# A spectral clustering approach for the evolution of the COVID-19 pandemic in the state of Rio Grande do Sul, Brazil

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**Abstract**. This file complements the paper of the same title. Regarding pendulum migration, we describe the dissemination of COVID-19 within each cluster. Regarding ICU beds, we give information about all clusterings. Regarding our SEIR model, we show the results of our simulations in several cities.

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# ${\bf 1.} \ \ {\bf COVID\ evolution\ in\ partition\ based\ on\ people\ pendular\ migration}$

The figures in this sections complement section 3.1 in the paper.

# 1.1. Clustering pendulum migration

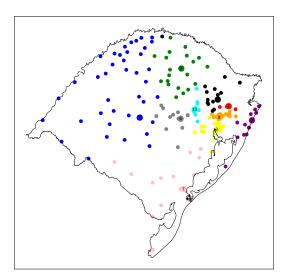


Figure 1: Same partition of Figure 1 in the paper, with clusters numbered in their largest cities.

#### 1.2. Cluster 1



Figure 2: March 7-13 (left) and March 14-20 (right).



Figure 3: March 21-27 (left) and March 28 to April 3 (right).



Figure 4: April 4-10 (left) and April 11-17 (right).



Figure 5: April 18-24 (left) and April 25 to May 1 (right).



Figure 6: May 2-8 (left) and May 9-15 (right).



Figure 7: May 16-22 (left) and May 23-29 (right).

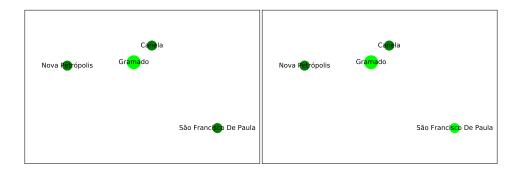


Figure 8: May 30 to June 5 (left) and June 6-12 (right).



Figure 9: June 13-19 (left) and June 20-26 (right).



Figure 10: June 27 to July 3

#### 1.3. Cluster 2

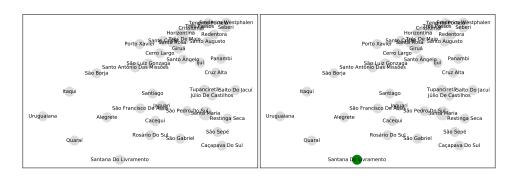


Figure 11: March 7-13 (left) and March 14-20 (right).

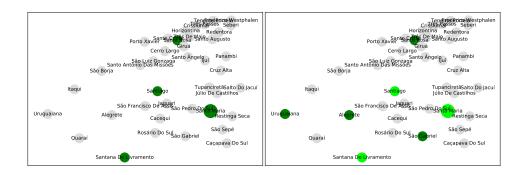


Figure 12: March 21-27 (left) and March 28 to April 3 (right).

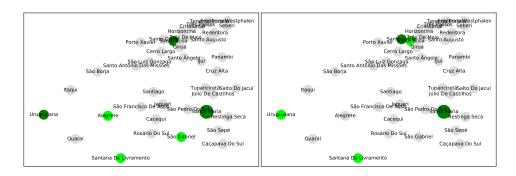


Figure 13: April 4-10 (left) and April 11-17 (right).

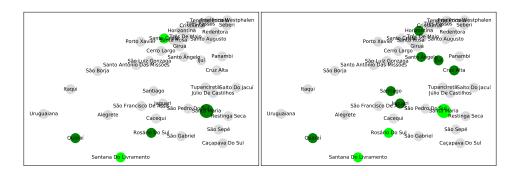


Figure 14: April 18-24 (left) and April 25 to May 1 (right).

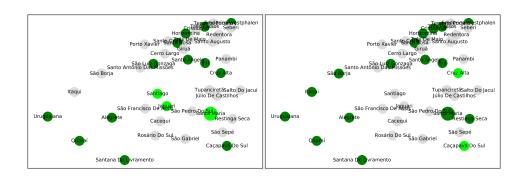


Figure 15: May 2-8 (left) and May 9-15 (right).

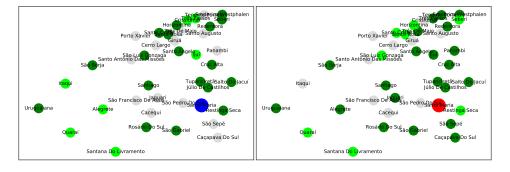


Figure 16: May 16-22 (left) and May 23-29 (right).

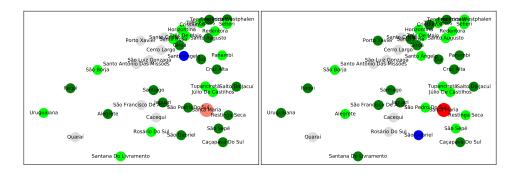


Figure 17: May 30 to June 5 (left) and June 6-12 (right).

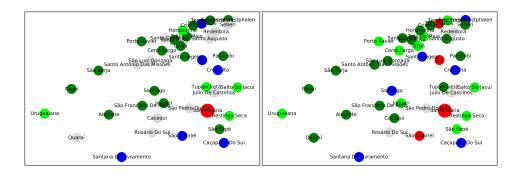


Figure 18: June 13-19 (left) and June 20-26 (right).

