Battery Modelling

An MMSC Case Study on Mathematical Modelling

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Abstract

This work will attempt to

Contents

1 Introduction

Clearly, electric batteries are largely important for various industries today and demand for them is ever-growing. This includes, especially, the renewable energy sector due to the unpredictability of energy supplies such as wind and solar power where short-term storage is a necessary evil. Similar relevance may be found in the car industry where one aims for highly (space-)efficient mobile storage of energy. In many countries and/or regions, Electric Vehicles (EVs) still lack a well-enlarged network of charging stations, for various reasons including incompatibilties between charging station suppliers.

In this report, we will consider how to model a Lithium-Ion battery, more specifically the Panasonic 18650PF for which there is some characteristic data available in the public domain (Kollmeyer 2018). To do this, we will consider an Equivalent Circuit Model (ECM), namely the Thevenin ECM, which may be found in Figure 2. The key idea here is to model the voltage output $V(t)$ given a current profile $I(t)$. Further

2 Problem Formulation

2.1 The Isolated Battery

Let $s \in [0,1]$ denote the *state of charge* (SOC) of the battery, $h \in [0,1]$ the *state of health* (SOH), $Q \in \mathbb{R}^+$ the charge, $Q_{00} \in \mathbb{R}^+$ the maximum possible charge at the time of production (in Coulombs), $V \in \mathbb{R}$ the voltage across the battery (in Volts) with $I \in \mathbb{R}$ the corresponding current (in Amperes) where $I > 0$ corresponds to discharging the battery. Then, per common definition, $s := \frac{Q}{Q_0}$ is the amount of charge currently present in the battery as compared to $Q_0 \in \mathbb{R}^+$ the current maximum capacity, which itself is dependent on the state of health, as given by $Q_0 := hQ_{00}$. Further let $T \in [-273.15, \infty)$ denote the temperature of the battery (in degrees Celsius) and let $t \in \mathbb{R}$ represent time (in seconds).

From the definition of current $I := \frac{\mathrm{d}Q}{\mathrm{d}t}$, we further have that for a single cycle,

$$s = 1 - \frac{1}{Q_0} \int_0^t I(\tau) \mathrm{d}\tau \,,$$

under the assumption that Q_0, and therefore h, stays constant during that cycle.

2.2 The Equivalent Circuit Model

As mentioned earlier, we want to model the battery as an electrical component which exerts specific behaviour in an electrical circuit. In electrical engineering, such models of more complicated components are frequently represented by equivalent circuits which only consist of basic components, mostly resistors, diodes, transistors, capacitors and inductors.

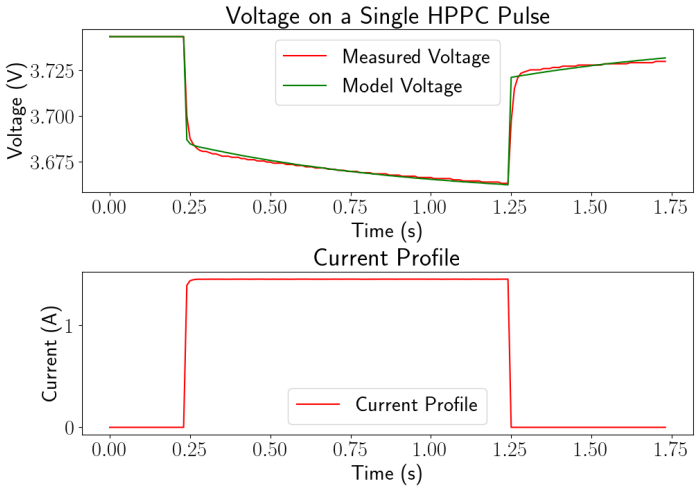


Figure 1: HPPC Pulse

Empirically, we know that a battery's voltage output looks roughly

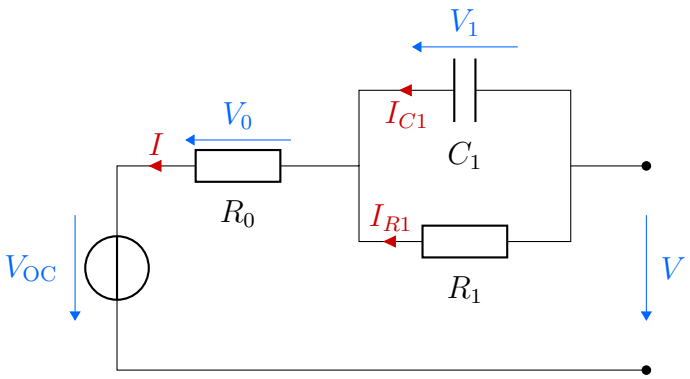


Figure 2: The Thevenin equivalent circuit model (ECM) with parameters $R_0 \in \mathbb{R}^+$, $R_1 \in \mathbb{R}^+$ and $C_1 \in \mathbb{R}^+$ and $V_{\text{OC}} \in \mathbb{R}^+$ the *open circuit voltage* which behaves according to a function $V_{\text{OC}}(s, h, T)$ dependent on s, h and T.

