Formal Specification

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(Source: Fundamentals of Software Engineering by Dr. RAJIB Mall)

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- **Algebraic specification:**
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Formal Definitions

- **Formal specification techniques:**
  - **model-oriented**
  - **property-oriented**

Formal Definitions

- **Property-oriented techniques:**
  - **axiomatic specification**
  - **algebraic specification**
- **Model-oriented techniques:**
  - **Z, VDM, Petri net, State machine, etc.**

Formal Definitions

- **Axiomatic techniques:**
  - **based on early work on program verification.**
  - **Use first-order predicate logic:**
    - **specify operations through pre and post conditions**

Axiomatic specification

- **Example**

$$F(x{:}\, real){:}\ real$$

Pre: $\boldsymbol{x} \in \boldsymbol{R}$

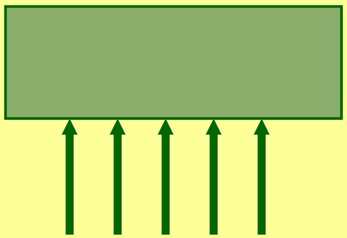
Post: $\left\{(\boldsymbol{x} \leq \mathbf{100}) \cap \left(\boldsymbol{f}(\boldsymbol{x}) = \frac{\boldsymbol{x}}{\mathbf{2}}\right)\right\} \cup$
$\{(\mathbf{x} > \mathbf{100}) \cap (\boldsymbol{f}(\boldsymbol{x}) = \mathbf{2} * \boldsymbol{x})\}$

Formal Definitions

- Algebraic technique:
  - data types are viewed as heterogeneous algebra
  - axioms are used to state properties of data type operations

Algebraic Specification

- **Using algebraic specification:**



  - **the meaning of a set of interface procedures is defined by using equations.**

Algebraic Specification

- Algebraic specifications are usually presented in four parts:
  - types section
  - exceptions section
  - signature section
  - rewrite rules section

Types Section

- **Types Section Lists:**
  - **sorts (or types) being specified**
  - **sorts being imported**
  - **Importing a sort:**
    - **makes it available in specification.**

Exception Section

- **Lists names of exceptional conditions used in later sections:**
  - **under exceptional conditions error should be indicated.**

Signature Section

- **Defines signatures of interface procedures:**
  - **e.g. PUSH takes a stack and an element and returns a new stack.**
  - **push:**
    - **stack $\times$ element $\rightarrow$ stack**

Rewrite rules section

- Lists the properties of the operators:
  - In the form of a set of axioms or rewrite rules.
    - allowed to have conditional expressions

Developing Algebraic Specification

- The first step in defining an algebraic specification:
  - identify the set of required operations.
  - e.g. for string identify operations:
    - create, compare, concatenate, length, etc.

Developing Algebraic Specification

- Generally operations fall into 2 classes:
- **Constructor Operations :**
  - Operations which create or modify entities of the sort e.g., create, update, add, etc.

Developing Algebraic Specification

- **Inspection Operations :**
  - **Operations which evaluate attributes of the sort, e.g., eval, get, etc.**

Developing Algebraic Specification

- A rule of thumb for writing algebraic specifications:
  - first establish constructor and inspection operations

Developing Algebraic Specifications

- **Next, write down axioms:**
  - **compose each inspection operator over each constructor operator.**

Developing Algebraic Specifications

- **If there are m constructors and n inspection operators:**
  - **we should normally have m*n axioms.**
  - **However, an exception to this rule exists.**

Developing Algebraic Specifications

- **If a constructor operation can be defined using other constructors:**
  - **we need to define inspection operations using only primitive constructors.**

Example: Stack

- **Let us specify an unbounded stack supporting:**
  - **push,**
  - **pop,**
  - **newstack,**
  - **top,**
  - **empty.**

Example: Stack

- **Types:**
- **defines stack**
- **uses boolean, element**
- **Exception:**
- **underflow, novalue**