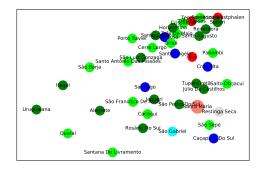


Figure 19: June 27 to July 3

#### 1.4. Cluster 3

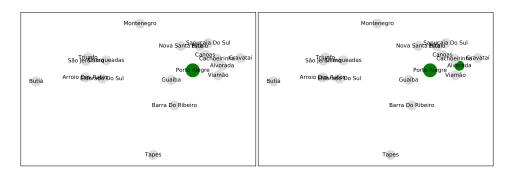


Figure 20: March 7-13 (left) and March 14-20 (right).



Figure 21: March 21-27 (left) and March 28 to April 3 (right).

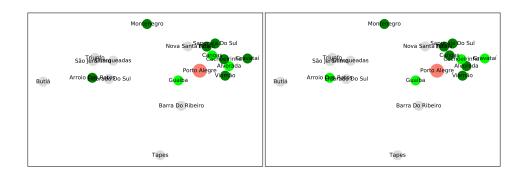


Figure 22: April 4-10 (left) and April 11-17 (right).

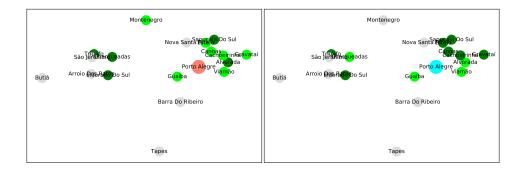


Figure 23: April 18-24 (left) and April 25 to May 1 (right).

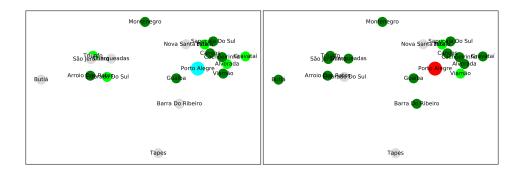


Figure 24: May 2-8 (left) and May 9-15 (right).

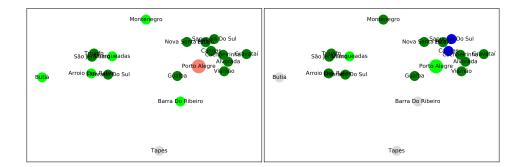


Figure 25: May 16-22 (left) and May 23-29 (right).

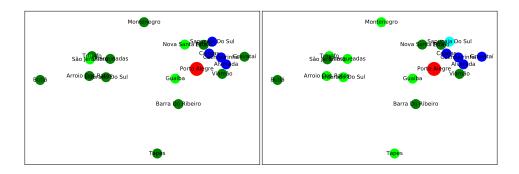


Figure 26: May 30 to June 5 (left) and June 6-12 (right).

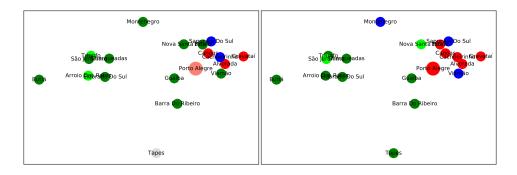


Figure 27: June 13-19 (left) and June 20-26 (right).

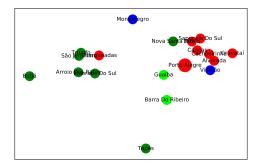


Figure 28: June 27 to July 3

#### 1.5. Cluster 4



Figure 29: March 7-13 (left) and March 14-20 (right).

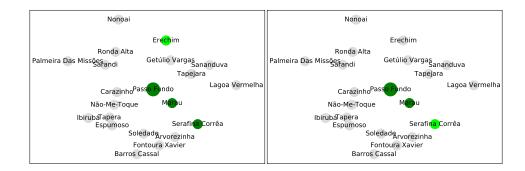


Figure 30: March 21-27 (left) and March 28 to April 3 (right).



Figure 31: April 4-10 (left) and April 11-17 (right).

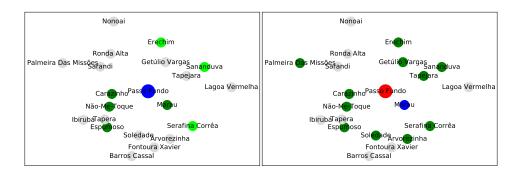


Figure 32: April 18-24 (left) and April 25 to May 1 (right).

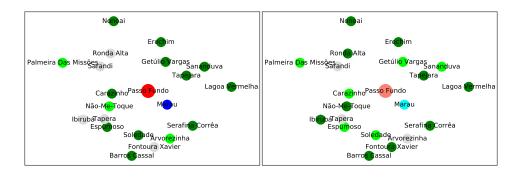


Figure 33: May 2-8 (left) and May 9-15 (right).

