Advanced Encryption Standard (AES)

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Advanced Encryption Standard AES

"It seems very simple."

"It is very simple. But if you don't know what the key is it's virtually indecipherable."

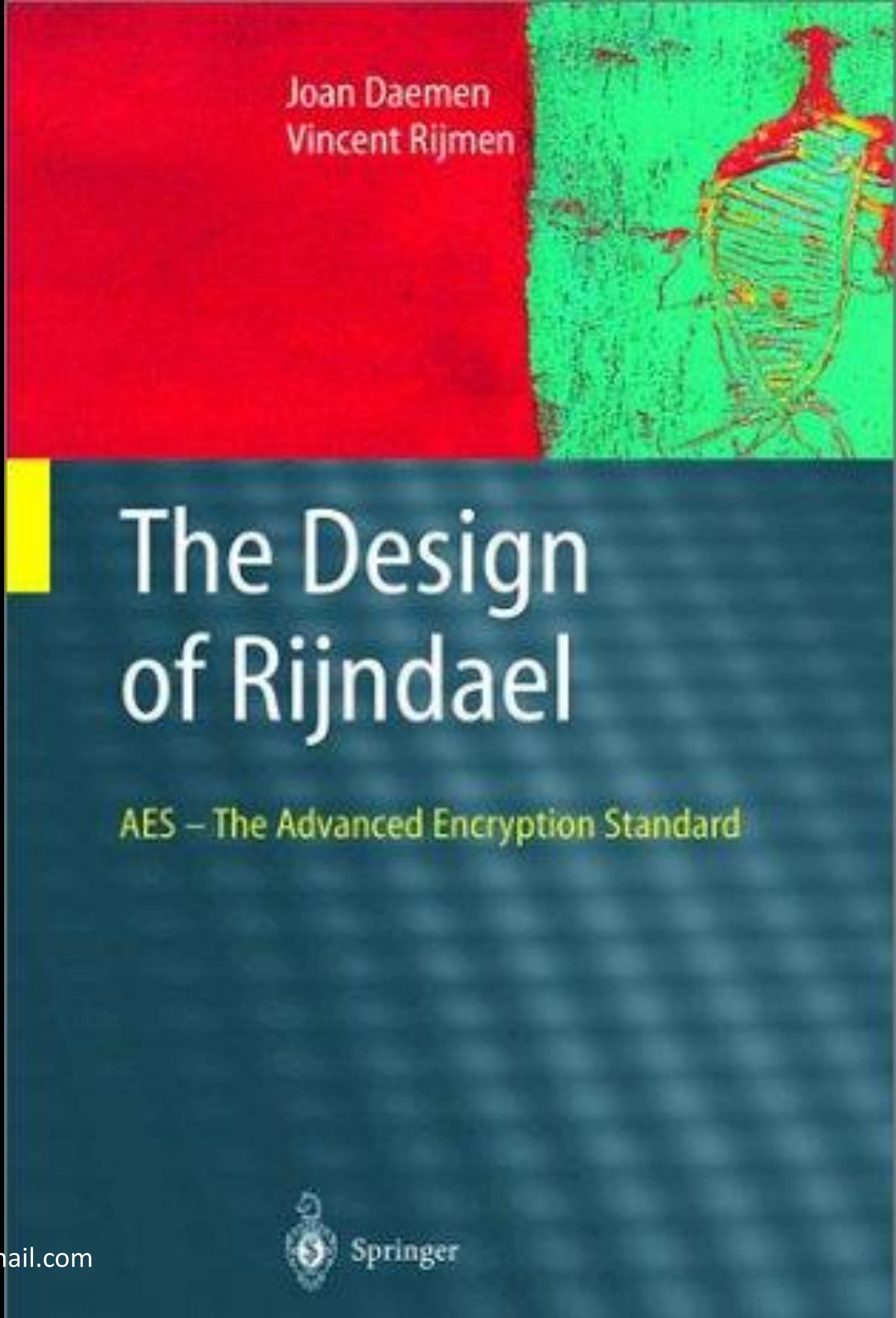
—*Talking to Strange Men,* Ruth Rendell

Advanced Encryption Standard

- In 1997, NIST made a formal call for algorithms stipulating that the AES would specify an unclassified, publicly disclosed encryption algorithm, available royalty-free, worldwide.
- Goal: replace DES for both government and private-sector encryption.
- The algorithm must implement symmetric key cryptography as a block cipher and (at a minimum) support block sizes of 128-bits and key sizes of 128-, 192-, and 256-bits.
- In 1998, NIST selected 15 AES candidate algorithms.
- On October 2, 2000, NIST selected **Rijndael** (invented by Joan Daemen and Vincent Rijmen) to as the AES.

AES Features

- Designed to be efficient in both hardware and software across a variety of platforms.
- Not a Feistel Network
- Block size: 128 bits
- Variable key size: **128, 192, or 256 bits.**
- Variable number of rounds (10, 12, 14):
  - 10 if K = 128 bits
  - 12 if K = 192 bits
  - 14 if K = 256 bits
- No known weaknesses



Origins

- a replacement for DES was needed
  - have theoretical attacks that can break it
  - have demonstrated exhaustive key search attacks
- can use Triple-DES – but slow with small blocks