Example: stack

- **Syntax:**
- **push:**
  - **stack $\times$ element $\rightarrow$ stack**
- **pop:**
  - **stack $\rightarrow$ stack+{underflow}**

Example: stack

- **top:**
  - **stack $\rightarrow$ element+{novalue}**
- **empty:**
  - **stack $\rightarrow$ boolean**
- **newstack:**
  - **$\phi \rightarrow$ stack**

Equations: stack

- pop(newstack)=underflow
- pop(push(s,e))=s
- top(newstack)=novalue
- top(push(s,e))=e
- empty(newstack)=true
- empty(push(s,e))=false

Rewrite rules

- **Rewrite rules let you determine:**
  - **the meaning of any sequence of calls on the stack functions.**

Rewrite rules

- **Empty(push(pop(push(newstack,e_1)),e_2)):**
  - **you can eliminate the call on pop by observing:**
    - **it is of the form pop(push(s,e)).**

Rewrite rules

- **After simplification:**
  - **empty(push(newstack,e_2))**
  - **false**

Two important questions

- **Finite termination property:**
  - **Does application of rewrite rules terminate after a finite number of steps?**
  - **We might endlessly go on applying rewrite rules without coming to any conclusion?**

Two important questions

- **Unique termination property:**
  - **Can different sequence in application of the rewrite rules always give the same answer?**
  - **If we choose to simplify different terms of the expression in different experiments:**
    - **shall we always get the same answer?**

Algebraic Specification

- **For arbitrary algebraic equations:**
  - **convergence is undecidable.**
- **If the r.h.s. of each rewrite rule has fewer terms than the left:**
  - **rewrite process must terminate.**

Auxiliary Functions

- Sometimes development of a specification requires:
  - extra functions not part of the system:
    - to define the meaning of some interface procedures.

Auxiliary Functions: Example

- To specify bounded stacks:
  - need to add a depth function:
    - push returns either a stack or
    - an exception "overflow" when depth is exceeded.

Bounded stack

- In order to specify a bounded stack:
  - we need to make changes to different sections to include auxiliary functions.

Auxiliary Functions

- **Syntax:**
- **push:**
  - **stack × element ==> stack**
- **depth:**
  - **stack → integer**

Auxiliary Functions

- **Equations:**
  - **depth(newstack)=0**
  - **depth(push(s,e))=depth(s)+1**
  - **push(s,e)=overflow if depth(s) >= Max**

Example 2: coord

- **Types:**
  - **sort coord**
  - **imports integer, boolean**

Example: coord

- **Signature:**
  - –**create(integer,integer) → coord**
  - –**X(coord) → integer**
  - –**Y(coord) → integer**
  - –**Eq(coord,coord) → boolean**

Example: coord

- **Rewrite rules:**
  - **X(create(x,y))=x**
  - **Y(create(x,y))=y**
  - **Eq(create(x1,y1),create(x2,y2)) = ((x1=x2) and (y1=y2))**

Structured Specifications

- Writing formal specifications is time consuming.
- To reduce effort, we need to reuse specifications:
  - instantiation of generic specifications
  - incremental development of specifications

Specification Instantiation

- **Take an existing specification:**
  - **specified with some generic parameter**
  - **Instantiate with some sort**

Incremental Development

- **Develop specifications for simple sorts:**
  - **using these specify more complex entities.**

Pros and Cons

- **Algebraic specifications have a strong mathematical basis:**
  - **can be viewed as heterogeneous algebra.**

Pros and Cons

- An important shortcoming of algebraic specifications:
  - cannot deal with side effects
  - difficult to use with common programming languages.

Pros and Cons

- **Algebraic specifications are hard to understand:**
  - **also changing a single property of the system**
    - **may require changing several equations.**

Specification of timing constraints

- **Timing constraints:**
  - **expressed in terms of occurrence of certain events.**

Events

- **A stimulus to the system from its environment.**
- **Can also be an externally observable response:**
  - **that the system makes to its environment**
- **Events can be instantaneous**
  - **or assumed to have a duration**

Types of timing constraints

- **Performance constraints:**
  - constraints imposed on the response of the system.
- **Behavioral constraints:**
  - constraints imposed on the action and reaction time of the environment (or the user).

