Network modeling of time-series data

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Network Analysis 2023

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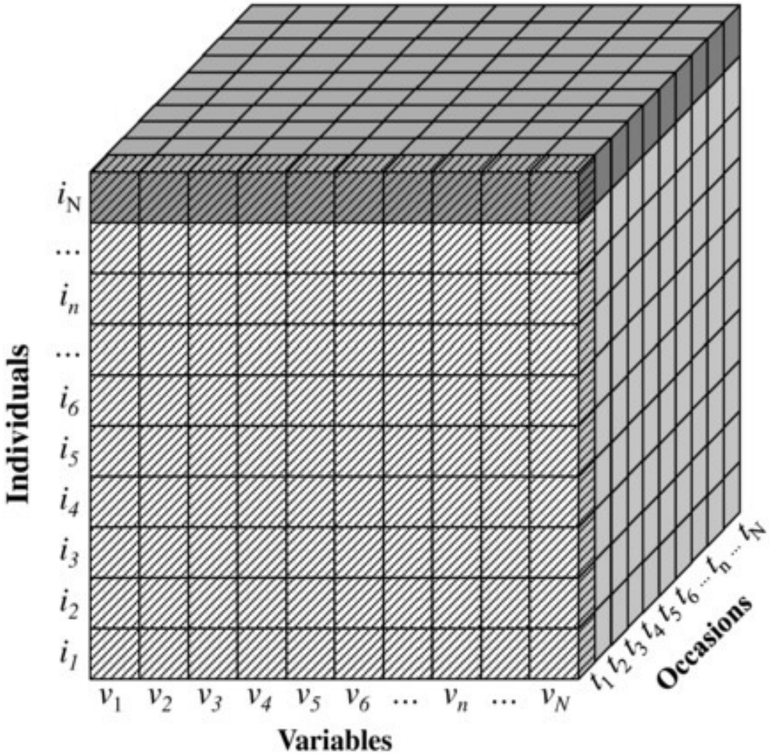
Cross-sectional versus time-series data



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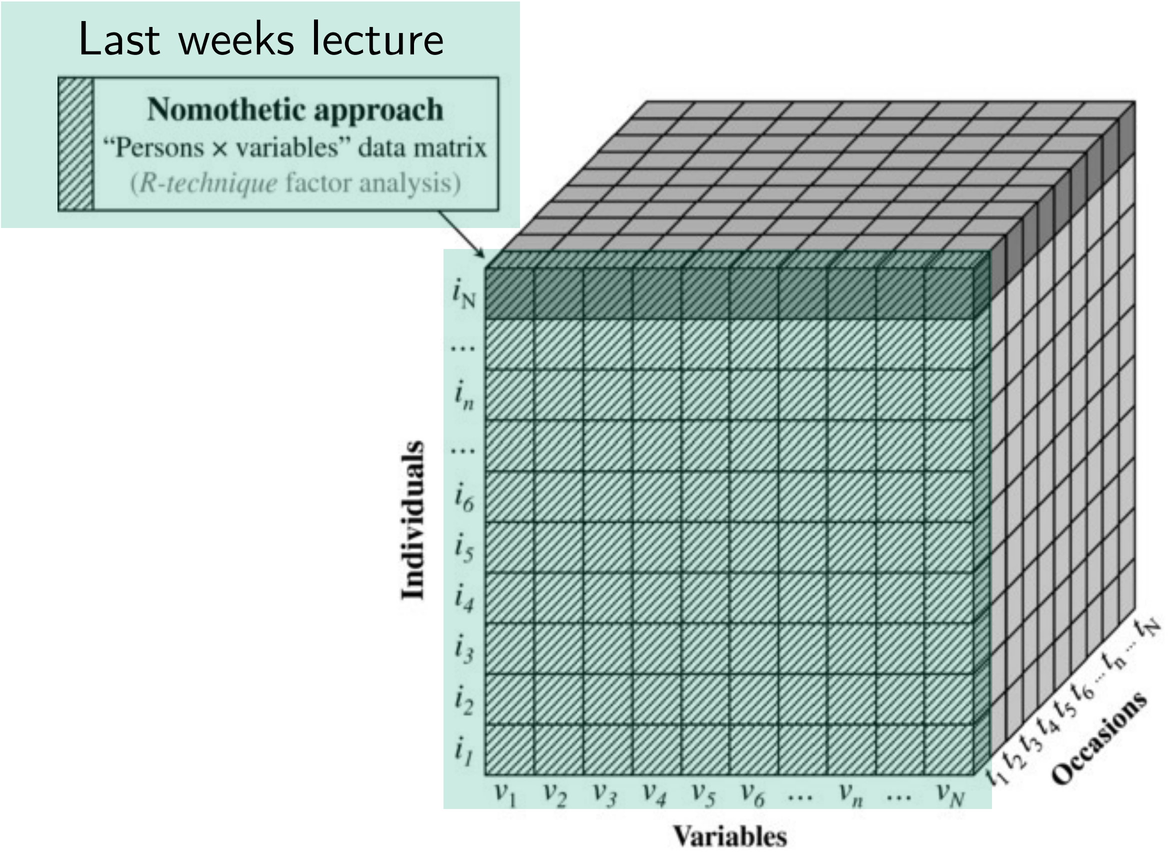

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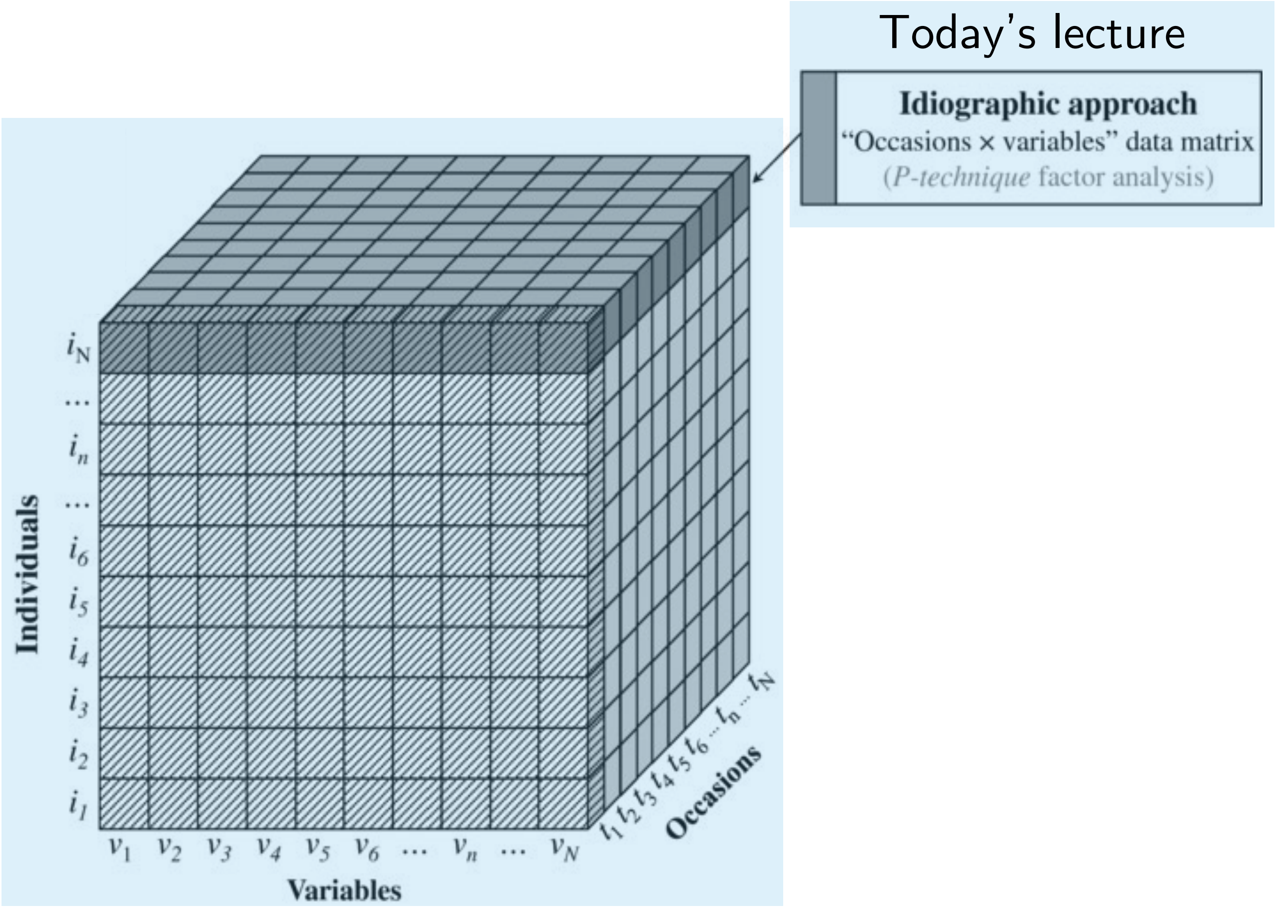

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What is time-series data?

- One or multiple individuals are measured multiple times

What is time-series data?

- Different forms of time-series data:
  - Daily Diary (DD)
  - Experience Sampling Method (ESM)
  - Ecological Momentary Assessment (EMA)

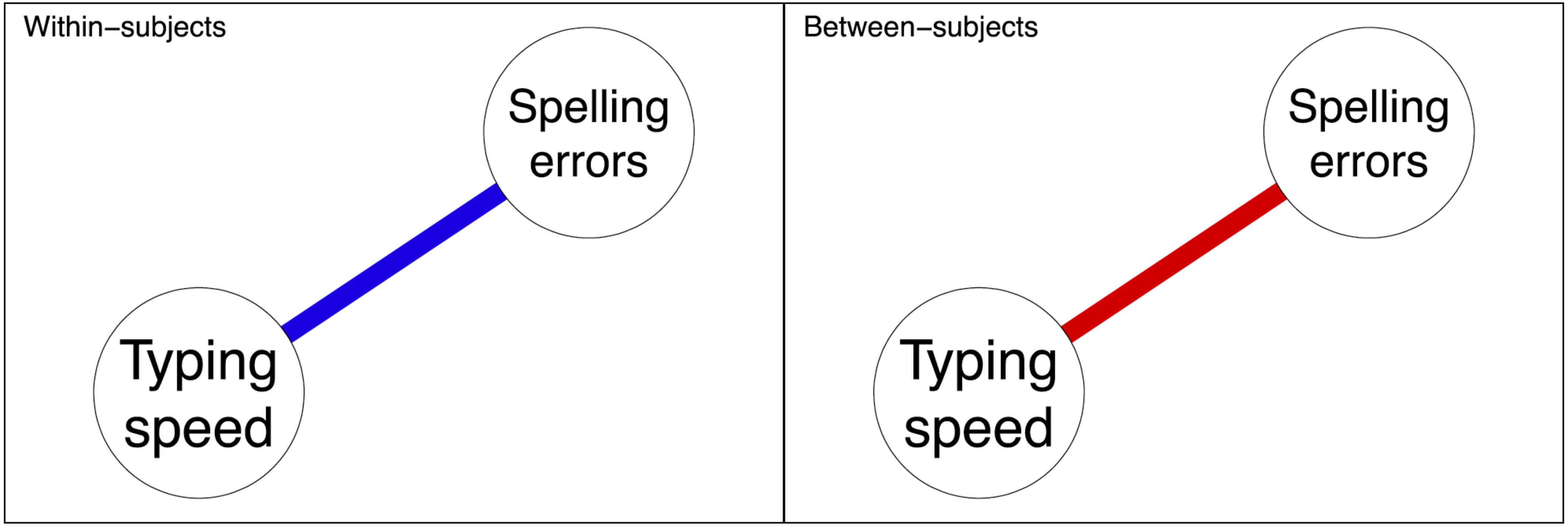
- Cases are not independent
  - There is a natural temporal ordering in the data
    - Knowing someone's mood at one time point helps predict their mood at a next time point
    - Observations closer together in time will be more closely related than observations further apart in time

Why time-series modeling?

- Gives insight into the **dynamic** relations between variables
  - Granger causality
  - Can be part of eHealth

- Difficult to translate results from cross-sectional analysis to individual
  - Results are only identical when a system is **ergodic**:
    - Stationarity
    - Homogenous
    - i.e., there are no between-person differences
  - Simpsons paradox

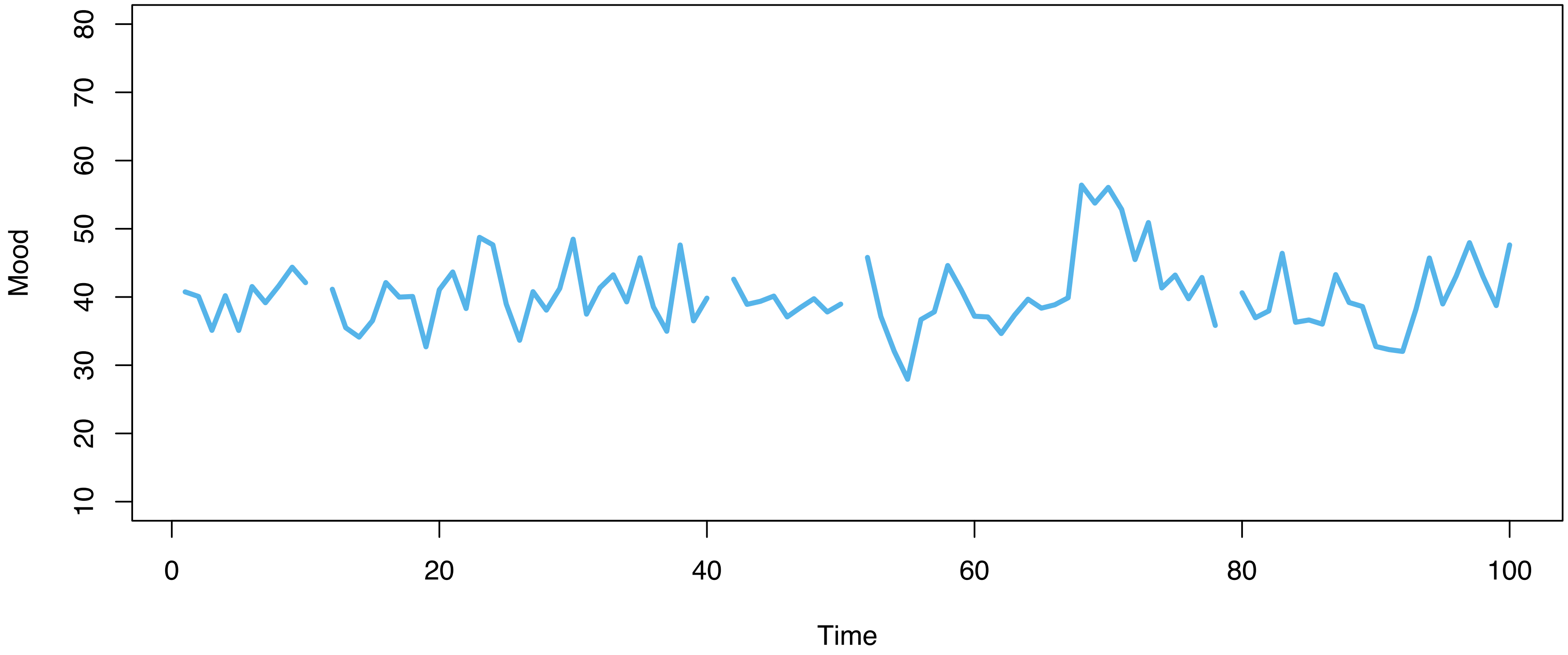
Simpsons Paradox



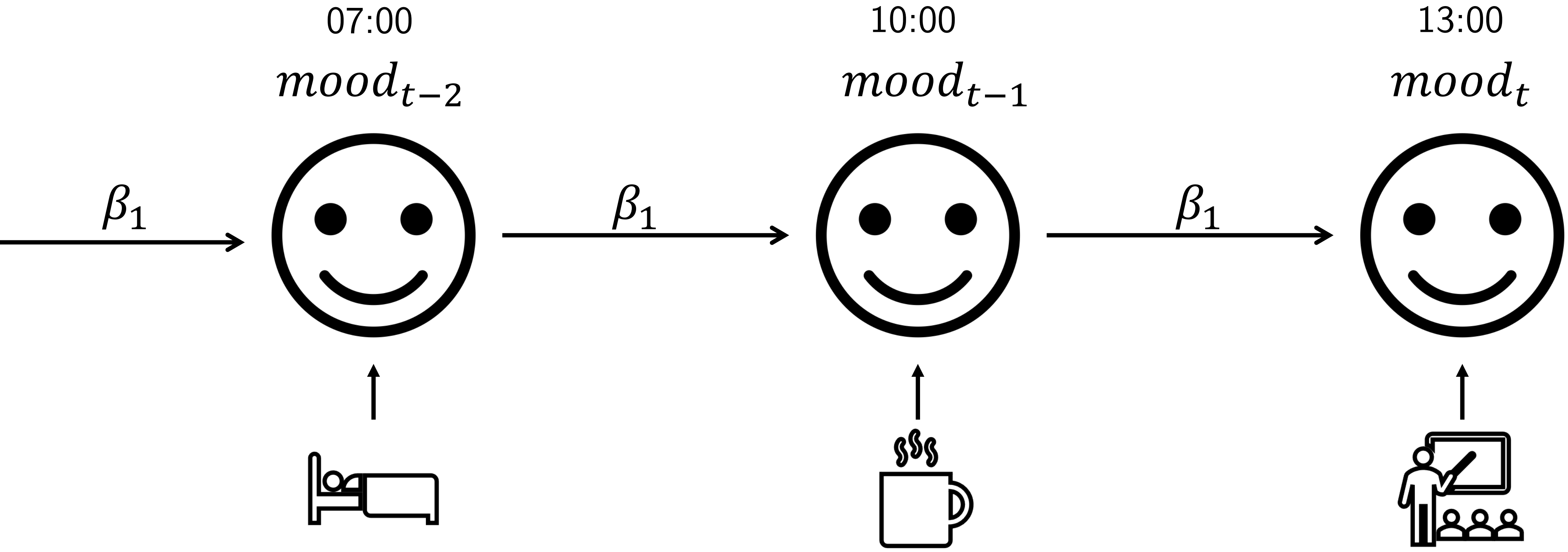
Example based on Hamaker, E. L. (2012). Why Researchers Should Think ‘Within-Person’: A Paradigmatic Rationale. Handbook of Research Methods for Studying Daily Life. The Guilford Press New York, NY, 43–61.

Modeling time-series data: $N = 1$ Networks

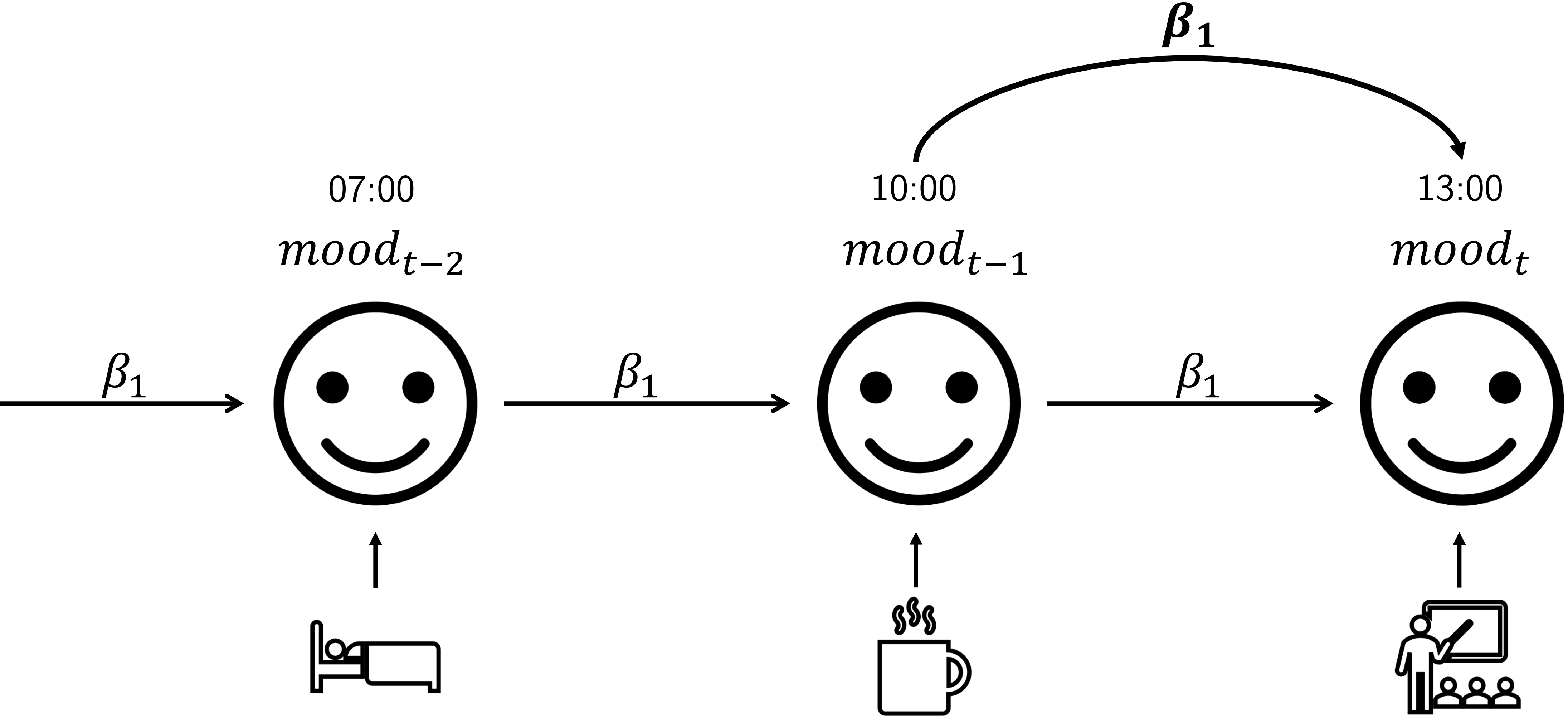
How to model **univariate** time-series?



