1. Find the state transition matrix for the following systems.

(a) Parametrically chirped harmonic oscillator.

$$\dot{x}(t) = \begin{bmatrix} 0 & \frac{1}{1+t} \\ -\frac{1}{1+t} & 0 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t).$$

A: The state transition matrix $\Phi(t,t_0)$ is defined as

$$\dot{x}(t) = \Phi(t,t_0)\, x(t_0)$$

with zero input, we have:

$$\dot{x}(t) = A(t)\, x(t) = \begin{bmatrix} 0 & \frac{1}{1+t} \\ -\frac{1}{1+t} & 0 \end{bmatrix} x(t)$$

The fundamental matrix $X(t) = e^{\int_0^t A(\tau)\, d\tau}$

where $\int_0^t A(\tau)\, d\tau = \begin{bmatrix} 0 & \ln(1+t) \\ -\ln(1+t) & 0 \end{bmatrix}$

For computing exponential simplier, we assume $a = \ln(1+t)$ and $a > 0, \forall t > 0$.
Then we compute exponential of matrix $\begin{bmatrix} 0 & a \\ -a & 0 \end{bmatrix}$

$\Rightarrow \det \begin{bmatrix} -\lambda & a \\ -a & -\lambda \end{bmatrix} = 0 \Rightarrow \lambda = \pm a i \quad (a > 0).$

$\Rightarrow$ For $\lambda_1 = ai$, $\bar{v}_1 = [ai \ \ -a]^T$

$\lambda_2 = -ai$, $\bar{v}_2 = [-ai \ \ -a]^T$

Thus $P = \begin{bmatrix} ai & -ai \\ -a & -a \end{bmatrix}$ and $P^{-1} = \begin{bmatrix} -a & ai \\ a & ai \end{bmatrix} \cdot \frac{-1}{2a^2 i}$

$\Rightarrow \exp\left(\int_0^t A(\tau)\, d\tau\right) = P \cdot \begin{bmatrix} e^{ai} & 0 \\ 0 & e^{-ai} \end{bmatrix} P^{-1}$

$\Rightarrow$ From Euler's Formula: $e^{ai} = \cos(a) + i\sin(a)$; $e^{-ai} = \cos(a) - i\sin(a)$.

$$\exp\left(\int_0^t A(\tau)\, d\tau\right) = \begin{bmatrix} ai & -ai \\ -a & -a \end{bmatrix} \cdot \begin{bmatrix} e^{ai} & 0 \\ 0 & e^{-ai} \end{bmatrix} \cdot \begin{bmatrix} -a & ai \\ a & ai \end{bmatrix} \frac{-1}{2a^2 i}$$

$$= \begin{bmatrix} a\cos(a)i - a\sin(a) & -a\cos(a)i - a\sin(a) \\ -a\cos(a) - a\sin(a)i & -a\cos(a) + a\sin(a)i \end{bmatrix} \cdot \begin{bmatrix} -a & ai \\ a & ai \end{bmatrix} \cdot \frac{-1}{2a^2 i}$$

$$= \begin{bmatrix} -2a^2\cos(a)i & -2a^2\sin(a)i \\ 2a^2\sin(a)i & -2a^2\cos(a)i \end{bmatrix} \cdot \frac{i}{2a^2} = \begin{bmatrix} \cos(a) & \sin(a) \\ -\sin(a) & \cos(a) \end{bmatrix}$$

$$\Rightarrow X(t) = \begin{bmatrix} \cos(\ln(1+t)) & \sin(\ln(1+t)) \\ -\sin(\ln(1+t)) & \cos(\ln(1+t)) \end{bmatrix} \Rightarrow X(0) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I.$$

$$\Rightarrow \Phi(t,t_0) = X(t)\cdot X(t_0)^{-1} = \begin{bmatrix} \cos(\ln(1+t)) & \sin(\ln(1+t)) \\ -\sin(\ln(1+t)) & \cos(\ln(1+t)) \end{bmatrix} \cdot \begin{bmatrix} \cos(\ln(1+t_0)) & -\sin(\ln(1+t_0)) \\ \sin(\ln(1+t_0)) & \cos(\ln(1+t_0)) \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\ln(1+t) - \ln(1+t_0)) & \sin(\ln(1+t) - \ln(1+t_0)) \\ -\sin(\ln(1+t) - \ln(1+t_0)) & \cos(\ln(1+t) - \ln(1+t_0)) \end{bmatrix}.$$

