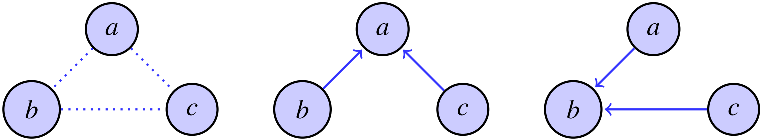
Directed and undirected networks
Causality and discovery



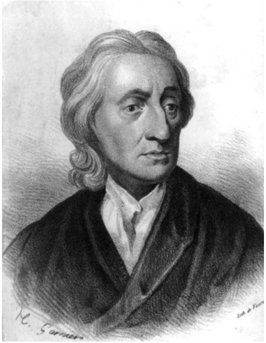
Network analysis

Lourens Waldorp
Universiteit van Amsterdam

Objectives

Get to know and have some intuition about

- Causality in philosophy (of science)
- Conditional independence
- Undirected and directed networks
- Causal discovery using `R`



David Hume (1711-1776)

Topics

Why causality is interesting

- Explanations
- Causality and probability

d-separation and conditional independence

- conditional independence
- d-separation
- When are models the same?

Discovering causal graphs

- Inductive Causation algorithm

Explanations

René Descartes
(1596 – 1650)

Explanations

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Naturalism:
Explanation of phenomena in terms of physical objects

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- Divide mechanism into parts

Explanations

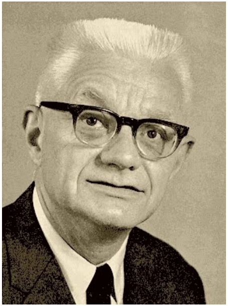
René Descartes

(1596 – 1650)

Naturalism:
Explanation of phenomena in terms of physical objects

- Example: the dials of a clock rotate by cogwheels and springs
- Divide mechanism into parts
- Not of mental processes like memory

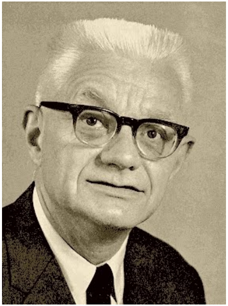
Explanations



Carl Hempel (1905 – 1977)

Logical positivists

Explanations



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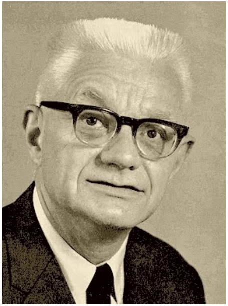
Deductive-nomological model

Explanations



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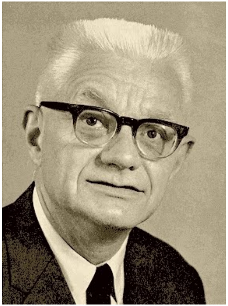
Deductive-nomological model

- $C_1, \ldots, C_k$ (circumstances)
- $L_1, \ldots, L_n$ (laws)

$\vdash E$ (explanandum)

Explanations



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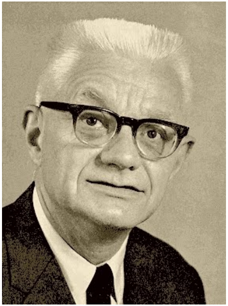

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Deductive-nomological model

- $C_1, \ldots, C_k$ (circumstances)
- $L_1, \ldots, L_n$ (laws)

$\vdash E$ (explanandum)

- Necessarily true if C and L are true.
- *The* basis for all sciences are laws and logical deductions.

Explanations

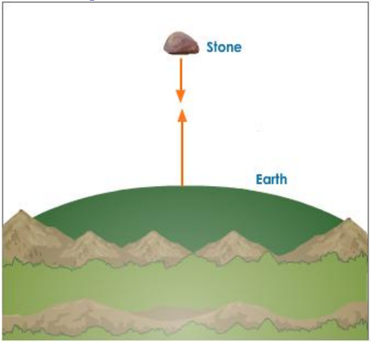
Isaac Newton (1643 – 1727)

Explanations

Isaac Newton $(1643 - 1727)$

You can fall back on the laws of Newton

Example



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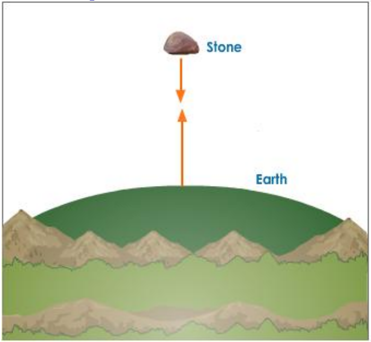
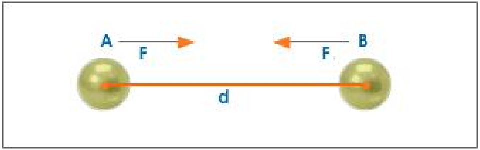
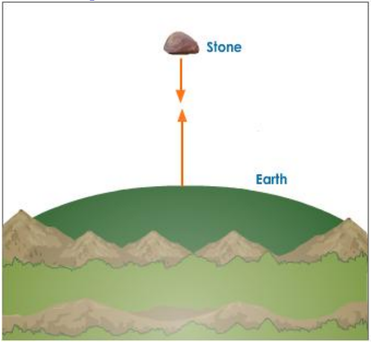
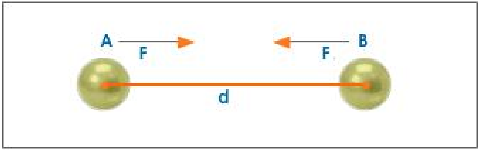




3rd law of Newton

Example

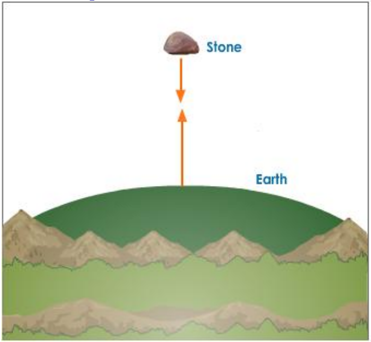


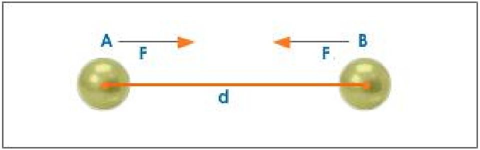


3rd law of Newton

- C : we are in the gravitational pull of the earth

Example





3rd law of Newton

- C : we are in the gravitational pull of the earth
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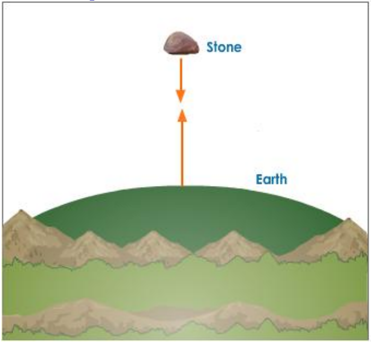

