Framework for solving PDEs of plasma fluid models

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Outline

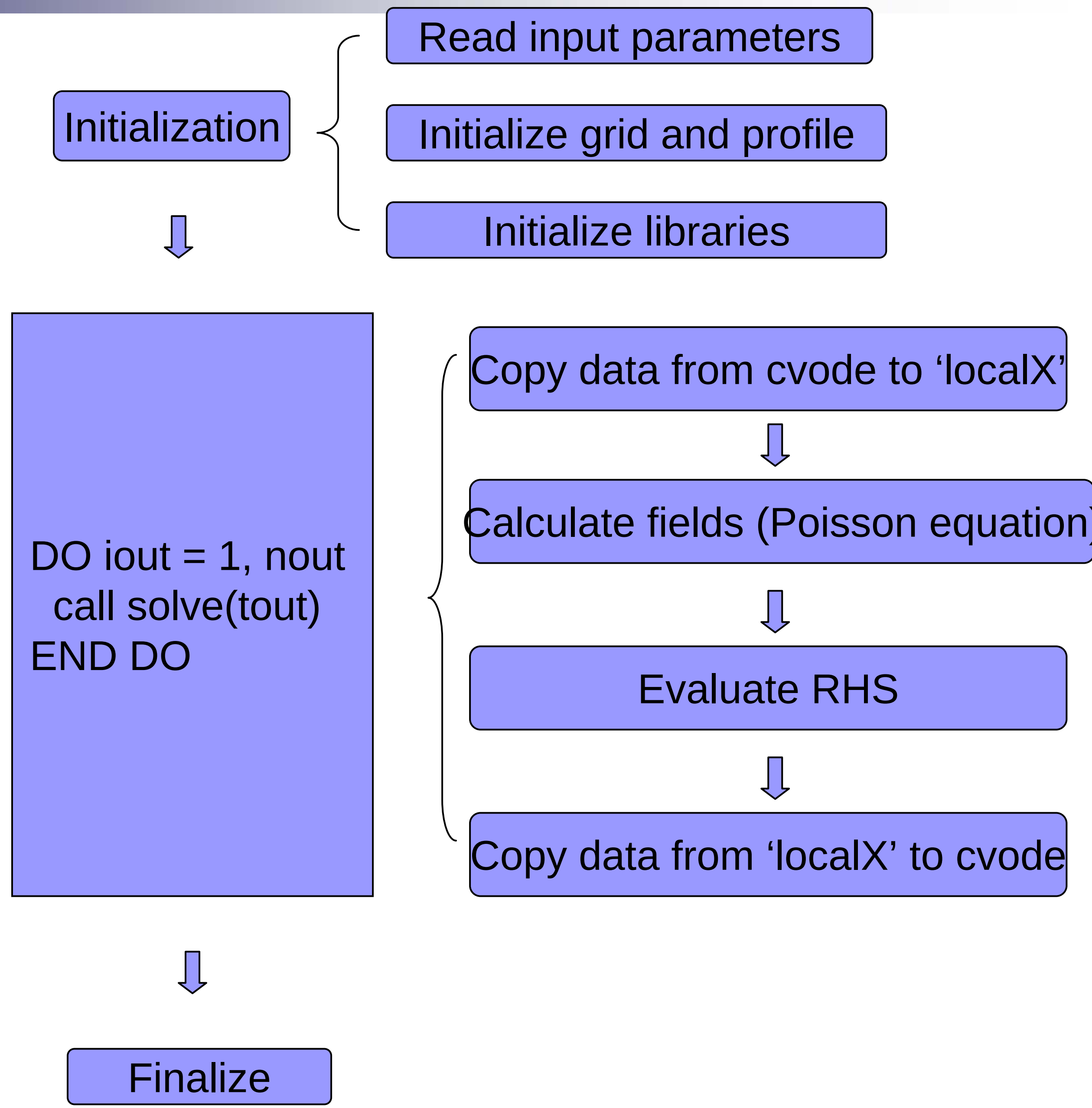
- Motivation
- Overview
- Important components
  - Time integrator： CVODE
  - Poisson solver： 2D and 3D
- Benchmark