Auto Regressive (AR) Model

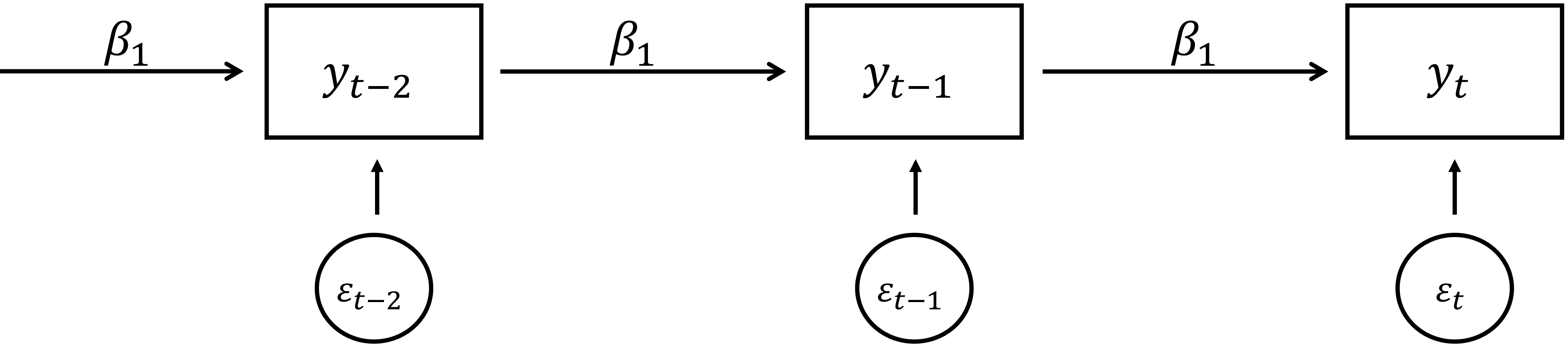


AR (1): Autoregressive **Lag 1** Model

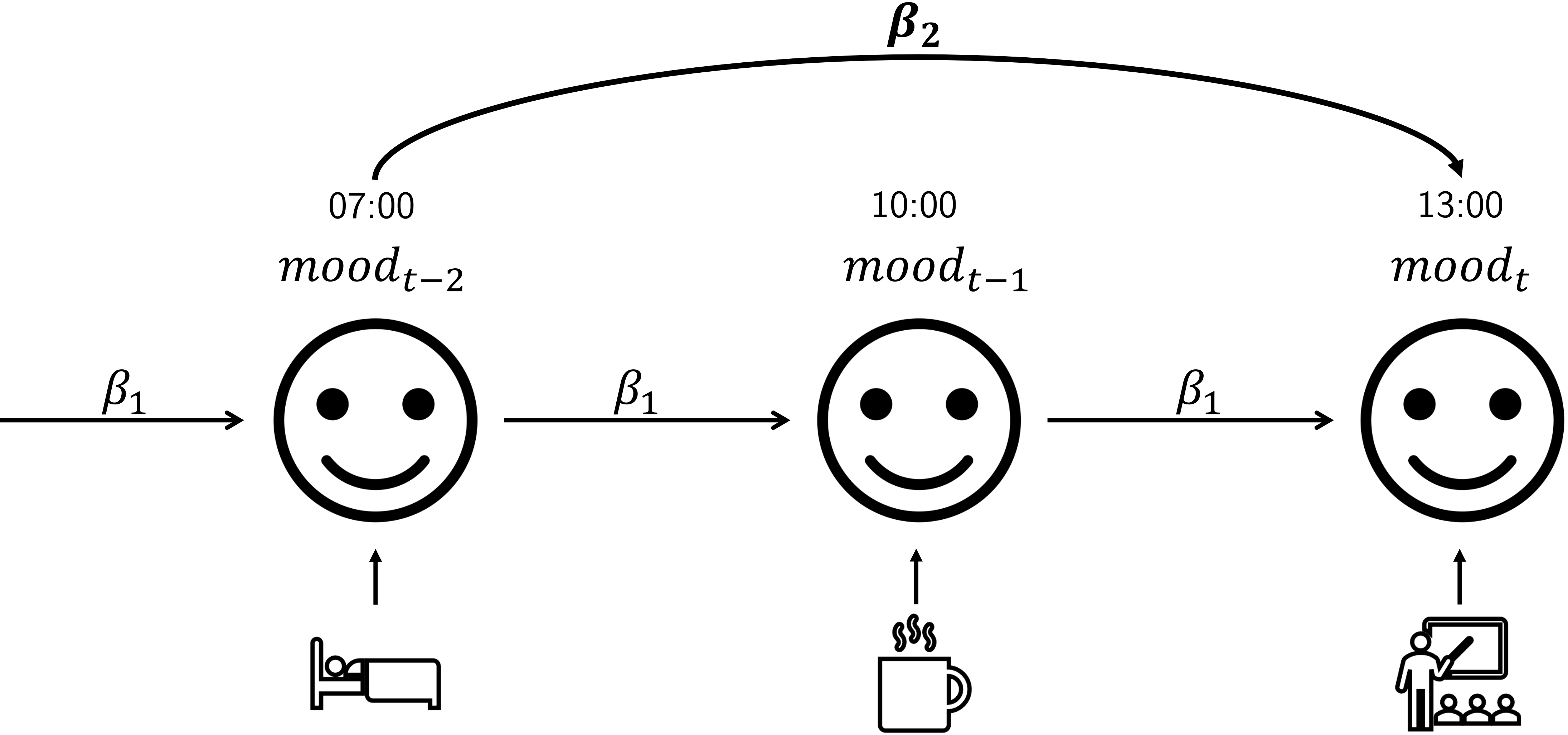


AR (1) Model:

$$y_t = \beta_1 y_{t-1} + \varepsilon_t$$

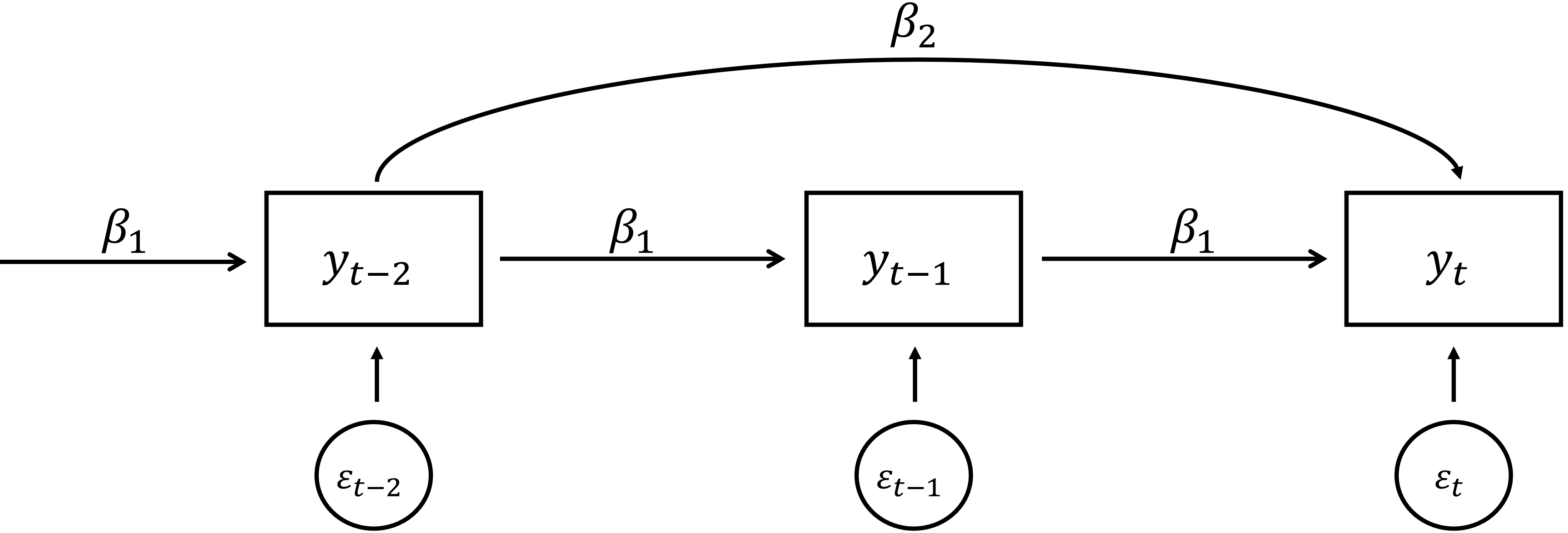


AR (2): Autoregressive **Lag 2** Model

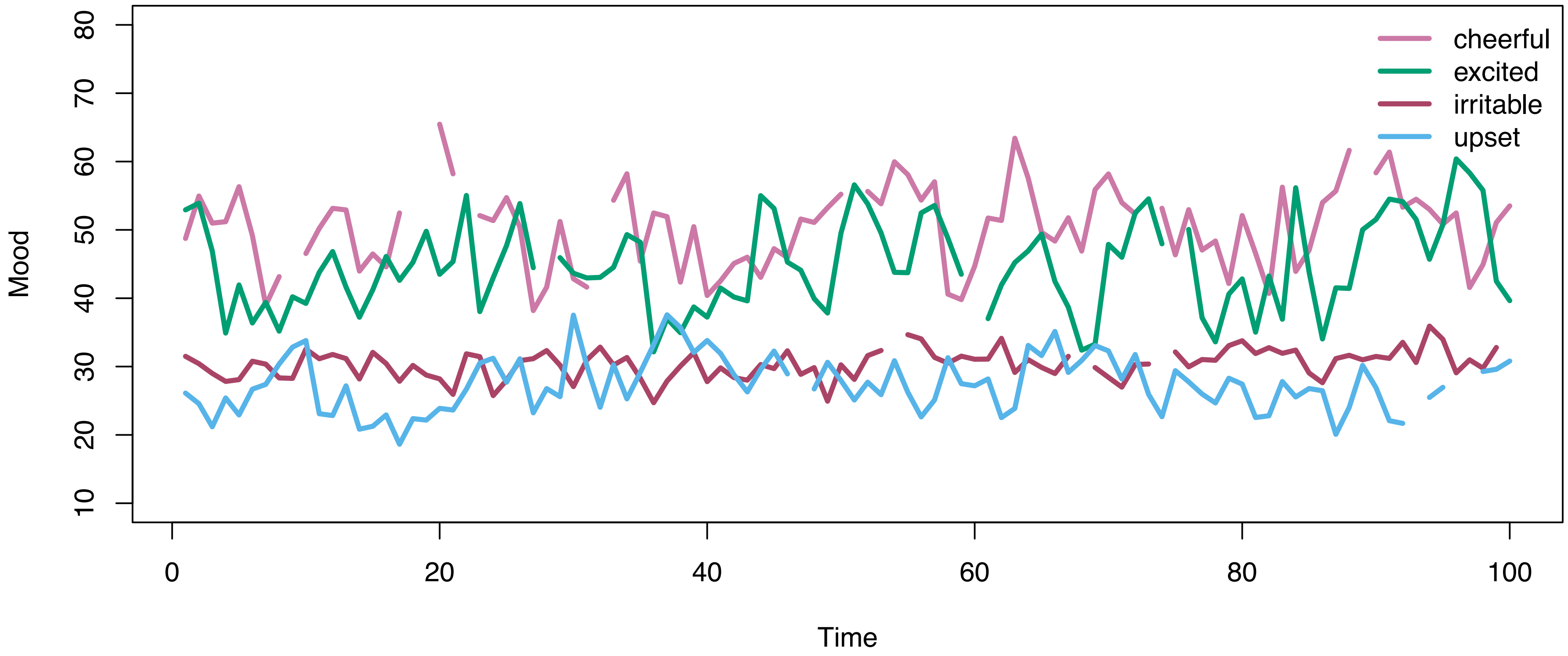


AR (2) model:

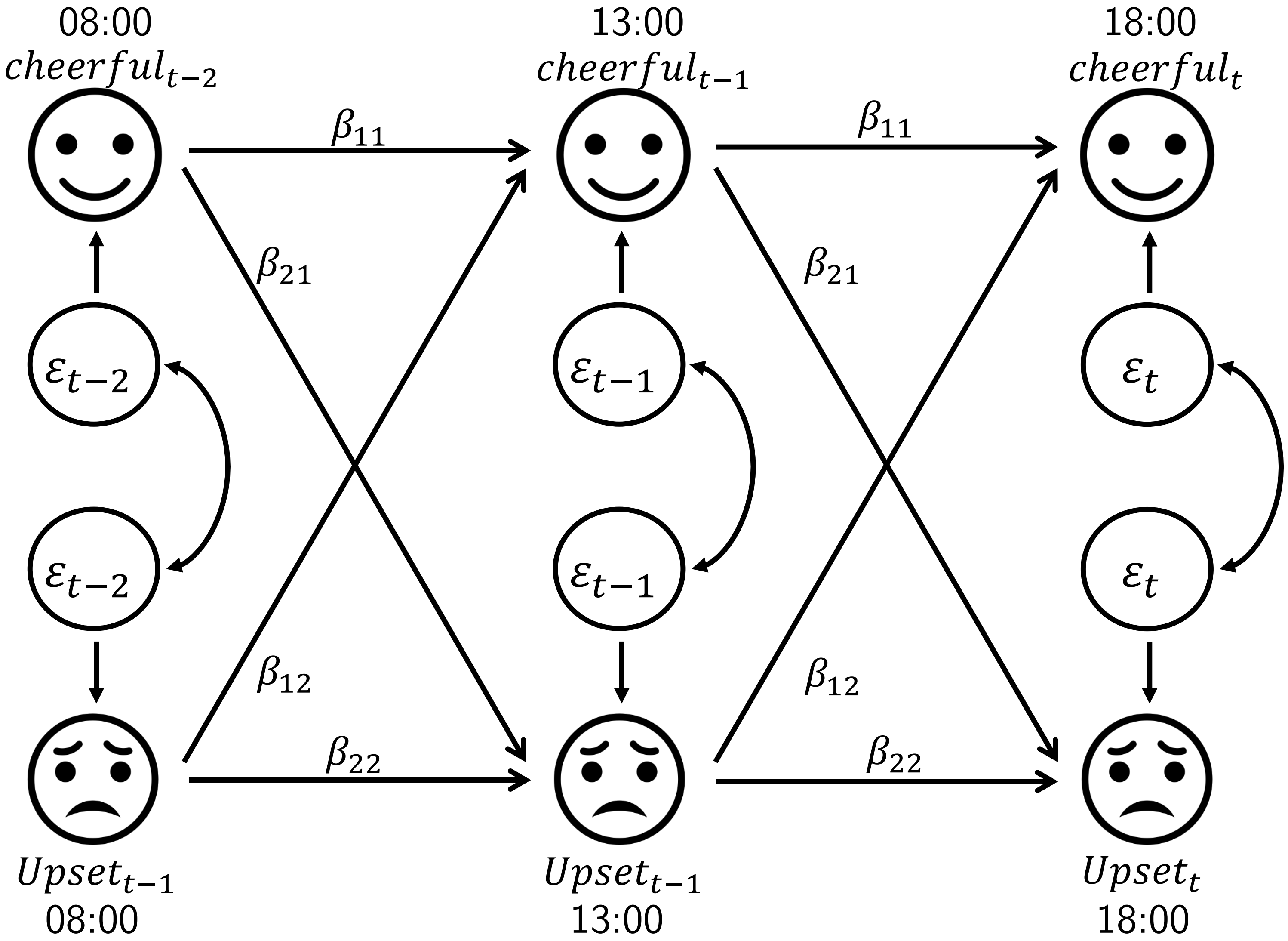
$$y_t = \beta_1 y_{t-1} + \beta_2 y_{t-2} + \varepsilon_t$$



How to model **multivariate** time-series?



Vector Auto Regressive (VAR) model



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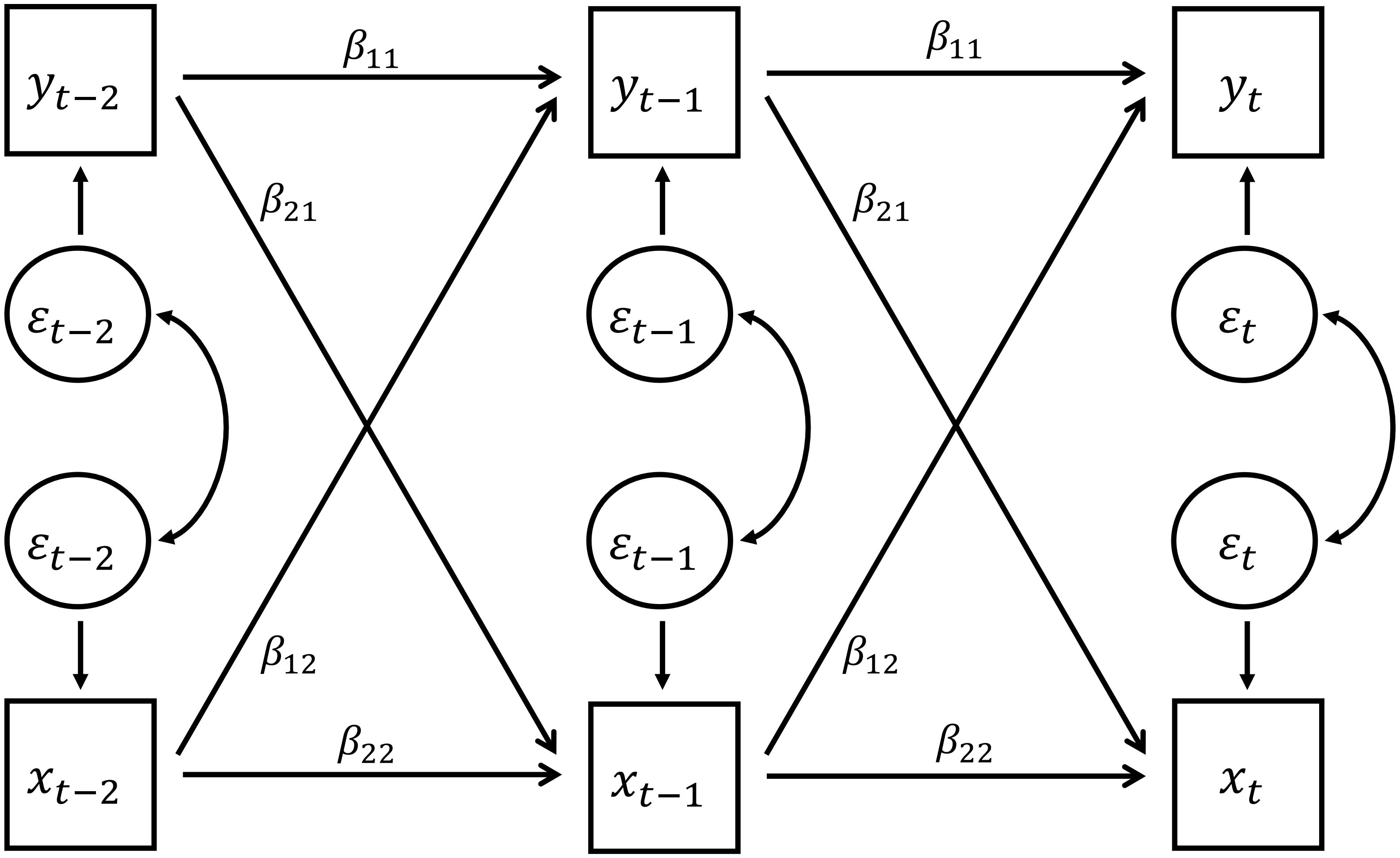

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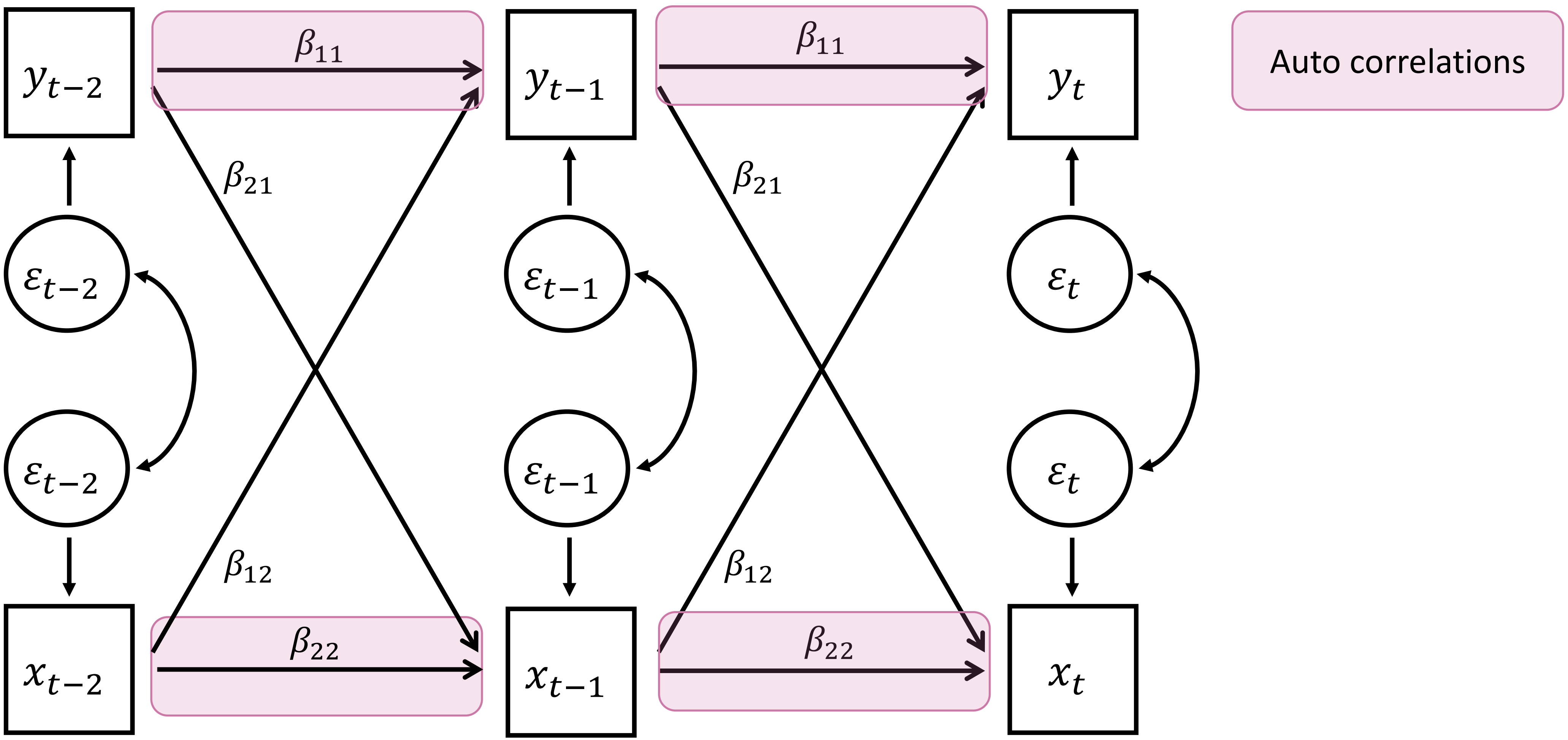