- US NIST issued call for ciphers in 1997
- 15 candidates accepted in Jun 98
- 5 were short-listed in Aug-99
- Rijndael was selected as the AES in Oct-2000
- issued as *Federal Information Processing Standards (FIPS)* PUB 197 standard in Nov-2001

AES Requirements

- private key symmetric block cipher
- 128-bit data, 128/192/256-bit keys
- stronger & faster than Triple-DES
- active life of 20-30 years (+ archival use)
- provide full specification & design details
- both C & Java implementations
- NIST have released all submissions & unclassified analyses

AES Evaluation Criteria

- initial criteria:
  - security – effort to practically cryptanalyse
  - cost – computational
  - algorithm & implementation characteristics
- final criteria
  - general security
  - software & hardware implementation ease
  - implementation attacks
  - flexibility (in en/decrypt, keying, other factors)

AES Shortlist

- after testing and evaluation, shortlist in Aug-99:
  - MARS (IBM) - complex, fast, high security margin
  - RC6 (USA) - v. simple, v. fast, low security margin
  - Rijndael (Belgium) - clean, fast, good security margin
  - Serpent (Euro) - slow, clean, v. high security margin
  - Twofish (USA) - complex, v. fast, high security margin
- then subject to further analysis & comment
- saw contrast between algorithms with
  - few complex rounds verses many simple rounds
  - which refined existing ciphers verses new proposals

