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1. The signal $u(t)$ is referred to as a *power* signal if its average power has a finite value. We define the function *pow* as the square root of the average power:

$$pow(u) := \left(\lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} u(t)^2 dt\right)^{\frac{1}{2}}$$

Although *pow* does not satisfy all the criteria of a norm, it can be used to establish an inclusion relation in the Venn diagram show in Fig.1a. Each set represents the set of signals that have a finite value according to the given norm definition (or *pow* definition).

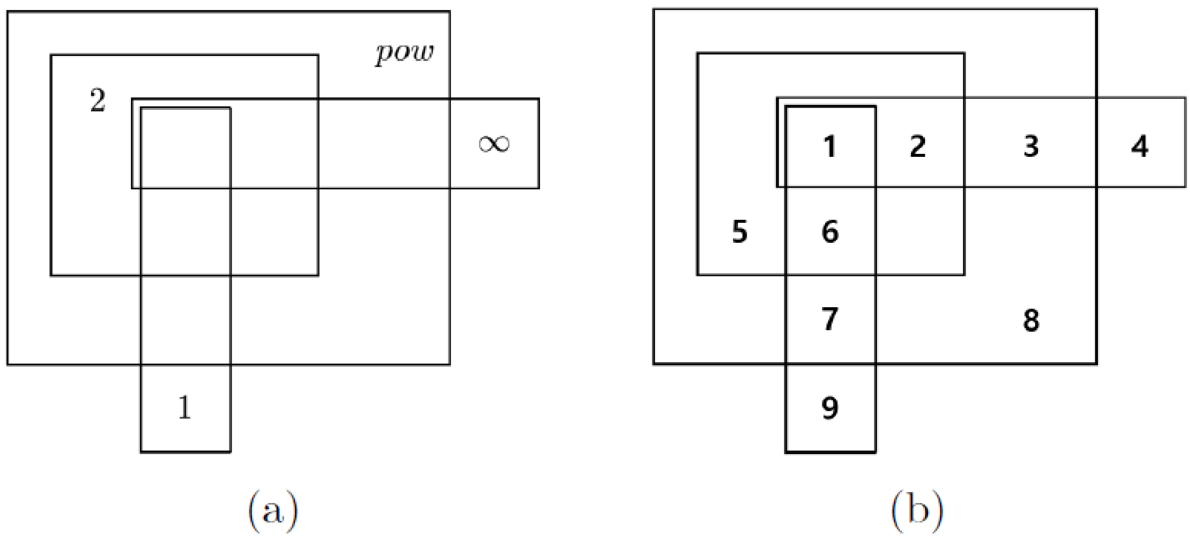

Figure 1: Set inclusions

(a) To prove the validity of Fig.1a, demonstrate the following statements:

i. If $\|u\|_2 < \infty$, then $u(t)$ is a *power signal* with $pow(u) = 0$

Because of $\|u\|_2 < \infty$, then we have $(\Sigma_{i=1}^{n} u_i^2)^{\frac{1}{2}} < \infty$. And we can assume that term is a constant C which is smaller than infinity. Then,

$$pow(u) := \left(\lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} u(t)^2 dt\right)^{\frac{1}{2}} = \left(\lim_{T\to\infty} (\frac{1}{2T})^{\frac{1}{2}} \int_{-T}^{T} \|u\|_2 dt\right) = \left(\lim_{T\to\infty} \frac{1}{2T}^{\frac{1}{2}} C\right) = 0$$

ii. If $\|u\|_1 < \infty$ and $\|u\|_\infty < \infty$, then $\|u\|_2 < \infty$

Because of $\|u\|_1 < \infty$, we have $\min_{i\in R} u_i < \infty$. And $\|u\|_\infty < \infty$, we have $\max_{i\in R} u_i < \infty$. Then, we have,

$$\|u\|_2 = (\Sigma_{i=1}^{n} u_i^2)^{\frac{1}{2}}$$

where the equation is bounded by the L_1 norm and L_∞ norm,

$$\infty \geq (\Sigma_{i=1}^{n} u_i^2)^{\frac{1}{2}} \geq (\Sigma_{i=1}^{n} \min u^2)^{\frac{1}{2}}$$
$$(\Sigma_{i=1}^{n} u_i^2)^{\frac{1}{2}} \leq (\Sigma_{i=1}^{n} \max u^2)^{\frac{1}{2}} < \infty$$

Then we can easily know that L_2 norm is bounded, it has a finite value.