Kirchhoff's law further tells us that the currents $I_{R1} \in \mathbb{R}$ and $I_{C1} \in \mathbb{R}$ add up to the total current $I = I_{R1} + I_{C1}$, and that the voltages $V_0 \in \mathbb{R}$, $V_1 \in \mathbb{R}$ and V_{OC} sum up

to $V = V_0 + V_1 + V_{OC}$. The capacitor behaves according to $I_{C1} = C_1 \frac{dV_1}{dt}$, while the resistors follow Ohm's law $V_0 = R_0 I$ and $V_1 = R_1 I_{R1}$.

2.3 Battery in an Electric Vehicle (EV)

On a graph (V_G, E) with edges $E = \{AB, AC, ...\} \subseteq V_G \times V_G$ and vertices $V_G = \{A, B, ...\}$, let $d_{AB} \in \mathbb{R}^+$ denote the distance between two vertices $A \in V_G$ and $B \in V_G$ (in meters), $x = x_{AB} \in [0, d_{AB}]$ the progress (current location) on the route from vertex A to B (in meters), $v := \frac{dx}{dt}$ denote the current velocity with $v_{\text{max,AB}} \in \mathbb{R}^+$ the maximum allowed velocity on AB (in meters per second). Then let $T_{\text{env}}(x) \in [-273.15, \infty)$ denote the temperature of the environment (in degrees Celsius) at location x.

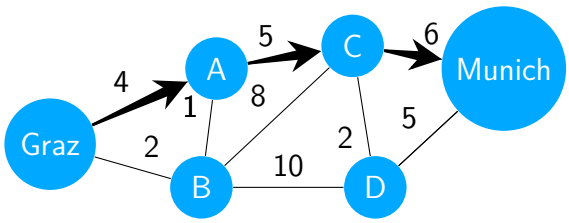


Figure 3: Path

Let $P \in \mathbb{R}$, $P := I \cdot V$ denote the (electrical) power the car draws from the battery (in Watts) so $P > 0$ corresponds to discharging the battery. This power is to be realised into a mechanical component $P_{\text{motor}} \in \mathbb{R}^+$ driving the car forwards, heating for the battery $P_{\text{heat}} \in \mathbb{R}^+$, $P_{\text{heat}} = c(T - T_{\text{env}})$, with $c \in \mathbb{R}^+$ the heat conduction constant describing the relation between the heater and battery, and power dissipation $P_{\text{diss}} \in \mathbb{R}^+$. While driving, $P = P_{\text{motor}} + P_{\text{heat}} + P_{\text{diss}}$. The acceleration of the car $a \in \mathbb{R}$ (in meters per second squared), where $a := \frac{dv}{dt} = \frac{d^2x}{dt}$ is decomposed into $a_m \in \mathbb{R}$, which directly impacts $P_{\text{motor}}(a_m)$, and the deceleration due to friction (air, etc.) $a_f(v) \in \mathbb{R}^-$, so that in total $a = a_m + a_f$.

On the graph (V_G, E) there exists a set of EV charging stations $V_{\text{charge}} \subseteq V_G$ where $P_{C,\text{charge}}$ denotes the possible charging power (in Watts) at the charging station vertex $C \in V_{\text{charge}}$ with $K_C \in \mathbb{R}^+$ the occuring costs per energy unit (in Euros per Watt second) and $t_C \in \mathbb{R}^+$ the charging time per charging station B (in seconds).

2.4 A Variational Optimisation Problem

Given source and destination vertices $A, Z \in V_G$ on the graph (V_G, E), which connected set of edges $E_R \subseteq E$ connecting A to Z, set of visited charging stations $V_C \subseteq V_{\text{charge}}$ and charging times $\{t_C\}_{C \in V_C}$ visited on the route E_R, and driving behaviour $a_m(x, v, t, s, h, T_{\text{env}}, ...)$, $a_m \in \mathcal{C}^1(\mathbb{P})$ with $\mathbb{P}$ the parameter space [1] minimises

1. the total travel time $t_{\text{total}} := \int_{V_R} \frac{1}{v} \mathrm{d}x + \sum_{C \in V_C} t_C$,
2. the total cost of travel $K := \sum_{C \in V_C} P_{C,\text{charge}} t_C K_C$,
3. $-N$ where N is the highest possible number of repetitions (commutes from A to Z) with the same battery (requiring $h > 0$).