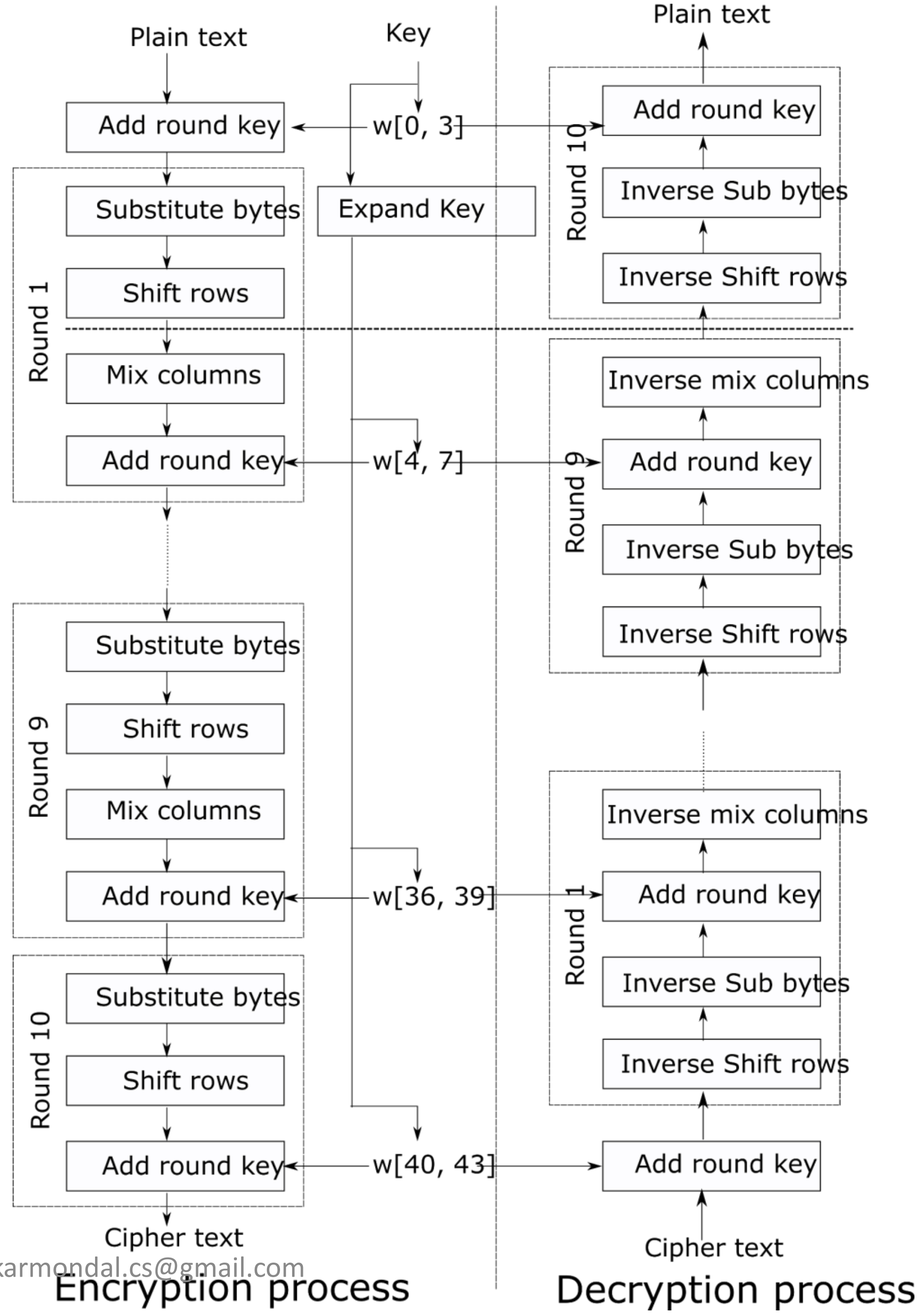
The AES Cipher - Rijndael

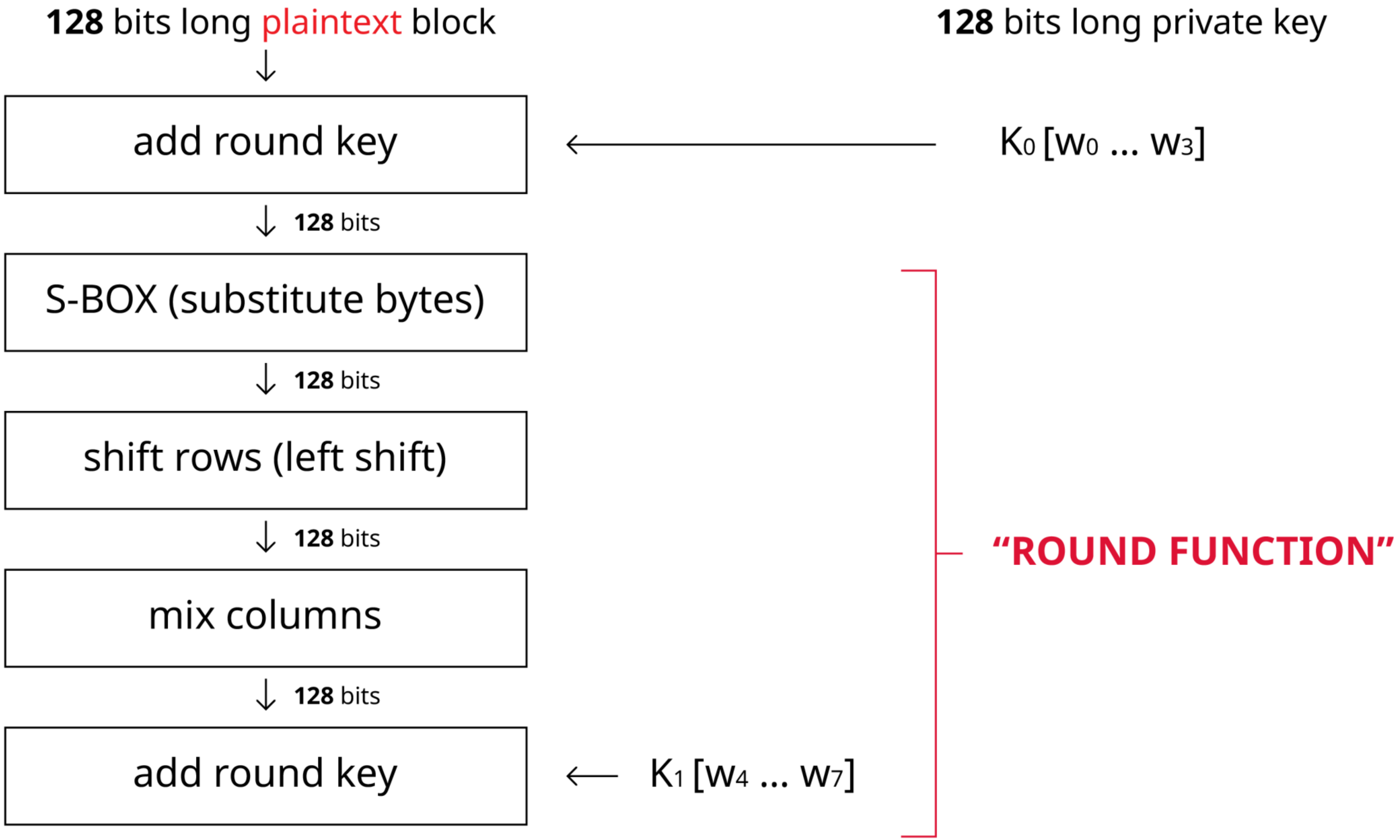
- designed by Rijmen-Daemen in Belgium
- has 128/192/256 bit keys, 128 bit data
- 10 rounds for 128-bit key length, 12 rounds for 192-bit key length, 14 rounds for 256-bit key length
- an **iterative** rather than **feistel** cipher
  - treats data in 4 groups of 4 bytes
  - operates an entire block in every round
- designed to be:
  - resistant against known attacks
  - speed and code compactness on many CPUs
  - design simplicity

Rijndael

- processes data as 4 groups of 4 bytes (state)
- has 9/11/13 rounds in which state undergoes:
  - byte substitution (1 S-box used on every byte)
  - shift rows (permute bytes between groups/columns)
  - mix columns (subs using matrix multiply of groups)
  - add round key (XOR state with key material)
- initial XOR key material & incomplete last round
- all operations can be combined into XOR and table lookups - hence very fast & efficient

Rijndael


Plain text
Key
Add round key
w[0, 3]
Substitute bytes
Expand Key
Shift rows
Round 1
Mix columns
Add round key
w[4, 7]
Substitute bytes
Shift rows
Round 9
Mix columns
Add round key
w[36, 39]
Substitute bytes
Round 10
Shift rows
Add round key
w[40, 43]
Cipher text
Encryption process
Plain text
Add round key
Round 10
Inverse Sub bytes
Inverse Shift rows
Inverse mix columns
Add round key
Round 9
Inverse Sub bytes
Inverse Shift rows
Inverse mix columns
Add round key
Round 1
Inverse Sub bytes
Inverse Shift rows
Add round key
Cipher text
Decryption process

