Stream Ciphers

Elements of Applied Data Security M

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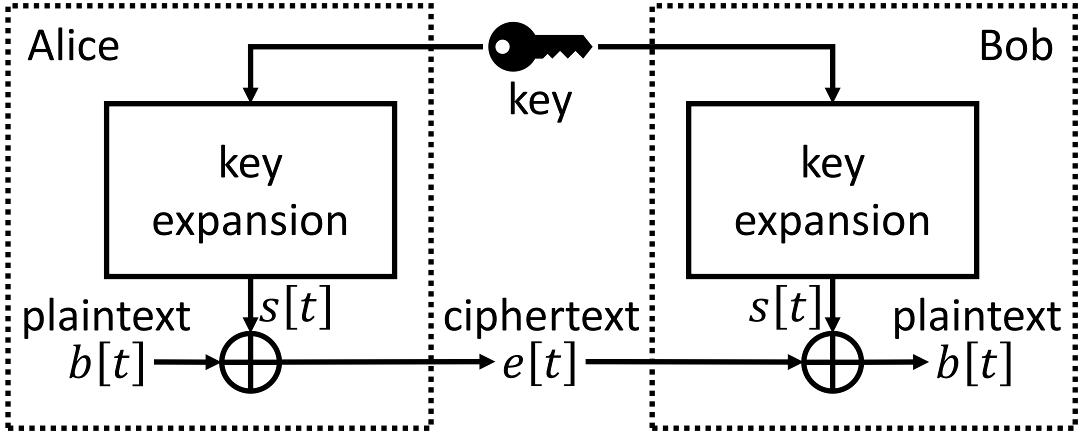
Stream Cipher

A stream cipher is a **symmetric key** cipher where the plaintext is encrypted (and ciphertext is decrypted) one digit at a time. A digit usually is either a bit or a byte.



Encryption (decryption) is achieved by xoring the plaintext (ciphertext) with a stream of pseudorandom digits obtained as an expansion of the key.

Tasks

1. LFSR
2. Berlekamp-Massey Algorithm
3. LFSR-based generator

Python Module

- A **module** is a `.py` file in which you can collect several definitions that you may want to use repeatedly.
- You can then **import** such definitions either into your notebook or even in other modules.

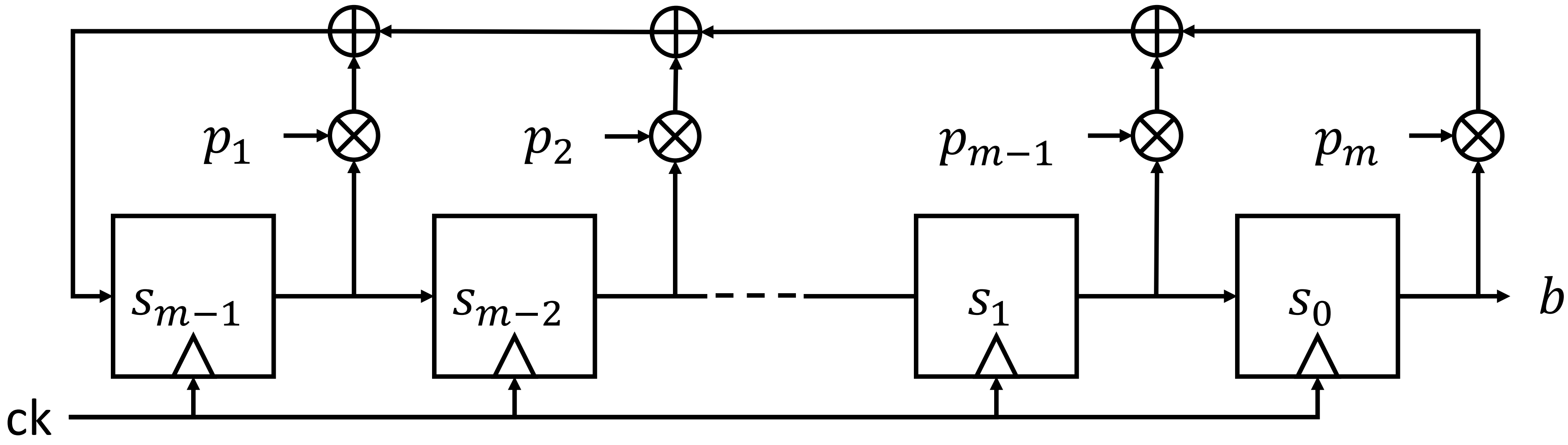
For the current assignment:

- You <u>must</u> collect all the functions and objects employed to implement the requested structures in a python module `streamcipher.py` following the naming conventions indicated in the following slides.

Task 1: LFSR

LFSR

In an LFSR, the output from a standard shift register is fed back into its input causing an endless cycle. The feedback bit is the result of a linear combination of the shift register content and the polynomial coefficients.



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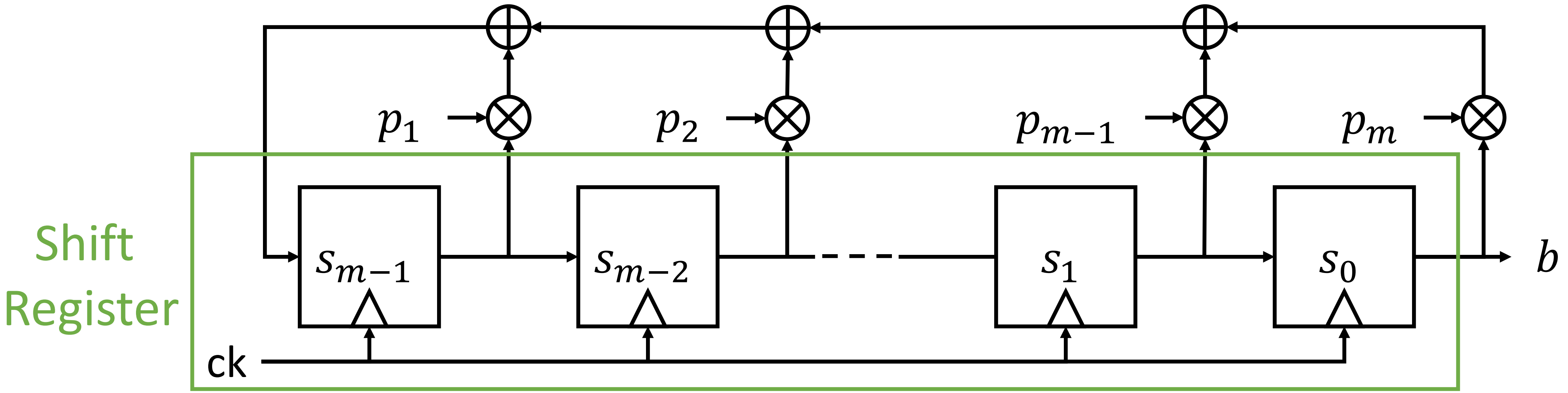