Motivation

- Solving problems related to tearing mode in cylindrical and torus geometry
- Implicit time integrator
- Fully parallelized with capacity of Poisson equations
- Try to write a code by myself as a practice

Overview

- Flow chart
  - CVODE based
  - Fortran
  - C



CVODE

- CVODE solves IVPs:

$$\frac{\partial f}{\partial t} = F(f,t) \qquad f(t_0) = f_0$$

- The method is based on the formula:

$$\sum_{i=0}^{K_1} \alpha_{n,i} f^{n-i} + h_n \sum_{i=0}^{K_2} \beta_{n,i} \left(\frac{\partial F^{n-i}}{\partial t}\right) = 0$$

$$\Downarrow$$

$$f^n - h_n \beta_{n,0} F(t_n, f^n) - a_n = 0$$

CVODE

- KAW

$$\partial_t \delta n_e = -\vec{b}_0 \cdot \nabla(n_0 \delta u_{e||})$$

$$\frac{1}{c}\partial_t \delta A_{||} = \frac{T_e}{n_0 e}\vec{b}_0 \cdot \nabla \delta n_e - \vec{b}_0 \cdot \nabla \delta \phi$$

$$\frac{e}{T_e}\rho_s^2 \nabla_\perp^2 \delta\phi = \frac{\delta n_e}{n_0}$$

$$\delta u_{e||} = \frac{c}{4\pi n_0 e}\nabla_\perp^2 \delta A_{||}$$

$$\omega^2 = \frac{2}{\beta}k_{||}^2(1 + k_\perp^2 \tau)$$

CVODE: Shear Alfven wave

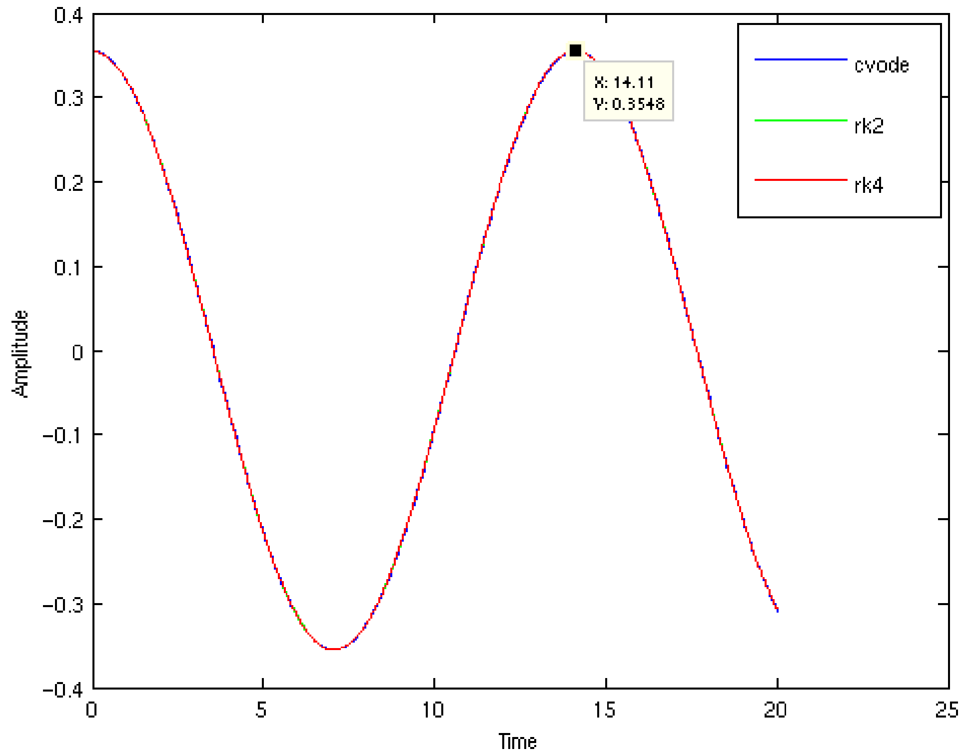
- 2^{nd} and 4^{th} order Runge-Kutta method
- CVODE(BDFs)

