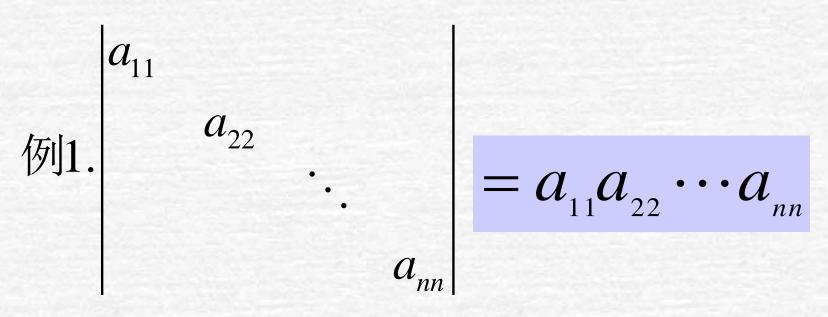
特殊行列式的计算



(未写出的为0)

这个行列式称为对角行列式。

例2.
$$\begin{vmatrix} a_{11} & a_{21} & a_{22} \\ \vdots & \vdots & \ddots \\ a_{n1} & a_{n2} & a_{nn} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{nn} & a_{nn} \end{vmatrix}$$

(未写出的为0)

$$=a_{11}a_{22}\cdots a_{nn}$$

分别称为下三角行列式和上三角行列式。统称为三角行列式。

$$\begin{vmatrix} a_{1n} \\ a_{2(n-1)} & a_{2n} \\ \vdots & \vdots \end{vmatrix} = \begin{vmatrix} a_{11} & \cdots & a_{1(n-1)} & a_{1n} \\ a_{21} & \cdots & a_{2(n-1)} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{n(n-1)} & a_{nn} \end{vmatrix}$$

(未写出的为0)

$$= (-1)^{\frac{n(n-1)}{2}} a_{1n} a_{2(n-1)} \cdots a_{n1}$$
 统称为斜三角行列式。

例3.计算行列式

$$D = \begin{bmatrix} x & y & 0 \\ 0 & x & y & \ddots \\ & \ddots & \ddots & \ddots & 0 \\ \vdots & & \ddots & x & y & 0 \\ 0 & & 0 & x & y \\ y & & \cdots & & 0 & x \end{bmatrix}$$

(未写出的为0)

按第一列展开:

$$D = \begin{bmatrix} x & y & 0 & & & \\ 0 & x & y & \ddots & & \\ & \ddots & \ddots & \ddots & 0 & \\ \vdots & & \ddots & x & y & 0 \\ 0 & & 0 & x & y \\ y & \cdots & 0 & x \end{bmatrix}$$

$$\begin{vmatrix} x & y & 0 \\ 0 & x & y & \ddots \\ \vdots & \ddots & \ddots & 0 \\ \vdots & \ddots & x & y & 0 \\ 0 & 0 & x & y \\ y & \cdots & 0 & x \end{vmatrix} = x \begin{vmatrix} x & y & & & \\ x & y & & & \\ \ddots & \ddots & & & \\ x & y & & \\ x & &$$

$$\begin{vmatrix} y \\ x & y \\ & x & \ddots \\ & & \ddots \\ & & y \\ & & x & y \end{vmatrix}$$

$$= x^{n} + (-1)^{n+1} y^{n}$$

例4.求
$$D_{2n}=$$

$$\begin{bmatrix} a & & & & & b \\ & a & & & \ddots \\ & & \ddots & & \ddots \\ & & c & d \\ & & \ddots & & \ddots \\ & & & c & d \\ & & & & d \end{bmatrix}.$$

(未写出的为0)

$$= \begin{vmatrix} a & 0 & \cdots & 0 & 0 & \cdots & 0 & b \\ 0 & a & \cdots & 0 & 0 & \cdots & b & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & a & b & \cdots & 0 & 0 \\ 0 & 0 & \cdots & c & d & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & c & \cdots & 0 & 0 & \cdots & d & 0 \\ c & 0 & \cdots & 0 & 0 & \cdots & 0 & d \end{vmatrix}$$

按第1行展开,有

$$= (ad - bc)D_{2n-2}$$

$$= (ad - bc)(ad - bc)D_{2n-4} = (ad - bc)^{2}D_{2(n-2)}$$

$$= \dots = (ad - bc)^{n-1} D_{2(n-(n-1))}$$

$$= (ad - bc)^{n-1} D_2.$$

$$D_2 = ad - bc,$$

$$D_{2n} = (ad - bc)^n.$$