Statistical Models with Variational Methods

End-of-degree project

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CONTENTS

1 PROBABILITY

1.1 *Notation*

Variables will be denoted with lower case x and a set of variables with a calligraphic symbol like $\mathcal{V}$.

The meaning of $p(state)$ will be clear without a reference to the variable. Otherwise $p(x = state)$ will be used.

We will denote $p(x)$ the probability of x taking a specific value, this means that

$$\int_x f(x) = \int_{dom(x)} f(x = s)ds$$

1.2 *Definitions*

We will define some concepts from a given joint distribution $p(x, y)$, this is, the probability of two events.

Definition 1. A **marginal** $p(x)$ of the joint distribution is the distribution of a single variable given by

$$p(x) = \sum_y p(x, y) \qquad\qquad p(x) = \int_y p(x, y)$$

We can undestand this as the probability of an event irrespective of the outcome of another variable.

Definition 2. The **conditional probability** of x given y is defined as

$$p(x|y) = \frac{p(x, y)}{p(y)}$$

If $p(y) = 0$ then it is not defined.

This formula is also known as **Bayes' rule**. With this definition the conditional probability is the probability of one event occurring in the presence of a second event.

Now suppose we have some observed data $\mathcal{D}$ and we want to learn about a set of parameters θ. Using Bayes' rule we got that

$$p(\theta|\mathcal{D}) = \frac{p(\mathcal{D}|\theta)p(\theta)}{p(\mathcal{D})} = \frac{p(\mathcal{D}|\theta)p(\theta)}{\int_\theta p(\mathcal{D}|\theta)p(\theta)}$$

This shows how from a *generative model* $p(\mathcal{D}|\theta)$ of the dataset and a *prior* belief $p(\theta)$, we can infer the *posterior* distribution $p(\theta|\mathcal{D})$.

Example 1. Consider a study where the relation of a disease D and an habit H is being investigated. Consider $p(D) = 10^{-5}$, $p(H) = 0.5$ and $p(H|D) = 0.9$. What is the probability that a person with habit H will have disease D?

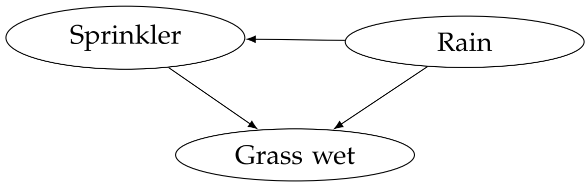
$$p(D|S) = \frac{p(D, H)}{p(D)} = \frac{p(H|D)p(D)}{p(H)} = \frac{0.9 \times 10^{-5}}{0.5} = 1.8 \times 10^{-5}$$

If we set the probability of having habit H to a much lower value as $p(H) = 0.001$, then the above calculation gives aproximately $1/100$.

Intuitively, a smaller number of people have the habit and most of them have the desease. This means that the relation between having the desease and the habit is stronger.

2 GRAPHICAL MODELS

	Sprinkler	
Rain	T	F
F	0.4	0.6
T	0.01	0.99



Sprinkler	
T	F
0.2	0.8

		Grass wet	
Sprinkler	rain	T	F
F	F	0.4	0.6
F	T	0.01	0.99
T	F	0.01	0.99
T	T	0.01	0.99