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(b). Jump parameter system. $\dot{x}(t) = A(t)\, x(t)$. where.

$$A(t) = \begin{cases} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} & \text{for } 0 \le t < 2 \\ \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} & \text{for } 2 \le t \end{cases}$$

A: Because of the Jump system. there are four different conditions to consider. $\{ (t, t_0 \in [0,2))$, $(t, t_0 \in [2, +\infty))$, $(t \in [0,2), t_0 \in [2, +\infty))$, $(t \in [2, +\infty), t_0$

$\Rightarrow$ For the first two conditions. the state transition matrix can be computed by $\exp\left(\int_{t_0}^{t} A_1\, dt\right) = \exp(A_1 (t - t_0))$ and $\exp\left(\int_{t_0}^{t} A_2\, dt\right) = \exp(A_2 (t - t_0))$

$\Rightarrow$ $\exp(A_1(t-t_0))$:

$$\det \begin{bmatrix} -\lambda & 0 \\ 0 & 1-\lambda \end{bmatrix} = 0 \Rightarrow \begin{matrix} \lambda_1 = 0 & \bar{v}_1 = [1 \;\; 0]^T \\ \lambda_2 = 1 & \bar{v}_2 = [0 \;\; 1]^T \end{matrix}$$

Thus. $P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $P^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\Rightarrow \exp(A_1(t-t_0)) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} e^{0\cdot(t-t_0)} & 0 \\ 0 & e^{1(t-t_0)} \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow$ $\exp(A_2(t-t_0))$:

$$\det \begin{bmatrix} -\lambda & 1 \\ 0 & 1-\lambda \end{bmatrix} = 0 \Rightarrow \begin{matrix} \lambda_1 = 0 & \bar{v}_1 = [1 \;\; 0]^T \\ \lambda_2 = 1 & \bar{v}_2 = [1 \;\; 1]^T \end{matrix}$$

Thus. $P = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ $P^{-1} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$ $\Rightarrow \exp(A_2(t-t_0)) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} e^{0(t-t_0)} & 0 \\ 0 & e^{1\cdot(t-t_0)} \end{bmatrix} \cdot \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1 & e^{(t-t_0)} - 1 \\ 0 & e^{(t-t_0)} \end{bmatrix}$$

$\Rightarrow$ The last two conditions, the state transmition matrix will cross the jump point. thus the $t \in [0,2)$, $t_0 \in [2, +\infty)$ will be considered as.

- $\exp(A_1(2-t)) = \begin{bmatrix} 1 & 0 \\ 0 & e^{(2-t)} \end{bmatrix}$ $\exp\cdot(A_2(t_0-2)) = \begin{bmatrix} 1 & e^{(t_0-2)} - 1 \\ 0 & e^{(t_0-2)} \end{bmatrix}$

$$\Phi(t, t_0) = \exp(A_1(2-t)) \cdot \exp(A_2(t_0-2)) = \begin{bmatrix} 1 & e^{(t_0-2)} - 1 \\ 0 & e^{(t_0-t)} \end{bmatrix}$$

- $t \in [2, +\infty)$, $t_0 \in [0, 2)$.

$\exp(A_2(t-2)) = \begin{bmatrix} 1 & e^{(t-2)} - 1 \\ 0 & e^{(t-2)} \end{bmatrix}$ $\exp\cdot(A_1(2-t_0)) = \begin{bmatrix} 1 & 0 \\ 0 & e^{(2-t_0)} \end{bmatrix}$

$$\Phi(t, t_0) = \exp(A_2(t-2)) \cdot \exp(A_1(2-t_0)) = \begin{bmatrix} 1 & e^{(t-t_0)} - e^{(2-t_0)} \\ 0 & e^{(t-t_0)} \end{bmatrix}$$

$\Rightarrow$ Finally. in summary

$$\Phi(t, t_0) = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & e^{(t-t_0)} \end{bmatrix}, t, t_0 \in [0,2); \begin{bmatrix} 1 & e^{(t-t_0)} - 1 \\ 0 & e^{(t-t_0)} \end{bmatrix}, t, t_0 \in [2, +\infty); \begin{bmatrix} 1 & e^{(t_0-2)} - 1 \\ 0 & e^{(t_0-t)} \end{bmatrix}, t \in [0,2). t_0 \in [2, \right.$$

$$\left. \begin{bmatrix} 1 & e^{(t-t_0)} - e^{(2-t_0)} \\ 0 & e^{(t-t_0)} \end{bmatrix}, t \in [2, +\infty), t_0 \in [0, 2). \right\}$$

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2. Answer the following questions about stability.

