Acoustic FWI in the frequency domain

1. Abstract

This is an instruction on the FWI procedure in the frequency domain.

2. Isotropic acoustic wave equation

The acoustic wave equation in isotropic media is given by (Kosloff & Baysah, 1983)

$$\frac{1}{v^2}\frac{\partial^2 p(x,z,t)}{\partial t^2} = \nabla^2 p(x,z,t) + f(x,z,t) \tag{2.1}$$

where p is the pressure field and f is the source term.

The frequency domain of (2.1) is

$$\frac{\omega}{v^2}P(x,z,\omega) + \nabla^2 P(x,z,\omega) = -F(x,z,\omega) \tag{2.2}$$

Applying discretization to (2.2) yields

$$\begin{aligned} &\frac{\omega}{v^2}P(x,z,\omega) \\ &+\frac{P(x+h,z,\omega)-2P(x,z,\omega)+P(x-h,z,\omega)}{h^2} \\ &+\frac{P(x,z+h,\omega)-2P(x,z,\omega)+P(x,z-h,\omega)}{h^2} = -F(x,z,\omega) \end{aligned} \tag{2.3}$$

where the solution of P is independent in each frequency.

And (2.3) can be linearized in a matrix form

$$A(\omega,v)P = -F \tag{2.4}$$

where A is the forward propagator which can be obtained from (2.3).

3. FWI procedure

The algorithm of FWI is described as below (Li, Lin, Zhang, Li, & Yu, 2018).

1. For each frequency ω_k
    a. Forward propagate P_k with an initial velocity model v by (2.4).
    b. Generate synthetic recordings d_k by

$$d_k = RP_k \tag{3.1}$$

where R is the sampling operator.

c. Compute misfit E

$$E = d_k - D_k \tag{3.2}$$

where D_k is the true recordings.

d. Simulate adjoint wavefield $P_k^\star$ by

$$A(\omega, v)^* P_k^\star = P_k^\star E \tag{3.3}$$

e. The gradient G is given by

$$G = \sum_k \omega_k{}^2 diag(P_k)^* P^\star \tag{3.4}$$

2. Sum G along all of the frequencies.
3. Update v with G

$$v \leftarrow v - \alpha G \tag{3.5}$$

4. Repeat 1- 3.

4. Implementation

The FWI is performed with Perfectly mached layer and absorbing boundary condition introduced to attenuate the reflection from the simulation boundaries.

Figure 4-1 and Figure 4-2 give more details on the input velocity model and output model for this implementation.

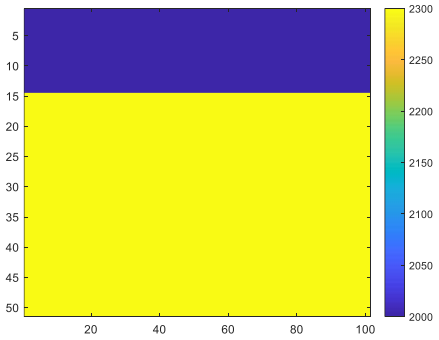

Figure 4-1. Initial velocity model.

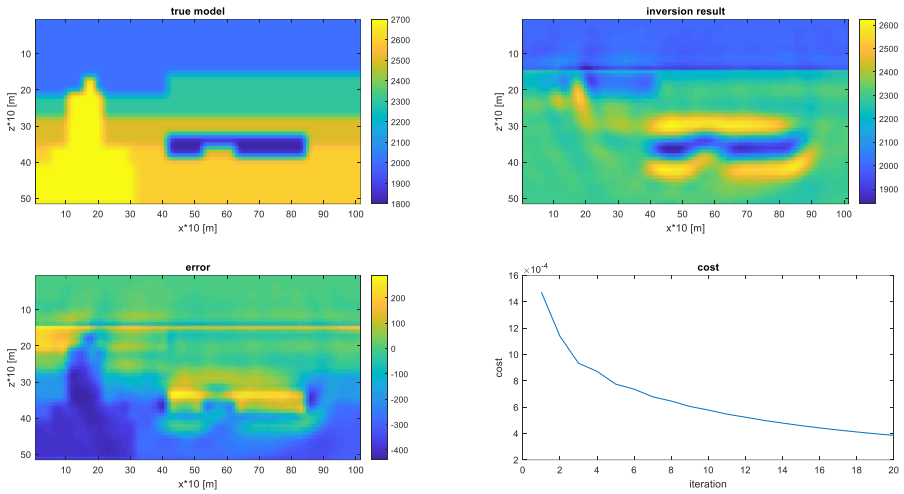

Figure 4-2. Result after 10 iterations with 10 Hz source.

5. References

Kosloff, D. D., & Baysah, E. (1983). Migration with the full acoustic wave equation. *48*(6), 677-687.

Li, Z.-C., Lin, Y.-Z., Zhang, K., Li, Y.-Y., & Yu, Z.-N. (2018). Time-domain wavefield reconstruction inversion. *Applied Geophysics*, *14*(4), 523-528. doi:10.1007/s11770-017-0629-6