伴随矩阵

$$A=\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix},$$

A_{ij}为a_{ij}的代数余子式，

$$A^{*}=\begin{pmatrix} A_{11} & A_{21} & \cdots & A_{n1} \\ A_{12} & A_{22} & \cdots & A_{n2} \\ \vdots & \vdots & \cdots & \vdots \\ A_{1n} & A_{2n} & \cdots & A_{nn} \end{pmatrix}$$

伴随矩阵

写A^{*}时要注意什么？ **代数余子式的顺序！**

$$A^* = \begin{pmatrix} A_{11} & A_{21} & \cdots & A_{n1} \\ A_{12} & A_{22} & \cdots & A_{n2} \\ \vdots & \vdots & \cdots & \vdots \\ A_{1n} & A_{2n} & \cdots & A_{nn} \end{pmatrix}$$

二阶矩阵$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$的伴随矩阵=？

$$A_{11} = d, \quad A_{12} = -c, \quad A_{21} = -b, \quad A_{22} = a.$$

$$A^* = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

你发现规律了吗?

记住了吗?

练习

$$A=\begin{pmatrix}1 & 0\\ -1 & 1\end{pmatrix}, A^{*}=\underline{\qquad\qquad\qquad},$$

$$B=\begin{pmatrix}1 & -2\\ 3 & 1\end{pmatrix}, B^{*}=\underline{\qquad\qquad\qquad},$$

$$C=\begin{pmatrix}2 & -3\\ 1 & 4\end{pmatrix}, C^{*}=\underline{\qquad\qquad\qquad}.$$

练习答案

$$A^{*}=\begin{pmatrix}1 & 0\\ 1 & 1\end{pmatrix},\quad B^{*}=\begin{pmatrix}1 & 2\\ -3 & 1\end{pmatrix},\quad C^{*}=\begin{pmatrix}4 & 3\\ -1 & 2\end{pmatrix}.$$

复习

$$a_{j1}A_{i1}+a_{j2}A_{i2}+\cdots+a_{jn}A_{in}=\begin{cases}D, & (i=j).\\ 0, & (i\neq j).\end{cases}$$

$$a_{1j}A_{1i}+a_{2j}A_{2i}+\cdots+a_{nj}A_{ni}=\begin{cases}D, & (i=j).\\ 0, & (i\neq j).\end{cases}$$

$$AA^* = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix} \begin{pmatrix} A_{11} & A_{21} & \cdots & A_{n1} \\ A_{12} & A_{22} & \cdots & A_{n2} \\ \vdots & \vdots & \cdots & \vdots \\ A_{1n} & A_{2n} & \cdots & A_{nn} \end{pmatrix}$$

$$= \begin{pmatrix} |A| & & \\ & \ddots & \\ & & |A| \end{pmatrix}$$

$$= |A| E$$

$$A^*A=\begin{pmatrix} A_{11} & A_{21} & \cdots & A_{n1} \\ A_{12} & A_{22} & \cdots & A_{n2} \\ \vdots & \vdots & \cdots & \vdots \\ A_{1n} & A_{2n} & \cdots & A_{nn} \end{pmatrix}\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}$$

$$=\begin{pmatrix} |A| & & \\ & \ddots & \\ & & |A| \end{pmatrix}=|A|E$$

$$\boxed{AA^* = A^*A = |A|E}$$

一个很重要的公式