Comparison of Gaussian Graphical Models (GGM) and Directed *Cyclic* Graphs (DCG) as Causal Discovery Tools

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1. Introduction

Background

- Network theory of psychopathology suggests mental disorder is produced by causal interactions among symptoms that reinforce each other via *feedback loops* or *cycles*[2].
- Empirical researchers often fit statistical network models to observational data in order to explore these causal relations.

Limitations

- The utility of statistical network models as causal discovery tools compared to causal graphical models is in general unclear.
- Past research is limited to comparisons between statistical network models and directed *acyclic* graphs (DAG), which is at odds with network theory of psychopathology where the cycle plays a central role.

Goal

- To investigate the utility of statistical network models as causal discovery tools in cyclic settings compared to the *directed cyclic graph* models (DCG).

2. Methods

Data: data are simulated from several different cyclic models with *linear* causal relationships and *independent Gaussian* error terms, which are commonly assumed in psychological research.

Causal discovery algorithm: we use *cyclic causal discovery* (CCD) algorithm[1], which can handle cycles.

- Output: *partial ancestral graph* (PAG) that represents a set of *statistically-equivalent DCGs*.

Statistical Network model: we focus on *Gaussian graphical models* (GGM), as the simulated data are *continuous* and correspond to the *observational (cross-sectional)* data type.

Evaluation Metrics: we gauge how much the GGM and DCG deviate from the true cyclic model, based on the following two metrics:

- Overall density
- Degree centrality

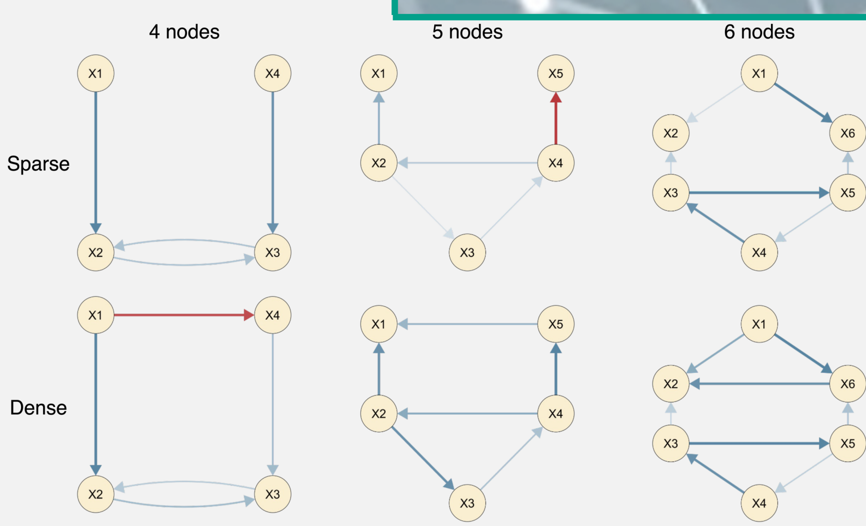

Figure 1. Simulation Design

we vary the number of variables (columns of Figure 1), and density (rows of Figure1)

3. Results

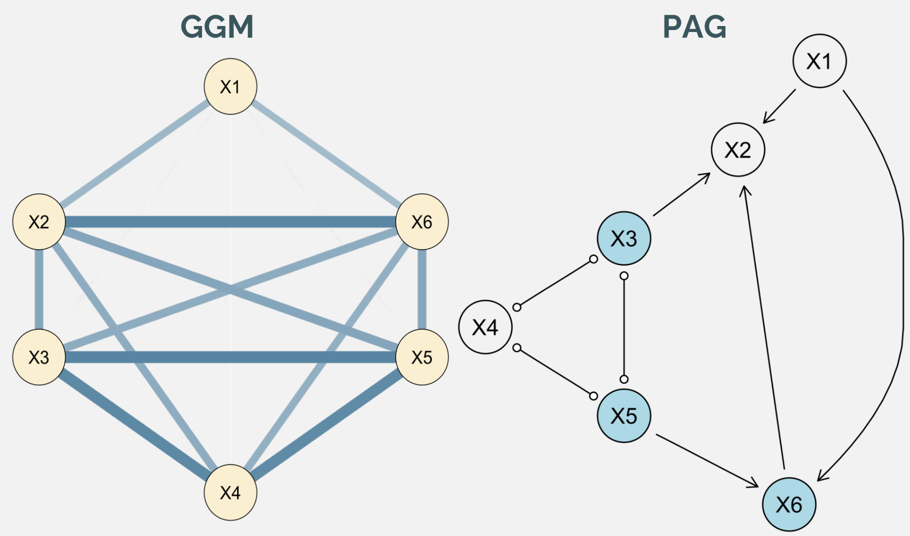

Figure 2. 6-Node Dense Condition

Note that in the PAG, circle (o) edge endpoint means that the algorithm does not know the direction of that corresponding edge. A colored node (in blue) indicates a solid underlining in triples in the PAG representation. See Richardson (1996)[1] for further details on the PAG representation.