$$\omega^2 = \frac{2}{\beta} k_{//}^2$$

- For

$$k_{//} = 0.005, \beta = 0.01$$

T=14.14



2D Poisson solver

With finite difference method, the Poisson equation can be written in matrix form

$$\frac{e}{T_e}\rho_s^2\nabla_\perp^2\delta\phi = \frac{\delta n_e}{n_0}$$

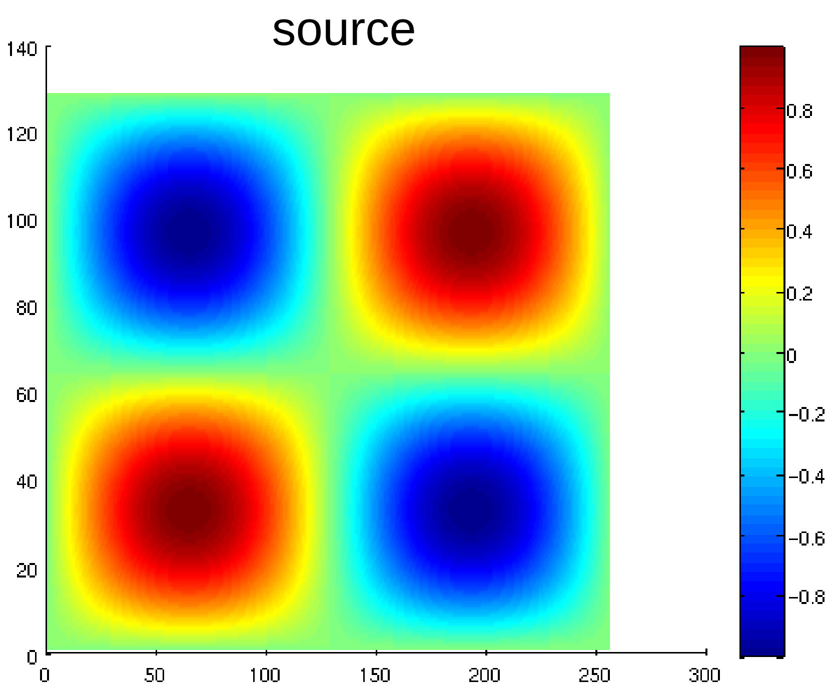
$$[A]\,[U] = [b]$$

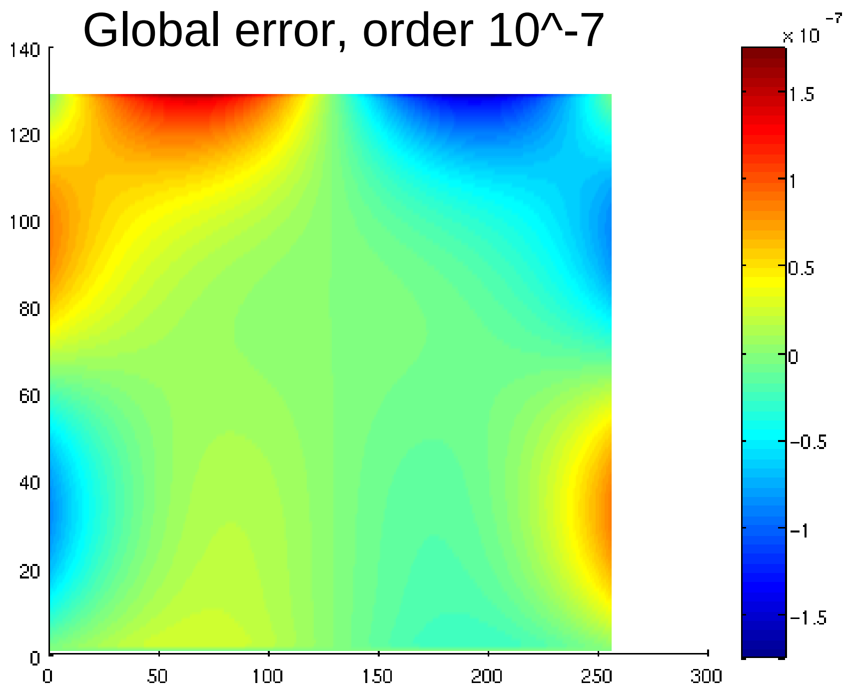
$$A = \begin{bmatrix} D & -I & 0 & 0 & 0 & \ldots & 0 \\ -I & D & -I & 0 & 0 & \ldots & 0 \\ 0 & -I & D & -I & 0 & \ldots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \ldots & 0 & -I & D & -I & 0 \\ 0 & \ldots & \ldots & 0 & -I & D & -I \\ 0 & \ldots & \ldots & \ldots & 0 & -I & D \end{bmatrix}$$