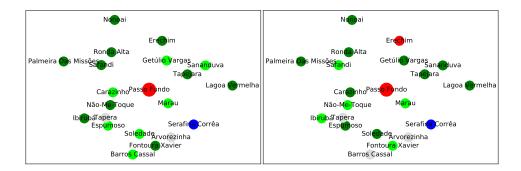


Figure 34: May 16-22 (left) and May 23-29 (right).

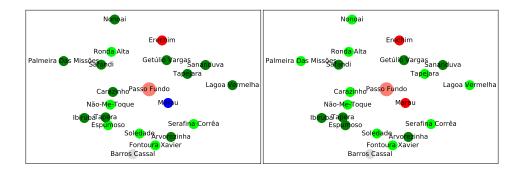


Figure 35: May 30 to June 5 (left) and June 6-12 (right).

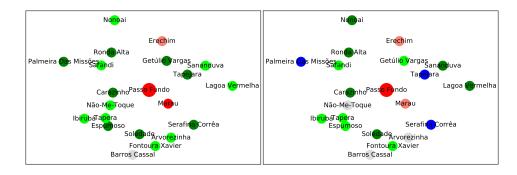


Figure 36: June 13-19 (left) and June 20-26 (right).

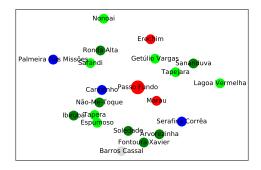


Figure 37: June 27 to July 3

#### 1.6. Cluster 5



Figure 38: March 7-13 (left) and March 14-20 (right).



Figure 39: March 21-27 (left) and March 28 to April 3 (right).



Figure 40: April 4-10 (left) and April 11-17 (right).



Figure 41: April 18-24 (left) and April 25 to May 1 (right).



Figure 42: May 2-8 (left) and May 9-15 (right).



Figure 43: May 16-22 (left) and May 23-29 (right).



Figure 44: May 30 to June 5 (left) and June 6-12 (right).



Figure 45: June 13-19 (left) and June 20-26 (right).

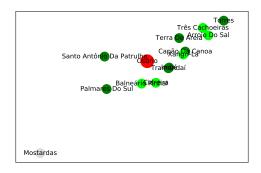


Figure 46: June 27 to July 3

#### 1.7. Cluster 6



Figure 47: March 7-13 (left) and March 14-20 (right).



Figure 48: March 21-27 (left) and March 28 to April 3 (right).



Figure 49: April 4-10 (left) and April 11-17 (right).



Figure 50: April 18-24 (left) and April 25 to May 1 (right).



Figure 51: May 2-8 (left) and May 9-15 (right).

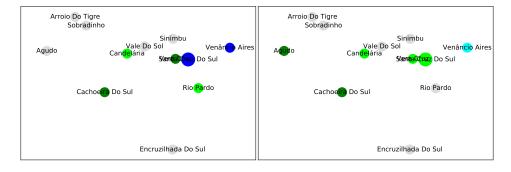


Figure 52: May 16-22 (left) and May 23-29 (right).

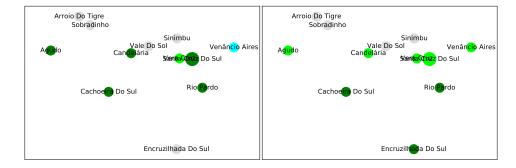


Figure 53: May 30 to June 5 (left) and June 6-12 (right).

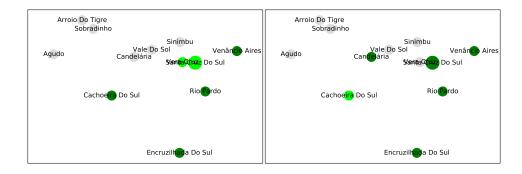


Figure 54: June 13-19 (left) and June 20-26 (right).

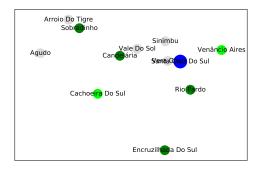


Figure 55: June 27 to July 3

#### 1.8. Cluster 7

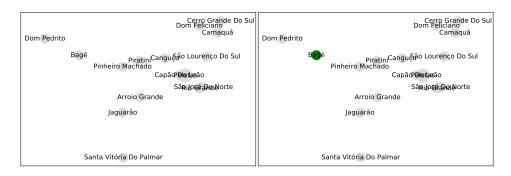


Figure 56: March 7-13 (left) and March 14-20 (right).

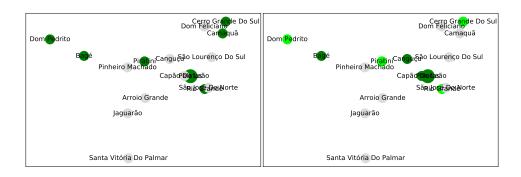


Figure 57: March 21-27 (left) and March 28 to April 3 (right).

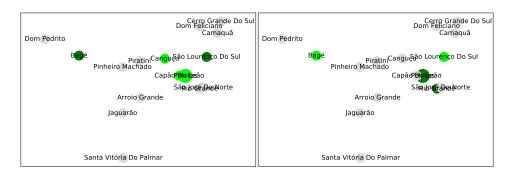


Figure 58: April 4-10 (left) and April 11-17 (right).



