$MPA_1A_{2,}A_3$

$\underbrace{\alpha, \alpha \to \beta}_{\beta}$

$\vdash \alpha$

α

$\alpha_1, \ldots, \underbrace{a_n}_{\alpha}$

$1 \leq i \leq n$

MP"a_i

$XA_1 \cup A_2 \cup A_3 \cup X$

MP

$X \vdash \alpha$

$y \vdash \alpha X \subseteq YX$

MPA_1, A_2

$\alpha, \beta X$

$X \vdash \alpha \to \beta \ \Leftrightarrow \ X, \alpha \vdash \beta$

$X \vdash \alpha \to \beta \Rightarrow$

$X, \alpha \vdash \beta$

$X, \alpha \vdash \beta \Leftarrow$

$X \vdash \alpha \to \beta$"

$X, \alpha \vdash \beta$

$a_1, \ldots, a_n$

i

$a_n = \beta MP\alpha X a_i$

$1 \leq i \leq n a_i$

$X \vdash \alpha \to a_i$

$a_n = \beta$

$X \vdash \alpha \to \beta$

i

$X \vdash \alpha \to a_i$

a_i

α

X

a_1

$a_1 \to (\alpha \to a_i) \ A_1$

$\alpha \to a_1 \ MP_{1,2}$

$\vdash \alpha \to a_1$
$X \vdash (\alpha \to a_i)$

$\vdash \alpha \to \alpha$
$X \vdash \alpha \to \alpha$

$a_1 X$

$a_1 \to (\alpha \to a_1)$

$\alpha \to a_1 \ MP_{1,2}$

$X \vdash \alpha \to a_1$
$a_i j < i$

a_i

α

X

$m, l < i a_m a_l MP a_i$

$a_1 = \delta \to a_i$

$a_m = \delta$

$a_i \qquad MP \ a_l, a_m$

$X \vdash (\alpha \to (\delta \to a_i))$
$X \vdash (\alpha \to \delta)$

$\alpha \to (\delta \to a_i) X$

$(\alpha \to \delta) X$

$(((\alpha \to (\delta \to a_i)) \to$
$((\alpha \to \delta) \to (\alpha \to a_i))) \ A_2$

$((\alpha \to \delta) \to (\alpha \to a_i)) \ MP_{1,3}$

$(\alpha \to a_i) \ MP_{2,4}$
$X \vdash (\alpha \to a_i)$
$\blacksquare$

$\vdash (\alpha \to (\beta \to \gamma)) \to (\beta \to (\alpha \to \gamma))$

$$(\alpha \to (\beta \to \gamma))$$

$$\beta$$

$$\alpha$$

$$(\beta \to \gamma)\ MP_{1,3}$$

$$\gamma\ MP_{4,2}$$

$$(\alpha \to (\beta \to \gamma)), \beta, \gamma \vdash \alpha$$

$$(\alpha \to (\beta \to \gamma)), \beta \vdash (\alpha \to \gamma)$$

““$(\alpha \to (\beta \to \gamma)) \vdash (\beta \to \alpha \to \gamma)$

““$\vdash (\alpha \to (\beta \to \gamma)) \to (\beta \to (\alpha \to \gamma))$

$$\vdash (\neg\neg(\alpha \to \alpha)$$

$$\neg\neg\alpha$$

$$(\neg\neg\alpha \to (\neg\alpha \to (\neg\neg\neg\alpha)) \ \vdash (\neg\alpha \to (\alpha \to \beta)$$

$$(\neg\alpha \to \neg\neg\neg\alpha)\ MP_{1,2}$$

$$(\underbrace{\neg\alpha}_{\beta} \to \underbrace{\neg\neg\neg\alpha}_{\neg\alpha}) \to (\underbrace{\neg\neg\alpha}_{\alpha} \to \underbrace{\alpha}_{\beta})\ A_3$$

$$(\neg\neg\alpha \to \alpha)\ MP_{3,4}$$

$$\alpha\ MP_{5,1}$$

$$\neg\neg\alpha \vdash \alpha$$

$$\vdash (\neg\neg\alpha \to \alpha)$$

$$A_3 : (\neg\beta \to \neg\alpha) \to (\alpha \to \beta)$$

$$\vdash (\neg \underbrace{\alpha}_{(\neg\alpha)} \to (\underbrace{\alpha}_{(\neg\alpha)} \to \underbrace{\beta}_{(\neg\neg\neg\alpha)}))$$

$$\neg\neg\alpha \vdash (\neg\neg\alpha \to \alpha)$$

$$\{\neg\neg a, \neg\neg\alpha\} \vdash \alpha \to$$

$$\neg\neg\alpha \vdash \alpha$$

$$\vdash (\neg\neg\alpha \to \alpha)$$

$$\vDash \alpha \quad \Leftarrow \quad \vdash \alpha$$

$$X \vDash \alpha \quad \Leftarrow \quad X \vdash \alpha$$

$$vX \vDash \alpha$$

$$v \vDash \alpha\beta \in Xv \vDash \beta v \vDash X$$

$$X \vdash \alpha \Leftarrow X \vDash \alpha$$

$$\vdash \alpha \quad \Leftarrow \quad \vDash \alpha$$

$x \vdash \alpha$
$X \vDash \alpha$“

$a_1, \dots, \underbrace{a_n}_{\alpha}$

Xa_1

a_1

Xa_1
$X \vDash a_1$
$v \vDash a_1 \beta \in X v \vDash \beta v \vDash X$

a_1
$\vDash a_1$
$X \vDash a_1$

$X \vDash a_j$
$X \vDash a_K$
Xa_i
$j, k < i a_k, a_j MP$
$a_k = \beta a_j = \beta \to a_i$
$X \vDash \beta \to a_i$
$X \vDash \beta$
$X \nvDash a_i$
v
$v \vDash X$
$v \nvDash a_i$

$v \vDash \beta \to a_i$
$v \vDash \beta$
$: \to$
$v \vDash a_i$
$X \vDash \alpha X \vdash \alpha$
$X \nvdash \alpha X \nvDash \alpha$

$X \vdash \neg \alpha X \vdash \alpha \alpha X$

$X \vdash \beta \beta X X$

$X = \{\alpha, \neg \alpha\}$
$X \vdash \alpha$
$X \vdash \neg \alpha$

$$X = \{\alpha \to \beta, \alpha, \neg\beta\}$$
$$X \vdash \neg\beta$$
$$X \vdash \beta$$

$$\Leftarrow$$
$$X \vdash \neg\alpha X \nvdash \alpha, \alpha$$
$$X \nvdash \neg\alpha X \vdash \alpha\alpha$$
$$\Leftarrow$$
$$X \nvdash \beta\beta X X$$