Laboratory work №2

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Task №1.1

a)

Type:

The equation in full differentials

$$\frac{\partial P}{\partial y} = 4xy - \frac{6x^2}{y^3}$$
$$\frac{\partial Q}{\partial x} = 4xy - \frac{6x^2}{y^3}$$
$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$$

Method:

$$\int_{x_0}^{x} \left(2xy^2 + 3x^2 + \frac{1}{x^2} + \frac{3x^2}{y^3}\right) dx + \int_{y_0}^{y} \left(2x_0 y + 3y^2 + \frac{1}{y^2} - \frac{3x_0^3}{y^3}\right) dy = u(x,y)$$

b)

Type:

equation with separable variables

Method:

$$\int \frac{xdx}{1+x^2} = -\int \frac{dy}{y(1+y^2)}$$

c)

Type:

homogeneous equation

Method:

replacing variables

$$\begin{cases} \frac{y}{x} = u \\ x = x \end{cases}$$
$$y = ux$$
$$dy = xdu + udx$$

$$
\begin{gathered}
2x^2udx - (x^2 - u^2x^2)(xdu + udx) = 0 \\
2x^2udx - x^3du - x^2udx + u^2x^3du + u^3x^2dx = 0 \\
(x^2u + u^3x^2)dx + (-x^3 + u^2x^3)du = 0 \quad | : x^2
\end{gathered}
$$

$$
x'(u + u^3) + x(-1 + u^2) = 0 \tag{1}
$$

(1) - linear in x

$$
\begin{gathered}
\frac{dx}{du}(u + u^3) + x(-1 + u^2) = 0 \\
\int \frac{dx}{du} = -\int \frac{(u^2 - 1)du}{u + u^3}
\end{gathered}
$$

d)

Type:

linear equation in y

Method:

Lagrange method

e)

Type:

Bernoulli 's equation in y, $m = -2$

Method:

replacing variables

$$
\begin{gathered}
u = y^3 \\
du = 3y^2dy
\end{gathered}
$$

f)

Type:

the Riccati equation

Method:

not solved in the general case

g)

Type:

linear equation in x

Method:

Lagrange method

Task №1.2

$$(x^3 + x^3 \ln x + 2y)dx + (3y^2x^3 - x)dy = 0$$

$$\Psi(y): \quad \frac{Q'_x - P'_y}{P} = \frac{9x^2y^2 - 3}{x^3 + x^3\ln(x) + 2y} = f(x,y) \Rightarrow \Psi \neq \Psi(y)$$

$$\Psi(x): \quad \frac{P'_y - Q'_x}{Q} = \frac{3 - 9x^2y^2}{3x^3y^2 - x} = \frac{3(1 - 3x^2y^2)}{-x(1 - 3x^2y^2)} = -\frac{3}{x} \Rightarrow \Psi = \Psi(x)$$

$$\mu(x) = e^{\int \Psi(x)dx} = e^{-\int \frac{3}{x}dx} = e^{-3\ln x} = x^{-3}$$

$$\left(1 + \ln(x) + \frac{2y}{x^3}\right)dx + \left(3y^2 + -\frac{1}{x^2}\right)dy = 0, \quad x \neq 0 \tag{2}$$

$$\frac{\partial P_1}{\partial y} = \frac{\partial Q_1}{\partial x} = \frac{2}{x^3}$$

(2) - equation in full differentials

$$\frac{\partial u}{\partial x} = 1 + \ln x + \frac{2y}{x^3}$$

$$u(x,y) = \int \left(1 + \ln x + \frac{2y}{x^3}\right)dx$$

$$u(x,y) = x\ln x - \frac{y}{x^2} + c(y)$$

$$\frac{\partial u}{\partial y} = -\frac{1}{x^2} + c'(y)$$

$$c'(y) = 3y^2$$

$$c(y) = y^3$$

$$c = x\ln x - \frac{y}{x^2} + y^3$$

$$x = 0 \Rightarrow u'_x dx + u'_y dy = 0 \Rightarrow x = 0 \text{ - solution.}$$

Answer: $c = x\ln x - \frac{y}{x^2} + y^3$, $x = 0$.