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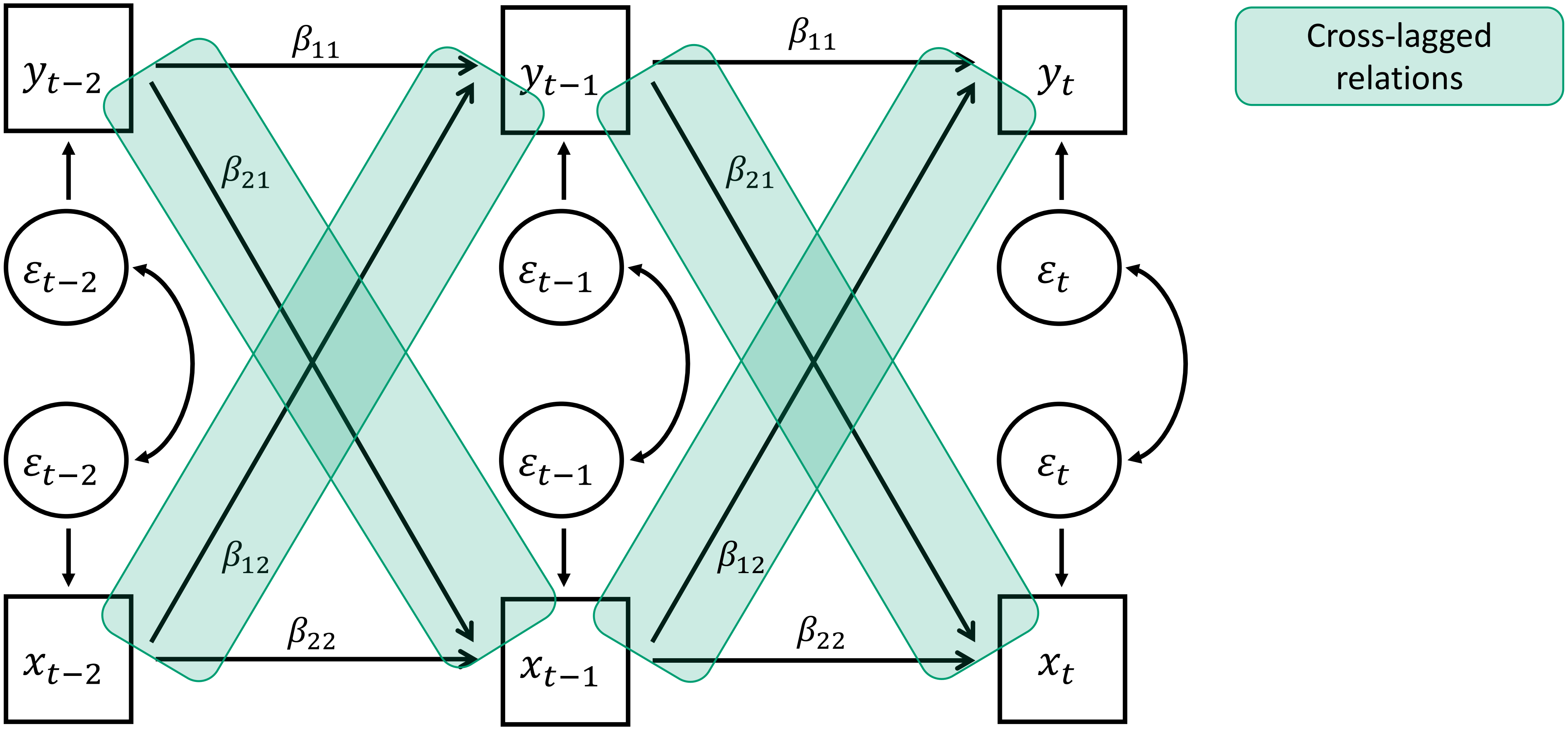

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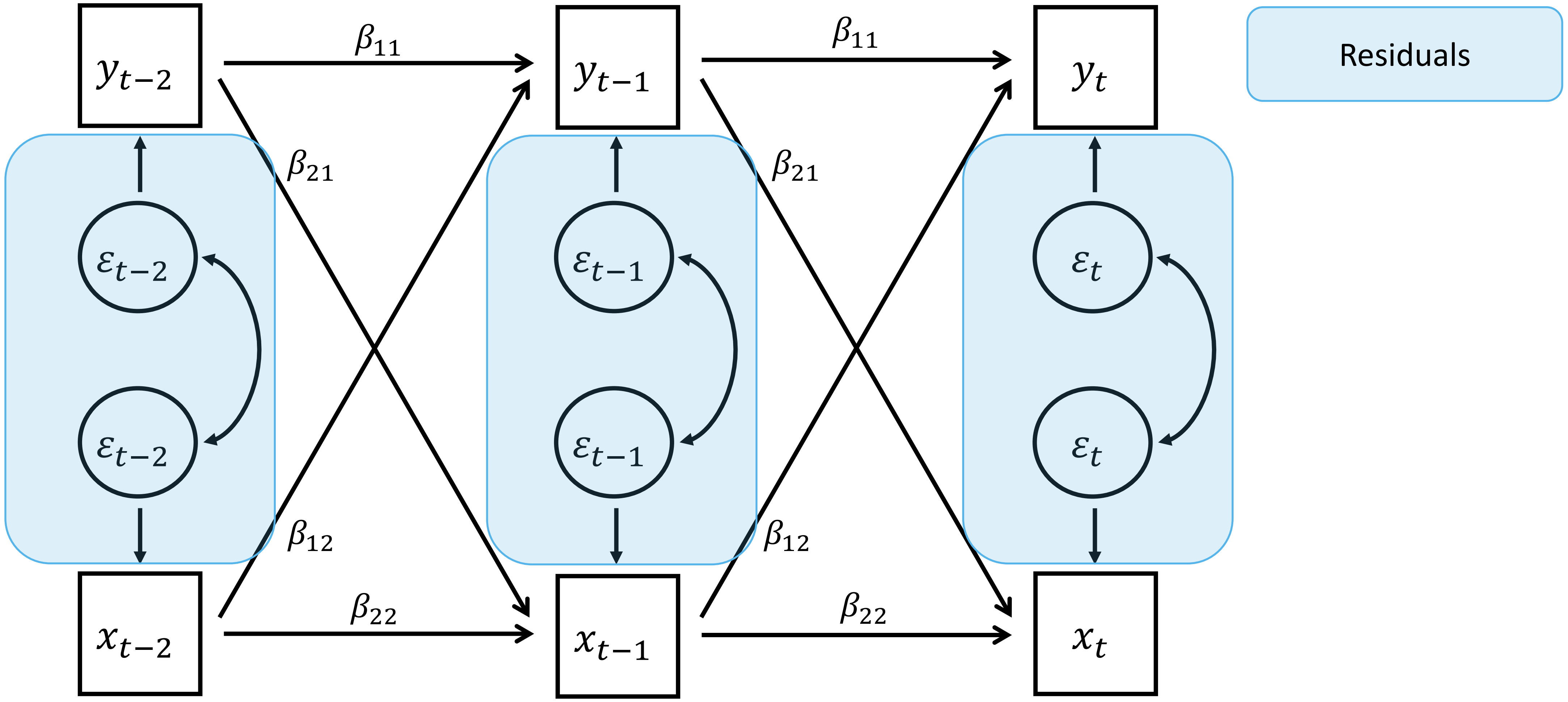

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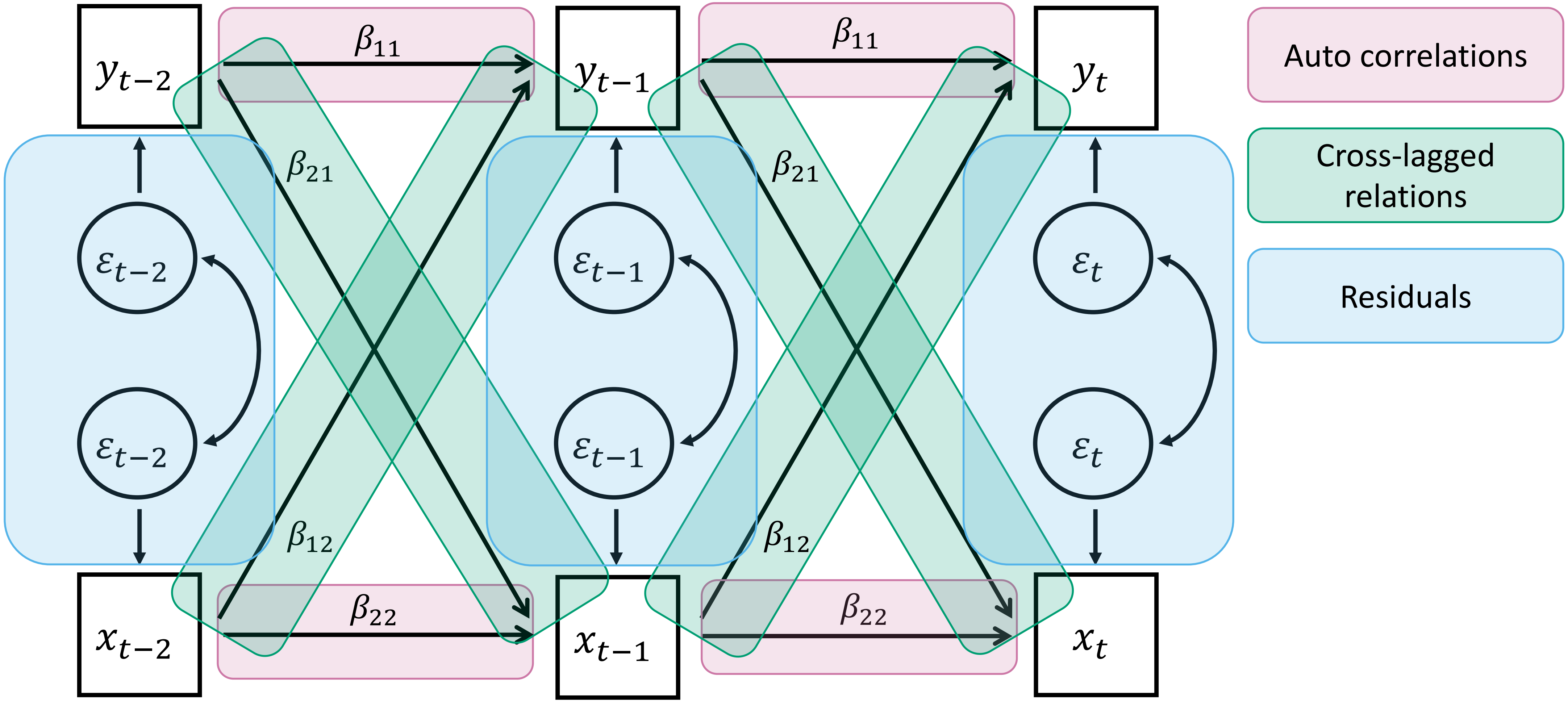

From VAR to **Graphical** VAR (GVAR)

- **Temporal network:** shows that one variable predicts another variable in the next measurement occasion
  - Granger causality

	relaxed	sad	nervous	concentration	tired	rumination	bodily.discomfort	time
31	5	6	4	5	6	4	4	2014-05-01 10:15:00
32	3	5	4	5	6	4	5	2014-05-01 13:15:00
33	NA	NA	NA	NA	NA	NA	NA	2014-05-01 16:15:00
34	3	5	4	5	6	4	4	2014-05-01 19:15:00
35	4	6	4	4	7	5	4	2014-05-01 22:15:00
36	4	3	3	5	5	2	3	2014-05-02 10:15:00
37	4	3	4	5	5	3	2	2014-05-02 13:15:00

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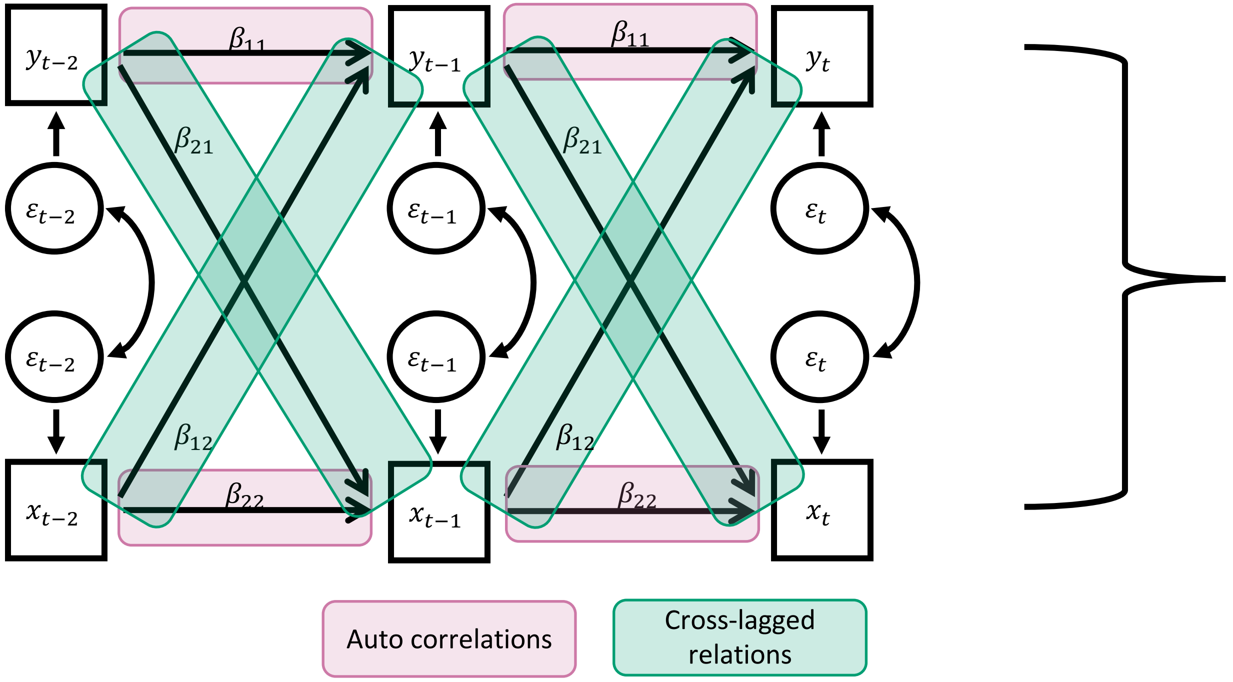

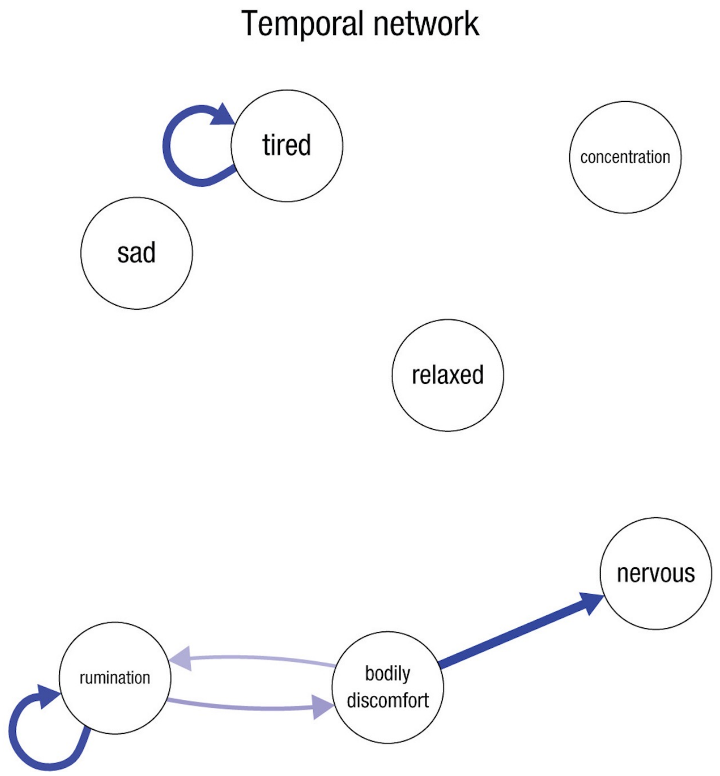

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From VAR to **Graphical** VAR (GVAR)

- **Contemporaneous network:** shows if two variables predict one another after taking temporal information into account
  - Contains *effects faster* than the time-window of measurement

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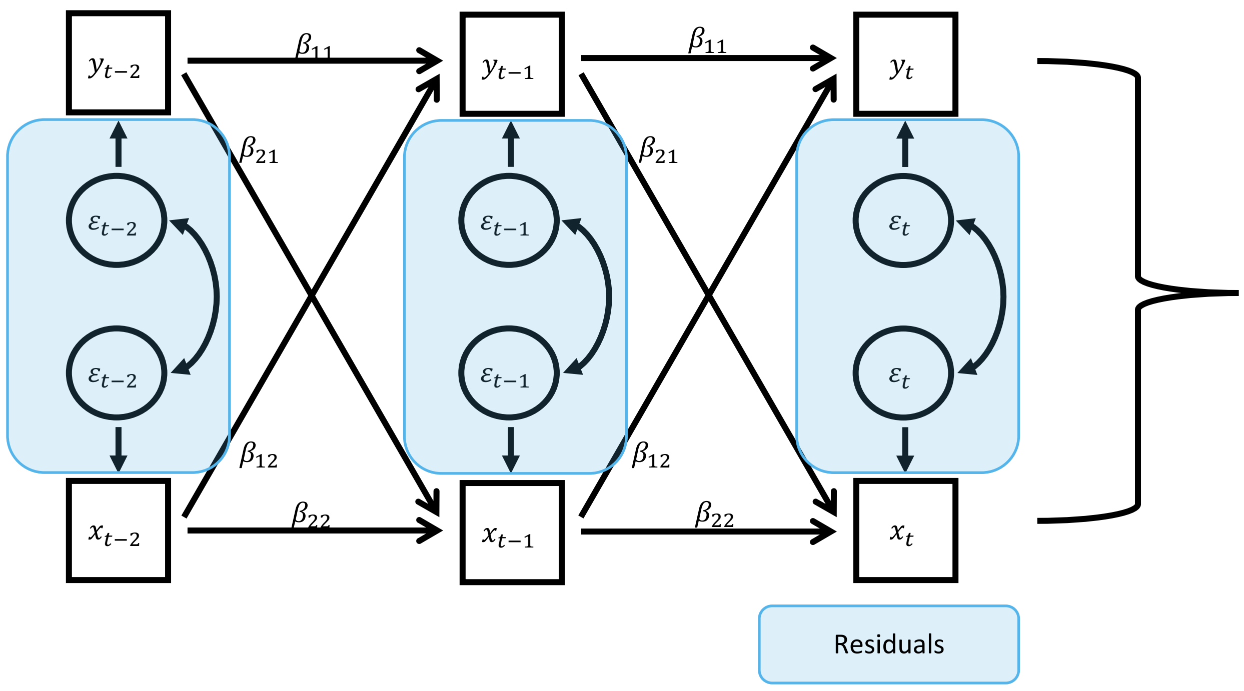

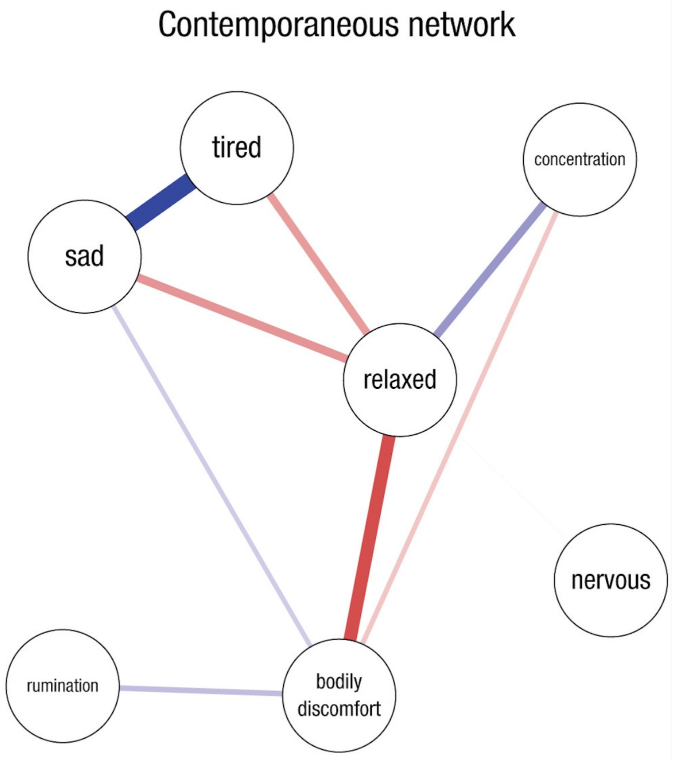

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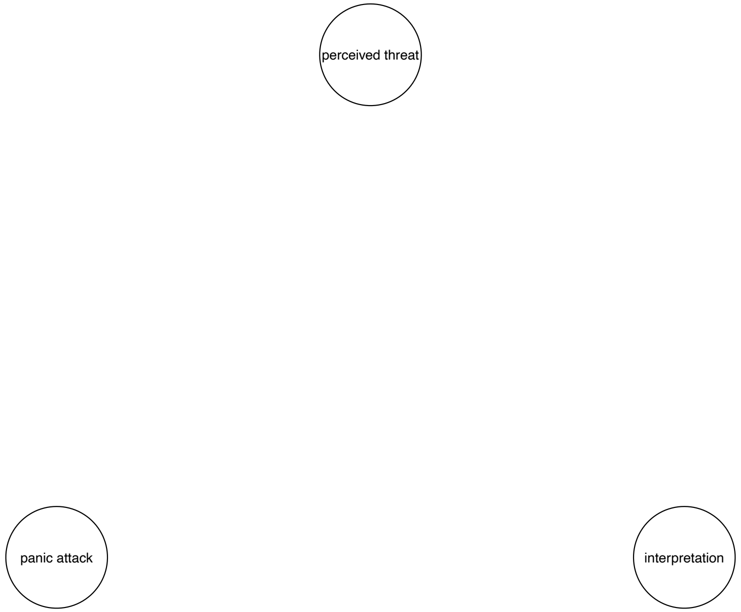
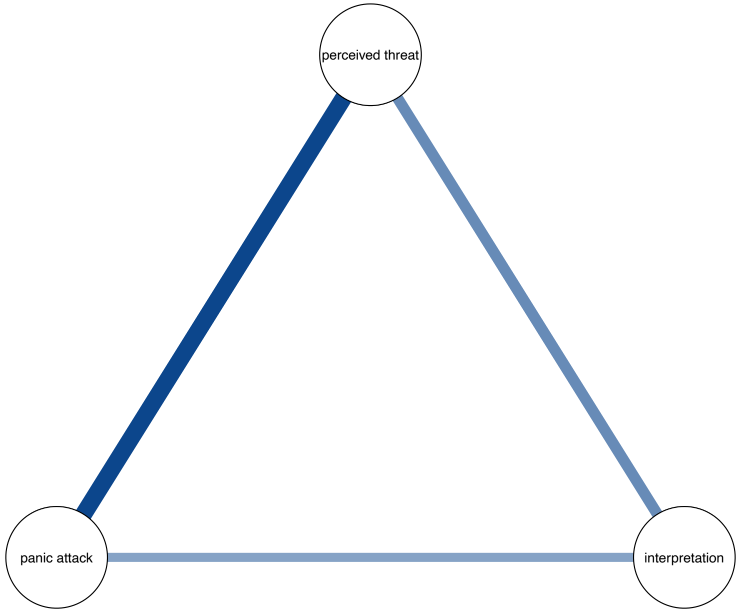
From VAR to **Graphical** VAR (GVAR)