Figure 59: April 18-24 (left) and April 25 to May 1 (right).

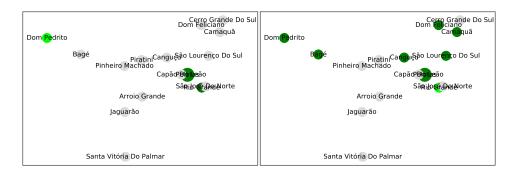


Figure 60: May 2-8 (left) and May 9-15 (right).



Figure 61: May 16-22 (left) and May 23-29 (right).

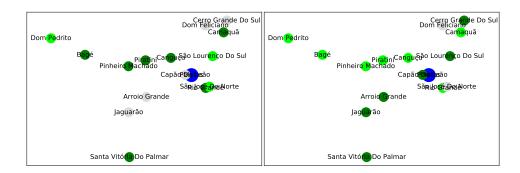


Figure 62: May 30 to June 5 (left) and June 6-12 (right).

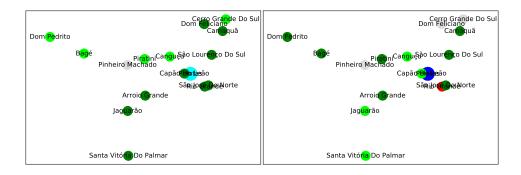


Figure 63: June 13-19 (left) and June 20-26 (right).

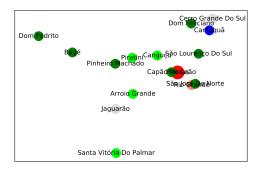


Figure 64: June 27 to July 3

#### 1.9. Cluster 8



Figure 65: March 7-13 (left) and March 14-20 (right).

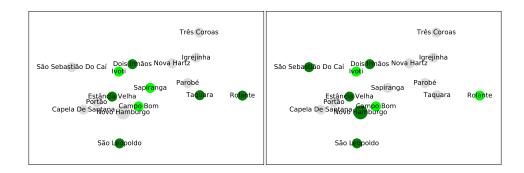


Figure 66: March 21-27 (left) and March 28 to April 3 (right).

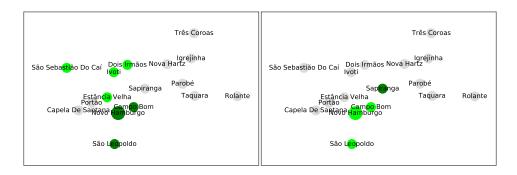


Figure 67: April 4-10 (left) and April 11-17 (right).

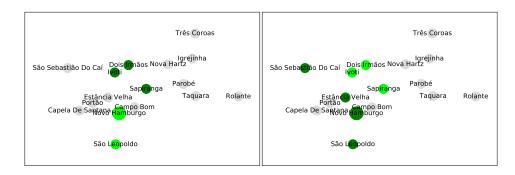


Figure 68: April 18-24 (left) and April 25 to May 1 (right).



Figure 69: May 2-8 (left) and May 9-15 (right).

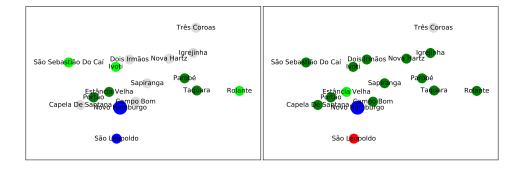


Figure 70: May 16-22 (left) and May 23-29 (right).

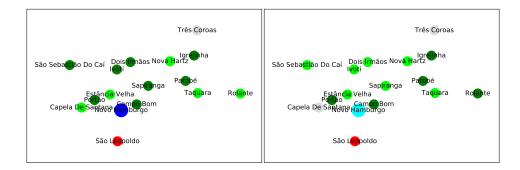


Figure 71: May 30 to June 5 (left) and June 6-12 (right).

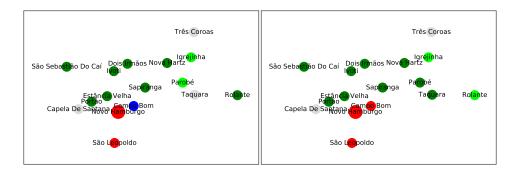


Figure 72: June 13-19 (left) and June 20-26 (right).

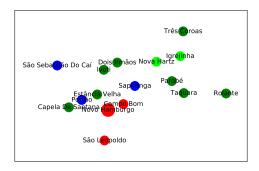


Figure 73: June 27 to July 3

#### 1.10. Cluster 9

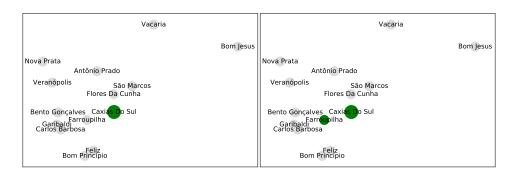


Figure 74: March 7-13 (left) and March 14-20 (right).