Formulated differently, we aim to minimise the functional $F \in \mathcal{C}(\mathbb{P})^*, F : \mathcal{C}(\mathbb{P}) \mapsto \mathbb{R}$ where either $F[\chi] = t_{\text{total}}$ or $F[\chi] = K$.

Charging station + street data could be obtained from OpenStreetMap by calling `osmfilter england-latest.o5m keep="amenity=charging_station"` [2].

2.5 Battery Aging

$$Q(t, c, s, I) = Q_0 - \frac{F_{acycle}(c)}{F_{current}(I)} - F_{cal}$$

By $F_{acycle}(c)$ we denote the *Current Agnostic Cycle Degradation Factor* models cycle aging at a single current. Also $F_{current}(I)$ represents the *Current Scaling Factor*, a value $0 < F_{current} \leq 1$ that incorporates behaviour where higher currents are worse for cycle aging. Finally, $F_{cal}(s, t)$, the *Calendar Degradation Factor*, ages the battery over time and accounts for suboptimal storage in terms of state of charge.

[1] to be defined.. TODO

[2] OSM's public map data may be obtained from https://download.geofabrik.de/

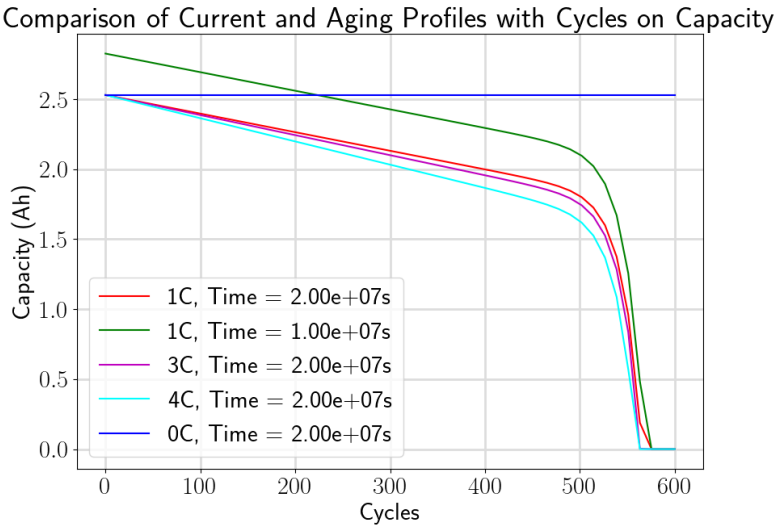


Figure 4: Aging

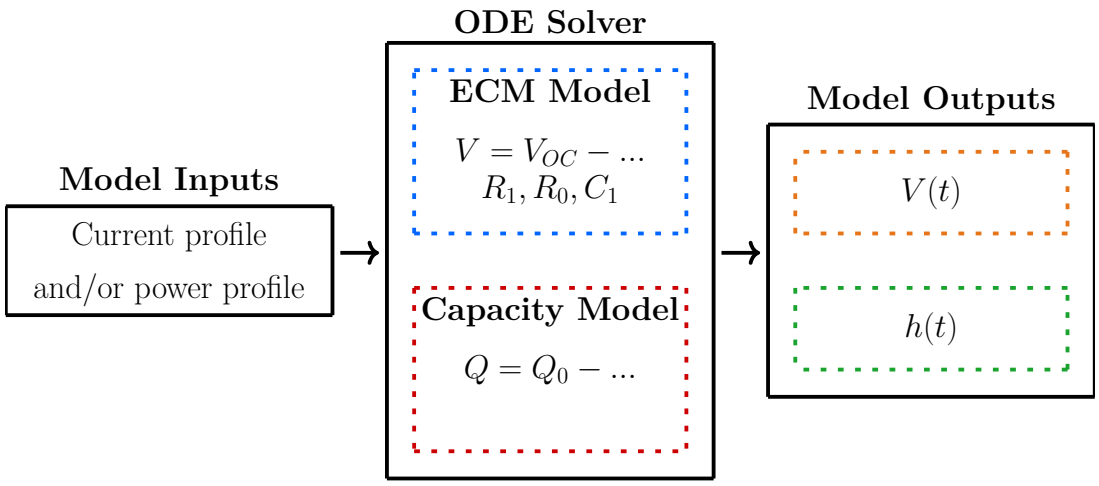
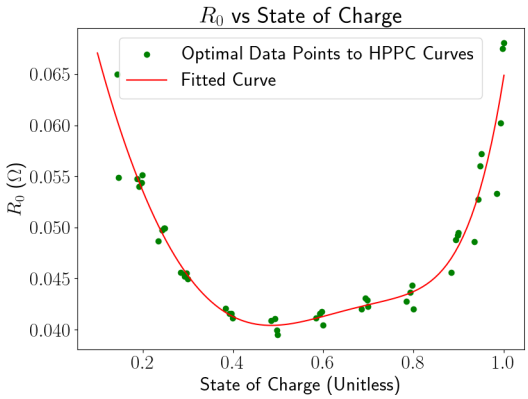