(b) Determine the location $(1, \ldots, 9)$ where u should be included in Fig.1b. Given that $u(t) = 0$ for $t < 0$, consider the following cases:

i. $u(t) = 1$

$pow(u) = 1$ for $\forall t$ 5

ii. $u(t) = \begin{cases} \frac{1}{\sqrt{t}} & t \leq 1 \\ 0 & t > 1 \end{cases}$

$pow(u) \leq 1$ 6

iii. $u(t) = \begin{cases} t^{-\frac{1}{4}} & t \leq 1 \\ 0 & t > 1 \end{cases}$

$pow(u) \leq 1$ 2

iv. $u(t) = \sum_{k=1}^{\infty} v_k(t)$ **where** $v_k(t) = \begin{cases} k & k < t < k + k^{-3} \\ 0 & otherwise \end{cases}$

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2. Consider a plant $P(s) = 1/(s-1)$ with a unity feedback system. Suppose a disturbance $\omega(t) = asin(2t + \theta)$, with unknown amplitude a and phase θ, enters the plant as shown in Fig.2. Design a compensator of degree 3 that is proper (but not strictly proper) in such a way that the output asymptotically tracks any step reference input and rejects the disturbance. Place the poles at $-1 \pm 2j$ and $-2 \pm 1j$. Verify your results using MATLAB.

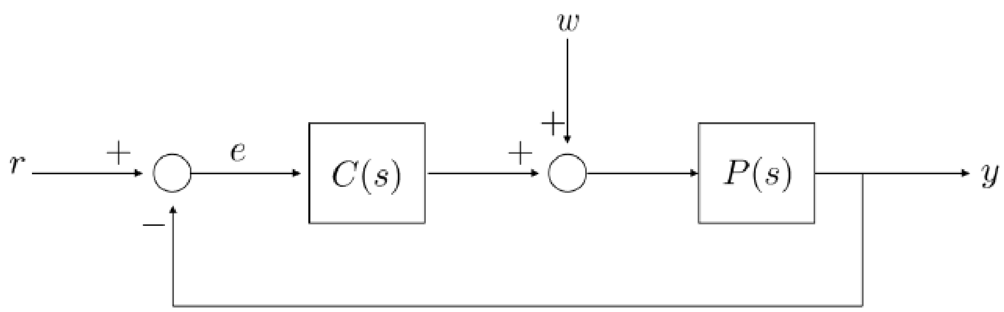


Figure 2: A unity feedback system

Assume that the disturbance $\omega(t) = 0$, then we can compute the transfer function $G_1(s)$ with input signal $r(t)$ and output signal $y(t)$:

$$G_1(s) = \frac{C(s)P(s)}{1 + C(s)P(s)}$$

And assume that the input signal $r(t) = 0$, then we can compute the transfer function $G_2(s)$ with the disturbance signal $\omega(t)$ and output signal $y(t)$:

$$G_2(s) = \frac{P(s)}{1 + C(s)P(s)}$$

Then the output can be represented with above transfer functions:

$$y(s) = G_1(s)r(s) + G_2(s)\omega(s)$$

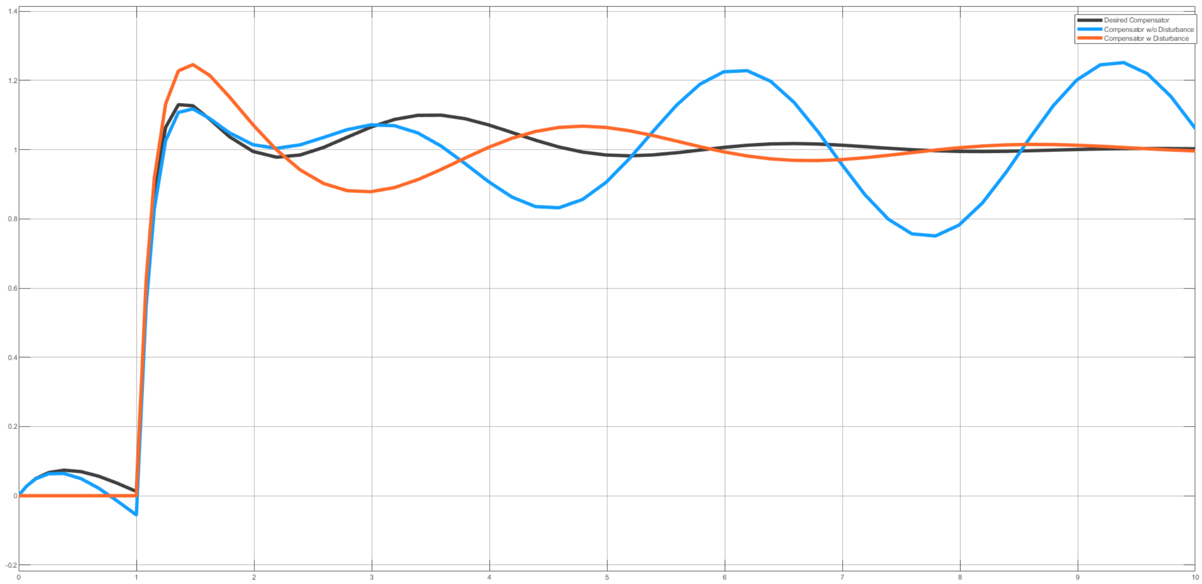
For the internal model, the $\phi(s) = \frac{1}{r(s)\omega(s)} = s^3 + 4s$, Then $B(s)/A(s)$ can be solved from:

$$B(s) + A(s)(s-4)\phi(s) = F(s)$$

where
$F(s) = (s + (-1+2j))(s + (-1-2j))(s + (-2+1j))(s + (-2-1j)) = s^4 + 6s^3 + 18s^2 + 30s + 25$
.

Because the compensator of degree is 3 and the degree of $\phi(s)$ is 3 now, thus $A(s) = 1$. And $B(s) = 10s^3 + 14s^2 + 46s + 25$ by solving the equation. Then the compensator is:

$$C(s) = \frac{B(s)}{A(s)\phi(s)} = \frac{10s^3 + 14s^2 + 46s + 25}{s^3 + 4s}$$



3. Given the plant transfer function $P(s)$, implement the model H_0:

$$P(s) = \frac{s^2 - 1}{s^3 + 2s^2 + 3s + 4}, \quad H_0(s) = \frac{(s-1)(2s+1)}{(s+2)^2(s^2+2s+2)}$$

by designing a feedforward pre-compensator $C_1(s)$ and a feedback controller $C_2(s)$. Determine if the resulting system is stable and check desired model matching is achieved by using MATLAB.

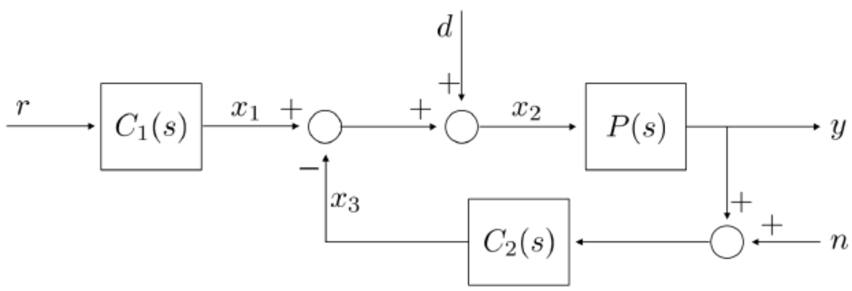


Figure 3: A system configuration for model matching

$H_0(s)$ is implementable:

$$\frac{H_0}{N_P} = \frac{2s+1}{(s+1)(s+2)^2(s^2+2s+2)} = \frac{\bar{E}}{\bar{F}}$$

