$\underbrace{X \vDash \alpha}$ $\underbrace{X \vdash \alpha}$
$\forall v$if v$\vDash$ X then v$\vDash$ αthere is proof. sequence for α using X assumes

$\boxed{\text{X}\nvdash \alpha}$ $X \nvDash \alpha$

$X \vdash \alpha X \vDash \alpha$
$X \vdash \neg\alpha X \vdash \alpha\alpha X$
$X \nvdash \beta\beta X$
$X \vdash \neg\alpha X \vdash \alpha\alpha\Leftarrow$
$X\neg\alpha\alpha$
$X \vdash \neg\alpha X \vdash \alpha\alpha X \nvdash \beta\beta\Leftarrow$

- $\vdash \neg\alpha \to (\alpha \to \beta)$

$\vdash \neg\alpha \to (\alpha \to \beta)$

$\neg\alpha X$

αX

$\alpha \to \beta$MP 1,2

βMP 3,4

$X \vdash \beta$
$X \nvdash \beta$
$X \vdash \neg\alpha X \vdash \alpha\alpha$
X
$X \vDash \neg\alpha X \vDash \alpha$
v

- $v \vDash \alpha v \vDash X$
- $v \vDash \neg\alpha v \vDash X$

Xv
X

- $X = \{p_1, \neg p_1\}$
- $X = \{\underbrace{\alpha \to \beta}_{\beta}, \alpha, \neg\beta\}$

$X\Leftarrow X$
$X\Leftarrow X$

$X\Leftarrow$
$Y \subseteq X$
$Y \vdash \neg\alpha Y \vdash \alpha\alpha$
$XX \vdash \neg\alpha X \vdash \alpha$
$\Rightarrow$
X
$X \vdash \neg\alpha X \vdash \alpha$
$X'' \vdash \neg\alpha \quad x' \vdash \alpha$
$X''X'$
$X' \cup X''$
$X' \cup X'' \vdash \neg\alpha X' \cup X'' \vdash \alpha$

$$X \nvdash \neg\alpha X \cup \{\alpha\}$$

$$X \nvdash \alpha X \cup \{\neg\alpha\}$$

$X \cup \{\alpha\}\Leftarrow$
$X \vdash \neg\alpha$
$$X \cup \{\alpha\}\begin{cases} X \cup \{\alpha\} \vdash \alpha \\ X \cup \{\alpha\} \vdash \neg\alpha \end{cases}$$
$X \vdash \neg\alpha$
$X \cup \{\alpha\} X \nvdash \neg\alpha\Rightarrow$
$\alpha\neg\alpha\beta X \cup \{\alpha\} \vdash \neg\alpha$
$\Downarrow$
$X \vdash \alpha \to \neg\alpha$
"$\vdash (\alpha \to \neg\alpha) \to \neg\alpha$"

$$(\alpha \to \neg\alpha) \to \neg\alpha$$

$$(\alpha \to \neg\alpha) X$$

$$\neg\alpha$$

$X \vdash \neg\alpha$
XX
$X \vDash \neg\alpha X \vDash \alpha X \vdash \neg\alpha X \vdash \alpha XX$
$X\Leftarrow X$
$X \vdash \neg\alpha X \vdash \alpha\alpha X$
v
p
$v(p) = TX \vdash p$
$v(p) = FX \vdash \neg p$
$vX \nvdash \neg p X \nvdash p$
$X \nvdash pv$
$X \nvdash \neg p$

$X = \{\overbrace{p_o \vee p_1}^{F}\}$

$p_0 \vee p_1 \not\models \overbrace{p_0}^{F} \Rightarrow p_0 \vee p_1 \nvdash p_0$

$p_0 \vee p_1 \not\models \neg \overbrace{p_0}^{F}$

$p_0 \vee p_1 \not\models \overbrace{p_1}^{F}$

$\not\models \neg \overbrace{p_1}^{T}$

αX

$X \vdash \alpha$

$X \vdash \neg\alpha$

$Y \cup \{\alpha\} Y \vdash \alpha \alpha Y Y$
$Y \Leftarrow$
Y
$Y \vdash \neg\alpha Y \nvdash \alpha$

$$Y \cup \{\alpha\} \Leftarrow \begin{cases} Y \cup \{\alpha\} \vdash \alpha \\ Y \cup \{\alpha\} \vdash \neg\alpha \end{cases}$$

$Y \cup \{\alpha\} y \nvdash \alpha \alpha Y \Rightarrow$
Y

$Y \vdash \alpha$

$Y \cup \{\alpha\} Y \nvdash \alpha$

$Y \vdash \neg\alpha$
$X \subseteq Y Y X$

$\alpha_1, \alpha_2, \alpha_3 \ldots$

X

$$\begin{array}{l} X_0 \subseteq X_1 \subseteq X_2 \subseteq \ldots \\ \shortparallel \\ X \end{array}$$

$X_n n$

$X_{n+1} = X_n X_n \vdash \neg\alpha_n$
$X_{n+1} = X_n \cup \{\alpha_n\} X_n \nvdash \neg\alpha_n$
$Y = \cup X_n$
XY

$X \subseteq Y$

X_n

n

- XX_0

- $X_{n+1} X_n$

$X_{n+1} X_n = X_{n+1}$

Y
YW
$W \subseteq X_k$
$W \subseteq Y w_i \in X_i w_i \in W$
$w_i m$
$W \subseteq X_m$

$y \vdash \neg\alpha_n Y \vdash \alpha_n \alpha_n Y$