(a). Does a system with an impulse response $g(t) = \frac{1}{1+t}$ exhibit BIBO sta

A: For the BIBO stability, we need to show.

$$\int_0^\infty \| H(t, t_0) \| \, dt < \infty \quad \forall t \in (-\infty, +\infty).$$

where $H(t, t_0)$ is impulse response matrix.

The impulse response is given and it is scalar.

$$\Rightarrow \int_0^\infty \left| \frac{1}{1+t} \right| \cdot dt = \ln(1+t) \Big|_0^\infty = \infty$$

Thus this system is not BIBO stable.

(b). Does a system

$$\dot{x}(t) = \begin{bmatrix} -1 & 10 \\ 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} -2 \\ 0 \end{bmatrix} u(t), \quad y(t) = [-2 \quad 3] x(t) - 2u(t).$$

exhibit BIBO stable?

A: The impulse response matrix can be computed as

$$H(t, t_0) = C(t)\Phi(t, t_0) \cdot B(t) + D(t) \cdot \delta(t - t_0).$$

For comput simplier, we can use Laplace transform.

$$\mathcal{L}[H(t, t_0)] = C \cdot (sI - A)^{-1} \cdot B + D \cdot \frac{1}{s}$$

$$= \begin{bmatrix} -2 & 3 \end{bmatrix} \cdot \begin{bmatrix} s+1 & -10 \\ 0 & s-1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} -2 \\ 0 \end{bmatrix} + 2 \cdot \frac{1}{s}$$

$$= \frac{4}{s+1} + \frac{2}{s}.$$

And the inverse Laplace transform of it is.

$$H(t, t_0) = \mathcal{L}^{-1}\left[\frac{4}{s+1} + \frac{2}{s}\right] = 4e^{-t} + 2\delta(t - t_0).$$

$$\Rightarrow \int_0^\infty H(t, t_0) = \int_0^\infty (4e^{-t} + 2\delta(t - t_0)) dt = (-4e^{-t}) \Big|_0^\infty + 2 = 6 < \infty$$

Thus this system is BIBO stable.

(c). Consider the CT-LTI system:

$$\dot{x}(t) = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} x(t)$$

Is the system marginally stable and asymptotically stable?

A: $\det \begin{bmatrix} \lambda+1 & 0 & -1 \\ 0 & \lambda & -1 \\ 0 & 0 & \lambda \end{bmatrix} = 0 \Rightarrow \lambda_1 = -1 ; \lambda_2 = 0 ; \lambda_3 = 0.$

The system is not asymptotically stable, because the eigen values of A are not all negative. (λ_2 & λ_3).

And then $\lambda_1 = 0$. we have the matrix with

$$(A - \lambda I) = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} v = 0.$$

From above, we can know it only have one linearly independent

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eigen vector. Then the Jordan Form is.

$$J = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

It has order 2 and the system is ~~not~~ marginally stable. Because all the $\lambda_i \in LHP$ except some non-deficient λ_i on $j\omega$-axis.

(e). Find the state-response of the system

$$x_{k+1} = \frac{1}{2}\begin{bmatrix} -1 & 2 & 2 \\ -1 & 0 & 1 \\ -1 & 3 & 2 \end{bmatrix} x_k + \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} u_k.$$

Then, show the system is asymptotically stable.

A: For diagolization, we need to find eigen values.

$$\Rightarrow \det(\lambda I - A) = \begin{bmatrix} \lambda+0.5 & -1 & -1 \\ 0.5 & \lambda & -0.5 \\ 0.5 & -1.5 & \lambda-1 \end{bmatrix}$$

$$= \lambda^3 - 0.5\lambda^2 - 0.25\lambda + 0.125 = (\lambda-0.5)(\lambda-0.5)(\lambda+0.5).$$

Thus $\lambda_1 = 0.5$, $\lambda_2 = 0.5$, $\lambda_3 = -0.5$.