Specification of timing constraints

- We will specify timing constraints:
  - in terms of stimuli and response
  - modelled as FSMs (Finite State Machines)

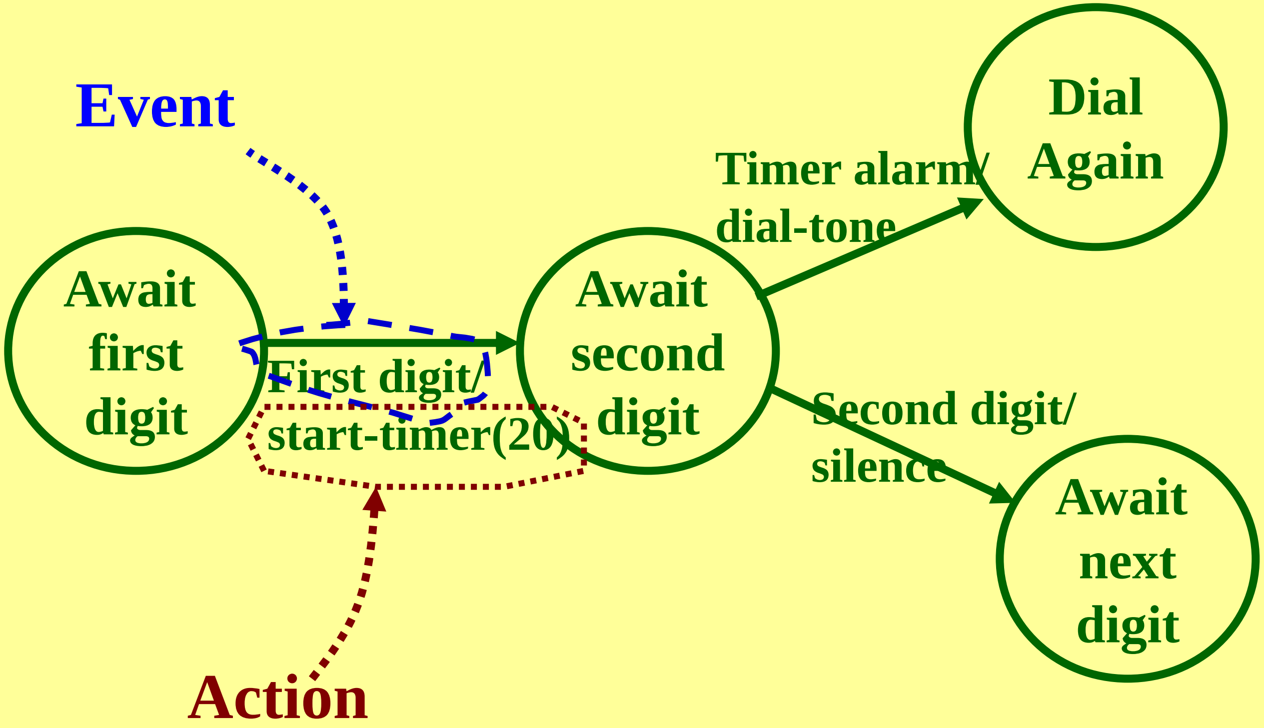
State machine modelling

- Assumes that at any time:
  - the system is in one of a number of possible states.
- When a stimulus is received,
  - it may cause a transition to a different state.

