University of Helsinki



## **UNIVERSITY OF HELSINKI**



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$13~{ m src/geometry/closest points.cpp}$	11	<pre>// TCR #include <bits stdc++.h=""></bits></pre>	
$14~{ m src/datastructure/orderedset.cpp}$	11	using namespace std;	
$15~{ m src/datastructure/dynamichull.cpp}$	11	<pre>int main(){     // Iterate all submasks in increasing order. Does not list 0.</pre>	
$16~{ m src/datastructure/linkcut.cpp}$	12	<pre>int mask=13; for (int sub=0;(sub=(sub-mask)&amp;mask);) {</pre>	
$17~{ m src/datastructure/treap.cpp}$	13	<pre>cout&lt;<sub<<endl; 1="" 12="" 13="" 4="" 5="" 8="" 9="" pre="" print="" should="" }cout<<endl;<=""></sub<<endl;></pre>	
18 src/datastructure/HLD.cpp	13	<pre>// Iterate all submasks in decreasing order. Does not list 0. for (int sub=mask;sub;sub=(sub-1)&amp;mask) {</pre>	
19 src/math/berlekampmassey.cpp	14	<pre>cout&lt;<sub<<endl; 12="" 13="" 1<="" 4="" 5="" 8="" 9="" pre="" print="" should=""></sub<<endl;></pre>	
20 src/math/crt.cpp	15	<pre>}cout &lt;&lt; endl; int n=24;</pre>	
21 src/math/fftmod.cpp	16	<pre>cout&lt;&lt;(n&amp;-n)&lt;<end1; 11="" 64bit<="" 8="" bit="" cout<<builtin_popcountll(n)<<end1;="" pre="" print="" remember="" set.="" should="" smallest="" using="" when=""></end1;></pre>	
, , , ===	10	// Compute the next number that has the same number of bits set as n	
$22~{ m src/math/gaussjordan.cpp}$	17	// Returns -1 for 0 int t=n (n-1);	
$23 \ \mathrm{src/math/simplex.cpp}$	18	int w=(t + 1)   ((("t & -"t) - 1) >> (_builtin_ctz(n) + 1));	
24 src/math/fft.cpp	19	<pre>cout&lt;<w<<endl; 33="" pre="" print="" should="" }<=""></w<<endl;></pre>	

```
src/flags.txt
// TCR
Warnings: -Wall -Wextra -pedantic -Wshadow -Wformat=2 -Wfloat-equal -Wconversion
-Wlogical-op -Wcast-qual -Wcast-align
Runtime checks, these might make the code much slower: -D_GLIBCXX_DEBUG
-D_GLIBCXX_DEBUG_PEDANTIC -D_FORTIFY_SOURCE=2 -fsanitize=address -fsanitize=undefined
-fno-sanitize-recover -fstack-protector
Use these: -std=c++11 -02 -Wall -Wextra -Wshadow
   src/string/suffixarray.cpp
// Suffix array in O((n+S) log n)
// S is the size of alphabet, meaning that 0 <= s[i] < S for all i
// You can change vector<int> s to string s. In that case S is 256
#include <bits/stdc++.h>
using namespace std;
vector<int> suffixArray(vector<int> s, int S) {
    int n=s.size();int N=n+S;
    vector<int> sa(n), ra(n);
    for(int i=0;i<n;i++) {sa[i]=i;ra[i]=s[i];}
    for(int k=0; k< n; k?k*=2:k++) {
        vector<int> nsa(sa), nra(n), cnt(N);
        for(int i=0;i< n;i++) nsa[i]=(nsa[i]-k+n)%n;
        for(int i=0;i<n;i++) cnt[ra[i]]++;
        for(int i=1;i<N;i++) cnt[i]+=cnt[i-1];</pre>
        for(int i=n-1;i>=0;i--) sa[--cnt[ra[nsa[i]]]]=nsa[i];
        int r=0:
        for(int i=1;i<n;i++) {</pre>
            if(ra[sa[i]]!=ra[sa[i-1]]) r++;
            else if(ra[(sa[i]+k)\%n]!=ra[(sa[i-1]+k)\%n]) r++;
            nra[sa[i]]=r;
        ra=nra;
    return sa;
   src/string/lcparray.cpp
// Constructs LCP array from suffix array in O(n) time
// You can change vector<int> s to string s
#include <bits/stdc++.h>
```

```
using namespace std;
vector<int> lcpArray(vector<int> s, vector<int> sa) {
    int n=s.size(), k=0;
    vector<int> ra(n), lcp(n);
    for (int i=0;i<n;i++) ra[sa[i]]=i;</pre>
    for (int i=0; i< n; i++) {
        if (k) k--:
        if (ra[i]==n-1) {
            k=0;
            continue:
        int j=sa[ra[i]+1];
        while (k < n \& \& s[(i+k)%n] == s[(j+k)%n]) k++;
        lcp[ra[i]]=k;
        if (ra[(sa[ra[i]]+1)%n]>ra[(sa[ra[j]]+1)%n]) k=0;
    return lcp;
   src/string/aho-corasick.cpp
// TCR
// Aho-Corasick algorithm
// Building of automaton is O(L) where L is total length of dictionary
// Matching is O(n + number of matches), O(n sqrt(L)) in the worst case
// Add dictionary using addString and then use pushLinks
#include <bits/stdc++.h>
#define F first
#define S second
using namespace std;
struct AhoCorasick {
    vector<map<char, int> > g;
    vector<int> link, tlink, te;
    // Use 1-indexing in id
    void addString(const string& s, int id) 
        int tn=0;
        for (int i=0;i<(int)s.size();i++) {</pre>
            if (g[tn][s[i]]==0) {
                g[tn][s[i]]=g.size();
                g.push_back(map<char, int>());
                link.push_back(0);
                tlink.push_back(0);
                te.push_back(0);
            tn=g[tn][s[i]];
```

```
te[tn]=id:
    void pushLinks() -
        queue<int> bfs;
        bfs.push(0);
        while (!bfs.empty()) {
            int x=bfs.front();
            bfs.pop();
            for (auto nx:g[x]) {
                int l=link[x]:
                while (1!=-1\&\&g[1].count(nx.F)==0) l=link[1];
                if (l!=-1) link[nx.S]=g[l][nx.F];
                bfs.push(nx.S);
                if (te[link[nx.S]]) tlink[nx.S]=link[nx.S];
                else tlink[nx.S]=tlink[link[nx.S]];
    // Returns matches {id, endpos}
    vector<pair<int, int> > match(const string& s) {
        int tn=0;
        vector<pair<int, int> > re;
        for (int i=0;i<(int)s.size();i++) {</pre>
            while (tn!=-1\&\&g[tn].count(s[i])==0) tn=link[tn];
            if (tn==-1) tn=0;
            tn=g[tn][s[i]];
            int f=tlink[tn];
            if (te[tn]) re.push_back({te[tn], i});
            while (f) {
                re.push_back({te[f], i});
                f=tlink[f];
        return re;
    AhoCorasick() {
        g.push_back(map<char, int>());
        link.push_back(-1);
        tlink.push_back(0);
        te.push_back(0);
   src/string/z.cpp
// TCR
```

```
// Computes the Z array in linear time
// z[i] is the length of the longest common prefix of substring
// starting at i and the string
// You can use string s instead of vector<int> s
// z[0]=0 by definition
#include <bits/stdc++.h>
using namespace std;
vector<int> zAlgo(vector<int> s) {
    int n=s.size();
    vector<int> z(n):
    int l=0; int r=0;
    for (int i=1;i<n;i++) {
        z[i]=max(0, min(z[i-1], r-i));
        while (i+z[i] < n\&\&s[z[i]] == s[i+z[i]]) z[i] ++;
        if (i+z[i]>r) {
            l=i;r=i+z[i];
    return z;
7 src/string/suffixautomaton.cpp
// TCR
// Online suffix automaton construction algorithm
// Time complexity of adding one character is amortized O(1)
#include <bits/stdc++.h>
using namespace std;
struct SuffixAutomaton {
    vector<map<char, int> > g;
    vector<int> link, len;
    int last;
    void addC(char c) {
        int p=last;int t=link.size();
        link.push_back(0);
        len.push_back(len[last]+1);
        g.push_back(map<char, int>());
        while (p!=-1\&\&g[p].count(c)==0) {
            g[p][c]=t;p=link[p];
        if (p!=-1) {
            int q=g[p][c];
            if (len[p]+1==len[q]) {
                link[t]=q;
            else {
```

typedef complex<CT> co;

```
int qq=link.size();
                link.push_back(link[q]);
                len.push_back(len[p]+1);
                g.push_back(g[q]);
                while (p!=-1\&\&g[p][c]==q) {
                    g[p][c]=qq;p=link[p];
                link[q]=qq;link[t]=qq;
        last=t;
    SuffixAutomaton() : SuffixAutomaton("") {}
    SuffixAutomaton(string s) {
        last=0;
        g.push_back(map<char, int>());
        link.push_back(-1);
        len.push_back(0);
        for (int i=0;i<(int)s.size();i++) addC(s[i]);</pre>
    vector<int> terminals()
        vector<int> t;int p=last;
        while (p>0) {
            t.push_back(p);p=link[p];
        return t;
};
   src/geometry/convexhull.cpp
// Computes the convex hull of given set of points in O(n log n)
// Uses Andrew's algorithm
// The points on the edges of the hull are not listed
// Change > to >= in ccw function to list the points on the edges
// Returns points in counterclockwise order