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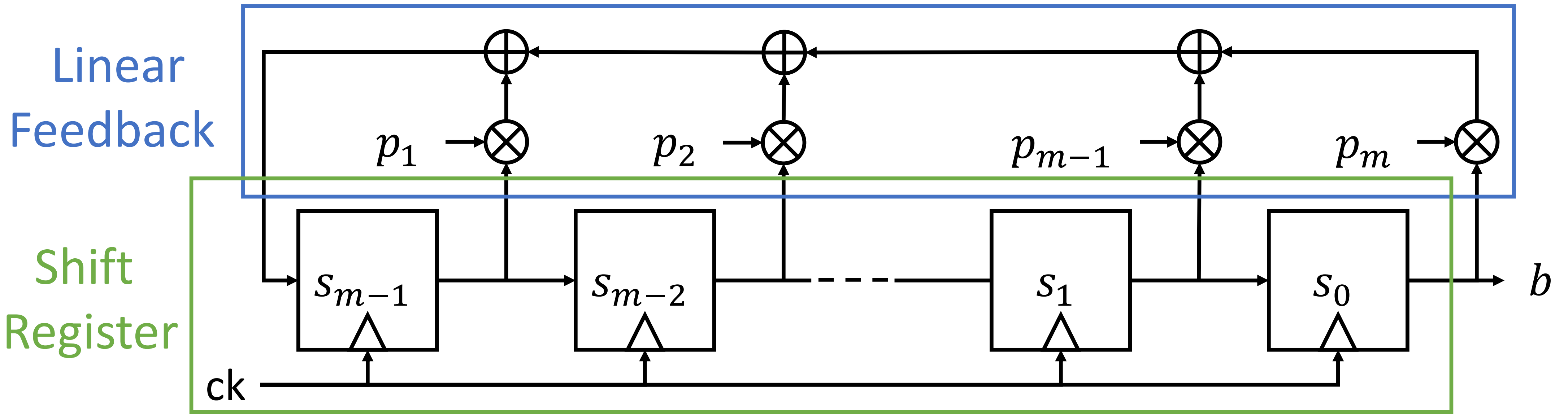
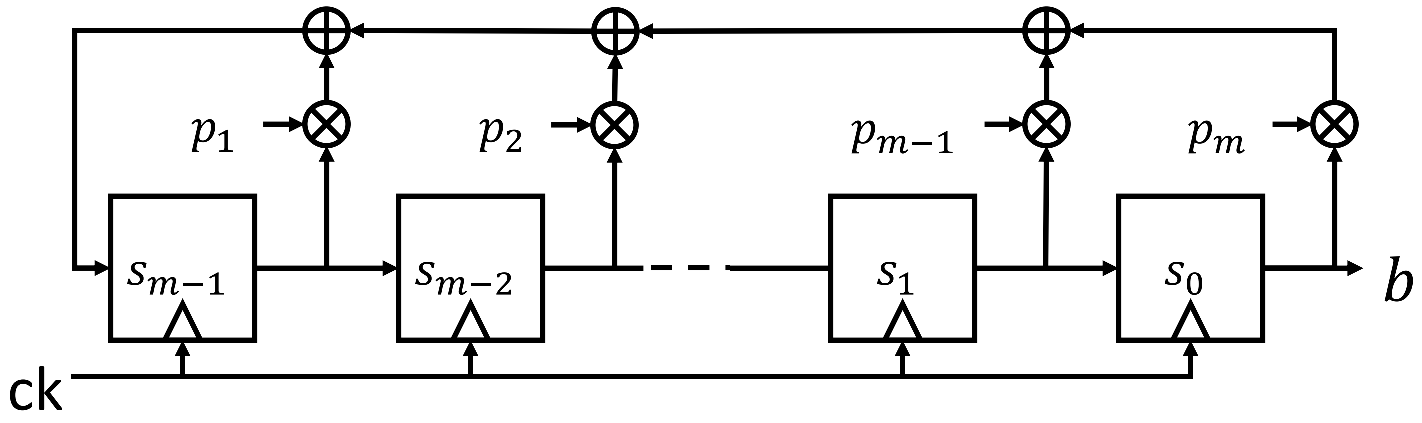


LFSR

From the block scheme:

$$\begin{cases} s_j[t] = s_{j+1}[t-1], \qquad j = 0,1,\dots,m-2 \\ s_{m-1}[t] = \bigoplus_{j=0}^{m-1} p_{m-j} \otimes s_j[t-1] \\ b[t] = s_0[t] \end{cases}$$



LFSR

From the block scheme:

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\begin{cases}
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s_{m-1}[t] = \bigoplus_{j=0}^{m-1} p_{m-j} \otimes s_j[t-1] \\
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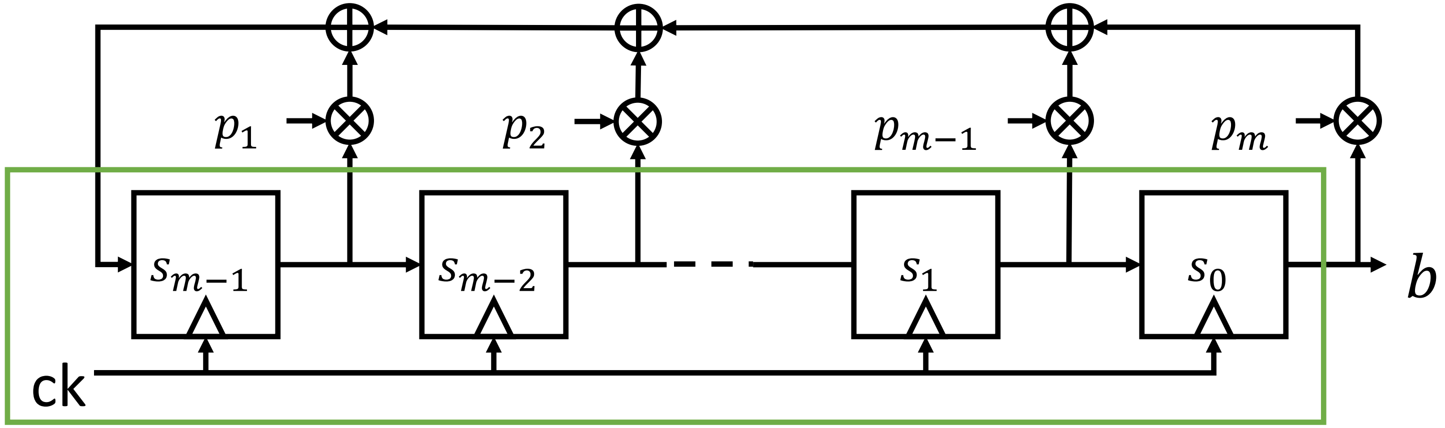
\end{cases}
$$