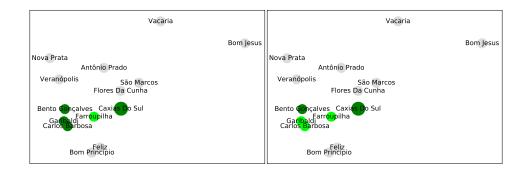


Figure 75: March 21-27 (left) and March 28 to April 3 (right).

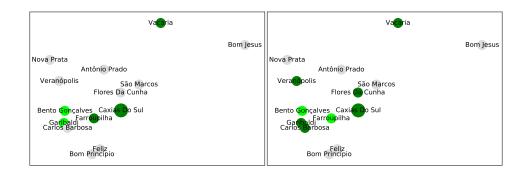


Figure 76: April 4-10 (left) and April 11-17 (right).

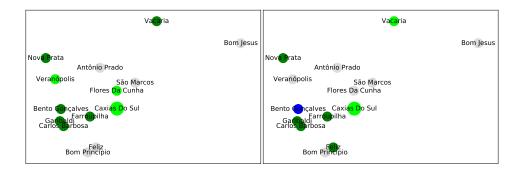


Figure 77: April 18-24 (left) and April 25 to May 1 (right).

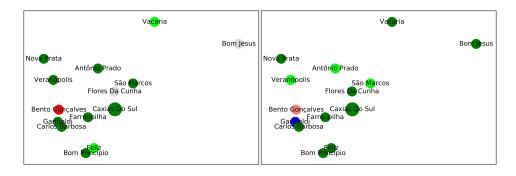


Figure 78: May 2-8 (left) and May 9-15 (right).

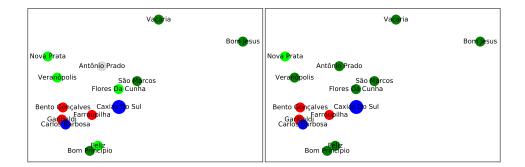


Figure 79: May 16-22 (left) and May 23-29 (right).

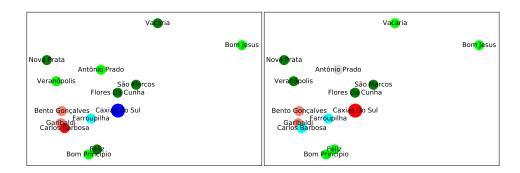


Figure 80: May 30 to June 5 (left) and June 6-12 (right).

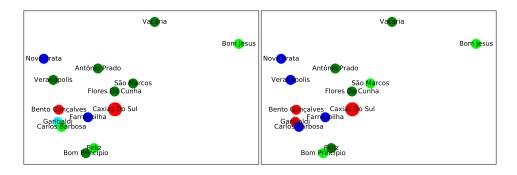


Figure 81: June 13-19 (left) and June 20-26 (right).

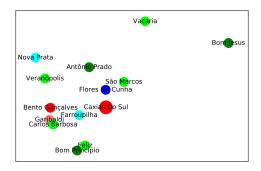


Figure 82: June 27 to July 3

## 1.11. Cluster 10



Figure 83: March 7-13 (left) and March 14-20 (right).

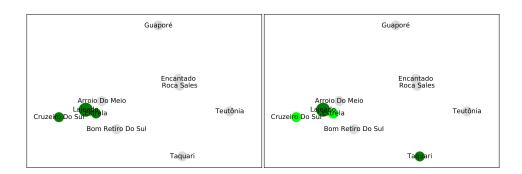


Figure 84: March 21-27 (left) and March 28 to April 3 (right).

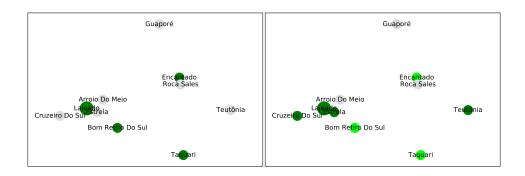


Figure 85: April 4-10 (left) and April 11-17 (right).

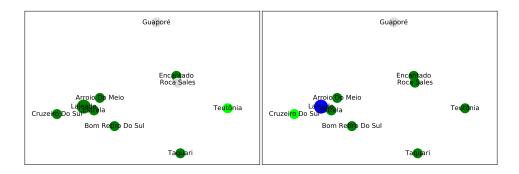


Figure 86: April 18-24 (left) and April 25 to May 1 (right).

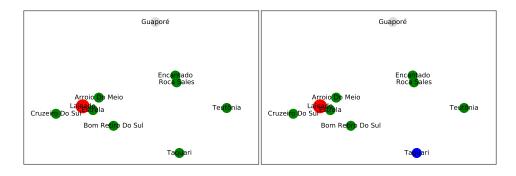


Figure 87: May 2-8 (left) and May 9-15 (right).

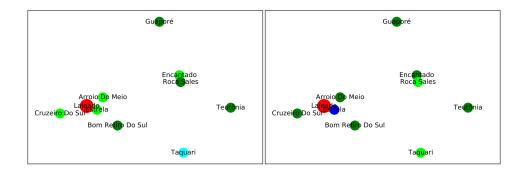


Figure 88: May 16-22 (left) and May 23-29 (right).

