(III) Air resistance acting on a falling body can be taken into account by the approximate relation for the acceleration:

$$a = \frac{dv}{dt} = g - kv,$$

where k is a constant. (*a*) Derive a formula for the velocity of the body as a function of time assuming it starts from rest ($v = 0$ at $t = 0$). [*Hint*: Change variables by setting $u = g - kv$.] (*b*) Determine an expression for the terminal velocity, which is the maximum value the velocity reaches.

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a) $\exists\, u = g - kv$, тогда

$dv = u\,dt$; $\frac{du}{dv} = -k \to dv = \frac{du}{-k}$

т.е. $\int dt = \int \frac{du}{-ku}$

тогда $t + C_1 = \frac{-1}{k}\ln|u| + C_2$

(1) $t = -\frac{1}{k}\ln|u| + C$, $C = C_2 - C_1 \in \mathbb{R}$

начальное условие: $\frac{1}{k}\ln|g - k\cdot 0| = C$

т.е. $C = \frac{\ln g}{k}$

подставим константу в (1):

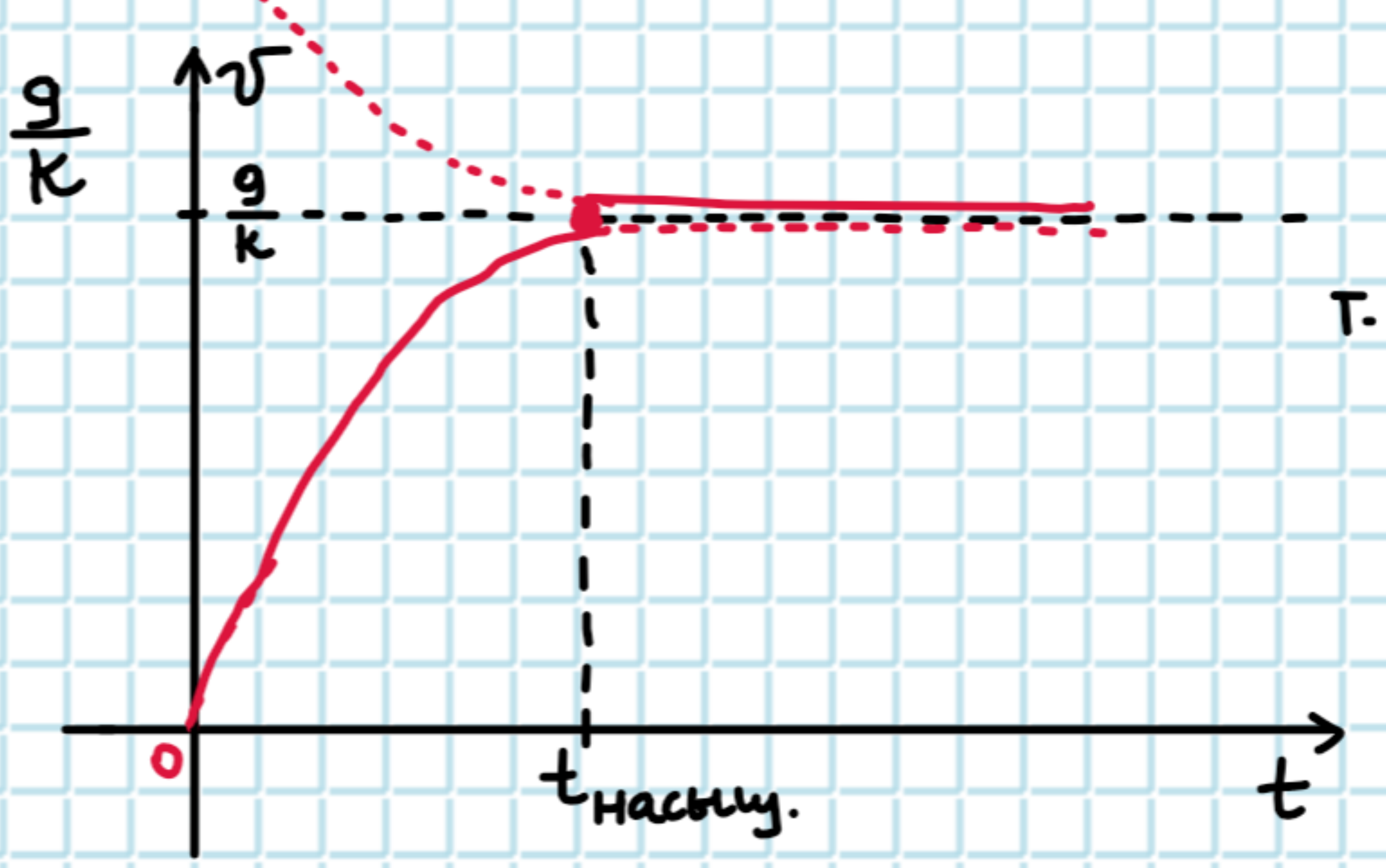
$\to t = -\frac{1}{k}\ln|g-kv| + \frac{\ln|g|}{k} = \frac{\ln\left|\frac{g}{g-kv}\right|}{k}$

т.е. $e^{kt} = \left|\frac{g}{g-kv}\right| \longrightarrow$ $\boxed{\begin{aligned} v<\tfrac{g}{k}&:\ v = \tfrac{g}{k}\left(1-e^{-kt}\right) \\ v>\tfrac{g}{k}&:\ v = \tfrac{g}{k}\left(1+e^{-kt}\right) \end{aligned}}$ – ОТВЕТ.

b) просят найти v_{max} или значение в момент насыщения

заметим, что график функции $v(t)$ монотонно возрастает (часть склейки $v < \frac{g}{k}$)

Нарисуем график вблизи $v = \frac{g}{k}$



т.е. $\boxed{v_{max} = \frac{g}{k}}$ – ОТВЕТ.