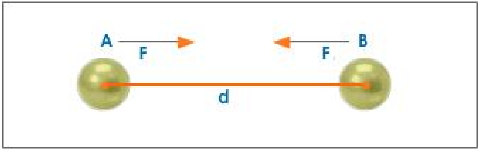

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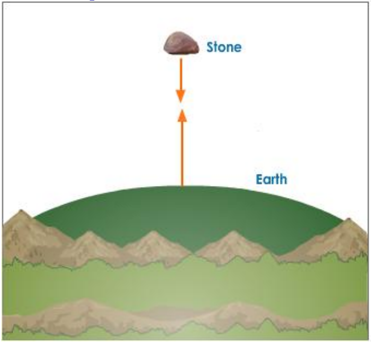
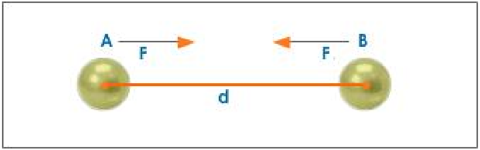
$$F = ma \text{ and } F = \frac{gm_1m_2}{d^2}$$

Example





3rd law of Newton

- C : we are in the gravitational pull of the earth
- L : 2nd and 3rd laws of Newton

$$F = ma \text{ and } F = \frac{gm_1m_2}{d^2}$$

$\vdash$ the rock falls to the earth

Explanations

So, what are these laws?

Explanations

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- Does the effect really always follow the cause?

Explanations

So, what are these laws?

- Does the effect really always follow the cause? No.

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Explanations

So, what are these laws?

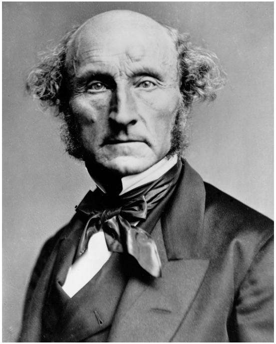
- Does the effect really always follow the cause? No.
- So, it is a regularity? Well, sort of...
- But it is a causal relation? That could be, but what is causality....
- This is what we need to figure out. It is the key to providing an explanation.

Causal relation and laws

John Stuart Mill's conditions

- A always co-occurs with B
- A occurs before B
- There is no alternative explanation for the co-occurrence of A and B

But this was unsatisfactory because A does not always occur with B; no universality.



John Stuart Mill (1806-1973)

Probabilities enter the stage

- Probability is essential to replace 'universal laws'
  Example

$$P(\text{catch fish} \mid \text{fishing rod}) \geq P(\text{catch fish})$$

- Conditional independence relations are the key to causal relations
  Example

$$P(\text{shark bite} \mid \text{ice cream}) \geq P(\text{shark bite})$$

$$P(\text{shark bite} \mid \text{ice cream, hot weather})$$

$$= P(\text{shark bite} \mid \text{hot weather})$$

Hans Reichenbach (1891-1953)

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$$P(\text{shark bite} \mid \text{ice cream, hot weather})$$
$$= P(\text{shark bite} \mid \text{hot weather})$$

- ‘Screening off’: search for nonredundent and sufficient ‘variables’ that define the relation (statistically relevant)

Wesley Salmon (1925-2001)

Probabilities enter the stage

- Spirtes and Glymour: Use screening off principle to identify unique relations from data
- Pearl: Semantics for difference between 'do' and 'see'
  Example

$$P(\text{catch fish} \mid \text{rod}) \neq P(\text{catch fish} \mid \text{do(rod)})$$

- Causality definition:

$$E(\text{catch fish}) \neq E(\text{catch fish} \mid \text{do(rod)})$$

Judea Pearl (1936-)

Peter Spirtes (1956-)

Clark Glymour (1942-)

Topics

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Discovering causal graphs

- Inductive Causation algorithm

What is a causal graph?

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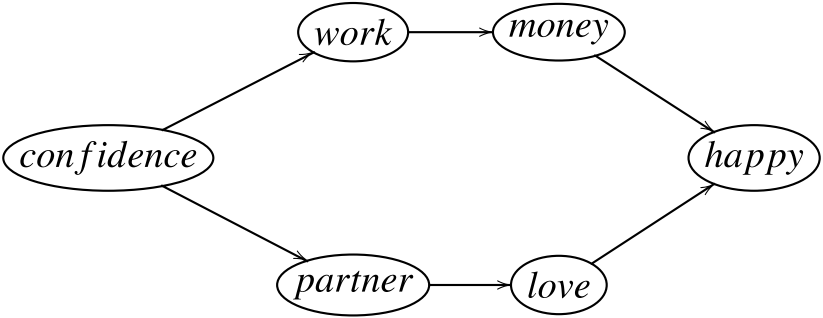

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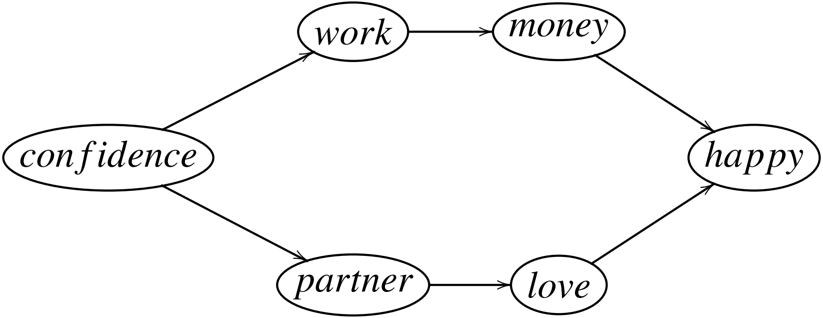

John Stuart Mill

- A (always) co-occurs with B
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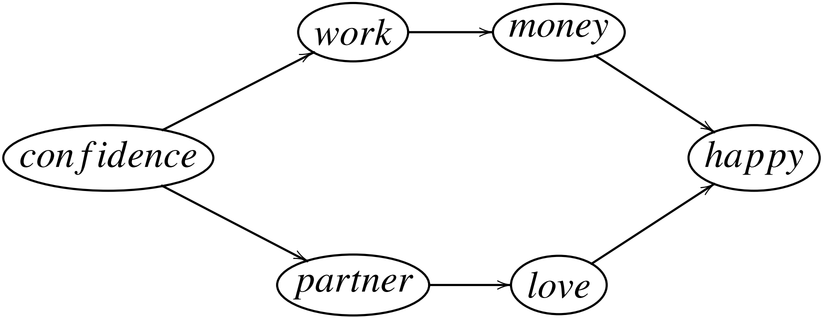