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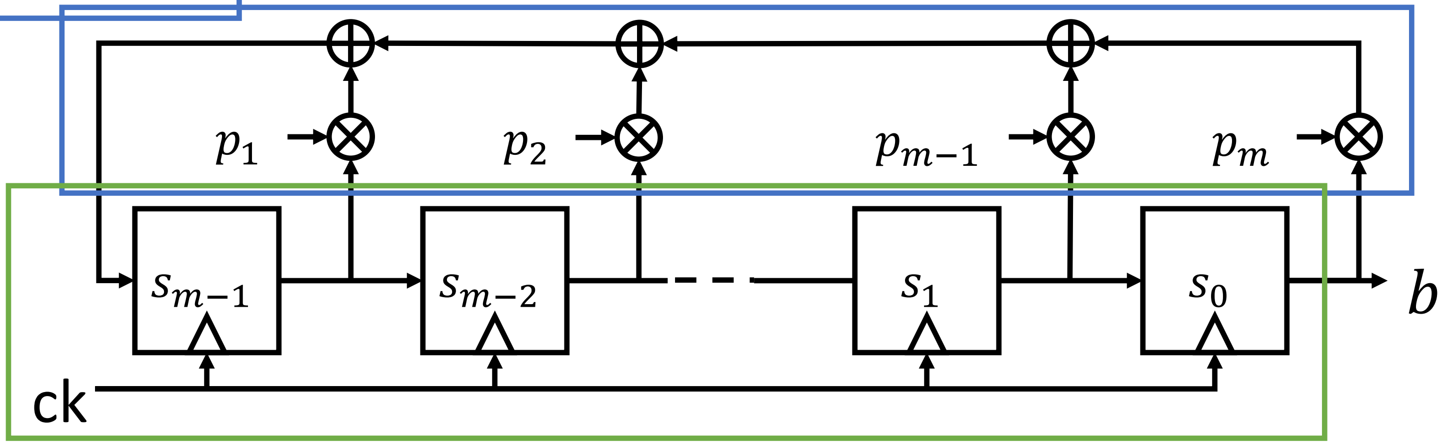

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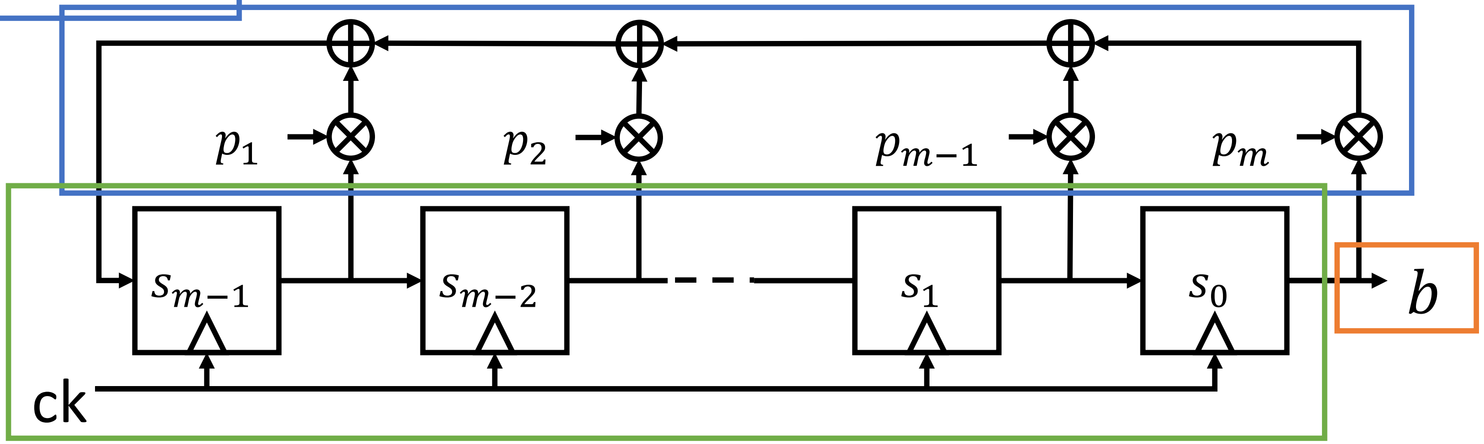
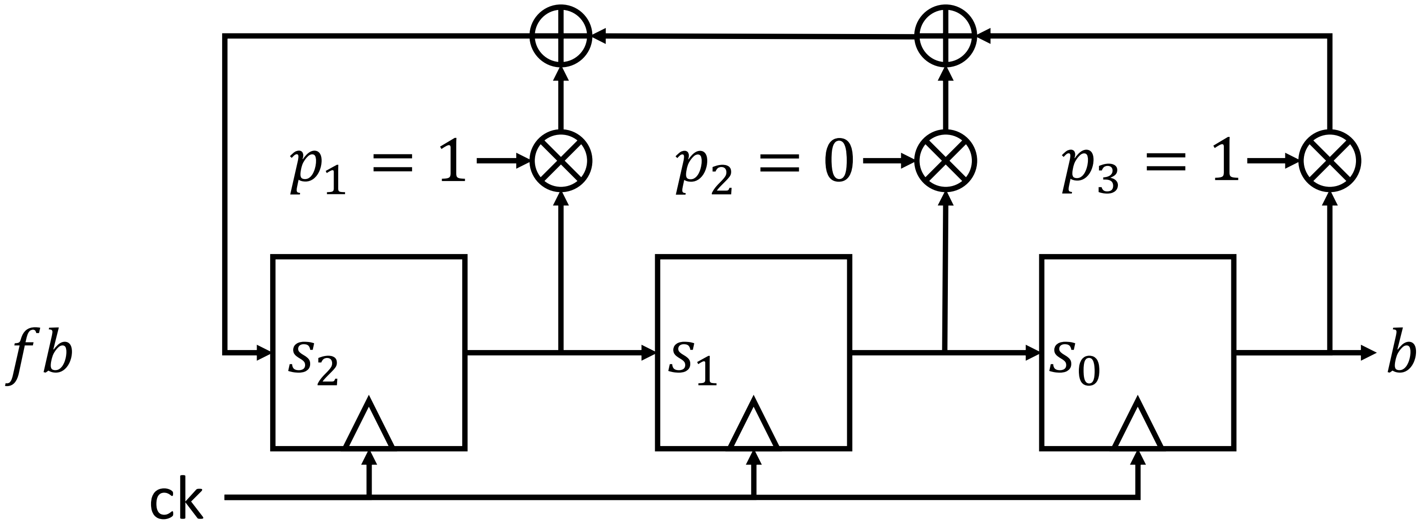


