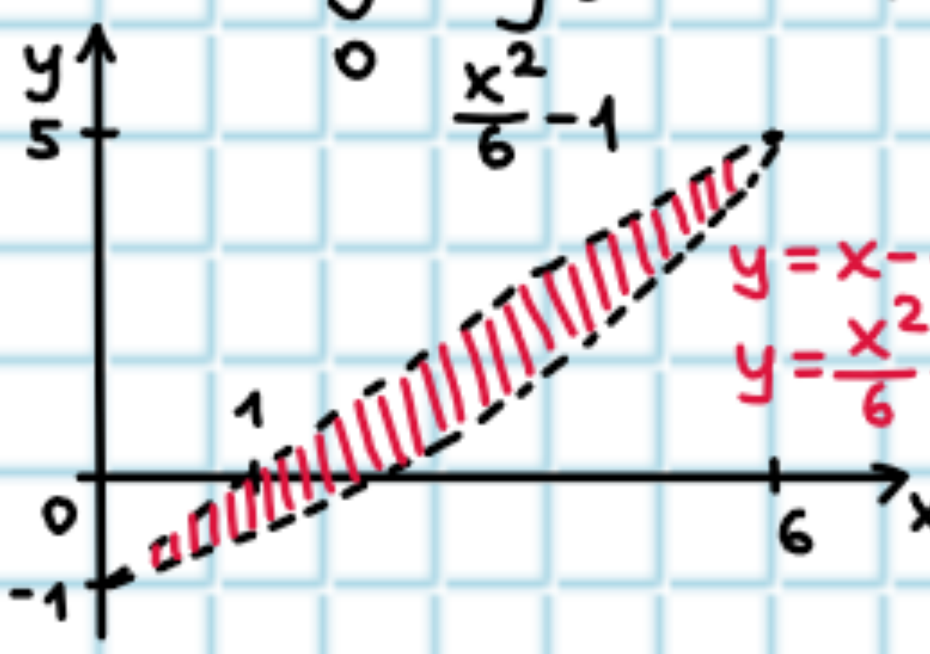
Лопатенко Георгий М3102

Изменить порядок интегрирования в интеграле:

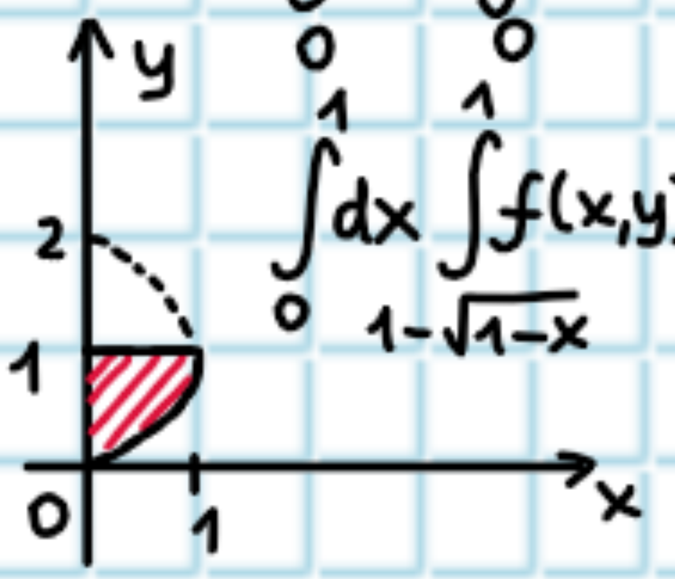
15.37 $$\int_0^6 dx \int_{\frac{x^2}{6}-1}^{x-1} f(x,y)\,dy = \int_{-1}^{5} dy \int_{1+y}^{\sqrt{6y+6}} f(x,y)\,dx$$