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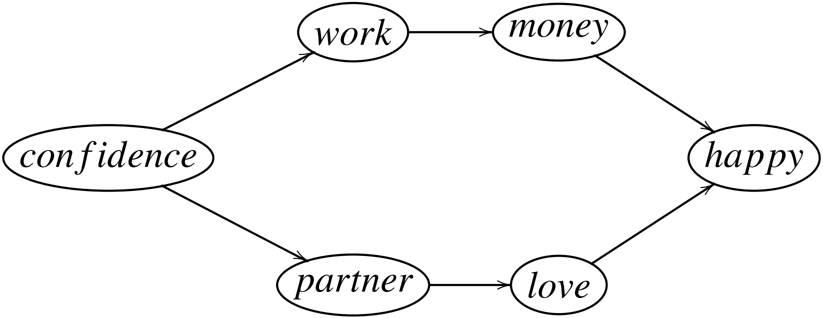

Causality by intervention

- If I wiggle A then B wiggles too (manipulation)
- There is no alternative explanation for the change in B as a result of the change in A

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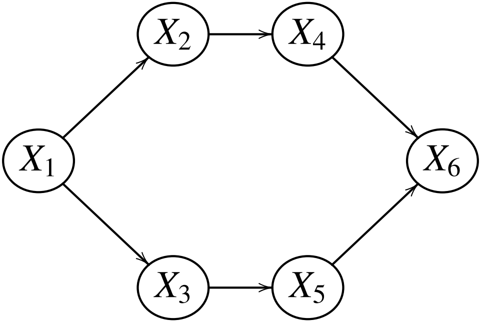

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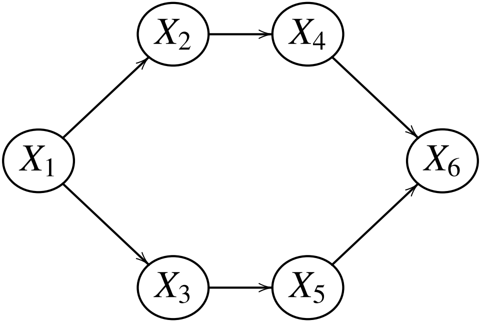

- A directed acyclic graph (DAG)
- A probability distribution over the nodes

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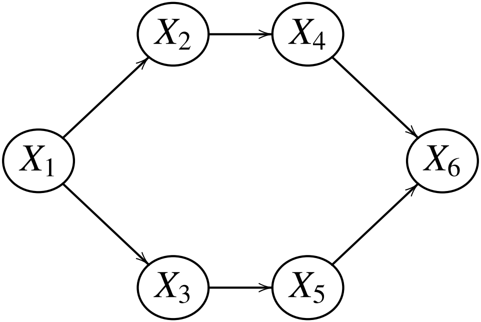

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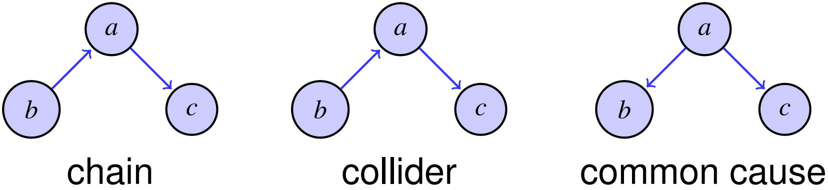
This forms the basis to infer causal relations between variables

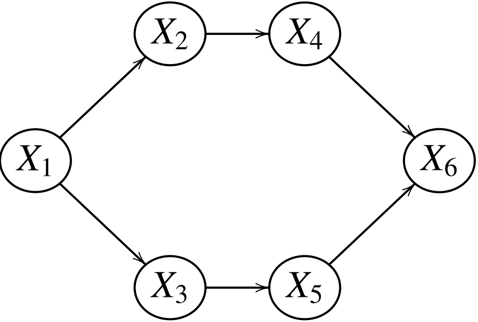
d-separation in directed graphs



examples

- chain: healthy life style → exercise → recovery
- collider: healthy life style → recovery ← medication
- com. cause: exercise ← healthy life style → recovery

Graphs and probability



$$\Longleftrightarrow \quad P(X_1, X_2, X_3, X_4, X_5, X_6)$$

Graphs and probability

Theorem 1.2.5 (Pearl, 2000, p. 18)
For any three nodes (X, Y, Z) in a DAG G and for all probability functions P, we have

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Topics

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conditional independence

d-separation

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conditional independence

Affliction vitamin K

	A	$\neg A$	total
K	14	56	70
$\neg K$	6	24	30
total	20	80	100

frequencies

	A	$\neg A$	total
K	0.14	0.56	0.67
$\neg K$	0.06	0.24	0.30
total	0.20	0.80	1.00

probabilities

$P(A) = 0.20 \qquad P(A, K) = 0.14$

conditional independence

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$$P(A \mid K) = \frac{P(A, K)}{P(K)} = \frac{0.14}{0.70} = 0.20$$

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$$P(A \mid K) = P(A \mid \neg K) = 0.20$$

So, A and K are independent or $A \perp\!\!\!\perp K$

conditional independence

Combined	A	$\neg A$		$P(A \mid (\neg)K)$	
Vit (K)	16	74	90	18%	$P(A \mid K) \neq P(A \mid \neg K)$
No vit ($\neg K$)	14	96	110	13%	
	30	170	200		

Gene C	A	$\neg A$		$P(A \mid C, (\neg)K)$	
Vit (K)	2	18	20	10%	$P(A \mid C, K) = P(A \mid C, \neg K)$
No vit ($\neg C$)	8	72	80	10%	
	10	90	100		

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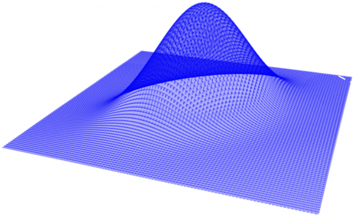
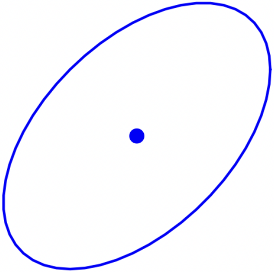
$P(A \mid \neg C, K) = P(A \mid \neg C, \neg K)$