- **Contemporaneous network:** shows if two variables predict one another after taking temporal information into account
  - Contains *effects faster* than the time-window of measurement
  - Example: Dynamics of panic disorder



Temporal network



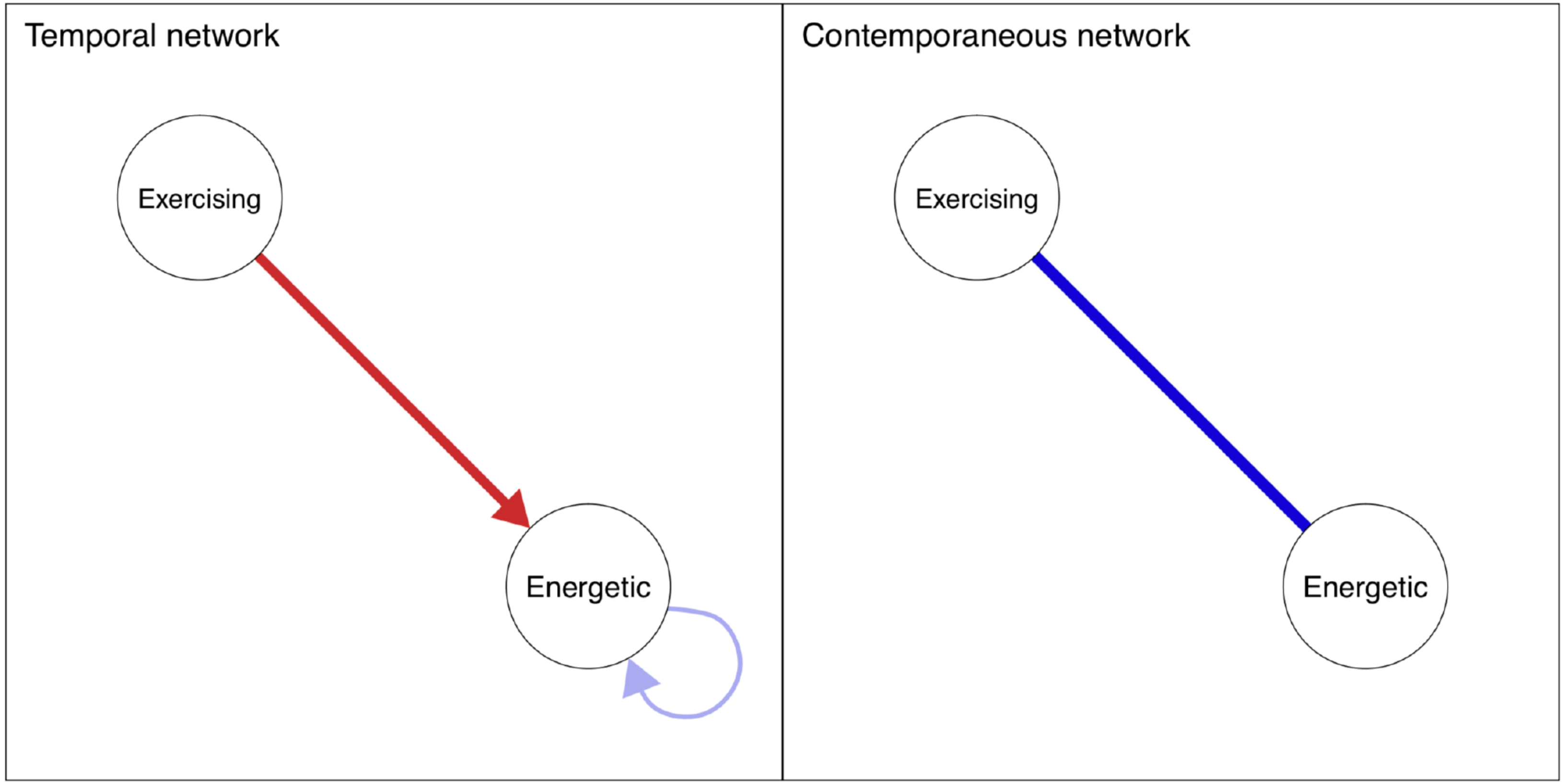
Contemporaneous network

From VAR to **Graphical** VAR (GVAR)

- **Contemporaneous network:** shows if two variables predict one another after taking temporal information into account
  - Contains *effects faster* than the time-window of measurement
  - Example: Dynamics of panic disorder

- Note: not equal to **lag (0)**
  - Takes the temporal information into account
  - GGM of residuals

Temporal versus contemporaneous effects



"Exercising now will lead me to be less energetic in the next time frame. I am energetic now, I will probably still be energetic in the next time frame."

"While I am exercising, I feel energetic."

Graphical VAR model:

$$\boldsymbol{y}_t = \boldsymbol{B}\boldsymbol{y}_{t-1} + \boldsymbol{\varepsilon}_t$$

$$\boldsymbol{\varepsilon}_t \sim N(0, \boldsymbol{\Theta})$$

- $\boldsymbol{y}$ represents a p-variate vector of scores on p measures for an individual at time t
- $\boldsymbol{B}$ is the p × p matrix containing the lagged regression coefficients
    - encodes the **temporal network**
- $\boldsymbol{\varepsilon}$ is a p-variate vector of residuals with a mean of zero and a time-invariant p × p dimensional covariance
- $\boldsymbol{K} = \boldsymbol{\Theta}^{-1}$
    - encodes the **contemporaneous network**

Graphical VAR estimation

- Estimation without regularization/model selection is possible by using multiple regression
- Estimation with regularization: finding sparse solutions for $\boldsymbol{B}$ and $\boldsymbol{\Theta}^{-1}$
  - LASSO estimation with EBIC model selection implemented in the `graphicalVAR` package
    - Missings can be handled using the Kahlman filter in the `impute.ts` package
  - Stepwise non-regularized estimation implemented in the `psychonetrics` package
    - Includes full information maximum likelihood (FIML) for missing data handling

Graphical VAR estimation in R using `graphicalVAR` package

`graphicalVAR()`

- `gamma`: EBIC hyper-parameter
- `vars`: vector of variables to include in the analysis
- `beepvar`: string indicating assessment beep per day
- `dayvar`: string indicating assessment day

Graphical VAR estimation in R using `graphicalVAR` package

```
res_gvar <- graphicalVAR(data, gamma = 0, vars =
vars, dayvar = dayvar, beepvar = beepvar)
```

- Estimated **temporal** network:

  ```
  res_gvar$PDC
  ```

- Estimated **contemporaneous** network:

  ```
  res_gvar$PCC
  ```

Graphical VAR estimation in R using `psychonetrics` package

- Specify model:

```
mod <- gVAR(data, vars = vars, dayvar = dayvar, beepvar = beepvar,
estimator = "FIML")
```

- Unregularized model estimation:

```
mod <- mod %>% runmodel()
```

- "Regularization" i.e., pruning the model:

```
mod <- mod %>% prune(alpha = 0.05, recursive = FALSE, runmodel = FALSE)
```

- Evaluate model fit:

```
mod %>% fit()
```

- Estimated **contemporaneous** network:

```
mod %>% getmatrix("omega_zeta")
```

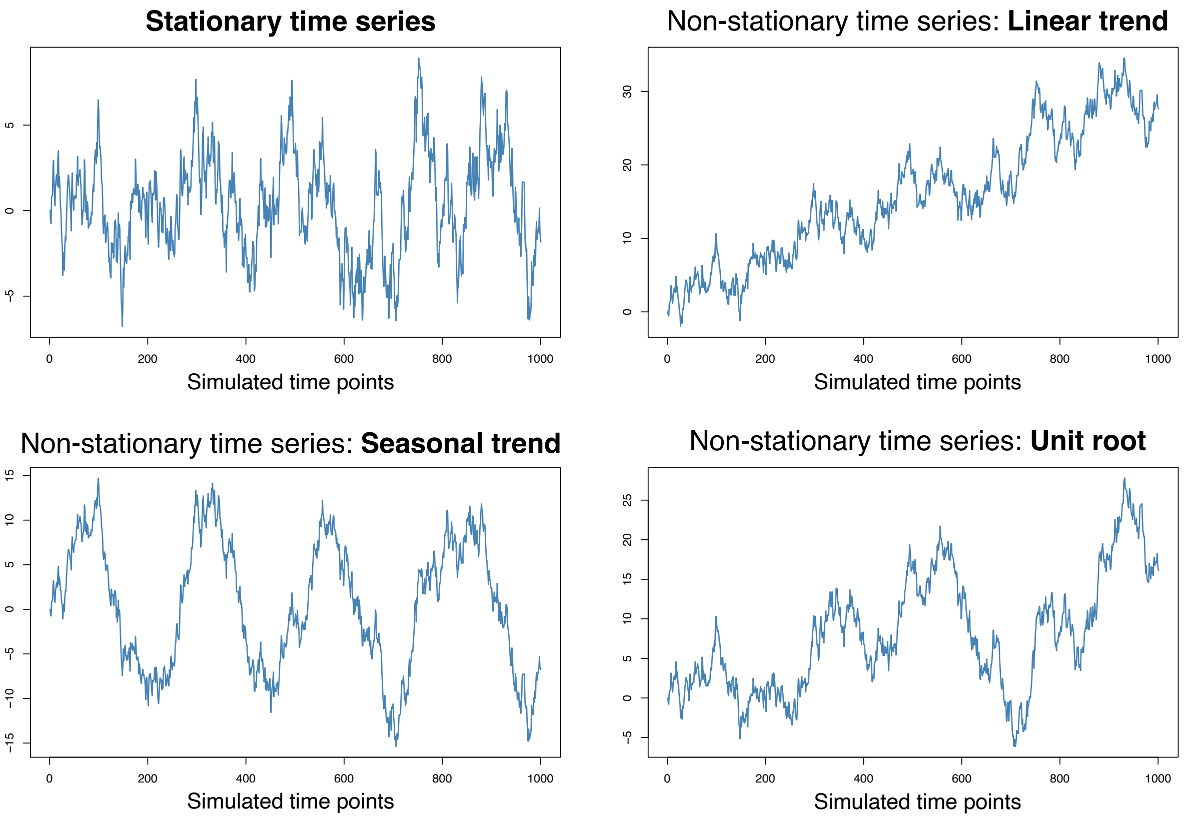
- Estimated **temporal** network:

```
mod %>% getmatrix("PDC")
```

Note: different pruning options here such as FDR-thresholding, greedy set up search and within each of these pruning options you can make different choices again such as correcting for multiple testing

Assumptions of VAR (1): **stationarity**

- We speak of stationary time series when statistical properties such as mean, variance, autocorrelation, etc. are all *constant* over time



Assumptions of VAR (1): **stationarity**

- We speak of stationary time series when statistical properties such as mean, variance, autocorrelation, etc. are all *constant* over time
  - Plausible when data is obtained within a short time span but is often not what people are interested in
  - Can be assessed by regressing each time-series on itself as predictor
    - `kpp.test(data)` in R

- Solution:
  - Detrending is possible: for example you can remove linear trends from the data
    - However often it is these trends that people are interested in
  - Non stationary network model estimation such as time-varying network models (Haslbeck, Bringmann, & Warldorp, 2021)

Assumptions of VAR (1): **stationarity**

- Check for linear trend in R
  - Regress variable var1 on time:

```
lm_var1 <- lm(var1 ~ time, data = data)
Summary(lm_var1)
```

  - If significant, detrend data and take residuals as data:

```
Data$var1[!is.na(data$var1)] <- residuals(lm_var1)
```

Note this is not a magic trick, there can still be non-linear trends in your data, or trends in your means, variances etc.

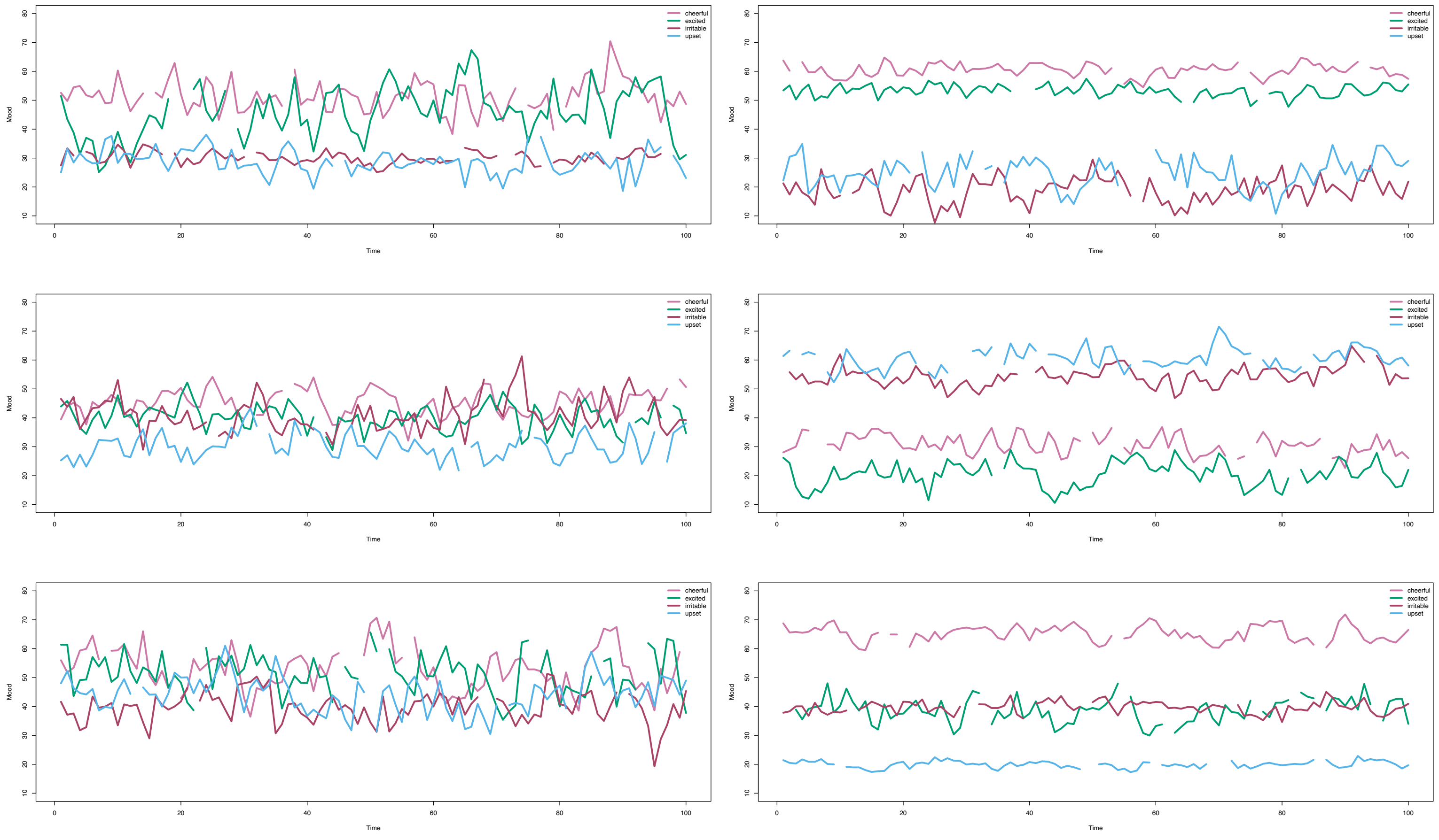
Assumptions of VAR (2): **Equidistant measures**

- Equal distance between measures
  - When measuring over multiple days this assumption is by definition violated due to the nights

- Possible solutions:
  - Remove nights, or model nights as missing observations (can be done in `graphicalVAR` package)
  - Continuous time-series modeling (Ryan & Hamaker, 2022)

Modeling time-series data: $N > 1$ Networks

How to model **multivariate** time-series for **multiple** people?



Extend VAR for multiple people

- With time-series data of multiple persons, a GVAR model could be estimated per person
  - $N = 1$ analyses are typically low-powered → might be lacking the power to pick up effects in the data
- Multilevel VAR modeling allows us to use all information in a single analysis
  - Instead of many under powered analyses → 1 powerful analysis

Why multilevel VAR modeling?

- Several reasons:
  - People are similar, people are different
  - Borrowing information from group-level data
  - Separate *between* from *within* variance
  - Requires fewer observations (compared to averaging the parameters for each individual network)

- Shrinkage
  - Estimates of individuals are somewhat “pulled” together

- Less parameters need to be estimated
  - Only fixed effects and variance–covariance of the random effects need to be estimated

How to extend the VAR model over multiple people?

- We add an extra level to the measurements

- Level 1:
  - Variation *within* an individual over time
  - **Random effects**

- Level 2:
  - Variation *between* individuals over time
  - **Fixed effects**

Multilevel VAR model

- **Level 1:** Each individual is assumed to have their own temporal and contemporaneous VAR model:

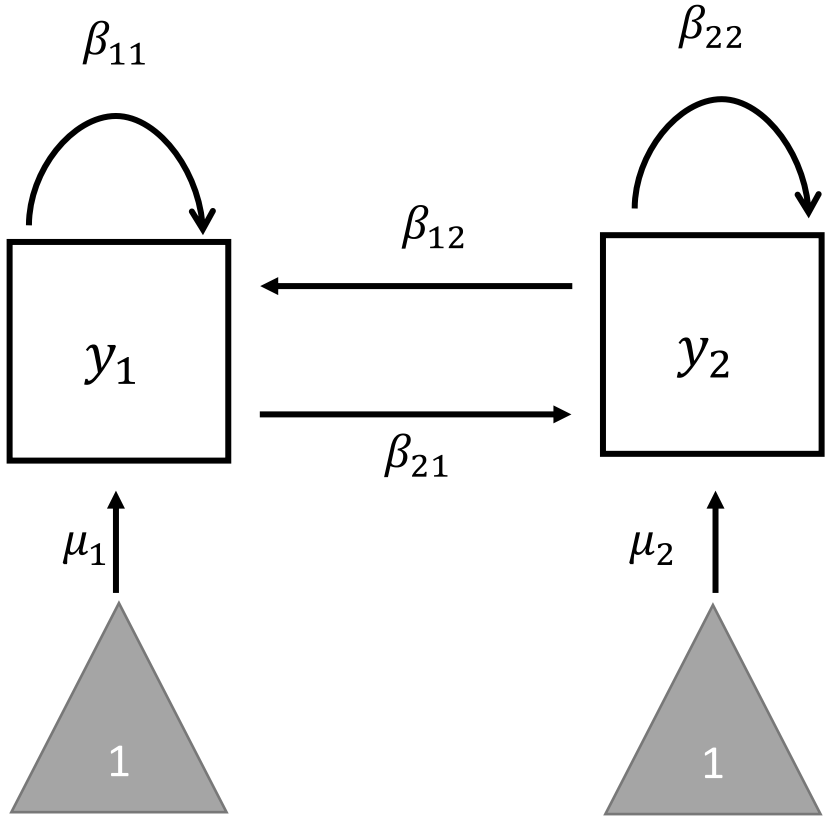
$$\boldsymbol{Y}_t^{(p)} | \boldsymbol{y}_t^{(p)} = N(\boldsymbol{\mu}^{(p)} + \boldsymbol{B}^{(p)}\left(\boldsymbol{y}_{t-1}^{(p)} - \boldsymbol{\mu}^{(p)}\right), \boldsymbol{\Theta}^{(p)})$$

- **Level 2:** Between-subject effects model:

$$\begin{bmatrix} \boldsymbol{\mu}^{(p)} \\ Vec(\boldsymbol{B}^{(p)}) \end{bmatrix} \sim N(\boldsymbol{f}, \boldsymbol{\Omega})$$

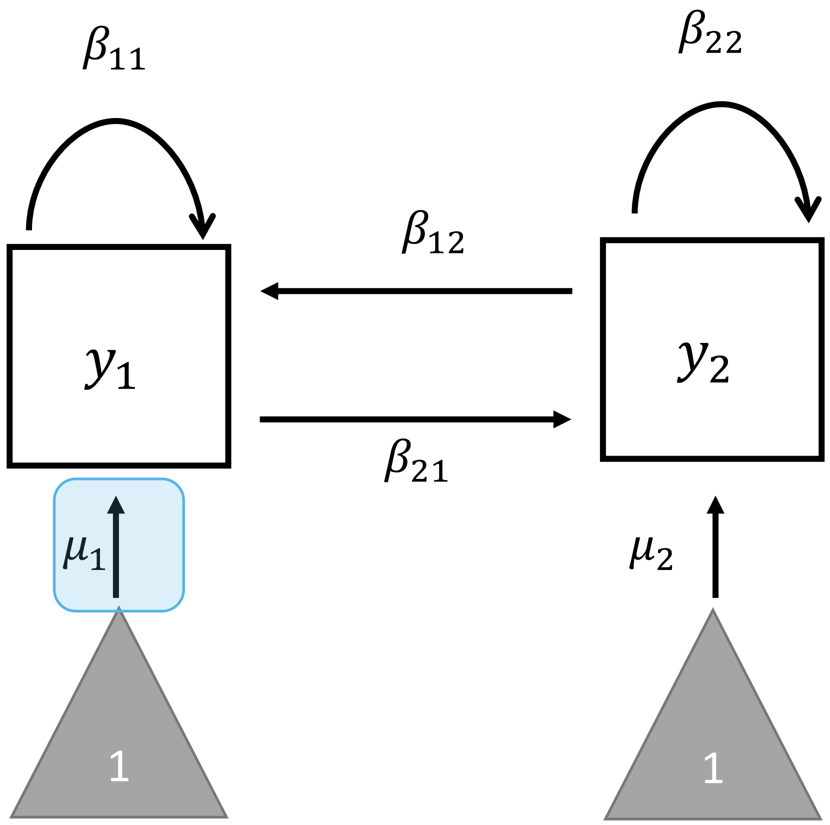
- $\boldsymbol{f}$ encodes fixed effects and Ω the distribution of random effects