$y = x-1 \longrightarrow x = 1+y$

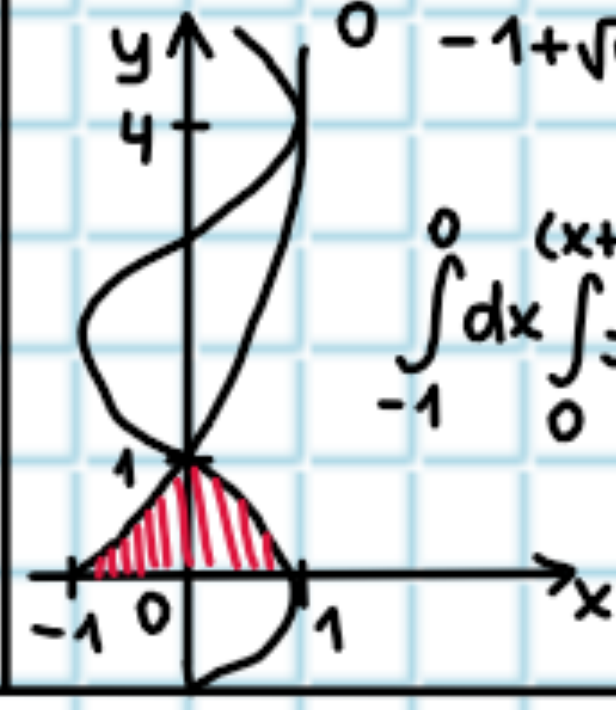
$y = \frac{x^2}{6} - 1 \longrightarrow x = \pm\sqrt{6y+6}$

15.40 $$\int_0^1 dy \int_0^{2y-y^2} f(x,y)\,dx$$



$$\int_0^1 dx \int_{1-\sqrt{1-x}}^{1} f(x,y)\,dy$$

15.41 $$\int_0^1 dy \int_{-1+\sqrt{y}}^{\cos(\frac{\pi y}{2})} f(x,y)\,dx$$

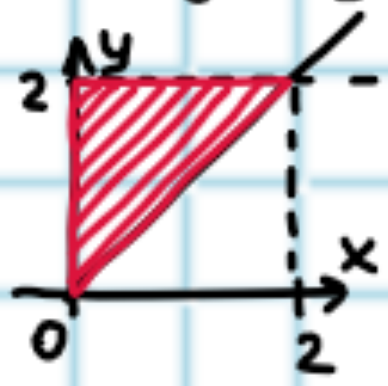


$$\int_{-1}^{0} dx \int_0^{(x+1)^2} f\,dy + \int_0^1 dx \int_0^{\frac{2\arccos x}{\pi}} f\,dy$$

Изменив порядок интегрирования, вычислить интеграл:

15.60 $$\int_0^1 dy \int_0^{1-y} e^{-x^2+2x+1}dx = \int_0^1 dx \int_0^{1-x} e^{-x^2+2x+1}dy = \int_0^1 (1-x)e^{-x^2+2x+1}dx = \frac{e^{-x^2+2x+1}}{2}\Big|_0^1 = \frac{e^2-e}{2}$$

15.61 $$\int_0^2 x^2dx \int_x^2 \ln(1+y^2)\,dy = \int_0^2 dy \int_0^y x^2\ln(1+y^2)\,dx = \int_0^2 \frac{y^3}{3}\ln(1+y^2)\,dy = \dots$$



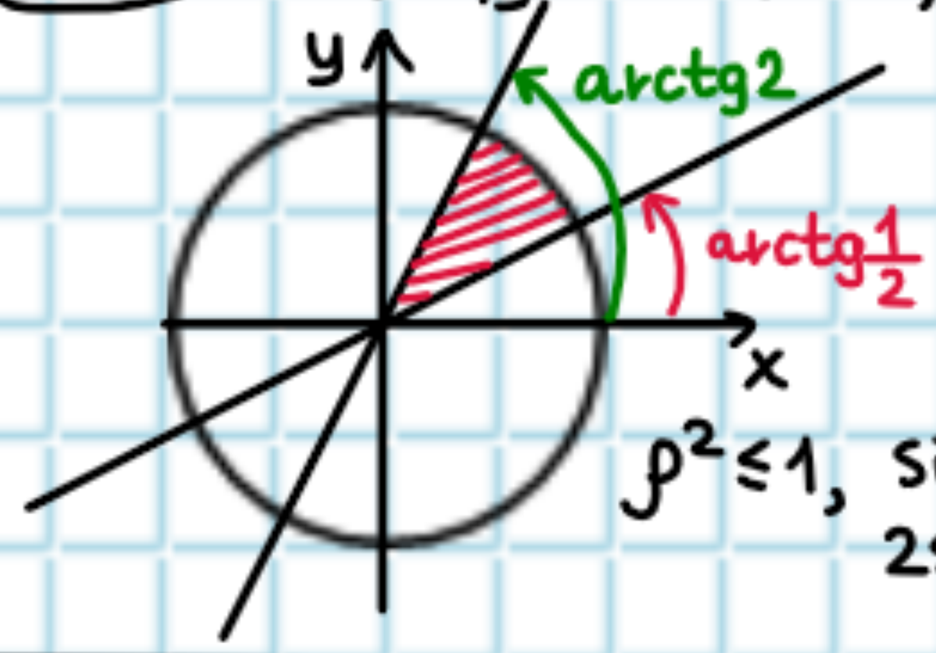
15.62 $$\int_0^\pi x\,dx \int_x^\pi \frac{\sin y}{y}dy = \int_0^\pi dy \int_0^y x\cdot\frac{\sin y}{y}dx = \int_0^\pi \frac{y^2\sin y}{2y} = \frac{-y\cos y+\sin y}{2}\Big|_0^\pi = \frac{\pi}{2}$$