So: A is independent of K conditional on C, or $A \perp\!\!\!\perp K \mid C$

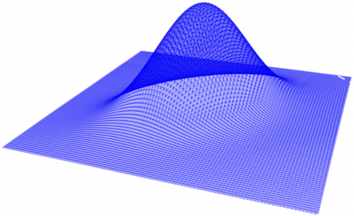
conditional independence

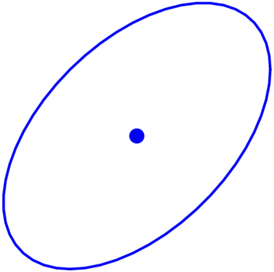




If the data are multivariate normal then we can use partial correlations for conditional independence

conditional independence





for multivariate normal data, the following are equivalent

(a) X and Y are independent conditional on Z

(b) the partial correlation between X and Y is 0 given Z ($\omega_{XY|Z} = 0$)

conditional independence

If the data are multivariate normal then we can use partial correlations for conditional independence

$$\Sigma = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 1.5 & 0.5 \\ -1 & 0.5 & 1.5 \end{pmatrix} \qquad \Sigma^{-1} = K = \begin{pmatrix} 1 & 0.5 & 0.5 \\ 0.5 & 1 & 0 \\ 0.5 & 0 & 1 \end{pmatrix}$$

covariances partial covariances

partial correlation: $\omega_{XY|Z} = -\frac{\kappa_{XY}}{\sqrt{\kappa_{XX}\kappa_{YY}}}$ or $X \perp\!\!\!\perp Y \mid Z$

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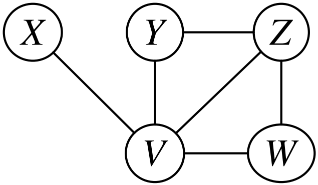
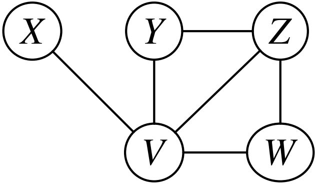
d-separation undirected graphs



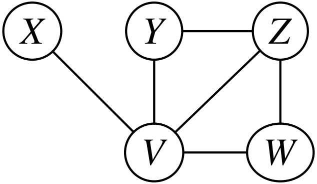
1. d-separation is a graphical (pictorial) tool to determine if variables are or can be separated.
2. For undirected graphs we simply check whether we can get from one node to another given a third (set).

d-separation undirected graphs



$(X \perp\!\!\!\perp Z \mid YW)$

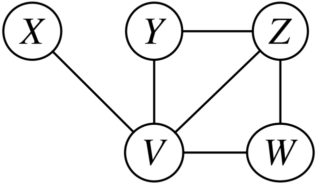
d-separation undirected graphs



$(X \perp\!\!\!\perp Z \mid YW)$

1. We block paths between X and Z with Y and W.

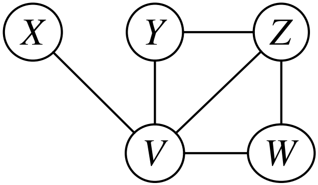
d-separation undirected graphs



$(X \perp\!\!\!\perp Z \mid YW)$

1. We block paths between X and Z with Y and W.
2. There is a path $X - V - Z$ that remains unblocked.

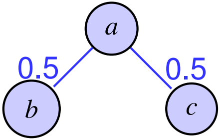
d-separation undirected graphs



$(X \perp\!\!\!\perp Z \mid YW)$

1. We block paths between X and Z with Y and W.
2. There is a path $X - V - Z$ that remains unblocked.
3. So: X and Z are not d-separated when blocking on Y and W $\cancel{(X \perp\!\!\!\perp Z \mid YW)}$.

graphs and conditional independence



$G_{(a,b),(a,c)}$

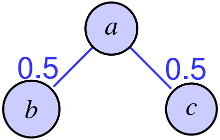
$$\Sigma = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 1.5 & 0.5 \\ -1 & 0.5 & 1.5 \end{pmatrix} \qquad \Sigma^{-1} = K = \begin{pmatrix} 1 & 0.5 & 0.5 \\ 0.5 & 1 & 0 \\ 0.5 & 0 & 1 \end{pmatrix}$$

covariances

partial covariances

for multivariate normal data (Gaussian graphical model, ggm)

graphs and conditional independence



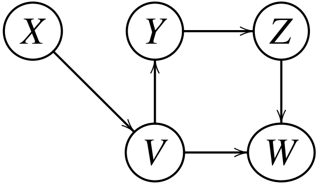
$G_{(a,b),(a,c)}$

$$\Omega = \begin{pmatrix} 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0 \\ 0.5 & 0 & 0 \end{pmatrix}$$

connectivity parameters

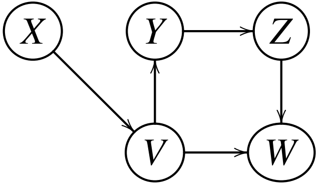
for multivariate binary data (Ising model, IsingFit)

d-separation in directed graphs



Lauritzen $(X \perp\!\!\!\perp Z \mid YW)$

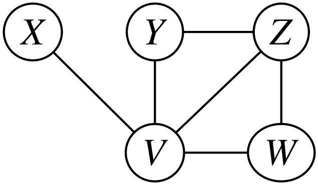
d-separation in directed graphs



Lauritzen $(X \perp\!\!\!\perp Z \mid YW)$

1. Make the *ancestral graph* (G_{an}): the variables of interest and all variables that have a directed path to those variables $\{W, X, Y, Z\}$.

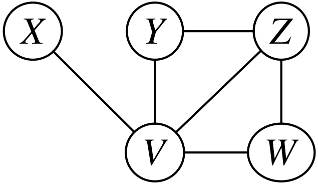
d-separation in directed graphs



Lauritzen $(X \perp\!\!\!\perp Z \mid YW)$

1. Make the *ancestral graph* (G_{an}): the variables of interest and all variables that have a directed path to those variables $\{W, X, Y, Z\}$.
2. Make the *ancestral graph moral* (G^m_{an}): marry all the parents that have a child in common *and* convert all arrows into undirected edges.