http://en.wikipedia.org/wiki/Discrete_Poisson_equation

2D Poisson solver

- KSP from PETSC
  - scalable linear equations solvers (KSP)





http://www.mcs.anl.gov/petsc/petsc-as/

2D Poisson solver

- Divide the prime communicator into ‘npz’ sub-communicators
- call MPI_COMM_SPLIT(comm,ipz,irank,commz,ierr)
- This can dramatically reduce the time of communication and programming complexity

3D Poisson solver

- The source is periodic in two of the three dimensions
- Use the library fftw3 to carry out 2D DFT in parallel
- Use the KSP of PETSC to solve the resulting my*mz 1D BVPs
- Still under construction

Benchmark

- Shear Alfven wave package propagation

$$\partial_t \delta n_e = -\vec{b}_0 \cdot \nabla (n_0 \delta u_{e||})$$

$$\frac{1}{c}\partial_t \delta A_{||} = \frac{T_e}{n_0 e}\vec{b}_0 \cdot \nabla \delta n_e - \vec{b}_0 \cdot \nabla \delta \phi$$

$$\frac{e}{T_e}\rho_s^2 \nabla_\perp^2 \delta \phi = \frac{\delta n_e}{n_0}$$

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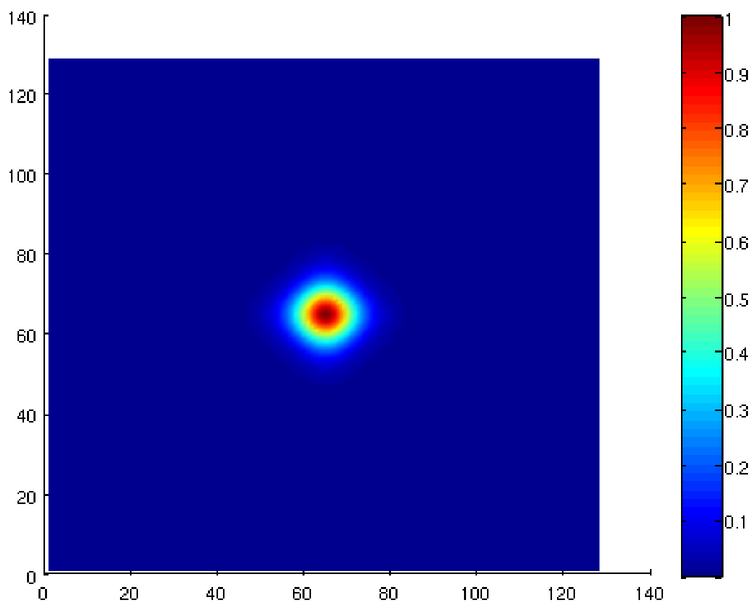
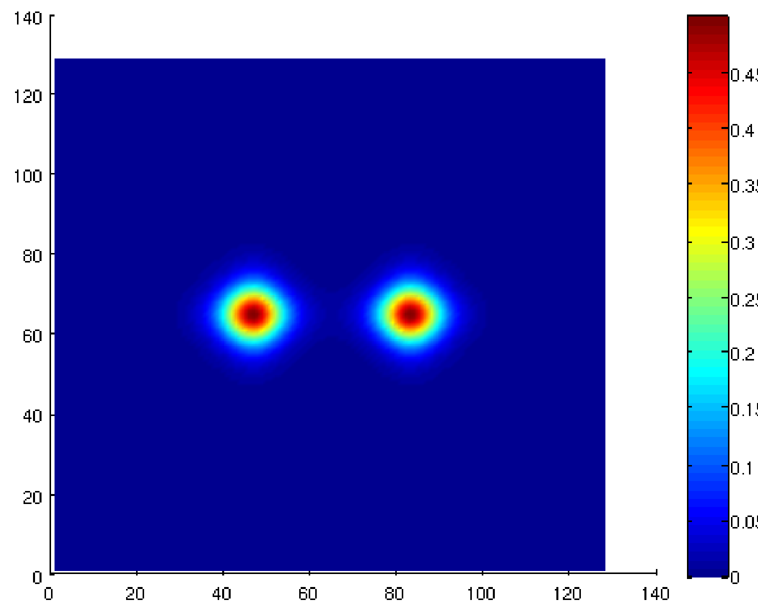
$$\omega^2 = \frac{2}{\beta} k_{||}^2 (1 + k_\perp^2 \tau)$$