#include <bits/stdc++.h>
#define X real()
#define Y imag()
using namespace std;
typedef long double ld;
typedef long long 11;
// Coordinate type
typedef 11 CT;
```

```
bool ccw(co a, co b, co c) {
    return ((c-a)*conj(b-a)).Y>0;
vector<co> convexHull(vector<co> ps) {
    auto cmp = [] (co a, co b) {
        if (a.X==b.X) return a.Y<b.Y;
        else return a.X<b.X;</pre>
    };
    sort(ps.begin(), ps.end(), cmp);
    ps.erase(unique(ps.begin(), ps.end()), ps.end());
    int n=ps.size();
    if (n<=2) return ps;
    vector<co> hull;hull.push_back(ps[0]);
    for (int d=0;d<2;d++) {
        if (d) reverse(ps.begin(), ps.end());
        size_t s=hull.size();
        for (int i=1;i<n;i++) {
            while (hull.size()>s&&!ccw(hull[hull.size()-2],hull.back(),ps[i])) {
                hull.pop_back();
            hull.push_back(ps[i]);
    hull.pop_back();
    return hull:
   src/geometry/anglesort.cpp
// Comparasion function for sorting points around origin
// Points are sorted in clockwise order
// Works with integers and doubles
/*122
  143
  443*/
#include <bits/stdc++.h>
#define X real()
#define Y imag()
using namespace std;
typedef long double ld;
typedef long long 11;
typedef complex<11> co;
bool ccw(co a, co b, co c) {
    return ((c-a)*conj(b-a)).Y>0;
```

```
int ar(co x) {
    if (x.Y)=0&&x.X<0 return 1;
    if (x.X)=0&&x.Y>0) return 2;
    if (x.Y \le 0 \& x.X > 0) return 3;
    return 4;
bool cp(co p1, co p2)
    if (ar(p1)!=ar(p2)) {
        return ar(p1) < ar(p2);</pre>
    return ccw(\{0, 0\}, p2, p1)>0;
10 src/geometry/hullhulltan.cpp
// TCR
// O(log n log m)
// poinHullTan
// Finds the common tangents of a convex polygon and a point
// The polygon should be strictly convex and in counterclockwise order
// Pointhulltan returns {-1, -1} if the point is inside the polygon, otherwise
// it returs {maximal, minimal} vertices in terms of visibility from point p
// Remember to implement the special case n \le 2
// Points on the boundary are considered to be inside
// hullHullTan
// Finds the common tangents of two convex polygons
// All of the conditions as above and it probably does not work if n \le 2 or m \le 2
// 1 is maximal and -1 is minimal
#include <bits/stdc++.h>
#define X real()
#define Y imag()
using namespace std;
typedef long long 11;
typedef complex<11> co;
11 ccw(co a, co b, co c) {
    return ((c-a)*conj(b-a)).Y;
bool up(co p, vector<co>& h, int a, int b, int d) {
    int n=h.size();
    return (ll)d*ccw(p, h[(a+n)\%n], h[(b+n)\%n])<=0;
int getTanP(co p, vector<co>& h, int d) {
    int n=h.size();int mi=0;int ma=n;
    while (mi+1<ma) {
        int mid=(mi+ma)/2;
        if (up(p, h, mi, mi+1, d)) {
```

```
if (up(p, h, mid+1, mid, d)) ma=mid;
            else if(up(p, h, mi, mid, d)) mi=mid;
            else ma=mid:
        else
            if (up(p, h, mid, mid+1, d)) mi=mid;
            else if(up(p, h, mid, mi, d)) mi=mid;
            else ma=mid:
    int step=0;
    if (d==1) {
        ma\%=n;
        while (up(p, h, ma, ma+1, d)) {
            ma=(ma+1)%n; step++;
            assert(step<2);
        return ma;
    else
        while (up(p, h, mi, mi+1, d)) {
            mi=(mi+1)%n;step++;
            if (step>=3) return -1;
       if (up(p, h, mi, mi-1, d)) mi=(mi-1+n)%n;
        return mi:
pair<int, int> pointHullTan(co p, vector<co>& h) {
    if ((int)h.size()<=2) return {0, 0};
    int t1=getTanP(p, h, -1);
    if (t1==-1) return {-1, -1};
    return {getTanP(p, h, 1), t1};
bool up2(vector<co>& h1, vector<co>& h, int a, int b, int d1, int d2) {
    int n=h.size();int k=getTanP(h[(b+n)%n], h1, d1);
    return (11)d2*ccw(h[(a+n)%n], h[(b+n)%n], h1[k]) <=0;
pair<int, int> getTanH(vector<co>& h1, vector<co>& h, int d1, int d2) {
    int n=h.size();int mi=0;int ma=n;
    while (mi+1<ma) {
       int mid=(mi+ma)/2;
       if (up2(h1, h, mi, mi+1, d1, d2)) {
            if (up2(h1, h, mid+1, mid, d1, d2)) ma=mid;
            else if(up2(h1, h, mi, mid, d1, d2)) mi=mid;
            else ma=mid:
```

```
else
            if (up2(h1, h, mid, mid+1, d1, d2)) mi=mid;
            else if (up2(h1, h, mid, mi, d1, d2)) mi=mid;
            else ma=mid;
    int step=0;
    if (d2==1)
        ma%=n:
        while (up2(h1, h, ma, ma+1, d1, d2)) {
            ma=(ma+1)%n; step++;
            assert(step<2);</pre>
        return {getTanP(h[ma], h1, d1), ma};
    else {
        while (up2(h1, h, mi, mi+1, d1, d2)) {
            mi=(mi+1)%n;step++;
            assert(step<3);</pre>
        if (up2(h1, h, mi, mi-1, d1, d2)) mi=(mi-1+n)%n;
        return {getTanP(h[mi], h1, d1), mi};
vector<pair<int, int> > hullHullTan(vector<co>& h1, vector<co>& h2) {
    vector<pair<int, int> > ret;
    ret.push_back(getTanH(h1, h2, 1, 1));
    ret.push_back(getTanH(h1, h2, 1, -1));
    ret.push_back(getTanH(h1, h2, -1, 1));
   ret.push_back(getTanH(h1, h2, -1, -1));
    return ret;
11 src/geometry/minkowskisum.cpp
// Computes the Minkowski sum of 2 convex polygons in O(n+m log n+m)
// Returns convex polygon in counterclockwise order
// The points on the edges of the hull are listed
// The convex hulls must be in counterclockwise order
#include <bits/stdc++.h>
#define X real()
#define Y imag()
#define F first
#define S second
```

```
using namespace std;
typedef long double ld;
typedef long long 11;
typedef complex<11> co;
11 ccw(co a, co b, co c) {
    return ((c-a)*conj(b-a)).Y;
int ar(co x) {
    if (x.Y)=0\&\&x.X<0 return 1;
    if (x.X)=0&&x.Y>0) return 2:
    if (x.Y \le 0 \& x.X > 0) return 3;
    return 4;
bool cp(pair<co, pair<int, int> > p1, pair<co, pair<int, int> > p2) {
    if (ar(p1.F)!=ar(p2.F))
       return ar(p1.F) < ar(p2.F);
    assert((ccw({0, 0}, p1.F, p2.F)=0)==(ccw({0, 0}, p2.F, p1.F)=0));
    if (ccw({0, 0}, p1.F, p2.F)==0) {
        return p1.S>p2.S;
    return ccw(\{0, 0\}, p2.F, p1.F)>0;
vector<co> minkowski(vector<co>& a, vector<co>& b) {
    int n=a.size();
    int m=b.size():
    if (n==0) return b;
    if (m==0) return a;
    if (n==1) {
        vector<co> ret(m);
       for (int i=0;i<m;i++) {
           ret[i]=b[i]+a[0];
        return ret;
    if (m==1) {
        vector<co> ret(n);
       for (int i=0;i<n;i++) {
           ret[i]=a[i]+b[0];
       return ret;
    vector<pair<co, pair<int, int> > > pp;
    int f1=0:
    int f2=0;
    for (int i=0;i<n;i++) {
```

```
if (ccw(a[(i-1+n)%n], a[i], a[(i+1)%n])!=0) {
            f1=i;break;
    for (int i=0;i<n;i++) {
        pp.push_back({a[(i+1+f1)%n]-a[(i+f1)%n], {1, i}});
    for (int i=0;i<m;i++) {
        if (ccw(b[(i-1+m)\%m], b[i], b[(i+1)\%m])!=0) {
            f2=i:break:
    for (int i=0;i<m;i++) {
        pp.push_back(\{b[(i+1+f2)\%m]-b[(i+f2)\%m], \{2, i\}\});
    sort(pp.rbegin(), pp.rend(), cp);
    co s={0, 0};
    co ad=\{0, 0\};
    for (int i=0;i<(int)pp.size();i++) {</pre>
        s+=pp[i].F;
        if (pp[i].S.F!=pp[i+1].S.F) {
            if (pp[i].S.F==1) ad=a[(pp[i].S.S+1+f1)%n]+b[(pp[i+1].S.S+f2)%m];
            else ad=b[(pp[i].S.S+1+f2)\mm]+a[(pp[i+1].S.S+f1)\mm];
            ad-=s:break:
    s=ad;
    vector<co> ret(pp.size());
    for (int i=0;i<(int)pp.size();i++) {</pre>
        ret[i]=s;s+=pp[i].F;
    return ret;
    src/geometry/basic.cpp
// Basic geometry functions using complex numbers
// Mostly copied from https://github.com/ttalvitie/libcontest/
/* Useful functions of complex number class
    CT abs(co x): Length
    CT norm(co x): Square of length
    CT arg(co x): Angle
    co polar(CT length, CT angle): Complex from polar components*/
#include <bits/stdc++.h>
#define X real()
```

```
#define Y imag()
using namespace std;
typedef long double ld;
typedef long long 11;
// Coordinate type
typedef ld CT;
typedef complex<CT> co;
ld eps=1e-12;
// Return true iff points a, b, c are CCW oriented.