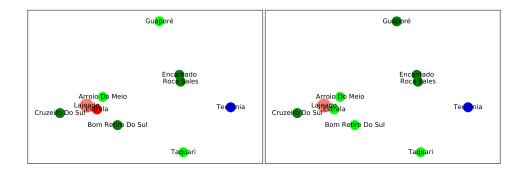


Figure 89: May 30 to June 5 (left) and June 6-12 (right).

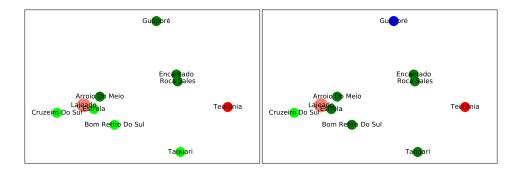


Figure 90: June 13-19 (left) and June 20-26 (right).

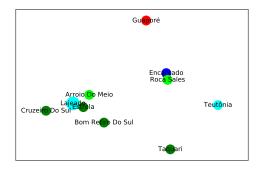


Figure 91: June 27 to July 3

# 2. Clustering based on available ICU beds

The figures in this sections complement section 3.2 in the paper.

## 2.1. Static partition based on total ICU beds

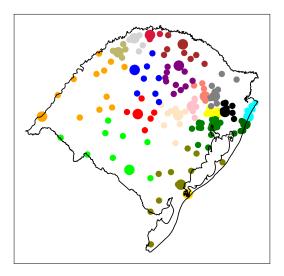


Figure 92: Partition obtained using total ICU beds in cities data, whose gap is  $\text{NCut}(\mathcal{P})/\text{ncut}_k^{rel}(G) \approx 1.0339$ .

## 2.2. Weekly partitions based on available ICU beds

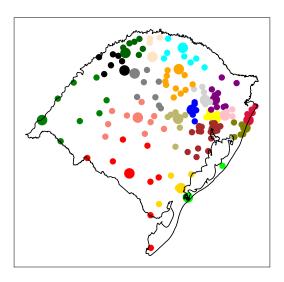


Figure 93: Partition obtained using data from week May 9-15, whose gap is  $\text{NCut}(\mathcal{P})/\text{ncut}_k^{rel}(G) \approx 1.0360$ .

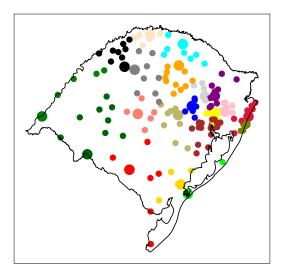


Figure 94: Partition obtained using data from week May 16-22, whose gap is  $\mathrm{NCut}(\mathcal{P})/\mathrm{ncut}_k^{rel}(G) \approx 1.0337$ .

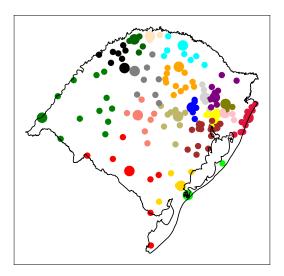


Figure 95: Partition obtained using data from week May 23-29, whose gap is  $\mathrm{NCut}(\mathcal{P})/\mathrm{ncut}_k^{rel}(G) \approx 1.0330$ .

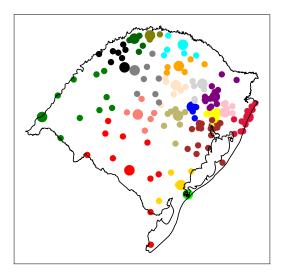


Figure 96: Partition obtained using data from week May 30 to June 5, whose gap is  $\text{NCut}(\mathcal{P})/\text{ncut}_k^{rel}(G) \approx 1.0330$ .

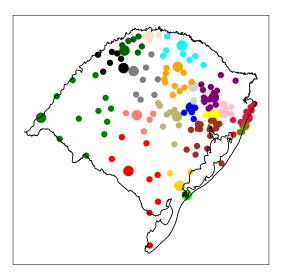


Figure 97: Partition obtained using data from week June 6-12, whose gap is  $\mathrm{NCut}(\mathcal{P})/\mathrm{ncut}_k^{rel}(G) \approx 1.0311$ .

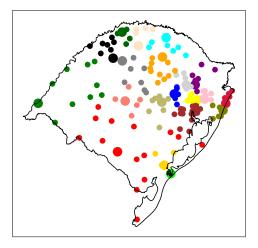


Figure 98: Partition obtained using data from week June 13-19, whose gap is  $\mathrm{NCut}(\mathcal{P})/\mathrm{ncut}_k^{rel}(G) \approx 1.0329$ .

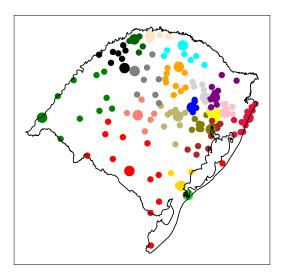


Figure 99: Partition obtained using data from week June 20-26, whose gap is  $\mathrm{NCut}(\mathcal{P})/\mathrm{ncut}_k^{rel}(G) \approx 1.0354$ .