Then the eigen vector:

$\lambda_1 = 0.5 \quad \bar{v}_1 = [0.7071 \quad 0 \quad 0.7071]^T$

$\lambda_2 = 0.5 \quad \bar{v}_2 = [-0.7071 \quad 0 \quad -0.7071]^T$

$\lambda_3 = -0.5 \quad \bar{v}_3 = [\,0 \quad -0.7071 \quad 0.7071]^T.$

Thus $A = Q \cdot \begin{bmatrix} 0.5 & 1 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & -0.5 \end{bmatrix} Q^{-1}$ where $Q = \begin{bmatrix} 0.7071 & -0.7071 & 0 \\ 0 & 0 & -0.7071 \\ 0.7071 & -0.7071 & 0.7071 \end{bmatrix}$

Then the state transition matrix $\Phi(k, k_0)$ is.

$$\Phi(k, k_0) = A^{k-k_0} = Q \cdot \begin{bmatrix} (0.5)^{k-k_0} & (k-k_0)(0.5)^{k-k_0-1} & 0. \\ 0 & (0.5)^{k-k_0} & 0 \\ 0 & 0 & (-0.5)^{k-k_0} \end{bmatrix} Q^{-1}.$$

Thus the state response of the system is.

$$x[k] = A^k x[0] + \sum_{m=0}^{k-1} A^{k-1-m} B u[m].$$

Because the eigenvalues are 0.5 and -0.5 which are smaller than 1 we can say the system is asymptotically stable.

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3. Consider the following DT-LTI system.

$$x_{k+1} = A x_k + B u_k . \quad A = \begin{bmatrix} 0.5 & 1 \\ 0 & -0.5 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

(a) Calculate the zero-input state response using z-transform when $x = [1 \ -1]^T$

A: As we know, For DT-LTI system.

$$\hat{x}(z) = z(zI - A)^{-1} x[0] + (zI - A)^{-1} \cdot B \hat{u}(z)$$

Because zero input, $u_k = 0$. for all k.

$$\Rightarrow \quad \hat{x}(z) = z(zI - A)^{-1} x[0].$$

where $z \cdot (zI - A)^{-1} = z \cdot \begin{bmatrix} z - 0.5 & -1 \\ 0 & z + 0.5 \end{bmatrix}^{-1}$

$$= \frac{z}{(z+0.5)(z-0.5)} \cdot \begin{bmatrix} z+0.5 & 1 \\ 0 & z-0.5 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{z}{z-0.5} & \frac{z}{(z+0.5)(z-0.5)} \\ 0 & \frac{z}{z+0.5} \end{bmatrix}$$

$$\Rightarrow \hat{x}(z) = z(zI - A)^{-1} x[0]$$

$$= \begin{bmatrix} \frac{z}{z-0.5} & \frac{z}{(z+0.5)(z-0.5)} \\ 0 & \frac{z}{z+0.5} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} \frac{z^2 + 0.5z - z}{(z+0.5)(z-0.5)} \\ \frac{-z}{z+0.5} \end{bmatrix} = \begin{bmatrix} \frac{z}{z+0.5} \\ \frac{-z}{z+0.5} \end{bmatrix}$$

Using z-inverse transform.

$$x(k) = \begin{bmatrix} (-0.5)^k \cdot (1)^k \\ (0.5)^k \cdot (-1)^k \end{bmatrix}$$

(b). Calculate the zero-state respose using z-transform when $u_k = \delta(k)$
What about then $u_k = \frac{9}{2} k$?

A: As mentioned above.

$$\hat{x}(z) = z(zI - A)^{-1} x[0] + (zI - A)^{-1} B \hat{u}(z)$$

Because zero-state, $x[0] = 0$.

$$\Rightarrow \quad \hat{x}(z) = (zI - A)^{-1} B \hat{u}(z).$$

$u_k = \delta(k) \Rightarrow \hat{u}(z) = 1$.