bool ccw(co a, co b, co c) {
    return ((c-a)*conj(b-a)).Y>0;
// Return true iff points a, b, c are collinear.
// Note: doesn't make much sense with non-integer CT.
bool collinear(co a, co b, co c) {
    return abs(((c-a)*conj(b-a)).Y)<eps;</pre>
// Rotate x with agle ang
co rotate(co x, CT ang)
    return x*polar((CT)1, ang);
// Check whether segments [a, b] and [c, d] intersect.
// The segments must not be collinear. Doesn't handle edge cases (endpoint of
// a segment on the other segment) consistently.
bool intersects(co a, co b, co c, co d) {
    return ccw(a, d, b)!=ccw(a, c, b)\&\&ccw(c, a, d)!=ccw(c, b, d);
// Interpolate between points a and b with parameter t.
co interpolate(CT t, co a, co b) {
    return a+t*(b-a);
// Return interpolation parameter between a and b of projection of v to the
// line defined by a and b.
// Note: no rounding behavior specified for integers.
CT projectionParam(co v, co a, co b) {
    return ((v-a)/(b-a)).X;
// Compute the distance of point v from line a..b.
// Note: Only for non-integers!
CT pointLineDistance(co p, co a, co b) {
    return abs(((p-a)/(b-a)).Y)*abs(b-a);
// Compute the distance of point v from segment a..b.
// Note: Only for non-integers!
CT pointSegmentDistance(co p, co a, co b) {
    co z=(p-a)/(b-a);
```

```
if(z.X<0) return abs(p-a);
    if(z.X>1) return abs(p-b);
    return abs(z.Y)*abs(b-a);
// Return interpolation parameter between a and b of the point that is also
// on line c..d.
// Note: Only for non-integers!
// x=a*(1-t)+b*t
CT intersectionParam(co a, co b, co c, co d) {
    u = (c-a)/(b-a):
    co v=(d-a)/(b-a);
    return (u.X*v.Y-u.Y*v.X)/(v.Y-u.Y);
// Intersection points of circles with centers p1 and p2 with radiuses r1 and r2
// The first return value is the number of intersection points, 3 for infinite
pair<int, pair<co, co> > circleIntersection(co p1, CT r1, co p2, CT r2) {
    if (norm(p1-p2)>(r1+r2)*(r1+r2)||norm(p1-p2)<(r1-r2)*(r1-r2))
        return {0, {{0, 0}, {0, 0}}};
    if (abs(p1-p2) < eps \&\& abs(r1-r2) < eps)
        return {3, {{p1.X, p1.Y+r1}, {p1.X+r1, p1.Y}}};
    CT = a=abs(p1-p2);
    CT x=(r1*r1-r2*r2+a*a)/(2*a);
    co v1=\{x, sqrt(r1*r1-x*x)\};
    co v2=\{x, -sqrt(r1*r1-x*x)\};
    v1=v1*(p2-p1)/a+p1;
    v2=v2*(p2-p1)/a+p1;
    if (abs(v1-v2) < eps) return \{1, \{v1, v1\}\};
    return {2, {v1, v2}};
// Intersection of lines a..b and c..d
// Only for doubles
pair<int, co> lineIntersection(co a, co b, co c, co d) {
    if (collinear(a, b, c)&&collinear(a, b, d)) {
        return {2, a};
    else if(abs(((b-a)/(c-d)).Y)<eps) {
        return {0, {0, 0}};
    else {
        CT t=intersectionParam(a, b, c, d);
        return \{1, a*(1-t)+b*t\};
// Is b between a and c
// Only for doubles
int between(co a, co b, co c) {
```

```
return abs(abs(a-b)+abs(b-c)-abs(a-c))<eps;
// Intersection of segments a..b and c..d
// Only for doubles
// The first return value is the number of intersection points, 2 for infinite
// The second values are the endpoints of the intersection segment
pair<int, pair<co, co> > segmentIntersection(co a, co b, co c, co d)
    if (abs(a-b) < eps) {
        if (between(c, a, d)) return {1, {a, a}};
        else return \{0, \{0, 0\}\};
    else if (abs(c-d)<eps) {</pre>
        if (between(a, c, b)) return \{1, \{c, c\}\};
        else return \{0, \{0, 0\}\};
    else if (collinear(a, b, c)&&collinear(a, b, d)) {
        if (((b-a)/(d-c)).X<0) swap(c, d);
        co beg;
        if (between(a,c,b)) beg=c;
        else if (between(c,a,d)) beg=a;
        else return \{0, \{\{0, 0\}, \{0, 0\}\}\}\;
        co en=d;
        if (between(c, b, d)) en=b;
        if (abs(beg-en)<eps) return {1, {beg, beg}};</pre>
        return {2, {beg, en}};
    else if(abs(((b-a)/(c-d)).Y)<eps) {
        return {0, {0, 0}};
    else ·
        CT u=intersectionParam(a, b, c, d);
        CT v=intersectionParam(c, d, a, b);
        if (u < -eps||u > 1 + eps||v < -eps||v > 1 + eps)
            return {0, {{0, 0}, {0, 0}}};
        else {
            co p=a*(1-u)+b*u;
            return {1, {p, p}};
// Returns a point from the ray bisecting the non-reflex angle abc.
// Only for doubles. Returns 0 if the points are collinear.
pair<co,int> angleBisector(co a, co b, co c) {
    if (collinear(a,b,c)) return {{0, 0}, 0};
    co aa=(a-b)/abs(a-b):
```

```
co cc=(c-b)/abs(c-b):
    co bb=sqrt(aa/cc);
    return {b+bb*cc, 1};
     src/geometry/closestpoints.cpp
// TCR.
