Probing grids :

(probe_calibration_points)

- The smaller grids are for 0-point normalisation, 1-point and 4-point calibration.
- The larger grids probe multiple centre points, the tower positions and opposite tower positions and a number of intermediate points for 7-point calibration.
- The centre points are numerically averaged.
- The intermediate point are distributed to the nearest tower and opposite tower positions according a $\sin^2$ and $\cos^2$ distribution and then averaged. This is mathematically equivalent as construction higher order (13-point …) displacement matrices and inverting them (see below).
- In the largest grids probe points are taken on 3 different circles and averaged; the calibration radius is reduced in the calculations to that of the middle circle (x 0.9).

	Centre points	Radius Points	Total
P0	0	0	0^2 = 0
P1	1	0	1^2 = 1
P2	1	3	2^2 = 4
P3	3	6	3^2 = 9
P4	4	12	4^2 = 16
P5	7	18	5^2 = 25
P6	6	36	6^2 = 36
P7	7	42	7^2 = 49
P8	10	3 x 18	8^2 = 64
P9	9	3 x 24	9^2 = 81
P10	10	3 x 30	10^2 = 100

The more probe points the more small defects in the print surface will be averaged out. The user has to choose between most accuracy and speed. Calibration radius needs to be chosen as large as possible that give reliable results (e.g. inductive probes tend to be unreliable to close to the bed edge).

Calculations :

For each case: 1-point, 4-point and 7-point calibration a displacement matrix was made (see Excel sheet). For each parameter to be calibrated (the end-stop corrections Δx, Δy and Δz, the delta radius Δr and the tower angle corrections $\Delta\alpha$, $\Delta\beta$ and $\Delta\gamma$) one column in the matrix is made for the changes in height (displacement) of the 7 calibration points when the corresponding parameter is changed with one unit.

- 1-point calibration is done by $[\Delta x, \Delta y, \Delta z] = [1, 1, 1] \times \Delta z(C)$
- For the end-stop corrections the linear assumption is made that a change of one end-stop tilts the bed in such a way the centre point displaces with 1/3 of a unit, so when 3 end-stop corrections are made simultaneously by 1 unit, the centre point is displaced by one unit as well. The displacements of the other calibration points is calculated geometrically using the ratio of calibration radius and delta radius (CR in the Excel sheet). With a ratio of 1 the displacement will be 1 unit at the corresponding tower of the end-stop correction.
- These linear assumptions are also implemented as the (approximate) forward kinematic calculation (here moving one of the carriages by a unit) for the 7 calibration points (see Excel sheet for the formulas) (forward_kinematics_probe_points).
- The reverse kinematics are already implemented in Marlin. (reverse_kinematics_probe_points)
- The average displacements delta radius and tower angle corrections are calculated from the forward and revers kinematics above (Δz and Δz in the Excel sheet). (calc_kinematics_diff_probe_points)

The 4-point displacement matrices for tower and opposite tower positions are inverted and normalised using scale factors derived from CR and Δz (above) (auto_tune_h and auto_tune_r) and combined to for 7-point calibration. As such, the matrix is universal whatever the values of CR and Δz. (A)

For the tower angles a 3x3 displacement matrix is made by combining the changes for each tower with its opposite point. Only 2 tower angels need to be calibrated; as such the matrix is not invertible. First a very small number X is added to all the elements in the matrix, this one can be inverted, but than 1/X needs to be subtracted to obtain the correct result. Finally this 3x3 is split evenly over tower and opposite tower point. Again the obtained result is normalised using a scale factor derived from Δz (above) (auto_tune_a) to make it universal whatever value is obtained for Δz (B).

(A) and (B) combined gives the inverted displacement matrix for 7-point calibration: multiplied with the probe results it will calculate the required corrections to the calibration parameters (end-stops corrections, delta radius and tower angle corrections). However since delta kinematics are not linear (this was an assumption to simplify the math), the obtained solution can not be obtained in one go. One must iterate this inverted displacement matrix, but the above math will make the iterations converge to a solution the quickest way possible. (gcode_G33 – inverted matrices in switch (probe_points) see comment)

Finally the end-stop corrections are normalised by setting the smallest to zero, diminish the two others and increase the delta height by the same amount; and the tower angle corrections are normalised to be minimal by the least squares method.