Benchmark

- With beta=0.01, Vgroup=14.14, t=7, S=98.98, L= -100, 100





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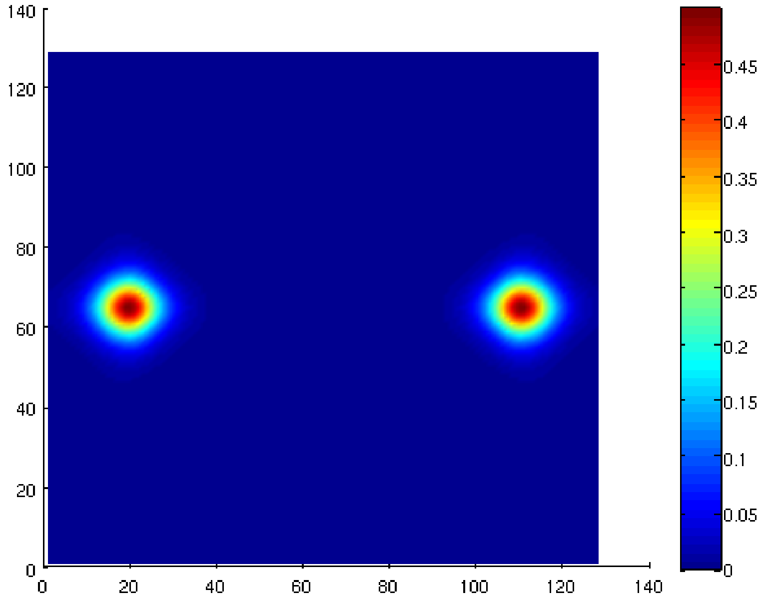