15.75 $D=\{(x,y):\ x^2+y^2\le a^2\}$

$$\begin{cases} x=\rho\cos\varphi \\ y=\rho\sin\varphi \end{cases} \quad \iint_D f(x,y)\,dxdy = \int_0^{2\pi} d\varphi \int_0^{|a|} f(\rho,\varphi)\cdot\rho\, d\rho$$

Якобиан

15.83 $D=\{(x,y):\ x^2+y^2\le 1,\ y-2x\le 0,\ 2y-x\ge 0,\ x\ge 0\}$



$$\int_{\operatorname{arctg}\frac{1}{2}}^{\operatorname{arctg}2} d\varphi \int_0^1 f(\rho,\varphi)\rho\, d\rho$$

$\rho^2\le 1,\quad \sin\varphi \le 2\cos\varphi$

$2\sin\varphi \ge \cos\varphi$

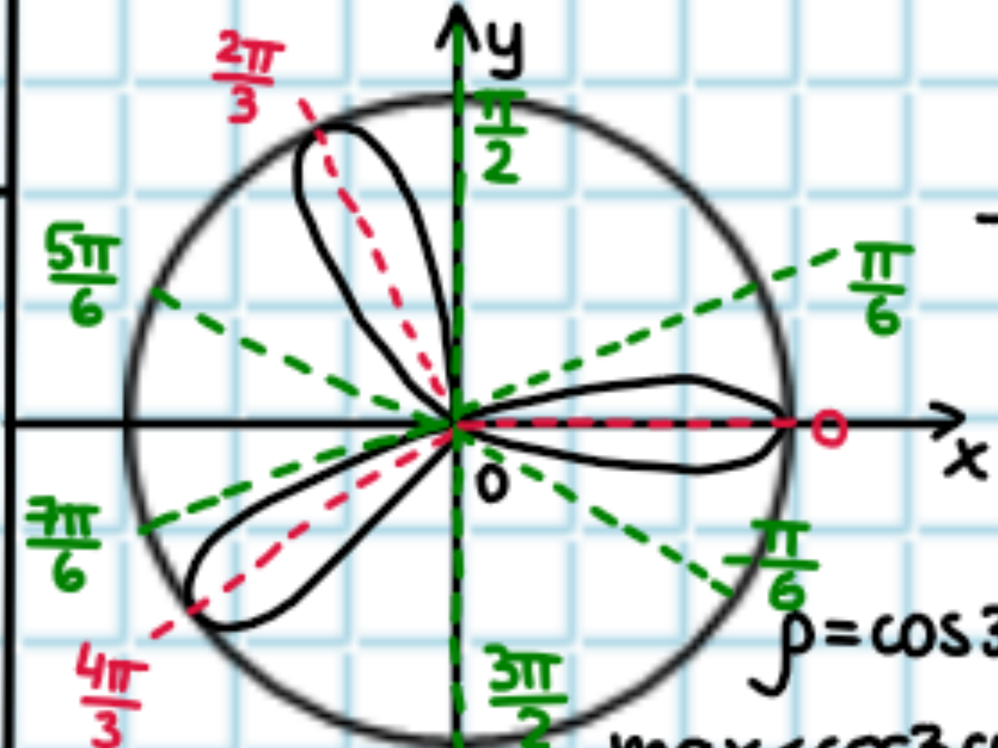
15.86 $D=\{(x,y):\ (x-1)^2+y^2\le 1,\ x\in[1,2]\}$

$\rho^2\cos^2\varphi - 2\rho\cos\varphi + 1 + \rho^2\sin^2\varphi \le 1 \Rightarrow \rho \le 2\cos\varphi$