Finite Automaton with Output

- A set of states
- final state:
  - some states designated as final states
- an alphabet of input symbols
- an alphabet of output symbols
- a transition function:
  - maps a combination of states and input symbols to states

Representation



Types of finite automaton

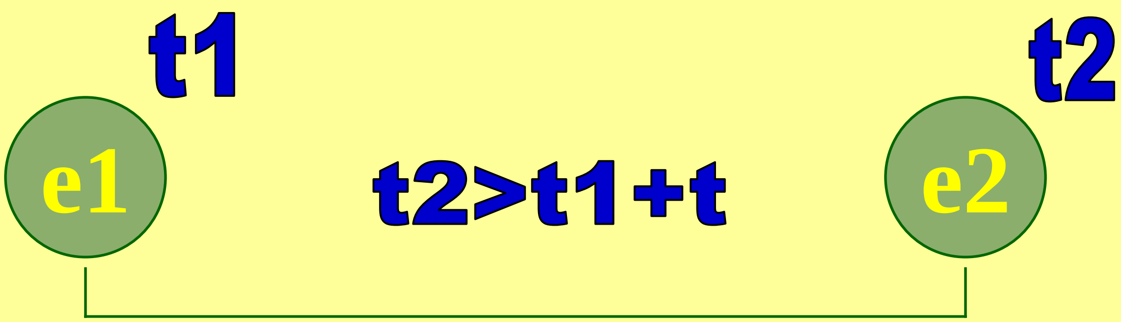
- **A finite automaton with output may be organized in two ways:**
  - **A Moore machine is an automaton**
    - **each state is associated with an output symbol**
  - **A Mealy machine associates each transition with an output symbol**

Classification:

- **Three types of timing constraints:**
  - **Minimum**
  - **Maximum**
  - **Durational**

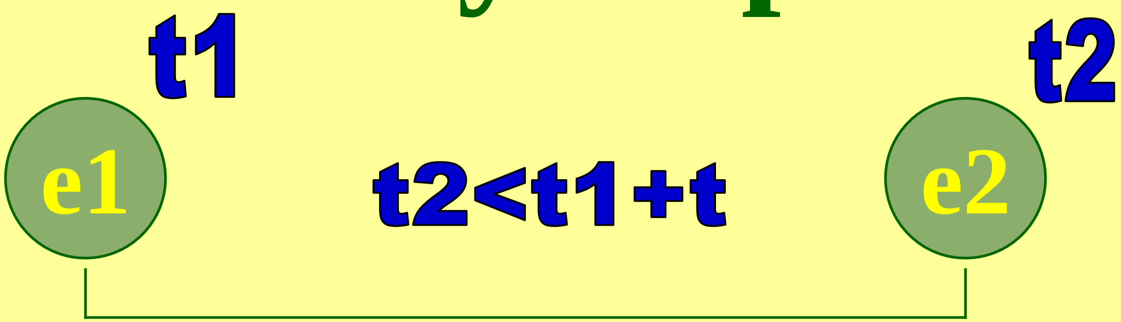
Minimum

- **Between two events:**
  - **No less than t time units may elapse**



Maximum

- **Between two events:**
  - **No more than t time units may elapse**



Durational

- **An event must occur for t units of time**

Maximum

- S-S (stimulus-stimulus)
- S-R (stimulus-response)
- R-S (response-stimulus)
- R-R (response - response)