// Returns square of distance between closest 2 points
// O(n log n)
#include <bits/stdc++.h>
#define X real()
#define Y imag()
#define F first
#define S second
using namespace std;
typedef long long 11;
typedef complex<11> co;
const 11 inf=2e18;
11 csqrt(ll x) {
    11 r=sqrt(x);
    while (r*r < x) r++;
    while (r*r>x) r--;
    return r;
ll sq(ll x) {
    return x*x;
11 closestPoints(vector<co> points) {
    int n=points.size();
    vector<pair<11, 11> > ps(n);
    for (int i=0; i< n; i++) ps[i]=\{points[i].X, points[i].Y\};
    sort(ps.begin(), ps.end());
    int i2=0;11 d=inf;
    set<pair<11, 11> > pss;
    for (int i=0;i<n;i++)
        while (i2 < i \& sq(ps[i].F-ps[i2].F) > d) {
            pss.erase({ps[i2].S, ps[i2].F});i2++;
        auto it=pss.lower_bound({ps[i].S-csqrt(d), -inf});
        for (;it!=pss.end();it++) {
            if (sq(it->F-ps[i].S)>d) break;
            d=min(d, sq(it->F-ps[i].S)+sq(it->S-ps[i].F));
        pss.insert({ps[i].S, ps[i].F});
```

```
return d;
14 src/datastructure/orderedset.cpp
// TCR
// Sample code on how to use g++ ordered set
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
#include <ext/pb_ds/detail/standard_policies.hpp>
using namespace std;
using namespace __gnu_pbds;
//using namespace pb_ds;
typedef tree<int, null_type, less<int>, rb_tree_tag,
tree_order_statistics_node_update> ordered_set;
int main() {
    ordered_set X;
    X.insert(1); X.insert(4);
    cout<<*X.find_by_order(1)<<endl; // 4</pre>
    cout << X.order_of_key(3) << endl; // 1
     src/datastructure/dynamichull.cpp
// Data structure that maintains a set of lines in O(log n) query time
// Operations: insert line, find the highest line at {\tt x} coordinate {\tt x}
// Works with integers and doubles
// Cast too large integers to doubles when comparing to avoid overflow
#include <bits/stdc++.h>
using namespace std;
typedef long long 11;
const 11 isQuery=-(1LL<<62);</pre>
struct Line {
    ll m. b:int id:
    Line(ll m_, ll b_, int id_) : m(m_{-}), b(b_{-}), id(id_{-}) {}
    mutable multiset<Line>::iterator it,e;
    const Line* succ() const {
        return next(it) == e ? 0 : &*next(it);
    bool operator < (const Line& rhs) const
        if (rhs.b!=isQuery) return m<rhs.m;
        const Line* s=succ();
        if (!s) return 0:
```

```
11 x=rhs.m;
        return b-s->b<(s->m-m)*x;
};
struct DynamicHull : public multiset<Line> {
    bool bad(iterator y) {
        auto z=next(y);
        if (y==begin()) {
            if (z==end()) return 0;
            return y->m==z->m\&\&y->b<=z->b;
        auto x=prev(y);
        if (z==end()) return y->m==x->m\&\&y->b<=x->b;
        return (x->b-y->b)*(z->m-y->m)>=(y->b-z->b)*(y->m-x->m);
    void insertLine(ll m, ll b, int id) {
        auto y=insert({m, b, id});
        y->it=y;y->e=end();
        if (bad(y)) {erase(y);return;}
        while (next(y)!=end()&&bad(next(y))) erase(next(y));
        while (y!=begin()&&bad(prev(y))) erase(prev(y));
    pair<11, int> getMax(11 x) {
        auto l=*lower_bound({x, isQuery, 0});
        return {1.m*x+1.b, 1.id};
};
     src/datastructure/linkcut.cpp
// Link/cut tree. All operations are amortized O(log n) time
// Use functions link, cut and rootid for black box forest dynamic connectivity
#include <bits/stdc++.h>
using namespace std;
struct Node {
    Node* c[2], *p;
    int id, rev;
    int isr() {
        return |\mathbf{p}||(\mathbf{p}-\mathbf{c}[0]!=\mathbf{this\&\&p->c[1]}!=\mathbf{this});
    int dir() {
        return p->c[1]==this;
    void setc(Node* s, int d) {
        c[d]=s;
```

```
if (s) s->p=this;
    void push() {
        if (rev) {
            swap(c[0], c[1]);
            if (c[0]) c[0]->rev^=1;
           if (c[1]) c[1]->rev^=1;
           rev=0:
   Node(int i) : id(i) {
        c[0]=0;c[1]=0;p=0;rev=0;
};
struct LinkCut
    void rot(Node* x) {
        Node* p=x->p;int d=x->dir();
        if (!p->isr()) {
           p->p->setc(x, p->dir());
        else {
            x->p=p->p;
        p->setc(x->c[!d], d);x->setc(p, !d);
    void pp(Node* x) {
        if (!x->isr()) pp(x->p);
        x->push();
    void splay(Node* x)
       pp(x);
        while (!x->isr()) {
            if (x->p->isr()) rot(x);
            else if(x->dir()==x->p->dir()) {
                rot(x->p);rot(x);
            else {
                rot(x);rot(x);
    Node* expose(Node* x) {
        Node* q=0;
        for (;x;x=x->p)
            splay(x); x->c[1]=q; q=x;
```

```
return q;
    void evert(Node* x)
        x=expose(x); x->rev^=1; x->push();
    void link(Node* x, Node* y) {
        evert(x);evert(y);splay(y);x->setc(y, 1);
    void cut(Node* x, Node* y) {
        evert(x); expose(y); splay(x); x->c[1]=0; y->p=0;
    int rootid(Node* x) {
        expose(x);splay(x);
        while (x->c[0]) {
            x=x->c[0];x->push();
        splay(x);
        return x->id;
};
     src/datastructure/treap.cpp
// Treap implementation with pointers
// Expected running time of split and merge is O(log n)
#include <bits/stdc++.h>
using namespace std;
typedef struct node* pnode;
struct node {
    pnode l,r;
    int pr,c;
    node()
        l=0;r=0;c=1;pr=rand();
// Returns the size of the subtree t
int cnt(pnode t)
    if (t) return t->c;
    return 0;
// Updates the size of the subtree t
void upd(pnode t) -
    if (t) t->c=cnt(t->1)+cnt(t->r)+1;
// Put lazy updates here
```

```
void push(pnode t) {
    if (t) {}// Lazy update
// Merges trees 1 and r into tree t
void merg(pnode& t, pnode l, pnode r) {
    push(1);push(r);
    if (!1) t=r;
    else if(!r) t=1:
    else
        if (l->pr>r->pr) {
            merg(1->r, 1->r, r); t=1;
        else {
            merg(r->1, 1, r->1); t=r;
    upd(t);
// Splits tree t into trees 1 and r
// Size of tree 1 will be k
void split(pnode t, pnode& 1, pnode& r, int k) {
    if (!t) {
        l=0;r=0;return;
    else
        push(t);
        if (cnt(t->1)>=k)
            split(t->1, 1, t->1, k); r=t;
        else {
            split(t->r, t->r, r, k-cnt(t->l)-1); l=t;
    upd(t);
    src/datastructure/HLD.cpp
// TCR
// Builds Heavy-light decomposition of tree in O(n) time
// getPath returns decomposed path from a to b in a vector which contains
// \{\{u, v\}, \{index[u], index[v]\}\} index[u] <= index[v], depth[u] <= depth[v]
// lca(a, b) is in the last path of the vector
// Uses 1-indexing
#include <bits/stdc++.h>
using namespace std;
```

```
struct HLD
    vector<int> aps, pRoot, pLI, pRI, nPath, nPathId, p;
    int index;
   void dfs1(vector<int>* g, int x) {
       aps[x]=1;
       for (int nx:g[x])
           if (nx!=p[x])
               p[nx]=x;dfs1(g, nx);
               aps[x] += aps[nx];
   void dfs2(vector<int>* g, int x, int path, int pi) {
       if (path==-1)
           path=pRoot.size();
           pRoot.push_back(x);
           pLI.push_back(index);
           pRI.push_back(index);
       nPath[x]=path;
       nPathId[x]=pi;
       pRI[path]=index++;
       int ma=0;
       for (int nx:g[x]){
           if (nx!=p[x]&&aps[nx]>aps[ma]) ma=nx;
       if (ma) dfs2(g, ma, path, pi+1);
       for (int nx:g[x]){
           if (nx!=p[x]\&\&nx!=ma) dfs2(g, nx, -1, 0);
   HLD(vector < int > * g, int n) : aps(n+1), nPath(n+1), nPathId(n+1), p(n+1) 
       index=0;dfs1(g, 1);
       dfs2(g, 1, -1, 0);
   vector<pair<pair<int, int>, pair<int, int> > getPath(int a, int b) {
       vector<pair<int, int>, pair<int, int> > ret;
       while (nPath[a]!=nPath[b]) {
           int pa=nPath[a];
           int pb=nPath[b];
           if (pa>pb)
               ret.push_back({{pRoot[pa], a}, {pLI[pa], pLI[pa]+nPathId[a]}});
               a=p[pRoot[pa]];
           else ·
               ret.push_back({{pRoot[pb], b}, {pLI[pb], pLI[pb]+nPathId[b]}});
```

```
b=p[pRoot[pb]];
        int pa=nPath[a];
        if (nPathId[a]>nPathId[b]) swap(a, b);
        ret.push_back({{a, b}, {pLI[pa]+nPathId[a], pLI[pa]+nPathId[b]}});
        return ret;
};
     src/math/berlekampmassey.cpp
// TCR
// Berlekamp massey
// Give a sequence of integers in constructor and query with get(index)
#include <bits/stdc++.h>
using namespace std;
typedef long long 11;
ll powmod(ll a, ll p, ll modd)
   if (p==0) return 1;
    if (p\%2==0)
        a=powmod(a, p/2, modd);
        return (a*a) %modd;
    return (a*powmod(a, p-1, modd))%modd;
ll invp(ll a, ll p) {
    return powmod(a, p - 2, p);
vector<11> solve(vector<11> S, 11 mod) {
    vector < 11 > C = \{1\};
    vector < 11 > B = \{1\};
    11 L = 0; 11 m = 1; 11 b = 1; 11 N = S.size();
    for (11 i = 0; i < N; i++) {
       11 d = S[i]:
       for (ll j = 1; j <= L; j++) {
            d += C[j]*S[i - j];d %= mod;