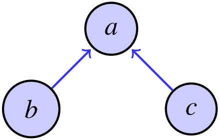
d-separation in directed graphs



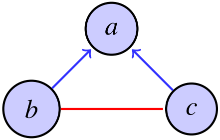
Lauritzen $(X \perp\!\!\!\perp Z \mid YW)$

1. Make the *ancestral graph* (G_{an}): the variables of interest and all variables that have a directed path to those variables $\{W, X, Y, Z\}$.
2. Make the *ancestral graph moral* (G_{an}^m): marry all the parents that have a child in common *and* convert all arrows into undirected edges.
3. Consider separating all paths in G_{an}^m between X and Z.

d-separation in directed graphs



true



spurious

example

- suppose that recovery (a) is because of healthy life style (b) or homeopathy (c)
- we know you are recovering, and you do not take homeopathy
- then, it must be a healthy life style that causes you to recover

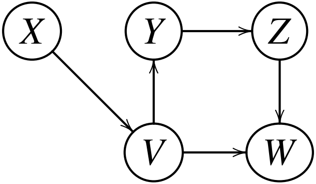
d-separation in directed graphs

$(X \perp\!\!\!\perp Z \mid YW)$

d-separation in directed graphs

$(X \perp\!\!\!\perp Z \mid YW)$

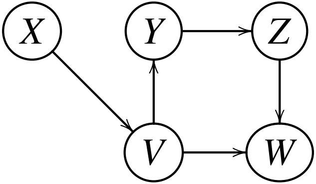
G



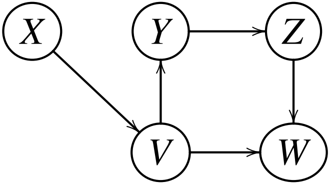
d-separation in directed graphs

$(X \perp\!\!\!\perp Z \mid YW)$

G



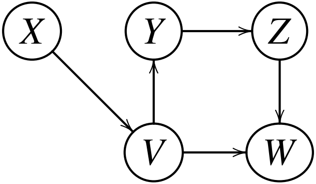
G_{an}



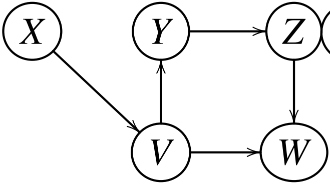
d-separation in directed graphs

$(X \perp\!\!\!\perp Z \mid YW)$

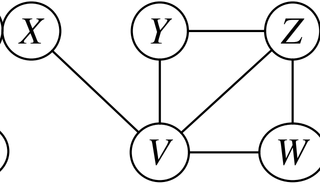
G



G_{an}



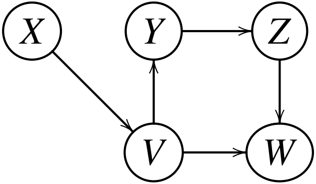
G_{an}^{m}



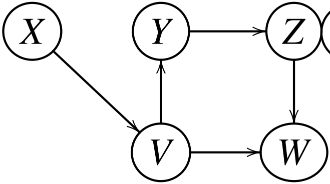
d-separation in directed graphs

$(X \perp\!\!\!\perp Z \mid YW)$

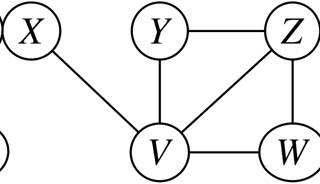
G



G_{an}



G_{an}^{m}



$\cancel{(X \perp\!\!\!\perp Z \mid YW)}$

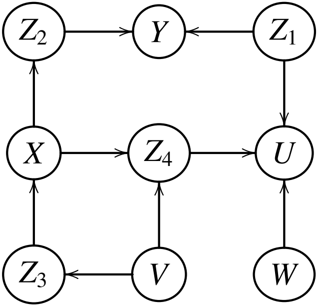
d-separation in directed graphs

$(X \perp\!\!\!\perp Y \mid Z_2, Z_3)$

d-separation in directed graphs

$(X \perp\!\!\!\perp Y \mid Z_2, Z_3)$

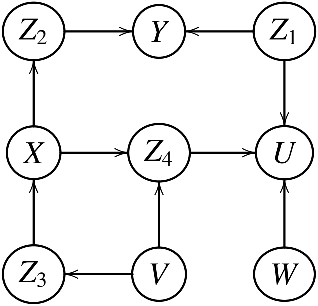
G

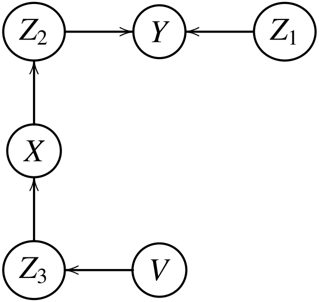


d-separation in directed graphs

$(X \perp\!\!\!\perp Y \mid Z_2, Z_3)$

G $\qquad\qquad\qquad$ G_{an}

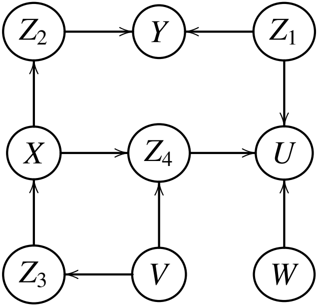




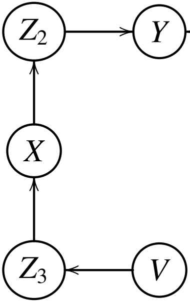
d-separation in directed graphs

$(X \perp\!\!\!\perp Y \mid Z_2, Z_3)$

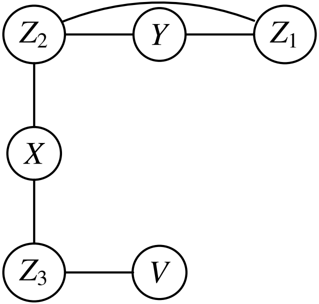
G



G_{an}

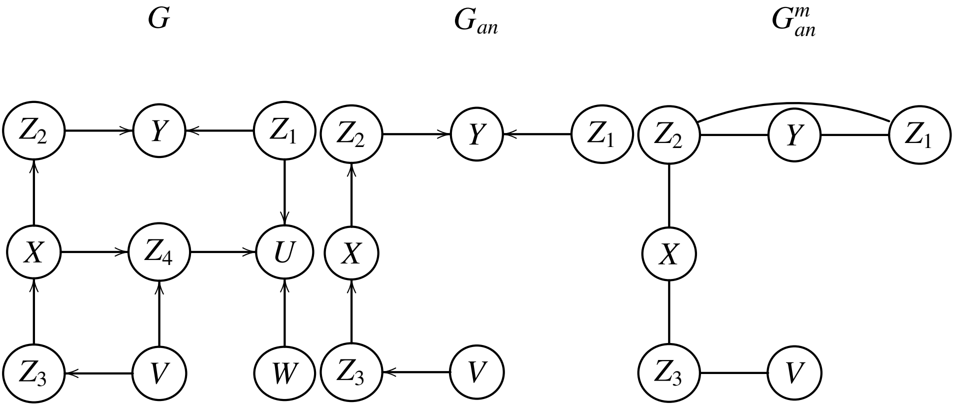


G_{an}^m



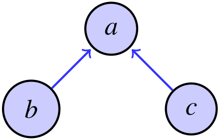
d-separation in directed graphs

$(X \perp\!\!\!\perp Y \mid Z_2, Z_3)$

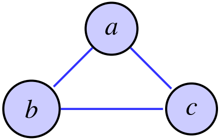


$(X \perp\!\!\!\perp Y \mid Z_2, Z_3)$

d-separation in directed graphs



directed



undirected

edge in directed graph

- causal relation $b \to a$

edge in undirected graph

- causal relation $a \to b$
- causal relation $b \to a$
- reciprocal $a - b$

Today

Why causality is interesting

- Explanations
- Causality and probability

d-separation and conditional independence

- conditional independence
- d-separation
- When are models the same?