# ${\bf 2.3.}$ Comparison of state's flags with our flags in state's regions

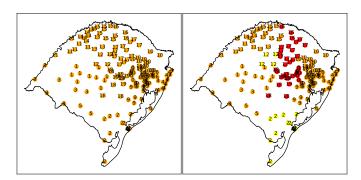


Figure 100: Flags assigned by the state formula (left) and by our formula (right) on the week May 9-15.

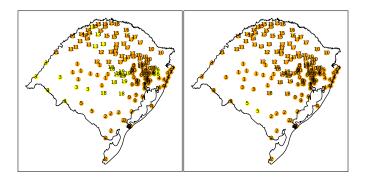


Figure 101: Flags assigned by the state formula (left) and by our formula (right) on the week May 16-22.

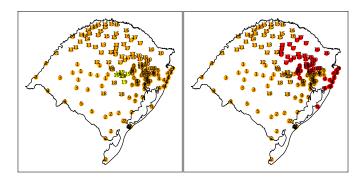


Figure 102: Flags assigned by the state formula (left) and by our formula (right) on the week May 23-29.

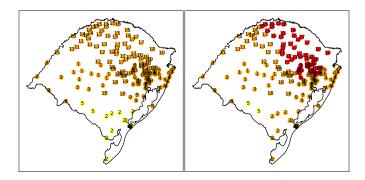


Figure 103: Flags assigned by the state formula (left) and by our formula (right) on the week May 30 to June 5.

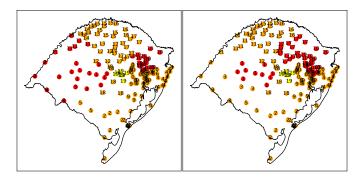


Figure 104: Flags assigned by the state formula (left) and by our formula (right) on the week June 6-12.

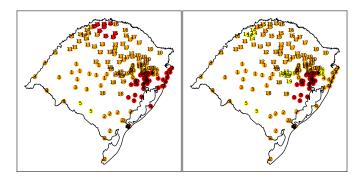


Figure 105: Flags assigned by the state formula (left) and by our formula (right) on the week June 13-19.

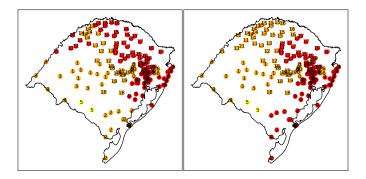


Figure 106: Flags assigned by the state formula (left) and by our formula (right) on the week June 20-26.

## 2.4. Our flags assigned to state's regions and to dynamic regions along seven weeks

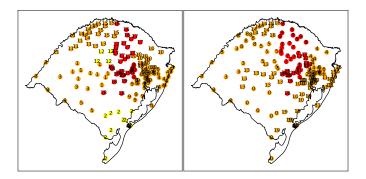


Figure 107: Flags assigned by our formula to the state's regions (left) and to the dynamic regions (right) on the week May 9-15.

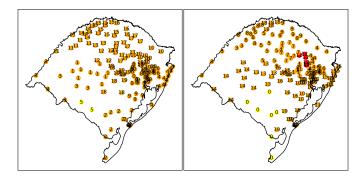


Figure 108: Flags assigned by our formula to the state's regions (left) and to the dynamic regions (right) on the week May 16-22.

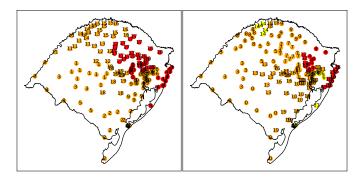


Figure 109: Flags assigned by our formula to the state's regions (left) and to the dynamic regions (right) on the week May 23-29.

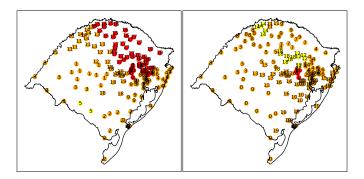


Figure 110: Flags assigned by our formula to the state's regions (left) and to the dynamic regions (right) on the week May 30 to June 5.

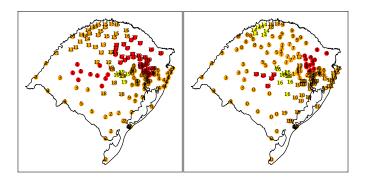


Figure 111: Flags assigned by our formula to the state's regions (left) and to the dynamic regions (right) on the week June 6-12.

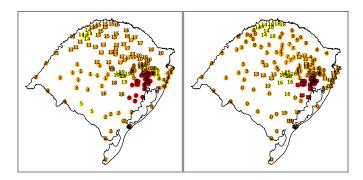


Figure 112: Flags assigned by our formula to the state's regions (left) and to the dynamic regions (right) on the week June 13-19.

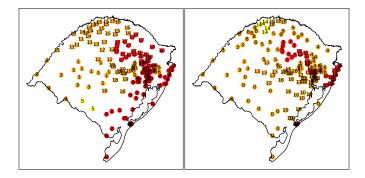


Figure 113: Flags assigned by our formula to the state's regions (left) and to the dynamic regions (right) on the week June 20-26.