        if (d == 0)
            m++;
        } else if (2*L <= i) {
            vector<ll> T = C;
            11 a = (invp(b, mod)*d)%mod;
            for (int j=0; j< i+1-2*L; j++) {
                C.push_back(0);
```

```
L=i+1-L:
            for (ll j = m; j \leq= L; j++) {
                C[j] -= a*B[j - m]; C[j] %= mod;
            B = T; b = d; m = 1;
       } else {
           11 a = (invp(b, mod)*d) \%mod;
           for (ll j = m; j < m+(int)B.size(); j++) {
                C[j] = a*B[j - m]; C[j] \% = mod;
            m++;
   for (ll i = 0; i <= L; i++) {
       C[i] += mod; C[i] %= mod;
   return C:
struct LinearRecurrence
    vector<vector<11> > mat;
   vector<11> seq;
   vector<vector<11> > mul(vector<vector<11> > a, vector<vector<11> > b) {
       int n=a.size():
       vector<vector<ll> > ret(n);
       for (int i=0; i< n; i++){
            ret[i].resize(n);
           for (int j=0; j< n; j++){
                ret[i][j]=0;
                for (int k=0; k< n; k++) {
                    ret[i][j]+=a[i][k]*b[k][j];
                    ret[i][j]%=mod;
       return ret;
    vector<vector<ll> > pot(vector<vector<ll> > m, ll p) {
       if (p==1) return m;
       if (p\%2==0) {
            m=pot(m, p/2);
            return mul(m, m);
       else
            return mul(m, pot(m, p-1));
```

```
11 get(11 index)
        if (index<(ll)mat.size()) {</pre>
            return seq[index];
        vector<vector<ll> > a=pot(mat, index-(ll)mat.size()+1);
        for (int i=0;i<(int)mat.size();i++) {</pre>
            v+=a[0][i]*seq[(int)mat.size()-i-1];
        return v;
    LinearRecurrence(vector<11> S, 11 mod_) {
        seq=S;
        mod=mod_;
        vector<11> C=solve(S, mod);
        int n=C.size()-1;
        mat.resize(n):
        for (int i=0;i<n;i++) {
            mat[i].resize(n);
        for (int i=0;i<n;i++) {
            mat[0][i]=(mod-C[i+1])%mod:
        for (int i=1;i<n;i++) {
            mat[i][i-1]=1;
};
     src/math/crt.cpp
// TCR
// (Generalised) Chinese remainder theorem (for arbitrary moduli):
// Solves x from system of equations x == a_i \pmod{m_i},
// giving answer modulo m = lcm(m_1, ..., m_n)
// Runs in O(\log(m)+n) time
// Overflows only if m overflows
// Returns \{1, \{x, m\}\} if solution exists, and \{-1, \{0, 0\}\} otherwise
#include <bits/stdc++.h>
using namespace std;
typedef long long 11;
typedef __int128 111;
ll ee(ll ca, ll cb, ll xa, ll xb, ll&x)
    if (cb) return ee(cb, ca\( cb, xb, xa-(ca/cb) * xb, x);
```

```
x = xa;
    return ca;
pair<int, pair<11, 11>> crt(vector<11> as, vector<11> ms) {
    11 aa = 0, mm = 1, d, a, x;
    for (int i = 0; i < (int) as.size(); i++) {
        d = ee(ms[i], mm, 1, 0, x);
        if ((aa-as[i])%d) return \{-1,\{0,0\}\};
        a = ms[i]/d;
        mm *= a:
        aa = (as[i] + (aa-as[i])*(((111)a*x)%mm))%mm;
    if (aa < 0) aa += mm;
    return {1, {aa, mm}};
     src/math/fftmod.cpp
// TCR
// Precise FFT modulo mod
#include <bits/stdc++.h>
using namespace std;
typedef long long 11;
// Number of form (2^20)*k+1
const 11 mod=1045430273;
// Number whose order mod mod is 2^20
const 11 root=363;
const 11 root_pw=1<<20;
// 128 bit
// typedef __int128 lll;
//const 111 mod=2097152000007340033;
//const lll root=2014907510281342032;
//const lll root_pw=1<<20;</pre>
ll pot(ll x, ll p) {
    if (p==0) return 1;
    if (p\%2==0) {
        x=pot(x, p/2);
        return (x*x)%mod;
    return (x*pot(x, p-1))%mod;
ll inv(ll x) {
    return pot(x, mod-2);
vector<ll> fft (vector<ll> a, int d) {
    11 root_1=inv(root):
```

```
int n=(int)a.size();
    for (int i=1, j=0; i < n; i++) {
        int bit=n>>1;
        for (; j>=bit; bit>>=1) j-=bit;
        j+=bit;
        if (i<j) swap (a[i], a[j]);</pre>
    for (int len=2;len<=n;len<<=1) {</pre>
        11 wlen=root;
        if (d==-1) wlen=root_1:
        for (int i=len;i<root_pw;i<<=1) wlen=(wlen*wlen)%mod;</pre>
        for (int i=0;i<n;i+=len) {</pre>
            11 w = 1;
            for (int j=0; j<len/2; j++)
                 11 u = a[i+j];
                11 v = (a[i+j+len/2]*w) \% mod;
                 if (u+v < mod) a[i+j]=u+v;
                 else a[i+j]=u+v-mod;
                 if (u-v)=0 a[i+j+len/2]=u-v;
                 else a[i+j+len/2]=u-v+mod;
                 w=(w*wlen)%mod;
    if (d==-1) {
        11 nrev=inv(n):
        for (int i=0;i<n;i++) a[i]=(a[i]*nrev)%mod;
    return a;
vector<11> conv(vector<11> a, vector<11> b) {
    int as=a.size();
    int bs=b.size();
    vector<11> aa(as);
    vector<11> bb(bs);
    for (int i=0;i<as;i++) aa[i]=a[i];
    for (int i=0;i<bs;i++) bb[i]=b[i];</pre>
    int n=1;
    while (n < as + bs - 1) n = 2;
    aa.resize(n*2);
    bb.resize(n*2);
    aa=fft(aa, 1);
    bb=fft(bb, 1);
    vector < 11 > c(2*n);
    for (int i=0;i<2*n;i++) c[i]=(aa[i]*bb[i])%mod;
    c=fft(c, -1):
```

```
c.resize(as+bs-1);
    return c:
int main() {
    // Shoud print 12 11 30 7
    vector<11> a={3, 2, 7};
   vector<11> b={4, 1};
    vector<11> c=conv(a, b);
    for (11 t:c) {
        cout << t << endl:
    src/math/gaussjordan.cpp
// TCR
// Solves system of linear equations in O(n m^2)
// Using doubles or mod 2
// Using doubles might have large precision errors or overflow
// Returns 0 if no solution exists, 1 if there is one solution
// or 2 if infinite number of solutions exists
// If at least one solution exists, it is returned in ans
// You can modify the general algorithm to work mod p by using modular inverse
#include <bits/stdc++.h>
using namespace std;
typedef long long 11;
typedef long double ld;
const ld eps=1e-12;
// Using doubles
int gaussD (vector<vector<ld> > a, vector<ld>& ans) {
    int n=(int)a.size();
    int m=(int)a[0].size()-1;
    vector<int> where(m,-1);
    for (int col=0,row=0;col<m\&\&row<n;col++) {
        int sel=row:
        for (int i=row;i<n;i++) {</pre>
            if (abs(a[i][col])>abs(a[sel][col])) sel=i;
        if (abs(a[sel][col]) < eps) continue;</pre>
        for (int i=col;i<=m;i++) swap (a[sel][i], a[row][i]);
        where [col] = row:
        for (int i=0;i<n;i++) {
            if (i!=row) {
                ld c=a[i][col]/a[row][col];
                for (int j=col; j<=m; j++) a[i][j]-=a[row][j]*c;
```

```
row++;
    ans.assign(m, 0);
    for (int i=0;i<m;i++) {</pre>
        if (where[i]!=-1) ans[i]=a[where[i]][m]/a[where[i]][i];
    for (int i=0;i<n;i++) {
        ld sum=0;
        for (int j=0;j<m;j++) sum+=ans[j]*a[i][j];</pre>
        if (abs(sum-a[i][m])>eps) return 0;
    for (int i=0;i<m;i++) {
        if (where[i]==-1) return 2;
    return 1;
// \mod 2
// n is number of rows m is number of variables
const int M=4;
int gaussM(vector<bitset<M> > a, int n, int m, bitset<M-1>& ans) {
    vector<int> where (m, -1);
    for (int col=0,row=0;col<m&&row<n;col++) {</pre>
        for (int i=row;i<n;i++) {</pre>
            if (a[i][col]) {
                swap (a[i], a[row]);
                break;
        if (!a[row][col]) continue;
        where[col]=row;
        for (int i=0;i<n;i++) {
            if (i!=row&&a[i][col]) a[i]^=a[row];
        row++;
    ans=0:
    for (int i=0;i<m;i++) {</pre>
        if (where[i]!=-1) ans[i]=a[where[i]][m];
    for (int i=0;i<n;i++) {
        int sum=0;
        for (int j=0; j<m; j++) sum^=ans[j]*a[i][j];
        if (sum!=a[i][m]) return 0;
    for (int i=0; i < m; i++) {
```

```
if (where[i]==-1) return 2;
   return 1;
int main() {
    // Should output 2, 1 2 0
    vector<vector<ld> > d(3);
    d[0]={3, 3, -15, 9};
    d[1] = \{1, 0, -2, 1\};
   d[2]={2, -1, -1, 0};
    vector<ld> da;
    cout<<gaussD(d, da)<<endl;</pre>
    cout < < da[0] << " " < < da[1] << " " < < da[2] << endl;
    // Should output 1, 110
    // Note that bitsets are printed in reverse order
    bitset<M> r1("0110");
    bitset<M> r2("1101");
    bitset<M> r3("0111");
    vector < bitset < M > m = \{r1, r2, r3\};
    bitset<M-1> ma;
    cout<<gaussM(m, 3, 3, ma)<<endl;</pre>
    cout << ma << endl;
    src/math/simplex.cpp
// Source: https://github.com/jaehyunp/stanfordacm/blob/master/code/Simplex.cc
// Two-phase simplex algorithm for solving linear programs of the form
       maximize
                    c^T x
      subject to Ax <= b
//
                    x >= 0
// INPUT: A -- an m x n matrix
         b -- an m-dimensional vector
          c -- an n-dimensional vector
         x -- a vector where the optimal solution will be stored
// OUTPUT: value of the optimal solution (inf if unbounded
           above, -inf if infeasible)
// To use this code, create an LPSolver object with A, b, and c as
// arguments. Then, call Solve(x).