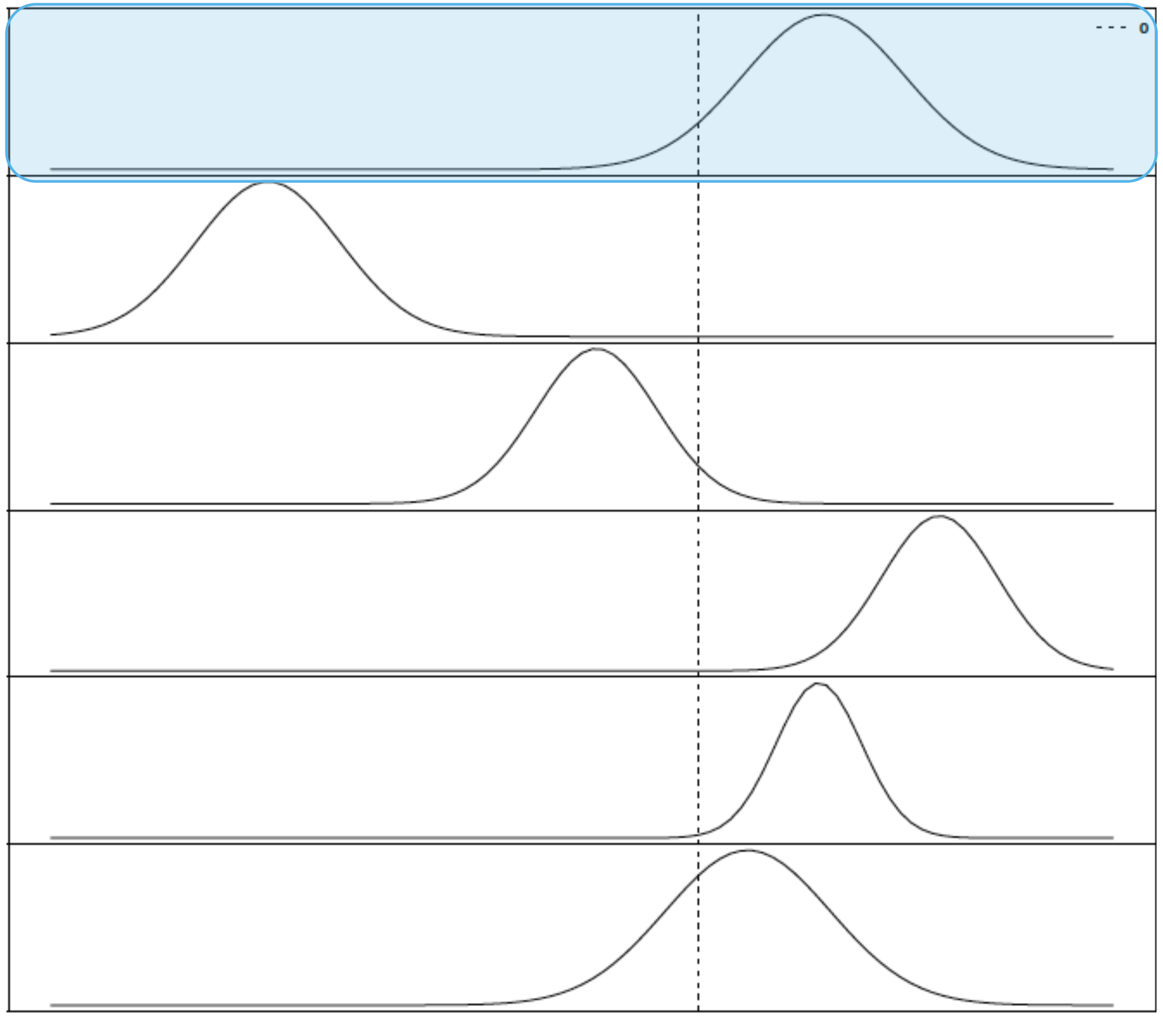
Multilevel VAR model: Basic idea



- Each individual has their own **mean** and **(cross-)lagged** correlations
- Individual parameters are sampled from a **shared probability distribution**

Each parameter has a distribution





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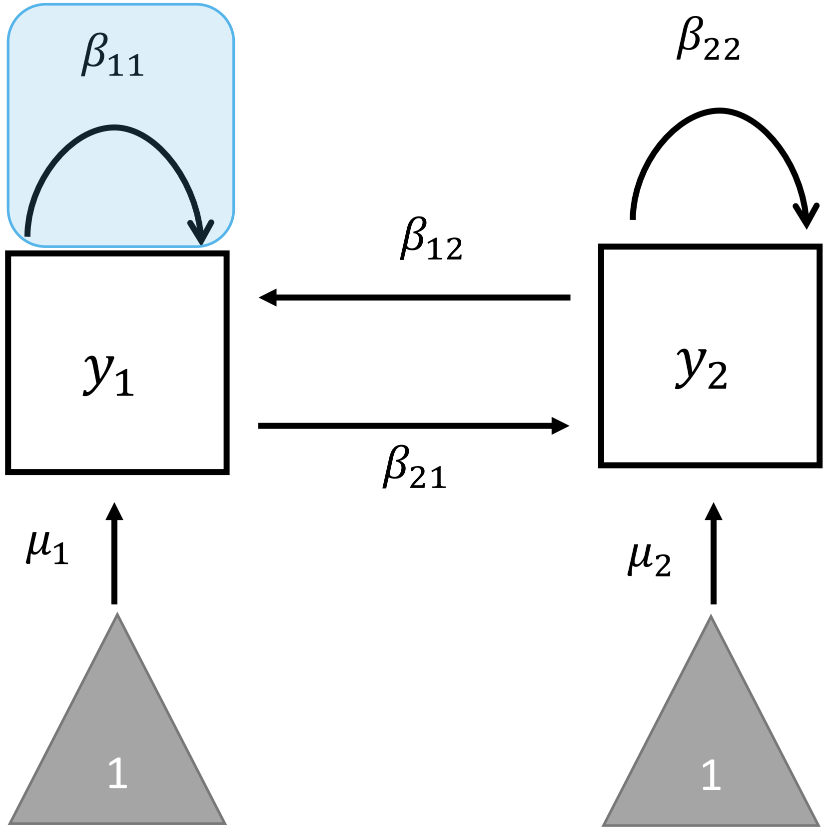

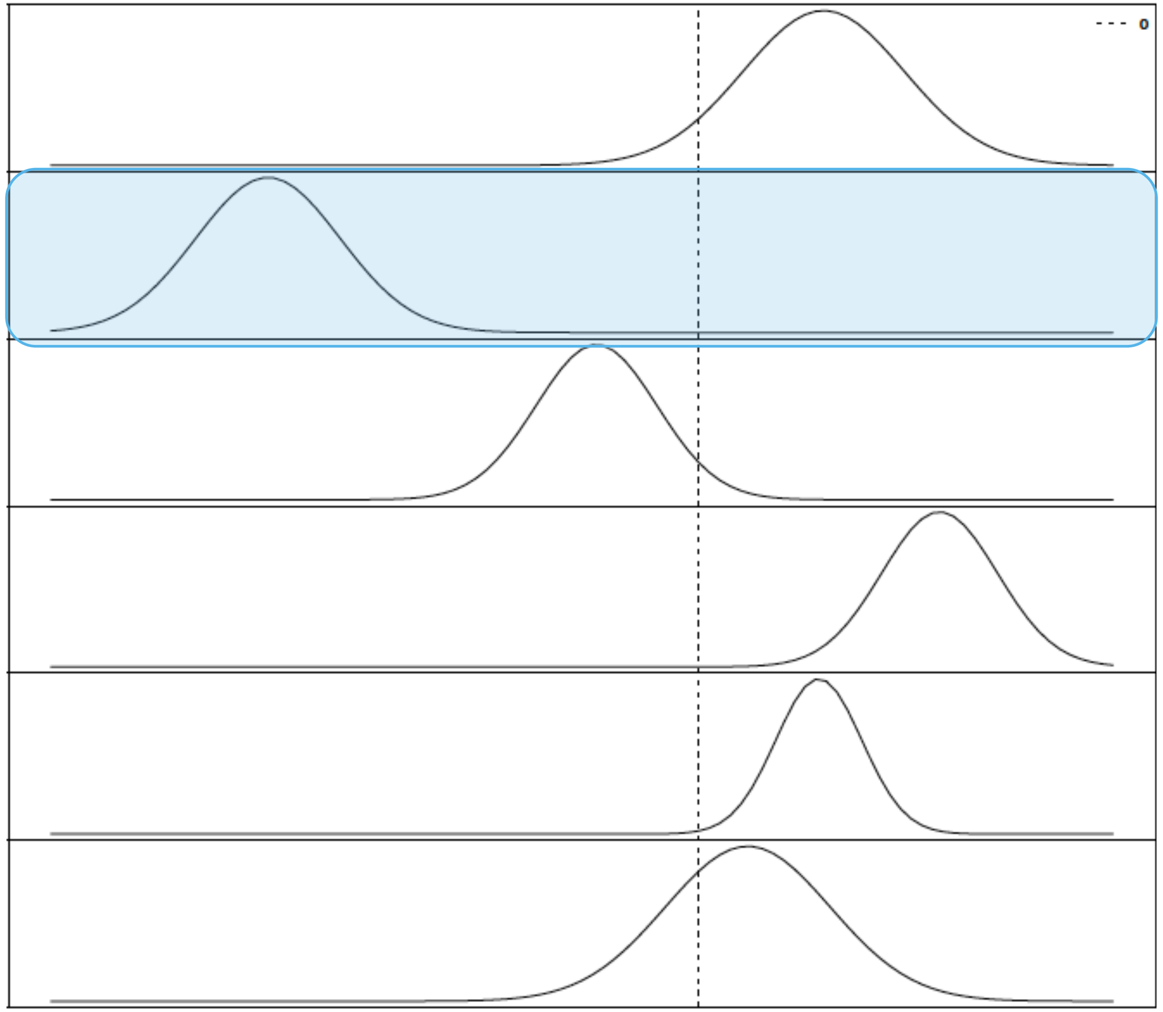

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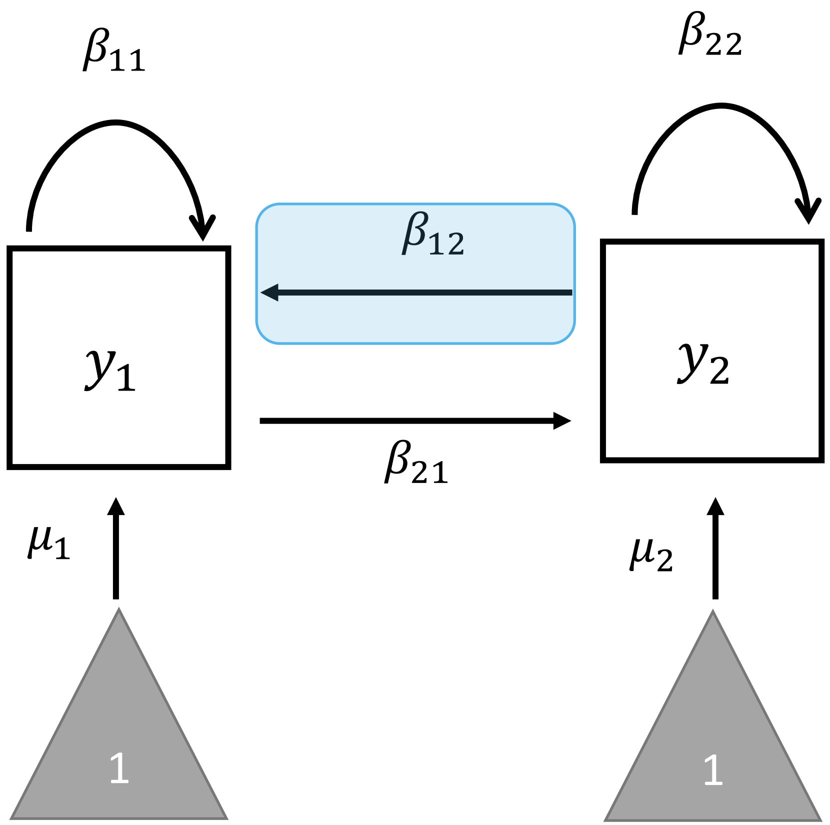

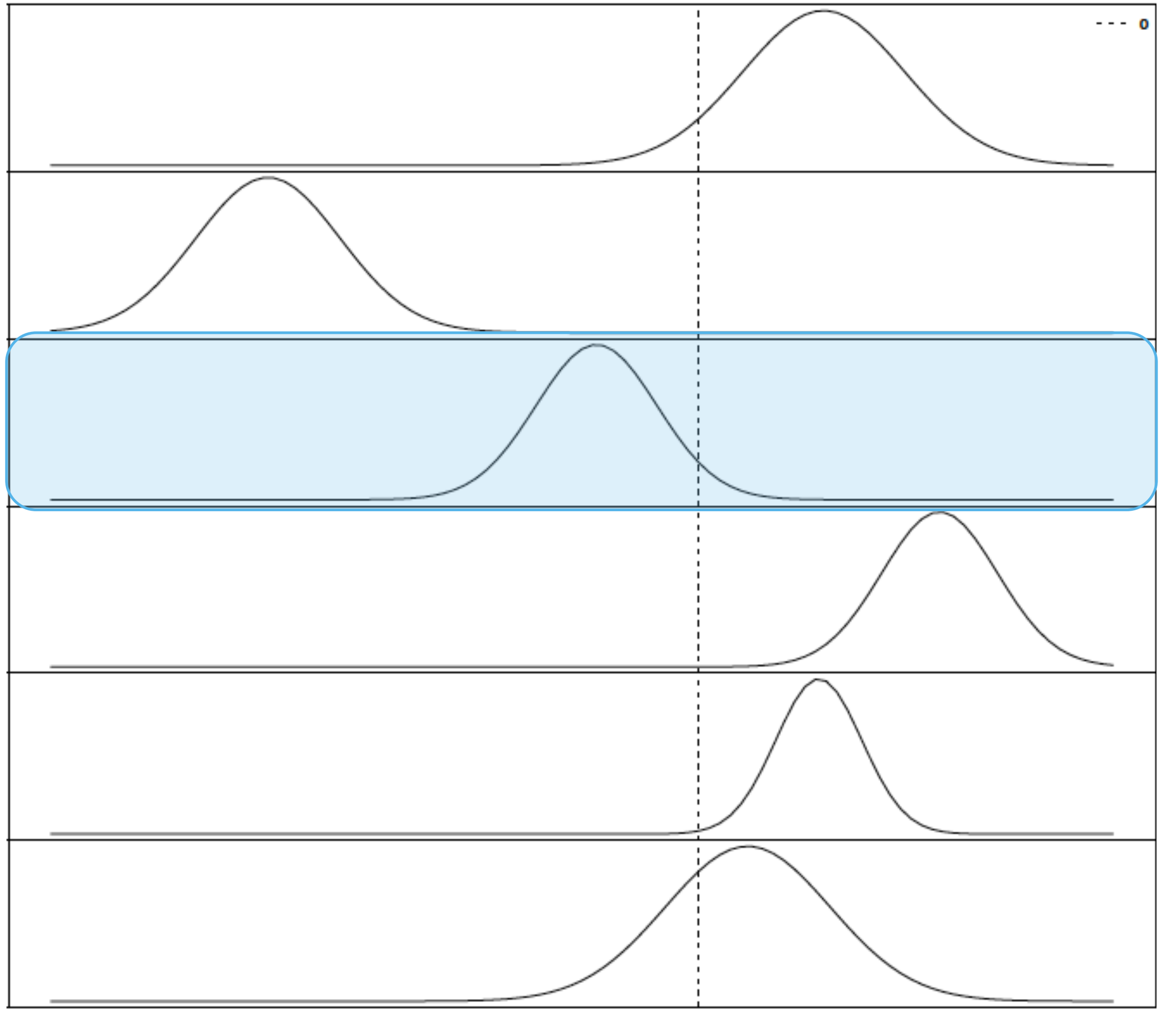

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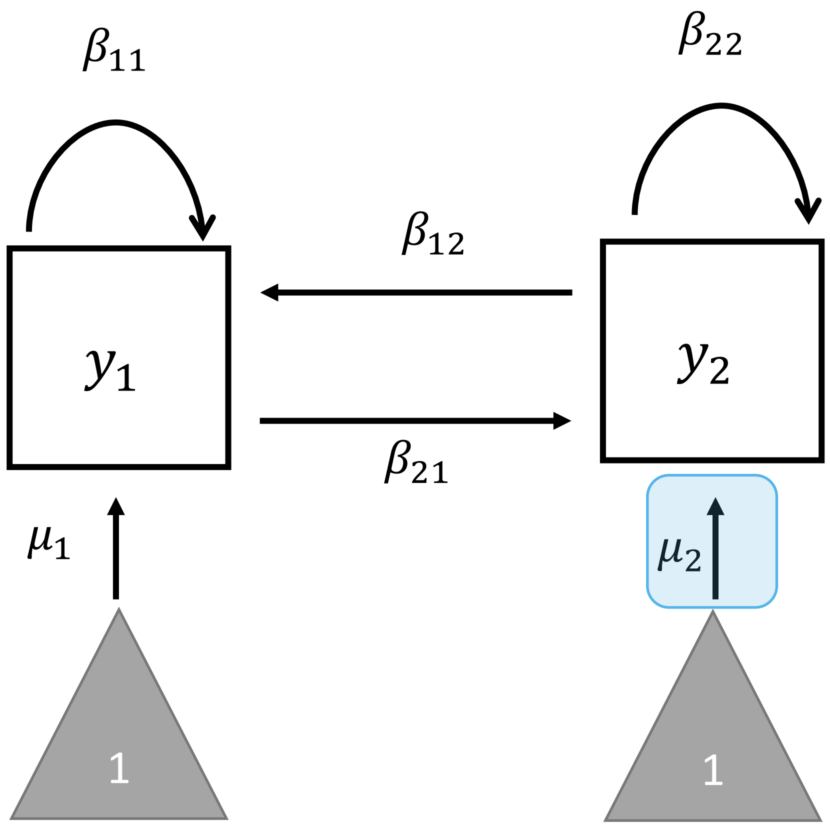

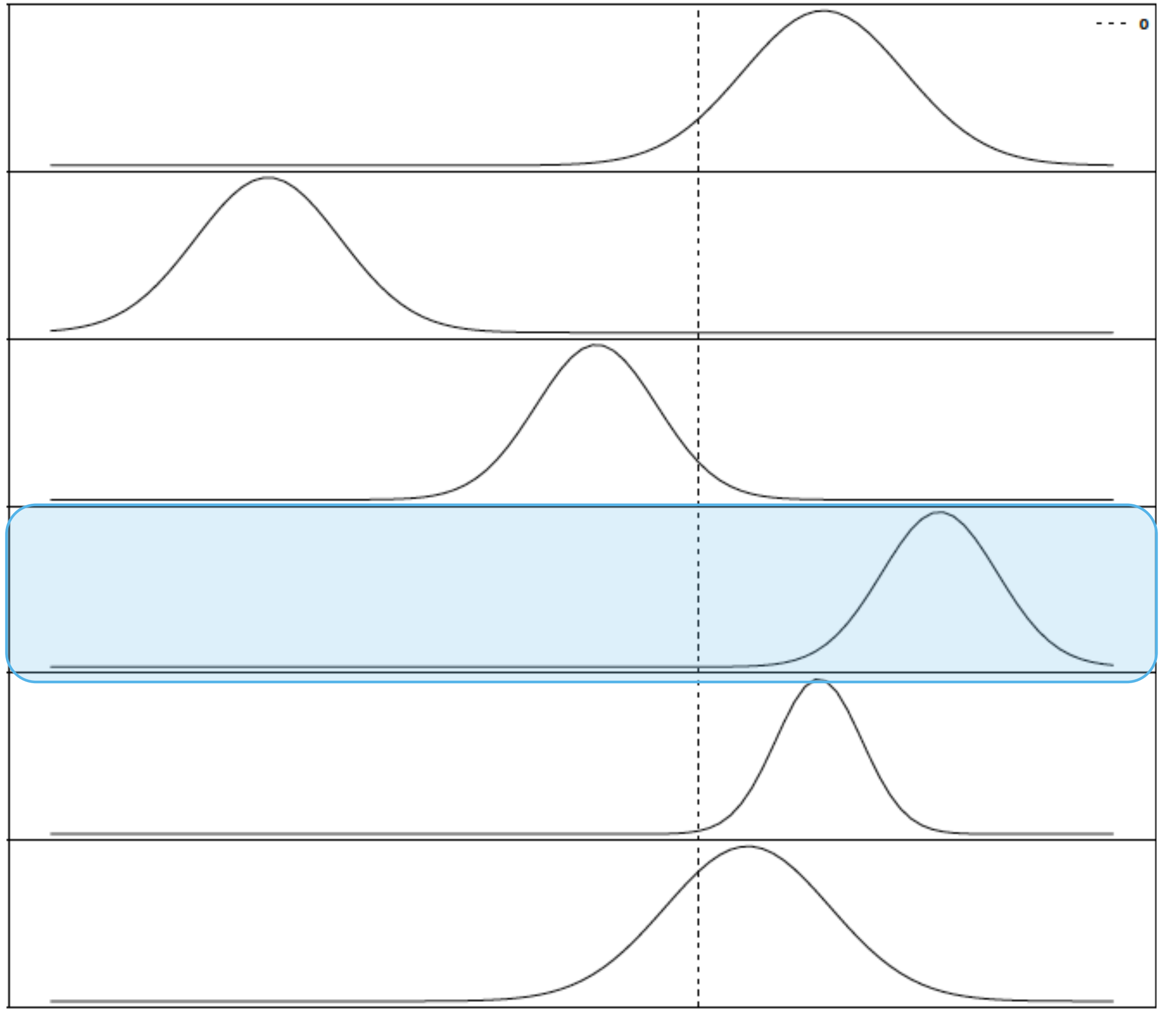

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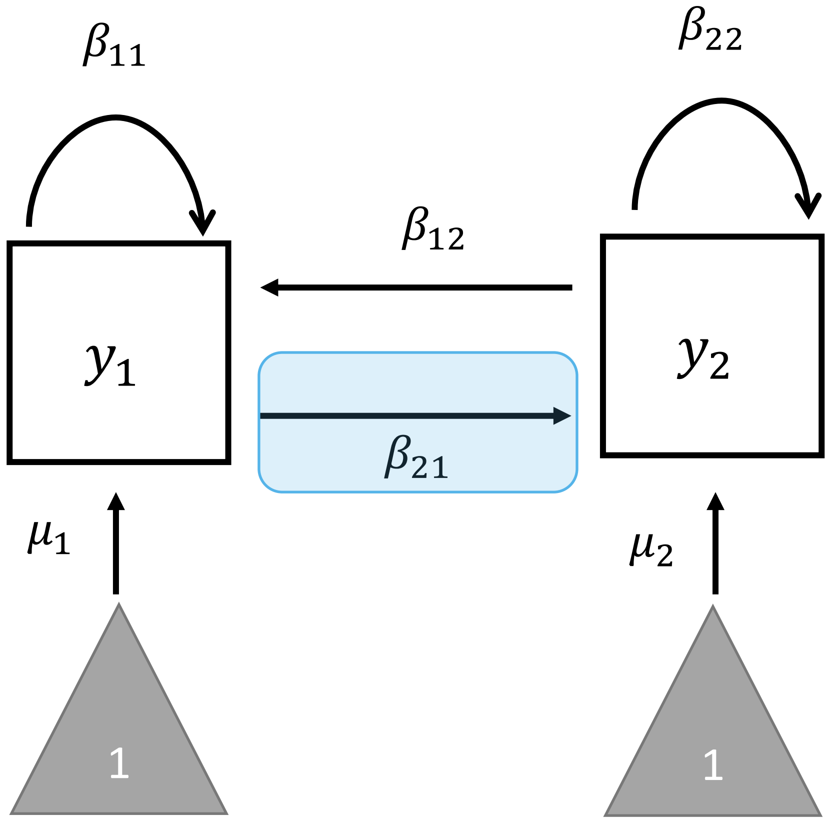