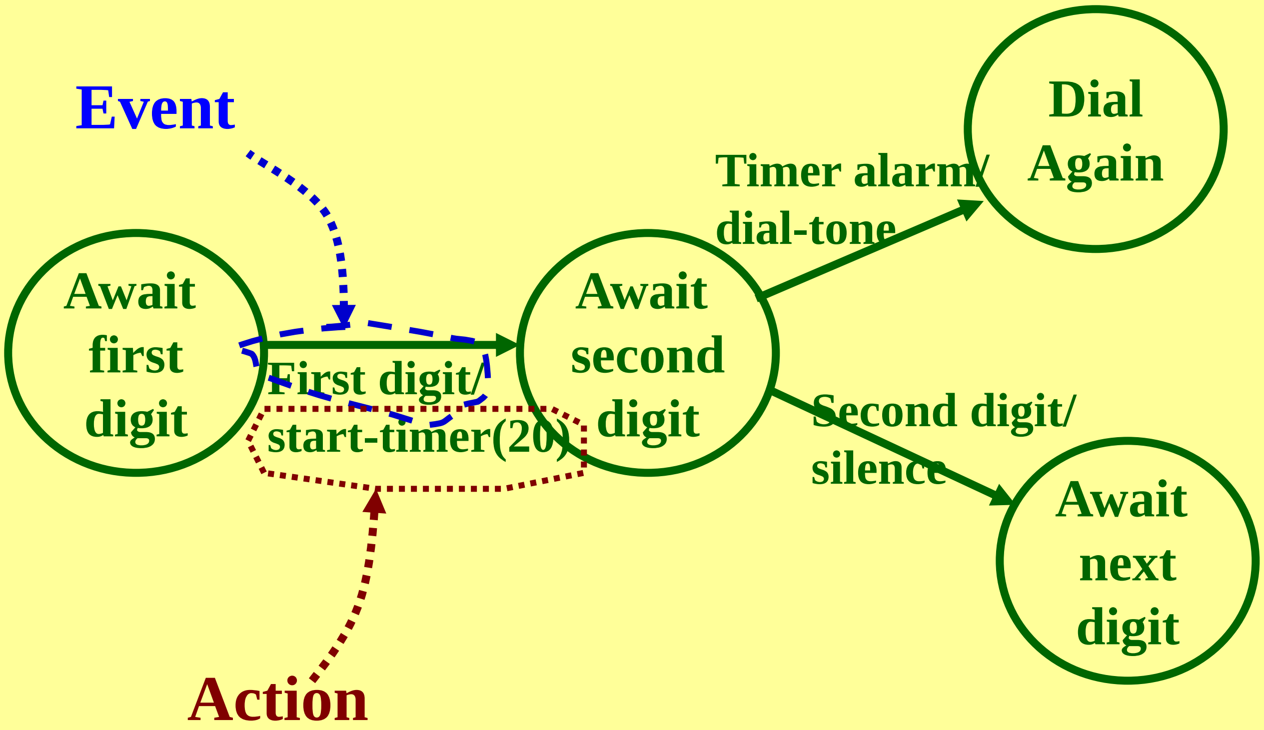
Maximum S-S

- **Maximum time between occurrence of two stimuli:**
  - **e.g. after dialling first digit,**
    - **the second digit should be dialled no more than 20 secs later.**

FSM Representation

- **To represent timing constraints in an FSM:**
  - **a timer alarm is used as an artificial stimulus.**

