Лопатенко Георгий М3102

6.3 $\int \frac{\sqrt{x}+\sqrt[3]{x}}{\sqrt[4]{x}}dx = \int\left(x^{1/4}+x^{1/12}\right)dx = \frac{x^{5/4}}{5/4}+\frac{x^{13/12}}{13/12}+C = \frac{4\sqrt[4]{x^5}}{5}+\frac{12\sqrt[12]{x^{13}}}{13}+C,\ C\in\mathbb{R}$

6.7 $\int\frac{4-x^2}{3+x^2}dx = \int\frac{-(3+x^2)+7}{3+x^2}dx = -x+\frac{7}{\sqrt3}\operatorname{arctg}\left(\frac{x}{\sqrt3}\right)+C,\ C\in\mathbb{R}$

6.9 $\int\frac{dx}{x^4-1} = \int\left(\frac{A}{x-1}+\frac{B}{x+1}+\frac{C}{x^2+1}\right)dx$

$A(x+1)(x^2+1)=1$ при $x=1$: $4A=1 \Rightarrow A=\frac14$

$B(x-1)(x^2+1)=1$ при $x=-1$: $-4B=1 \Rightarrow B=-\frac14$

$C(x+1)(x-1)=1$ при $x=i$: $-2C=1 \Rightarrow C=-\frac12$

т.е. $\int\left(\frac{1}{4(x-1)}-\frac{1}{4(x+1)}-\frac{1}{2(x^2+1)}\right)dx = \frac14\ln|x-1|-\frac14\ln|x+1|-\frac12\operatorname{arctg}x+F,\ F\in\mathbb{R}$

6.12 $\int\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\underbrace{\sqrt{1-x^4}}_{=\sqrt{(1+x^2)(1-x^2)}}}dx = \int\left(\frac{1}{\sqrt{1-x^2}}+\frac{1}{\sqrt{1+x^2}}\right)dx = \arcsin x+\ln\left(x+\sqrt{x^2+1}\right)+C,\ C\in\mathbb{R}$

$x=\operatorname{sh}t$, $dx=\operatorname{ch}t$: $\int\frac{1}{\sqrt{1+\operatorname{sh}^2t}}\operatorname{ch}t\,dt = \int\frac{\operatorname{ch}t}{\operatorname{ch}t}dt = t = \frac{e^x-e^{-x}}{2}$;

$\operatorname{ch}^2t-\operatorname{sh}^2t=1$

$2t\cdot e^x = e^{2x}-1$

$x=\ln\left(t+\sqrt{t^2+1}\right)$

6.17 $\int\frac{dx}{\sqrt{2x^2+5}} = \frac{1}{\sqrt2}\cdot\ln\left|x+\sqrt{x^2+\frac52}\right|+C,\ C\in\mathbb{R}$

$\sqrt{2x^2+5}=\sqrt2\left(\sqrt{x^2+\frac52}\right)$

6.24 $\int\frac{2^{2x}-1}{2^{x/2}}dx = \int\left(2^{\frac{3x}{2}}-2^{-\frac{x}{2}}\right)dx = \frac23\cdot\frac{2^{\frac{3x}{2}}}{\ln2}-2\cdot\frac{2^{-\frac{x}{2}}}{\ln2}+C,\ C\in\mathbb{R}$

6.25 $\int\sin^2\frac{x}{2}dx = \int\frac{1-\cos x}{2}dx = \frac12x-\frac{\sin x}{2}+C,\ C\in\mathbb{R}$ (✱)

6.28 $\int\frac{\cos2x}{\sin^22x}dx = \int\frac{1}{2t^2}dt = -\frac12t^{-1}+C = -\frac{1}{2\sin2x}+C,\ C\in\mathbb{R}$

$\sin2x=t$, $x=\frac{\arcsin t}{2}$, $dx=\frac{1}{2\sqrt{1-t^2}}dt=\frac{1}{2\cos2x}dt$

6.30 $\int\operatorname{ctg}^2x\,dx = \int\frac{t^2}{-(1+t^2)}dt = \int\left(-1+\frac{1}{1+t^2}\right)dt = -\operatorname{ctg}x-\operatorname{arctg}(\operatorname{ctg}x) = -\operatorname{ctg}x-x+C,\ C\in\mathbb{R}$

$t=\operatorname{ctg}x$; $x=\operatorname{arcctg}t$; $dx=-\frac{1}{1+t^2}dt$

6.38 $\int\frac{x+3}{(x+2)(x-1)}dx = \int\left(\frac{A}{x+2}+\frac{B}{x-1}\right)dx = -\frac13\ln|x+2|+\frac43\ln|x-1|+C,\ C\in\mathbb{R}$

$\begin{cases}A+B=1\\2B-A=3\end{cases} \to B=\frac43;\ A=-\frac13$

6.40 $\int(2x+5)^{17}dx = \int(2x+5)^{17}\cdot\frac{d(2x+5)}{2} = \frac{(2x+5)^{18}}{36}+C,\ C\in\mathbb{R}$

6.42 $\int\sqrt{4x+1}\,dx = \frac{(4x+1)^{3/2}}{4\cdot3/2}+C,\ C\in\mathbb{R}$

6.43 $\int\frac{dx}{\sqrt{1-2x}} = \frac{2}{(-2)}\cdot\sqrt{1-2x}+C,\ C\in\mathbb{R}$

