALSE

There is no correlation between 1 and 4, and so no (direct) connection $1-4$.
No collider since conditioning on 3 and 5 removes the correlation again.

Inductive Causation algorithm

IC-Algorithm Pearl (1988)

Inductive Causation algorithm

IC-Algorithm Pearl (1988)

Input $\hat{P}$ a sampled distribution

Inductive Causation algorithm

IC-Algorithm Pearl (1988)

Input $\hat{P}$ a sampled distribution

Output some acyclic graph for $\hat{P}$

Inductive Causation algorithm

IC-Algorithm Pearl (1988)

Input $\hat{P}$ a sampled distribution

Output some acyclic graph for $\hat{P}$

1. For each pair a and b, look for $(a \perp\!\!\!\perp b \mid S_{ab})$. If no such S_{ab} exists, then a and b are dependent.

Inductive Causation algorithm

IC-Algorithm Pearl (1988)

Input $\hat{P}$ a sampled distribution

Output some acyclic graph for $\hat{P}$

1. For each pair a and b, look for $(a \perp\!\!\!\perp b \mid S_{ab})$. If no such S_{ab} exists, then a and b are dependent.
2. For each trio (a, b, c) such that $a - c - b$ check if c belongs to S_{ab}. If so, then nothing. If c is not in S_{ab} then make a collider at c, i.e. $a \rightarrow c \leftarrow b$.

Inductive Causation algorithm

IC-Algorithm Pearl (1988)

Input $\hat{P}$ a sampled distribution

Output some acyclic graph for $\hat{P}$

1. For each pair a and b, look for $(a \perp\!\!\!\perp b \mid S_{ab})$. If no such S_{ab} exists, then a and b are dependent.
2. For each trio (a, b, c) such that $a - c - b$ check if c belongs to S_{ab}. If so, then nothing. If c is not in S_{ab} then make a collider at c, i.e. $a \rightarrow c \leftarrow b$.
3. Orient as many of the undirected edges as possible, subject to: (i) no new v-structures and (ii) no cycles.

directed and undirected networks, causality and discovery

- undirected networks represent reciprocal or unidirectional relations
- directed networks are often interpreted as causal networks
- d-separation is a pictorial (graphical) tool to determine conditional independencies
- causal statements using the idea of excluding alternative explanations are the same as the interventionist view on causality

Try all this in the practical