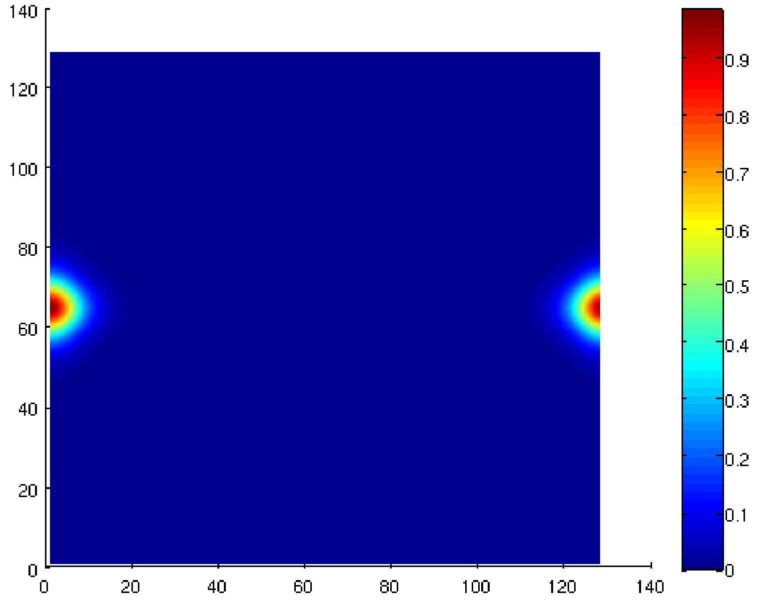
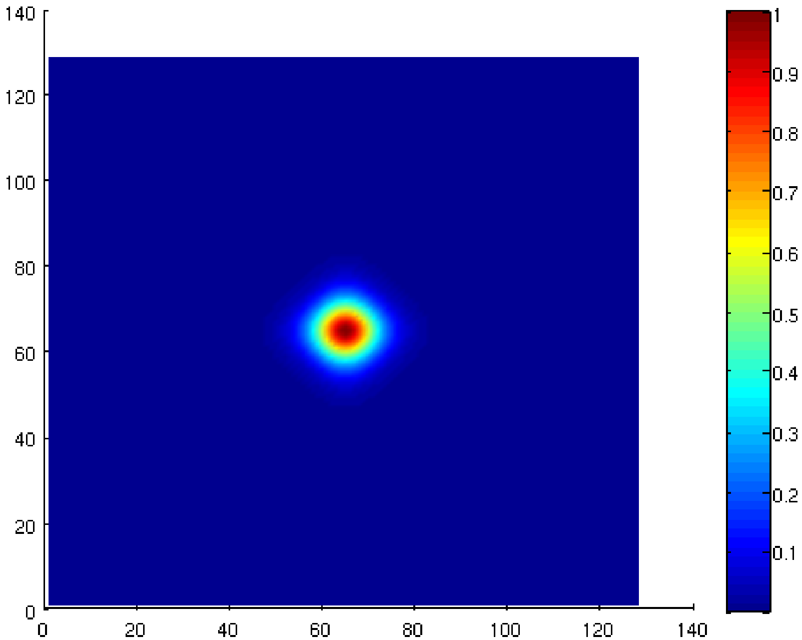
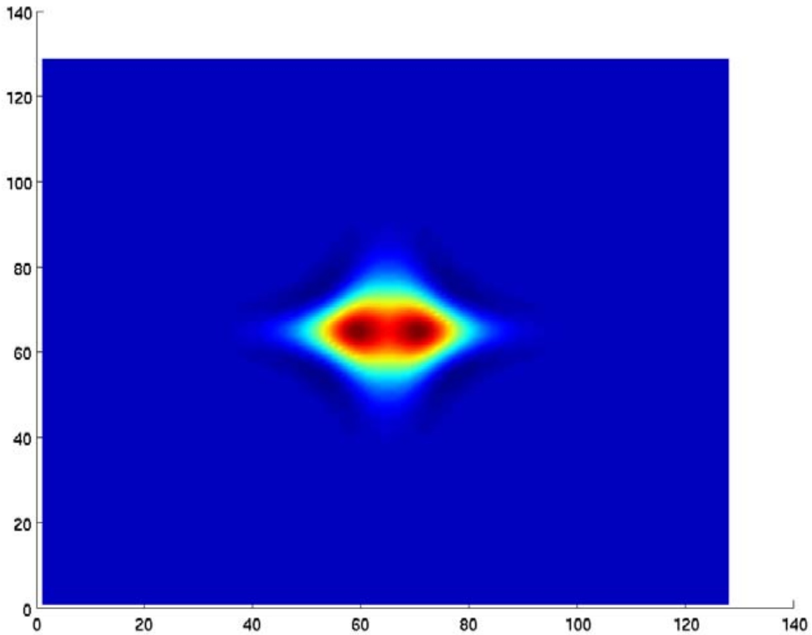