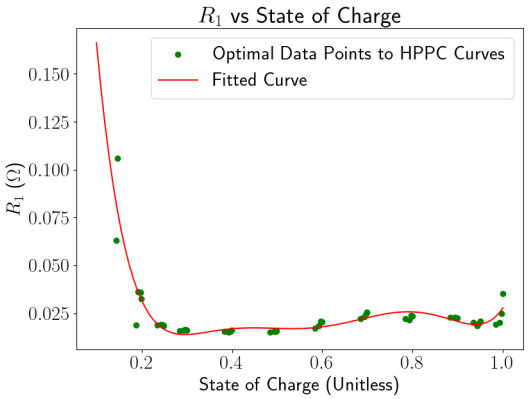
Figure 5: Overview

3 Numerical Simulation of a Car

3.1 Finding ECM Parameters



(a) Fit of R_0 as a function of the SOC (s).



(b) Fit of R_1 as a function of the SOC (s).

3.2 Forward Euler Simulation

4 Metropolis-Hastings and A-Star

4.1 Definition: Undirected Graph

A graph $G = (V, E)$ with vertices V and edges $E \subseteq V \times V$ is undirected if and only if $(v_i, v_j) \in E \Rightarrow (v_j, v_i) \in E \quad \forall\, v_i, v_j \in V$.

4.1 Shortest Path Finding

The graph data was retrieved using OSMNX (Boeing 2017) which itself is built on NetworkX (Hagberg, Schult and Swart 2008).

4.2 Monte-Carlo Optimisation

Optimise a large problem (huge state-space). Metric: Time! Could be anything. ⇒ Use Monte-Carlo Markov Chain Methods! Slightly perturb the route using a specific alteration technique. Metropolis-Hastings updates the state (route) based on

$$p_{\text{accept}} = \min\left(1, \mathrm{e}^{-\beta(T_{\text{next}} - T_{\text{current}})}\right), \qquad \text{with } \beta \in \mathbb{R}^+ \text{ a transition factor}.$$

Does a full numerical simulation of the drive. Stop to charge? Explore the state-space to some extent, and return the best route! Larger scales / maps (e.g. England) are not a problem!

```
newRoute = self.simulator.batgraph.perturbRoute(self.route)
delta = self.measureRoute(newRoute) - self.testedRoutes[self.route]
acceptanceProbability = min(1, math.exp(-delta / self.temperature))
if random.random() < acceptanceProbability:
    self.route = newRoute
    print(f"Accepted new route {newRoute} with delta: {delta:.2f}. Total:
    ↪ {self.testedRoutes[newRoute]:.2f}.")
else:
    print(f"Rejected route {newRoute} with delta: {delta:.2f}.")
```

4.3 Special Case: Granny's House

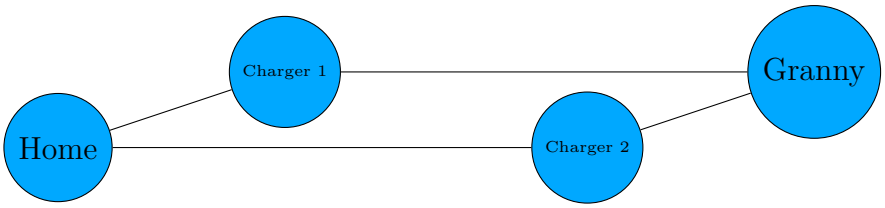


Figure 7: The exemplary problem "Granny's House", a special case of Problem (TODO).

