

$$\begin{array}{c}
MPA_1A_2,A_3 \\
\alpha,\alpha\rightarrow\beta \\
\beta \\
\vdash\alpha \\
\alpha \\
\alpha_1,\ldots,\underbrace{a_n}_{\alpha} \\
1\leq i\leq n \\
MP^{\alpha}a_i \\
XA_1\cup A_2\cup A_3\cup X \\
MP \\
X\vdash\alpha \\
y\vdash\alpha X\subseteq YX \\
MPA_1,A_2 \\
\alpha,\beta X \\
X\vdash\alpha\rightarrow\beta\iff X,\alpha\vdash\beta \\
X\vdash\alpha\rightarrow\beta\Rightarrow \\
X,\alpha\vdash\beta \\
X,\alpha\vdash\beta\Leftarrow \\
X\vdash\alpha\rightarrow\beta^{\alpha} \\
X,\alpha\vdash\beta \\
a_1,\ldots,a_n \\
i \\
a_n=\beta MP\alpha Xa_i \\
1\leq i\leq na_i \\
X\vdash\alpha\rightarrow a_i \\
a_n=\beta \\
X\vdash\alpha\rightarrow\beta \\
i \\
X\vdash\alpha\rightarrow a_i \\
a_i \\
\alpha \\
X \\
a_1 \\
a_1\rightarrow(\alpha\rightarrow a_i)\ A_1
\end{array}$$

$$\alpha \rightarrow a_1 \text{ } MP_{1,2}$$

$$\begin{array}{c} \vdash \alpha \rightarrow a_1 \\ X \vdash (\alpha \rightarrow a_i) \end{array}$$

$$\begin{array}{c} \vdash \alpha \rightarrow \alpha \\ X \vdash \alpha \rightarrow \alpha \end{array}$$

$$a_1 X$$

$$a_1 \rightarrow (\alpha \rightarrow a_1)$$

$$\alpha \rightarrow a_1 \text{ } MP_{1,2}$$

$$\begin{array}{c} X \vdash \alpha \rightarrow a_1 \\ a_i j < i \end{array}$$

$$a_i$$

$$\alpha$$

$$X$$

$$m, l < i a_m a_l MP a_i$$

$$a_1 = \delta \rightarrow a_i$$

$$a_m = \delta$$

$$a_i \quad MP \ a_l, a_m$$

$$\begin{array}{c} X \vdash (\alpha \rightarrow (\delta \rightarrow a_i)) \\ X \vdash (\alpha \rightarrow \delta) \end{array}$$

$$\alpha \rightarrow (\delta \rightarrow a_i) X$$

$$(\alpha \rightarrow \delta) X$$

$$\begin{array}{c} (((\alpha \rightarrow (\delta \rightarrow a_i)) \rightarrow \\ ((\alpha \rightarrow \delta) \rightarrow (\alpha \rightarrow a_i))) \ A_2 \end{array}$$

$$((\alpha \rightarrow \delta) \rightarrow (\alpha \rightarrow a_i)) \text{ } MP_{1,3}$$

$$\begin{array}{c} (\alpha \rightarrow a_i) \text{ } MP_{2,4} \\ X \vdash (\alpha \rightarrow a_i) \end{array}$$

■

$$\vdash (\alpha \rightarrow (\beta \rightarrow \gamma)) \rightarrow (\beta \rightarrow (\alpha \rightarrow \gamma))$$

$$(\alpha \rightarrow (\beta \rightarrow \gamma))$$

$$\beta$$

$$\alpha$$

$$(\beta \rightarrow \gamma) MP_{1,3}$$

$$\gamma MP_{4,2}$$

$$(\alpha \rightarrow (\beta \rightarrow \gamma)), \beta, \gamma \vdash \alpha$$

$$(\alpha \rightarrow (\beta \rightarrow \gamma)), \beta \vdash (\alpha \rightarrow \gamma)$$

$$“(\alpha \rightarrow (\beta \rightarrow \gamma)) \vdash (\beta \rightarrow \alpha \rightarrow \gamma)”$$

$$“\vdash (\alpha \rightarrow (\beta \rightarrow \gamma)) \rightarrow (\beta \rightarrow (\alpha \rightarrow \gamma))”$$