LFSR example

- length = 3
- polynomial = $x^3 + x + 1$ ($p = 0b1011$)
- initial state $0b111$

s	b	fb



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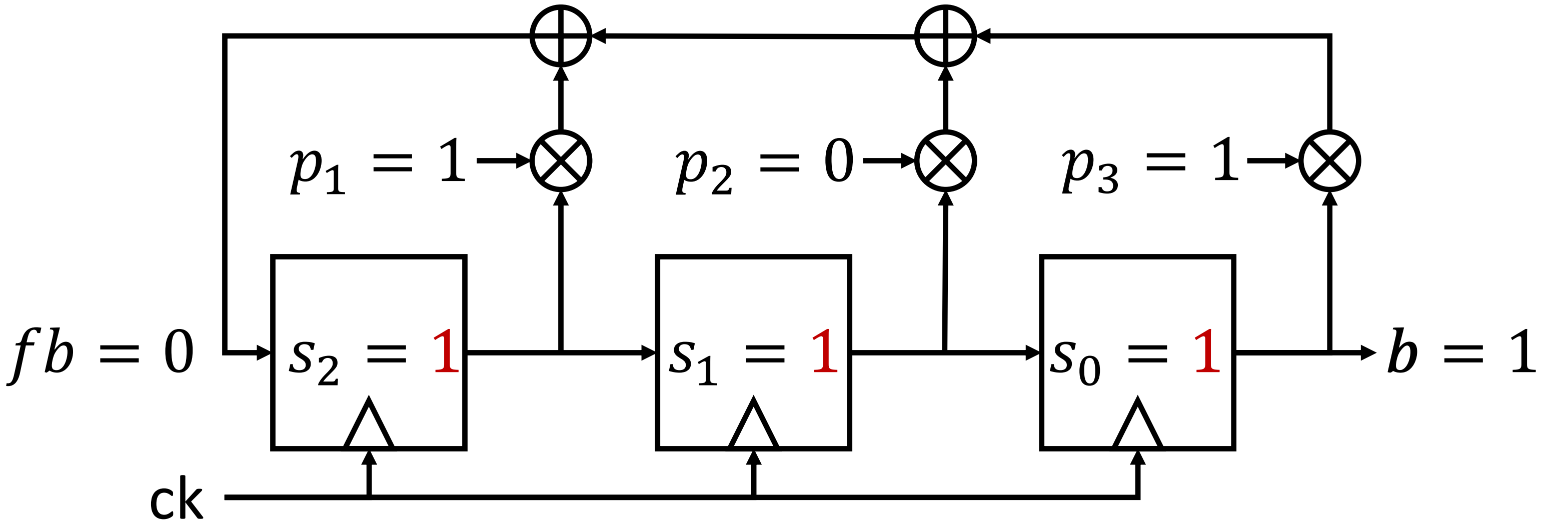
111 (7)	1	0



LFSR example

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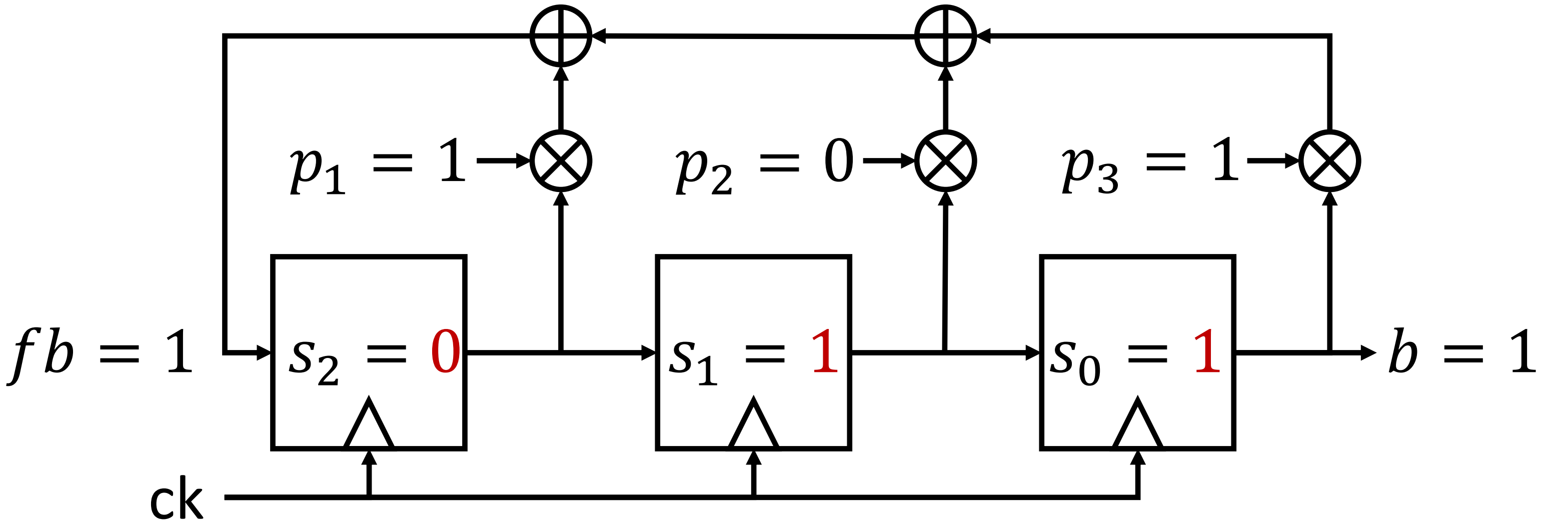
s	b	fb
111 (7)	1	0
011 (3)	1	1