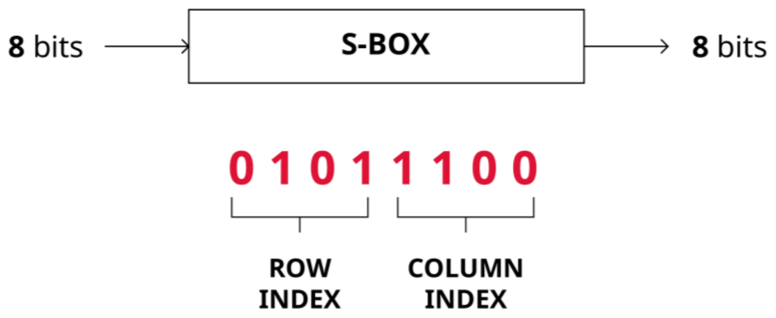

128 bits long plaintext block
128 bits long private key
add round key
K_0 [w_0 ... w_3]
128 bits
S-BOX (substitute bytes)
128 bits
shift rows (left shift)
128 bits
mix columns
128 bits
add round key
K_1 [w_4 ... w_7]
"ROUND FUNCTION"

We represent the data (plaintext, ciphertext and key) as metrixes

p_0	p_4	p_8	p_{12}
p_1	p_5	p_9	p_{13}
p_2	p_6	p_{10}	p_{14}
p_3	p_7	p_{11}	p_{15}

k_0	k_4	k_8	k_{12}
k_1	k_5	k_9	k_{13}
k_2	k_6	k_{10}	k_{14}
k_3	k_7	k_{11}	k_{15}

	00	01	02	03	04	05	06	07	08	09	0a	0b	0c	0d	0e	0f
00	63	7c	77	7b	f2	6b	6f	c5	30	01	67	2b	fe	d7	ab	76
10	ca	82	c9	7d	fa	59	47	f0	ad	d4	a2	af	9c	a4	72	c0
20	b7	fd	93	26	36	3f	f7	cc	34	a5	e5	f1	71	d8	31	15
30	04	c7	23	c3	18	96	05	9a	07	12	80	e2	eb	27	b2	75
40	09	83	2c	1a	1b	6e	5a	a0	52	3b	d6	b3	29	e3	2f	84
50	53	d1	00	ed	20	fc	b1	5b	6a	cb	be	39	4a	4c	58	cf
60	d0	ef	aa	fb	43	4d	33	85	45	f9	02	7f	50	3c	9f	a8
70	51	a3	40	8f	92	9d	38	f5	bc	b6	da	21	10	ff	f3	d2
80	cd	0c	13	ec	5f	97	44	17	c4	a7	7e	3d	64	5d	19	73
90	60	81	4f	dc	22	2a	90	88	46	ee	b8	14	de	5e	0b	db
a0	e0	32	3a	0a	49	06	24	5c	c2	d3	ac	62	91	95	e4	79
b0	e7	c8	37	6d	8d	d5	4e	a9	6c	56	f4	ea	65	7a	ae	08
c0	ba	78	25	2e	1c	a6	b4	c6	e8	dd	74	1f	4b	bd	8b	8a
d0	70	3e	b5	66	48	03	f6	0e	61	35	57	b9	86	c1	1d	9e
e0	e1	f8	98	11	69	d9	8e	94	9b	1e	87	e9	ce	55	28	df
f0	8c	a1	89	0d	bf	e6	42	68	41	99	2d	0f	b0	54	bb	16



Byte Substitution

- a simple substitution of each byte
- uses one table of 16x16 bytes containing a permutation of all 256 8-bit values
- each byte of state is replaced by byte in row (left 4-bits) & column (right 4-bits)
  - eg. byte {95} is replaced by row 9 col 5 byte
  - which is the value {2A}
- S-box is constructed using a defined transformation of the values in GF(2^8)
- designed to be resistant to all known attacks