Benchmark

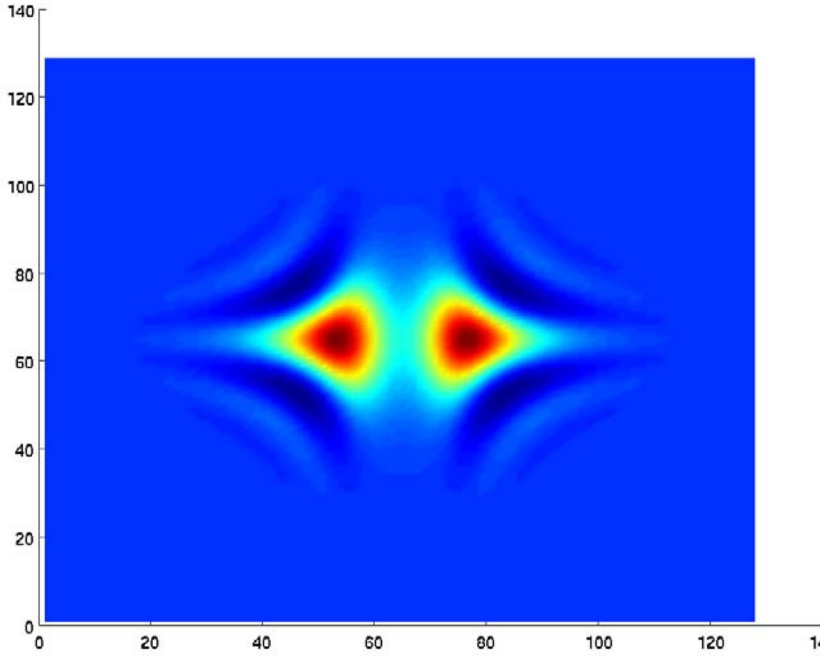
- With $\beta = 0.01, \tau = 1$

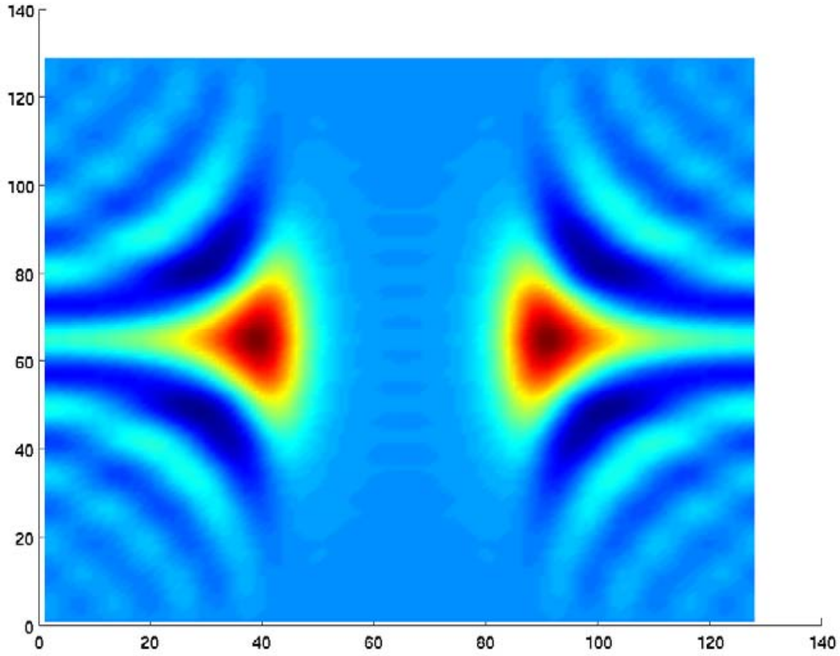




Benchmark

- KAW





To be resolved

- 3D Poisson solver
- Magnetic shear in field-line-align coordinate system
- Proper radius boundary

Thanks!