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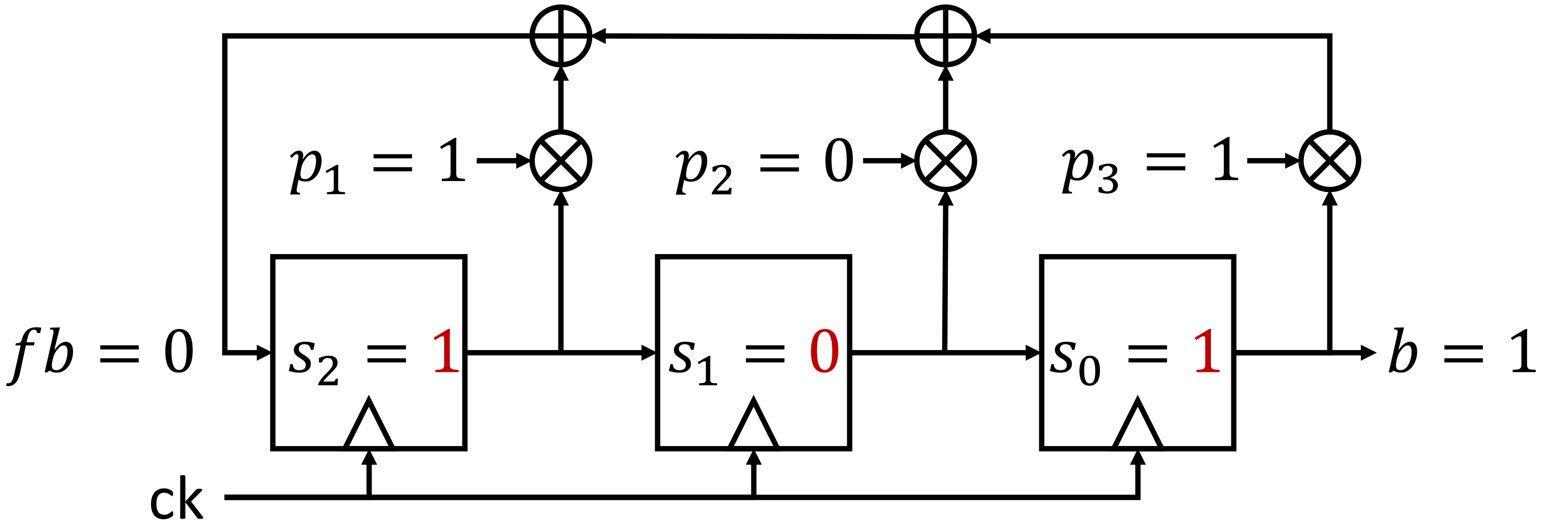

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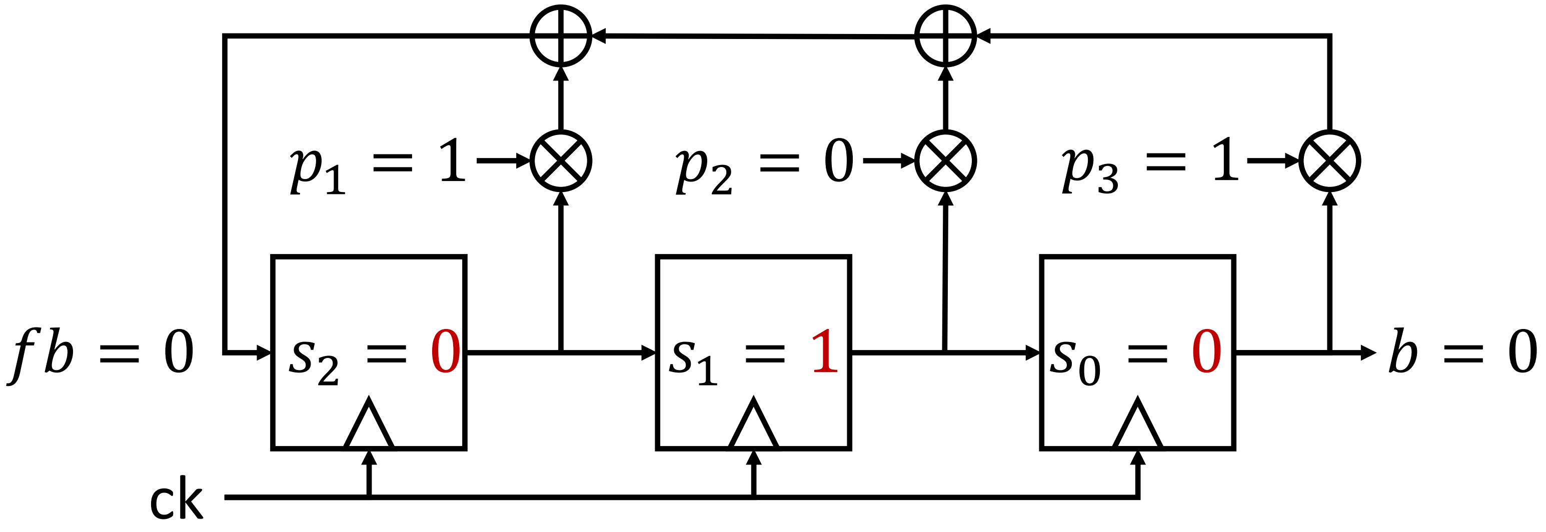

s	b	fb
111 (7)	1	0
011 (3)	1	1
101 (5)	1	0
010 (2)	0	0

LFSR example

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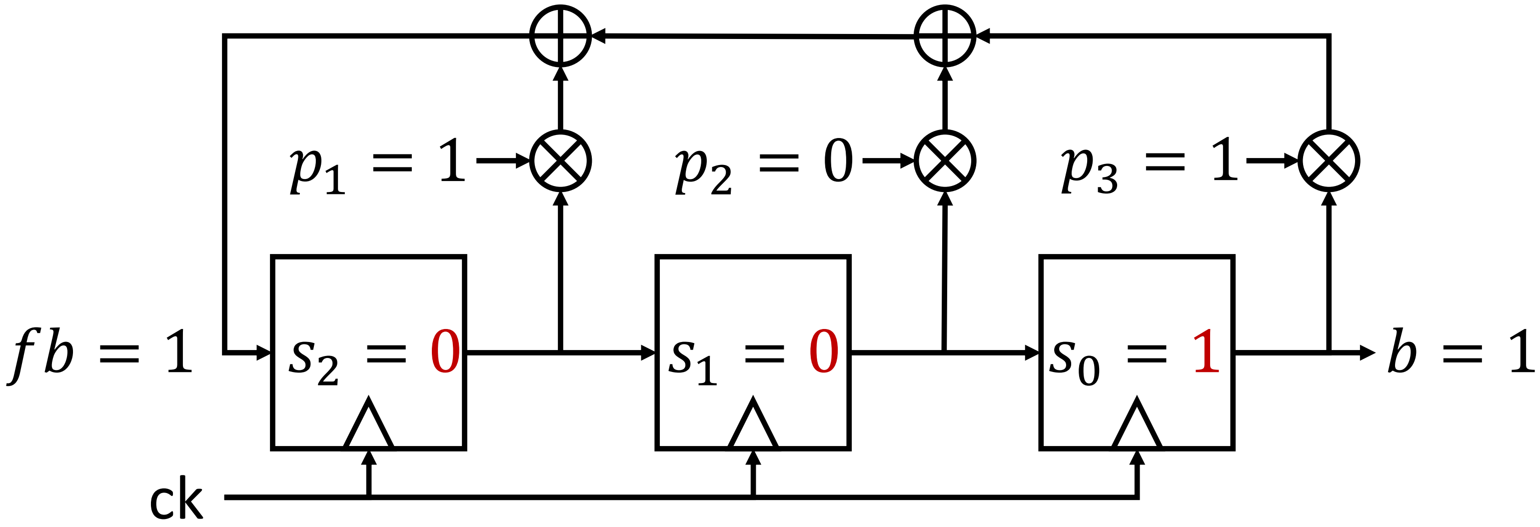
101 (5)	1	0
010 (2)	0	0
001 (1)	1	1