Shift Rows

a circular byte shift in each row

- 1st row is unchanged
- 2nd row does 1 byte circular shift to left
- 3rd row does 2 byte circular shift to left
- 4th row does 3 byte circular shift to left

decrypt does shifts to right

since state is processed by columns, this step permutes bytes between the columns

Shift Rows

S_0	S_4	S_8	S_{12}	⟵ circular left shift with 0 step
S_1	S_5	S_9	S_{13}	⟵ circular left shift with 1 steps
S_2	S_6	S_{10}	S_{14}	⟵ circular left shift with 2 steps
S_3	S_7	S_{11}	S_{15}	⟵ circular left shift with 3 steps

S_0	S_4	S_8	S_{12}
S_1	S_5	S_9	S_{13}
S_2	S_6	S_{10}	S_{14}
S_3	S_7	S_{11}	S_{15}

⟶

S_0	S_4	S_8	S_{12}
S_5	S_9	S_{13}	S_1
S_{10}	S_{14}	S_2	S_6
S	S_3	S_7	S_{11}

Mix Columns

- each column is processed separately
- each byte is replaced by a value dependent on all 4 bytes in the column
- effectively a matrix multiplication in GF(2^8) using prime poly $m(x) = x^8+x^4+x^3+x+1$

$$\begin{bmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{bmatrix} \begin{bmatrix} s_{0,0} & s_{0,1} & s_{0,2} & s_{0,3} \\ s_{1,0} & s_{1,1} & s_{1,2} & s_{1,3} \\ s_{2,0} & s_{2,1} & s_{2,2} & s_{2,3} \\ s_{3,0} & s_{3,1} & s_{3,2} & s_{3,3} \end{bmatrix} = \begin{bmatrix} s'_{0,0} & s'_{0,1} & s'_{0,2} & s'_{0,3} \\ s'_{1,0} & s'_{1,1} & s'_{1,2} & s'_{1,3} \\ s'_{2,0} & s'_{2,1} & s'_{2,2} & s'_{2,3} \\ s'_{3,0} & s'_{3,1} & s'_{3,2} & s'_{3,3} \end{bmatrix}$$

Mix Columns

$$\begin{bmatrix} 2 & 3 & 1 & 1 \\ 1 & 2 & 3 & 1 \\ 1 & 1 & 2 & 3 \\ 3 & 1 & 1 & 2 \end{bmatrix} \times \begin{bmatrix} s_0 \\ s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} s'_0 \\ s'_1 \\ s'_2 \\ s'_3 \end{bmatrix}$$

$$\begin{bmatrix} s_0 & s_4 & s_8 & s_{12} \\ s_1 & s_5 & s_9 & s_{13} \\ s_2 & s_6 & s_{10} & s_{14} \\ s_3 & s_7 & s_{11} & s_{15} \end{bmatrix} \longrightarrow \begin{bmatrix} s'_0 & s'_4 & s'_8 & s'_{12} \\ s'_1 & s'_5 & s'_9 & s'_{13} \\ s'_2 & s'_6 & s'_{10} & s'_{14} \\ s'_3 & s'_7 & s'_{11} & s'_{15} \end{bmatrix}$$

Add Round Key

- XOR state with 128-bits of the round key
- again processed by column (though effectively a series of byte operations)
- inverse for decryption is identical since XOR is own inverse, just with correct round key
- designed to be as simple as possible

1b	22	cb	03
7c	ae	f4	ba
14	01	1b	4f
09	a6	88	4a

…

subkey generation

private key represented as two-dimensional array, and each block has 1byte.

1b	22	cb	03									
7c	ae	f4	ba							…		
14	01	1b	4f									
09	a6	88	4a									

ba
4f
4a
03

subkey generation

First take the right-most column, and execute circular upward shift

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K_{i-4}			K_{i-1}	K_i								
1b	22	cb	03									
7c	ae	f4	ba							...		
14	01	1b	4f									
09	a6	88	4a									

$$\begin{bmatrix} 1b \\ 7c \\ 14 \\ 09 \end{bmatrix} \textbf{ XOR } \begin{bmatrix} f4 \\ 84 \\ d6 \\ 7b \end{bmatrix} \textbf{ XOR } \begin{bmatrix} 01 \\ 00 \\ 00 \\ 00 \end{bmatrix} = \begin{bmatrix} 03 \\ ab \\ 4c \\ a5 \end{bmatrix}$$

subkey generation

XOR operation with K_(i-4) columns and take the predefined value from rcon table, and do XOR operation again. The result first column of current round subkey.