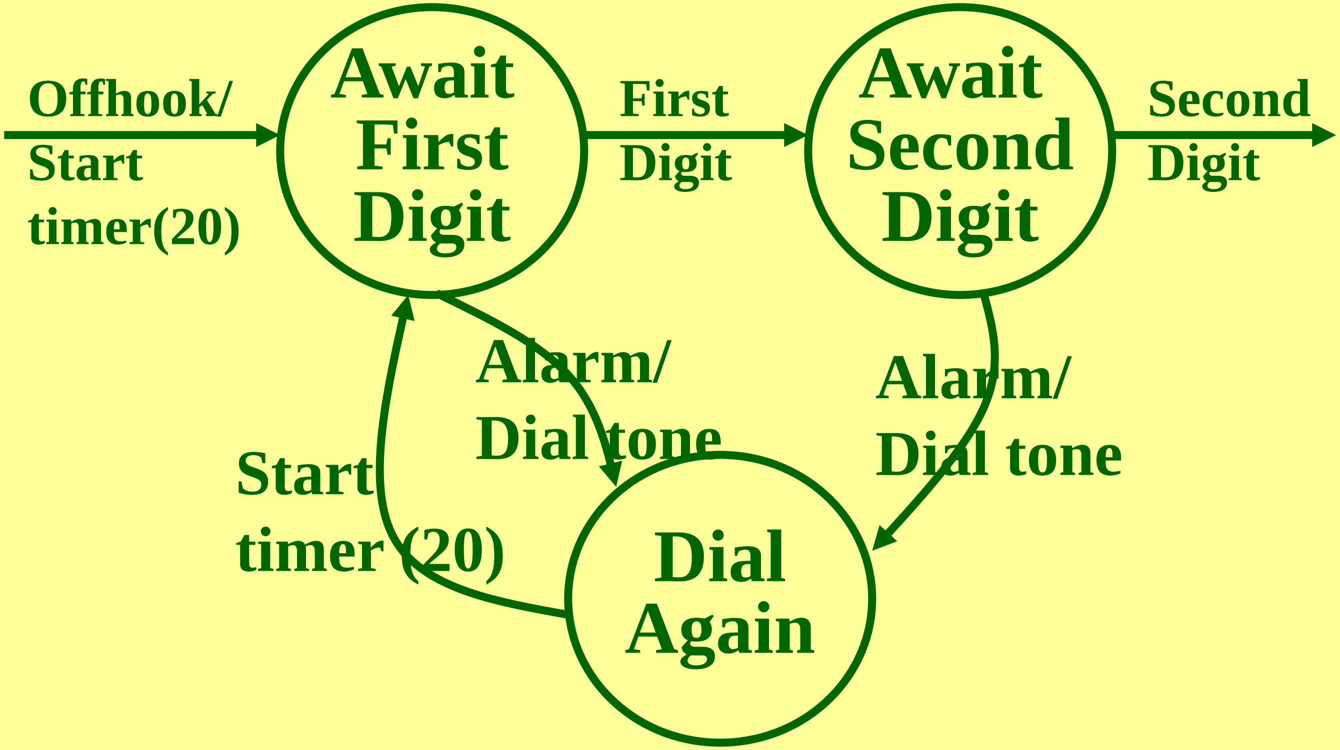
Representation



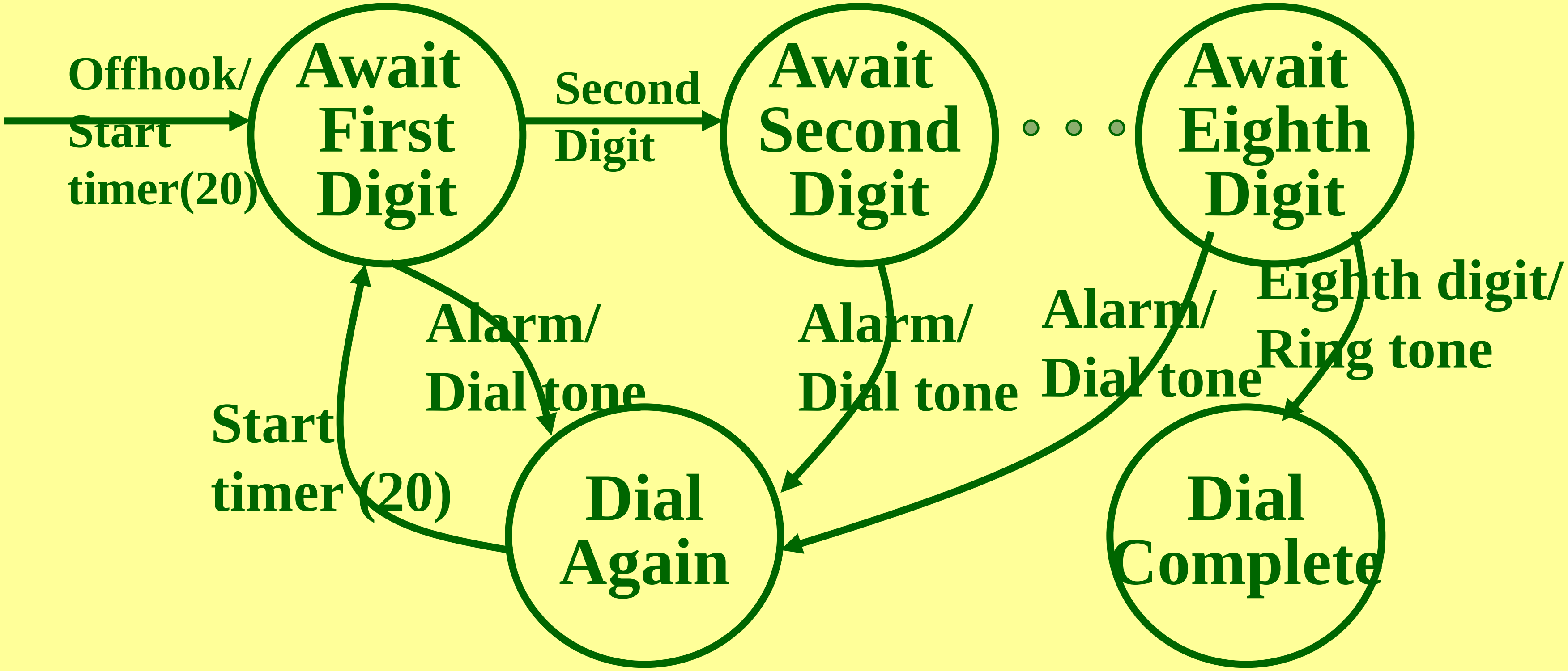
Maximum S-R

- **Maximum time between stimulus and response:**
  - **e.g. caller shall receive dial tone no later than 20 secs after lifting the receiver.**

Representation



Complete Representation



Maximum R-S

- **Maximum time between system's response and the next stimulus from the environment:**
  - **e.g after receiving the dial tone, the caller shall dial the first digit within 20 sec.**