Discovering causal graphs

- Inductive Causation algorithm

When are models the same?

$$X \rightarrow Z \rightarrow Y \qquad\qquad X \leftarrow Z \rightarrow Y$$

When are models the same?

$$X \rightarrow Z \rightarrow Y \qquad\qquad X \leftarrow Z \rightarrow Y$$

d-separation

When are models the same?

$$X \rightarrow Z \rightarrow Y \qquad\qquad X \leftarrow Z \rightarrow Y$$

d-separation

$$(X \perp\!\!\!\perp Y \mid Z)$$

When are models the same?

$$X \rightarrow Z \rightarrow Y \qquad X \leftarrow Z \rightarrow Y \qquad X \rightarrow Z \leftarrow Y$$

When are models the same?

$$X \rightarrow Z \rightarrow Y \qquad X \leftarrow Z \rightarrow Y \qquad X \rightarrow Z \leftarrow Y$$

$$(X \perp\!\!\!\perp Y \mid Z)$$

When are models the same?

$X \rightarrow Z \rightarrow Y$ $\qquad$ $X \leftarrow Z \rightarrow Y$ $\qquad$ $X \rightarrow Z \leftarrow Y$

$(X \perp\!\!\!\perp Y \mid Z)$ $\qquad$ $(X \perp\!\!\!\perp Y \mid Z)$

When are models the same?

$$X \rightarrow Z \rightarrow Y \qquad X \leftarrow Z \rightarrow Y \qquad X \rightarrow Z \leftarrow Y$$

$$(X \perp\!\!\!\perp Y \mid Z) \qquad (X \perp\!\!\!\perp Y \mid Z) \qquad (X \not\!\perp\!\!\!\perp Y \mid Z)$$

When are models the same?

$$X \rightarrow Z \rightarrow Y \qquad X \leftarrow Z \rightarrow Y \qquad X \rightarrow Z \leftarrow Y$$

$(X \perp\!\!\!\perp Y \mid Z)$

$(X \perp\!\!\!\perp Y \mid Z)$

$(X \not\perp\!\!\!\perp Y \mid Z)$

$(X \perp\!\!\!\perp Y)$

When are models the same?

$$X \rightarrow Z \rightarrow Y \qquad X \leftarrow Z \rightarrow Y \qquad X \rightarrow Z \leftarrow Y$$

$(X \perp\!\!\!\perp Y \mid Z)$	$(X \perp\!\!\!\perp Y \mid Z)$	$(X \not\perp\!\!\!\perp Y \mid Z)$
$(X \not\perp\!\!\!\perp Y)$	$(X \not\perp\!\!\!\perp Y)$	$(X \perp\!\!\!\perp Y)$

When are models the same?

$$X \rightarrow Z \rightarrow Y \qquad X \leftarrow Z \rightarrow Y \qquad X \rightarrow Z \leftarrow Y$$

$(X \perp\!\!\!\perp Y \mid Z)$	$(X \perp\!\!\!\perp Y \mid Z)$	$(X \not\!\perp\!\!\!\perp Y \mid Z)$
$(X \not\!\perp\!\!\!\perp Y)$	$(X \not\!\perp\!\!\!\perp Y)$	$(X \perp\!\!\!\perp Y)$

Models are the same when the d-separations are the same!

Today

Why causality is interesting

Explanations

Causality and probability

d-separation and conditional independence

conditional independence

d-separation

When are models the same?

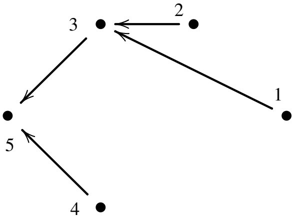
Discovering causal graphs

Inductive Causation algorithm

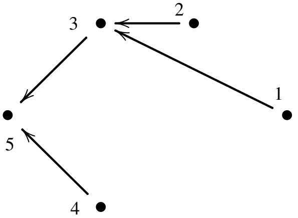
Inductive Causation algorithm

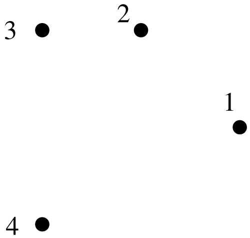
Inductive Causation algorithm

Inductive Causation algorithm

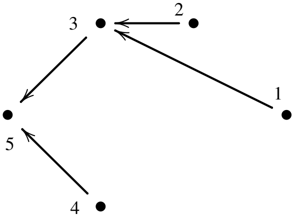


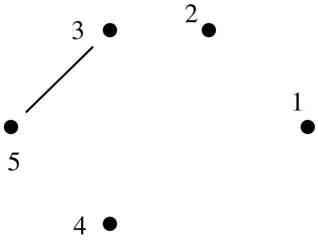
Inductive Causation algorithm



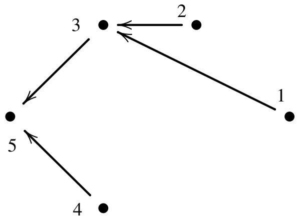


Inductive Causation algorithm

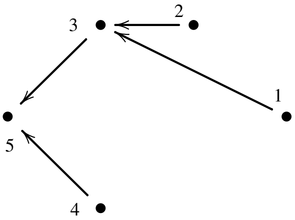




Inductive Causation algorithm

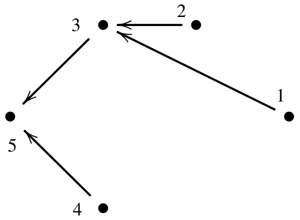


Inductive Causation algorithm



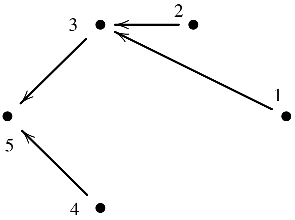
	$r(3,5)$	$r(3,5\|1)$	$r(3,5\|2)$	$r(3,5\|4)$	$r(3,5\|1,2)$	$\cdots$
$3-5$						