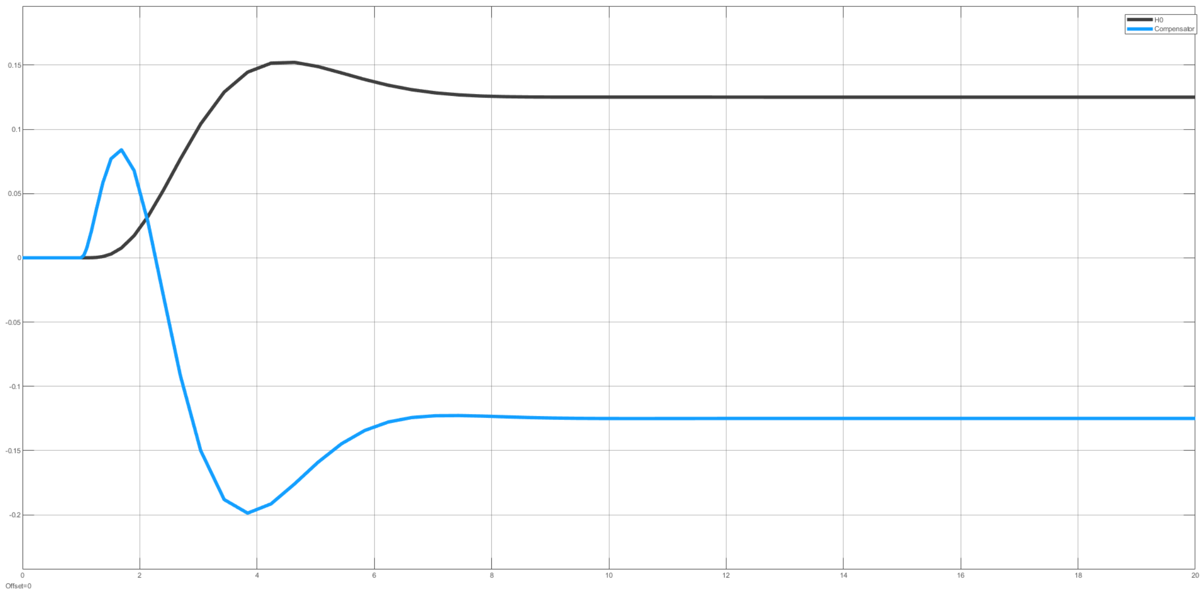
Then we need to find M_c and N_2 that satisfy:

$$M_p M_c + N_p N_2 = \bar{F}$$

Using `solve` function in MATLAB, we can solve this equation with,

```
syms a1 a2 b1 b2 b3
eqn1 = 2+a1+b1 == 7;
eqn2 = 3+2*a1+a2+b2 == 20;
eqn3 = 3*a1+2*a2+4-b1+b3==30;
eqn4 = 4*a1+3*a2-b2 == 24;
eqn5 = 4*a2-b3 == 8;
sol = solve(eqn1,eqn2, eqn3,eqn4, eqn5,a1,a2, b1, b2,b3);
```

Then, $a_1 = 4.5$, $a_2 = 3.5$, $b_1 = 0.5$, $b_2 = 4.5$, $b_3 = 6$.



4. Consider the pitch rate control of aircraft $P(s)$ where reference r is pitch rate command, and output y is pitch rate of the aircraft, $P_b(s)$ is bending mode is considered model:

$$P(s) = \frac{s+1}{s^2+7s+25}$$

$$P_b(s) = P(s)\frac{s^2+3s+30^2}{s^2+0.9s+45^2}$$

(a) Consider a controller $C(s)$ obtained using the loop transfer function $L(s) = \omega_c/s$ without considering bending. Use the maximum value of ω_c that satisfies $|L_b(j\omega)| < 0.5$ for all $\omega \geq 45$. Plot the loop shape $|L_b(j\omega)|$ considering bending using MATLAB when using this controller.

