

$$(6.3) \int \frac{\sqrt{x} + \sqrt[3]{x}}{\sqrt[4]{x}} dx = \int (x^{1/4} + x^{1/12}) dx = \frac{x^{5/4}}{5/4} + \frac{x^{13/12}}{13/12} + C = \frac{\sqrt[4]{x^5}}{5} + \frac{12\sqrt[12]{x^{13}}}{13} + C, C \in \mathbb{R}$$

$$(6.7) \int \frac{4-x^2}{3+x^2} dx = \int \frac{-(3+x^2)+7}{3+x^2} dx = -x + \frac{7}{\sqrt{3}} \operatorname{arctg}\left(\frac{x}{\sqrt{3}}\right) + C, C \in \mathbb{R}$$

$$(6.9) \int \frac{dx}{x^4-1} = \int \left(\frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x^2+1} \right) dx$$

$A(x+1)(x^2+1)=1$ при $x=1$: $4A=1 \Rightarrow A=\frac{1}{4}$
 $B(x-1)(x^2+1)=1$ при $x=-1$: $-4B=1 \Rightarrow B=-\frac{1}{4}$
 $C(x+1)(x-1)=1$ при $x=i$: $-2C=1 \Rightarrow C=-\frac{1}{2}$

т.е. $\int \left(\frac{1}{4(x-1)} - \frac{1}{4(x+1)} - \frac{1}{2(x^2+1)} \right) dx = \frac{1}{4} \ln|x-1| - \frac{1}{4} \ln|x+1| - \frac{1}{2} \operatorname{arctg} x + F, F \in \mathbb{R}$

$$(6.12) \int \frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1-x^4}} dx = \int \left(\frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1+x^2}} \right) dx = \operatorname{arcsinh} x + \ln(x + \sqrt{x^2+1}) + C, C \in \mathbb{R}$$

$\sqrt{(1+x^2)(1-x^2)}$
 $x = \operatorname{sh} t, dx = \operatorname{ch} t, \int \frac{1}{\sqrt{1+\operatorname{sh}^2 t}} \operatorname{ch} t dt = \int \frac{\operatorname{ch} t}{\operatorname{ch} t} dt = t = \frac{e^x - e^{-x}}{2};$
 $\operatorname{ch}^2 t - \operatorname{sh}^2 t = 1$
 $2t \cdot e^x = e^{2x} - 1$
 $x = \ln(t + \sqrt{t^2+1})$

$$(6.17) \int \frac{dx}{\sqrt{2x^2+5}} = \frac{1}{\sqrt{2}} \cdot \ln|x + \sqrt{x^2 + \frac{5}{2}}| + C, C \in \mathbb{R}$$

$\sqrt{2}(\sqrt{x^2 + \frac{5}{2}})$

$$(6.24) \int \frac{2^{2x}-1}{2^{x/2}} dx = \int \left(2^{\frac{3x}{2}} - 2^{-\frac{x}{2}} \right) dx = \frac{2}{3} \cdot \frac{2^{\frac{3x}{2}}}{\ln 2} - 2 \cdot \frac{2^{-\frac{x}{2}}}{\ln 2} + C, C \in \mathbb{R}$$

$$(6.25) \int \sin^2 \frac{x}{2} dx = \int \frac{1-\cos x}{2} dx = \frac{1}{2} x - \frac{\sin x}{2} + C, C \in \mathbb{R}$$

$$(6.28) \int \frac{\cos 2x}{\sin^2 2x} dx = \int \frac{1}{2t^2} dt = -\frac{1}{2} t^{-1} + C = -\frac{1}{2 \sin 2x} + C, C \in \mathbb{R}$$

$\sin 2x = t, dx = \frac{1}{2\sqrt{1-t^2}} dt = \frac{1}{2 \cos 2x} dt$
 $x = \frac{\operatorname{arcsin} t}{2}$

$$(6.30) \int \operatorname{ctg}^2 x dx = \int \frac{t^2}{-(1+t^2)} dt = \int \left(-1 + \frac{1}{1+t^2} \right) dt = -\operatorname{ctg} x - \operatorname{arctg}(\operatorname{ctg} x) = -\operatorname{ctg} x - x + C, C \in \mathbb{R}$$

$t = \operatorname{ctg} x; x = \operatorname{arccot} t$
 $dx = -\frac{1}{1+t^2} dt$

38	47	82	99
40	61	83	271
42	66	84	269
43	75	89	272
44	81	98	288