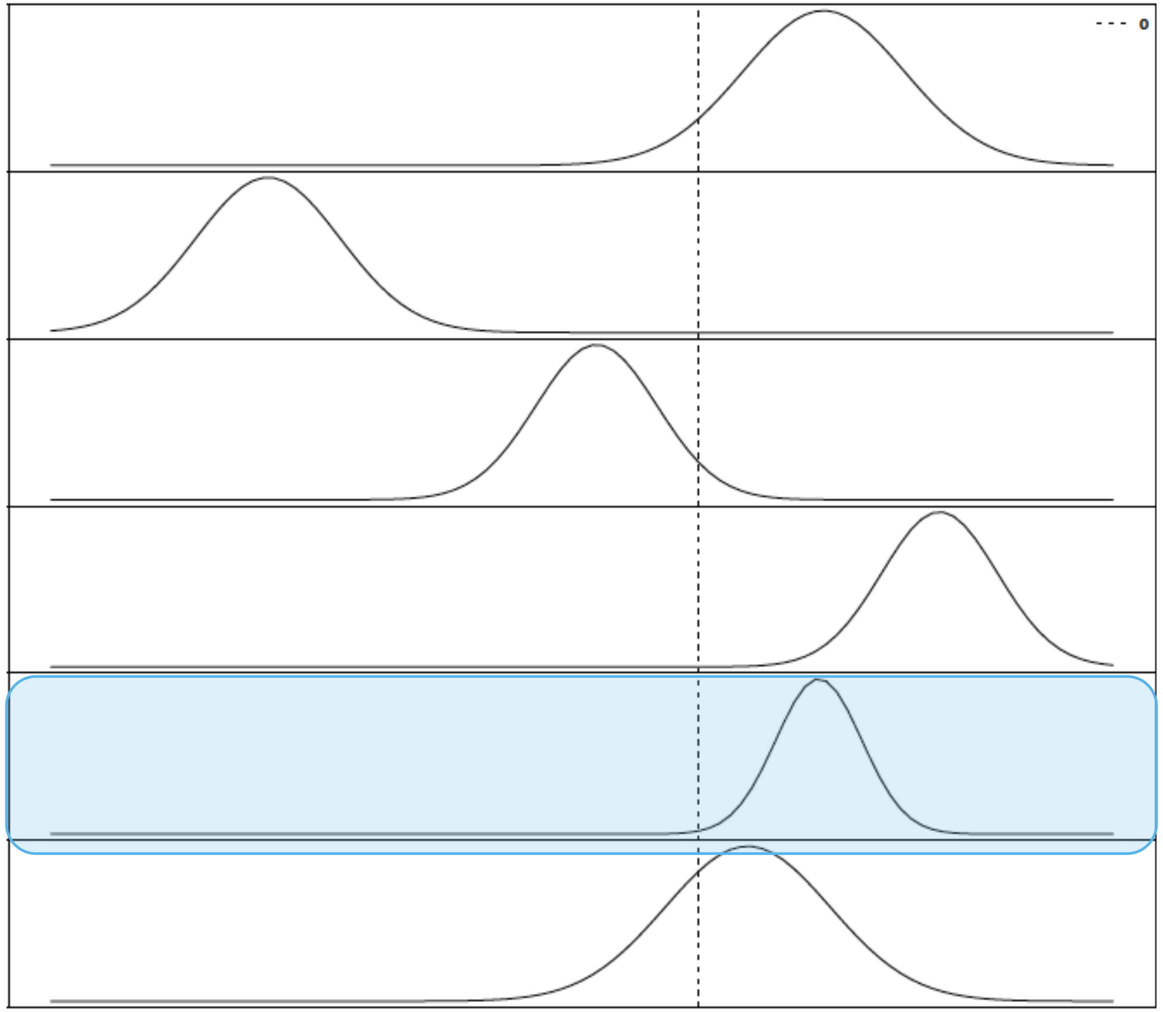

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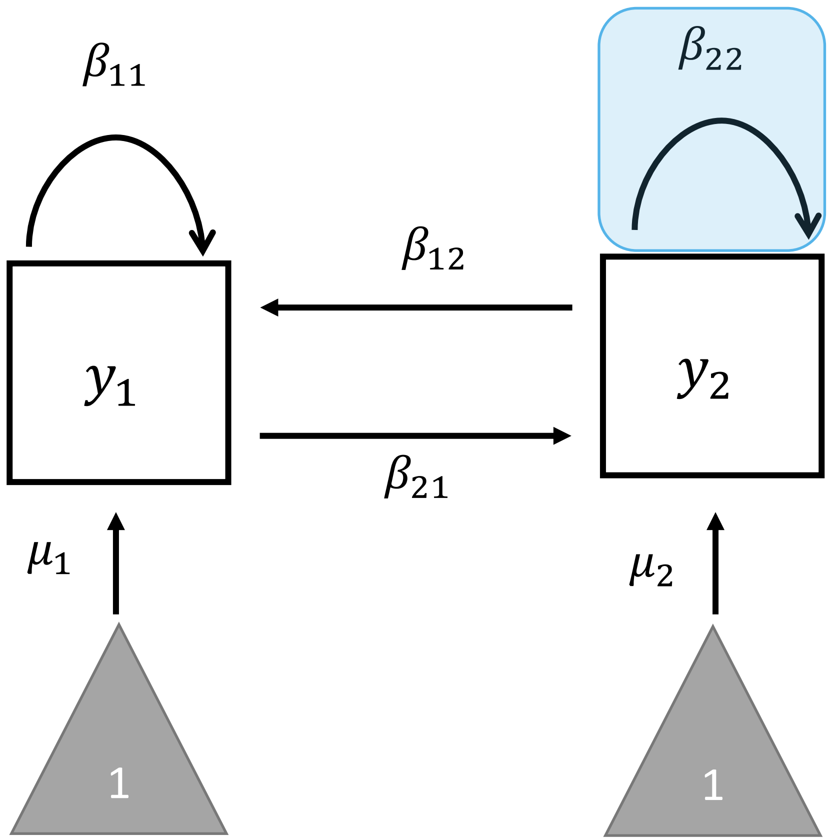
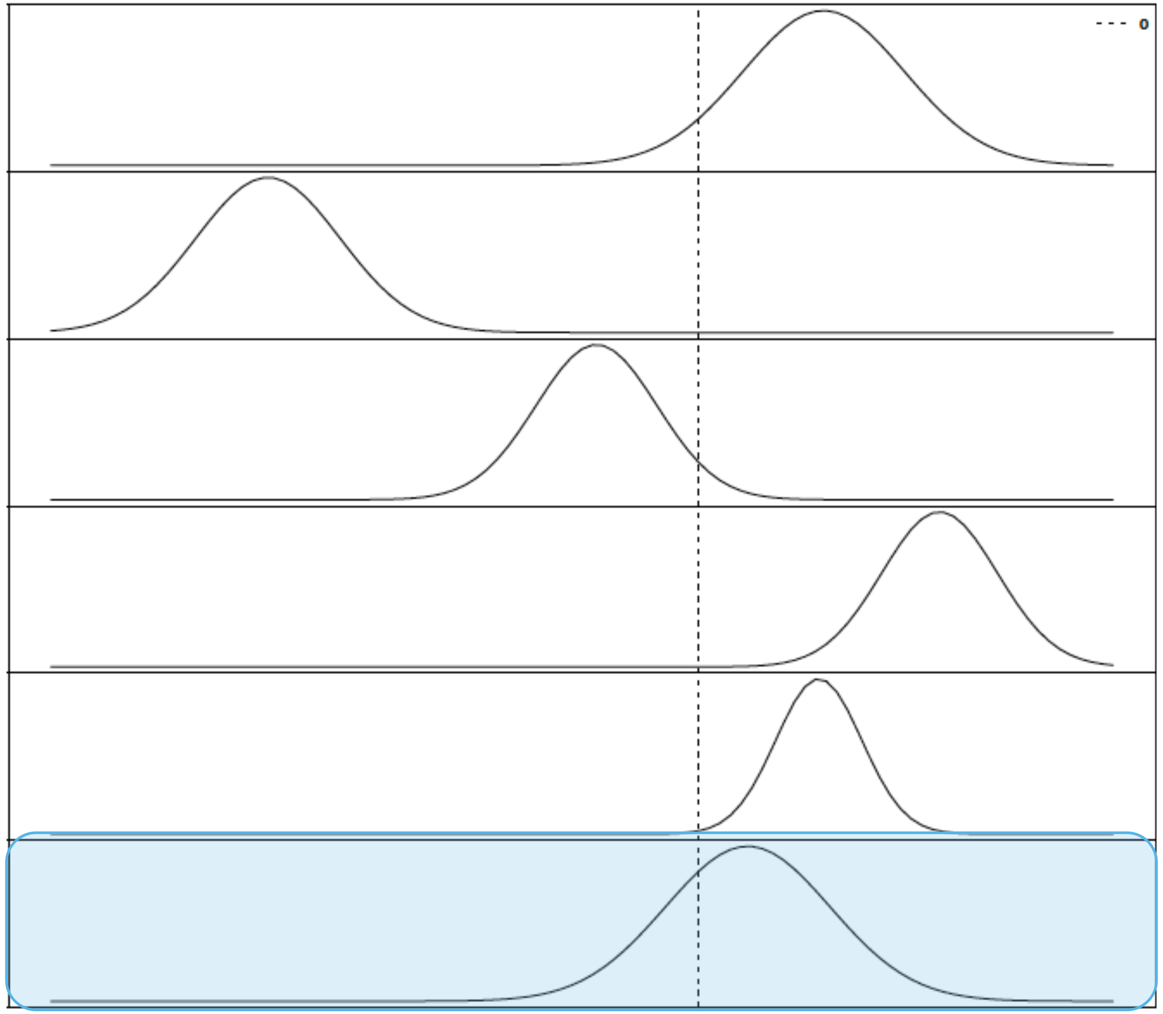
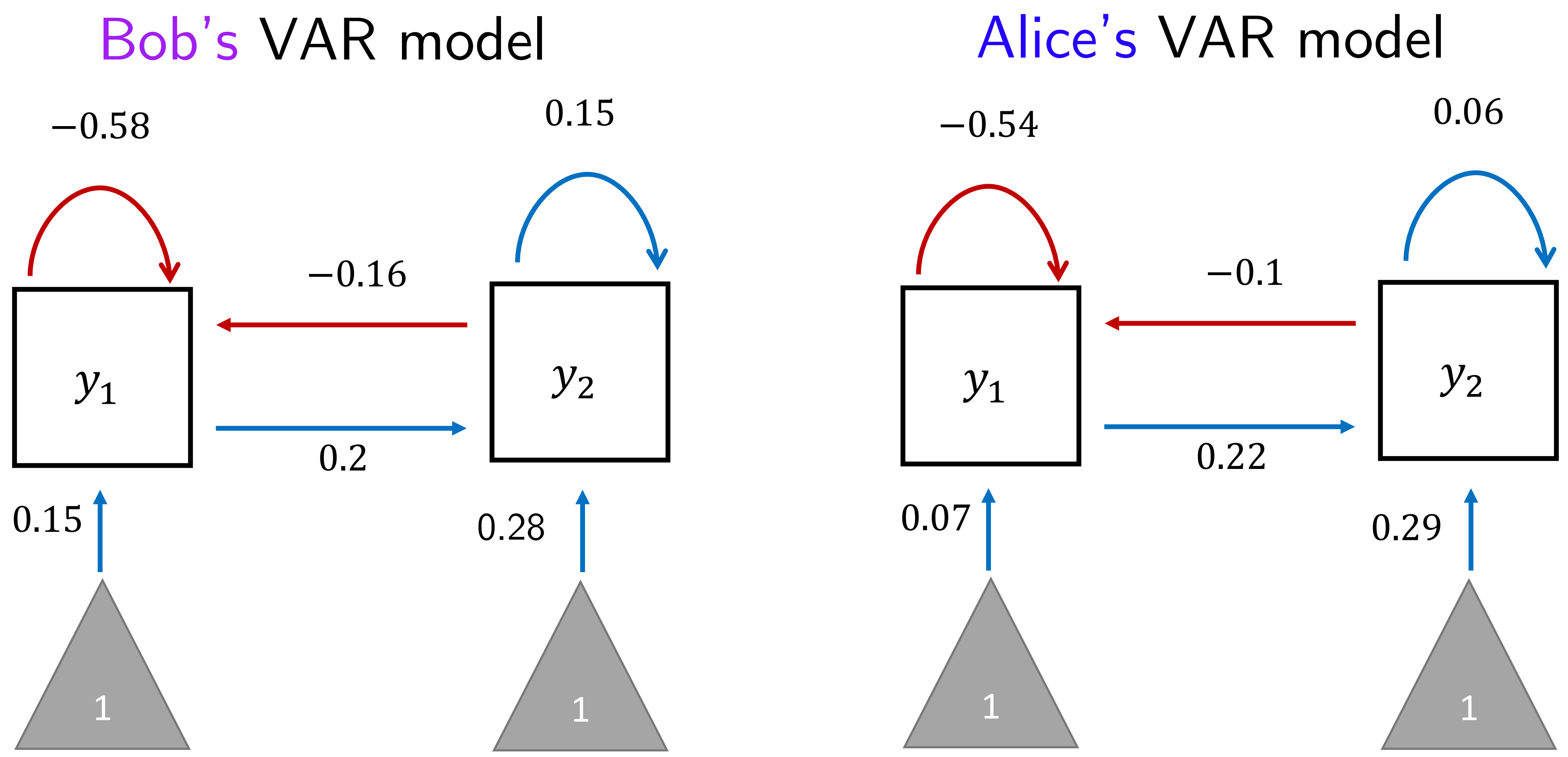
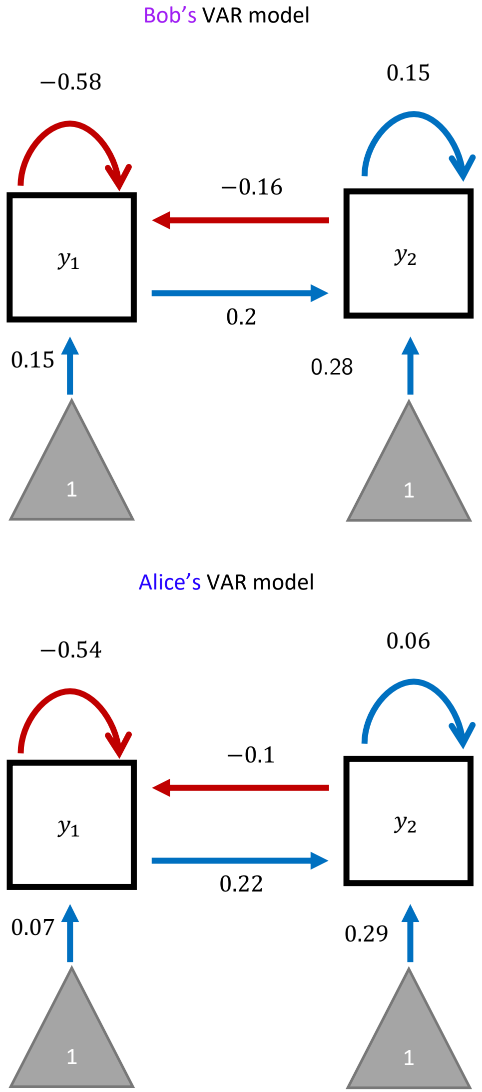