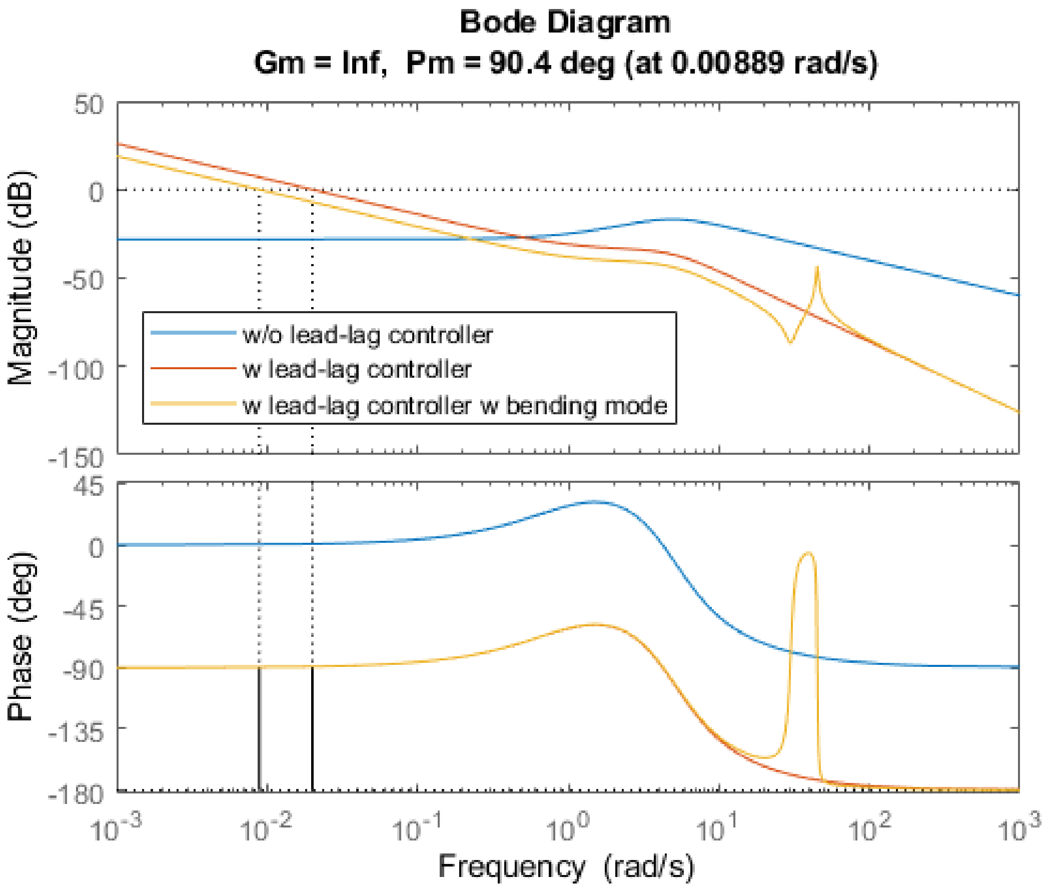
I plotted three systems:

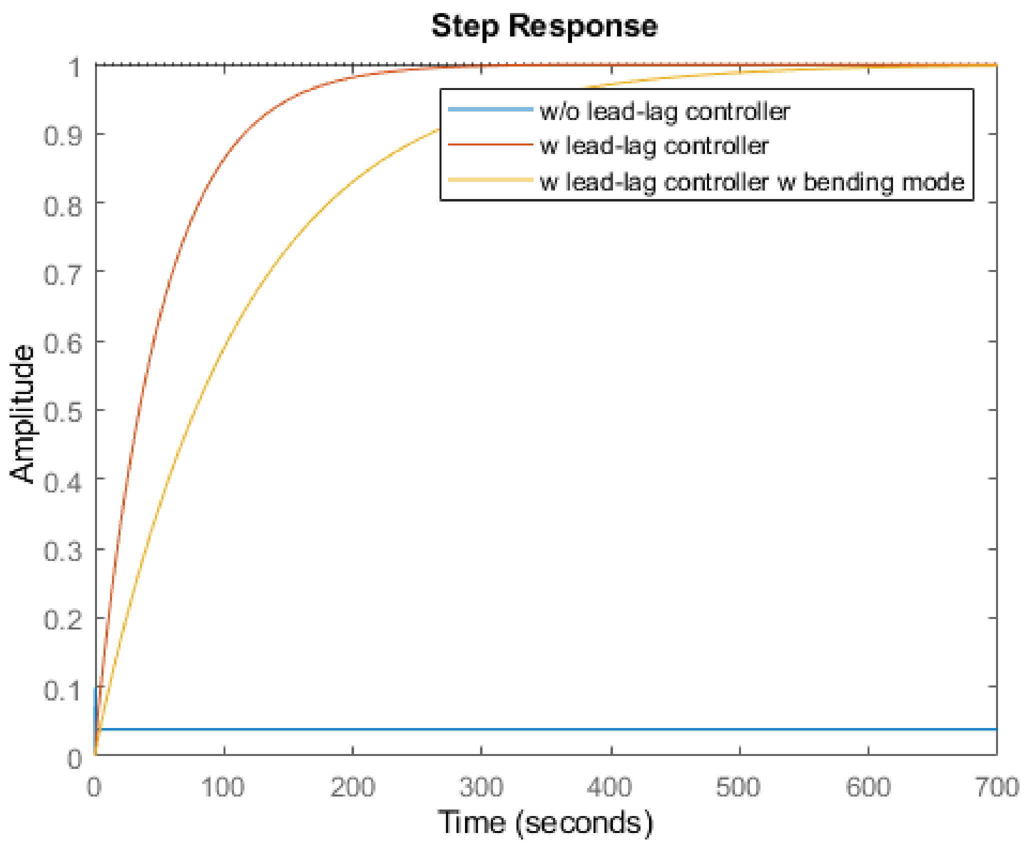
```
1. without lead-lag compensator and just original $P(s)$
1. with lead-lag compensator and just original $P(s)$
1. with lead-lag compensator and considered model $P_b(s)$
```

And I also plotted the Bode figure in the MATLAB. I tried to design the parameter w_c which is desired the condition in this question.

```
clear
clc
close all
s = tf('s');
P = (s+1)/(s^2+7*s+25);
Pb = P * (s^2+3*s+900)/(s^2+0.9*s+2025);
%% Lead-lag controller
wc = 0.5;
F = wc/s;
%% plot
figure(1)
margin(P)
hold on
margin(P*F)
hold on
margin(Pb*F)
legend('w/o lead-lag controller','w lead-lag controller','w lead-lag controller w
bending mode');
hold off

figure(2)
stepplot(feedback(P,1))
stepinfo(feedback(P,1))
hold on
stepplot(feedback(P*F,1))
stepinfo(feedback(P*F,1))
hold on
stepplot(feedback(Pb*F,1))
stepinfo(feedback(Pb*F,1))
legend('w/o lead-lag controller','w lead-lag controller','w lead-lag controller w
bending mode');
```


Bode Diagram
Gm = Inf, Pm = 90.4 deg (at 0.00889 rad/s)
50
0
-50
-100
-150
Magnitude (dB)
w/o lead-lag controller
w lead-lag controller
w lead-lag controller w bending mode
45
0
-45
-90
-135
-180
Phase (deg)
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Frequency (rad/s)


Step Response
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Amplitude
w/o lead-lag controller
w lead-lag controller
w lead-lag controller w bending mode
0
100
200
300
400
500
600
700
Time (seconds)

(b) Design a proper controller $C_b(s)$ to achieve a loop shape with the following properties: a) The loop shape should have a similar value to the original loop shape ω/c at low frequencies, and b) The loop shape should satisfy $|L_b(j\omega)| < 0.5$ for all $\omega \geq 45$. Find the cut off frequency ω_c for controller C_b and compare the loop shape obtained in Problem (4a) using MATLAB.

I am so sorry for this question. I have no idea.

MingshanHe

淡泊名利，矢志不渝；

上下求索，苦心孤诣；

百折不挠，夜以继日；

宁静致远，知行合一；