行列式概念的引进

$$\begin{cases} a_{11}x_1 + a_{12}x_2 = b_1, \\ a_{21}x_1 + a_{22}x_2 = b_2. \end{cases}$$

$$a_{11}a_{22} - a_{12}a_{21} \neq 0.$$

求这个方程组的解。

我们一般是用高斯消元法解这个方程组的。

$$\begin{cases} a_{11}x_1 + a_{12}x_2 = b_1, \\ a_{21}x_1 + a_{22}x_2 = b_2. \end{cases}$$

$$x_1 = \frac{a_{22}b_1 - a_{12}b_2}{a_{11}a_{22} - a_{12}a_{21}},$$

$$x_2 = \frac{a_{11}b_2 - a_{21}b_1}{a_{11}a_{22} - a_{12}a_{21}}.$$

你能在十秒钟内记住这个解的公式吗？

为了记忆，我们引进记号

$$a_{11}a_{22} - a_{12}a_{21}$$

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \overset{\text{约定}}{=} a_{11}a_{22} - a_{12}a_{21}.$$

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21} \, .$$

$$a_{22} \, b_1 - a_{12} \, b_2 \, .$$

$$\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix} = a_{22}b_1 - a_{12}b_2 \, .$$

$$a_{11} \, b_2 - a_{21} \, b_1 \, .$$

$$\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix} = a_{11} \, b_2 - a_{21} \, b_1 \, .$$

$$x_1 = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad x_2 = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}.$$

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}.$$

你记住这个解的公式了吗？

二阶行列式

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}.$$

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1, \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2, \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3. \end{cases}$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{aligned} & a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ & -a_{13}a_{22}a_{31} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33}. \end{aligned}$$
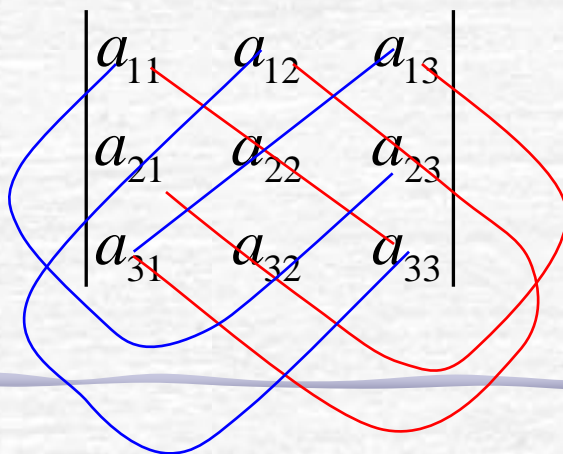
1.六项的代数和。恰好是1，2，3这3个数的全排列的个数。

2.每项都是3个元素的乘积，分析这3个元素的下标：

它们取自不同行不同列。

123，231，312

321，132，213

$$\begin{vmatrix} 1 & 4 & 3 \\ -5 & 2 & 1 \\ 3 & 6 & 1 \end{vmatrix} = \underline{\qquad\qquad}.$$

$$\begin{vmatrix} 1 & 0 & 0 \\ -5 & 2 & 3 \\ 3 & 3 & 5 \end{vmatrix} = \underline{\qquad\qquad}.$$

$$\begin{vmatrix} 1 & 4 & 3 \\ -5 & 2 & 1 \\ 3 & 6 & 1 \end{vmatrix} = (2+12-90)-(18+6-20) = -80.$$

$$\begin{vmatrix} 1 & 0 & 0 \\ -5 & 2 & 3 \\ 3 & 3 & 5 \end{vmatrix} = (10+0+0)-(0+9+0) = 1.$$

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