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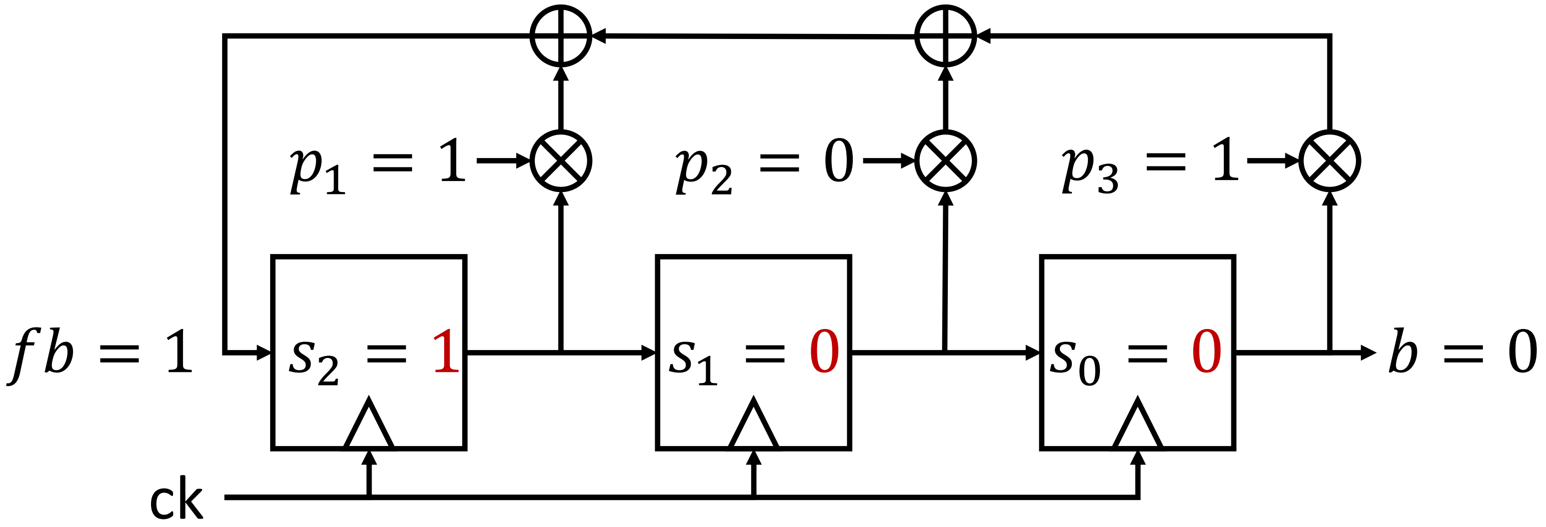

s	b	fb
111 (7)	1	0
011 (3)	1	1
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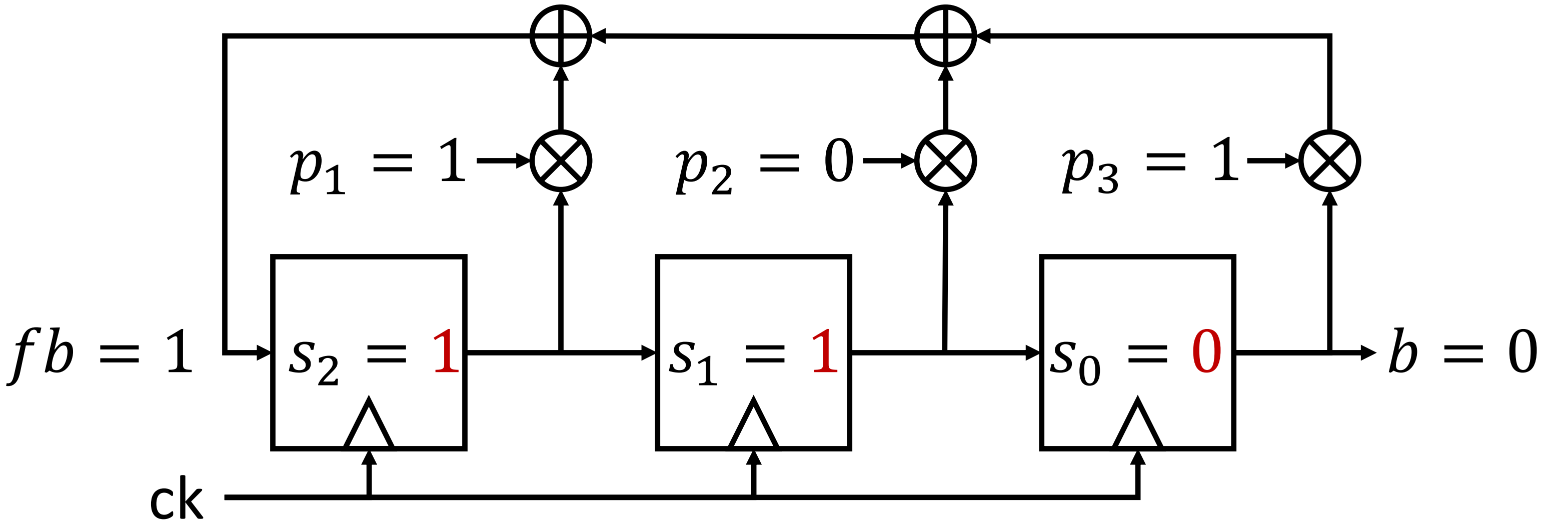

