Battery Modelling

An MMSC Case Study on Mathematical Modelling
Candidate Number: 1072462

Abstract

This work will attempt to

Contents

1 Introduction

Clearly, electric batteries are largely important for various industries and demand for them is ever-growing. This includes, especially, the renewable energy sector due to the unpredictability of energy supplies such as wind and solar power where short-term storage is a necessary evil. Similar relevance may be found in the car industry where one aims for highly (space-)efficient mobile storage of energy. In many countries and/or regions, Electric Vehicles (EVs) still lack a well-enlarged network of charging stations, for various reasons including incompatibilties between charging station suppliers.

In this report, we will consider how to model a battery

2 Problem Formulation

2.1 The Isolated Battery

Let $s \in [0, 1]$ denote the *state of charge* (SOC) of the battery, $h \in [0, 1]$ the *state of health* (SOH), $Q \in \mathbb{R}^+$ the charge, $Q_{00} \in \mathbb{R}^+$ the maximum possible charge at the time of production (in Coulombs), $V \in \mathbb{R}$ the voltage across the battery (in Volts) with $I \in \mathbb{R}$ the corresponding current (in Amperes) where $I > 0$ corresponds to discharging the battery. Then, per common definition, $s := \frac{Q}{Q_0}$ is the amount of charge currently present in the battery as compared to $Q_0 \in \mathbb{R}^+$ the current maximum capacity, which itself is dependent on the state of health, as given by $Q_0 := hQ_{00}$. Further let $T \in [-273.15, \infty)$ denote the temperature of the battery (in degrees Celsius) and let $t \in \mathbb{R}$ represent time (in seconds).

From the definition of current $I := \frac{\mathrm{d}Q}{\mathrm{d}t}$, we further have that for a single cycle,

$$s = 1 - \frac{1}{Q_0} \int_0^t I(\tau) \mathrm{d}\tau \,,$$

under the assumption that Q_0, and therefore h, stays constant during that cycle.

2.2 The Equivalent Circuit Model

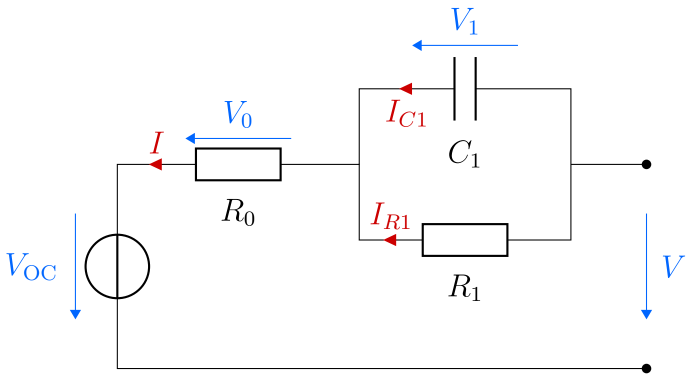


Figure 1: The Thevenin equivalent circuit model (ECM) with parameters $R_0 \in \mathbb{R}^+$, $R_1 \in \mathbb{R}^+$ and $C_1 \in \mathbb{R}^+$ and $V_{\mathrm{OC}} \in \mathbb{R}^+$ the *open circuit voltage* which behaves according to a function $V_{\mathrm{OC}}(s, h, T)$ dependent on s, h and T.

Kirchhoff's law further tells us that the currents $I_{R1} \in \mathbb{R}$ and $I_{C1} \in \mathbb{R}$ add up to the total current $I = I_{R1} + I_{C1}$, and that the voltages $V_0 \in \mathbb{R}$, $V_1 \in \mathbb{R}$ and V_{OC} sum up to $V = V_0 + V_1 + V_{\mathrm{OC}}$. The capacitor behaves according to $I_{C1} = C_1 \frac{\mathrm{d}V_1}{\mathrm{d}t}$, while the resistors follow Ohm's law $V_0 = R_0 I$ and $V_1 = R_1 I_{R1}$.