	K_{i-4}			K_{i-1}	K_i							
1b	22	cb	03	03						…		
7c	ae	f4	ba	ab								
14	01	1b	4f	4c								
09	a6	88	4a	a5								

$$\begin{bmatrix} 22 \\ ae \\ 01 \\ a6 \end{bmatrix} \ \textbf{XOR} \ \begin{bmatrix} 03 \\ ab \\ 4c \\ a5 \end{bmatrix} = \begin{bmatrix} 01 \\ 22 \\ a3 \\ 88 \end{bmatrix}$$

subkey generation

Generating 2nd, 3rd and last column of subkey is rather simple, just do XOR operation on K_(i-1) and K_(i-4) column.

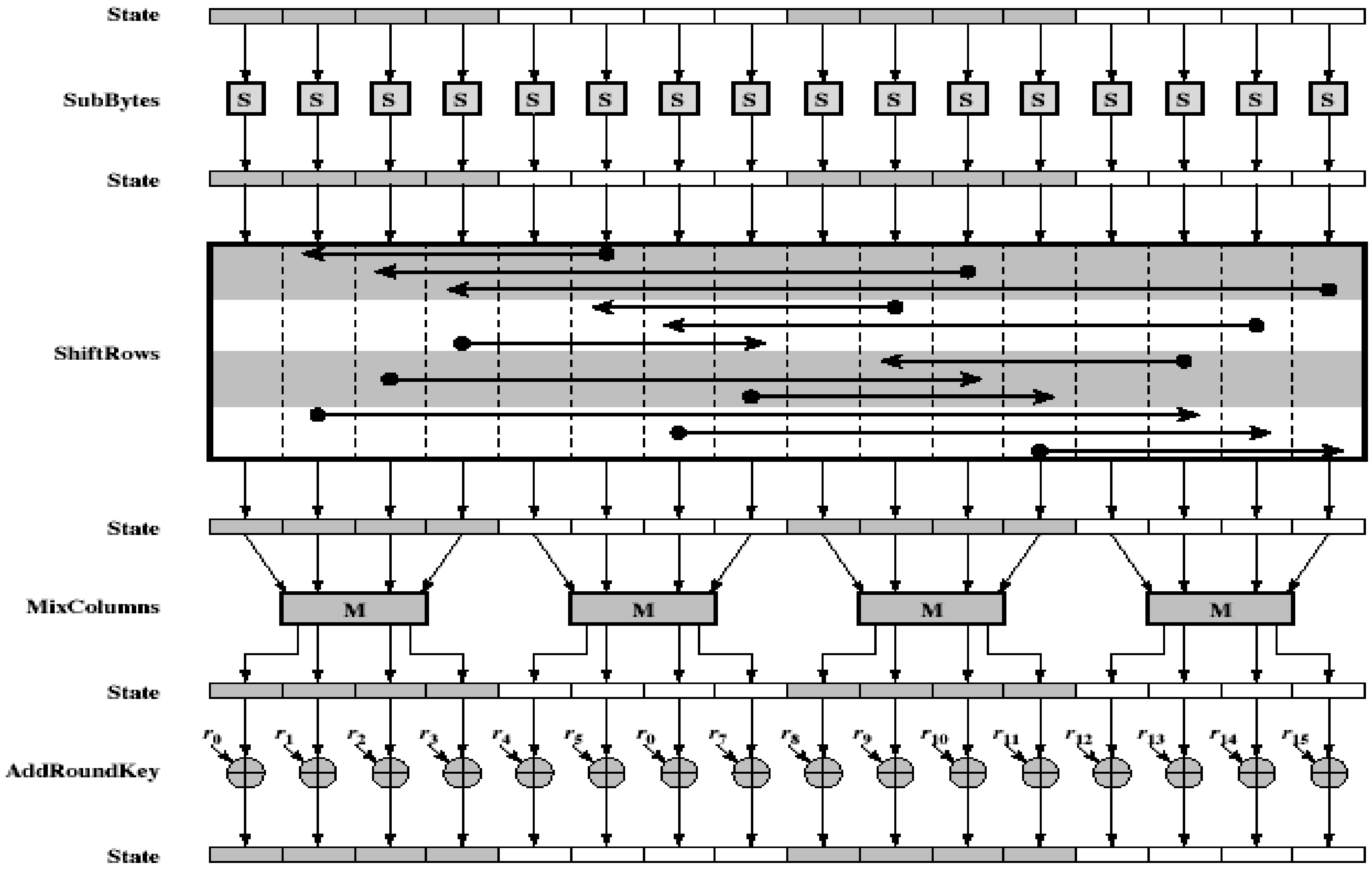
1b	22	cb	03	03	01	f1	23		
7c	ae	f4	ba	ab	22	ac	a3		
14	01	1b	4f	4c	03	02	39		
09	a6	88	4a	a5	88	22	39		