и $\varphi\in[-\frac{\pi}{4};\frac{\pi}{4}]$

$$\int_{-\pi/4}^{\pi/4} d\varphi \int_{1/\cos\varphi}^{2\cos\varphi} f(\rho,\varphi)\rho\, d\rho$$

15.88 D — фигура, лежащая внутри окружности $x^2+y^2=1$ и вне кривой $\rho=\cos 3\varphi$



$\rho = \cos 3\varphi$

$$\int_{-\pi/6}^{\pi/6} d\varphi \int_{\cos 3\varphi}^{1} f(r,\varphi)\rho\, d\rho + \int_{\pi/6}^{\pi/2}\dots\int_0^1\dots + \int_{\pi/2}^{5\pi/6}\dots\int_{\cos3\varphi}^{1}\dots + \int_{5\pi/6}^{7\pi/6}\dots\int_0^1\dots + \int_{7\pi/6}^{3\pi/2}\dots\int_{\cos3\varphi}^{1}\dots + \int_{3\pi/2}^{11\pi/6}\dots\int_0^1\dots$$

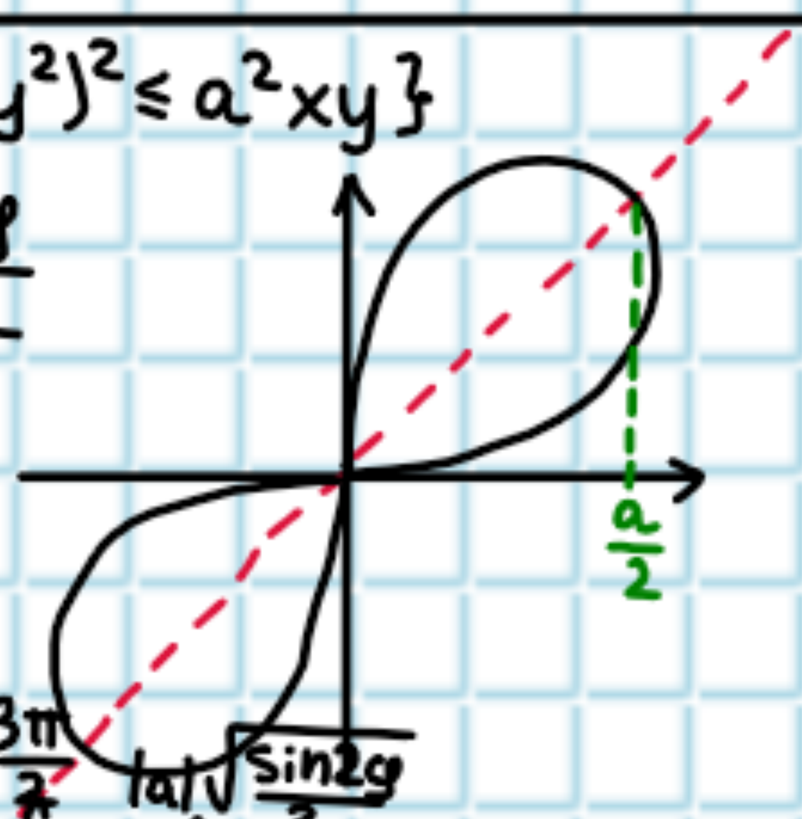
$\max\langle\cos 3\varphi\rangle := \varphi = \frac{2\pi k}{3}$

$\min\langle\cos 3\varphi\rangle := \varphi = \frac{\pi}{6} + \frac{\pi k}{3}$

15.95 $D=\{(x,y):\ (x^2+y^2)^2\le a^2xy\}$

$\rho^4 \le a^2\rho^2\cos\varphi\sin\varphi$

$\rho \le |a|\sqrt{\frac{\sin 2\varphi}{2}}$



$\sin 2\varphi \ge 0$

$\varphi\in[\pi k;\ \frac{\pi}{2}+\pi k]$

$$\int_0^{\pi/2} d\varphi \int_0^{|a|\sqrt{\frac{\sin 2\varphi}{2}}} f(\rho,\varphi)\rho\, d\rho + \int_\pi^{3\pi/2} d\varphi \int_0^{|a|\sqrt{\frac{\sin 2\varphi}{2}}} f(\rho,\varphi)\rho\, d\rho$$