#include <bits/stdc++.h>
using namespace std;
typedef long double ld;
struct LPSolver {
    const ld eps=1e-9;
    const ld inf=1e30;
```

```
int m, n;
vector<int> N, B;
vector<vector<ld> > D;
LPSolver(vector<vector<ld> >& A, vector<ld>& b, vector<ld>& c) :
m(b.size()), n(c.size()), N(n + 1), B(m), D(m + 2, vector < ld > (n + 2)) 
    for (int i = 0; i < m; i++) {
        for (int j = 0; j < n; j++) D[i][j] = A[i][j];
   for (int i = 0; i < m; i++) {
        B[i] = n + i; D[i][n] = -1; D[i][n + 1] = b[i];
   for (int j = 0; j < n; j++) { N[j] = j; D[m][j] = -c[j]; }
   N[n] = -1; D[m + 1][n] = 1;
void Pivot(int r, int s) {
   ld inv = 1.0 / D[r][s];
   for (int i = 0; i < m + 2; i++) if (i != r)
        for (int j = 0; j < n + 2; j++) if (j != s)
            D[i][j] = D[r][j] * D[i][s] * inv;
    for (int j = 0; j < n + 2; j++) if (j != s) D[r][j] *= inv;
    for (int i = 0; i < m + 2; i++) if (i != r) D[i][s] *= -inv;
   D[r][s] = inv;
    swap(B[r], N[s]);
bool Simplex(int phase)
    int x = phase == 1 ? m + 1 : m;
    while (true) {
        int s = -1:
        for (int j = 0; j <= n; j++) {
            if (phase == 2 && N[j] == -1) continue;
            if (s=-1||D[x][j]<D[x][s]||(D[x][j]==D[x][s]&&N[j]<N[s])) s=j;
        if (D[x][s] > -eps) return true;
        int r = -1;
       for (int i = 0; i < m; i++) {
            if (D[i][s] < eps) continue;
            if (r == -1 \parallel D[i][n + 1] / D[i][s] < D[r][n + 1] / D[r][s] \parallel
                ((D[i][n + 1]/D[i][s]) == (D[r][n+1]/D[r][s]) \&\&B[i] < B[r]))r = i;
        if (r == -1) return false;
        Pivot(r, s);
ld Solve(vector<ld>& x) {
    int r = 0;
    for (int i = 1; i < m; i++) if (D[i][n+1] < D[r][n+1]) r = i;
    if (D[r][n + 1] < -eps) {
        Pivot(r, n);
```

```
if (!Simplex(1) || D[m + 1][n + 1] < -eps) return -inf;
            for (int i = 0; i < m; i++) if (B[i] == -1) {
                int s = -1;
                for (int j = 0; j <= n; j++)
                if (s=-1||D[i][j]<D[i][s]||(D[i][j]==D[i][s]&&N[j]<N[s])) s=j;
                Pivot(i, s);
        if (!Simplex(2)) return inf;
        x = vector < ld > (n):
        for (int i = 0; i < m; i++) if (B[i] < n) x[B[i]] = D[i][n + 1];
        return D[m][n + 1];
};
int main(){
    const int m = 4; const int n = 3;
    1d A[m][n] = \{ \{ 6, -1, 0 \}, \{ -1, -5, 0 \}, \{ 1, 5, 1 \}, \{ -1, -5, -1 \} \};
   ld _b[m] = { 10, -4, 5, -5 };
   ld _c[n] = \{ 1, -1, 0 \};
    vector < vector < 1d > A(m); vector < 1d > b(_b, _b + m); vector < 1d > c(_c, _c + n);
    for (int i = 0; i < m; i++) A[i] = vector < ld > (A[i], A[i] + n);
   LPSolver solver(A, b, c);
    vector<ld> x;
    ld value = solver.Solve(x):
    cerr << "VALUE: " << value << endl; // VALUE: 1.29032
    cerr << "SOLUTION:"; // SOLUTION: 1.74194 0.451613 1</pre>
    for (size_t i = 0; i < x.size(); i++) cerr << " " << x[i];
    src/math/fft.cpp
// TCR
// Fast Fourier transform and convolution using it
// O(n log n)
#include <bits/stdc++.h>
using namespace std;
typedef long double ld;
typedef long long 11;
typedef complex<ld> co;
const ld PI=atan2(0, -1);
vector<co> fft(vector<co> x, int d) {
    int n=x.size();
   for (int i=0;i<n;i++) {
        int u=0:
        for (int j=1; j < n; j*=2) {
            u*=2:
```

```
if (i&j) u++;
        if (i < u) swap(x[i], x[u]);
    for (int m=2; m<=n; m*=2) {
        co wm=exp(co\{0, d*2*PI/m\});
        for (int k=0; k< n; k+=m)
            co w=1:
            for (int j=0; j < m/2; j++) {
                co t=w*x[k+j+m/2];
                co u=x[k+j];
                x[k+j]=u+t;
                x[k+j+m/2]=u-t;
                w*=wm;
    if (d==-1) for (int i=0; i< n; i++) x[i]/=n;
    return x:
vector<ll> conv(vector<ll> a, vector<ll> b){
    int as=a.size(), bs=b.size();
    int n=1;
    while (n < as + bs - 1) n = 2;
    vector < co > aa(n*2), bb(n*2);
    for (int i=0;i<as;i++) aa[i]=a[i];
    for (int i=0;i<bs;i++) bb[i]=b[i];
    aa=fft(aa, 1);bb=fft(bb, 1);
    vector<co> c(2*n);
    for (int i=0;i<2*n;i++) c[i]=aa[i]*bb[i];
    c=fft(c, -1);
    c.resize(as+bs-1);
    vector<11> r(as+bs-1);
    for (int i=0;i<as+bs-1;i++) r[i]=(l1)round(c[i].real());
    return r:
// Double FFT trick, not necessary
pair<vector<co>, vector<co> > tfft(vector<co>& a, vector<co>& b, int d) {
    vector<co> fv(a.size());
    for (int i=0;i<(int)a.size();i++) fv[i]=a[i]+co(0, 1)*b[i];
    vector<co> r=fft(fv, d);
    vector<co> r1(a.size()), r2(a.size());
    for (int i=0;i<(int)a.size();i++) {
        if (d=-1||i==0||i==(int)a.size()/2)
            r1[i]=r[i].real();r2[i]=r[i].imag();
        } else {
```

```
co t=r[i]-r[(int)a.size()-i];
            r1[i] = \{r[i] . real() - t . real()/2, t . imag()/2\};
            r2[i]=(r[i]-r1[i])*co(0, -1);
    return {r1, r2};
int main() {
    // Shoud print 12 11 30 7
   vector < 11 > a = {3, 2, 7};
   vector<11> b={4, 1};
    vector<11> c=conv(a, b);
    for (11 t:c) cout<<t<<endl;</pre>
    src/math/pollard-rho.cpp
// TCR
// Pollard Rho Integer factorization
// Support of 128 bit integers is required to factor over 32 bit integers
// requires isPrime function
// expected time complexity is O(n^{(1/4)})
#include <bits/stdc++.h>
using namespace std;
typedef long long 11;
typedef __int128 lll;
void step(11 \& x, 11 n, 11 c) {x=(111)((111)x*(111)x+(111)c)\%n;}
void rFactor(11 n, map<11, 11>& r) {
    while (n\%2==0) {
        n/=2;r[2]++;
   if (n==1) return;
    if (isPrime(n)) r[n]++;
    else {
        while (1) {
            ll x=rand()%n;ll y=x;
            11 c=rand()%n:
            for (ll i=0;i*i<=n;i++) {
                step(x, n, c); step(x, n, c); step(y, n, c);
                11 g = gcd(max(x, y) - min(x, y), n);
                if (g==n) break;
                else if(g>1) {
                    rFactor(n/g, r);
                    rFactor(g, r);
                    return;
```

```
map<11, 11> factor(11 n) {
    map<11, 11> ret;
    if (n>1) rFactor(n, ret);
    return ret:
26 src/math/miller-rabin.cpp
// TCR
// Deterministic Miller-Rabin primality test
// Works for all 64 bit integers
// Support of 128 bit integers is required to test over 32 bit integers
#include <bits/stdc++.h>
using namespace std;
typedef long long 11;
typedef __int128 lll;
111 powmod(111 a, 111 p, 111 mod) {
    if (p==0) return 1;
    if (p\%2==0)
        a=powmod(a, p/2, mod);
        return (a*a)%mod;
    return (a*powmod(a, p-1, mod))%mod;
bool is_w(ll a, ll even, ll odd, ll p) {
    111 u = powmod(a, odd, p);
    if (u==1) return 0;
    for (ll j=1;j<even;j*=2) {
        if (u==p-1) return 0;