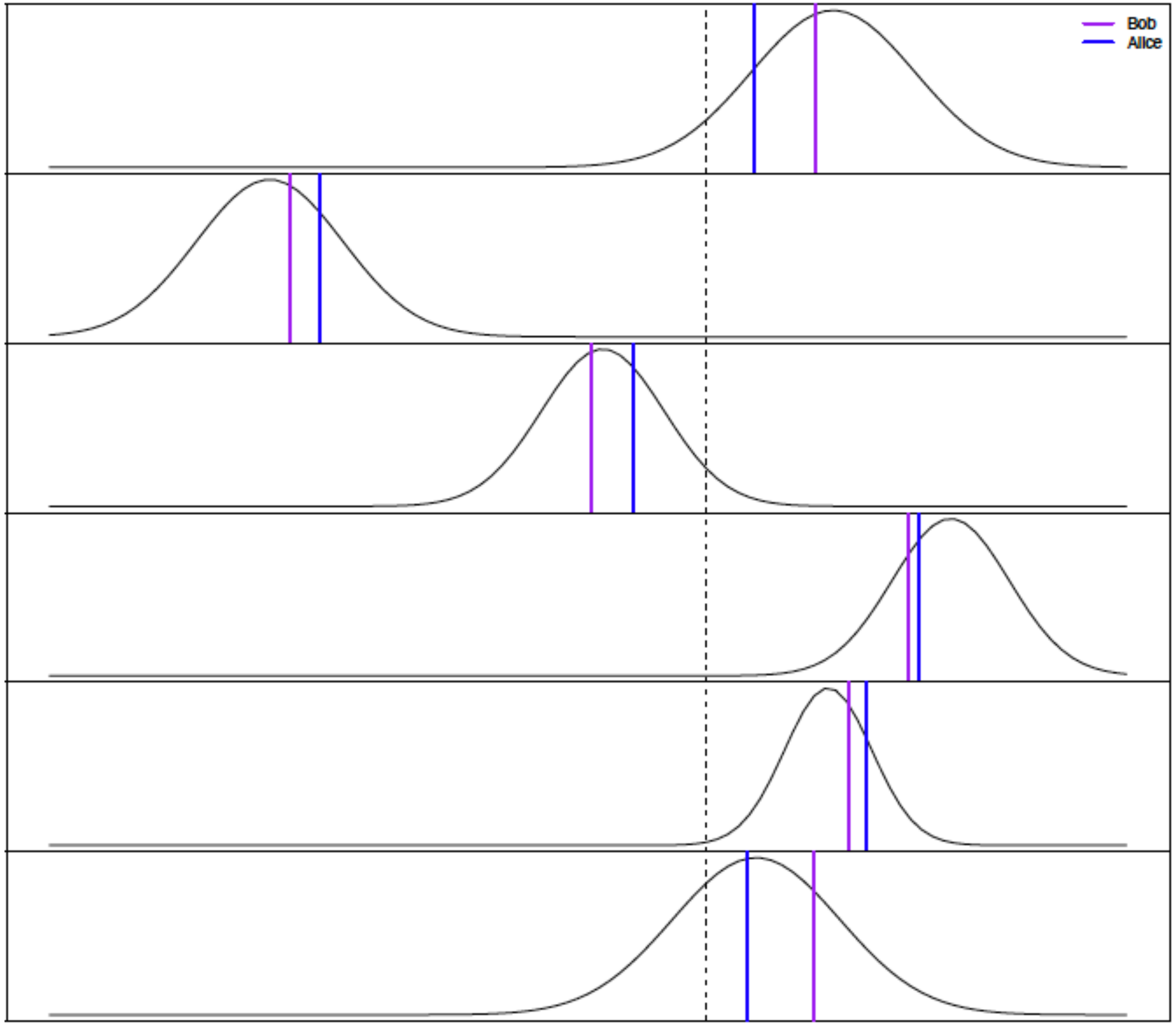


Individual VAR models

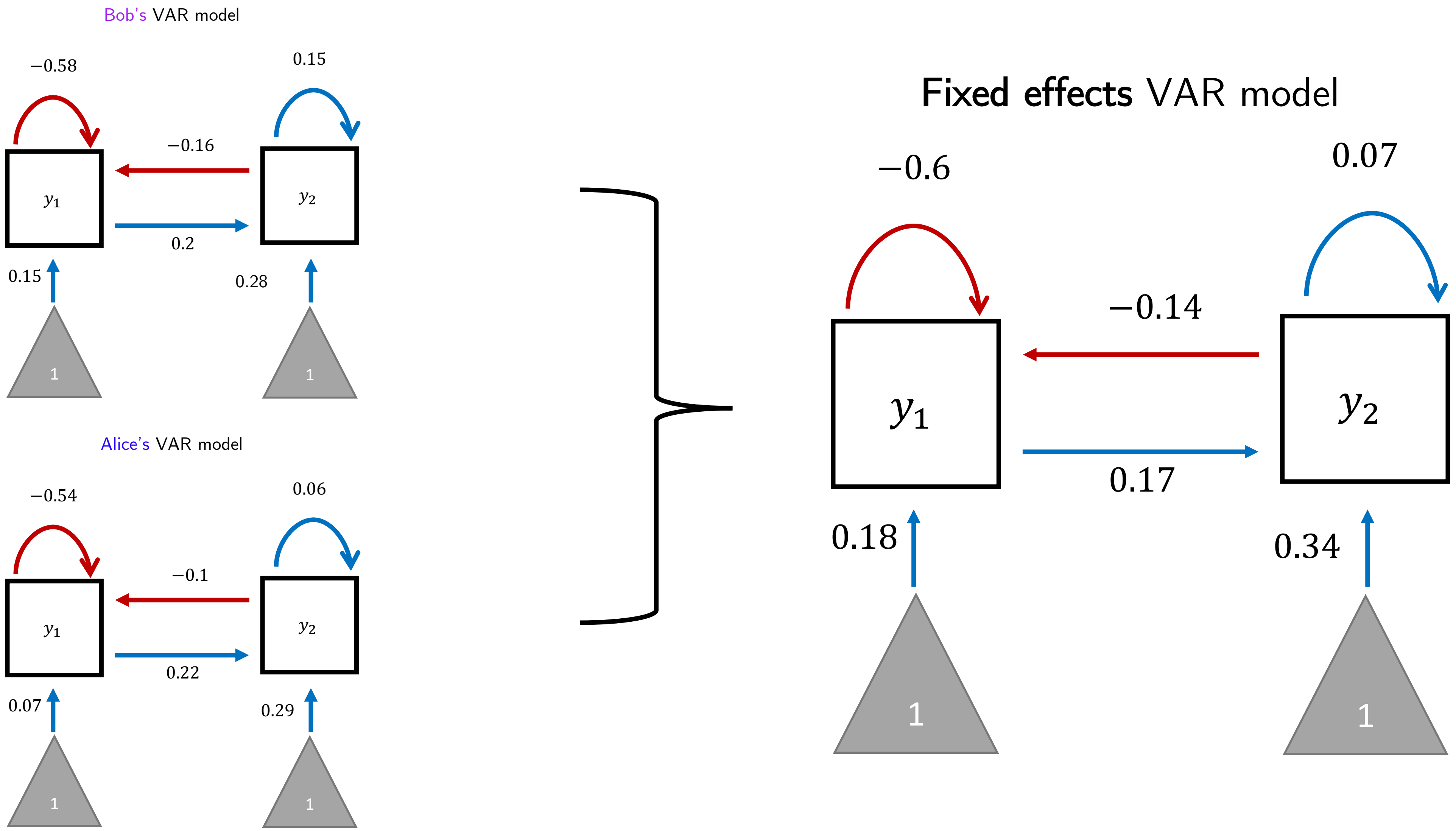


mlVAR: Random effects

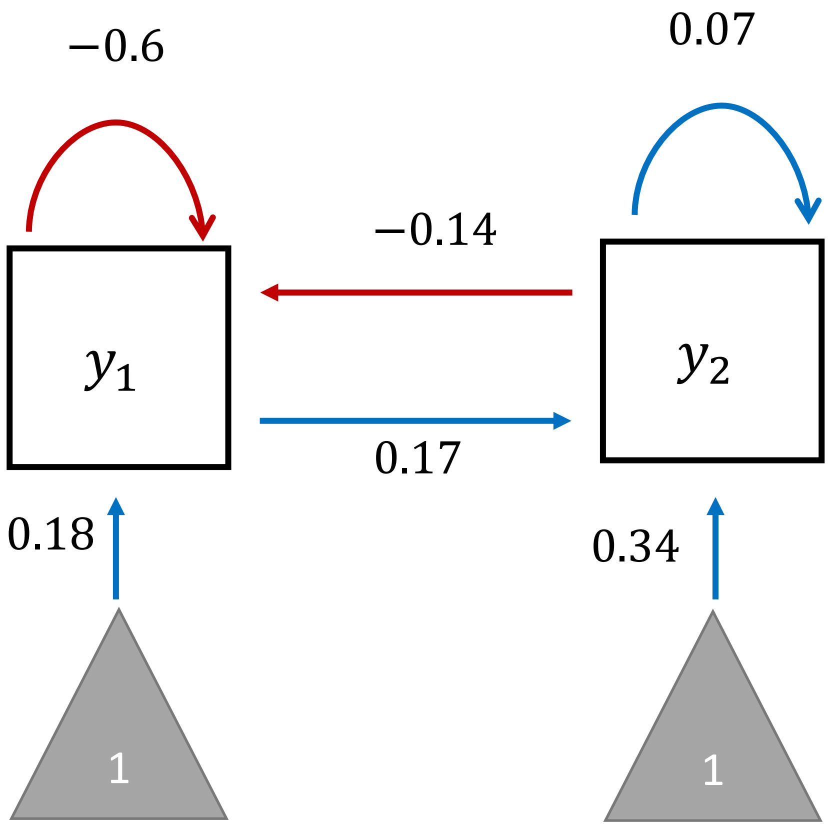


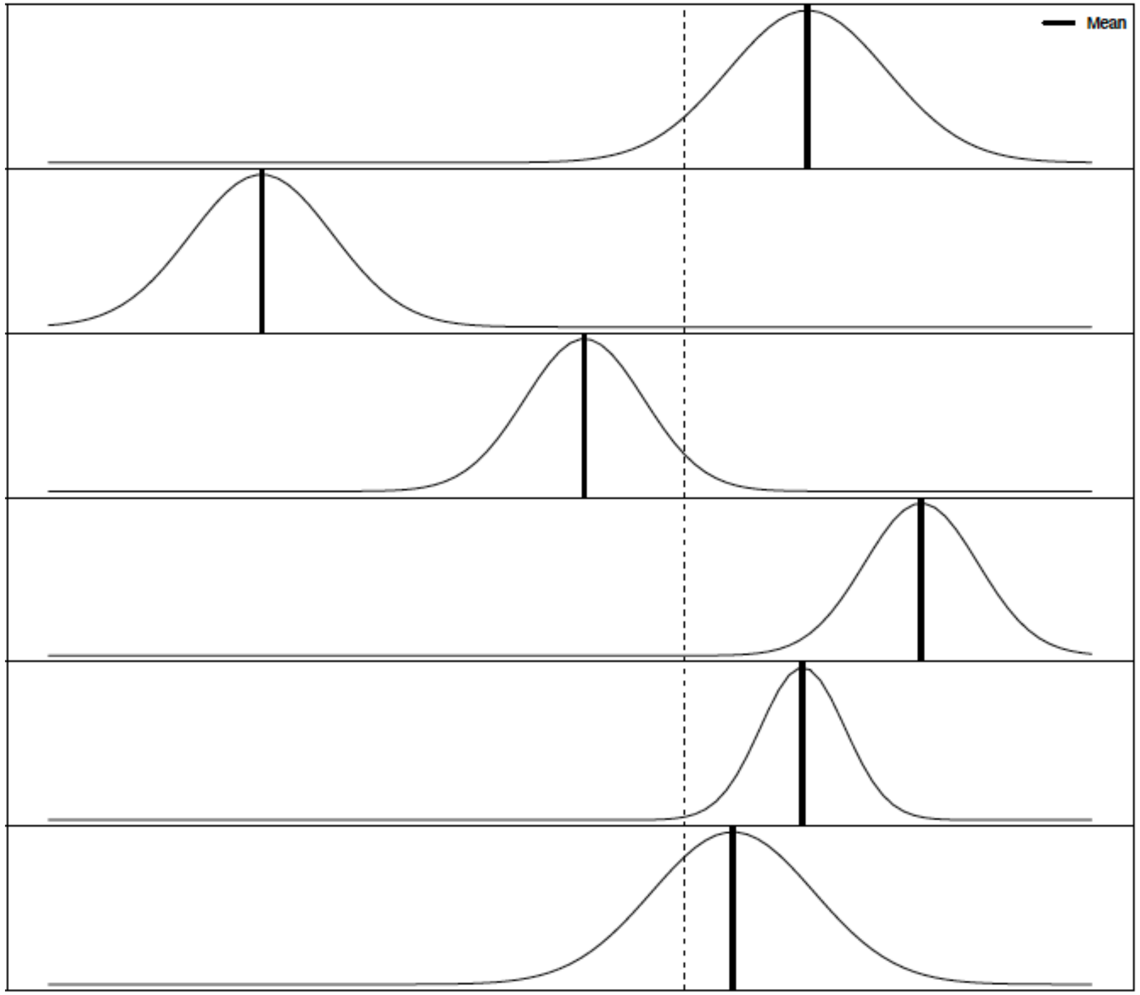


mlVAR: Random effects vs fixed effects

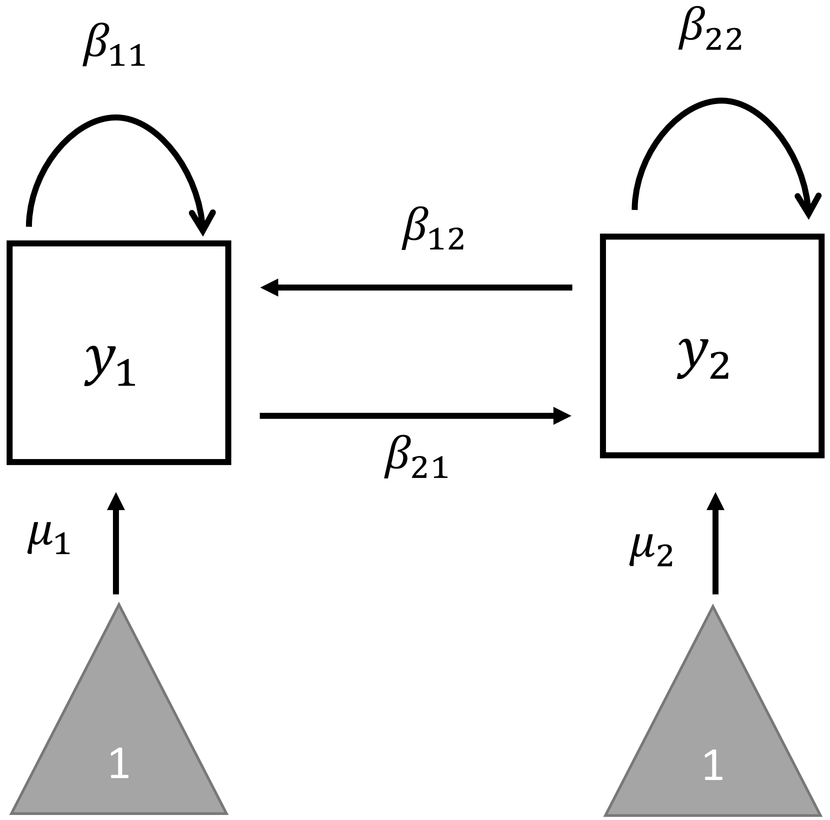


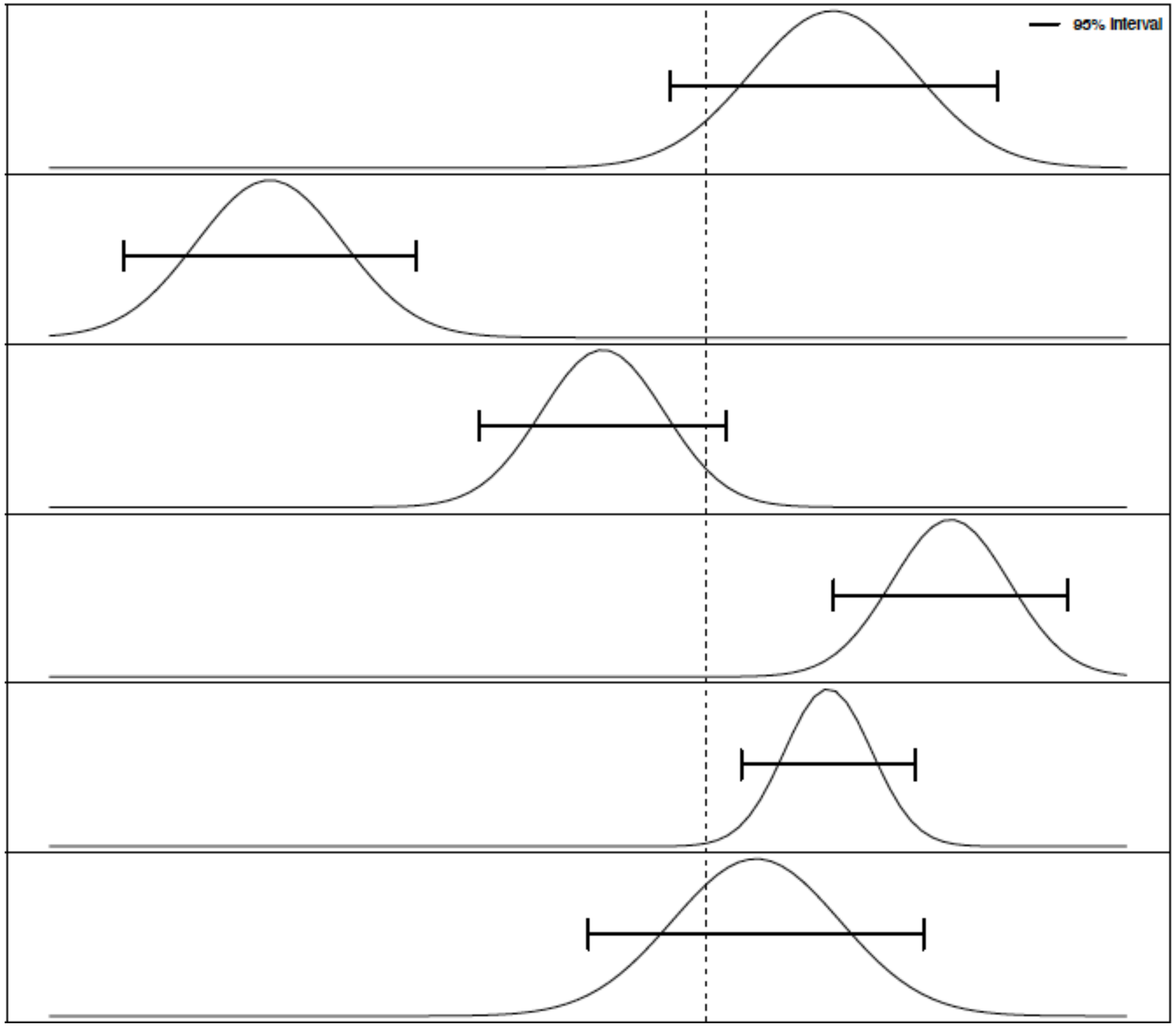
mlVAR: Fixed effects





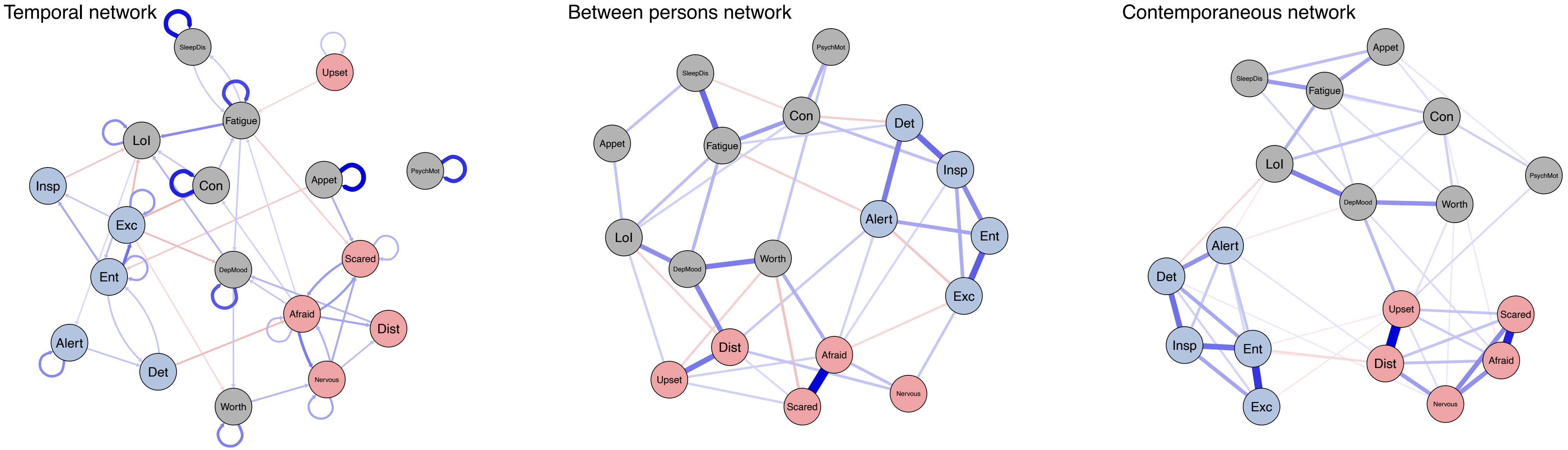
mlVAR: Individual differences





Two step multilevel VAR estimation

- **Step 1:** Node-wise multilevel regressions of variables on within-person centered lagged predictors (temporal effects) and person-wise means (between-subject effects)
- **Step 2:** Node-wise multilevel regressions using residuals from step 1 (contemporaneous effects)
- Implemented in `mlVAR` package



Lunansky, G.,* Hoekstra, R. H. A.,* & Blanken, T. F. (2022). Disentangling the role of affect in the evolution of depressive complaints using complex dynamical networks.

Two step multilevel VAR estimation

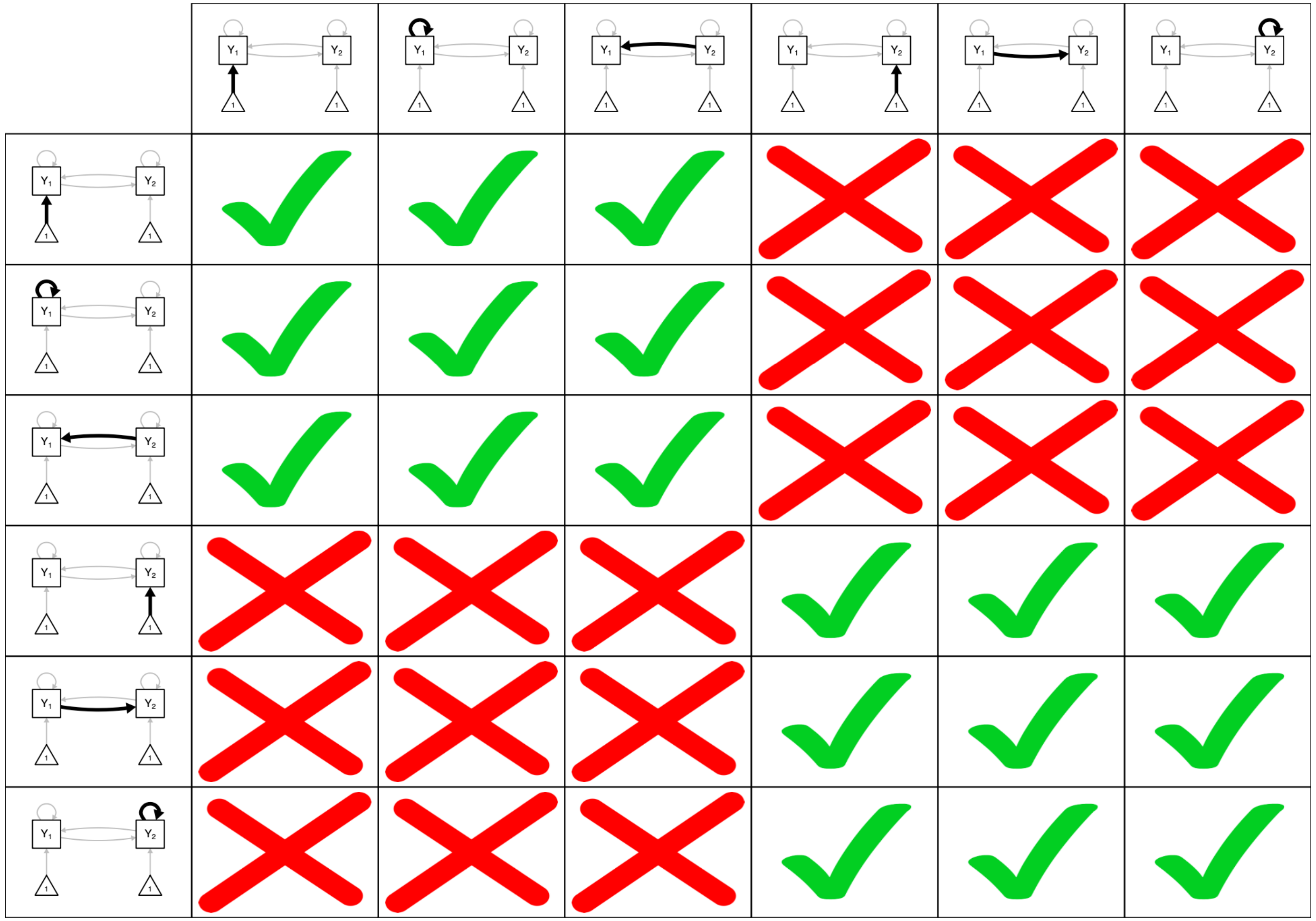
- Full multivariate estimation of the ml VAR model is not possible in open source software

- Solution:
  - Estimating sequentially univariate models: estimate all incoming edges, per node (bringmann et al., 2013)

- Two options for **univariate** multilevel regression models:
  - **Correlated** estimation
  - **Orthogonal** estimation

Correlated versus orthogonal estimation

- **Correlated** estimation in mlVAR

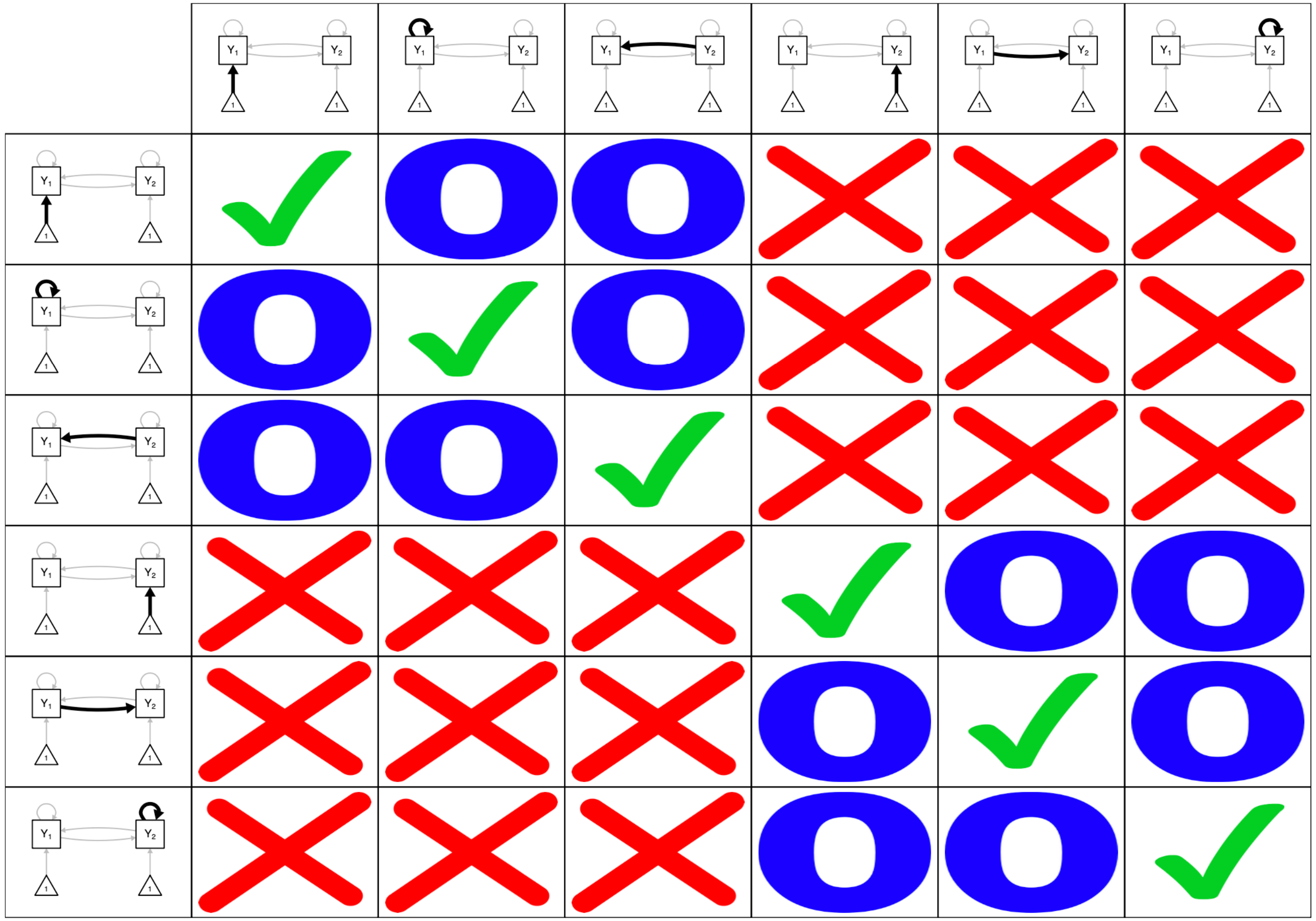


Correlated versus orthogonal estimation

- **Correlated** estimation in mlVAR
  - Some (not all) parameter covariances are being estimated
  - This needs to be integrated out a high-dimensional distribution over parameters
  - Only feasible for up to 6 nodes

Correlated versus orthogonal estimation

- **Orthogonal** estimation in mlVAR



Correlated versus orthogonal estimation

- **Orthogonal** estimation in mlVAR
  - Parameter covariances can be fixed to zero
  - Fast and works for many variables
  - Doesn't return any parameter co-dependencies

Multilevel VAR estimation in R

`mlVAR()`

- `estimator`: "LMER" for sequential univariate multi-level estimation, "mplus" for multivariate Bayesian estimation (requires Mplus), and "lm" for fixed effects estimation
- `contemporaneous/temporal`: estimating correlated or orthogonal contemporaneous/temporal networks?