Then $\hat{x}(z) = (zI - A)^{-1} B = \begin{bmatrix} \frac{1}{z-0.5} & \frac{1}{(z+0.5)(z-0.5)} \\ 0 & \frac{1}{z+0.5} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$= \begin{bmatrix} \frac{1}{(z+0.5)(z-0.5)} \\ \frac{1}{z+0.5} \end{bmatrix}$$

Then using z-inverse transform.

$$x(k) = \begin{bmatrix} 2^{1-k}((-1)^k + 1) \\ (-0.5)^{k-1} \end{bmatrix}.$$

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For $u_k = \frac{9}{2}k$

$\Rightarrow$ z Transform : $\hat{u}(z) = \frac{9}{2} \cdot \frac{z}{(z-1)^2}$

Then $\hat{x}(z) = (zI - A)^{-1} B\hat{u}(z)$.

$$= \begin{bmatrix} \frac{1}{(z+0.5)(z-0.5)} \\ \frac{1}{z+0.5} \end{bmatrix} \cdot \frac{9}{2} \frac{z}{(z-1)^2}$$

$$= \begin{bmatrix} \frac{9}{2} \frac{z}{(z+0.5)(z-0.5)(z-1)^2} \\ \frac{9}{2} \frac{z}{(z+0.5)(z-1)^2} \end{bmatrix}$$

$\Rightarrow$ z Inverse transform :

$$x(k) = \begin{bmatrix} 18\left(\frac{2}{9}n + 2^{-n} - \frac{1}{9}(-0.5)^n - \frac{8}{9}\right) \\ 2\left((-0.5)^n - 1\right) + 3n \end{bmatrix}.$$