        u*=u;u%=p;
    return 1;
bool isPrime(ll p) {
    if (p==2) return 1;
    if (p<=1||p\%2==0) return 0;
    11 odd=p-1;11 even=1;
    while (odd\%2==0) {
        even*=2;odd/=2;
    11 b[7]={2, 325, 9375, 28178, 450775, 9780504, 1795265022};
    for (ll i=0; i<7; i++) {
```

```
11 a=b[i]%p;
       if (a==0) return 1;
       if (is_w(a, even, odd, p)) return 0;
   return 1;
    src/math/primitiveroot.cpp
// Computes primitive root
// O(sqrt(n))
#include <bits/stdc++.h>
using namespace std;
typedef long long 11;
11 pot(11 x, 11 p, 11 mod)
   if (p==0) return 1;
    if (p\%2==0) {
       x=pot(x, p/2, mod);
       return (x*x)%mod;
   return (x*pot(x, p-1, mod))%mod;
11 primitiveRoot(11 p) {
    vector<11> fact;
   11 phi=p-1;11 n=phi;
   for (11 i=2;i*i<=n;i++) {
       if (n%i==0) {
           fact.push_back(i);
           while (n\%i==0) n/=i;
    if (n>1) fact.push_back (n);
    for (11 res=2;res<=p;res++) {
       bool ok = true;
       for (int i=0;i<(int)fact.size()&&ok;i++)ok&=pot(res, phi/fact[i], p)!=1;</pre>
       if (ok) return res;
   return -1;
int main() {
    cout<<pre>cout<<pre>cout
```

## 28 src/math/diophantine.cpp

// TCR

```
// Solves ax+by=c in O(log a+b) time
// Returns {is, {x, y}}, is=0 if there is no solution
// Use __int128 for 64 bit numbers
#include <bits/stdc++.h>
#define F first
#define S second
using namespace std;
typedef long long 11;
ll ee(ll a, ll b, ll ca, ll cb, ll xa, ll xb, ll&x, ll&y) {
    if (cb==0) {
       x=xa;
        if (b==0) y=0;
        else y=(ca-a*xa)/b;
       return ca;
    else return ee(a, b, cb, ca%cb, xb, xa-(ca/cb)*xb, x, y);
pair<int, pair<11, 11> > solve(11 a, 11 b, 11 c) {
    if (c==0) return {1, {0, 0}};
    if (a==0&&b==0) return {0, {0, 0}};
    11 x, y;
    ll g=ee(a, b, a, b, 1, 0, x, y);
    if (abs(c)\%g>0) return \{0, \{0, 0\}\};
    return \{1, \{x*(c/g), y*(c/g)\}\};
    src/graph/stronglyconnected.cpp
// TCR
// Kosaraju's algorithm for strongly connected components O(V+E)
// Components will be returned in topological order
// Uses 1-indexing
// Returns strongly connected components of the graph in vector ret
// n is the size of the graph, g is the adjacency list
#include <bits/stdc++.h>
using namespace std;
struct SCC
    vector<int> used;
    vector<vector<int> > g2;
    void dfs1(vector<int>* g, int x, vector<int>& ns) {
        if (used[x]==1) return;
        used[x]=1;
        for (int nx:g[x]) {
```

void dfs(int x, vector<int>& ret) {

```
g2[nx].push_back(x);
            dfs1(g, nx, ns);
       ns.push_back(x);
    void dfs2(int x, vector<int>& co) {
       if (used[x]==2) return;
       used[x]=2:
       co.push_back(x);
       for (int nx:g2[x]) dfs2(nx, co);
   SCC(vector<int>* g, int n, vector<vector<int> >& ret) : used(n+1), g2(n+1) {
       vector<int> ns;
       for (int i=1;i<=n;i++) dfs1(g, i, ns);
       for (int i=n-1;i>=0;i--) {
            if (used[ns[i]]!=2) {
               ret.push_back(vector<int>());
                dfs2(ns[i], ret.back());
    src/graph/eulertour.cpp
// NOT TESTED PROPERLY??
// Finds Euler tour of graph in O(E) time
// Parameters are the adjacency list, number of nodes, return value vector,
// and d=1 if the graph is directed
// Return array contains E+1 elements, the first and last elements are same
// Undefined behavior if Euler tour doesn't exist
// Note that Eulerian path can be reduced to Euler tour by adding an edge from
// the last vertex to the first
// In bidirectional graph edges must be in both direction
// Be careful to not add loops twice in case of bidirectional graph
#include <bits/stdc++.h>
#define F first
#define S second
using namespace std;
struct EulerTour
    int dir;
    vector<vector<pair<int, int> > > g;
   vector<int> used;
```

```
int t=x;vector<int> c;
        while (1) {
            while (used[g[t].back().S]) g[t].pop_back();
            auto nx=g[t].back();
            g[t].pop_back();
            used[nx.S]=1;t=nx.F;
            c.push_back(t);
            if (t==x) break;
        for (int a:c) {
            ret.push_back(a);
            while (g[a].size()>0&&used[g[a].back().S]) g[a].pop_back();
            if (g[a].size()>0) dfs(a, ret);
    EulerTour(vector<int>* og, int n, vector<int>& ret, int d=0):dir(d),g(n+1) {
        int i2=0;
        for (int i=1;i<=n;i++) {
            for (int nx:og[i]) {
                if (d==1||nx<=i) {
                    if (d==0&&nx<i) g[nx].push_back({i, i2});</pre>
                    g[i].push_back({nx, i2++});
        used.resize(i2);
        for (int i=1;i<=n;i++)
            if (g[i].size()>0)
                ret.push_back(i);
                dfs(i, ret);
                break;
};
    src/graph/cutvertices.cpp
// TCR
// Finds cutvertices and 2-vertex-connected components of graph
// 2-vertex-connected components are stored in bg
// Uses 1-indexing
#include <bits/stdc++.h>
#define F first
#define S second
using namespace std;
```

```
struct Biconnected {
    vector<int> cut, h, d, used;
    vector<map<int, vector<int> > bg;
    vector<pair<int, int> > es;
   int cc;
   void dfs(vector<int>* g, int x, int p) {
       h[x]=d[x];
       int f=0:
       for (int nx:g[x]) {
           if (nx!=p)
               if (!used[nx]) es.push_back({x, nx});
               if (d[nx]==0) {
                   f++;d[nx]=d[x]+1;
                   int ts=es.size();
                   dfs(g, nx, x);
                   h[x]=min(h[x], h[nx]);
                   if (h[nx] > = d[x])
                        cut[x]=1;
                        while ((int)es.size()>=ts) {
                            auto e=es.back();
                            bg[e.F][cc].push_back(e.S);
                            bg[e.S][cc].push_back(e.F);
                            used[e.S]=1;used[e.F]=1;
                            es.pop_back();
                        used[x]=0;cc++;
               h[x]=min(h[x], d[nx]);
       if (p==0) {
           if (f>1) cut[x]=1;
            else cut[x]=0;
    Biconnected(vector<int>* g, int n):cut(n+1),h(n+1),d(n+1),used(n+1),bg(n+1){
       cc=1;
       for (int i=1;i<=n;i++) {
           if (d[i]==0) {
               d[i]=1;dfs(g, i, 0);
```

## 32 src/graph/scalingflow.cpp

```
// TCR
// Scaling flow algorithm for maxflow
// O(E^2 log U), where U is maximum possible flow
// In practice O(E^2)
// Uses 1-indexing
#include <bits/stdc++.h>
#define F first
#define S second
using namespace std;
typedef long long 11;
struct MaxFlow {
    // Use vector<map<int, ll> > for sparse graphs
    vector<vector<ll> > f;
    vector<vector<int> > g;
    vector<int> used;
    int cc;