6.44 $\int\frac{dx}{\sqrt[5]{(3x+1)^4}} = \frac{1\cdot5}{3\cdot1}\sqrt[5]{3x+1}+C,\ C\in\mathbb{R}$

6.47 $\int\frac{dx}{\sin^27x} = \frac{-1}{7}\operatorname{ctg}7x+C,\ C\in\mathbb{R}$

6.61 $\int x\underbrace{\sqrt{1-2x}}\,dx = \int\frac{\sqrt t-\sqrt{t^3}}{2}\cdot\left(-\frac12\right)dt = -\frac{1\cdot2}{4\cdot3}t^{3/2}+\frac{1\cdot2}{4\cdot5}t^{5/2} = -\frac16\sqrt{(1-2x)^3}+\frac{1}{10}\sqrt{(1-2x)^5}+C,\ C\in\mathbb{R}$

$t=1-2x$, $x=\frac{1-t}{2}$; $dx=-\frac12$

6.66 $\int\frac{x^2dx}{x^6-5} = \int\frac{\frac13d(x^3)}{(x^3)^2-5} = \frac{1}{6\sqrt5}\ln\left|\frac{x^3-\sqrt5}{x^3+\sqrt5}\right|+C,\ C\in\mathbb{R}$

6.75 $\int\frac{dx}{x\ln^5x} = \int\frac{d(\ln x)}{\ln^5x} = \frac{\ln^{-4}x}{-4}+C,\ C\in\mathbb{R}$

6.81 $\int\frac{e^x+e^{2x}}{1-e^x}dx = \int\frac{1+e^x}{1-e^x}d(e^x) = \int\left(-1+\frac{2}{1-e^x}\right)d(e^x) = -e^x-2\ln|1-e^x|+C,\ C\in\mathbb{R}$

6.82 $\int\sqrt{e^{3x}+e^{2x}}dx = \frac{\sqrt{(e^x+1)^3}}{3/2}+C,\ C\in\mathbb{R}$

$\int\sqrt{e^x+1}\,d(e^x)$

6.83 $\int\underbrace{x}_{u}\underbrace{\sin^2x\,dx}_{dv} = \frac{x^2}{2}-\frac{x\sin2x}{4}-\int\left(\frac{x}{2}-\frac{\sin2x}{4}\right)dx = \frac{x^2}{4}-\frac{x\sin2x}{4}-\frac{\cos2x}{8}+C,\ C\in\mathbb{R}$

$du=dx$; $v=\frac12x-\frac{\sin2x}{4}$ (✱)

6.84 $\int\frac{1}{\sqrt x}\cos\sqrt x\,dx = \int2\cos t\,dt = 2\sin\sqrt x+C,\ C\in\mathbb{R}$

$t=\sqrt x$; $dx=2t\,dt$

6.89 $\int\operatorname{ctg}x\,dx = \int\frac{d(\sin x)}{\sin x} = \ln|\sin x|+C,\ C\in\mathbb{R}$

6.98 $\int\frac{x+\sqrt{\operatorname{arctg}2x}}{(2x)^2+1} = \int\frac{d(4x^2+1)}{8(4x^2+1)}+\int\sqrt{\operatorname{arctg}2x}\,d(\operatorname{arctg}2x) = \frac{\ln|4x^2+1|}{8}+\frac{2\sqrt{\operatorname{arctg}^3x}}{3}+C,\ C\in\mathbb{R}$

6.99 $\int\frac{\arcsin x}{\sqrt{1-x^2}}dx+\int\frac{\arccos x}{-\sqrt{1-x^2}}dx = \frac{\arcsin^2x+\arccos^2x}{2}+C,\ C\in\mathbb{R}$

6.271 $\int\frac{dx}{(1+x)^3(1-x)^2} = \frac{3\ln(|1+x||1-x|)}{16}-\frac{1}{4(1+x)}-\frac{1}{8(1+x)^2}+\frac{1}{16(1-x)^2}+F,\ F\in\mathbb{R}$

$\frac{A}{1+x}+\frac{B}{(1+x)^2}+\frac{C}{(1+x)^3}+\frac{D}{1-x}+\frac{E}{(1-x)^2}$; $A=\frac{3}{16}$, $B=\frac14=C$, $D=\frac{3}{16}$, $E=\frac18$.

6.269 $\int\frac{x^3+1}{x^2(1-x)}dx = \int\left(\underbrace{\frac1x+\frac{1}{x^2}+\frac{1}{1-x}}_{\frac{1}{x^2-x^3}}\underbrace{-1+\frac{1}{1-x}}_{\frac{x}{1-x}}\right)dx = \ln|x|-2\ln|1-x|-x-\frac1x+C,\ C\in\mathbb{R}$

6.272 $\int\frac{dx}{(x+1)(x^2+1)^2} = \frac{\ln|x+1|}{4}-\frac{\ln|x^2+1|}{8}+\frac{x+1}{4(x^2+1)}+\frac{\operatorname{arctg}x}{2}+C,\ C\in\mathbb{R}$

$\frac{A}{x+1}+\frac{Bx+C}{x^2+1}+\frac{Dx+E}{(x^2+1)^2}$; $A=\frac14$; $B=-\frac14$; $C=\frac14$; $D=-\frac12$; $E=\frac12$

6.288 $\int\frac{3x^2+x-2}{(x-1)^3(x^2+1)}dx = \frac{\ln|x^2+1|}{4}-\frac{\ln|x-1|}{2}-\frac{x+2}{2(x-1)^2}+C,\ C\in\mathbb{R}$