### 3. SEIR model

The figures in this sections complement section 4 in the paper.

#### 3.1. Largest city from each cluster in section 1

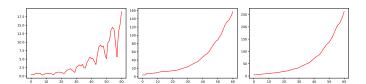


Figure 114: Gramado (center of cluster 1 in section 1,  $62^{th}$  largest city), number of new cases assuming the actual isolation data (left). Number of active cases (center). Overall number of cases (right).

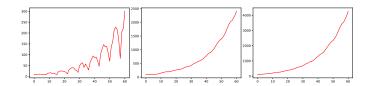


Figure 115: Santa Maria (center of cluster 2 in section 1,  $5^{th}$  largest city), number of new cases assuming the actual isolation data (left). Number of active cases (center). Overall number of cases (right).

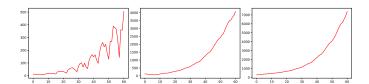


Figure 116: Porto Alegre (center of cluster 3 in section 1, largest city of the state), number of new cases assuming the actual isolation data (left). Number of active cases (center). Overall number of cases (right).

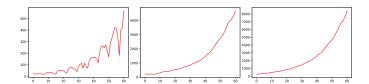


Figure 117: Passo Fundo (center of cluster 4 in section 1,  $12^{th}$  largest city), number of new cases assuming the actual isolation data (left). Number of active cases (center). Overall number of cases (right).

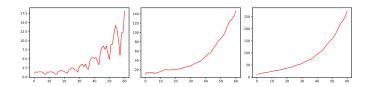


Figure 118: Osório (center of cluster 5 in section 1,  $47^{th}$  largest city), number of new cases assuming the actual isolation data (left). Number of active cases (center). Overall number of cases (right).

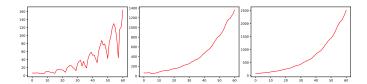


Figure 119: Santa Cruz do Sul (center of cluster 6 in section 1,  $14^{th}$  largest city), number of new cases assuming the actual isolation data (left). Number of active cases (center). Overall number of cases (right).

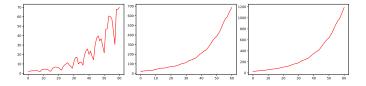


Figure 120: Pelotas (center of cluster 7 in section 1,  $4^{th}$  largest city), number of new cases assuming the actual isolation data (left). Number of active cases (center). Overall number of cases (right).

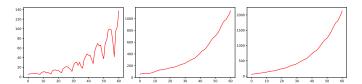


Figure 121: Novo Hamburgo (center of cluster 8 in section 1,  $8^{th}$  largest city), number of new cases assuming the actual isolation data (left). Number of active cases (center). Overall number of cases (right).

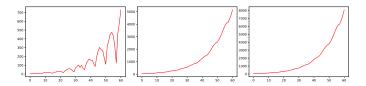


Figure 122: Caxias do Sul (center of cluster 9 in section 1,  $2^{nd}$  largest city), number of new cases assuming the actual isolation data (left). Number of active cases (center). Overall number of cases (right).

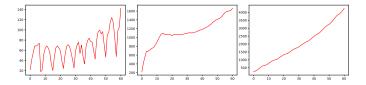


Figure 123: Lajeado (center of cluster 10 in section 1,  $21^{th}$  largest city), number of new cases assuming the actual isolation data (left). Number of active cases (center). Overall number of cases (right).

#### 3.2. Other large cities

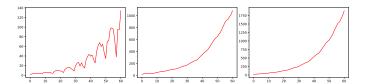


Figure 124: Canoas (member of cluster 3 in section 1,  $3^{rd}$  largest city), number of new cases assuming the actual isolation data (left). Number of active cases (center). Overall number of cases (right).

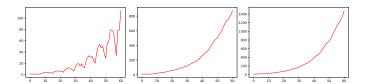


Figure 125: Gravataí (member of cluster 3 in section 1,  $6^{th}$  largest city), number of new cases assuming the actual isolation data (left). Number of active cases (center). Overall number of cases (right).

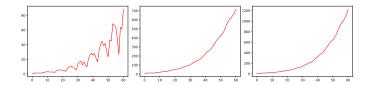


Figure 126: Viamão (member of cluster 3 in section 1,  $7^{th}$  largest city), number of new cases assuming the actual isolation data (left). Number of active cases (center). Overall number of cases (right).

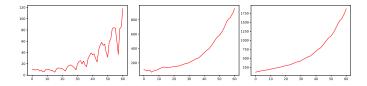


Figure 127: São Leopoldo (member of cluster 8 in section 1,  $9^{th}$  largest city), number of new cases assuming the actual isolation data (left). Number of active cases (center). Overall number of cases (right).

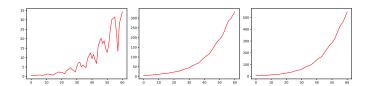


Figure 128: Rio Grande (member of cluster 7 in section 1,  $10^{th}$  largest city), number of new cases assuming the actual isolation data (left). Number of active cases (center). Overall number of cases (right).