    11 flow(int x, int t, ll fl, ll miv)
        if (x==t) return fl;
        used[x]=cc;
        for (int nx:g[x]) {
            if (used[nx]!=cc\&\&f[x][nx]>=miv)
                11 r=flow(nx, t, min(fl, f[x][nx]), miv);
                if (r>0) {
                    f[x][nx]=r; f[nx][x]+=r;
                    return r;
        return 0;
    // maxv is maximum expected maxflow
    11 getMaxFlow(int source, int sink, 11 maxv) {
        cc=1;ll r=0;ll k=1;
        while (k*2 <= maxv) k*=2;
        for (;k>0;k/=2) {
            while (ll t=flow(source, sink, maxv, k)) {
                r+=t;cc++;
            cc++;
        return r;
    void addEdge(int a, int b, ll c) {
        if (f[a][b]==0&&f[b][a]==0) {
```

```
g[a].push_back(b);
            g[b].push_back(a);
       f[a][b]+=c;
   MaxFlow(int n) : f(n+1), g(n+1), used(n+1) 
       for (int i=1;i<=n;i++)
           f[i]=vector<ll>(n+1):
};
    src/graph/bridges.cpp
// Finds bridges and 2-edge connected components of graph
// Component of vertex x is c[x]
// Edge is a bridge iff its endpoints are in different components
// Graph in form {adjacent vertex, edge id}
// Uses 1-indexing
#include <bits/stdc++.h>
#define F first
#define S second
using namespace std;
struct Bridges {
   vector<int> c, h;
   void dfs(vector<pair<int, int> >* g, int x, int pe, int d, vector<int>& ns){
       if (h[x]) return;
       h[x]=d;ns.push_back(x);
       for (auto nx:g[x])
           if (nx.S!=pe) {
               dfs(g, nx.F, nx.S, d+1, ns);
               h[x]=min(h[x], h[nx.F]);
       if (h[x]==d) {
           while (ns.size()>0) {
               int t=ns.back();c[t]=x;
               ns.pop_back();
               if (t==x) break;
    Bridges(vector<pair<int, int> >* g, int n) : c(n+1), h(n+1) {
       vector<int> ns;
       for (int i=1;i<=n;i++) dfs(g, i, -1, 1, ns);
```

```
};
34 src/graph/mincostflow.cpp
// TCR
// Find minimum-cost k-flow
// O(VE) normalizing and O(E log V) for each augmenting path
// getKFlow augments at most k flow and returns {flow, cost}
// Uses 1-indexing
#include <bits/stdc++.h>
#define F first
#define S second
using namespace std;
typedef long long 11;
const ll inf=1e18;
template<int V, int E> struct MinCostFlow {
    struct Edge {
        int a, b;
       11 ca, co;
    } es[E*2]:
    int eu=0,nmz=0;
    vector<int> g[V+1];
    ll p[V+1],d[V+1];
    int fr[V+1],u[V+1];
    void addEdge(int a, int b, ll ca, ll co) {
        nmz=0;
        es[eu++]={a, b, ca, co};
        es[eu++]=\{b, a, 0, -co\};
        g[a].push_back(eu-2);
        g[b].push_back(eu-1);
    void normalize(int source)
        if (nmz) return;nmz=1;
        for (int i=1;i<=V;i++)
            p[i]=inf;u[i]=0;
        p[source]=0;
        queue<int> q;q.push(source);
        while (!q.empty()){
            int x=q.front();
            u[x]=0;q.pop();
            for (int e:g[x]) {
                if (es[e].ca>0&&p[x]+es[e].co<p[es[e].b]) {
                    p[es[e].b]=p[x]+es[e].co;
                    if (!u[es[e].b]) {
```

```
u[es[e].b]=1:
                    q.push(es[e].b);
11 augment(int x, ll fl) {
    if (fr[x]==-1) return fl;
    11 r=augment(es[fr[x]].a, min(fl, es[fr[x]].ca));
    es[fr[x]].ca-=r;
    es[fr[x]^1].ca+=r;
    return r;
pair<11, 11> flow(int source, int sink, 11 mf) {
    priority_queue<pair<11, int> > dij;
    for (int i=1;i<=V;i++) {
        u[i]=0;fr[i]=-1;d[i]=inf;
    d[source]=0;
    dij.push({0, source});
    while (!dij.empty())
        auto x=dij.top();dij.pop();
        if (u[x.S]) continue;
       u[x.S]=1;
       for (int e:g[x.S]) {
           11 nd=d[x.S]+es[e].co+p[x.S]-p[es[e].b];
            if (es[e].ca>0&&nd<d[es[e].b]) {
                d[es[e].b]=nd;
                fr[es[e].b]=e;
                dij.push({-nd, es[e].b});
    11 co=d[sink]+p[sink];
    for (int i=1;i<=V;i++) {
        if (fr[i]!=-1) p[i]+=d[i];
    if (u[sink]) {
        11 fl=augment(sink, mf);
        return {fl, fl*co};
    else return {0, 0};
pair<11, 11> getKFlow(int source, int sink, 11 k) {
    11 fl=0;11 co=0;
```

```
normalize(source):
        while (1) {
            pair<11, 11> t=flow(source, sink, k);
            fl+=t.F;k-=t.F;co+=t.S;
            if (k==0||t.F==0) break;
        return {fl, co};
};
    src/graph/dynamicconnectivity.cpp
// TCR
// O(n log n) offline solution for dynamic connectivity problem.
// Query types:
// {1, {a, b}} add edge. If edge already exists nothing happns.
// {2, {a, b}} remove edge. If no edge exists nothing happens.
// {3, {0, 0}} count number of connected components.
// Uses 1-indexing
#include <bits/stdc++.h>
#define F first
#define S second
using namespace std;
struct DynamicConnectivity {
    struct Edge {
        int a, b, 1, r;
    vector<int> ret, tq, id, is;
    vector<vector<int> > g;
    int dfs(int x, int c) {
        id[x]=c;int r=is[x];
        for (int nx:g[x]) if (!id[nx]) r = dfs(nx, c);
        return r;
    void go(int 1, int r, int n, int out, vector<Edge>& es) {
        vector<Edge> nes;
        for (int i=1;i<=n;i++) {
            g[i].clear();
            id[i]=0;is[i]=0;
        for (auto e:es) {
            if (e.1>r||e.r<1||e.a==e.b) continue;
            if (e.l<=l&&r<=e.r) {
                g[e.a].push_back(e.b);
                g[e.b].push_back(e.a);
```

```
else
           nes.push_back(e);
           is[e.a]=1;is[e.b]=1;
   int i2=1;
   for (int i=1;i<=n;i++) {
       if ((int)g[i].size()>0||is[i]) {
           if (!id[i]) {
               int a=dfs(i, i2);
               if (!a) out++;
               else i2++;
       else out++;
   for (auto&e:nes) {
        e.a=id[e.a];e.b=id[e.b];
   if (l==r) {
       if (tq[1]) ret[tq[1]-1]=out+i2-1;
   else {
       int m=(1+r)/2;
       go(1, m, i2-1, out, nes);
       go(m+1, r, i2-1, out, nes);
vector<int> solve(int n, vector<pair<int, pair<int, int> > queries) {
   map<pair<int, int>, int> ae;
   tq.resize(queries.size());
   id.resize(n+1);
   is.resize(n+1);
   g.resize(n+1);
   int qs=0;vector<Edge> es;
   for (int i=0;i<(int)queries.size();i++) {</pre>
       auto q=queries[i];
       if (q.S.F>q.S.S) swap(q.S.F, q.S.S);
       if (q.F==1) {
           if (ae[q.S]==0) ae[q.S]=i+1;
        else if(q.F==2) {
           if (ae[q.S])
               es.push_back({q.S.F, q.S.S, ae[q.S]-1, i});
               ae[q.S]=0;
```

```
}
    else if (q.F==3) {
        tq[i]=1+qs++;
    }
}
for (auto e:ae) {
        if (e.S) es.push_back({e.F.F, e.F.S, e.S-1, (int)queries.size()});
    }
    ret.resize(qs);
    if ((int)queries.size()>0) go(0, (int)queries.size()-1, n, 0, es);
    return ret;
}
```