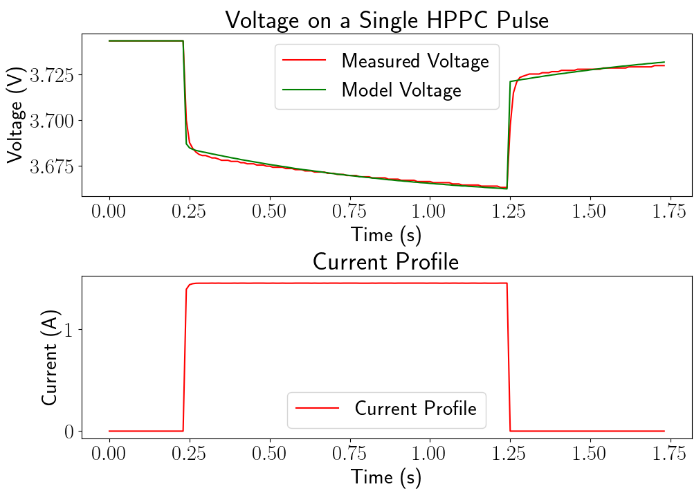


Figure 2: HPPC Pulse

2.3 Battery in an Electric Vehicle (EV)

On a graph (V_G, E) with edges $E = \{AB, AC, ...\} \subseteq V_G \times V_G$ and vertices $V_G = \{A, B, ...\}$, let $d_{AB} \in \mathbb{R}^+$ denote the distance between two vertices $A \in V_G$ and $B \in V_G$ (in meters), $x = x_{AB} \in [0, d_{AB}]$ the progress (current location) on the route from vertex A to B (in meters), $v := \frac{\mathrm{d}x}{\mathrm{d}t}$ denote the current velocity with $v_{\text{max,AB}} \in \mathbb{R}^+$ the maximum allowed velocity on AB (in meters per second). Then let $T_{\text{env}}(x) \in [-273.15, \infty)$ denote the temperature of the environment (in degrees Celsius) at location x.

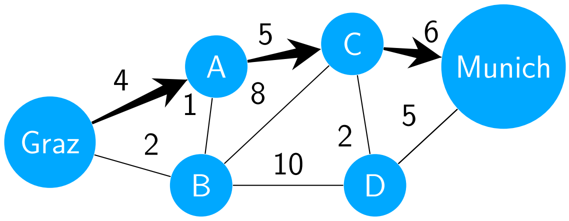


Figure 3: Path

Let $P \in \mathbb{R}$, $P := I \cdot V$ denote the (electrical) power the car draws from the battery (in Watts) so $P > 0$ corresponds to discharging the battery. This power is to be realised into a mechanical component $P_{\text{motor}} \in \mathbb{R}^+$ driving the car forwards, heating for the battery $P_{\text{heat}} \in \mathbb{R}^+$, $P_{\text{heat}} = c(T - T_{\text{env}})$, with $c \in \mathbb{R}^+$ the heat conduction constant describing the relation between the heater and battery, and power dissipation $P_{\text{diss}} \in \mathbb{R}^+$. While driving, $P = P_{\text{motor}} + P_{\text{heat}} + P_{\text{diss}}$. The acceleration of the car $a \in \mathbb{R}$ (in meters per second squared), where $a := \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}^2x}{\mathrm{d}t}$ is decomposed into $a_m \in \mathbb{R}$, which directly impacts $P_{\text{motor}}(a_m)$, and the deceleration due to friction (air, etc.) $a_f(v) \in \mathbb{R}^-$, so that in total $a = a_m + a_f$.

On the graph (V_G, E) there exists a set of EV charging stations $V_{\text{charge}} \subseteq V_G$ where $P_{C,\text{charge}}$ denotes the possible charging power (in Watts) at the charging station vertex $C \in V_{\text{charge}}$ with $K_C \in \mathbb{R}^+$ the occuring costs per energy unit (in Euros per Watt second) and $t_C \in \mathbb{R}^+$ the charging time per charging station B (in seconds).

2.4 A Variational Optimisation Problem

Given source and destination vertices $A, Z \in V_G$ on the graph (V_G, E), which connected set of edges $E_R \subseteq E$ connecting A to Z, set of visited charging stations $V_C \subseteq V_{\text{charge}}$ and charging times $\{t_C\}_{C \in V_C}$ visited on the route E_R, and driving behaviour $a_m(x, v, t, s, h, T_{\text{env}}, ...)$, $a_m \in \mathcal{C}^1(\mathbb{P})$ with $\mathbb{P}$ the parameter space [1] minimises

1. the total travel time $t_{\text{total}} := \int_{V_R} \frac{1}{v} \mathrm{d}x + \sum_{C \in V_C} t_C$,
2. the total cost of travel $K := \sum_{C \in V_C} P_{C,\text{charge}} t_C K_C$,

[1] to be defined.. TODO

3. $-N$ where N is the highest possible number of repetitions (commutes from A to Z) with the same battery (requiring $h > 0$).

Formulated differently, we aim to minimise the functional $F \in \mathcal{C}(\mathbb{P})^*, F : \mathcal{C}(\mathbb{P}) \mapsto \mathbb{R}$ where either $F[\chi] = t_{\text{total}}$ or $F[\chi] = K$.

Charging station + street data could be obtained from OpenStreetMap by calling `osmfilter england-latest.o5m keep="amenity=charging_station"` [2].