s	b	fb
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110 (6)	0	1

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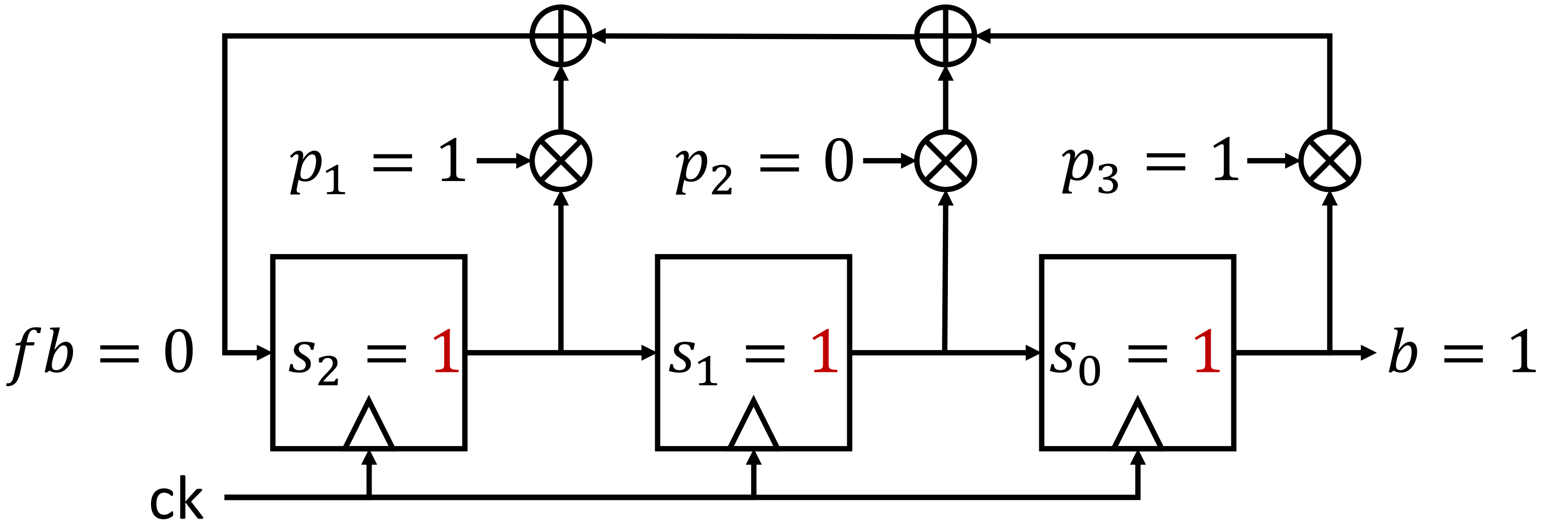


s	b	fb
111 (7)	1	0
011 (3)	1	1
101 (5)	1	0
010 (2)	0	0
001 (1)	1	1
100 (4)	0	1
110 (6)	0	1
111 (7)	1	0

LFSR Iterable

Inputs:

- **Feedback Polynomial**:
  list of integers representing the degrees of the non-zero coefficients.
  Example: $[12, 6, 4, 1, 0]$ represents $P(x) = x^{12} + x^6 + x^4 + x^1 + 1$
- **LFSR state** (optional, default all bits to 1)
  bytes object representing the LFSR initial state. Note that bytes are converted into bits as big-endian (most significant bit first).
  Example: `b'e\xa0'` (`[0x65, 0xa0]`) represents the state 0110 0101 1010

LFSR Iterable

Attributes:

- **poly**: list of the polynomial coefficients (list of int)
- **length**: polynomial degree and length of the shift register (int)
- **state**: LFSR state (bytes)
- **output**: output bit (bool)
- **feedback**: last feedback bit (bool)

LFSR Iterable

Methods:

- **__init__**: class constructor;
- **__iter__**: necessary to be an iterable;
- **__next__**: update LFSR state and returns output bit;
- **cycle**: returns a list of bool representing the full LFSR cycle ;
- **run_steps**: execute N LFSR steps and returns the corresponding output list of bool (N is a input parameter, default N=1);
- **__str__**: return a string describing the LFSR class instance.

LFSR Iterable

```python
class LFSR:
  ''' class docstring '''

  def __init__(self, poly, state=None):
    ''' constructor docstring '''
    ...
    self.poly = ...
    self.length = ...
    self.state = ...
    self.output = ...
    self.feedback = ...

  def __iter__(self):
    return self

  def __next__(self):
    ''' next docstring '''
    ...
    return self.output

  def run_steps(self, N=1):
    ''' run_steps docstring '''
    ...
    return list_of_bool

  def cycle(self, state=None):
    ''' cycle docstring '''
    ...
    return list_of_bool
```

Hints

There are several ways to implement an LFSR in Python.

The first choice to make is how to store the internal state and the polynomial. I suggest two types:

- **list of bool**: it is the most straightforward choice as it directly maps the LFSR block scheme, but bit-wise logical operation may not be as easy.
- **integer**: bit-wise logical operation, as well as bit-shift, are easy to perform on integers, while XOR of multiple bits or reversing the bit order are less straightforward.

Useful functions

- **XOR**: In Python bit-wise xor between two integers is implemented with the `^` mark. It is also implemented as function (`xor`) in the built-in module [`operator`](). Example: `xor(5,4) -> 5^4 -> 0b101^0b100 -> 0b001 -> 1`
- **reduce**: available from the built-in module [`functools`](), apply a function of two arguments cumulatively to the items of an iterable so as to reduce the iterable to a single value.
  Example: `reduce(xor, [True, False, True, False]) -> False`
- **compress**: available from the built-in module [`itertools`](), make an iterator that filters elements from data returning only those that have a corresponding element in selectors that evaluates to True.
  Example: `compress([3, 7, 5], [True, False, True]) -> [3, 5]`

Task 2: Berlekamp-Massey Algorithm

Berlekamp-Massey Algorithm

Find the shortest LFSR for a given binary sequence.

- **Inputs**: sequence of bit b of length N
- **Outputs**: feedback polynomial $P(x)$.

```python
def berlekamp_massey(b):
  ''' function docstring '''
  # algorithm implementation
  return poly
```

Input $b = [b_0, b_1, \dots, b_N]$
$P(x) \leftarrow 1, m \leftarrow 0$
$Q(x) \leftarrow 1, r \leftarrow 1$
For $\tau = 0,1,\dots,N-1$

$$d \leftarrow \bigoplus_{j=0}^{m} p_j \otimes b[\tau - j]$$

  If $d = 1$ **then**
    If $2m \leq \tau$ **then**
      $R(x) \leftarrow P(x)$
      $P(x) \leftarrow P(x) + Q(x)x^r$
      $Q(x) \leftarrow R(x)$
      $m \leftarrow \tau + 1 - m$
      $r \leftarrow 0$
    else
      $P(x) \leftarrow P(x) + Q(x)x^r$
    endif
  endif
  $r \leftarrow r + 1$
endfor
Output $P(x)$

Berlekamp-Massey Algorithm

τ	b_τ	d		$P(x)$	m	$Q(x)$	r
-	-	-		1	0	1	1
0	1	1	A	$1+x$	1	1	1
1	0	1	B	1	1	1	2
2	1	1	A	$1+x^2$	2	1	1
3	0	0		$1+x^2$	2	1	2
4	0	1	A	1	3	$1+x^2$	1
5	1	1	B	$1+x+x^3$	3	$1+x^2$	2
6	1	0		$1+x+x^3$	3	$1+x^2$	3
7	1	0		$1+x+x^3$	3	$1+x^2$	4