Inductive Causation algorithm



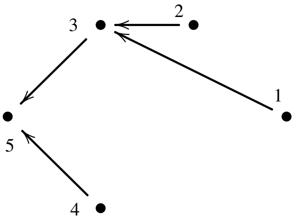
	$r(3,5)$	$r(3,5\|1)$	$r(3,5\|2)$	$r(3,5\|4)$	$r(3,5\|1,2)$	$\cdots$
$3-5$	TRUE					

Inductive Causation algorithm



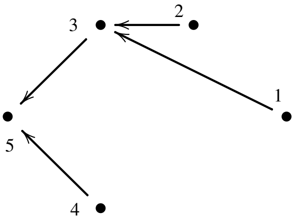
	$r(3,5)$	$r(3,5\|1)$	$r(3,5\|2)$	$r(3,5\|4)$	$r(3,5\|1,2)$	$\cdots$
$3-5$	TRUE	TRUE				

Inductive Causation algorithm



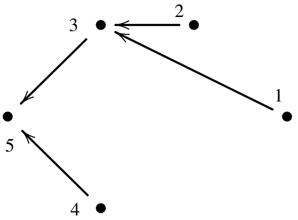
	$r(3,5)$	$r(3,5\|1)$	$r(3,5\|2)$	$r(3,5\|4)$	$r(3,5\|1,2)$	$\cdots$
$3-5$	TRUE	TRUE	TRUE			

Inductive Causation algorithm



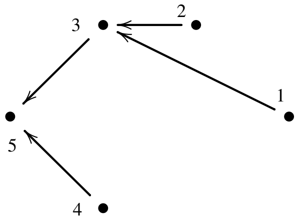
	$r(3,5)$	$r(3,5\|1)$	$r(3,5\|2)$	$r(3,5\|4)$	$r(3,5\|1,2)$	$\cdots$
$3-5$	TRUE	TRUE	TRUE	TRUE		

Inductive Causation algorithm



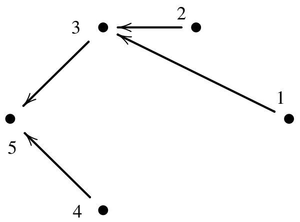
	$r(3,5)$	$r(3,5\|1)$	$r(3,5\|2)$	$r(3,5\|4)$	$r(3,5\|1,2)$	$\cdots$
$3 - 5$	TRUE	TRUE	TRUE	TRUE	TRUE	

Inductive Causation algorithm



	$r(3,5)$	$r(3,5\|1)$	$r(3,5\|2)$	$r(3,5\|4)$	$r(3,5\|1,2)$	$\cdots$
$3-5$	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE

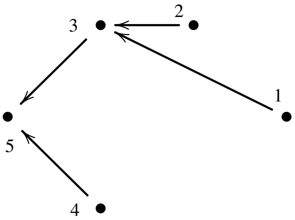
Inductive Causation algorithm



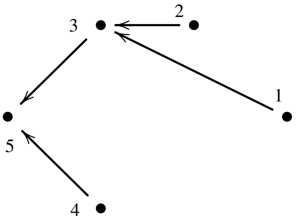
	$r(3,5)$	$r(3,5\|1)$	$r(3,5\|2)$	$r(3,5\|4)$	$r(3,5\|1,2)$	$\cdots$
$3 - 5$	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE

There is no (set of) node(s) x such that $(3 \perp\!\!\!\perp 5 \mid x)$ holds; and so connection $3 - 5$ is TRUE

Inductive Causation algorithm

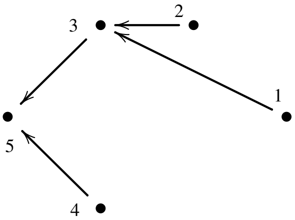


Inductive Causation algorithm



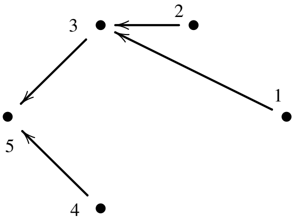
	$r(3,4)$	$r(3,4\|1)$	$r(3,4\|2)$	$r(3,4\|5)$	$r(3,4\|1,2)$	$\cdots$
$3-4$						

Inductive Causation algorithm



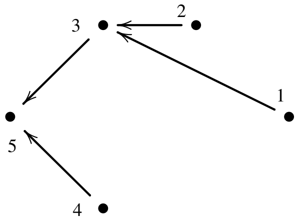
	$r(3,4)$	$r(3,4\|1)$	$r(3,4\|2)$	$r(3,4\|5)$	$r(3,4\|1,2)$	$\cdots$
3 – 4	FALSE					

Inductive Causation algorithm



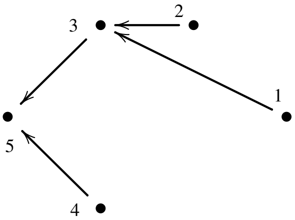
	$r(3,4)$	$r(3,4\|1)$	$r(3,4\|2)$	$r(3,4\|5)$	$r(3,4\|1,2)$	$\cdots$
$3 - 4$	FALSE	FALSE				

Inductive Causation algorithm



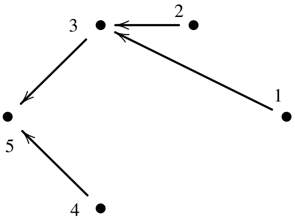
	$r(3,4)$	$r(3,4\|1)$	$r(3,4\|2)$	$r(3,4\|5)$	$r(3,4\|1,2)$	$\cdots$
3 – 4	FALSE	FALSE	FALSE			

Inductive Causation algorithm



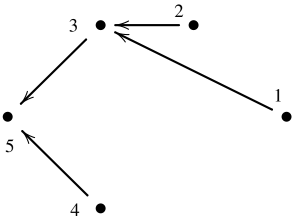
	$r(3,4)$	$r(3,4\|1)$	$r(3,4\|2)$	$r(3,4\|5)$	$r(3,4\|1,2)$	$\cdots$
3 – 4	FALSE	FALSE	FALSE	TRUE		

Inductive Causation algorithm



	$r(3,4)$	$r(3,4\|1)$	$r(3,4\|2)$	$r(3,4\|5)$	$r(3,4\|1,2)$	$\cdots$
3 – 4	FALSE	FALSE	FALSE	TRUE	FALSE	

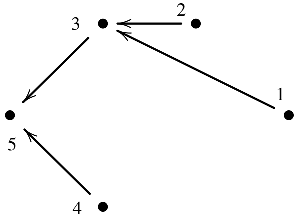
Inductive Causation algorithm



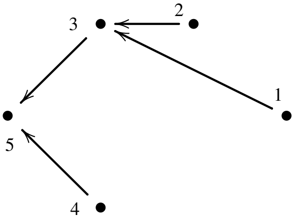
	$r(3,4)$	$r(3,4\|1)$	$r(3,4\|2)$	$r(3,4\|5)$	$r(3,4\|1,2)$	$\cdots$
$3 - 4$	FALSE	FALSE	FALSE	TRUE	FALSE	

No correlation between 3 and 4 and so no (direct) connection $3 - 4$, but conditioning on 5 gives correlation $r(3,4)$ and so a collider $3 \rightarrow 5 \leftarrow 4$.