Above inverse z transformations are referenced from WolframAlpha website

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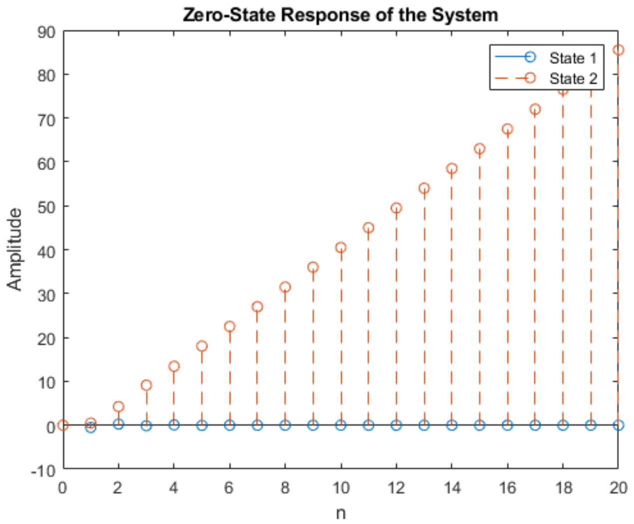
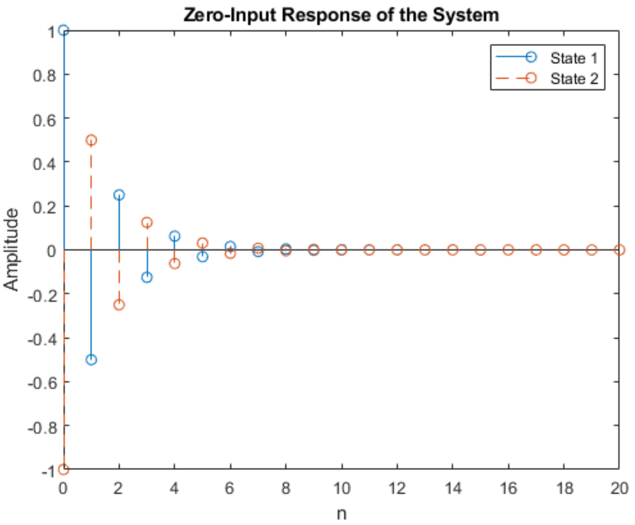
```
>> HW1

Phi =

[ cos(log(t + 1) - log(t0 + 1)), sin(log(t + 1) - log(t0 + 1))]
[-sin(log(t + 1) - log(t0 + 1)), cos(log(t + 1) - log(t0 + 1))]


Phi =

[1, exp(t - t0) - 1]
[0,      exp(t - t0)]

The system g(t) = 1/(1+t) is not BIBO stable.
System is BIBO stable
System is not marginally stable
System is not asymptotically stable
     0     1     0
     0     0     0
     0     0    -1


is_marginally_stable =

  logical

   1

System is not asymptotically stable
```





```matlab
%%

% HOMWORK1-1(A)
syms t t0 real
assume(t > t0 > 0)
A = [0 1/(1+t); -1/(1+t) 0];
Phi = expm(int(A,t0, t));
display(Phi);

% HOMEWORK1-1(B)
syms t t0 real
assume(t0 > 5)
assume(t > 5)
A1 = [0 0; 0 1]; % for 0 <= t <2
A2 = [0 1; 0 1]; % for 2<= t
A = piecewise(t<2, A1, t>=2, A2);
Phi = expm(int(A, t0, t));
display(Phi);

%%

% HOMEWORK1-2(A)
syms t
h = (1+t);
L1_norm = int(h, 0, Inf);
if isfinite(L1_norm)
    disp('The system g(t) = 1/(1+t) is BIBO stable.');
else
    disp('The system g(t) = 1/(1+t) is not BIBO stable.');
end

%%

% HOMEWORK1-2(B)
% Define the system matrices
A = [-1 10; 0 1];
B = [-2; 0];
C = [-2 3];
D = 2;

sys = ss(A,B,C,D);


h = impulse(sys);

tf_sys = tf(sys);

w = linspace(0,10,1000);

H = freqresp(tf_sys,w);

mag = abs(H);

% Check if the system is BIBO stable
if max(mag(:)) < Inf
    disp('System is BIBO stable');
else
    disp('System is not BIBO stable');
end

%%

% HOMEWORK1-2(C)
% Define the system matrices
```

```
A = [-1 0 1; 0 0 1; 0 0 0];
B = [0; 0; 0];
C = [0 0 0];
D = 0;
sys = ss(A, B, C, D);

% Compute the system eigenvalues
e = eig(A);

% Check for marginal stability
if all(real(e) == 0)
    disp('System is marginally stable');
else
    disp('System is not marginally stable');
end


% Check for asymptotic stability
if all(real(e) < 0)
    disp('System is asymptotically stable');
else
    disp('System is not asymptotically stable');
end

%%

% HOMEWORK1-2(D)
% Define the LTI system
A = [-1 0 1; 0 0 1; 0 0 0];
B = [0; 0; 0];
C = [0 0 0];
D = 0;
sys = ss(A, B, C, D);

% Compute the Jordan form of A
J = jordan(A);
disp(J)
% Check if the system is marginally stable
is_marginally_stable = any(diag(J) == 0)

%%

% HOMEWORK1-2(E)
% Define the system matrices
A = [-1 0 1; 0 0 1; 0 0 0];
B = [0; 0; 0];
C = [0 0 0];
D = 0;
sys = ss(A, B, C, D);

% Compute the system eigenvalues
e = eig(A);

% Check for asymptotic stability
if all(real(e) < 0)
    disp('System is asymptotically stable');
else
    disp('System is not asymptotically stable');
end

%%

% HOMEWORK1-3(A)
% Define the system
A = [0.5 1; 0 -0.5];
```

```matlab
x0 = [1; -1];

% Compute the zero-input response for the first state
n = 0:20;
x1 = zeros(2,length(n));
x1(:,1) = x0;
for i = 2:length(n)
    x1(:,i) = A*x1(:,i-1);
end

% Plot the two states in the same figure
figure;
stem(n,x1(1,:));
hold on;
stem(n,x1(2,:),'--');
legend('State 1', 'State 2');
xlabel('n');
ylabel('Amplitude');
title('Zero-Input Response of the System');



%%

% HOMEWORK1-3(B)
% Define the system
A = [0.5 1; 0 -0.5];
B = [0; 1];
x0 = [0; 0];

% Compute the zero-input response for the first state
n = 0:20;
x = zeros(2,length(n));
u = (9/2)*n;
x(:,1) = x0;
for i = 2:length(n)
    x(:,i) = A*x1(:,i-1)+B*u(:,i-1);
end

% Plot the two states in the same figure
figure;
stem(n,x(1,:));
hold on;
stem(n,x(2,:),'--');
legend('State 1', 'State 2');
xlabel('n');
ylabel('Amplitude');
title('Zero-State Response of the System');
```