$$\vdash (\neg\neg(\alpha \rightarrow \alpha))$$

$$\neg\neg\alpha$$

$$(\neg\neg\alpha \rightarrow (\neg\alpha \rightarrow (\neg\neg\neg\alpha))) \vdash (\neg\alpha \rightarrow (\alpha \rightarrow \beta))$$

$$(\neg\alpha \rightarrow \neg\neg\neg\alpha) MP_{1,2}$$

$$\underbrace{(\neg\alpha)}_{\beta} \rightarrow \underbrace{(\neg\neg\neg\alpha)}_{\neg\alpha} \rightarrow \underbrace{(\neg\neg\alpha)}_{\alpha} \rightarrow \underbrace{\alpha}_{\beta} A_3$$

$$(\neg\neg\alpha \rightarrow \alpha) MP_{3,4}$$

$$\alpha MP_{5,1}$$

$$\neg\neg\alpha \vdash \alpha$$

$$\vdash (\neg\neg\alpha \rightarrow \alpha)$$

$$A_3 : (\neg\beta \rightarrow \neg\alpha) \rightarrow (\alpha \rightarrow \beta)$$

$$\vdash (\neg \underbrace{\alpha}_{(\neg\alpha)} \rightarrow (\underbrace{\alpha}_{(\neg\alpha)} \rightarrow \underbrace{\beta}_{(\neg\neg\alpha)}))$$

$$\neg\neg\alpha \vdash (\neg\neg\alpha \rightarrow \alpha)$$

$$\{\neg\neg\alpha, \neg\neg\alpha\} \vdash \alpha \rightarrow$$

$$\neg\neg\alpha \vdash \alpha$$

$$\vdash (\neg\neg\alpha \rightarrow \alpha)$$

$$\models \alpha \Leftarrow \vdash \alpha$$

$$X \models \alpha \Leftarrow X \vdash \alpha$$

$$vX \models \alpha$$

$$v \models \alpha \beta \in X v \models \beta v \models X$$

$$X \vdash \alpha \Leftarrow X \models \alpha$$

$$\vdash \alpha \Leftarrow \models \alpha$$

$$\begin{array}{c} x \vdash \alpha \\ X \models \alpha \end{array}$$

$$a_1, \dots, \underbrace{a_n}_{\alpha}$$

$$Xa_1$$

$$a_1$$

$$\begin{array}{c} Xa_1 \\ X \models a_1 \\ v \models a_1 \beta \in X v \models \beta v \models X \end{array}$$

$$\begin{array}{c} a_1 \\ \models a_1 \\ X \models a_1 \end{array}$$

$$\begin{array}{c} X \models a_j \\ X \models a_K \\ Xa_i \end{array}$$

$$\begin{array}{c} j,k < i a_k, a_j MP \\ a_k = \beta a_j = \beta \rightarrow a_i \\ X \models \beta \rightarrow a_i \\ X \models \beta \\ X \not\models a_i \end{array}$$

$$\begin{array}{c} v \\ v \models X \\ v \not\models a_i \end{array}$$

$$\begin{array}{c} v \models \beta \rightarrow a_i \\ v \models \beta \\ \vdots \rightarrow \\ v \models a_i \\ X \models \alpha X \vdash \alpha \\ X \not\models \alpha X \not\models \alpha \end{array}$$

$$X \vdash \neg \alpha X \vdash \alpha \alpha X$$

$$X \vdash \beta \beta X X$$

$$\begin{array}{c} X = \{ \alpha, \neg \alpha \} \\ X \vdash \alpha \\ X \vdash \neg \alpha \end{array}$$

$$\begin{array}{l}
X = \{\alpha \rightarrow \beta, \alpha, \neg\beta\} \\
X \vdash \neg\beta \\
X \vdash \beta
\end{array}$$

$$\begin{array}{l}
\Leftarrow \\
X \vdash \neg\alpha \quad X \not\vdash \alpha, \alpha \\
X \not\vdash \neg\alpha \quad X \vdash \alpha \alpha \\
\Leftarrow \\
X \not\vdash \beta \beta X X
\end{array}$$