Multilevel VAR estimation in R

```
res_ml <- mlVAR(data, vars = vars, idvar = idvar,
contemporaneous = ”correlated”, temporal =
“correlated”)
```

- Fixed effects temporal network:

  ```
  plot(res_ml , “temporal”, title = “Temporal Network”)
  ```
- Fixed effects contemporaneous network:
  - `plot(res_ml , “Contemporaneous”, title = “Contemporaneous network”)`
- Between-subjects network:

  ```
  plot(res_ml , “between”, title = “Between-subjects
  network”)
  ```

Multilevel VAR estimation in R

```
res_ml <- mlVAR(data, vars = vars, idvar = idvar,
contemporaneous = ”correlated”, temporal = “correlated”)
```

- Random effects temporal network:

  ```
  plot(res_ml , type= “temporal”, subject = 1, title = “Temporal
  Network”)
  ```

- Random effects contemporaneous network:

  ```
  plot(res_ml , type = Contemporaneous”, subject = 1, title =
  “Contemporaneous network”)
  ```

- Individual differences network:

  ```
  plot(res_ml , type = “temporal”, SD = TRUE, title = “Individual
  differences temporal network”)
  plot(res_ml , type = “contemporaneous”, SD = TRUE, title =
  “Individual differences contemporaneous network”)
  ```

Disadvantages of multilevel VAR

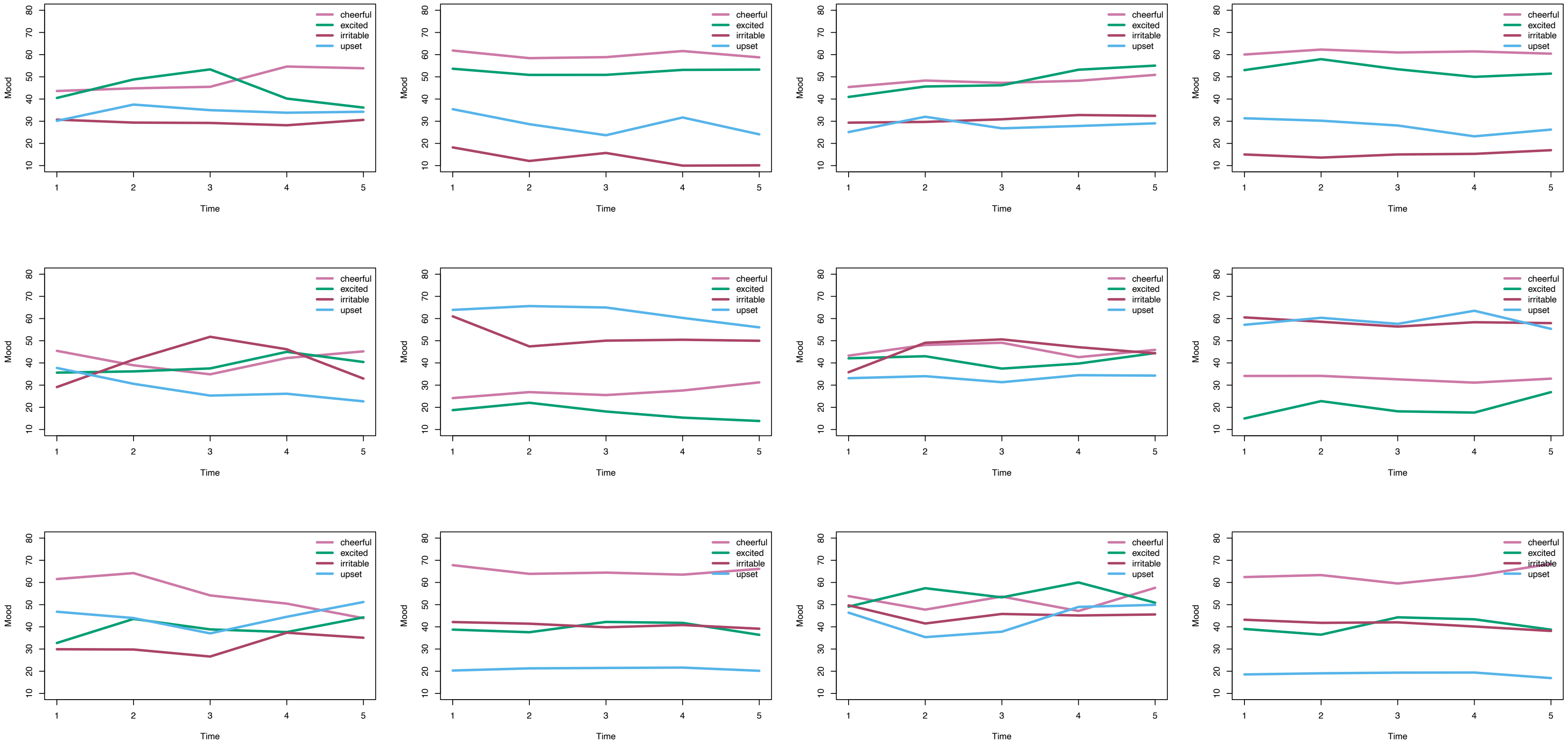
- Some parameters in the model might be correlated.
  - For example: People who have a high mean of worry could also have a high auto-correlation of worry

- This can be modeled in a joint multivariate distribution.
  - However, multi-level VAR uses univariate multilevel regression models $\rightarrow$ forces some random effects to be uncorrelated.

+ Computationally less expensive

- Not “fully” multivariate

Extending VAR over multiple people with fewer time-points



Panel VAR model

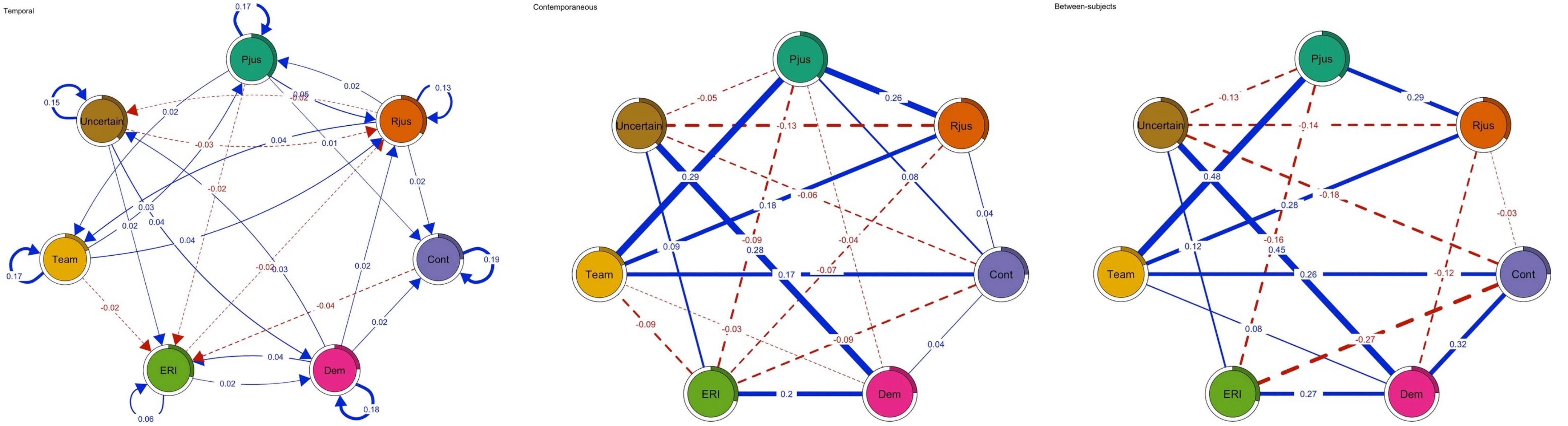
- An often occurring datatype between cross-sectional and time-series data is panel data
  - Many people measured on a few repeated occasions ($t = 3$ or $t = 5$)

- Estimating a VAR model with panel data cannot be done in mlVAR estimation
  - Recently the ml VAR framework of contemporaneous, temporal and between-subjects network is extended to model panel VAR models

- **Advantage**: if interested in the fixed effect structures, fewer data-points are required

Panel VAR model estimation

- Fixed effect within-subject temporal network modeled with $\boldsymbol{B}$
- Fixed-effect within-subject contemporaneous network modeled with $\boldsymbol{\Theta}^{-1}$
- Between-subject network modeled with $\boldsymbol{\Omega}$



Elovainio, M., Hakulinen, C., Komulainen, K., Kivimäki, M., Virtanen, M., Ervasti, J., & Oksanen, T. (2022). Psychosocial work environment as a dynamic network: a multi-wave cohort study. *Scientific reports*, *12*(1), 1-11.

Panel VAR model estimation

- Fixed effect within-subject temporal network modeled with $\boldsymbol{B}$
- Fixed-effect within-subject contemporaneous network modeled with $\boldsymbol{\Theta}^{-1}$
- Between-subject network modeled with Ω

- Temporal, contemporaneous and between-subject network are estimated using FIML estimation
  - Implemented in `psychometrics` package

Panel VAR model estimation in R

- Form model:
  ```
  mod <- panelgvar(data, vars = vars)
  ```

- Estimate model:
  ```
  mod <- mod %>% runmodel()
  ```
- Estimate pruned model:
  ```
  mod %>% prune(alpha = 0.05, recursive = FALSE)
  ```
- Evaluate model fit:
  ```
  Mod %>% fit()
  ```

- Estimated fixed effects **contemporaneous** network:
  ```
  mod %>% getmatrix(”omega_zeta_within")
  ```
- Estimated fixed effects **temporal** network:
  ```
  mod %>% getmatrix(”PDC")
  ```
- Estimated **between-subjects** network:
  ```
  mod %>% getmatrix(”omega_zeta_between”)
  ```

Note: different pruning options here such as FDR-thresholding, greedy set up search and within each of these pruning options you can make different choices again such as correcting for multiple testing

Challenges in time-series modeling

Challenge: **Measurement**

- Response scales:
  - What scales should we use and how do they impact measurement?
    - VAS-scales vs. Likert-type ordinal scales
  - Recent study showing that different scale types seem to affect distribution (Haslbeck et al., 2022)

- Response shifts:
  - Will measurement of a construct stay the same across the assessment period? If not, why not? How to integrate it into assessments and analyses?
    - If people change their internal standards or redefine constructs, then variability in scores may reflect response shifts in measurement, not changes in the actual construct

Challenge: **Measurement**

- Does continuous administration of intensive longitudinal questionnaires evoke any kind of answer tendencies?
  - If measurement leads to changes in these construct, we don't measure ecologically valid behavior, we measure a "new reality"(Ram et al., 2017)

- Inter-individual differences in scale use:
  - How do people actually decide on their score? (e.g., do they compare with previous scores or personal anchors?)
    - Differences in response strategies may bias measurement in construct-irrelevant ways.
  - How do we take into account inter-individual differences in scale use/interpretation?
    - Recent study: a 50 on a scale of 0-100 meant fundamentally different things to individuals (based on qualitative interviews)

Challenge: **No covariance without variance**

- **Variance** in a variable is necessary in order to show **covariance** with another variable
  - Note: no variance means no connections in the VAR network

- Not all variables we are interested in show variance (within the measurement period)
  - e.g., childhood trauma, economic status, income

- As a consequence: some of the variables that we might be interested in, we cannot model in time-series models

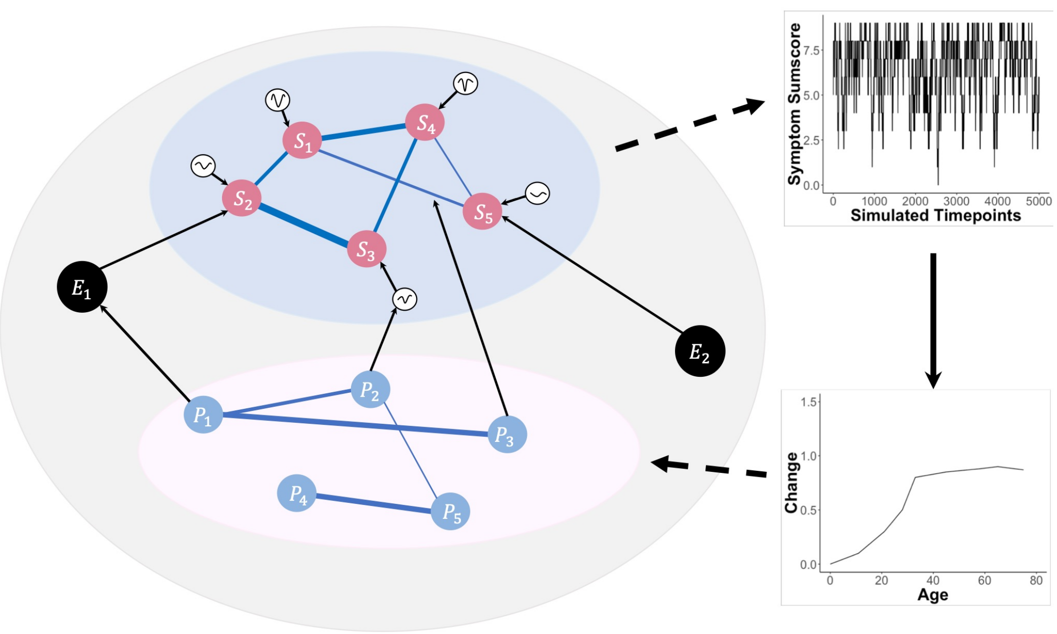
Challenge: **Time-scale**

- Did you get the time scale right?
  - In many cases, we lack knowledge regarding the exact time-scale on which processes of interest operate
  - For practical considerations, ESM questionnaires are often administered every 2-4 hours, or once a day

- Choosing the lag
  - Most commonly, we use lag1 for modeling in idiographic networks
  - This raises the question: Are we capturing the dynamics, or are they faster/slower than our assessment?

Challenge: **Time-scale**

- One time scale for all variables?
  - Even if we capture some of the relevant processes with our measurements, it seems unlikely to assume that all processes within one network operate on the same time scale:
    - E.g., clin … combination with slow processe



Lunansky, G., van Borkulo, C., & Borsboom, D. (2020). Personality, resilience, and psychopathology: A model for the interaction between slow and fast network processes in the context of mental health. *European Journal of Personality*, *34*(6), 969-987.

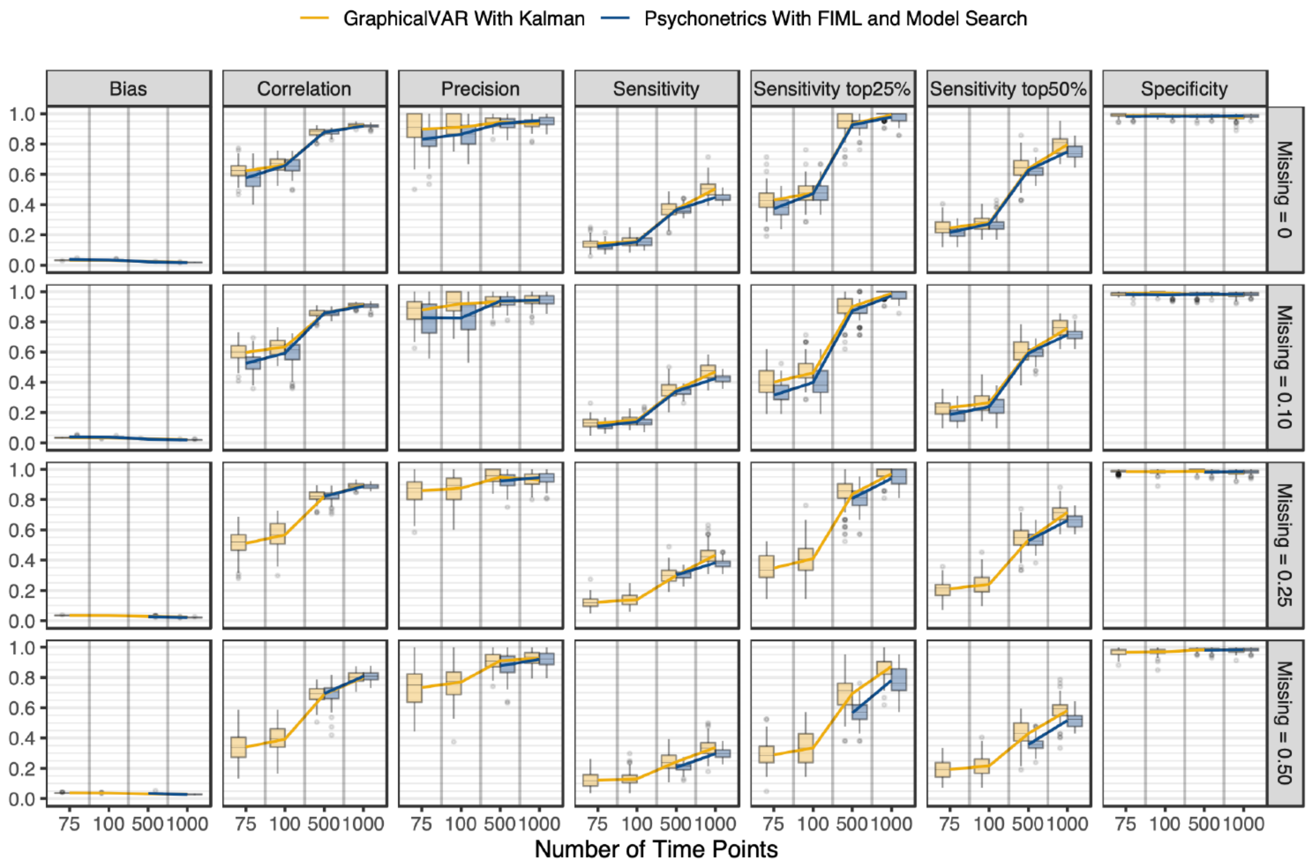
Challenge: **Systematic missingness**

- Often time-series data contains a lot of missingness
  - Is the data missing completely at random?
  - Missing at random?
  - Missing not at random?

- How to handle missing data?
  - Imputation?
    - Which imputation technique to use?
  - FIML estimation
- What percentage of missing data is acceptable?

Challenge: **Systematic missingness**

- What percentage of missing data is acceptable?



Mansueto, A. C., Wiers, R. W., van Weert, J., Schouten, B. C., & Epskamp, S. (2022). Investigating the feasibility of idiographic network models. *Psychological methods*.

Challenge: **Trends in the data**

- How to account for trends in the data?
  - We saw an example of how to detrend for linear trends, but what about all other possible trends in the data?
  - What effect has detrending on the **interpretation**?

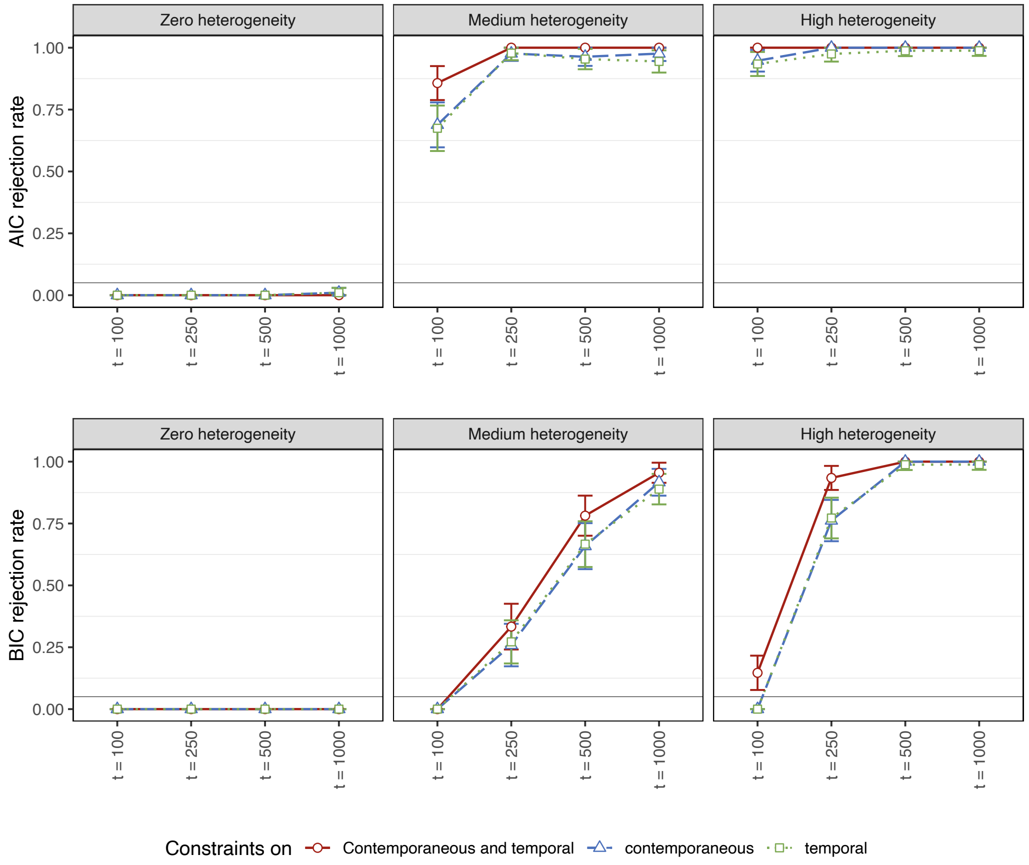
- What if our research question implies non-stationarity?
  - Often we are interested in the trend in the data (e.g., change after treatment)

- **Possible alternative**:
  - Time-varying VAR (Haslbeck, Bringmann, & Waldorp, 2021)

Challenge: **Heterogeneity**

- How to test heterogeneity *between* individuals?
- How to test heterogeneity *within* individuals?

- **Individual Network Invariance Test**: place equality constraints on network models and compare model fit (Hoekstra et al. in preparation)



Hoekstra, R. H. A., Epskamp., S, Niergenberg, A. A., Borsboom, D. & McNally, R. (in preparation). Testing similarity in longitudinal networks: The Individual Network Invariance Test (INIT)

Thank you for your attention!