2.5 Battery Aging

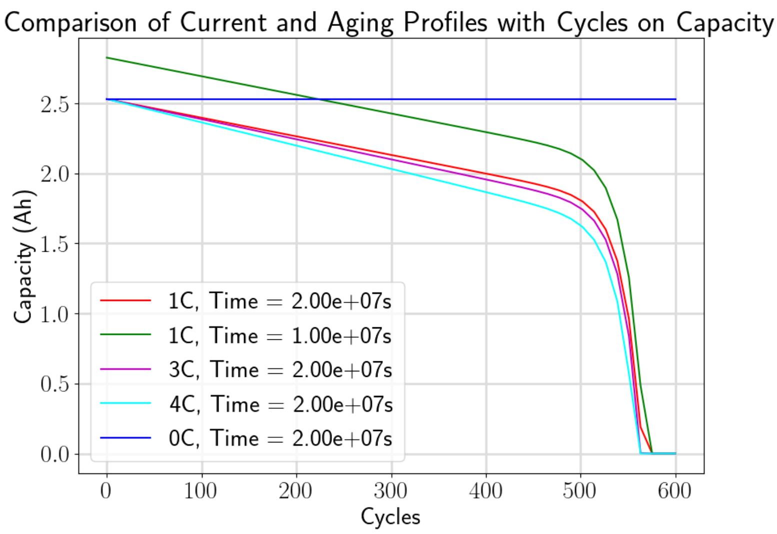


Figure 4: Aging

[2]OSM's public map data may be obtained from https://download.geofabrik.de/

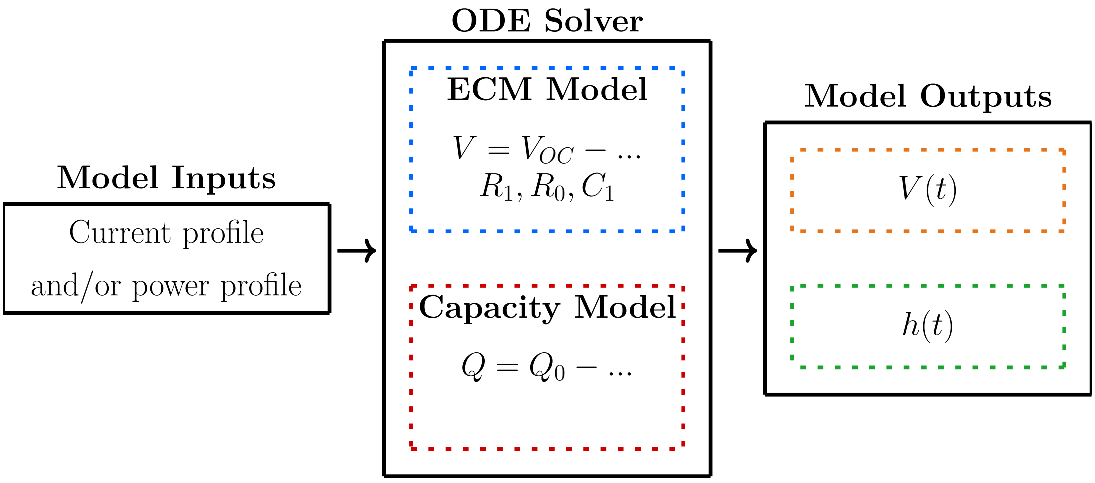
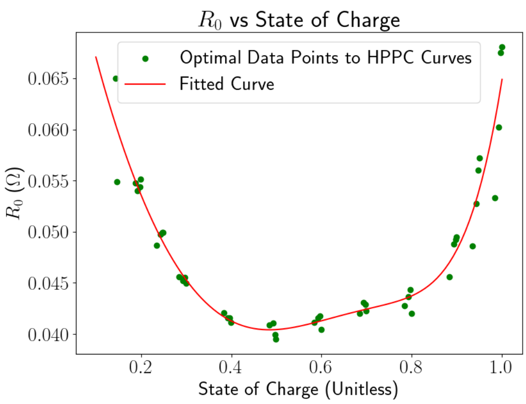


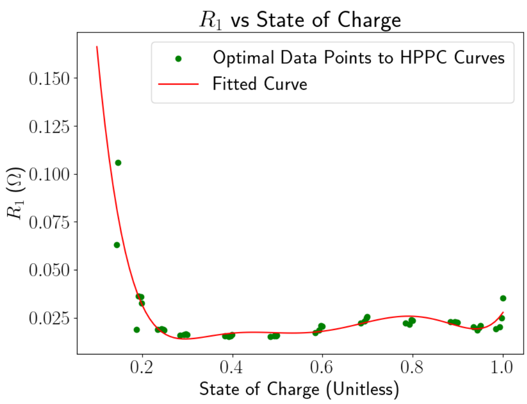
Figure 5: Overview

3 Numerical Simulation of a Car

3.1 Finding ECM Parameters



(a) Fit of R_0 as a function of the SOC (s).