Task №1.3

$$y(x+\ln y)+(x-\ln y)y'=0 \quad \ln y=\eta \quad d\eta=\frac{dy}{y}$$

$$\frac{dy}{d\eta}(x+\eta)+(x-\eta)\frac{dy}{dx}=0 \quad |:dy$$

$$(x+\eta)dx+(x-\eta)d\eta=0$$

$$\frac{\partial P}{\partial \eta}=\frac{\partial Q}{\partial x}=1 \Rightarrow \text{the equation in full differentials}$$

solution method:

$$\int_{x_0}^{x}(x+\eta)dx+\int_{\eta_0}^{\eta}(x-\eta)d\eta=u(x,y)$$

$$\frac{x^2}{2}+x\eta-\frac{\eta^2}{2}=c$$

$$\frac{x^2}{2}+x\ln y-\frac{\ln^2 y}{2}=c$$

Answer: $\frac{x^2}{2}+x\ln y-\frac{\ln^2 y}{2}=c$

Task №1.4

$$dy=(y-2)^{\frac{2}{3}}dx, \quad y|_{x=1}=2$$

$$d(y-2)(y-2)^{-\frac{2}{3}}=dx, \quad y-2=t$$

$$\int t^{-\frac{2}{3}}dt=\int dx$$

$$3t^{\frac{1}{3}}=x+c$$

$$3(y-2)^{\frac{1}{3}}=x+c$$

$$0=1+c \Rightarrow c=-1$$

$$3(y-2)^{\frac{1}{3}}=x-1$$

Answer: $3(y-2)^{\frac{1}{3}}=x-1$.

Task №2.1

a)

Type:

equation with separable variables

Method:

$$\frac{x}{1+x^2}dx - \frac{1}{y(1+y^2)}dy = 0$$

b)

Type:

homogeneous of degree 0

Method:

$$\frac{x^2dy - y^2dx}{(x-y)^2} = 0$$
$$\frac{(tx)^2}{(tx-ty)^2}dy - \frac{(ty)^2}{(tx-ty)^2}dx = 0$$
$$\frac{t^2x^2}{t^2(x-y)^2}dy - \frac{t^2y^2}{(x-y)^2t^2}dx = 0$$

c)

Type:

homogeneous of degree 2

Method:

$$(x^2+y^2)dx - 2xy = 0$$
$$((xt)^2 + (yt)^2)dx - 2(tx)(ty) = 0$$

d)

Type:

linear in y

Method:

$$y' + \frac{x}{4-x^2}y = 4$$

e)

Type:

Bernoulli equation

Method:

$$y' \tan(x) + 2y\tan(x)^2 = ay^2$$

f)

Type:

the Riccati equation

Method:

$$y' + \frac{1}{x}y = xy^2 + \frac{1}{x}$$

g)

Type:

linear by x

Method:

$$x' + x = -y^2$$

Task №2.2

$$y^2(x-y)dx + (1-xy^2)dy = 0$$

$$\int y^2(x-y)dx + \int (1-xy^2)dy = C$$

$$x^2 - 2xy - \frac{2}{y} = C$$

$$y = 0$$

$$\mu = \mu(y) : \frac{Q'_x - P'_y}{P} = \frac{-y^2 - (2xy - 3y^2)}{y^2(x-y)}$$

$$\mu = \mu(x) : \frac{P'_y - Q'_x}{Q} = \frac{2xy - 3y^2 - y^2}{1 - xy^2} = \frac{2y(x-2y)}{1-xy^2}$$

$$\mu' = \frac{2}{y^2}\mu$$

$$\frac{d\mu}{\mu} = -\frac{2dy}{y^2}$$

$$\ln \mu = \ln y^{-2}$$

$$\mu = y^{-2}$$

Answer: $x^2 - 2xy - \frac{2}{y} = C, y = 0$

Task №2.3

$$y' = x + e^{x+2y}$$

$$\eta = e^{-2y}$$

$$d\eta = -2e^{-2y}dy$$

$$\frac{dy}{dx} = x + e^{x+2y}$$

$$e^{-2y}dy = (xe^{-2y} + e^x)dx$$

$$-\frac{1}{2}d\eta = (x\eta + e^x)dx$$

$$\eta' + 2x\eta = -2e^x \text{ - linear by } \eta$$

Answer: $\eta' + 2x\eta + ae^x = 0$

Task №2.4

$$dy = x\sqrt{y}dx, y|_{x=1} = 0$$

$$xdx = \frac{dy}{\sqrt{y}}$$

$$\frac{x^2}{2} = 2\sqrt{y} + C$$

$$C = \frac{1}{2}$$

$$x^2 - 4\sqrt{y} = 1$$

Answer: $x^2 - 4\sqrt{y} = 1$