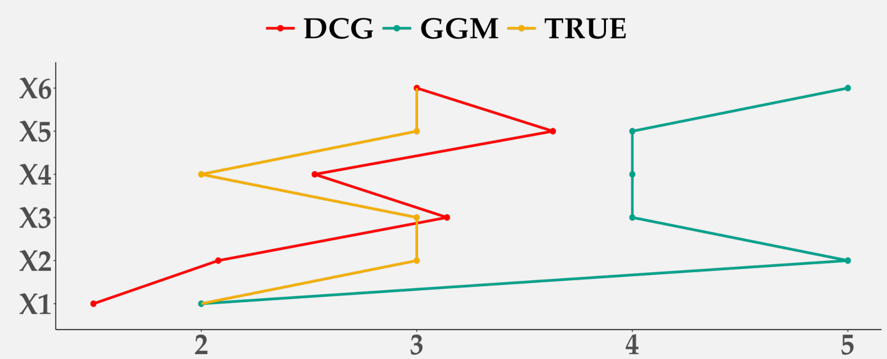

Figure 3. Degree Centrality of the 6-Node Dense Case

In the other simulation conditions, similar patterns were observed. See the supplementary material for more information.

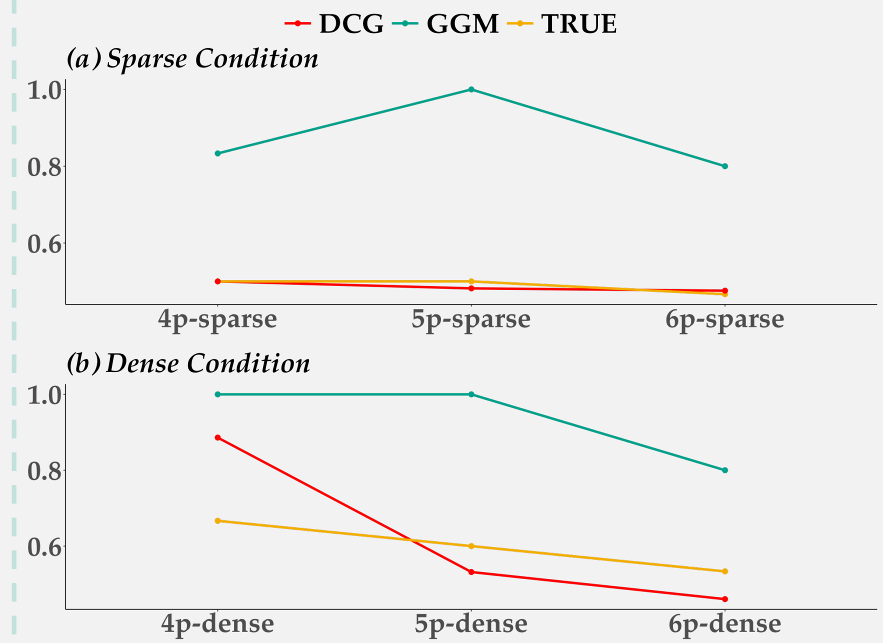

Figure 4. Overall Density of all Models

(a) In low-density condition. (b) In high-density condition.

- *GGMs* often *overestimated* the *density* as shown in Figure 4, which also resulted in *high degree for* almost every node in the model (see Figure 3).
- Figure 4 shows that *DCGs* more clearly *outperformed* GGMs, when the true causal models were *sparse*.
- Overall, the DCGs more closely approximated the true cyclic models compared to the GGMs in terms of both density and degree centrality.
- For the complete results, check the *supplementary material* here.

4. Conclusions

- Statistical network models perform more poorly as causal discovery tools compared to directed cyclic graphs when the true system contains feedback loops.
- The estimation accuracy of causal discovery algorithm (e.g., CCD) tends to drop when the true model is dense, but it still outperforms statistical network models.
- We recommend using the purpose-built cyclic causal discovery algorithms such as the CCD if causal hypotheses are of interest.

References

[1] Richardson, T.: (1996) A discovery algorithm for directed cyclic graphs. *In: Proceedings of the Twelfth International Conference on Uncertainty in Artificial Intelligence*, Morgan Kaufmann Publishers Inc., San Francisco, CA, USA,

[2] Borsboom, D., & Cramer, A. O. (2013). Network analysis: an integrative approach to the structure of psychopathology. *Annual review of clinical psychology*, *9*, 91–121.