Inductive Causation algorithm

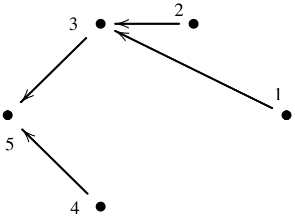


Inductive Causation algorithm



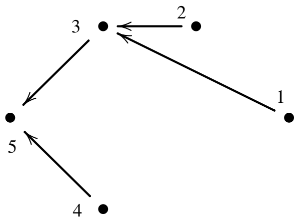
	$r(1,4)$	$r(1,4\vert 2)$	$r(1,4\vert 3)$	$r(1,4\vert 5)$	$r(1,4\vert 3,5)$
$1-4$					

Inductive Causation algorithm



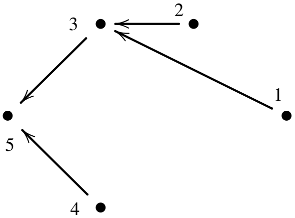
	$r(1,4)$	$r(1,4\|2)$	$r(1,4\|3)$	$r(1,4\|5)$	$r(1,4\|3,5)$
$1-4$	FALSE				

Inductive Causation algorithm



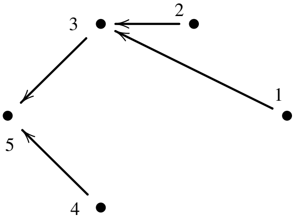
	$r(1,4)$	$r(1,4\|2)$	$r(1,4\|3)$	$r(1,4\|5)$	$r(1,4\|3,5)$
$1 - 4$	FALSE	FALSE			

Inductive Causation algorithm



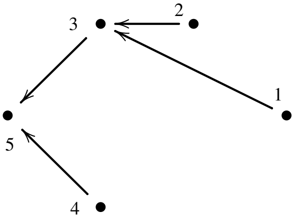
	$r(1,4)$	$r(1,4\|2)$	$r(1,4\|3)$	$r(1,4\|5)$	$r(1,4\|3,5)$
$1-4$	FALSE	FALSE	FALSE		

Inductive Causation algorithm



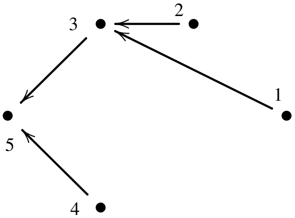
	$r(1,4)$	$r(1,4\|2)$	$r(1,4\|3)$	$r(1,4\|5)$	$r(1,4\|3,5)$
$1-4$	FALSE	FALSE	FALSE	TRUE	

Inductive Causation algorithm



	$r(1,4)$	$r(1,4\|2)$	$r(1,4\|3)$	$r(1,4\|5)$	$r(1,4\|3,5)$
$1-4$	FALSE	FALSE	FALSE	TRUE	

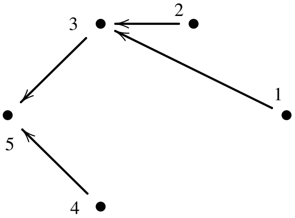
Inductive Causation algorithm



	$r(1,4)$	$r(1,4\|2)$	$r(1,4\|3)$	$r(1,4\|5)$	$r(1,4\|3,5)$
$1-4$	FALSE	FALSE	FALSE	TRUE	

There is no correlation between 1 and 4, and so no (direct) connection $1-4$.

Inductive Causation algorithm



	$r(1,4)$	$r(1,4\|2)$	$r(1,4\|3)$	$r(1,4\|5)$	$r(1,4\|3,5)$
$1-4$	FALSE	FALSE	FALSE	TRUE	FALSE

There is no correlation between 1 and 4, and so no (direct) connection $1-4$.
No collider since conditioning on 3 and 5 removes the correlation again.

Inductive Causation algorithm

IC-Algorithm Pearl (1988)

Inductive Causation algorithm

IC-Algorithm Pearl (1988)

Input $\hat{P}$ a sampled distribution

Inductive Causation algorithm

IC-Algorithm Pearl (1988)

Input $\hat{P}$ a sampled distribution
Output some acyclic graph for $\hat{P}$

Inductive Causation algorithm

IC-Algorithm Pearl (1988)

Input $\hat{P}$ a sampled distribution

Output some acyclic graph for $\hat{P}$

1. For each pair a and b, look for $(a \perp\!\!\!\perp b \mid S_{ab})$. If no such S_{ab} exists, then a and b are dependent.

Inductive Causation algorithm

IC-Algorithm Pearl (1988)

Input $\hat{P}$ a sampled distribution

Output some acyclic graph for $\hat{P}$

1. For each pair a and b, look for $(a \perp\!\!\!\perp b \mid S_{ab})$. If no such S_{ab} exists, then a and b are dependent.
2. For each trio (a, b, c) such that $a - c - b$ check if c belongs to S_{ab}. If so, then nothing. If c is not in S_{ab} then make a collider at c, i.e. $a \rightarrow c \leftarrow b$.

Inductive Causation algorithm

IC-Algorithm Pearl (1988)

Input $\hat{P}$ a sampled distribution

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1. For each pair a and b, look for $(a \perp\!\!\!\perp b \mid S_{ab})$. If no such S_{ab} exists, then a and b are dependent.
2. For each trio (a, b, c) such that $a - c - b$ check if c belongs to S_{ab}. If so, then nothing. If c is not in S_{ab} then make a collider at c, i.e. $a \rightarrow c \leftarrow b$.
3. Orient as many of the undirected edges as possible, subject to: (i) no new v-structures and (ii) no cycles.

directed and undirected networks, causality and discovery

- undirected networks represent reciprocal or unidirectional relations
- directed networks are often interpreted as causal networks
- d-separation is a pictorial (graphical) tool to determine conditional independencies
- causal statements using the idea of excluding alternative explanations are the same as the interventionist view on causality

Try all this in the practical