Input $b = [b_0, b_1, \ldots, b_N]$
$P(x) \leftarrow 1, m \leftarrow 0$
$Q(x) \leftarrow 1, r \leftarrow 1$
For $\tau = 0,1,\ldots,N-1$

$$d \leftarrow \bigoplus_{j=0}^{m} p_j \otimes b[\tau - j]$$

If $d = 1$ **then**
If $2m \leq \tau$ **then**

A
$R(x) \leftarrow P(x)$
$P(x) \leftarrow P(x) + Q(x)x^r$
$Q(x) \leftarrow R(x)$
$m \leftarrow \tau + 1 - m$
$r \leftarrow 0$

else

B
$P(x) \leftarrow P(x) + Q(x)x^r$

endif
endif
$r \leftarrow r + 1$
endfor
Output $P(x)$

Berlekamp-Massey Algorithm Task

- Implement the Berlekamp-Massey Algorithm
- Test the algorithm implementation
- Apply the Berlekamp-Massey Algorithm to the bit sequence stored in the file `binary_sequence.bin` to compute:
  - The polynomial of the shortest LFSR that can produce that sequence
  - The linear complexity of the bit sequence

Task 3: LFSR-based generator

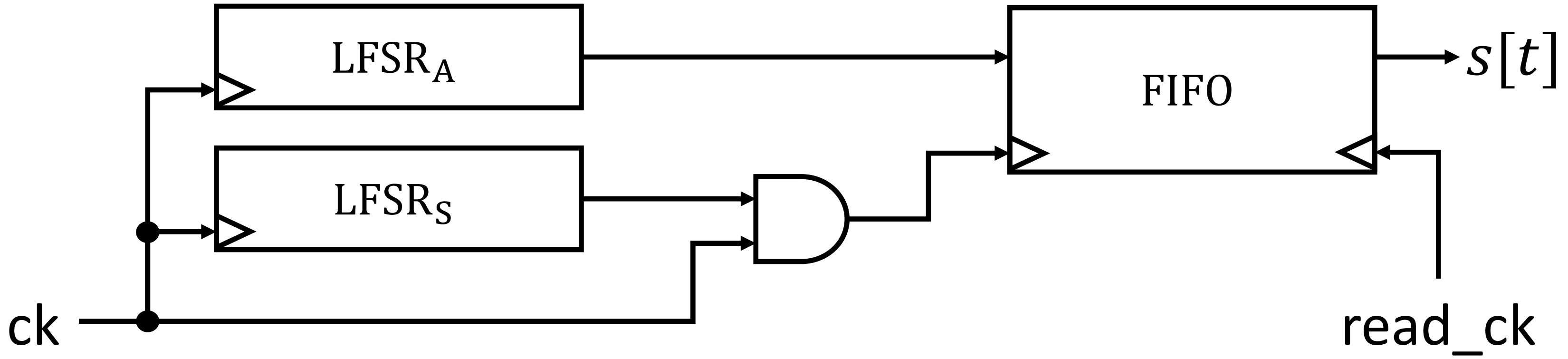
Generator Type Assignment

- Select *one of the two* possible generators according to the procedure described below
- Procedure:
  1. Transform the **name of your group** into a bytes object (big-endian)
  2. Transform the bytes object into a sequence of booleans (big-endian)
  3. Compute the **parity bit**

Parity bit	Assigned generator
0	Shrinking Generator
1	Self-Shrinking Generator

Shrinking Generator

The shrinking generator comprises $\mathrm{LFSR_A}$ that produces bits and $\mathrm{LFSR_S}$ that selects the produced bits. Since bit selection is irregular, a FIFO is necessary for a regular output rate.



Shrinking Generator Class

- **Inputs**
  - Polynomials `polyA` and `polyS` (optional, default are
    - $P_A(x) = x^5 + x^2 + 1$
    - $P_S(x) = x^3 + x + 1$
  - `stateA` and `stateS` (optional, default all bits of the LFSR states at 1)
- **Attributes**
  - `lfsrA`: the LFSR class instance for $\mathrm{LFSR_A}$.
  - `lfsrS`: the LFSR class instance for $\mathrm{LFSR_S}$.
  - `output`: boolean storing the last produced output bit

Shrinking Generator Task

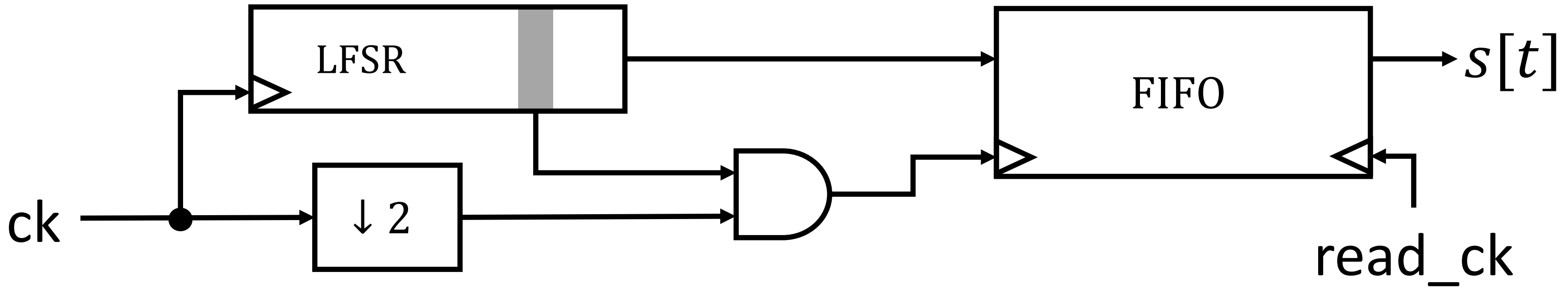
- Implement the Shrinking Generator Class

- Testing the functionality of the implemented class
  - Test that all methods and attributes work as expected

- Decrypt the ciphertext `chipertext_shrinking.bin` given:
  - $P_A(x) = x^{16} + x^{15} + x^{12} + x^{10} + 1$, stateA: `b'\xc5\xd7'`
  - $P_S(x) = x^{24} + x^{11} + x^{5} + x^{2} + 1$, stateS: `b'\x14\x84\xf8'`

Self-Shrinking Generator

The Self-Shrinking Generator is composed by a single LFSR that produce bits and selects them depending on the value of the internal state. A decimator and a FIFO are necessary for a regular output rate.



Self-Shrinking Generator Class

- **Inputs**
  - Polynomial `poly` (optional, default is $P(x) = x^5 + x^2 + 1$)
  - Index of the selection bit `selection_bit` (optional, default 3)
  - State `state` (optional, default all bits of the LFSR state at 1)
- **Attributes**
  - `lfsr`: the LFSR class instance for LFSR.
  - `sbit`: integer storing the index of the selection bit.
  - `output`: boolean storing the last produced output bit

Self-Shrinking Generator Task

- Implement the Self-Shrinking Generator Class

- Testing the functionality of the implemented class
  - Test that all methods and attributes work as expected

- Decrypt the ciphertext `chipertext_selfshrinking.bin` given:
  - $P(x) = x^{32} + x^{16} + x^7 + x^2 + 1$
  - Selection bit: 4
  - state: `b'mJ\x9by'`

Deadline

Tuesday, April 23rd at 12PM (noon)