(b) Fit of R_1 as a function of the SOC (s).

3.2 Forward Euler Simulation

4 Metropolis-Hastings and A-Star

4.1 Definition: Undirected Graph

A graph $G = (V, E)$ with vertices V and edges $E \subseteq V \times V$ is undirected if and only if $(v_i, v_j) \in E \Rightarrow (v_j, v_i) \in E \quad \forall\, v_i, v_j \in V$.

4.1 Shortest Path Finding

The graph data was retrieved using OSMNX (Boeing 2017) which itself is built on NetworkX (Hagberg, Schult and Swart 2008).

4.2 Monte-Carlo Optimisation

Optimise a large problem (huge state-space). Metric: Time! Could be anything. $\Rightarrow$ Use Monte-Carlo Markov Chain Methods! Slightly perturb the route using a specific alteration technique. Metropolis-Hastings updates the state (route) based on

$$p_{\text{accept}} = \min\left(1, \mathrm{e}^{-\beta(T_{\text{next}} - T_{\text{current}})}\right), \qquad \text{with } \beta \in \mathbb{R}^+ \text{ a transition factor}.$$

Does a full numerical simulation of the drive. Stop to charge? Explore the state-space to some extent, and return the best route! Larger scales / maps (e.g. England) are not a problem!

4.3 Special Case: Granny's House

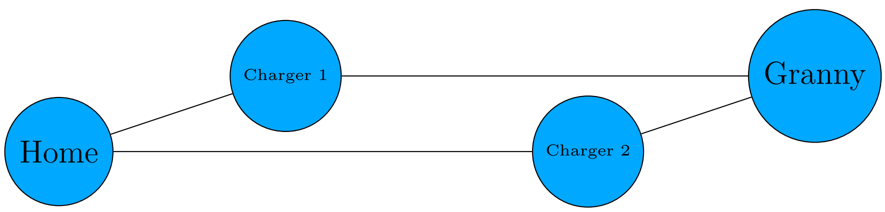


Figure 7: The exemplary problem "Granny's House", a special case of Problem (TODO).

4.4 Routing in Jericho

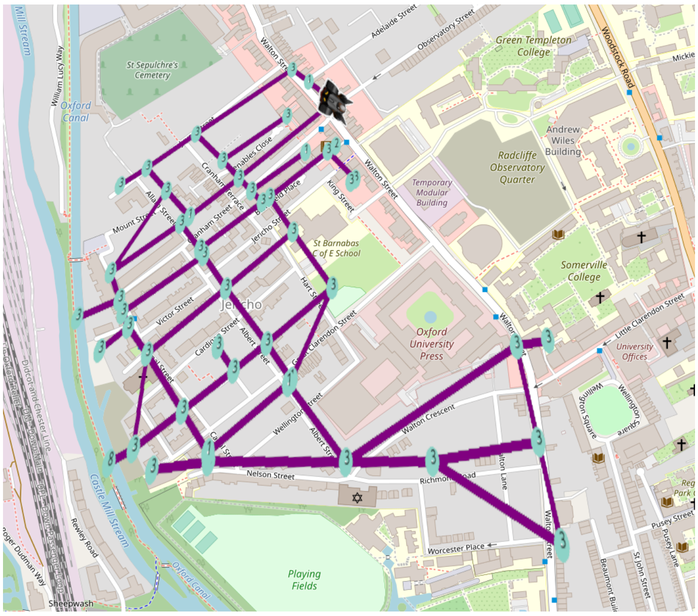


Figure 8: Overlay of the routing graph on a map of Jericho (OpenStreetMap contributors 2023), without adjusting for the Merkator projection, which leads to a slightly skewed appearance. The underlying data is exactly the same.

5 Conclusion

References

Boeing, Geoff (2017). 'OSMnx: New methods for acquiring, constructing, analyzing, and visualizing complex street networks'. In: *Computers, Environment and Urban Systems* 65, pp. 126–139. ISSN: 0198-9715. DOI: 10.1016/j.compenvurbsys.2017.05.004.

Hagberg, Aric A., Daniel A. Schult and Pieter J. Swart (2008). 'Exploring Network Structure, Dynamics, and Function using NetworkX'. In: *Proceedings of the 7th Python in Science Conference*. Ed. by Gaël Varoquaux, Travis Vaught and Jarrod Millman. Pasadena, CA USA, pp. 11–15.

OpenStreetMap contributors (2023). *Planet dump retrieved from planet.osm.org*. URL: %5Curl%7Bhttps://www.openstreetmap.org%7D (visited on 08/03/2023).