Minimum constraints

- S-S
- R-S
- R-R

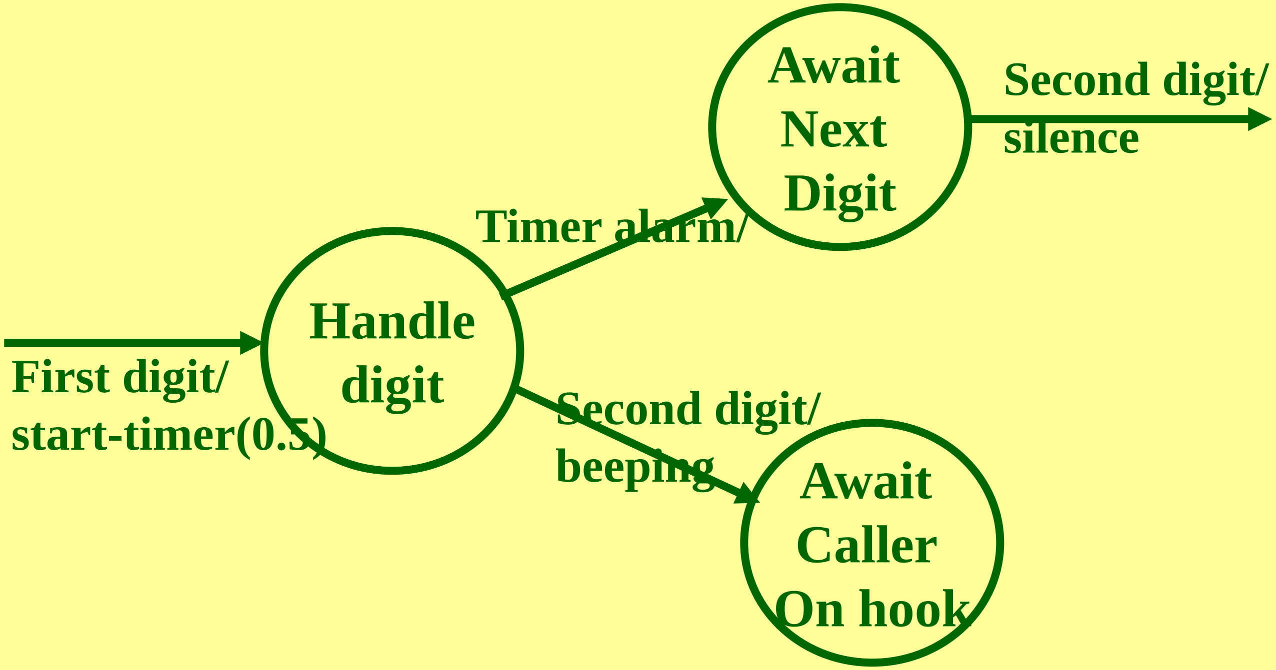
Minimum S-S

- **A minimum time is required between two stimuli:**
  - **e.g. a minimum of 0.5 sec must elapse between the dialling of one digit and the dialling of the next.**

Minimum S-S

- This is an example of behavioral constraints on system users:
  - the complete specification should include
    - response the system should make if the user responds too soon.

Representation



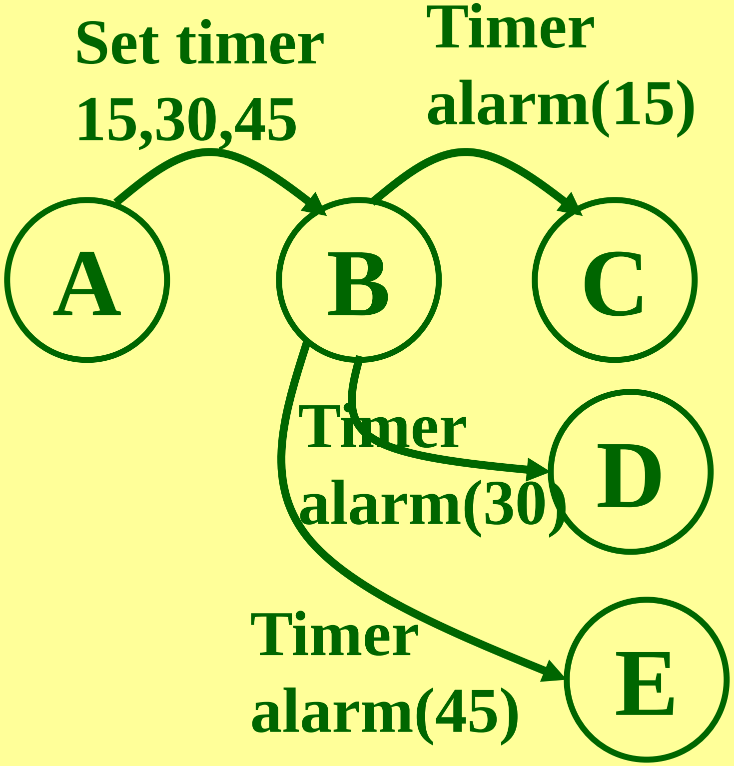
Durational timing constraints

- **To go back to the international operator**
  - **press the button for at least 15 secs (but no more than 30 secs)**

Durational timing constraints

- **To get back to the local operator:**
  - **press the button for at least 30 secs, but no more than 45 secs**
- **To receive a dial tone press the button for at least 45 secs.**

Representation



Reference

- Ian Somerville, Chapter Chapter 10
- R. Mall, Chapter 4
- B. Dasarathy, “Timing constraints of R-T systems,” IEEE TSE, 1985, pp. 80--86.

Summary

- **We started by discussing some general concepts in:**
  - **formal specification techniques.**

Summary

- **Formal requirements specifications:**
  - **have several positive characteristics.**
  - **But the major shortcoming is that they are hard to use.**

Summary

- **It is possible that formal techniques will become more usable in future**
  - **with the development of suitable front-ends.**

Summary

- We discussed a sample specification technique,
  - algebraic specification
  - gives us a flavour of the issues involved in formal specification.

Summary

- We discussed specification of timing constraints:
  - classification
  - modelling using FSMs

Next Lecture

- Real-time system design:
  - Structured analysis of system behavior
  - Ward and Mellor technique