…

subkey generation

Key generated

AES Round



State
SubBytes
State
ShiftRows
State
MixColumns
State
AddRoundKey
State

AES Key Expansion

- takes 128-bit (16-byte) key and expands into array of 44/52/60 32-bit words
- start by copying key into first 4 words
- then loop creating words that depend on values in previous & 4 places back
  - in 3 of 4 cases just XOR these together
  - every 4th has S-box + rotate + XOR constant of previous before XOR together
- designed to resist known attacks

AES Decryption

- AES decryption is not identical to encryption since steps done in reverse
- but can define an equivalent inverse cipher with steps as for encryption
  - but using inverses of each step
  - with a different key schedule
- works since result is unchanged when
  - swap byte substitution & shift rows
  - swap mix columns & add (tweaked) round key

Implementation Aspects

- can efficiently implement on 8-bit CPU
  - byte substitution works on bytes using a table of 256 entries
  - shift rows is simple byte shifting
  - add round key works on byte XORs
  - mix columns requires matrix multiply in $GF(2^8)$ which works on byte values, can be simplified to use a table lookup

Implementation Aspects

- can efficiently implement on 32-bit CPU
  - redefine steps to use 32-bit words
  - can pre-compute 4 tables of 256-words
  - then each column in each round can be computed using 4 table lookups + 4 XORs
  - at a cost of 16Kb to store tables
- designers believe this very efficient implementation was a key factor in its selection as the AES cipher

Summary

- have considered:
  - the AES selection process
  - the details of Rijndael – the AES cipher
  - looked at the steps in each round
  - the key expansion
  - implementation aspects

brute force exhaustive search of AES-256

Computing power	Average time to crack using exhaustive search
High-end PC	27,337,893,038,406,611,194,430,009,974,922,940,323,611,067,429,756,962,487, 493,203 years. 27 trillion trillion trillion trillion trillion years
Fastest supercomputer	27,337,893,038,406,611,194,430,009,974,922,940,323,611,067,429,756,962,487 years. 27,337,893 trillion trillion trillion trillion years
2 billion high-end PCs	13,668,946,519,203,305,597,215,004,987,461,470,161,805,533,714,878,481 years 13,689 trillion trillion trillion trillion years
Age of the universe	15,000,000,000 years 15 billion years

https://scrambox.com/article/brute-force-aes/#:~:text=On%20average%2C%20to%20brute%2Dforce,simply%202255%20%2F%202%2C117.8%20trillion.

References

- William Stallings, Network Security Essentials : Applications and Standards, ISBN: 9788131761755, 8131761754

- Thanks to the many unknown sources from where some information is adopted.