4.4 Routing in Jericho

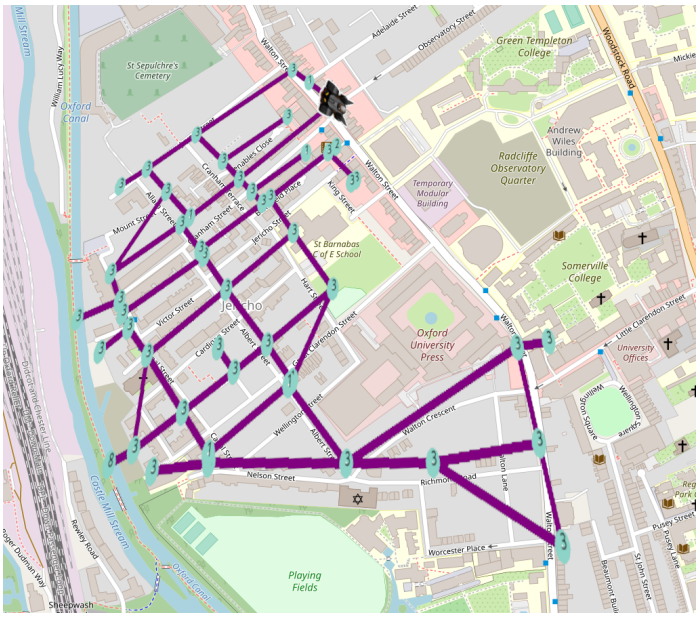


Figure 8: Overlay of the routing graph on a map of Jericho (OpenStreetMap contributors 2023), without adjusting for the Merkator projection, which leads to a slightly skewed appearance. The underlying data is exactly the same.

5 Conclusion

References

Boeing, Geoff (2017). 'OSMnx: New methods for acquiring, constructing, analyzing, and visualizing complex street networks'. In: *Computers, Environment and Urban Systems* 65, pp. 126–139. ISSN: 0198-9715. DOI: 10.1016/j.compenvurbsys.2017.05.004.

Hagberg, Aric A., Daniel A. Schult and Pieter J. Swart (2008). 'Exploring Network Structure, Dynamics, and Function using NetworkX'. In: *Proceedings of the 7th Python in Science Conference*. Ed. by Gaël Varoquaux, Travis Vaught and Jarrod Millman. Pasadena, CA USA, pp. 11–15.

Kollmeyer, Phillip (21st June 2018). 'Panasonic 18650PF Li-ion Battery Data'. In: 1. Publisher: Mendeley Data. DOI: 10.17632/wykht8y7tg.1. URL: https://data.mendeley.com/datasets/wykht8y7tg/1 (visited on 02/03/2023).

OpenStreetMap contributors (2023). *Planet dump retrieved from planet.osm.org*. URL: https://www.openstreetmap.org (visited on 08/03/2023).

A Simulation Code

```python
from typing import Optional

from simulator.battery import Battery


class BatMobile:
    """Represents the battery car ("Bat-Mobile") driving on an edge of the BatGraph. Models everything."""

    accelerationPerWatt = 100
    friction = -120.0
    currentJump = 0.1

    def __init__(self):
        self.battery = Battery()

        self.position: float = 0  # meters
        self.velocity: float = 0  # meters / second
        self.acceleration: float = 0  # true acceleration that determines velocity. in meters / second².
        self.motorAcceleration: float = 0  # goal acceleration, positively affects the true acceleration
        self.dragAcceleration: float = self.friction  # acceleration caused by drag, negatively affects the true accel,

        self.P_motor: float = 0

        self.sourceNode: Optional[str] = None  # one of the nodes of the BatGraph
        self.destinationNode: Optional[str] = None  # another node of the BatGraph

    def iterate(self, dt):
        self.battery.iterate(dt)

        self.motorAcceleration = self.accelerationPerWatt * self.battery.voltage * self.battery.current
        self.simulateDrag()

        self.P_motor = self.battery.voltage * self.battery.current

        self.acceleration = self.motorAcceleration + self.dragAcceleration
        self.velocity += self.acceleration * dt
        self.velocity = max(self.velocity, 0)  # velocity is at least 0..
        self.position += self.velocity * dt

    def accelerate(self, currentIncrease):
        """Increases the current through the motor by the given amount in Amps, therefore accelerating the car based on battery output.
        """
        self.battery.current += currentIncrease
```

```
45
46      def simulateDrag(self):
47          """Computes the drag acceleration which increases with v².""""
48          self.dragAcceleration = -6e-3 * self.velocity**2 + self.friction
49
50      def halt(self):
51          self.battery.current = 0
52          self.velocity = 0
53
54      def chargeUp(self):
55          self.battery.soc = 1
56          print("Charged up!")
```